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AF MANUAL 52-8
VOLUME I

188

ELECTRONIC

CIRCUIT ANALYSIS

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CHANGE **AFM 52-8A Vol I**

AIR FORCE MANUAL
NUMBER 52-8A, Vol I

DEPARTMENT OF THE AIR FORCE
Washington 25, D.C.
1 October 1964

Technical Training

ELECTRONIC CIRCUIT ANALYSIS

AFM 52-8, Volume I, 1 November 1962, is changed as follows:

1. Make the following changes by pen and ink:

Page	*Column*	*Line*	*Action To Be Taken*
1-4	1	4	Move quotient to the left so that the first figure is directly above the first figure in the dividend
1-10	1	21	Change "$\sqrt[3]{18\frac{1}{8}}$" to "$\sqrt[3]{16\frac{7}{8}}$"
1-14	1	23	Change ".0009." to ".0011."
1-14	1	24	Change "$\frac{.0009}{.0016}$ of 10, or 6" to "$\frac{.0011}{.0016}$ of 10, or 7"
1-14	1	25	Change "2706." to "2707."
1-14	1	26	Change "270.6." to "270.7."
1-15	1	Solution to second example	Change last three lines of solution to read: $=3.9949+3.3140-2.6131$ $=4.6958$ Antilog. 4.6958=109.5
1-21	2	36	Change "$81.5\underline{/48.6°}$." to "$81.3\underline{/48.6°}$."
2-6	2	13	Change "electrons" to "atoms"
2-6	2	14	Change "a molecule" to "an atom"
2-17	2	boldface heading	Change "Law" to "Laws"
2-55	2	27, 28	Change "capacity" to "capacitance"
2-55	2	last	Change to read "farad, also called a picofarad (pf)."

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Page	*Column*	*Line*	*Action To Be Taken*
2–76	1	Fig. 2–86	On right side of top illustration change "L=.95hy" to "L=.0095hy"
2–76	2	2	Change "0.95" to ".0095"
2–77	1	19	Change "$Z_t=\sqrt{R^2+X_c^2}$" to "$Z_t=\sqrt{R^2+X_L^2}$"
2–96	2	Fig. 2–116	Change "$\frac{(wM)^2}{Z_2+Z_s}$" to "$\frac{Z_m^2}{Z_2+Z_s}$"
3–24	Figure 3–29, title		Change "2:3" to "3:2"
3–26	1	38	Change "ratio of 2:3" to "ratio of 3:2"
4–4	1	Fig. 4–4	Change "T1" and "T2" to "V1" and "V2"
4–4	1	5th from bottom	Change "to" to "toward"
4–6	2	Fig. 4–7	Add labels "PLATE CURRENT (MA)" at left side of graph, and "GRID VOLTS" below graph
4–6	2	Fig. 4–8	Add labels "PLATE VOLTS" at left side of graph, and "GRID VOLTS" below graph
5–2	2	Fig. 5–1	On second line, change "$f_2+\Delta f$" to "$f_1+\Delta f$"
5–8	2	33, 34	Change "wider pulse width" to "narrower bandwidth"
6–6	Figure 6–8		Label circuits at left "A"
6–85	2	6	Change "and capacitor C_2 charges" to "as soon as capacitor C_2 has charged"
7–7	1	Fig. 7–6	Change "CURRENT PLATE" to "PLATE CURRENT"
7–35		Fig. 7–48	To circuit A, add grounding symbol (⏚) below the dot on the lower right-hand side. To circuit B, add grounding symbol (⏚) below the dot on the center horizontal line, right-hand side.
7–42	1	9	Change "negative" to "positive"
7–42	1	10	After "coefficient." insert the following sentence "If the frequency decreases with the temperature rise, the crystal has a negative temperature coefficient."
8–35	Fig. 8–47		Lower the top left dot along the solid diagonal line of graph so that it covers the connecting point of line $1_B=350\mu A$. Draw new horizontal and vertical dash lines to newly positioned dot.

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Page	*Column*	*Line*	*Action To Be Taken*
8–43	1	1, 2	Change "the oscillator to block." to "distortion."
8–43	Fig 8–62		Lower right-hand corner, on bottom line delete last two dots and the line connecting them
8–46	2	18, 19	Delete "(the capacitance is increased)"
8–61	Fig. 8–87		On lower right side opposite L2, change "C4" to "C9"

2. After posting the changes, staple this change sheet in the back of the manual.

BY ORDER OF THE SECRETARY OF THE AIR FORCE

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Director of Administrative Services

AIR FORCE MANUAL
NUMBER 52-8

DEPARTMENT OF THE AIR FORCE
WASHINGTON 25, D.C.
1 November 1962

Technical Training

ELECTRONIC CIRCUIT ANALYSIS

This manual, published in two volumes, of which this is Volume I, provides the graduate electronics repairman or technician with reference information on the fundamentals of electronics. It also makes available a basic coverage of the subject to the student repairman. A prerequisite to understanding these basic concepts is a knowledge of mathematics and electricity. Students who are not too familiar with these subjects may have to consult other sources of information for greater detail. Both Volume I and Volume II include the same attachments of Symbols and Formulas, and Tables, preceding the Index. The Index pertains only to that volume in which it appears.

CONTENTS

—VOLUME I—

Chapter *Page*
1 Mathematics for Electronics 1–1
2 DC and AC Circuits 2–1
3 Measuring Instruments 3–1
4 Electron Tubes and Power Supplies 4–1
5 Principles of Radar 5–1
6 Timing Circuits 6–1
7 Amplifiers and Oscillators 7–1
8 Transistors 8–1
Attachments
1 Symbols and Formulas *1*
2 Tables *7*
3 Index (Volume I) *23*

—VOLUME II—

Chapter *Page*
9 Modulation, Detection, and Single Sideband Transmission . . 9–1
10 Transmission Lines 10–1
11 Waveguides, Cavity Resonators, and Duplexers 11–1
12 UHF Oscillators 12–1

This manual supersedes AFM 52-8, Radar Circuit Analysis, 30 June 1951

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This training manual is published by Air Training Command in accordance with AFR 5-5.

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CONTENTS (Continued)

Chapter *Page*

13 Antennas 13–1

14 Synchros and Servomechanisms 14–1

15 Special Circuits 15–1

Attachments

  1 Symbols and Formulas *1*

  2 Tables *7*

  3 Index (Volume II) *23*

Comments or recommendations for the improvement of this manual should be sent to Headquarters Air Training Command, Randolph Air Force Base, Texas.

CHAPTER 1

Mathematics for Electronics

Learning to understand the operation of many electrical and electronic circuits can be accomplished by a study of current flow and by mechanical analogy, but for you to understand how and why the circuits actually work requires the application of mathematics. Algebra, a common mathematical subject, is applied frequently in the analysis and study of electronic circuits. You probably studied this subject in high school, but may have forgotten many of the principles and thus need a review. Logarithms, trigonometry, vector algebra, and the slide rule are not commonly taught in high school, but also are very useful tools for analyzing electronic circuits.

This chapter deals with the mathematics you need in understanding the electronic circuits in this manual. Algebra is reviewed rapidly. Logarithms, trigonometry, vector algebra and the slide rule, new subjects to many, are treated less rapidly and with emphasis on the type of problems you will encounter in the manual. Each mathematical principle stated is followed by examples and solutions to give you a better understanding of its application.

ALGEBRA

Definitions

An *algebraic expression* is any combination of signs, numerals, and letters used for numbers, written according to the rules of algebra. For example, $a + b$, l, lw and $-\frac{x}{2Y}$ are algebraic expressions. Numbers represented by letters are called *literal* numbers. Thus a, b, l, w, x, and y are literal numbers. Signs may indicate whether the numbers are positive or negative, or they may indicate operations such as addition, subtraction, multiplication, division, extracting a root, or raising to a power.

Multiplication of two algebraic numbers need not be indicated by the symbols for multiplication, the times sign (x) or the dot (•). Just putting them together indicates multiplication. Thus, the product of a and b may be written ab. In the expression ab, a and b are called *factors* of the product. Likewise 6, a, and b are factors of $6ab$.

Each factor of a product is the *coefficient* of the other factors. In the algebraic expression $6ab$, 6 is the coefficient of ab, $6a$ of b, and $6b$ of a. In this expression, 6 is called the numerical coefficient and b the literal coefficient of a. The word coefficient is usually restricted to mean only the numerical, or arithmetic, coefficient.

The size, or magnitude, of a number is called its *absolute value.* Absolute value refers only to the magnitude with no consideration for the sign preceding the number. For example, the absolute value of $+7$ is 7, and the absolute value of -9 is 9. Both $+3$ and -3 have the same absolute value, 3.

An *exponent* is a number or letter which indicates the power to which a quantity is to be raised, or the number of times the quantity (called the base) is multiplied by itself. For example, 4^3 is read *4 to the third power,* and means $4 \times 4 \times 4$. The expression e^x is read *e to the x power,* and means that e is multiplied by itself x times.

Any arithmetic or literal number, or the product or the quotient of these numbers, is called a *term*. For example, 4, x, y, $25b$, ab, $\frac{x}{y}$, and $\frac{3a^2b}{c}$ are terms.

Terms that have the same literal parts are called *similar* or *like* terms. Thus a, $7a$, and $12a$ are like terms. Terms such as x^2, y, *and* lh, which do not have the same literal parts, are called *unlike* terms.

You can add or subtract two or more numbers if they are like terms. For example, you can add $4x$ to $5x$ to produce the sum $9x$. Likewise, you can add $5x^2$ to $10x^2$, since the literal parts x^2 are alike. (Literal numbers raised to the same power are like terms.) The sum of numbers such as $4x$ and $5m^2$ must be indicated as $4x + 5m^2$ since x and m^2 are not like terms.

An algebraic expression containing two or more terms joined together by a plus or minus sign is called a *polynomial*. Thus, the expressions $a + b - c$ and $ab - ac + \frac{a}{c}$ are polynomials. A polynomial with just two terms, such as $5x + y$, is a *binomial*. A polynomial with only three terms is a *trinomial*. The expression $5x^2 + ab + c$ is a trinomial. A single term, such as a, x, or $10x^2y$, is a *monomial*.

Algebraic Addition and Subtraction

To add two numbers having the same sign, add their absolute values and prefix the common sign.

Thus, $+6$ added to $+3$ equals $+9$
-6 added to -3 equals -9
$+6a^2$ added to $+3a^2$ equals $+9a^2$
$-6m^2n$ added to $-3m^2n$ equals $-9m^2n$

To add two numbers having opposite signs, take the difference between their absolute values and prefix the sign of the number with the larger absolute value.

Thus, $+6$ added to -3 equals $+3$
-6 added to $+3$ equals -3
$+6a^2$ added to $-3a^2$ equals $+3a^2$
$-3m^2n$ added to $+6m^2n$ equals $+3m^2n$

The process of subtraction is the opposite of the process of addition. To subtract one quantity from another, change the sign of the quantity to be subtracted; then add the quantities, following the rules of addition.

Example

$$(-6) - (-3) = (-6) + (+3) = -3$$
$$(-3) - (-6) = (-3) + (+6) = +3$$
$$(+6) - (-3) = (+6) + (+3) = +9$$
$$(+6) - (+3) = (+6) + (-3) = +3$$

Polynomials are added and subtracted in the same way as single terms (monomials).

For example,

Addition

$$\begin{array}{rrrr} & -4a^2 & +3az & -3y \\ 3a^3 & & -2az & +5y \\ \hline 3a^3 & -4a^2 & +\ az & +2y \end{array}$$

Subtraction

$$\begin{array}{rr} 3m^2 & -6n^2 \\ -3m^2 & -8n^2 \\ \hline 6m^2 & +2n^2 \end{array}$$

Note in these examples that the plus sign is omitted before $3a^3$, $3m^2$ and $6m^2$. Algebraic practice permits the omission of the sign before the first term of an expression if it is positive. Likewise, the numerical coefficient 1 is omitted before a literal number, such as before az in the answer to the addition problem.

Signs of Grouping

Certain symbols, called *signs of grouping*, tie, or group together, several quantities affected by the same operation. The most commonly used symbols of groupings are parentheses ( ), braces { }, brackets [ ], and the vinculum $\overline{\quad}$. All these symbols indicate that the quantities affected by them are to be treated as single quantities. For example, the expression $(4a^2 - 3ab) - (3a^2 + 2ab)$ means that the quantity $3a^2 + 2\,ab$ is to be subtracted from the quantity $4a^2 - 3ab$. To perform this operation, change the signs of the expression $3a^2 + 2ab$, and add algebraically,

$$\begin{array}{rr} 4a^2 & -3ab \\ -3a^2 & -2ab \\ \hline a^2 & -5ab \end{array}$$

In removing a sign of grouping preceded by a minus sign, change the signs of all terms included by the symbol. When one symbol of grouping occurs within another, remove the innermost symbol first. You may remove both symbols at the same time if you are quite careful, but this practice can easily lead to errors of sign. Therefore, it is best to remove only one symbol at a time. The following example illustrates the procedure for removing a symbol of grouping occurring within another and combining like terms:

$$3a^2 - [2am - (2a^2 + 5am) + a^2]$$
$$3a^2 - [2am - 2a^2 - 5am + a^2]$$
$$3a^2 - 2am + 2a^2 + 5am - a^2$$
$$4a^2 + 3am$$

The vinculum is used in fractions to indicate division and with a radical sign$\sqrt{}$

Multiplication

In the multiplication of algebraic terms, there are three things to consider — the sign, the exponent, and the coefficient.

The product of two terms with like signs is positive. The product of two terms with unlike signs is negative.

The *exponent* of any letter in the product is the sum of the exponents of the factors with the same base.

Thus, $x^3 \times x^5 = x^{(3+5)}$, or x^8
Likewise, $y^3 \times y^7 = y^{10}$

The arithmetic *coefficient* of the product is the product of the absolute values of the coefficients of the terms being multiplied.

Thus, $6x^3 \times 2x^5 = 12x^8$
Likewise, $3y^3 \times 3y^7 = 9y^{10}$

Examples

$$(3a^2) \times (5a^3) = 15a^5$$
$$(2ax) \times (-3a^2xy) = -6a^3x^2y$$
$$(m^2n) \times (-7mn^3) = -7m^3n^4$$
$$(4xy) \times (-xy^3) = -4x^2y^4$$
$$(-a) \times (-5a^3) = 5a^4$$

A literal quantity with no written exponents means the quantity is to the first power; that is, it has an exponent of one.

Multiplication of polynomials is similar to the multiplication of numbers consisting of several digits in arithmetic. Multiply each term of the multiplier by every term of the multiplicand and add the partial products. Thus, in multiplying $2e^2 - 3e - 5$ by $-4e + 7$, proceed as follows:

$$\begin{array}{rrrr} & 2e^2 & -3e & -5 \\ & & -4e & +7 \\ \hline -8e^3 & +12e^2 & +20e & \\ & +14e^2 & -21e & -35 \\ \hline -8e^3 & +26e^2 & -e & -35 \end{array}$$

Division

In the division of one term by another, the sign, the coefficient, and the exponent must be considered in obtaining the quotient.

The quotient of two positive or two negative quantities is positive; the quotient of a positive and a negative quantity is negative. Division by zero is excluded.

The quotient of powers having like bases has the same base as the given powers, and an *exponent* which is the difference of the exponents of the given powers.

Thus, $\frac{x^4}{x^2} = x^2$

Likewise, $\frac{x^{10}}{x^3} = x^7$

To obtain the *coefficient* of the quotient, divide the absolute value of the coefficient of the dividend by the absolute value of the coefficient of the divisor.

Thus, $\frac{6x^4}{3x^2} = 2x^2$

Likewise, $\frac{8x^{10}}{2x^3} = 4x^7$

In division there is the possibility of zero and negative exponents, as the following examples indicate.

$$\frac{a^5}{a^5} = a^{(5-5)} \text{ or } a^0$$

$$\frac{a^4}{a^5} = a^{-1}$$

$$\frac{a^3}{a^5} = a^{-2}$$

Any quantity with a zero exponent is equal to one.

Thus, $\frac{a^5}{a^5} = a^0 = 1$

Likewise, $\frac{x^3}{x^3} = a^0 = 1$

Any quantity with a negative exponent is equal to the reciprocal of that quantity with the corresponding positive exponent.

Thus, $\frac{a^3}{a^5} = a^{-2} = \frac{1}{a^2}$

Likewise, $\frac{1}{x^{-3}} = x^3$

Any factor can be moved from the numerator to the denominator of a fraction, or vice versa, without changing the value of the fraction if the sign of the exponent is changed.

Thus, $\frac{3x^2y}{a^3b} = \frac{3x^2ya^{-3}}{b} = \frac{3a^{-3}x^2y}{b}$

Division of one polynomial by another is similar to long division in arithmetic. One difference is that the dividend, divisor, and the remainder (if there is one) must be arranged in order of ascending or descending powers of some letter.

Example

Divide $30c^4+3-82c^2-5c+11c^3$ by $3c^2-4+2c$

Solution

$$\begin{array}{r|rrrrrl}
 & & & 10c^2 & -3c & -12 & \\ \cline{2-6}
3c^2+2c-4 & 30c^4 & +11c^3 & -82c^2 & -5c & +3 & \\
 & 30c^4 & +20c^3 & -40c^2 & & & \\ \cline{2-4}
 & & -9c^3 & -42c^2 & -5c & & \\
 & & -9c^3 & -6c^2 & +12c & & \\ \cline{3-5}
 & & & -36c^2 & -17c & +3 & \\
 & & & -36c^2 & -24c & +48 & \\ \cline{4-6}
 & & & & 7c & -45 & \text{remainder}
\end{array}$$

Solution of Equations

Any equation is a statement that two quantities are equal. To solve an equation means to find the value, or values, of the literal numbers for which the equation holds true. In solving an equation, you will have to use *axioms*, statements accepted as true without proof. Here are a few of the more commonly used axioms:

1. If the same number is added to, or subtracted from, both sides of an equation, the result is still an equation.
2. If both sides of an equation are multiplied or divided by the same quantity (not zero), the result is still an equation.
3. If like roots or powers are taken of both sides of an equation, the result is still an equation.

Other processes that are based upon these axioms and that are often applied in the solution of equations are *transposition*, *changing signs*, and *cancellation*. Transposition is the transferring of a term from one side of an equation to the other and the changing of its sign. This is merely another way of saying that the term was added to, or subtracted from, both sides (axiom 1). Changing signs of terms on both sides of an equation is merely another way of multiplying or dividing both sides by -1 (axiom 2). Cancellation is the means of collecting terms, or transposing and collecting terms.

Examples

1. Given $x - 5 = 3$; find x.

Solution

Add 5 to both sides of the equation (axiom 1),

$$x - 5 + 5 = 3 + 5$$

Then collect terms.

$$x = 8$$

2. Given $5x - 4 = 21$; find x.

Solution

Add 4 to both sides of the equation (axiom 1),

$$5x - 4 + 4 = 21 + 4$$

Then collect terms on both sides,

$$5x = 25$$

Divide both sides by 5 (axiom 2),

$$x = 5$$

3. Given $\frac{1}{3}x + 5 = 8$; find x.

Solution

Subtract 5 from both sides of the equation (axiom 1),

$$\frac{1}{3}x + 5 - 5 = 8 - 5$$

Collect terms,

$$\frac{1}{3}x = 3$$

Then multiply both sides of the equation by 3 (axiom 2),

$$x = 9$$

4. Given $\frac{4}{5}x + 5 = 25 - 1\frac{1}{5}x$; find x.

Solution

Subtract 5 from both sides of the equation (axiom 1),

$$\frac{4}{5}x = 20 - 1\frac{1}{5}x$$

Then add $1\frac{1}{5}x$ to both sides (axiom 1),

$$\frac{4}{5}x + 1\frac{1}{5}x = 20 - 1\frac{1}{5}x + 1\frac{1}{5}x$$

Collect terms,

$$2x = 20$$

Divide both sides by 2 (axiom 2),

$$x = 10$$

5. Given $16 - 5\,(x+3) = 4(2x + 1) - 9\frac{1}{2}$; find x.

Solution

Remove the parentheses,

$$16 - 5x - 15 = 8x + 4 - 9\frac{1}{2}$$

Collect terms,

$$1 - 5x = 8x - 5\frac{1}{2}$$

Subtract 1 from both sides (axiom 1),

$$-5x = 8x - 6\frac{1}{2}$$

Subtract 8x from both sides (axiom 1),

$$-13x = -6\frac{1}{2}$$

Then divide both sides by -13 (axiom 2)

$$x = \frac{1}{2}$$

6. Given $\sqrt{x} + 2 = 4$; find x.

Solution

Subtract 2 from both sides of the equation,

$$\sqrt{x} = 2$$

Then square both sides (axiom 3),

$$x = 2^2 \text{ or } 4$$

7. Given $z^2 = r^2 + x^2$; find r.

Solution

Subtract x^2 from both sides of the equation,

$$z^2 - x^2 = r^2$$

Take the square root of both sides of the equation (axiom 3),

$$\sqrt{z^2 - x^2} = r$$

or

$$r = \sqrt{z^2 - x^2}$$

Monomial Square and Square Roots

When a monomial is multiplied by itself, the product is the *square* of the coefficient multiplied by each literal quantity with its exponent doubled.

Examples

$$(3a^2)^2 = 9a^4$$
$$(-5mn^3)^2 = 25m^2n^6$$

The square root of a monomial is the square root of the coefficient times each literal number with its exponent halved. There are two roots — a negative root and a positive root. This is true because the sign of the product of two negative quantities or of two positive quantities is positive.

Examples

$\sqrt{64a^4} = \pm 8a^2$ (The symbol ± means that it could be either $+8a^2$ or $-8a^2$)

$\sqrt{50a^3} = \pm 7.07a^{1.5}$

Special Products and Factoring

Factoring is the process of finding two or more quantities, each called a factor, whose product is equal to a given quantity. The simplest form of polynomial factoring is finding the common monomial factor. The expression, $5x + 5y$, is a polynomial in which 5 is a factor common to both monomials. Factor this expression by dividing $5x + 5y$ *by* 5. Write the result as $5(x + y)$.

Examples

$$ax + ab - az = a(x + b - z)$$
$$4mx - 8my + 2mz = 2m(2x - 4y + z)$$
$$5a^3b - 10a^2b^2 - 15ab^3 = 5ab(a^2 - 2ab - 3b^2)$$

A frequently encountered product has the form $(x + a)\ (x + b)$, which, when multiplied, gives $x^2 + x(a + b) + ab$. This relationship can be stated as a rule: The *product of two binomials having a common term* is the square of the common term, plus the product of the common term times the algebraic sum of the unlike terms, plus the algebraic product of the unlike terms.

Examples

$$(a + 3)\ (a + 4) = a^2 + a(3 + 4) + (3 \times 4)$$
$$= a^2 + 7a + 12$$
$$(a - 5)\ (a - 3) = a^2 - 8a + 15$$
$$(a - 7)\ (a + 3) = a^2 - 4a - 21$$
$$(x + 7)\ (x - 9) = x^2 - 2x - 63$$

A trinomial such as $x^2 + x(a + b) + ab$ is often expressed in the general form $x^2 + mx + n$ in which m represents the sum of the unlike terms and n, their product. *Factoring*, or finding the binomial factors, in trinomials of this form involves finding two quantities whose sum equals m and whose product equals n. After you determine these quantities, write the binomial factors by algebraically adding these quantities to the square root of the first term of the trinomial. Thus, for example, in $b^2 - 9b + 14$, the numbers -7 and -2 must be part of the binomial factors, for their sum is -9 and their product is $+ 14$. This means that the binomial factors are $(b - 7)$ and $(b - 2)$.

Examples

$$e^2 - 9e + 20 = (e - 5)\ (e - 4)$$
$$r^2 + 11r - 42 = (r + 14)\ (r - 3)$$
$$p^2 - p - 42 = (p - 7)\ (p + 6)$$
$$t^2 + 11t + 24 = (t + 3)\ (t + 8)$$

When the product is of the form $(x + a)\ (x + a)$ or $(x + a)^2$, the middle term of the product is $2ax$, and the final term is a^2. Thus, $(x + a)^2 = x^2 + 2ax + a^2$.

Expressed in rule form, this relation is: The *square of a binomial* is the sum of the squares of the two terms added to twice their algebraic product.

Examples

$$(r + t)^2 = r^2 + 2rt + t^2$$
$$(k - 2j)^2 = k^2 - 4jk + 4j^2$$
$$(3e - d)^2 = 9e^2 - 6ed + d^2$$

Factoring polynomials of the type $x^2 + 2ax + a^2$, called perfect trinomial squares, depends largely on your ability to recognize them as such. A trinomial is a perfect square if two of its terms are perfect squares and the third term is twice the product of their square roots.

Examples

$$g^2 + 2gk + k^2 = (g + k)^2$$
$$p^2 + 2pq + q^2 = (p + q)^2$$
$$r^2 - 10r + 25 = (r - 5)^2$$

The product of binomials of the type $mx + a$ and $nx + b$ is $mnx^2 + x\ (na + mb) + ab$; little time is saved by following a rule. However, knowing the rule for such multiplication may make it easier for you to factor the product of binomials of this type.

Rules for multiplying any two binomials are:

1. To find the first term of the product, multiply the first terms of the binomials by each other.
2. To find the second term of the product, obtain the product of the outer terms of the binomials and the product of the inner terms of the binomials and add them algebraically.
3. To find the third term of the product, multiply the second terms of the binomials by each other.

Thus, to find the product of $(2e - 3)(3e + 4)$, proceed as follows:

First term: $2e \times 3e$, or $6e^2$.

Second term: $(2e \times 4) + (-3 \times 3e) = 8e - 9e$ or $-e$.

Third term: -3×4, or -12.

The final product is: $6e^2 - e - 12$.

Factoring trinomials of the form $mx^2 + (mb + na)x + ab$ usually involves a trial-and-error method. As you saw in the rule just given, the factors must be such that the product of the first terms of the binomials equals the first term of the trinomial; the product of the second terms of the binomials equals the third term of the trinomial; and the algebraic sum of the cross products is the second term of the trinomial. The usual procedure is to try two binomials that satisfy the first and second conditions; then check to see if they also satisfy the third condition.

With practice you can eliminate some elements in guessing the right combination. For example, you can tell by the signs of the trinomial whether the signs of the binomials will be both plus, both minus, or one plus and one minus. You should remove monomial factors from the trinomial before extracting the binomial factors. Then you should not use a binomial factor that contains a monomial factor because there are no monomial factors left in the trinomial.

Example

Factor $6a^2 - 17a + 12$

(No monomials are present in the trinomial.)

Trial Factors		Product
$(6a - 1)(a - 12)$		$6a^2 - 73a + 12$ Wrong; middle term incorrect.
$(6a - 12)(a - 1)$	6 is a factor of $(6a - 12)$;	do not use.
$(6a - 3)(a - 4)$	3 is a factor of $(6a - 3)$;	do not use.
$(6a - 4)(a - 3)$	2 is a factor of $(6a - 4)$;	do not use.
$(3a - 2)(2a - 6)$	2 is a factor of $(2a - 6)$;	do not use.
$(3a - 6)(2a - 2)$	3 is a factor of $(3a - 6)$ and 2 is a factor of $(2a - 2)$;	do not use.
$(3a - 3)(2a - 4)$	3 is a factor of $(3a - 3)$ and 2 is a factor of $(2a - 4)$;	do not use.
$(3a - 4)(2a - 3)$		$6a^2 - 17a + 12$ Right.

Fractions and Fractional Equations

A fraction is an indicated division in which the *numerator* (number above the line) is the dividend and the *denominator* (number below the line) is the divisor. The ratio of two quantities (the value of the fraction) is unchanged if they are both multiplied or divided by the same number (not zero). Dividing both numerator and denominator by the same number is called reduction to lower terms. Sometimes it is easier to solve problems if you write them in factored form first.

Examples

1. $\frac{14}{21} = \frac{14 \div 7}{21 \div 7} = \frac{2}{3}$
2. $\frac{6ax}{9a} = \frac{(2x)(3a)}{(3a)(3a)} = \frac{2x}{3a}$
3. $\frac{a^2 - b^2}{a^2 - 2ab + b^2} = \frac{(a + b)(a - b)}{(a - b)(a - b)} = \frac{a + b}{a - b}$

Note that a quantity can be used as a divisor only if it is a *factor* of the *complete* numerator and *complete* denominator.

4. $\frac{e^2 - 14e - 51}{e^2 - 2e - 15} = \frac{(e - 17)(e + 3)}{(e - 5)(e + 3)} = \frac{e - 17}{e - 5}$

To change a given fraction to an equivalent fraction with a specific denominator, multiply both numerator and denominator by a number which will make the new denominator the desired value. For example, to change $\frac{1}{5}$ *to 25ths* multiply both the denominator and numerator by *5*. The equivalent fraction is $\frac{5}{25}$.

Examples

1. Given: $\frac{2a}{5b} = \frac{?}{10ab}$, find the equivalent numerator

Dividing 10ab by 5b gives 2a, so both terms of the fraction must be multiplied by 2a to change to the desired denominator.

Thus, $\frac{2a}{5b} \times \frac{2a}{2a} = \frac{4a^2}{10ab}$ and $\frac{2a}{5b} = \frac{4a^2}{10ab}$

2. Given: $\frac{2m + 1}{3m - 5} = \frac{?}{6m^2 - 7m - 5}$

The factors of $6m^2 - 7m - 5$ are $(3m - 5)\ (2m + 1)$

$$\frac{(6m^2 - 7m - 5)}{(3m - 5)} = (2m + 1)$$

Multiplying,

$$\frac{2m + 1}{3m - 5} \times \frac{2m + 1}{2m + 1} = \frac{(2m + 1)\ (2m + 1)}{(3m - 5)\ (2m + 1)}$$

$$= \frac{4m^2 + 4m + 1}{6m^2 - 7m - 5}$$

Fractions may be combined (added or subtracted) only if they have the same common denominator. When you need to combine fractions with different denominators, first change them to equivalent fractions with the same denominator (called *common denominator*). The least common denominator (LCD) is best since the use of any other denominator necessitates further reduction of the answer to lower terms. When dealing with polynomial denominators, factor each denominator as far as possible. The least common denominator is then the product of all the factors of the denominators, taking each factor the greatest number of times it occurs in any one denominator.

Examples

1. Combine: $\frac{x}{4} + \frac{3x}{5} - \frac{2x}{6}$

$$\text{LCD} = 2 \times 2 \times 5 \times 3 = 60$$

$$\frac{(15)x + (12)3x - 10(2x)}{60}$$

$$\frac{15x + 36x - 20x}{60} = \frac{31x}{60}$$

2. Combine: $\frac{3}{4xy} - \frac{2}{3x^2}$

$$\text{LCD} = 12x^2y$$

$$\frac{(3x)3 - (4y)2}{12x^2y} = \frac{9x - 8y}{12x^2y}$$

3. Combine: $\frac{1}{R^2 - 1} + \frac{3}{R^2 - 2R + 1}$

$$= \frac{1}{(R + 1)\ (R - 1)} + \frac{3}{(R - 1)\ (R - 1)}$$

$$= \frac{1(R - 1) + 3(R + 1)}{(R - 1)\ (R - 1)\ (R + 1)}$$

$$= \frac{R - 1 + 3R + 3}{(R - 1)\ (R - 1)\ (R + 1)}$$

$$= \frac{4R + 2}{(R - 1)^2\ (R + 1)}$$

4. Combine: $3R + 5 - \frac{5}{2R + 1}$

$$= \frac{3R + 5}{1} - \frac{5}{2R + 1}$$

$$= \frac{(3R + 5)\ (2R + 1) - 5}{2R + 1}$$

$$= \frac{6R^2 + 13R + 5 - 5}{2R + 1} = \frac{6R^2 + 13R}{2R + 1}$$

The expression combined in example 4 is known as a *mixed expression* and the answer as an *improper fraction.* Handle them as mixed numbers and improper fractions are handled in arithmetic.

The product of two fractions consists of the product of the numerators divided by the product of the denominators. Frequently it is possible to reduce the result to lower terms by dividing out *factors* common to both numerator and denominator. This is sometimes called *cancellation of common factors* or merely *cancellation.* To divide one fraction by another, invert the second fraction (or divisor) and multiply. Factoring of polynomial numerators and denominators is advisable in order to eliminate common factors from the result.

Examples

1. Perform the indicated operations: $\frac{4a}{3b} \times \frac{6ab}{2a^2} \div \frac{7ac}{9c^2}$

$$= \frac{4\cancel{a}}{\cancel{3}\cancel{b}} \times \frac{\overset{2}{\cancel{6}}\cancel{a}\cancel{b}}{\underset{\cancel{a}}{\cancel{2}\cancel{a^2}}} \times \frac{9\overset{c}{\cancel{c^2}}}{7a\cancel{c}} = \frac{36c}{7a}$$

Note the use of cancellation in this example. Wherever fractions are being multiplied, you can cancel out like terms. Note in the example that an *a* above the division line is cancelled and that a^2 below is replaced by an *a*. Remember,

$$\frac{a}{a^2} = \frac{a}{aa} = \frac{a}{a} \times \frac{1}{a} = \frac{1}{a}$$

Therefore, when you see an *a* above and a^2 below, you can mentally do the steps outlined, leaving in this case, an *a* below the line. Since there is an *a* above the line in the first fraction, this left-over *a* can be cancelled into it. Further cancellation removes practically two-thirds of the fraction.

2. Multiply: $\frac{m^2 + 5m + 6}{m^2 - 4} \times \frac{m^2 - 6m + 8}{m^2 - 9}$

$$= \frac{\cancel{(m + 2)}\ \cancel{(m + 3)}}{\cancel{(m + 2)}\ \cancel{(m - 2)}} \times \frac{\cancel{(m - 2)}\ (m - 4)}{\cancel{(m + 3)}\ (m - 3)}$$

$$= \frac{m - 4}{m - 3}$$

3. Perform the indicated operations:

$$\frac{5t^2+8t+3}{2t^2-6t} \times \frac{6t^2-18t}{5t^2-2t-3} \div \frac{t^2+5t+4}{t^2+3t-4}$$

$$= \frac{5t^2+8t+3}{2t^2-6t} \times \frac{6t^2-18t}{5t^2-2t-3} \times \frac{t^2+3t-4}{t^2+5t+4}$$

$$= \frac{\cancel{(5t+3)}\,\cancel{(t+1)}}{2t\cancel{(t-3)}} \times \frac{6t\cancel{(t-3)}}{\cancel{(5t+3)}\,\cancel{(t-1)}} \times \frac{\cancel{(t-1)}\,\cancel{(t+4)}}{\cancel{(t+1)}\,\cancel{(t+4)}}$$

$$= \frac{6t}{2t} = 3$$

Complex fractions are fractions whose numerator, denominator, or both, contain fractions. They can be simplified by treating them as division problems.

Example

Simplify:
$$\frac{R_1\left(\dfrac{R_2R_3}{R_2+R_3}\right)}{R_1+\dfrac{R_2R_3}{R_2+R_3}}$$

(The numbers 1, 2, 3 below and to the right of the R's are subscripts. They indicate that the R's are different quantities and are to be treated as if they were different letters.)

$$= R_1\left(\frac{R_2R_3}{R_2+R_3}\right) \div \left(R_1+\frac{R_2R_3}{R_2+R_3}\right)$$

$$= \frac{R_1R_2R_3}{R_2+R_3} \div \frac{R_1R_2+R_1R_3+R_2R_3}{R_2+R_3}$$

$$= \frac{R_1R_2R_3}{\cancel{R_2+R_3}} \times \frac{\cancel{R_2+R_3}}{R_1R_2+R_1R_3+R_2R_3}$$

$$= \frac{R_1R_2R_3}{R_1R_2+R_1R_3+R_2R_3}$$

The solution of fractional equations is not so difficult as you might think, for you can multiply both sides of an equation by the same number (axiom 2). Multiplying all the fractions by the common denominator, you obtain an equation involving whole numbers, which you can then solve as previously outlined.

Examples

1. Solve for a in the equation, $\frac{6a}{7} - \frac{1}{3} = \frac{5a}{14} + \frac{13}{6}$.

42 is the common denominator

Multiply both sides by 42,

$$36a - 14 = 15a + 91$$

Add 14 to both sides,

$$36a = 15a + 105$$

Subtract 15a from both sides,

$$21a = 105$$

Divide both sides by 21,

$$a = 5$$

2. Solve for x in the equation, $\frac{8}{x} + \frac{1}{2} = 2 + \frac{7}{x}$.

Multiply both sides by 2x,

$$16 + x = 4x - 14$$

Subtract 16 from both sides,

$$x = 4x - 30$$

Subtract 4x from both sides,

$$-3x = -30$$

Divide both sides by −3,

$$x = 10$$

3. Solve for m in the equation, $\frac{2m+1}{4m+1} = \frac{2m+5}{4m+7}$.

Multiply both sides by (4m + 1) (4m + 7)

$$(2m+1)(4m+7) = (2m+5)(4m+1)$$

$$8m^2 + 18m + 7 = 8m^2 + 22m + 5$$

Subtract $8m^2$ from both sides,

$$18m + 7 = 22m + 5$$

Subtract 7 from both sides,

$$18m = 22m - 2$$

Subtract 22m from both sides,

$$-4m = -2$$

Divide both sides by −4,

$$m = 0.5$$

4. Solve for r_t in the equation, $\frac{1}{r_t} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$.

Multiply both sides by (r_t) (r_1) (r_2) (r_3),

$$r_1r_2r_3 = r_2r_3r_t + r_1r_3r_t + r_1r_2r_t$$

Factor the right hand member,

$$r_1r_2r_3 = r_t(r_2r_3 + r_1r_3 + r_1r_2)$$

Divide both sides by $(r_2r_3 + r_1r_3 + r_1r_2)$,

$$\frac{r_1r_2r_3}{r_2r_3 + r_1r_3 + r_1r_2} = r_t$$

or

$$r_t = \frac{r_1r_2r_3}{r_2r_3 + r_1r_3 + r_1r_2}$$

Simultaneous Linear Equations

As long as equations contain only one unknown, the simple rules for solution apply, but when an equation has two unknowns, *simultaneous linear equations* must be used.

Simultaneous linear equations are two or more equations that contain only first powers of the unknown quantities and no products of unknowns. They are true for certain values of the unknowns. Among the various methods for finding the values of the unknowns are solution by graphing, elimination by addition or subtraction, elimination by substitution, and the use of determinants. Of these, the most common method is elimination by addition or subtraction. Therefore, it is well to understand this method.

If the unknown you wish to eliminate has the same coefficient in the two equations, merely add (or subtract) the two equations. If the coefficients are different, you can make them the same by multiplying both members of each equation by the right number as you can see in the sample problem.

Example

Solve for e and i in the following pair of equations,

$$2e + 10i = 25 \tag{1}$$

$$5e - \ \ 8i = 46 \tag{2}$$

Multiply equation (1) by 4,

$$8e + 40i = 100 \tag{3}$$

Multiply equation (2) by 5,

$$25e - 40i = 230 \tag{4}$$

Add equation (4) to equation (3),

$$33e = 330 \tag{5}$$

Divide both sides of equation (5) by 33,

$$e = 10 \tag{6}$$

Substitute 10 for e in equation (1),

$$20 + 10i = 25 \tag{7}$$

Subtract 20 from both sides of equation (7),

$$10i = 5 \tag{8}$$

Divide both sides of equation (8) by 10,

$$i = 0.5 \tag{9}$$

In this example, the coefficients of i were made equal by multiplying equation (1) by 4, and equation (2) by 5. You could have eliminated the e's by multiplying equation (1) by 5, and equation (2) by 2. While equations are not always in the form of (1) and (2), you can put them in such a form by simplifying and transposing.

If you have three equations with three unknown quantities, follow the same procedure of elimination. Eliminate one unknown by combining one pair of equations, then eliminate the same unknown from another pair of equations. This results in two equations with two unknowns, which you can solve as explained before.

Example

$$9i_1 - 4i_2 - \ \ 7i_3 = 4 \tag{1}$$

$$5i_1 + 3i_2 - 10i_3 = 3 \tag{2}$$

$$2i_1 - 5i_2 + \ \ 9i_3 = 2 \tag{3}$$

First, eliminate i_3 between (1) and (2)

Multiply equation (1) by 10,

$$90i_1 - 40i_2 - 70i_3 = 40 \tag{4}$$

Multiply equation (2) by 7,

$$35i_1 + 21i_2 - 70i_3 = 21 \tag{5}$$

Subtract equation (5) from equation (4),

$$55i_1 - 61i_2 = 19 \tag{6}$$

Next, eliminate i_3 from (1) and (3),

Multiply equation (1) by 9,

$$81i_1 - 36i_2 - 63i_3 = 36 \tag{7}$$

Multiply equation (3) by 7,

$$14i_1 - 35i_2 + 63i_3 = 14 \tag{8}$$

Add equation (7) and equation (8),

$$95i_1 - 71i_2 = 50 \tag{9}$$

Now you have two equations with two unknowns — equations (6) and (9).

$$55i_1 - 61i_2 = 19 \tag{6}$$

$$95i_1 - 71i_2 = 50 \tag{9}$$

Eliminate i_1

Multiply equation (6) by 19,

$$1045i_1 - 1159i_2 = 361 \tag{10}$$

Multiply equation (9) by 11,

$$1045i_1 - 781i_2 = 550 \tag{11}$$

Subtract equation (11) from equation (10),

$$-378i_2 = -189 \tag{12}$$

Divide equation (12) by −378,

$$i_2 = 0.5$$

Substitute 0.5 for i_2 in equation (6),

$$55i_1 - 30.5 = 19 \tag{13}$$

Add 30.5 to both sides of equation (13),

$$55i_1 = 49.5$$

Divide by 55,

$$i_1 = 0.9$$

Substitute 0.5 for i_2 and 0.9 for i_1 in equation (1),

$$8.1 - 2.0 - 7i_3 = 4$$

Collect terms,

$$6.1 - 7i_3 = 4$$

Subtract 6.1,

$$-7i_3 = -2.1$$

Divide by −7,

$$i_3 = 0.3$$

Check by substituting in equations (1), (2), and (3),

$$8.1 - 2.0 - 2.1 = 4$$

$$4 = 4$$

$$4.5 + 1.5 - 3.0 = 3$$

$$3 = 3$$

$$1.8 - 2.5 + 2.7 = 2$$

$$2 = 2$$

Exponents, Radicals, and Complex Numbers

You have already noted that a number raised to the zero power is equal to one, and a number raised to a negative exponent is the same as the reciprocal of the quantity to that same exponent with a plus sign. Now consider the significance of a fractional exponent. Squaring the quantity $x^{1/2}$ gives $(x^{1/2})\ (x^{1/2})$. By adding the exponents $x^{(1/2+1/2)}$, as in any other multiplication with exponents, you find that the result is x^1, or x. By this reasoning, you can see that $x^{1/2}$ is the square root of x. In like manner, $x^{1/3}$ is the cube root of x, and $x^{1/4}$ *is* $\sqrt[4]{x}$. The expression $x^{2/3}$ is read x to the two-thirds power. This means that x is raised to the power of the numerator of the fractional exponent, and reduced by the root of the denominator. Thus, $x^{2/3}$ is the cube root of x squared, or $\sqrt[3]{x^2}$.

Sometimes it is necessary to simplify an expression involving *radicals* (square roots, cube roots, etc.) without changing its value. Sometimes you can divide the quantity under the radical (called *radicand*) into two factors, and take the indicated root of one of the factors.

Examples

1. $\sqrt{80} = \sqrt{16 \cdot 5} = 4\sqrt{5}$
2. $\sqrt{54} = \sqrt{9 \cdot 6} = 3\sqrt{6}$
3. $\sqrt[3]{16m} = \sqrt[3]{8 \cdot 2m} = 2\sqrt[3]{2m}$

When the quantity under the radical sign is a fraction, multiply both numerator and denominator by a quantity which will make it possible to take the indicated root of the denominator.

Examples

1. $\sqrt{\frac{2}{3}} = \sqrt{\frac{2}{3} \cdot \frac{3}{3}} = \sqrt{\frac{6}{9}} = \sqrt{\frac{1}{9} \cdot 6} = \frac{1}{3}\sqrt{6}$
2. $\sqrt[3]{\frac{40}{x^2}} = \sqrt[3]{\frac{40}{x^2} \cdot \frac{x}{x}} = \sqrt[3]{\frac{40x}{x^3}} = \sqrt[3]{\frac{8}{x^3} \cdot 5x} = \frac{2}{x}\sqrt[3]{5x}$

You may combine (add or subtract) radicals if they differ only in the coefficient, that is, if they have the same radicand and root.

Examples

1. $4\sqrt{x} + 3\sqrt{x} - 2\sqrt{x} = 5\sqrt{x}$
2. $\sqrt{4x} + 5\sqrt{x} = 2\sqrt{x} + 5\sqrt{x} = 7\sqrt{x}$
3. $\sqrt[3]{40} + \sqrt[3]{18\frac{1}{8}} = \sqrt[3]{40} + \sqrt[3]{\frac{135}{8}}$

   $= \sqrt[3]{8 \cdot 5} + \sqrt[3]{\frac{27}{8} \cdot 5} = 2\sqrt[3]{5} + \frac{3}{2}\sqrt[3]{5}$

   $= 3\frac{1}{2}\sqrt[3]{5}$

You can multiply two radicals together provided they have the same index (indicated root). You can do this by multiplying the coefficients together for the coefficient of the product and multiplying the radicands together for the radicand of the product.

Examples

1. $(2\sqrt{3})\ (3\sqrt{2}) = 6\sqrt{6}$
2. $(7\sqrt{x})\ (2\sqrt{y}) = 14\sqrt{xy}$
3. $(2\sqrt{x})\ (3\sqrt{x}) = 6\sqrt{x^2} = 6x$
4. $a\sqrt{b}\ (2\sqrt{3} + 4) = 2a\sqrt{3b} + 4a\sqrt{b}$
5. $(5\sqrt{3} + 2\sqrt{6})\ (5\sqrt{3} + 4\sqrt{6})$

Multiply:

$$\begin{array}{r} 5\sqrt{3} + 2\sqrt{6} \\ \underline{5\sqrt{3} + 4\sqrt{6}} \\ 25\sqrt{9} + 10\sqrt{18} \\ \underline{20\sqrt{18} + 8\sqrt{36}} \\ 25\sqrt{9} + 30\sqrt{18} + 8\sqrt{36} \end{array}$$

$= 25 \cdot 3 + 30\sqrt{9 \cdot 2} + 8 \cdot 6$

$= 75 + 30 \cdot 3\sqrt{2} + 48$

$= 123 + 90\sqrt{2}$

To divide expressions containing radicals, use the opposite process; that is, divide coefficient by coefficient and radicand by radicand. In many cases the result of this process is an expression with a radical in the denominator. The radical can be eliminated by a special process called *rationalizing the denominator.* This process changes the denominator to a whole number, a fraction, or a mixed number.

You can rationalize the denominator by multiplying both the numerator and the denominator by a quantity that will eliminate radicals from the denominator. In the case of monomials this is quite simple, and is the same process as discussed previously under *simplification of fractional radicands.*

Example

$$\frac{\sqrt{5}}{\sqrt{6}} = \sqrt{\frac{5}{6}} = \sqrt{\frac{5}{6} \cdot \frac{6}{6}} = \sqrt{\frac{30}{36}} = \sqrt{\frac{1}{36} \cdot 30} = \frac{1}{6}\sqrt{30}$$

When the denominator is a binomial, rationalize by multiplying both the numerator and the denominator by the binomial with the sign between terms changed. This changed binomial is called the *conjugate* of the denominator.

Examples

1. $\frac{5}{3 - \sqrt{2}} = \frac{5}{3 - \sqrt{2}} \cdot \frac{3 + \sqrt{2}}{3 + \sqrt{2}} = \frac{15 + 5\sqrt{2}}{9 - \sqrt{4}} =$

   $\frac{15 + 5\sqrt{2}}{9 - 2} = \frac{15 + 5\sqrt{2}}{7}$

   Here the denominator was $3 - \sqrt{2}$ and the conjugate was $3 + \sqrt{2}$

2. $\frac{x}{\sqrt{x} + a} = \frac{x}{\sqrt{x} + a} \cdot \frac{\sqrt{x} - a}{\sqrt{x} - a} = \frac{x\sqrt{x} - ax}{\sqrt{x^2} - a^2}$

   $= \frac{x\sqrt{x} - ax}{x - a^2}$

3. $\frac{4 - \sqrt{m}}{2 + \sqrt{m}} = \frac{4 - \sqrt{m}}{2 + \sqrt{m}} \cdot \frac{2 - \sqrt{m}}{2 - \sqrt{m}} = \frac{8 - 6\sqrt{m} + \sqrt{m^2}}{4 - \sqrt{m^2}}$

   $= \frac{8 + m - 6\sqrt{m}}{4 - m}$

4. $\frac{\sqrt{r} - \sqrt{s}}{\sqrt{r} + 2\sqrt{s}} = \frac{\sqrt{r} - \sqrt{s}}{\sqrt{r} + 2\sqrt{s}} \cdot \frac{\sqrt{r} - 2\sqrt{s}}{\sqrt{r} - 2\sqrt{s}} =$

   $\frac{\sqrt{r^2} - 3\sqrt{rs} + 2\sqrt{s^2}}{\sqrt{r^2} - 4\sqrt{s^2}} = \frac{r + 2s - 3\sqrt{rs}}{r - 4s}$

Note that in each of the foregoing examples the final result has no radicals in the denominator.

Since nearly all applications of this process in electronic circuits deal with binomials, it is not necessary to discuss nor to give examples involving more terms.

Thus far you have dealt with roots of positive quantities. We say that P is a square root of B if $P^2 = B$. If B is positive, it has exactly two square roots, one positive and one negative, denoted by $\pm\sqrt{B}$. If a negative number $-A$ has R as square root, then $R^2 = -A$. But, if R is *either* positive or negative, R^2 is positive and cannot equal $-A$. Therefore, $-A$ has *no positive or negative square root*. In order for $-A$ to have square roots, the symbol $\sqrt{-1}$, with the property that $\sqrt{-1}\ \sqrt{-1} = -1$, is introduced. $\sqrt{-1}$ is *defined* as a new variety of number, called an *imaginary number*. The letter *i* stands for $\sqrt{-1}$ in mathematics. In electrical work the symbol *j* is used instead of *i* because the letter *i* is used to represent current. The quantity $\sqrt{-1}$ is usually referred to as the *j operator* and is used frequently in the solution of alternating current problems.

Now the indicated square roots of negative numbers can be treated as follows: $\sqrt{-x} = \sqrt{x} \cdot \sqrt{-1} = j\sqrt{x}$; $\sqrt{-3} = \sqrt{3} \cdot \sqrt{-1} = j\sqrt{3}$. In each case, *j* stands for the imaginary quantity, the square root of -1.

Since $j = \sqrt{-1}$, then $j^2 = -1$. Operations of addition, subtraction, multiplication, and division may be applied to combinations of *j* and real numbers as if *j* were an ordinary real literal number.

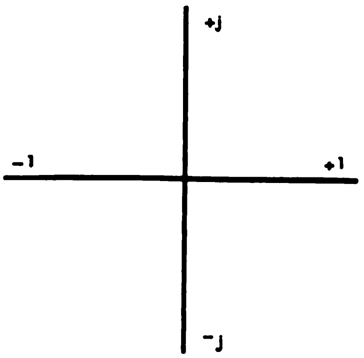


Figure 1—1. Positions of a Unit Vector

Real and imaginary quantities can be graphically represented by four positions of a unit vector, as shown in Figure 1-1. Positive real numbers are plotted to the right of the origin, and negative real numbers to the left along the horizontal axis, which is known as the *axis of reals*. Imaginaries that have *positive* signs are plotted *above* the origin and those that have negative signs, *below* the origin along the vertical axis, which is known as the axis of imaginaries.

Real and Imaginary Numbers

A *complex number* is the sum or difference of a real quantity and an imaginary quantity. For example, $5 - jb$ and $3 + j2$ are complex numbers. The term complex number is really inappropriate because the system is not complex at all. *Rectangular notation* is a better designation. It is not difficult to apply the fundamental processes to this special notation, for two or more complex numbers can be combined by combining the real portions and imaginary portions separately.

Examples

Addition. (1)
$$\begin{array}{r} 5 + j3 \\ 2 - j2 \\ \hline 7 + j \end{array}$$
(2)
$$\begin{array}{r} 6 - j4 \\ -5 - j3 \\ \hline 1 - j7 \end{array}$$

Subtraction. (1)
$$\begin{array}{r} 5 + j3 \\ 2 - j2 \\ \hline 3 + j5 \end{array}$$
(2)
$$\begin{array}{r} 6 - j4 \\ -5 - j3 \\ \hline 11 - j \end{array}$$

To multiply complex numbers, use the same procedure as with binomials except that, where j^2 occurs in the final result, you replace it by its equivalent, -1.

Example

Multiply $5 + j3$ by $2 - j2$

$$\begin{array}{r} 5 + j3 \phantom{{}-j^26} \\ 2 - j2 \phantom{{}-j^26} \\ \hline 10 + j6 \phantom{{}-j^26} \\ -j10 - j^26 \\ \hline 10 - j4 - j^26 \end{array}$$

$$= 10 - j4 - (-1)\ (6) = 10 - j4 + 6 = 16 - j4$$

Divide one rectangular quantity (complex number) by another by rationalizing the denominator, then dividing the real number into the numerator. Remember that you rationalize the denominator by multiplying both numerator and denominator by the conjugate of the denominator.

Examples

1. Divide (5 + j3) by (2 − j2)

$$\frac{5 + j3}{2 - j2} \cdot \frac{2 + j2}{2 + j2} = \frac{10 + j16 + j^2 6}{4 - j^2 4} = \frac{10 + j16 - 6}{4 + 4}$$

$$= \frac{4 + j16}{8} = \frac{4(1 + j4)}{8} = \frac{1 + j4}{2} \text{ or } 0.5 + j2$$

2. Divide (6 − j4) by (−5 − j3)

$$\frac{6 - j4}{-5 - j3} \cdot \frac{-5 + j3}{-5 + j3} = \frac{-30 + j38 - j^2 12}{25 - j^2 9}$$

$$= \frac{-30 + j38 + 12}{25 + 9} = \frac{-18 + j38}{34} = \frac{-9 + j19}{17}$$

Quadratic Equations

The degree of an equation in which the unknown has only positive integral exponents and does not appear in the denominator of a fraction is the same as its term of highest degree. The degree of a term in a letter means its exponent in that term. To illustrate, the degree of the term $4x^2y^3$ in x is the second. The equation $ax^2 + bx + c = 0$ is a second degree or quadratic equation in x. The following discusses the solution of equations of this form.

There are several methods you can use to solve a quadratic equation, such as graphing, completing the square, factoring, and using a formula derived from the general form $ax^2 + bx + c = 0$ by the completion of the square method. The formula is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where a, b, and c are, respectively the coefficient of x^2, the coefficient of x, and the term which does not contain x. Before applying the formula, always put the equation in the standard form $ax^2 + bx + c = 0$ first. The following examples illustrate the processes for solving quadratic equations by the formula.

Examples

1. Solve for x in the equation $3x^2 + 8x - 10 = 0$.

By the formula, $$x = \frac{-8 \pm \sqrt{64 + 120}}{6}$$

$$= \frac{-8 \pm \sqrt{184}}{6} = \frac{-8 \pm 13.56}{6}$$

$$= \frac{-21.56}{6} \text{ or } \frac{5.56}{6}$$

$$= -3.58 \text{ or } 0.93$$

2. Solve for a in the equation $2a = 3a^2 - 8$.

Subtract $3a^2$ from both sides

$$-3a^2 + 2a = -8$$

Add 8 to both sides

$$-3a^2 + 2a + 8 = 0$$

By formula

$$a = \frac{-2 \pm \sqrt{4 + 96}}{-6} = \frac{-2 \pm \sqrt{100}}{-6}$$

$$= \frac{-2 \pm 10}{-6}$$

$$= \frac{-12}{-6} \text{ or } \frac{8}{-6} = 2 \text{ or } -1.33$$

3. Solve for R in the equation $\frac{R^2}{R + 5} = \frac{R}{3} + \frac{5}{6}$

$$6R^2 = 2R\,(R + 5) + 5(R + 5)$$

$$6R^2 = 2R^2 + 10R + 5R + 25$$

$$6R^2 = 2R^2 + 15R + 25$$

$$4R^2 - 15R - 25 = 0$$

$$R = \frac{15 \pm \sqrt{225 + 400}}{8}$$

$$= \frac{15 \pm \sqrt{625}}{8} = \frac{15 \pm 25}{8}$$

$$= \frac{40}{8} \text{ or } \frac{-10}{8} = 5 \text{ or } -1.25$$

The quantity $b^2 - 4ac$ which appears under the radical in the formula is called the *discriminant*. It indicates the type of roots. If $b^2 - 4ac$ is positive, there are two real and unequal roots; if $b^2 - 4ac$ is negative, the roots are imaginary and unequal; if $b^2 - 4ac$ equals zero, the roots are real and equal.

LOGARITHMS

The *logarithm* of a quantity is the exponent (or the power) to which a given number (called the base) must be raised to equal that quantity. To illustrate, in the quantity $3^2 = 9$, the exponent, 2, is called the logarithm of 9 to the base 3. This relation is usually written $\log_3 9 = 2$. Any positive number greater than 1 might serve as a base. Two numbers have been selected, resulting in two systems of logarithms. One base, 2.718, usually indicated by the Greek letter epsilon (ϵ), or e, is used in the *natural* logarithm system. The other base is 10; it is used in the *common* system of logarithms. In the common system, the base 10 is usually omitted in the logarithmic expression. Thus $\log_{10} 1000 = 3$ is usually written $\log 1000 = 3$. In the natural system the base (e) may be written *ln*.

In the common system, logarithms that are exact powers of 10 are integers. Thus,

$\log 10000 = 4,$ since $10^4 = 10000$
$\log 1000 = 3,$ since $10^3 = 1000$
$\log 100 = 2,$ since $10^2 = 100$
$\log 10 = 1,$ since $10^1 = 10$
$\log 1 = 0,$ since $0 = 10^0$
$\log 0.1 = -1,$ since $0.1 = 10^{-1}$
$\log 0.01 = -2,$ since $0.01 = 10^{-2}$

For numbers not exact powers of ten, the logarithm consists of two parts, an integral part (whole number) and a decimal part. The integral part is called the *characteristic* and the decimal part the *mantissa*. Thus, for example, log *595* = *2.7745* (this means, the logarithm of *595 is 2.7745*), the characteristic is *2*, and the mantissa is *.7745*. The characteristic is found by inspection and the mantissa from logarithmic tables.

You can determine the characteristic by the following rules:

1. The characteristic of the logarithm of a number greater than 1 is positive and is one less than the number of digits to the left of the decimal point. For example, in the case of the log *595*, the characteristic is *2*, and for the log of *59.5*, the characteristic is *1*.
2. The characteristic of the logarithm of a number less than 1 is negative, and is equal to one more than the number of zeros immediately to the right of the decimal point. For example, for the log of *.0595*, the characteristic is *—2*, and for log *.00595*, the characteristic is *—3*.

When the characteristic is negative, do not put the minus sign in front of the logarithm, since it applies only to the characteristic and not to the entire logarithm. Instead, add *10* to the negative characteristic and indicate the subtraction of *10* at the end of the logarithm.

Thus the characteristic *—2* is written, *8. (mantissa) —10*, and the characteristic *—3* is written, *7. (mantissa) —10*.

Another method of indicating that the characteristic is negative is to place the minus sign above the characteristic. For example:

$\bar{2}$. (mantissa), and
$\bar{3}$. (mantissa)

The mantissa must be found from the tables of logarithms. Numbers which have the same figures in the same order and differ only in the position of the decimal point have the same mantissa in their logarithms. For example, the mantissa of *595* is *.7745;* the mantissa of *59.5* is also *.7745*.

To find the logarithm of a number:

1. Determine by inspection the characteristic of the number.
2. Find the mantissa from the tables. The mantissa of the number is independent of the position of the decimal point, so you can disregard the decimal point in the number when finding the mantissa. The mantissas in the table are the decimal part of the logarithm and therefore should be preceded by the decimal point.

In four-place logarithm tables, the first column in the table contains the first two digits of the numbers whose mantissas are given in the table, and the top row contains the third digit. Thus, to find the mantissa of *595*, find *59* in the left-hand column and *5* at the top. In the column under *5*, and opposite *59*, is *7745*, the mantissa. The logarithm of *595* is then *2.7745*. (See applicable Logarithm Table in Attachment 2.)

3. To find the logarithm of a quantity with more than three digits, use the process called interpolation. Suppose you want to find the logarithm of *5956*. The tables do not give the mantissa for *5956*. However, they give the mantissas for *5950* and *5960*.(The mantissa for *5950* is the same as that for *595*. Likewise, the mantissa for *5960* is the same as that for *596*.) Since *5956* lies between *5950* and *5960*, its mantissa must lie between the mantissa for these two numbers.

By arranging the mantissas in the following tabular form,

Mantissa of *5960* = *.7752*
Mantissa for *5956* = *?*
Mantissa for *5950* = *.7745*

you can see that *5956* is *6/10* of the way from *5950* to *5960*. We assume the mantissa of *5956* to be *6/10* of the way from *.7745* to *.7752*. Since the difference between the two is *.0007* and since *6/10* of *.0007* is *.00042*, add *.00042* to *.7745* (the mantissa of *5950*). The result, *.77492* is assumed to be the mantissa of 5956. Therefore, the logarithm of *5956* = *3.77492*.

The number corresponding to a given logarithm is called the *antilogarithm* of that number. It is written *antilog* or $\log^{-1}$. To find the antilog, reverse the process for finding logarithms.

Examples

1. To find the antilog (log^{-1}) of *1.8987*, first look in the logarithm table and locate the mantissa *.8987*. It is in line with the number *79* and under the column headed *2*. Thus the number corresponding with the mantissa *.8987* has the digits *792*.

To determine the location of the decimal point, reverse the rule for finding a characteristic. If the characteristic were zero, the decimal point would be placed after the first digit (*7.92*). Since the characteristic is *1*, count two places to the right and place the decimal point after the *9*. Thus the antilog of *1.8987* is *79.2*.

2. Find the antilog (log^{-1}) of *2.4325*.

The tables do not show the mantissa *.4325;* therefore you must interpolate. The mantissa *.4325* lies between the two mantissas *.4330* and *.4314*. These mantissas correspond to *2710* and *2700* respectively. The difference between *.4330* and *.4314* is *.0016*, and the difference between *.4325* (the given mantissa) and the mantissa for *2700* is *.0009*. Then find the number corresponding to *.4325* by adding $\frac{.0009}{.0016}$ of *10*, or *6*, to *2700*, thus giving *2706*. Since the given characteristic is *2*, the antilog of *2.4325* is *270.6*.

Computations with Logarithms

To multiply two quantities, add their logarithms and find the antilog of the result.

Example

Find the product of 6952 and 437.

Solution

Log (6952 × 437) = log 6952 + log 437.

Log 6952 = 3.8421

Log 437 = 2.6405

Adding, log 6952 + log 437 = 6.4826

Find the antilog 6.4826.

The antilog 6.4826 = 3,038,000.

Actual multiplication of *6952* by *437* would give the result *3,037,842*. This indicates an error of *176* in over *3,000,000* or *.006* of one percent. The error is because the logarithm tables go to four places only. Although greater accuracy would result from using five-place tables, usually the accuracy obtained with four-place tables is sufficient.

To divide two quantities, subtract the logarithm of the divisor from the logarithm of the dividend and find the antilog of the result.

Example

Find the quotient of 6952 divided by 437.

Solution

Log (6952 × 437) = log 6952 + log 437.

Log 6952 = 3.8421

Log 437 = 2.6405

Subtracting, log 6952 − log 437 = 1.2016

Find antilog 1.2016

Antilog 1.2016 = 15.908

To raise a quantity to any power, multiply the logarithm of the quantity by the exponent, or the power, and find the antilog of the result.

Examples

1. Find the value of $(5.2)^6$

Solution

$$\text{Log } (5.2)^6 = 6 \log (5.2)$$

$$\text{Log } (5.2) = 0.716$$

$$\text{Log } (5.2)^6 = 6(0.716) = 4.296$$

$$\text{Antilog } 4.296 = 19768$$

Therefore, $(5.2)^6 = 19768$

2. Find the value of $(3.7)^{1/5}$. (Exponent is fractional.)

Solution

$$\text{Log } (3.7)^{1/5} = \frac{1}{5} \log (3.7)$$

$$\text{Log } (3.7) = 0.5682$$

$$\text{Then log } (3.7)^{1/5} = \frac{1}{5}(0.5682) = 0.1136$$

$$\text{Antilog } 0.1136 = 1.2991$$

$$\text{Therefore, } (3.7)^{1/5} = 1.2991$$

3. Find the value of $(45.6)^{-4}$ (Exponent is negative.)

Solution

$$\text{Log } (45.6)^{-4} = -4 \log (45.6)$$

$$\text{Log } 45.6 = 1.659$$

$$\text{Log } (45.6)^{-4} = -4(1.659) = -6.636$$

Since logarithm tables list only positive values o mantissa, rewrite −6.636 as either 10 − 6.636 − 10 which is 3.364 − 10,

or $\overline{7}.364$.

$$\text{Antilog } (\overline{7}.364) = .0000002312$$

$$\text{Therefore, } (45.6)^{-4} = .0000002312$$

Alternate Solution

$$(45.6)^{-4} = \frac{1}{(45.6)^{4}}$$

$$\text{Log } 1/(45.6)^4 = \log 1 - 4 \log (45.6)$$

$$\text{Log } 1 = 1.000 \text{ or } 10.000 - 10$$

$$4 \log (45.6) = 4(1.659) = 6.636$$

$$\text{Log } (45.6)^{-4} = 10.000 - 6.636 - 10 = 3.364 - 10$$

$$\text{Antilog } (3.364 - 10) = .0000002312$$

$$\text{Therefore, } (45.6)^{-4} = .0000002312$$

To find the root of a quantity, obtain th logarithm of the quantity, divide it by the inde of the root, and find the antilog of the result.

Example

1. Find the value of $\sqrt[3]{1.572}$

Solution

$\text{Log } \sqrt[3]{1.572} = (\log 1.572) \div 3$

$\text{Log } 1.572 = 0.19646$

$\text{Then } \log \sqrt[3]{1.572} = \dfrac{(0.19646)}{3} = 0.06549$

$\text{Antilog } 0.06549 = 1.1611$

$\text{Therefore, } \sqrt[3]{1.572} = 1.1611$

NATURAL LOGARITHMS

The natural system of logarithms uses a base *e*, which is approximately *2.718*. Natural logarithm tables in Attachment 2 give the complete logarithm rather than the mantissas only.

To find the natural log of a quantity which does not appear in the table and cannot be obtained by interpolation, or if there is no natural logarithm table, use the common logarithm table and apply the following relationships:

$$\text{Log}_e N = 2.3026 \log_{10} N$$
$$\text{Log}_{10} N = 0.4343 \log_e N$$

In multiplying, dividing, raising to powers, or finding roots with the natural system, use the same rules as in the common logarithm system.

Example

Find the value of $\dfrac{54.3 \times 27.5}{13.65}$

Solution

$$\text{Log}_e \frac{54.3 \times 27.5}{13.65}$$
$$= \log_e 54.3 + \log_e 27.5 - \log_e 13.65$$
$$\text{Thus, } \log_e \frac{54.3 \times 27.5}{13.65}$$
$$= 3.9954 + 3.3190 - 2.6131$$
$$= 4.7004$$
$$\text{Antilog}_e\ 4.7004 = 109.9$$

Natural logarithms are frequently used in analyzing the charge and discharge action of capacitors. Analysis of capacitor action discussed in detail in Chapter 2 makes use of the expression e^{-x} in which *e* is the base in the natural logarithm system.

TRIGONOMETRIC FUNCTIONS

Several special relationships, called *trigonometric functions*, hold true in a right triangle. Electrical problems, when reduced to a right triangle, can be easily and quickly solved by use of tables based upon these functions.

In Figure 1–2 θ is the angle *ZOR*. *OR* is the projection of *OZ* on the horizontal axis; *OX* is the projection of *OZ* on the vertical axis. The length *ZR* is equal to the length *XO*. *R* is perpendicular to the horizontal axis, making *OZR* a right triangle. The letters *r*, *x*, and *z* represent the lengths of *OR*, *OX*, and *OZ*.

There are six different ratios associated with the three sides of a right triangle. Each ratio has been given a name. By definition, in a right triangle *ZOR* with the right angle at *R* (Figure 1–2):

$$\text{Sine of } \theta = \sin\theta = \frac{\text{side opposite the angle } \theta}{\text{hypotenuse}} = \frac{x}{z}$$

$$\text{Cosine of } \theta = \cos\theta = \frac{\text{side adjacent to the angle } \theta}{\text{hypotenuse}} = \frac{r}{z}$$

$$\text{Tangent of } \theta = \tan\theta = \frac{\text{side opposite the angle } \theta}{\text{side adjacent to the angle } \theta} = \frac{x}{r}$$

$$\text{Cotangent of } \theta = \cot\theta = \frac{\text{side adjacent to the angle } \theta}{\text{side opposite the angle } \theta} = \frac{r}{x}$$

$$\text{Secant of } \theta = \sec\theta = \frac{\text{hypotenuse}}{\text{side adjacent to the angle } \theta} = \frac{z}{r}$$

$$\text{Cosecant of } \theta = \csc\theta = \frac{\text{hypotenuse}}{\text{side opposite the angle } \theta} = \frac{z}{x}$$

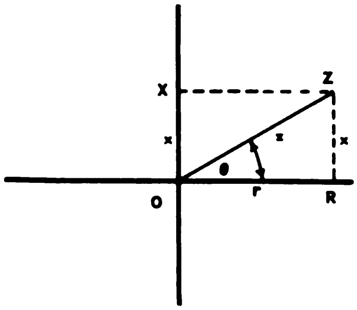


Figure 1–2. Trigonometric Functions

Notice that *sin θ* is the reciprocal of *csc θ*; *cos θ* is the reciprocal of *sec θ*, and *tan θ* is the reciprocal of *cot θ*. You will use sin θ, cos θ, and tan θ, very often in your work. You can save much time by memorizing these three ratios.

If you assume that in Figure 1–2, *OZ* has a unit length of 1 and is rotated in a counterclockwise direction beginning with angle θ at a value of 0° and continuing until it is 90°, then the functions will vary within the following limits:

$\sin\theta$ increases from 0 to 1
$\cos\theta$ decreases from 1 to 0
$\tan\theta$ increases from 0 to ∞
$\cot\theta$ decreases from ∞ to 0
$\sec\theta$ increases from 1 to ∞
$\csc\theta$ decreases from ∞ to 1

(∞ is not used as a number. It represents an *infinitely large* number and indicates that the function is very large.)

When θ varies between the values of 90° and 180° (second quadrant), the projection *r* is negative and the functions which involve *r* become negative. Thus, $\cos\theta$ and $\tan\theta$ are negative in this quadrant. In the third quadrant, both *r* and *x* are negative. Therefore, sine θ and $\cos\theta$, which involve only one or the other, are negative, while $\tan\theta$, which involves both *r* and *x*, is positive. In the fourth quadrant, *r* is positive, but *x* is still negative. For this reason, sine θ and $\tan\theta$ are negative in the fourth quadrant, while $\cos\theta$ is positive.

Variations in Value of Functions

Quadrant		*Sin θ*		*Cos θ*		*Tan θ*	
		From	*To*	*From*	*To*	*From*	*To*
I	(0°-90°)	0	1	1	0	0	∞
II	(90°-180°)	1	0	0	−1	$-\infty$	0
III	(180°-270°)	0	−1	−1	0	0	∞
IV	(270°-360°)	−1	0	0	1	$-\infty$	0

Trigonometric tables give functions up to 90° only. Therefore you will need to use special rules for finding values of functions of angles in the other quadrants.

The following rules apply for functions of angles greater than 90°.

In quadrant II: $\theta = 180°$ minus some acute angle which is designated α (alpha).

$\sin(180° - \alpha) = \sin(\alpha)$
$\cos(180° - \alpha) = -\cos(\alpha)$
$\tan(180° - \alpha) = -\tan(\alpha)$

In quadrant III: $\theta = 180° + \alpha$

$\sin(180° + \alpha) = -\sin\alpha$
$\cos(180° + \alpha) = -\cos\alpha$
$\tan(180° + \alpha) = \tan\alpha$

In quadrant IV: $\theta = 360° - \alpha$

$\sin(360° - \alpha) = -\sin\alpha$
$\cos(360° - \alpha) = \cos\alpha$
$\tan(360° - \alpha) = -\tan\alpha$

Examples

1. Find sin 29°.

Solution

Find the angle 29° in the table, (see Attachment 2). Opposite this angle and in the column headed sine is 0.4848. Therefore, sine 29° = 0.4848.

2. Find cos 129°.

Solution

Cos 129° = cos (180°-51°). According to the rules for angles larger than 90°, cos (180°-51°) = −cos 51°. The tables show that cos 51° is 0.6293. Therefore cos 129° = −0.6293.

Interpolation is necessary only occasionally, since the tables give functions of angles to tenths of a degree or to minutes.

The angle corresponding to a given function is called the inverse *trigonometric function.* The expression *arc sin h*, which is read *arc sine h*, means "an angle whose sine is h". By definition, then, these equations are equivalent:

$$h = \sin\theta \text{ and } \theta = \text{arc}\sin h.$$

Similar equations can be set up for each of the trigonometric functions. Another way of writing the inverse function is $\sin^{-1}h = \theta$.

There are two angles corresponding to any given function. For example, arc sin 0.7071 is 45° or 135°. To avoid confusion, certain values of arc sin, arc cos, arc tan, etc., are designated the principal values and are the values meant unless otherwise indicated. For arc sin and arc tan the principal values are those in the first and fourth quadrants; for arc cos the principal values are those in the first and second quadrants. Therefore, arc sin 0.7071 = 45°.

In finding the function of an angle, or vice versa, note that the functions repeat themselves every 360°. Thus $\sin 400° = \sin(400° - 360°) = \sin 40° = 0.6428$. You can say that a negative angle is equivalent to a positive angle which is 360° plus the negative angle. To illustrate, $-50° = 360° + (-50°) = 310°$. Hence $\sin(-50°) = \sin 310° = -0.7660$. Angles in the fourth quadrant are frequently expressed as negative angles.

USE OF THE SLIDE RULE

Paper and pencil methods of making computations are, at best, slow and often introduce unintentional errors. Mathematical computations can be done very quickly and quite accurately

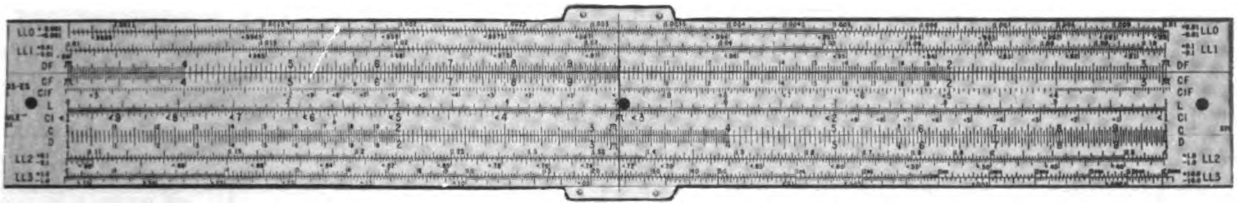


Figure 1–3. Slide Rule

with a slide rule. This instrument consists of two rules, one moving with respect to the other. It can be used in solving many types of problems. It is only a tool, however, and without you, it can do nothing.

The log-log decitrig slide rule shown in Figure 1–3 indicates the final answer of a computation in the form of three or four digits. For placing the decimal point there have been developed many rules. However, the surest method is to use the approximate answer to locate the decimal point in the answer provided by the slide rule. For example, in multiplying 513 by 13, on the slide rule, the digits in the answer are 667. To locate the decimal, round off the numbers 513 and 13 to 500 and 10, respectively, and mentally calculate the product. From this product, 5000, you can determine that the decimal point should be located so that the answer given by slide rule reads 6670. By adopting such a rule for locating the decimal point, you can avoid incorrect answers.

The slide rule consists of several logarithmic scales so arranged that you can make computations by setting a movable portion, the *slide*, in a certain position relative to the fixed part, the *body* of the slide rule, and reading the answer. The types of computations that are possible with a slide rule depend on the type of slide rule, but usually they include multiplication, division, squaring, cubing, extracting square and cube roots, and finding logarithms and trigonometric functions. These processes are discussed briefly here, but before taking up the processes, it is well to know the meaning of several terms. The slide and body have already been identified. The glass runner is called the *indicator* and the mark upon it is the *hairline*. The mark associated with the number one on the body or slide is the *index*. Two positions on different scales are opposite each other when the hairline covers both positions at the same time.

Multiplication

Use the C and D scales to find the product of two numbers. To perform this operation, set the index of the C scale opposite the multiplicand on the D scale, move the indicator to the

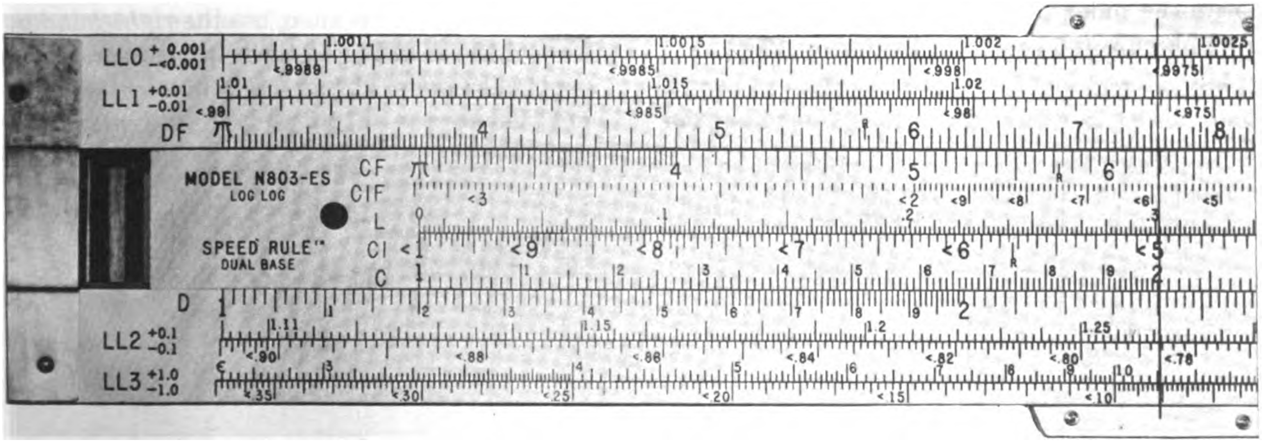


Figure 1–4. Multiplication (Slide Rule)

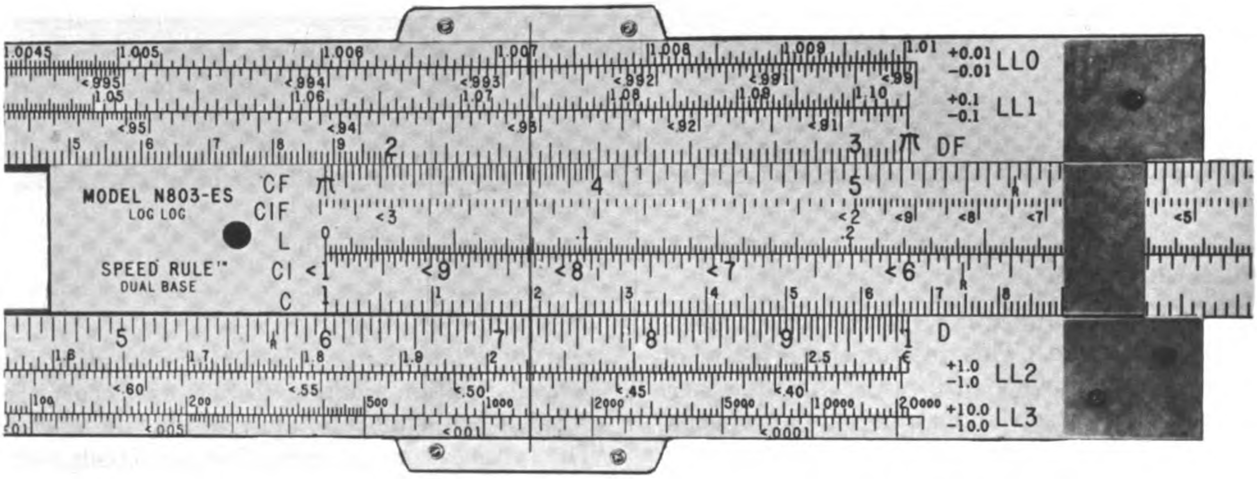


Figure 1–5. Division (Slide Rule)

multiplier on the C scale and read the product on the D scale, opposite the multiplier on the C scale under the hairline. This gives a number of three or four digits. Place the decimal point in accordance with a rough estimate. Note that the C scale has two indices, one at either end. If you find that when you set the left-hand index opposite the multiplicand, the multiplier on the C scale is beyond the index of the D scale, then use the right-hand index of the C scale and proceed as previously indicated. Figure 1–4 shows *12* multiplied by *2*.

Division

In dividing two numbers, set the slide so that the divisor on the C scale is opposite the dividend on the D scale. The number on the D scale opposite the index of the C scale is then the quotient. Figure 1–5 shows *72* divided by *12*.

Squaring and Extracting Square Root

To square a number, set the indicator so that the hairline covers, on the D scale, the number to be squared. Then read the result under the hairline on the A scale. The location of the decimal point can be found as previously explained.

To extract the square root of a number, set the indicator so that the hairline covers, on the A scale, the number whose root is to be extracted. Since the A scale is a double scale, the same number appears on it in two places. The *left-hand portion* of scale A is used for numbers between *1* and *10;* the *right-hand portion* for numbers between *10* and *100;* the *left-hand portion* for numbers between *100* and *1000*, and so on. If the number of digits to the left of the decimal point in the number is *odd*, use the *left-hand portion* of the A scale. If the number of digits to the left of the decimal point is *even*, use the *right-hand portion* of the A scale. If the number is a decimal,

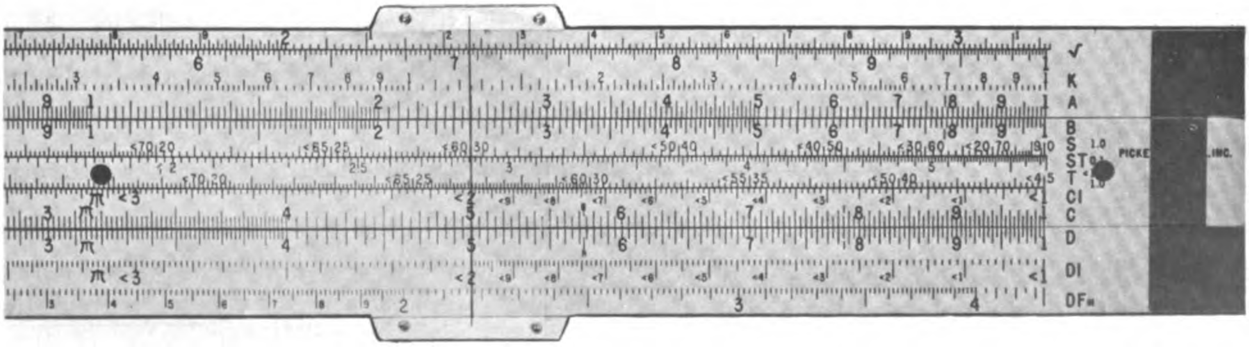


Figure 1–6. Square Root, Squaring (Slide Rule)

it can be changed to a whole number by rewriting it as a whole number times *10* raised to a negative *even* power. (When the root is extracted, remember to divide this negative exponent by two to find the root.) Now that the number is set up, read the root under the hairline on the D scale. A rough estimate will give you the location of the decimal point. Figure 1–6 illustrates the square of *5*, or reading in reverse, the square root of *25*.

Logarithms

The L scale gives the mantissa of the common logarithm for the opposite number on the D scale. Determine the characteristic in the usual manner from the number of digits. Find the antilog by locating the number on the D scale opposite the mantissa on the L scale and putting the decimal point in the place indicated by the characteristic.

Reciprocals

The CI scale is made up of the reciprocals of the opposite numbers on the C scale. The D and DI scales give the same information.

Trigonometric Functions

The process of finding the sines and tangents of angles differs from slide rule to slide rule. In some slide rules the S, ST, and T scales are on the body of the rule and the C scale is on the slide. In other rules the S, ST, and T scales are on the slide with the D scale on the body. On the S scale the numbers that increase from the left- to the right-hand end of the rule are used to find the value of the sine of the angle. On all slide rules these are black numerals. The numbers which decrease in value from the left-hand end to the right-hand end (red on some rules) can be used to find the cosine of the angle. For angles under 5.74° the ST scale is used for sines and tangents. The left index is 0.1 and the right index is 1.0 when using the C or D scale with the S scale. The left index is .01 and the right index is 0.1 when using either with the ST scale.

To find the sine of an angle over *5.74°*, set the hairline over the black number representing the size of the angle in degrees on the S scale, or over the number in the series which increases from left to right on the S scale. Then read the sine value from the C or the D scale. When the angle is less than *5.74°*, use the ST scale and follow the same procedure. When the value of the sine is given and the angle is sought, the process is reversed. Set the hairline over the number representing the sine of the angle on the C or the D scale and read the size of the angle from the S scale (sine values over 0.1) or the ST scale (sine values between .01 and 0.1).

Since $\sin \theta = \cos (90^\circ - \theta)$, it is possible to find the cosine of an angle using the S scale and the red numbers, or the numbers which increase in value from right to left, in the same way that the sine was found.

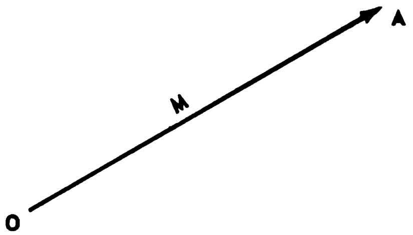
To find the tangent of angles less than *5.74°*, set the hairline to the angle on the ST scale and read the tangent on the C or the D scale. Remember that values on the C and D scales range from .01 at the left index to 0.1 at the right when used with the ST scale. For angles from *5.74°* to *45°*, set the hairline to the figure on the T scale and read the tangent from the C or the D scale. These values range from 0.1 to 1.0. For angles greater than *45°* some slide rules will have red figures. Again the hairline is set over the number on the T scale, but the results are read from the CI scale. On some slide rules you will need to turn the rule over and read from the opposite side, since the CI and DI scales are on just one side of the rule. These values range from 1 to 10. Tangents of angles above *84°* cannot be obtained directly from the slide rule.

Many other computations are possible with the slide rule. However, those given cover the bulk of calculations you will have to perform.

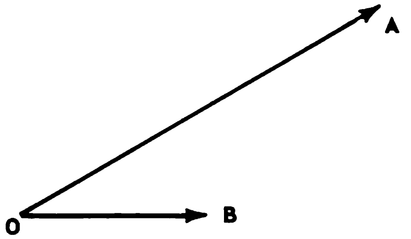
VECTOR QUANTITIES AND VECTOR ALGEBRA

Quantities that have magnitude only are known as *scalar* quantities. They may be added, subtracted, multiplied, etc., directly. Other quantities have both magnitude and direction. These are known as *vector* quantities. Any physical quantity possessing a magnitude and a direction can be represented as a vector. Forces, motion, acceleration, reactances, voltages, and currents are examples of vector quantities. Special methods must be used in dealing with such quantities, since they have direction as well as magnitude.

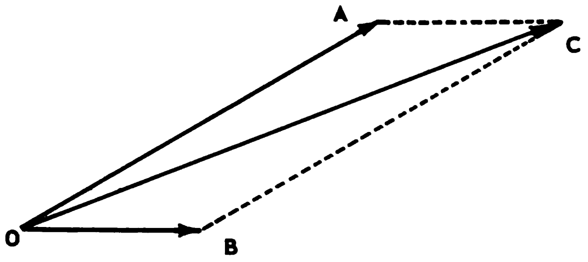
A vector quantity may be represented by a line segment with an arrow point to show the direction of the vector. This is shown in Figure 1–7A.



A REPRESENTING A VECTOR QUANTITY



B TWO FORCES ACTING ON AN OBJECT



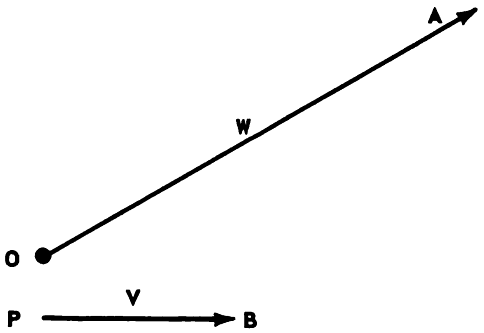
C RESULTANT VECTOR

Figure 1–7. Vector Quantities

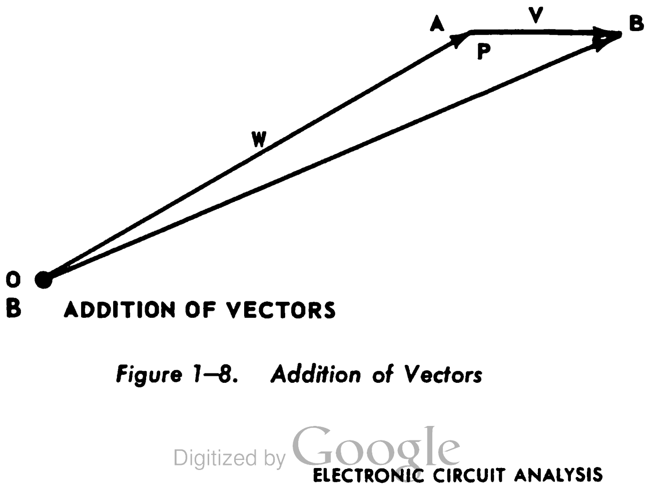
The length of the line indicates the magnitude, and the angular position of the line and the arrow point indicate the direction. Usually this line is spoken of as *vector OA*. However, you may find it written $\overrightarrow{\mathrm{OA}}$, $\overline{\mathrm{OA}}$, or $\underline{\mathrm{OA}}$. Other notations to indicate vector quantities are $\overline{\mathrm{M}}$, $\underset{\bullet}{\mathrm{M}}$, and $\dot{\mathrm{M}}$.

Since any force has a direction and a magnitude, it can be represented geometrically by a properly directed vector. The length of the vector in some linear unit represents the measure of the force in some physical unit. In Figure 1–7B vectors OA and OB represent two forces pulling (or acting) on an object at O simultaneously. It has been proved that the effect of the two forces acting on an object is the same as the effect of a single force of the magnitude and direction of the resultant vector OC shown in Figure 1–7C. This is referred to as the *parallelogram law for the composition of forces.* The resultant OC was found by completing the parallelogram AOBC (lines parallel to OA and OB were drawn) and drawing the diagonal from the origin. This law may be applied to the addition of any set of vector quantities. For more than two forces, the vectors must be taken two at a time in this method of addition.

Vector quantities may also be added as though they were not acting simultaneously. In Figure 1–8A, vectors OA and PB represent two forces acting on an object. Force W moves the object from the origin to point A (Figure 1–8B). Then force V is applied to the object and moves the object to point B (Figure 1–8B). The line OB represents the resultant of the two forces acting on the object. To add vectors in this way place the second vector (PB), without changing its magnitude or direction, so that P coincides with A. Then OB is the vector sum and represents the magnitude and direction of the resultant force. When three or more forces are applied to the same object, each succeeding vector is added to the preceding in this same way, without changing the size or direction of the vector. When all the vectors have been added to the drawing, a



A REPRESENTING TWO FORCES



B ADDITION OF VECTORS

Figure 1–8. Addition of Vectors

line drawn from the origin to the arrow point of the last vector will be the resultant, or vector sum, of the vectors.

Polar and Rectangular Forms

You can describe a vector in *polar* form or in *rectangular* form. The polar form shows the magnitude of the vector and the angle which it makes with the horizontal axis. For example, $10\angle 30^\circ$ describes a vector 10 units in length and at an angle of 30° with the horizontal axis. In rectangular form the vector is *resolved* into two components which are the projections of the vector on the horizontal and the vertical axes, and have as their origin the initial point of the vector. Figure 1–9 shows vector $z\angle\theta$ drawn as $\overline{OM}$. The projection of OM on the horizontal axis is $\overline{OP}$ and on the vertical axis is $\overline{ON}$. The lengths of these components are, respectively, r and x. In electrical work the horizontal component is designated as the *real* component and the vertical component as the *imaginary*, or *j*, component. From your study of trigonometric functions you can figure out the relationships between the vector and the components. You know that in Figure 1–9 $\sin\theta = x/z$ and $\cos\theta = r/z$. Solving these two equations for x and r results in

$$x = z \sin\theta \text{ and } r = z\cos\theta.$$

Thus $z\angle\theta = z\cos\theta + jz\sin\theta$. Hence the vector $10\angle 30^\circ = 8.66 + j5$.

Sometimes the vector is given in rectangular form. That is, the two components of the vector are given and the polar form of the vector must be found perhaps in order to draw it. Because triangle MPO in Figure 1–9 is a right triangle, $z^2 = r^2 + x^2$, and $\theta = \text{arc tan } \frac{x}{r}$. $3 + j4$ can be converted to polar form by using these formulas.

$$z = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

$$\theta = \text{arc tan } 4/3 = \text{arc tan } 1.333 = 53.2^\circ$$

In polar form, then, $3 + j4 = 5\angle 53.2^\circ$

Another way of finding z is by using either the formula for the sine or for the cosine of the angle and solving for z.

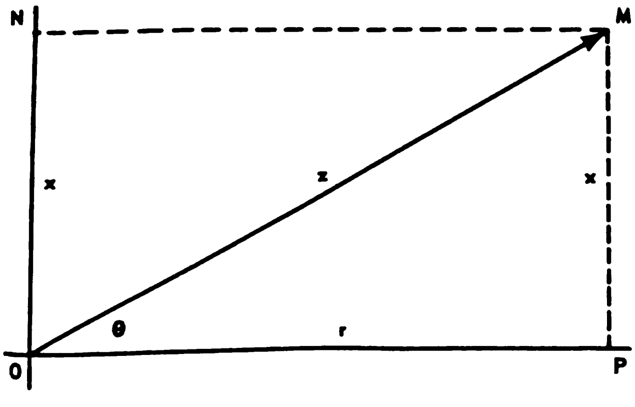


Figure 1–9. Components of a Vector

Addition and Subtraction

You have already observed how complex quantities (such as rectangular forms of vectors) can be added and subtracted. Since the addition of vectors by graphical means, as described before, is not satisfactory unless you use accurate instruments to construct the vectors, the usual method of combining two vectors is to convert them to rectangular form and to add them algebraically.

Examples

1. Add $35\angle 40^\circ$ and $47\angle 55^\circ$.

$$\begin{aligned} 35\angle 40^\circ &= 35\cos 40^\circ + j35\sin 40^\circ \\ &= 35\,(.7660) + j35(.6428) \\ &= 26.81 + j22.50 \\ 47\angle 55^\circ &= 47\cos 55^\circ + j47\sin 55^\circ \\ &= 47(.5736) + j47(.8192) \\ &= 26.96 + j38.50. \end{aligned}$$

Adding, $26.81 + j22.50$
$26.96 + j38.50$

The sum is $53.77 + j61.00$

The graphical addition of the vectors in this example is shown in Figure 1–10.

To convert this result to polar form, remember that $\frac{x}{r} = \tan\theta$. Tan θ in this case is $\frac{61.00}{53.77} = 1.134$, and $\theta =$ arc tan $1.1343 = 48.6^\circ$. Since $\cos\theta = \frac{r}{z}$, $z = \frac{r}{\cos\theta} = \frac{53.77}{\cos 48.6^\circ} = \frac{53.77}{.6613} = 81.3$. In polar form the answer is $81.5\angle 48.6^\circ$.

2. Subtract $45.6\angle 21.5^\circ$ from $51.4\angle -10.5^\circ$

$45.6\angle 21.5^\circ = 45.6\cos 21.5^\circ + j45.6\sin 21.5^\circ = 45.6\,(.9304) + j45.6\,(.3665) = 42.4 + j16.7$

$51.4\angle -10.5^\circ = 51.4\cos(-10.5^\circ) + j51.4\sin(-10.5^\circ) = 51.4\,(.9833) + j51.4\,(-.1822) = 50.5 - j9.36$

Subtracting, $50.5 - j9.36$
$42.4 + j16.7$

The remainder is $8.1 - j26.06$

$$\theta = \text{arc tan } \frac{-26.06}{8.1} = \text{arc tan } -3.217 = -72.7^\circ$$

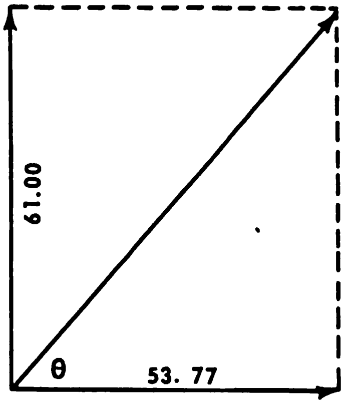


Figure 1–10. Graphic Example of Vectors

$$z = \frac{8.1}{\cos(-72.7^\circ)} = \frac{8.1}{.2957} = 27.1$$

In polar form the answer is $27.1\angle -72.7^\circ$

Multiplication

To multiply two vectors in polar form, multiply the magnitudes together and add the angles.

Example

Multiply $55\angle 40^\circ$ by $47\angle 55^\circ$.
$(55) \cdot (47) = 2585$
$40^\circ + 55^\circ = 95^\circ$
Hence $(55\angle 40^\circ)(47\angle 55^\circ) = 2585\angle 95^\circ$

You can check this result by converting from polar to rectangular form, multiplying and then converting back to polar form.

Example

$55/40^\circ = 42.13 + j35.35$
$47/55^\circ = 26.96 + j38.5$

$$\begin{array}{lr} & 42.130 + j35.354 \\ \text{Multiplying} & \underline{26.959 + j38.502} \\ & 1135.783 + j953.108 \\ & \underline{j1624.084 + j^2 1361.213} \end{array}$$

The product is $1135.783 + j2577.192 - 1361.213$

or $-225.43 + j2577.19$

$$\theta = \arctan \frac{2577.19}{-225.43} = \arctan(-11.43) = 95^\circ$$

$$z = \frac{-225.43}{\cos 95^\circ} = \frac{-225.43}{-.0872} = 2586$$

The answer is $2586\angle 95^\circ$, practically the same as obtained before.

Division

To divide one vector by another, divide the magnitudes and subtract angles.

Example

Divide $(55\angle 40^\circ)$ by $(47\angle 55^\circ)$
$55 \div 47 = 1.17$
$40^\circ - 55^\circ = -15^\circ$
Hence $\dfrac{55\angle 40^\circ}{47/55^\circ} = 1.17/-15^\circ$

Since a power of a quantity is a repeated multiplication process, note that $(15\angle 20^\circ)^2 = 225\angle 40^\circ$, $(15\angle 20^\circ)^3 = 3375\angle 60^\circ$, and so on. Raise the magnitude of the vector in polar form to the desired power and multiply the angle by the exponent.

Conversely, to extract a root of a vector in polar form, extract the required root of the magnitude and divide the angle by the index of the root taken.

Examples

$$\sqrt{10\angle 50^\circ} = 3.16\angle 25^\circ$$

$$\sqrt[3]{10\angle 50^\circ} = 2.15\angle 16.7^\circ$$

When either extracting a root or raising to a power, convert rectangular quantities to polar form before finding the root or power. Addition and subtraction are easier in the rectangular form. Multiplication, division, raising to a power, and extracting a root are easier in the polar form.

CHAPTER 2

DC and AC Circuits

ELECTRICAL FUNDAMENTALS

Radar, like all other applications of electricity from the simplest appliance to the most complicated piece of electrical machinery, is based primarily on a number of fundamental principles of electricity. Any attempt to study radar without first learning these fundamentals will at best be difficult.

The purpose of this chapter is to give you the more important fundamentals of electricity, those which you need in order to study radar profitably. This chapter discusses the basic concepts of electricity and magnetism, describes the behavior of direct and alternating current in a large variety of electrical circuits, and presents a large number of formulas and equations essential to your understanding of fundamental electricity.

ELECTRON THEORY

According to science, the smallest particle into which any substance can be divided and still retain its characteristics is the *molecule*. The substance might be anything, such as the air you breathe, the water you drink, or the food you eat. There are thousands of kinds of substances in the world. The molecule, in turn, is built of atoms; the molecule of water, for example, is made up of two hydrogen atoms and one oxygen atom. A substance that contains only atoms which have the same chemical properties is called an *element*. Oxygen is an element since it contains only oxygen atoms. Likewise, nitrogen is an element because it is composed only of nitrogen atoms. There are over 100 elements known to science. In addition to the 92 elements occurring in a natural state, scientists have synthesized ten more.

Finally, the atom itself consists of protons, electrons and various particles. The proton has a *positive* charge of electricity, and the electron has a *negative* charge of electricity. The nucleus in the center of the atom comprises protons, neutrons (another type of particle consisting of protons and electrons) and other particles having special qualities. It is thought that the electrons spin around this nucleus in the same manner as the planets move in orbits around the sun. It is impossible to subdivide electrons and protons further.

One of the fundamental laws of electricity is that *like* charges *repel* each other, and *unlike* charges *attract* each other. This law explains the bond that exists in the atom between the positively charged nucleus and the electrons revolving about the nucleus in planetary elliptical orbits. The radii of the orbits of the various electrons within an atom differ. Some electrons are close to the nucleus; others are farther away. Electrons in the inner orbits near the nucleus are bound tightly in the atom. Electrons in the outer orbits are rather loosely bound and when influenced by outside forces, may be torn away from the nucleus. This leaves the atom with a deficiency of electrons, a condition in which it is called an *ion*, or a *positively* charged atom. The electrons that are loosely bound to the nucleus are called *free electrons* and exist in greater numbers in the atoms of some elements than in others. Metals contain more free electrons than do such substances as rubber and glass.

When a metal bar or wire is connected between a point with an excessive number of electrons and another point with a normal amount, or with a deficiency of electrons, there will be a movement of the free electrons. The electrical pressure or potential that exists whenever there are more electrons at one point than at the other causes the movement of electrons. The greater the difference in the number of electrons, the greater the electrical potential and the greater the amount of electron flow. Likewise, the smaller the difference in the number of electrons the less the potential, and the less the amount of electron flow.

In addition to the electrical potential, the amount of electron flow depends upon the physical characteristics of the metal connector. A substance, such as a metal which contains many free electrons, is called a *conductor* and offers little opposition to the flow of electrons. Glass, which has few free electrons, is called an *insulator* and offers great opposition to electron flow. Actually, there is no such thing as a perfect conductor or a perfect insulator. A substance is a good conductor or a poor conductor, a good insulator or a poor insulator.

ELECTROSTATICS

Rods made of hard rubber or glass, when rubbed with fur or silk cloth, will attract small pieces of paper. The early Greeks were familiar with this phenomenon, and unknowingly discovered the type of electricity which they called *static* electricity, since it was considered electricity at rest. The Greeks knew that when they rubbed a piece of amber, which they called *elektron*, with a piece of cloth, it would attract other objects such as bits of cloth or pith, in a manner very much as a magnet attracts iron filings. From the Greek word for amber are derived the English words, *electron* and *electricity*.

In general, much of the knowledge of electricity has been obtained from experiments in static electricity. Among these, for example, is that of Guericke, who, in 1663, discovered that a globe of sulphur which was mounted on an axis became electrified when it was rotated while being pressed with the dry palm of the hand; and that of Benjamin Franklin, whose experiment with a kite in a thunderstorm led to the development of the lightning rod. Although radar deals with electricity in motion (dynamic electricity), an understanding of the effects of static electricity is basic to your study of radar.

Although static electricity was originally considered electricity at rest, with the acceptance of the electron theory that electrons are in continual motion in the atoms, static electricity is now thought of as electricity associated with insulators and dielectrics. A dielectric is the insulating material between a pair of oppositely charged bodies. Insulating materials are excellent dielectrics because they contain very few free electrons. Although there is little or no movement of free electrons in a dielectric, a dielectric is capable of transmitting a force from one body to another.

The electric capacitor, which you will study later, is an example of two oppositely charged bodies separated by a dielectric that, under certain conditions, transmits a force. A simple capacitor consists of two metal plates, or conductors, separated by air for the dielectric. When it is connected to a source of voltage, such as a battery, one plate will be charged negatively (excess of electrons), and the other plate positively (lack of electrons). No electrons move through the air (dielectric), unless the capacitor is charged too high. (A capacitor is charged when there are more electrons on one plate than the other.) In such instances the dielectric breaks down and a spark jumps between the plates, discharging the capacitor.

The clouds and earth form an excellent example of a charged capacitor in which the breakdown of the dielectric, which is air, manifests itself as lightning. The generally accepted theory of lightning is that the water droplets in the clouds have electrons on the outside. Turbulence in the clouds tears some electrons loose. The positive and negative charges are carried to different parts of the cloud. When these charges are large enough, the resistance of the air between them breaks down, and the electrons rush to the positively charged droplets. This produces the lightning flashes within clouds or from cloud to cloud.

When the bottom of a cloud has a strong negative or positive charge, an opposite charge is built up on the earth below it. The negative charge is usually in the cloud except in mountainous country. When the number of electrons on a cloud becomes excessively high or excessively low, the dielectric (air) breaks down and a spark, which is called lightning, jumps between the earth and the cloud.

Charged Bodies

A *charged* body is one that has more or less than the normal number of electrons. It may be either positively or negatively charged. A positively charged body is one in which some of the electrons have been removed from the atoms, and there is a deficiency of electrons, or fewer electrons than protons. A negatively charged body is one in which there are more than the normal number of electrons in each atom — that is, there are more electrons than protons. A body in which there is an equal number of electrons and protons in each atom is an *uncharged* body.

Removing electrons from a body involves physically attaching them to another body, and then moving the other body some distance away. The second body will have an excess of electrons, and thus will be negatively charged. The first body will have a deficiency in electrons and thus will be positively charged. This can be illustrated by rubbing glass with silk. Some of the electrons are rubbed off the glass onto the silk. This leaves the glass with a positive charge (lack of electrons) and the silk with a negative charge (surplus of electrons). As long as the two bodies, the silk and the glass, are not brought into contact, they will tend to retain the charges. However, when they are allowed to touch, the surplus of electrons on the silk will move onto the glass and neutralize the charge on the two bodies.

Force Between Charged Bodies

It has been proved experimentally that charged bodies act upon each other with a force of attraction when they are oppositely charged, and act upon each other with a force of repulsion when similarly charged. This is because electrons and protons *attract* each other, while electrons *repel* other electrons, and protons repel other protons. The forces of attraction or repulsion change with the magnitude of the charges and also with the distances between them. This is dealt with in Coulomb's Law, which states that charged bodies attract or repel each other with a force that is directly proportional to the product of the charges and inversely proportional to the square of the distance between them. Mathematically, Coulomb's Law can be expressed as follows:

$$F = \frac{Q \times Q'}{d^2}$$

where Q and Q' represent the charges and d the distance separating them.

The charge on one electron or proton might be used as the unit of electrical charge, but it would not be practical because of its small magnitude. Generally, the practical unit of charge used is the coulomb, which is equal to the charge of 6.28×10^{18} electrons.

Field of Force

The region of space around and between charged bodies where their influence is felt is called their dielectric field of force. The dielectric field requires no physical or mechanical connecting link, but can be applied through air or through a vacuum. Electrostatic field and electric field are other names given to this region of force.

Fields of force permeate the space surrounding certain objects and, in general, diminish in proportion to the square of the distance from their source or origin. The force of gravity is a field of force that permeates the space surrounding the earth, and acts through free space, causing all unsupported objects in its region to fall to the earth. Newton discovered the law of gravitation, which states that every object attracts every other object with a force that is directly proportional to the product of the masses and inversely proportional to the square of the distance between them. (Note the similarity between the law of gravitation and the law of attraction of charged bodies.)

The gravitational fields hold the universe together, for with no gravitational field, the planets, including the earth, would fly off at a tangent and travel through space instead of revolving around the sun. The moon would cease to revolve about the earth and, due to the earth's rotation, objects on the earth's surface would fly out into space like mud from a bicycle wheel. Electrons likewise revolve at a tremendous velocity around the positive protons of the atom, but they, like the planets revolving around the sun, do not fly off at a tangent. Therefore, there must be a field of force between them and the protons.

Relatively speaking, there are great distances between the electrons and the protons, even in apparently solid matter. If a copper one-cent piece were enlarged to the size of the earth's orbit around the sun (approximately 186,000,000 miles in diameter), the electrons in it would be the size of baseballs and they would be, on the average, three miles apart.

Since the field of force between the electrons and protons in the atom is the same as the dielectric field associated with charged bodies, any discussion of dielectric or electric fields is understood to mean the external fields about the charged bodies, unless specific reference is made to the field within the atom.

Lines of Force

Imaginary lines are used in diagrams to represent the direction and intensity of the field of force. The intensity (field strength) is indicated by the density (number of lines per unit area), and the direction of the field is indicated by arrowheads on lines drawn in the direction a small test charge moves, or tends to move, when acted upon by the field of force.

Either a small positive or a small negative test charge can be used to test the direction of force, because the force of a dielectric field will act on either. Arbitrarily, however, it has been agreed to use a small positive charge when determining the direction of a dielectric field. The test shows that the direction of the field about an isolated positive charge is *away* from the charge, for a positive test charge is *repelled;* and that the direction about an isolated negative charge is *toward* the charge, for a positive test charge is *attracted* (Figure 2–1). From this you can see that the direction of the field between the positive and the negative charge is from positive to negative.

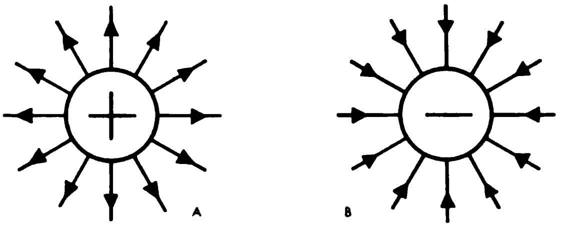
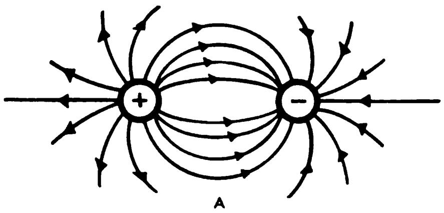
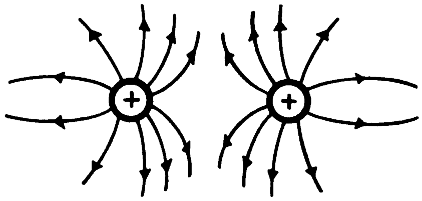


Figure 2–1. Direction of Fields about Isolated Charges

In Figure 2–2, notice that the lines of force apparently repel each other. Figure 2–2A shows the field about two unlike charges. The lines of force connecting the two charges are not parallel, but bulge out at the center as if they were repelling each other. The lines are in the same direction, that is, from left to right on the page.



A
TWO UNLIKE CHARGES



B
TWO LIKE CHARGES

Figure 2–2. Dielectric Fields about Like and Unlike Charges

In B, the lines of force located in the region between the charges apparently repel each other, as you can see by their bent appearance. They also are in the same direction. Thus, instead of saying that *like charges repel,* the law can be modified to say that *dielectric lines of force in the same direction repel each other.* This rule, you will find, is very convenient and useful in dealing with certain electrical phenomena.

Experimenting with Charged Bodies

When a rubber rod is briskly rubbed with a piece of woolen cloth, a number of electrons from the cloth adhere to the rubber rod. If the two objects are separated immediately, there will be an excess of electrons on the rubber rod. In other words, it is negatively charged. If two

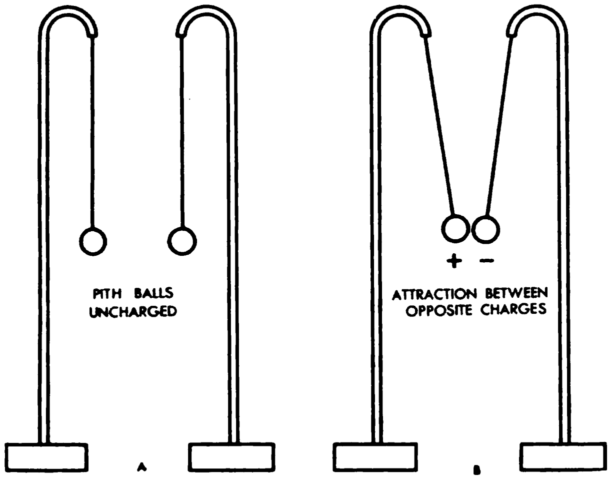


Figure 2–3. Experimenting with Dielectric Fields

pith balls, such as are shown in Figure 2–3, are oppositely charged by touching one of them with the rubber rod and the other with the cloth, they will have an attraction for each other. This shows that a force is present and indicates that a dielectric field has been established. In establishing this field, it was necessary to do work against the force of attraction in separating the charged bodies. If the bodies were allowed to come together as a result of the force of attraction between them, this energy would be regained. Hence, the conclusion is that energy can be stored in a dielectric field.

If you move the negatively-charged rubber rod an appreciable distance away from the cloth, a dielectric field will still exist in the space around it. You can prove that the field still exists by picking up bits of paper with the rod or by charging both of the pith balls with it. In this condition the pith balls show that a force of repulsion exists between them, a condition which indicates that the dielectric field still exists about the rod.

If an external force is used to bring the two charged pith balls closer, work is done and the force of repulsion is increased. The energy consumed in increasing the field is recovered whenever the external force is removed. It will be used up in returning the pith balls to their original positions. Here again, energy is necessary to establish or increase a field; force is necessary to maintain it, and recoverable energy is stored in the field.

If one negatively-charged pith ball is isolated and the negatively-charged rubber rod is brought toward it from any direction, a force of repulsion is present. If the pith ball is positively charged, there will be a force of attraction no matter from which direction you bring the negatively-charged rod toward the pith ball. The conclusion, then, is that a dielectric field entirely surrounds a charged body.

Distribution of Charges

Michael Faraday, the famous English scientist, and others proved that charges on various shaped objects distribute themselves according to a fixed pattern. For example, if a hollow sphere is charged either positively or negatively, there is no electric field on the inside, but the charge is uniformly distributed on the outside. On pointed objects, charges tend to accumulate on the pointed parts. Thus, on a charged tear-drop shaped object, the intensity of the electric field is greater in the region of the sharp point. Use of this fact is made in the design of spark gaps. In spark plugs, the shape of the electrodes determines the voltage at which a spark jumps across the spark gap. The sharper the points, the lower the voltage at which the spark will jump for a given separation of the electrodes.

Electroscope

While the detection of a charge can often be made by the ability of a charged body to attract other light bodies, this method is not sensitive enough to be of practical use. A simple and very sensitive instrument for detecting the presence of a charge of electricity is the *electroscope*. It consists of two very thin strips of metal, called leaves, suspended from one end of a metal rod whose other end terminates in a ball. The leaves are usually enclosed in glass to prevent their being affected by air currents. The electroscope serves to detect the kind of charge (positive or negative) and, to some extent, the amount of charge on a body.

The thinness and type of material used for the leaves chiefly determine the sensitivity of the instrument.

ELECTRICAL QUANTITIES

Charge

The unit of electrical charge or the unit quantity of excess electrons is the coulomb. The coulomb is the charge on a single charged body containing 6.28×10^{18} free electrons. For practical purposes this unit is quite large, but it is important in that it serves to define other units.

Current

Current is the movement of electrons through a conductor. The total amount of charge transferred by moving electrons is measured in coulombs or fractions of coulombs. The rate at which they flow is measured in amperes. An ampere is equal to one coulomb passing a given point per second. An ampere, for example, is approximately equal to the current flowing in a 100-watt lamp.

Resistance

The opposition that a conductor (or insulator) offers to the flow of electrons is called *resistance*. For any given conductor, the resistance depends upon the cross-sectional area, the length, and the relative resistance of the material.

The following are the relative resistances of several different materials for the same length and cross section, with silver as the standard.

Silver	1.00	Aluminum	1.8
Copper	1.08	Lead	13.5
Gold	1.4	Platinum	7.0

The numbers in this table are only approximate, for the resistance of each metal depends not only on its purity, but also on its condition, that is, whether the metal is hard drawn or annealed. Resistance is also affected by temperature. Generally, the higher the temperature the greater the resistance of the wire. Each kind of metal has its own temperature coefficient. Temperature coefficient is the change in resistance per degree change in temperature. For very accurate work, the temperature at which a resistor has a certain value must be specified. To calculate the resistance of wire at any temperature use the formula,

$$R_t = R_o (1 + \alpha T)$$

where R_t is the resistance in ohms, R_0 the resistance at 0° C, α the temperature coefficient, and T the centigrade temperature.

The following are the temperature coefficients for some of the common conductors:

Aluminum	0.00423	Manganese	0.00001
Copper (hard-drawn)	0.00408	Silver	0.0040
Tungsten	0.0051	Platinum	0.0051

Ordinarily, gases are considered to have a high resistance, but under certain conditions, such as ionization, they may be fairly good conductors. A gas is ionized when a relatively small number of its electrons are changed into ions. An ion is an electrified particle formed when a molecule loses or gains an electron. Gases may be ionized while under the influence of an electrostatic field or by bombardment of energy rays, such as light or X-rays.

The unit of resistance to electron flow is the ohm. The ohm is specified by international agreement as the resistance which a column of mercury of uniform cross section with a length of 106.3 cm and a mass of 14.45 grams offers to the flow of electrons at a temperature of 0° C. The cross section of the column is approximately a square millimeter. A copper wire 14700 yards long and one square centimeter in area has a resistance of one ohm. An iron rod 1093 yards long and one centimeter square has a resistance of one ohm, while a Nichrome rod 233 yards long and one centimeter square has a resistance of one ohm.

Electromotive Force

The external force or electrical pressure which tends to produce a flow of electrons is known as electromotive force (abbreviated as emf), potential difference, or voltage. All these terms mean practically the same thing and are used interchangeably.

The unit of electromotive force is the volt. The volt is the amount of electrical pressure required to maintain a current of one ampere through a resistance of one ohm. A dry cell produces 1.5 volts. Each cell of a lead-acid storage battery supplies slightly over 2.0 volts.

Another term relative to electromotive force is *voltage drop*. This is the voltage developed across each of the resistances to electron flow in a circuit.

Ohm's Law

A very useful relationship exists between the volt, the ampere, and the ohm. This relationship is stated in various ways and each way is sometimes used to define one of the three units. The relation between the volt, the ampere, and the ohm, is known as Ohm's law, in honor of the discoverer. It states that in any circuit, or part of a circuit, the current in amperes is equal to the electromotive force in volts divided by the resistance in ohms. Mathematically, this is stated, $I = E/R$, where I represents the current in amperes, E represents the voltage in volts, and R the resistance in ohms. Other ways of stating this formula are $E = IR$ and $R = E/I$.

Power

Another factor to be considered in electricity is power. Power is the rate of doing work. The watt is the unit of electrical power and is equal to work done at the rate of one joule per second. (A joule is the practical metric unit of work.) One joule equals 10^{17} ergs (the basic unit of work). Power in an electrical circuit is equal to the product of voltage and current and is expressed by the formula,

$$P = EI$$

where P equals the power in watts, and I the current in amperes, and E the electromotive force in volts.

Since, according to Ohm's law, $E = IR$, this equation can also be written $P = I^2R$. Likewise, since $I = E/R$, the formula is also written $P = E^2/R$. You will use all three forms frequently.

PRODUCING ELECTROMOTIVE FORCE BY CHEMICAL ACTION

There are two common methods of producing an electromotive force or voltage. One method is mechanical and involves rotating electrical conductors in a magnetic field in such a manner that an electrical voltage is generated. Such an arrangement is called a generator. (Generators are discussed later in this chapter.) The other method is chemical and involves transforming chemical energy into electrical energy by utilizing the chemical-electrical phenomena displayed by dissimilar substances when immersed in a chemical solution.

Primary Cell

If two dissimilar metals (or a metal and carbon) are immersed in a solution that produces greater chemical action on one metal than on the other, a difference of potential will exist between the two. If a conductor is connected between them, a current will flow. This arrangement is called a *primary cell;* the two metals are known as *electrodes*, and the solution is called the *electrolyte.* The difference of potential results from the fact that material from one or both of the electrodes goes into solution in the electrolyte, and in the process, ions form in the vicinity of the electrodes. Due to the electric field associated with the charged ions, the electrodes acquire charges.

The amount of difference in potential between the electrodes depends principally on the metals used. The type of electrolyte and the size of the cell have little or no effect on the potential difference produced.

There are two types of primary cells, the wet cell and the dry cell. In a wet cell the electrolyte is a liquid. A cell with a liquid electrolyte must remain in an upright position and is not readily transportable. The dry cell, much more commonly used than the wet cell, is not actually dry, but contains an electrolyte mixed with other materials to form a paste. Figure 2–4 shows the construction of a dry cell. The top of the cell is sealed by a substance such as sealing wax, or by a metal cap. This arrangement prevents the paste from drying out and the electrolyte from spilling, and thus makes it possible to use the cell in any position.

The container is zinc and acts as the negative electrode. Next to the zinc container is a layer of blotting paper which is saturated with the electrolyte, usually a dilute solution of ammonium chloride. The positive electrode is a carbon rod. The space between the positive electrode and the blotting paper is filled with a mixture of carbon, manganese dioxide, and the electrolyte itself. The voltage of a dry cell is 1.5 volts.

Internal Resistance

In any conductor there is a certain amount of resistance. Similarly there is some resistance in a cell. This resistance is largely due to the resistance in the electrodes and in the electrolyte. It depends upon the kind of materials used and the size and spacing of the electrodes. Large cells with close spacing of electrodes have less internal resistance than smaller cells made of the same materials with the same spacing.

Battery

A collection of cells connected together is called a *battery*. When connected in series, the individual cell voltages add to produce a greater voltage than a single cell. When connected in parallel, the maximum usable current is increased without changing the voltage. Portable radio sets use batteries containing small cells connected in series to furnish the high voltages required for the operation of the tubes. The battery designed for this purpose is called a *B* battery.

Primary Cell Faults

If the primary cell is to give satisfactory service, it must be guarded against two ill effects of chemical action. These are *local action* and *polarization*. Although primarily problems of the manufacturer, you should understand the effects of these undesirable actions in a cell.

Local action is identical to the chemical action occurring in the cell as a whole. However, it takes place only between the zinc electrode and impurities in the zinc. Chemical action between these elements forms small scale cells in which a voltage causing a minute current flow is set up. This causes the zinc to be consumed which, in turn, causes an increase in the internal resistance of the cell itself and lowers the voltage output.

Local action can take place even when the cell is not in use. For this reason, some cells have a date stamped on them; they should be put into service before this date. The remedy for local action is to clean the zinc electrode thoroughly and coat its surface with mercury. The alloy of mercury and zinc thus formed, called amalgam, covers the impurities, and thereby retards local action until the time when enough material has been consumed in the cell to expose the impurities.

Polarization takes place only while the cell is in use. It manifests itself in the formation of hydrogen bubbles on the positive electrode. These bubbles are nonconducting, and tend to increase the internal resistance of the cell, reducing the current flow. Current may cease to flow entirely if the entire electrode becomes coated. If the bubbles are removed, or prevented from forming, polarization is reduced and current can flow until the negative electrode does not contain sufficient material to furnish the energy. A chemical which prevents the formation of hydrogen bubbles is called a *depolarizer*. An example of a depolarizer is the chemical, manganese dioxide.

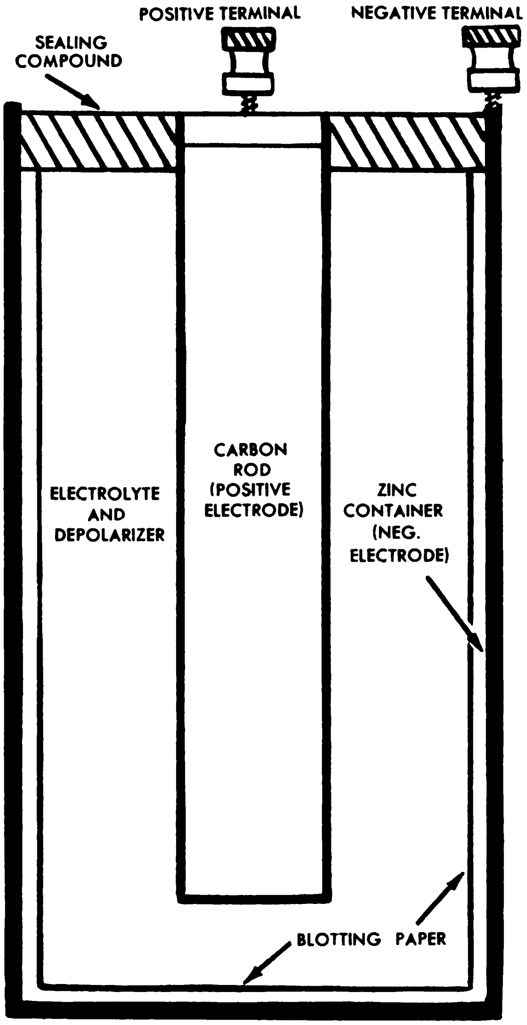

POSITIVE TERMINAL
NEGATIVE TERMINAL
SEALING COMPOUND
CARBON ROD (POSITIVE ELECTRODE)
ELECTROLYTE AND DEPOLARIZER
ZINC CONTAINER (NEG. ELECTRODE)
BLOTTING PAPER

Figure 2–4. Construction of a Dry Cell

Secondary Cell

A primary cell delivers electrical energy directly from the reaction of chemicals mixed together. However, a *secondary* or *storage* cell

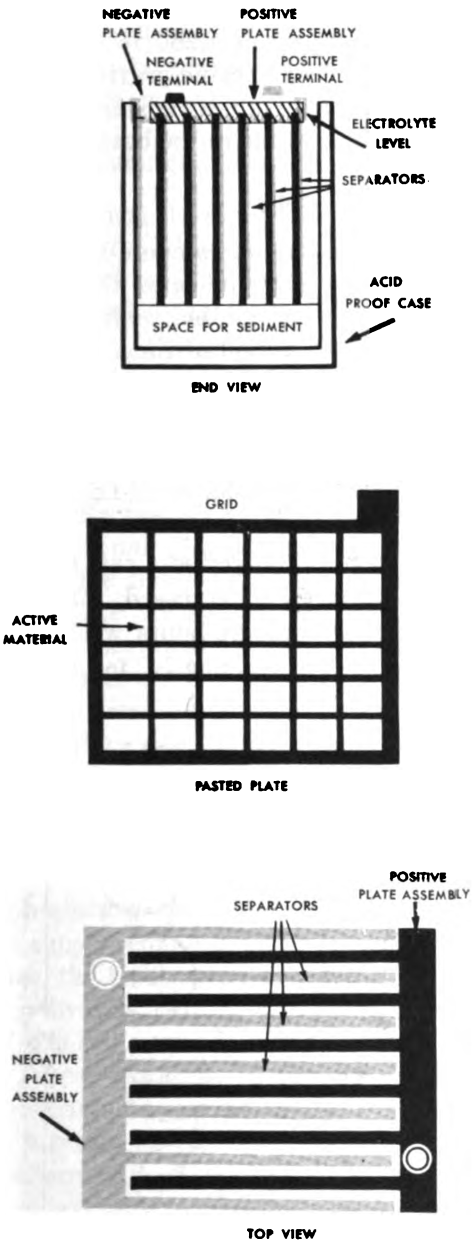


Figure 2–5. Secondary Cell Construction

must have energy stored in it by a process called *charging* before it will deliver energy. Charging the cell consists in forcing a current to flow through it in a direction opposite to that in which it flows when the cell is delivering energy (discharging) to the circuit. During charge electrical energy is converted to chemical energy and stored in the cell. During discharge the chemical energy is reconverted to electrical energy.

A simple secondary cell contains two lead electrodes immersed in a dilute solution of sulphuric acid. If a current is forced through the cell, the surface of the electrode which is connected to the positive terminal of the charging voltage will be changed to lead peroxide and the surface of the other electrode will be changed to spongy lead. When fully charged, a lead-acid cell will have a voltage of approximately 2.2 volts. It will then furnish current to an external circuit for a period of time depending upon the amount of current drawn. This discharge process is the result of chemical action which forms lead sulphate on both electrodes.

When the lead peroxide and spongy lead are converted to lead sulphate, the cell is discharged, and will not furnish more current until charged again.

Storage Battery

The automobile storage battery is a typical battery of secondary cells, consisting of three, or six, with such cells connected electrically and assembled in a single container. While the principle of operation is the same as that of the simple secondary cell, the construction is somewhat more complicated. For the cells to furnish large amounts of current, the surface area of the electrodes must be great, and the electrodes very closely spaced. The electrodes are rectangular, flat plates consisting of latticework, or grids, of lead alloy. The holes in the grids are filled with a paste of lead oxide (Figure 2–5B). The electrolyte is a weak solution of sulphuric acid. To increase the area of each electrode, several grids or plates are connected together.

The positive plates are connected in parallel by one plate strap, and the negative plates are connected in parallel to another strap. In every cell there is always one more negative plate than positive plates. When the plates are assembled, the positive plates are placed between the negative plates so that both sides of each positive plate have a negative plate opposite. During discharge, the material formed on the plates occupies more space than the original material.

With the chemical action on both sides of the positive plate the same, buckling of the plates is prevented.

Separators, or spacers, are put between the plates (Figure 2–5C) to keep them from touching each other and short-circuiting the cell. Separators may be made of any one of several insulating materials. Highly porous wood sheets, treated to remove any wood acids, are commonly used in the smaller cells. The separators must allow the electrolyte to pass through freely and not impede the flow of electrons.

Cell containers generally have ribs in the bottom to support the plates. They provide a space at the bottom so that active material which falls off the positive plates will not touch the bottom ends of the plates, cause a short circuit between the positive and negative plates, and discharge the cell.

The state of charge or discharge of a battery can be determined by a *hydrometer*, an instrument for measuring the specific gravity of a liquid. During the charging process, water in the electrolyte is used up and the sulphuric acid solution becomes more concentrated; that is, its specific gravity is increased. A fully charged cell has a specific gravity of 1.280 to 1.300 while a discharged cell has a specific gravity of 1.120 to 1.150. Intermediate values indicate the relative proportion of full charge in the battery.

Rating of Storage Cells

A storage cell is rated in ampere-hour capacity. Ampere-hour capacity indicates the number of amperes that the cell is able to furnish continuously for a stated period of time. For example, a battery rated at 100 amp-hours at an 8-hour discharge rate can deliver 12.5 amperes continuously for 8 hours. Generally, the normal discharge rate is based upon an 8-hour discharge period.

If a battery is discharged at a rate higher than normal, its capacity is decreased. Thus, a battery rated at 100 ampere-hours would not furnish 50 amperes continuously for two hours.

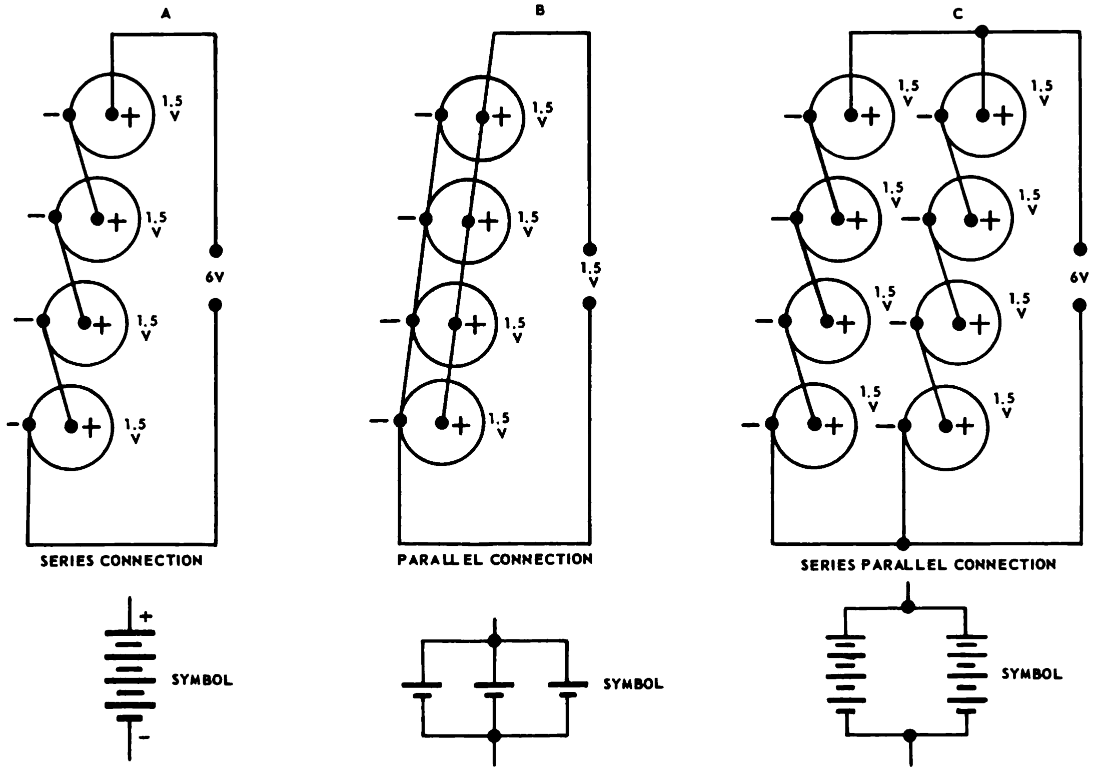


Figure 2–6. Methods of Connecting Primary Cells

Conversely, if discharged at a lower than normal rate, the capacity would be increased. The ampere-hour rating of a battery depends principally upon the size and number of plates. The number of plates gives an approximate indication of the battery's capacity.

Series and Parallel Connection of Cells

Many electrical devices require higher voltage or higher current than a single cell is able to furnish. Therefore, it is often necessary to connect several cells together in the form of a storage battery. The manner in which cells, primary or secondary, are connected depends upon whether you want to increase the voltage or the current.

If *greater voltage* is needed, connect the cells *in series*. To connect cells in series, connect the negative terminal of each cell to the positive terminal of the succeeding cell as shown in Figure 2–6A. The voltage of the battery is then equal to the total of the voltages of the separate cells. The same current, however, flows through the cells in succession. Therefore, the current that the battery can supply is equal to the current rating of a single cell. (The current rating of a cell depends on the physical size.) Thus, a battery composed of cells in series provides a higher voltage, but not a greater current capacity.

To obtain a greater *current* than one cell is able to supply, connect the cells in *parallel*. In this case, the total current available is the sum of the individual current ratings, since the current of one cell does not flow through the other cells. To connect cells in parallel, connect all positive terminals together and all negative terminals together as shown in Figure 2–6B. The positive terminals of the cells will form the positive terminal of the battery, and the negative terminals of the cells, the negative terminal.

Each cell must have the same voltage, otherwise a cell with higher voltage will force current through the lower voltage cells and carry the greater part of the load. The output voltage of the battery made up of cells connected in parallel is, therefore, the same as that of a single cell.

Another method of arranging cells is to connect them in series-parallel. In this method, groups of cells are connected in series and the groups are connected in parallel as shown in Figure 2–6C. This connection provides both a greater voltage and a greater current output.

CURRENT, VOLTAGE AND POWER RELATIONS IN SERIES CIRCUITS

Resistors

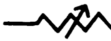
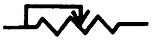
A resistor is a circuit element designed to offer a certain amount of opposition to current flow. Usually a resistor is cylindrical in shape and is constructed in one of two forms. Some resistors are sections of carbon rod or some combination of carbon and other substances molded into cylindrical form. Others are wire-wound, which makes possible more precise manufacture of the desired resistance. Wire-wound resistors are capable of carrying, without damage, heavier current than carbon resistors.

Another factor associated with resistors is power rating. Power rating indicates the maximum current that can flow through the resistor without danger of burning it out because of overheating.

Colored bands in combination, or some other arrangement of colors, are used to indicate the resistance value of resistors. (See a resistor color code for explanation of procedure for determining value of resistors.) Each resistor is marked with a certain value along with a percent of tolerance, since it is difficult to manufacture a resistor to exact value. Carbon type resistors usually have a tolerance of 5%, 10%, or 20%. Wire-wound resistors can have much lower tolerance. The tolerance in this type may be 1% or 2%.

A 2000-ohm resistor with a 10% tolerance, for example, has an actual value of 2000 ohms ±10%, or a range of 1800 to 2200 ohms. Although this may seem like a wide range of values, the majority of radio circuits will function satisfactorily if the circuit components are within 10% or even 20% of the desired value. Where precision is required, resistors of lower tolerance are used. Circuit problems in this manual assume the value of resistance to be exact, and do not require your considering tolerance unless specifically indicated.

Variable Resistors

Some circuits require a resistor which can be varied between limits. Such a resistor is called a *variable resistor.* Schematically, it is represented by the symbols  or  . Variable resistors are either tubular or circular in form and are constructed with one permanently connected contact and one movable contact. In the circular form the movable contact is connected to a shaft which, when rotated, moves the contact along the arc of the resistor. Tubular form resistors are wire-wound and have a sliding strap contact. They are designed to carry an appreciable amount of current. The circular type may be wire-wound or carbon. Variable resistors are known as *rheostats.*

Series Circuit

The simplest form of electrical circuit is the series circuit. In the series circuit current flow is the same at all points. Figure 2–7 shows a series circuit. The arrows indicate the direction of electron flow. At the positive terminal of the battery there is a deficiency of electrons, while at the negative terminal there is a surplus. The emf of the battery forces electrons to flow through the circuit in the direction indicated from the negative terminal to the positive terminal and thus neutralizes the charges of both electrodes. Since continued action within the cells of the battery maintains the emf, current flow is continuous.

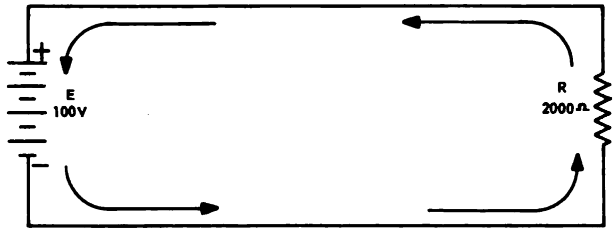


Figure 2–7. Series Circuit

To find the current flow in a series circuit, use the Ohm's law formula, $I = E/R$. In the circuit of Figure 2–7 the applied voltage E is 100 volts and the resistance of the resistor R is 2000 ohms. Substitute the given quantities in the formula and solve as follows:

$$I = \frac{100}{2000}$$

The current $I = .05$ ampere.

When this current flows through R, power is expended. To find the power expended, substitute in the power formula, $P = I^2R$ and solve as follows:

$$P = .05 \times .05 \times 2000$$

The power expended $P = 5$ watts.

Figure 2–8 is a series circuit containing three resistances. The resistors are connected in series, and the current in the circuit flows through each resistor in succession, as well as through the ammeter represented by an encircled *A*. The current is equal in value at all points so that the ammeter could have been inserted anywhere in the circuit. (The construction and operation of an ammeter, the instrument for measuring current flow, is described in detail in Chapter 3.) The encircled *V* represents a voltmeter, an instrument for measuring potential difference. (The construction and operation of a voltmeter is described in detail in Chapter 3.) A voltmeter must always be connected in parallel with that part of the circuit developing the voltage to be measured. Voltmeters can be used to measure the emf of a battery or the voltage drop across a resistor or across other circuit elements.

In the series circuit in Figure 2–8 with the values indicated, calculate the current flow and the power rating of each resistor. Since all the current flows through each resistor, the total opposition (resistance) is the sum of the individual oppositions (resistors). Mathematically, this is expressed by the equation

$$R_t = R_1 + R_2 + R_3$$

where R_t represents the total resistance.

By substituting in the equation

$$R_t = 1000 + 2000 + 5000$$

The total resistance $R_t = 8000$ ohms.

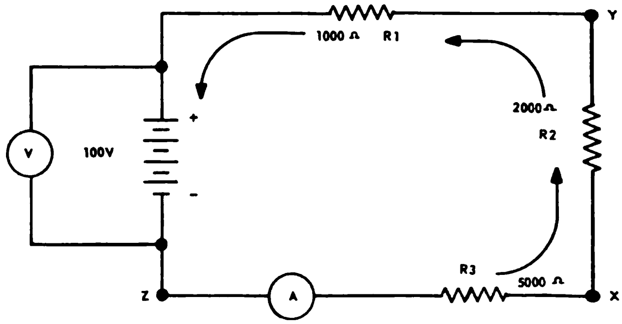


Figure 2–8. Series Resistors

To find the total current, use the Ohm's law formula $I = E/R$.

Thus, $I = \frac{100}{8000}$

The total current $I = .0125$ ampere.

Because the ampere is rather a large unit, current is often measured in *milliamperes* (abbreviated ma) — thousandths of an ampere — or in *microamperes* (abbreviated μa) — millionths of an ampere. The milliampere is widely used in radar. To find the power rating of each resistor, substitute in the power formula $P = I^2R$ and solve as follows:

$$P_1 = I^2 \times R_1 = (.0125)^2 \times 1000 = 0.15625 \text{ watt}$$
$$P_2 = I^2 \times R_2 = (.0125)^2 \times 2000 = 0.3125 \text{ watt}$$
$$P_3 = I^2 \times R_3 = (.0125)^2 \times 5000 = 0.78125 \text{ watt}$$

Manufacturers do not generally make resistors with the wattages determined in this problem. You will find wattages of ⅛-watt, ¼-watt, ½-watt, and integral values. For these three resistors, ignoring safety factor (which depends upon the specific application), you would select a resistor with the next higher wattage rating. For R_1 you would use ¼-watt; for R_2, ½-watt; and for R_3, one watt.

To find the voltage across each resistor, apply the formula $E = IR$. Thus,

$$E_1 = I \times R_1 = .0125 \times 1000 = 12.5 \text{ volts}$$
$$E_2 = I \times R_2 = .0125 \times 2000 = 25 \text{ volts}$$
$$E_3 = I \times R_3 = .0125 \times 5000 = 62.5 \text{ volts}$$

Notice that the sum of the voltages across the three resistors is 100 volts, a sum equal to the applied voltage. In any series circuit, the sum of the voltages across the individual resistors is always equal to the applied voltage.

The total power P_t expended by the battery is equal to the sum of the power expended by the individual resistors, $0.15625 + 0.3125 + 0.78125 = 1.25$ watts. The total power is also the total voltage multiplied by the total current

$$P_t = E_t \times I_t$$

Substituting values

$$P_t = 100 \times .0125 = 1.25 \text{ watts}$$

This is equal to the sum of the individual wattages of each resistor.

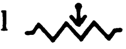
Polarity of IR Drop Across Resistances

When current flows through a resistance, it produces a potential difference between the terminals of the resistor. This is often called the *IR* drop. The magnitude of this voltage is found by applying Ohm's law to the circuit. The polarity of the voltage across a resistor is due to the current flow through it. *The end of the resistor at which the current enters is negative with respect to the end of the resistor the current leaves.* If you refer to Figure 2–7, you will see that the applied voltage across *R* is positive at the top and negative at the bottom. The current flow is up through *R*. Applying this rule you have just learned, you will see that the voltage across *R* due to the current through *R* makes the bottom of *R* negative with respect to the top.

In Figure 2–8 the polarity of the voltage drop across R_1 makes the left end of R_1 positive with respect to the right end; the IR drop across R_2 is positive at the top with respect to the bottom, and the polarity of the voltage drop across R_3 makes point *X* positive with respect to point *Z*. This means of figuring the polarity of voltage drops across circuit components is of great importance when it comes to connecting a voltmeter across a component to measure the voltage.

Voltage Dividers

The arrangement of resistors shown in Figure 2–8 is called a voltage divider, a device very useful for supplying voltages of various values. If, for some purpose, you want a voltage of 25 volts, you can use the voltage across R_2. If you need 30 volts instead of 25, you would have to select other values of resistance in the circuit, so that one resistor would have 3/10th of the total resistance. (The total voltage across the circuit is 100 volts.)

To make the voltage across a resistor variable between fixed limits, a kind of voltage divider called a *potentiometer* must be used. A potentiometer is similar to a variable resistor except that there are fixed contacts at both ends. The movable contact is located between the fixed contacts. Schematically, the potentiometer is represented by the symbol  .

To illustrate the use of a potentiometer, assume that R_2 in the circuit of Figure 2–8 is a 2000-ohm potentiometer. In this condition, the voltage at *X* (with respect to *Z*) is 62.5 volts. At *Y* the voltage (with respect to *Z*) is 87.5 volts.

By correctly setting the variable tap, it is possible to obtain any voltage between these two limits.

Summary

The current, voltage and power relations in a series circuit having *n* resistors are:

1. $R_t = R_1 + R_2 + R_3 + \ldots + R_n$
2. $I_t = I_1 = I_2 = I_3 = \ldots = I_n$
3. $E_t = E_1 + E_2 + E_3 + \ldots + E_n$
4. $P_t = P_1 + P_2 + P_3 + \ldots + P_n$

CURRENT, VOLTAGE AND POWER RELATIONS IN A PARALLEL CIRCUIT

In the parallel circuit shown in Figure 2–9, the resistors R_1, R_2, and R_3 have their terminals joined together at points *X* and *Y*. Current divides at point *X*, with part of it going through R_1, part through R_2, and the remainder through R_3. All of the individual currents recombine at point *Y* and flow to the positive terminal of the battery. The voltage across each of the three resistors is equal to the potential difference between points *X* and *Y*, which is equal to the voltage of the battery. Therefore, the voltage across each resistor is 100v ($E_1 = E_2 = E_3 = 100v$).

To find the total current supplied by the battery to the combination of parallel resistors, first find the current through each individual resistor, and then find the sum of the individual currents. You can readily find the individual currents by Ohm's law ($I = E/R$).

1. Current through R_1

$$I_1 = \frac{E_1}{R_1}$$

$$I_1 = \frac{100}{1000}$$

$$I_1 = 0.1 \text{ ampere}$$

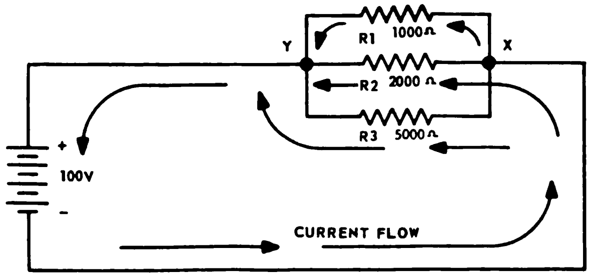


Figure 2–9. Parallel Circuit

2. Current through R_2

$$I_2 = \frac{E_2}{R_2}$$

$$I_2 = \frac{100}{2000}$$

$$I_2 = .05 \text{ ampere}$$

3. Current through R_3

$$I_3 = \frac{E_3}{R_3}$$

$$I_3 = \frac{100}{5000}$$

$$I_3 = .02 \text{ ampere}$$

4. The total current

$$I_t = I_1 + I_2 + I_3$$

$$I_t = 0.1 + .05 + .02$$

$$I_t = 0.17 \text{ ampere}$$

The total power expended by the battery i equal to the sum of the power expended in th individual resistors. This can be found by th power formula, $P = EI$.

1. Power in R_1

$$P_1 = E_1 \times I_1$$

$$P_1 = 100 \times 0.1$$

$$P_1 = 10 \text{ watts}$$

2. Power in R_2

$$P_2 = E_2 \times I_2$$

$$P_2 = 100 \times .05$$

$$P_2 = 5 \text{ watts}$$

3. Power in R_3

$$P_3 = E_3 \times I_3$$

$$P_3 = 100 \times .02$$

$$P_3 = 2 \text{ watts}$$

4. The total power

$$P_t = P_1 + P_2 + P_3$$

$$P_t = 10 + 5 + 2$$

$$P_t = 17 \text{ watts}$$

Since you already know the total current, yo can find the total power also by substituting t total current I_t in the power formula. Therefor

$$P_t = E_t \times I_t$$

$$P_t = 100 \times 0.17$$

$$P_t = 17 \text{ watts}$$

Resistance in Parallel Circuits

When the total current is known, the tot or equivalent, resistance in a parallel circuit m be found by the Ohm's law formula, $R = E/I$.

Thus, the equivalent resistance in the circuit of Figure 2–9 is,

$$R_e \text{ (equivalent)} = \frac{E_t}{I_t}$$

$$R_e = \frac{100}{0.17}$$

$$R_e = 588 \text{ ohms}$$

Note that the equivalent resistance 588 ohms is less than any one of the individual resistors R_1, R_2, or R_3. The equivalent resistance of two or more resistors in parallel is always less than that of any of the resistors alone.

The relationship between the resistances R_1, R_2, and R_3, and the equivalent resistance in the parallel circuit is developed as follows:

$$I_t = I_1 + I_2 + I_3 \qquad (1)$$

$$I_t = \frac{E_t}{R_e}, I_1 = \frac{E_1}{R_1}, I_2 = \frac{E_2}{R_2}, I_3 = \frac{E_3}{R_3}$$

Substituting these values of I in equation (1), it becomes

$$\frac{E_t}{R_e} = \frac{E_1}{R_1} + \frac{E^2}{R_2} + \frac{E_3}{R_3} \qquad (2)$$

Since $E_t = E_1 = E_2 = E_3$, equation (2) becomes

$$\frac{E_t}{R_e} = \frac{E_t}{R_1} + \frac{E_t}{R_2} + \frac{E_t}{R_3} \qquad (3)$$

Dividing by E_t, equation (3) becomes

$$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad (4)$$

Dividing both sides of equation (4) into 1 gives,

$$R_e = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \qquad (5)$$

For n resistors, the general formula is

$$R_e = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}}$$

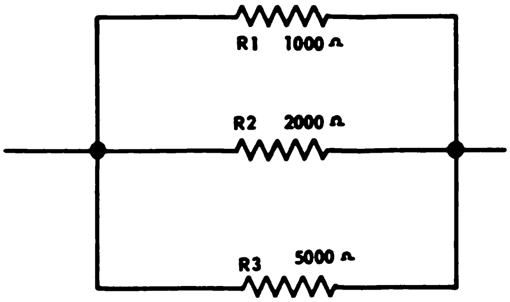


Figure 2–10. Reducing Parallel Resistances to Equivalent Resistance

This expression states that the combined resistance of a parallel circuit is equal to the reciprocal of the sum of the reciprocals of the individual resistances.

The circuit of Figure 2–10 shows three resistors, R_1, R_2 and R_3, with values of 1000 ohms, 2000 ohms, and 5000 ohms connected in parallel. Solve for the total resistance by formula (4):

$$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_e} = \frac{1}{1000} + \frac{1}{2000} + \frac{1}{5000}$$

$$\frac{1}{R_e} = \frac{10 + 5 + 2}{10{,}000} = \frac{17}{10{,}000}$$

$$R_e = \frac{10{,}000}{17}$$

$$R_e = 588 \text{ ohms}$$

When only two parallel resistors are involved, a simpler equation can be developed,

$$R_e = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$R_e = \frac{1}{\frac{R_2 + R_1}{R_1 R_2}}$$

$$R_e = \frac{R_1 R_2}{R_1 + R_2}$$

Stated as a rule, this reads, "The equivalent resistance of two parallel branches is the product of the two resistors divided by their sum." When there are three or more branches, you can use this formula by combining resistors two at a time until a single equivalent resistance is found.

Example

Find the equivalent resistance in Figure 2–10 using this method.

Solution

1. Combine R_1 and R_2

$$R_{e1} = \frac{1000 \times 2000}{1000 + 2000}$$

$$R_{e1} = 667 \text{ ohms}$$

2. Then find the equivalent resistance of this resistance and R_3

$$R_{e2} = \frac{667 \times 5000}{667 + 5000} = \frac{3335000}{5667} = 588 \text{ ohms}$$

When two equal resistors are connected in parallel, the combined resistance of the two resistors equals one-half of either resistor. Similarly, the total resistance of three equal resistors in parallel is one-third the resistance of one resistor.

Conductance

The reciprocal of resistance is conductance. Conductance is the property of a conductor that makes it possible for current to flow. It is represented by the letter *G*. The unit of conductance is the *mho* (ohm spelled backward). For example, the conductance of a conductor with a resistance of 1000 ohms equals .001 mho (1000 micromhos) since $G = \frac{1}{R} = \frac{1}{1000}$.

In a parallel circuit with several branches, the total conductance is the sum of the conductances of the several branches. In formula, it can be written,

$$G_t = G_1 + G_2 + G_3 + \ldots + G_n$$

Division of Current in Parallel Branches

The total current in a parallel circuit divides between the branches. Each branch current is proportional to the conductance in that branch, or inversely proportional to the resistance of that branch. In a circuit with two branches containing R_1 and R_2, the current flowing in R_1 is expressed as

$$I_1 = I_t \times \frac{R_2}{R_1 + R_2}$$

The current flowing in R_2 is expressed as

$$I_2 = I_t \times \frac{R_1}{R_1 + R_2}$$

Summary

The following are the general relationships of current, voltage, and power in a parallel circuit:

1. In a parallel circuit the same voltage appears across each element ($E_t = E_1 = E_2 = E_3 = \ldots = E_n$).

2. The total current in a parallel circuit is equal to the sum of the currents flowing in the individual branches ($I_t = I_1 + I_2 + I_3 + \ldots + I_n$).

3. The equivalent resistance is equal to the reciprocal of the sum of the reciprocals of the individual resistances,

$$R_e = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}}$$

4. The total conductance of the resistances is equal to the sum of the conductances of the individual resistors, ($G_t = G_1 + G_2 + G_3 + \ldots + G_n$).

5. The total power is the sum of the power expended in each individual resistor ($P_t = P_1 + P_2 + P_3 + \ldots + P_n$).

SERIES-PARALLEL CIRCUITS

Frequently, you will find circuits having combinations of resistors in series and in parallel. To solve problems of this type, apply the principles you learned in circuits having only resistors in series and those having only resistors in parallel.

Example 1

In the circuit in Figure 2–11, find the total current, the current through the 10-ohm resistor, and the voltage drop across the 5-ohm resistor.

Solution

1. First find the equivalent resistance (R_e) of the 10-ohm and the 15-ohm resistors in parallel.

$$R_e = \frac{10 \times 15}{10 + 15}$$

$$R_e = \frac{150}{25}$$

$$R_e = 6 \text{ ohms}$$

2. R_e is now in series with the 5-ohm resistor. Therefore, $R_t = 6 + 5 = 11$ ohms.

3. The total current

$$I_t = \frac{E_t}{R}$$

$$I_t = \frac{110}{11} = 10 \text{ amperes}$$

4. The portion of the total current flowing through the 10-ohm resistor can be found by the formula,

$$I = I_t \times \frac{R_2}{R_1 + R_2}$$

$$\text{Therefore, } I = 10 \times \frac{15}{10 + 15}$$

$$I = 10 \times \frac{15}{25} \text{ or 6 amperes.}$$

5. The voltage drop across the 5-ohm resistor can b found by the formula, $E_3 = I_t \times R_3$.

Therefore, $E_3 = 10 \times 5 = 50$ volts.

Example 2

Given the circuit in Figure 2–12A with the indicate values, find the total current and the current I_2 throug R_2.

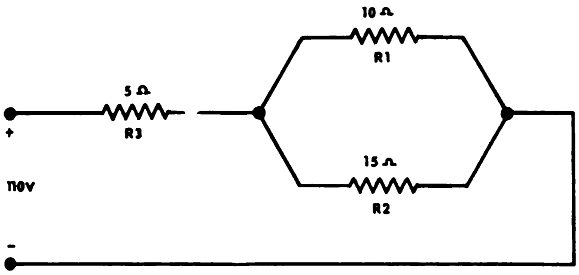


Figure 2–11. Single Series-Parallel Circuit

Solution

1. Resolving R_1 and R_2 into an equivalent resistance,

$$R_e = \frac{20 \times 5}{20 + 5} = \frac{100}{25}$$

$$R_e = 4 \text{ ohms}$$

giving the equivalent circuit in Figure 2–12B.

2. Combining the 4 ohms with the 16 ohms in series, the total resistance of R_1, R_2, and R_3 = 20 ohms, as shown in Figure 2–12C.

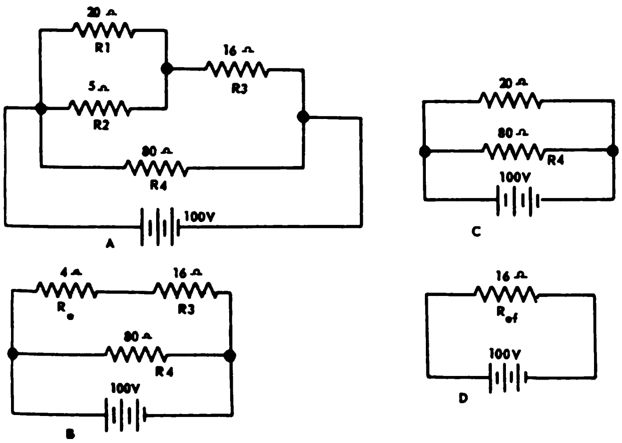


Figure 2–12. Series-Parallel Circuit

3. Since R_4 is in parallel with the combination having a resistance of 20 ohms,

$$R_{eq} = \frac{20 \times 80}{20 + 80}$$

$$R_{eq} = \frac{1600}{100} = 16 \text{ ohms, as shown in Figure 2–12D.}$$

4. The total current

$$I_t = \frac{E_t}{R_{eq}}$$

$$I_t = \frac{100}{16}$$

$$I_t = 6.25 \text{ amperes.}$$

5. Current I_3 through R_3 equals E_t divided by 20 ohms, the resistance of R_1, R_2 and R_3 in combination. Therefore, $I_3 = \frac{100}{20}$ or 5 amperes.

The 5-ampere current divides between R_1 and R_2, thus,

$$I_2 = 5 \times \frac{R_1}{R_1 + R_2}$$

$$I_2 = 5 \times \frac{20}{25} = 4 \text{ amperes.}$$

Kirchhoff's Law

More involved combinations of resistances and voltages are called networks. The solution of network problems is best handled by using Kirchhoff's laws which, in turn, are extensions of Ohm's law. Kirchhoff's laws may be stated for DC circuits as follows:

1. The algebraic sum of the currents flowing toward any point in an electric circuit is zero. In simpler form we say that the sum of the currents entering any point in an electric circuit equals the sum of the currents leaving that point.

2. The algebraic sum of the electromotive forces and the IR drops around any closed loop of an electric circuit is zero. That is, the sum of the battery or generator voltages around any closed loop of an electric circuit is equal to the sum of the voltage drops across the resistances in that closed loop.

Examples

Problem 1

In the circuit in Figure 2–13 find the current through each resistor, the voltage drop across each resistor, and the resistance of the network. Use Kirchhoff's laws.

Solution

Let I_1 be the current through R_5, I_2 the current through R_2 and I_3 the current through R_3. Assume that electron flow through R_3 is from f to c. (This assumption, if correct, will not affect the solution. I_3 will be positive if the assumption is correct, and negative if the assumption is incorrect.)

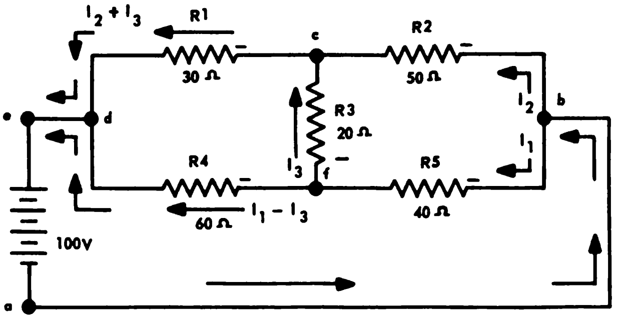


Figure 2–13. Current in a Network

Then, by Kirchhoff's first law, which states that the current leaving a point must equal the current flowing into that point, current I_1, flowing to f through R_5, must equal the current I_3 plus the current flowing through R_4. Or, the current flowing through R_4 must equal $(I_1 - I_3)$. Likewise, the sum of the currents entering point c $(I_2 + I_3)$ must equal the current leaving point c and flowing through R_1.

Since there are three unknown quantities (I_1, I_2, and I_3), there must be three equations in these unknowns in order to solve the problem. Using Kirchhoff's second law, you can set up equations for the voltages in three different closed loops. Remember that the polarity of the voltage drop across a resistor depends upon the direction of the assumed current through that resistor. Here are three equations that can be written.

In loop edfbae $60\,(I_1 - I_3) + 40\,I_1 - 100 = 0$

or $100\,I_1 - 60\,I_3 = 100 \qquad (1)$

In loop edcfbae $20\,I_3 + 30\,(I_2 + I_3) + 40\,I_1 - 100 = 0$

or $40\,I_1 + 30\,I_2 + 50\,I_3 = 100 \qquad (2)$

In loop edcbae $30\,(I_2 + I_3) + 50\,I_2 - 100 = 0$

or $80\,I_2 + 30\,I_3 = 100 \qquad (3)$

Multiply (1) by 2.

$200\,I_1 - 120\,I_3 = 200 \qquad (4)$

Multiply (2) by 5.

$200\,I_1 + 150\,I_2 + 250\,I_3 = 500 \qquad (5)$

Subtract (4) from (5).

$150\,I_2 + 370\,I_3 = 300 \qquad (6)$

Multiply (6) by 8.

$1200\,I_2 + 2960\,I_3 = 2400 \qquad (7)$

Multiply (3) by 15.

$1200\,I_2 + 450\,I_3 = 1500 \qquad (8)$

Subtract (8) from (7).

$2510\,I_3 = 900$

Divide by 2510.

$I_3 = 0.359$ ampere

Substitute 0.359 for I_3 in (1) and solve for I_1.

$100\,I_1 - 60\,(0.359) = 100$

$100\,I_1 - 21.5 = 100$

$100\,I_1 = 121.5$

$I_1 = 1.215$ amperes

Substitute 0.359 for I_3 in (3) and solve for I_2.

$80\,I_2 + 30\,(0.359) = 100$

$80\,I_2 + 10.77 = 100$

$80\,I_2 = 89.23$

$I_2 = 1.115$ amperes

then,

$I_1 - I_3 = 1.215 - 0.359 = 0.856$ a.

and

$I_2 + I_3 = 1.115 + 0.359 = 1.474$ a.

The voltage drops across R_1, R_2, R_3, R_4 and R_5 respectively are E_1, E_2, E_3, E_4 and E_5. By Ohm's law, $E = IR$, thus,

$E_1 = 30\,(1.474) = 44.22$ v.

$E_2 = 50\,(1.115) = 55.75$ v.

$E_3 = 20\,(0.359) = 7.18$ v.

$E_4 = 60\,(0.856) = 51.36$ v.

$E_5 = 40\,(1.215) = 48.60$ v.

Find the total current I_t at b or d and the total resistance.

At b $\quad I_t = I_1 + I_2 = 1.215 + 1.115 = 2.33$ amps, and the total resistance

$R_t = E_t/I_t = 100/2.33 = 42.9$ ohms

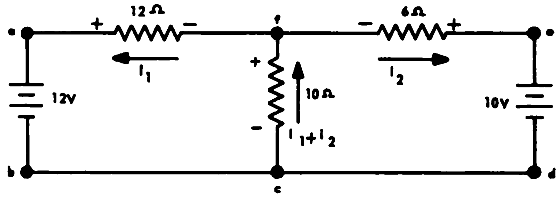


Figure 2–14. Multiple Battery Circuit

Problem 2

In the circuit shown in Figure 2–14, find the current and voltage drop across each resistor.

Solution

Let I_1 be the current through the 12-ohm and I_2 the current through the 6-ohm resistors. By Kirchhoff's first law the current from point c to point f is $I_1 + I_2$. Thus, the voltage equations from the second law are the following:

Around loop efcde,

$6\,I_2 + 10\,I_1 + 10\,I_2 - 10 = 0$

or

$10\,I_1 + 16\,I_2 = 10 \qquad (1)$

Around loop afcba,

$12\,I_1 + 10\,I_1 + 10\,I_2 - 12 = 0$

or

$22\,I_1 + 10\,I_2 = 12 \qquad (2)$

Multiply (2) by 8.

$176\,I_1 + 80\,I_2 = 96 \qquad (3)$

Multiply (1) by 5.

$50\,I_1 + 80\,I_2 = 50 \qquad (4)$

Subtract (4) from (3).

$126\,I_1 = 46$

Divide by 126.

$I_1 = 0.365$ ampere

Substitute for I_1 in (2) and solve for I_2.

$8.03 + 10\,I_2 = 12$

$10\,I_2 = 3.97$

$I_2 = 0.397$ ampere

Therefore,

$I_1 + I_2 = 0.762$ ampere.

The various voltage drops are the following:

10-ohm resistor = 10 × 0.762 or 7.62 volts

6-ohm resistor = 6 × 0.397 or 2.38 volts

12-ohm resistor = 12 × 0.365 or 4.38 volts

Superposition Method

A second method for solving networks containing more than one source of voltage is the superposition method. This method is based on the principle that the current in any part of a circuit is the sum of the currents produced by each voltage acting separately and with the other voltages shorted out.

Example

Solve the circuit illustrated in Figure 2–14, with the values indicated, by superposition. First, draw the equivalent circuit as shown in Figure 2–15 with the 10-volt battery shorted out, and find the total current in the equivalent circuit.

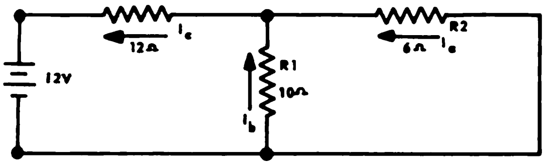


Figure 2–15. First Equivalent Circuit

1. To find total current I_c, calculate the equivalent resistance of the 6-ohm and 10-ohm resistors

$$R_e = \frac{R_1 \times R_2}{R_1 + R_2}$$

$$R_e = \frac{6 \times 10}{6 + 10} = \frac{60}{16} = 3.75 \text{ ohms}$$

Total resistance = 12 + 3.75 or 15.75 ohms

$$I_c = E/R$$

$$I_c = \frac{12}{15.75} = 0.762 \text{ ampere}$$

2. Find the current in each branch of the equivalent circuit.

$$I_a = I_t \times \frac{R_1}{R_1 + R_2}$$

$$I_a = 0.762 \times \frac{10}{10 + 6}$$

$$I_a = 0.762 \times \frac{10}{16} = 0.476 \text{ ampere}$$

$$I_b = I_t \times \frac{R_2}{R_1 + R_2}$$

$$I_b = 0.762 \times \frac{6}{10 + 6}$$

$$I_b = 0.762 \times \frac{6}{16} = 0.286 \text{ ampere}$$

Now, draw the equivalent circuit as shown in Figure 2–16 with the 12-volt battery shorted out and find the total current and the current in the branches.

1. Find the total current.
The equivalent resistance of the 12-ohm and 10-ohm resistors in parallel is $\frac{10 \times 12}{10 + 12}$ or 5.45 ohms.

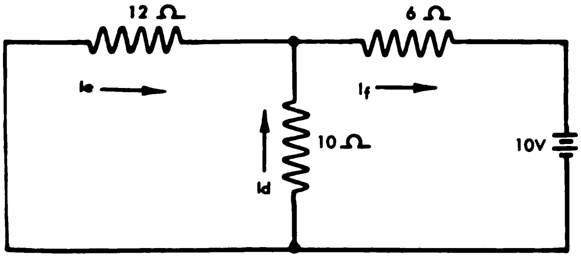


Figure 2–16. Second Equivalent Circuit

The total resistance in the circuit is 6 + 5.45 or 11.45 ohms.

The total current, $I_f = \frac{10}{11.45}$ or 0.873 amp.

2. Find the current in branches

$$I_d = 0.873 \times \frac{12}{10 + 12}$$

$$I_d = 0.873 \times \frac{6}{11} \text{ or } 0.476 \text{ ampere}$$

$$I_e = 0.873 \times \frac{10}{10 + 12}$$

$$I_e = 0.873 \times \frac{5}{11} \text{ or } 0.397 \text{ ampere.}$$

Next, combine the results of the two sets of computations.

1. The current through the 12-ohm resistor is $I_c - I_e$. Therefore, this current equals 0.365 ampere (0.762 − 0.397). (These currents are subtracted since they flow in reverse directions, I_c to the right; I_e to the left.) The difference is their net result.

2. The current through the 10-ohm resistor equals $I_b + I_d$ or 0.286 + 0.476 = 0.762 ampere.

3. The current through the 6-ohm resistor equals $I_f - I_d$ = 0.873 − 0.476 or 0.397 ampere.

On comparing these results with those obtained in the Kirchhoff law problem, note that the results obtained here are identical.

MAGNETISM

A magnet is an object which has the property of attracting iron and steel and which, if permitted to turn freely, will rotate to a definite direction. Magnets attract other materials such as nickel and cobalt, but not with as much force as iron and steel. Materials which can be attracted by magnets are called *magnetic substances.* There are two principal kinds of magnets — *natural* magnets and *artificial* magnets. Natural magnets are found in nature already magnetized, while artificial magnets must be made by magnetizing iron or steel. Artificial magnets can be either permanent or temporary.

Natural Magnets

A black mineral ore called lodestone, or magnetite, found in plentiful supplies in Asia Minor exhibits magnetic properties. Lodestone is called a natural magnet because it exists in nature already magnetized. Historically, lodestone has played an interesting role, being used in the middle ages to magnetize compass needles. Today, however, lodestone has very little value as a magnet chiefly because of its unstable physical structure and low magnetic strength.

Artificial Magnets

Artificial magnets are made of iron and steel. They are magnetized by induction from some exterior object, by stroking with some other artificial magnet or by being placed in the field of an electromagnet. (Electromagnets will be described later.) A bar of hard steel will hold magnetism for a long period of time and, for this reason, is called a permanent magnet. Soft iron can be magnetized easily but loses its magnetism quite rapidly.

The effectiveness of a material as a path for magnetic lines of force is its *permeability*. The property of a substance which causes it to remain magnetized is called *retentivity*, and the magnetism which remains is called *residual magnetism*. Steel has high retentivity, while soft iron has low.

Magnetic Poles

When a bar magnet is dipped into iron filings, a large number of filings will cling to the magnet near its ends, but few will attach themselves to the magnet near its center. This action indicates that the magnetism is concentrated at the two ends. These ends are called the poles of the magnet. The magnetic strength of the two poles of any magnet is equal.

A magnet, free to rotate, will always turn to a north-south direction, aligning itself with the earth's magnetic field. The pole of the magnet which always turns toward the north is called the *north-seeking pole*, or simply the north pole (N); and the pole at the opposite end, the *south-seeking pole*, or south pole (S).

Force Between Poles

Experiments show that if you bring the south pole of one bar magnet near the north pole of another, there will be an attraction between the two poles. If you bring two north poles or two south poles together, there will be a force of repulsion between them. This action is summed up in the basic law of magnetism, which states that *unlike* poles *attract* each other and *like* poles *repel* each other.

The force of a reaction (attraction or repulsion) between two poles varies directly as the product of the strength of the poles and inversely as the square of the distance between them. This relation is expressed mathematically as:

$$F = \frac{M_1 \times M_2}{\mu R^2}$$

where M_1 and M_2 are the strengths of the two poles, and R is the distance between them. The Greek letter μ is the symbol for permeability, whose value depends upon the medium in which the poles are located. If the space between the poles is a vacuum, μ is unity. Air is approximately the same. If the space between the poles contains a magnetic material, μ is some number greater than one. (This is discussed further under permeability.)

The unit of pole strength (referred to as a unit pole) is derived from the equation for the force between two poles. A unit magnetic pole has such strength that it will exert a force of one dyne upon an equal pole in a vacuum (or air) when placed one cm away from it.

Magnetic Field

The region around a magnet in which its effect can be detected is known as its magnetic field. When a magnetic pole is moved about the magnetic field, forces exerted on the pole vary both in direction and intensity. At any point near a magnet, the field has a certain direction and intensity. The direction of a magnetic field at a given point is the direction in which a unit north magnetic pole would be moved by the force applied to it. The intensity, or strength, (number of lines of force per unit area) at a point is equal to the force which the field exerts on a unit north pole placed at that point. This relation is represented by the equation,

$$H = \frac{F}{M}$$

where H is strength of the field in oersteds, M the strength of the pole brought into the field, and F the force which the field exerts on this pole.

Example

Find the strength of a magnetic field at a point where it exerts a force of 50 dynes, acting on a pole to the right. Assume the strength of the pole to be 10 units.

Solution

$$H = \frac{F}{M}$$

$$H = \frac{50}{10}$$

$$H = 5$$

Therefore, the strength of the field at the point in question is 5 oersteds to the right.

In the region around a pole the intensity of the field is not uniform, but diminishes rapidly as the distance from the pole increases. Experiments have shown that the field of intensity around a pole varies directly as the pole strength and inversely as the square of the distance from the pole. This relationship takes the following form:

$$H = \frac{M}{\mu r^2}$$

or in a vacuum

$$H = \frac{M}{r^2}$$

M represents the strength of the pole, r represents the distance of the place from the pole, and H the force in oersteds at that place.

Experiments show that the direction of the magnetic field about a magnet is radially outward from the north pole and radially inward toward the south pole. To find the intensity and direction at any point about the magnet, apply the equation $H = \frac{M}{\mu r^2}$ to both poles and add the results vectorially. The resultant vector direction is the direction of the field.

Lines of Force

Place a sheet of glass or some other nonmagnetic substance over a magnet and sprinkle iron filings on it. This will provide a means of observing the configuration of the magnetic field. Tapping the glass causes the filings to align themselves into chains or lines, producing a facsimile of the magnetic field itself. At any particular point in the space around the magnet there is a state of stress which exerts a force on any pole brought into the vicinity of the magnet. The direction that such a force takes on a unit north pole indicates the direction of the magnetic field at that point, and lines connecting the direction of the field at a series of points form lines called *lines of force.* It is along these lines of force that iron filings align themselves.

Figure 2–17 shows that the lines of force about a bar magnet travel from the north pole to the south pole. They have only one direction at a given point; do not cross each other, and form closed loops. The path of each loop is from the north pole to the south pole in space and thence through the magnet to the north pole.

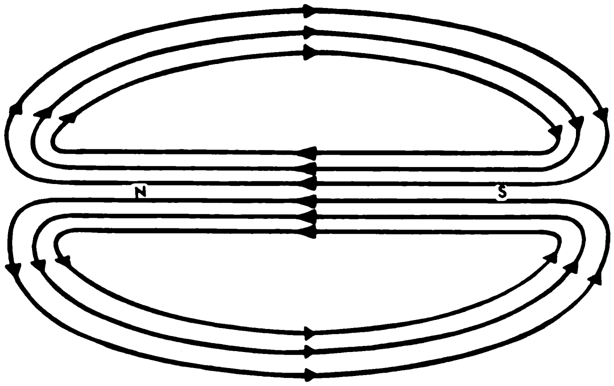


Figure 2–17. Magnetic Field about a Bar Magnet

A convenient way of regarding lines of force is to think of them as rubber bands that, in trying to shorten themselves, establish a stress field of mutual repulsion. When two magnets are brought together, the fields of stress interact, causing repulsion or attraction, depending upon the polarity of the poles.

Magnetic Flux

The total number of lines of force occupying a region of magnetic activity is called the *magnetic flux*, and is represented by the Greek letter ϕ (*phi*). The unit is the maxwell, which is one line of magnetic flux. The number of lines passing perpendicularly through a square centimeter is called *flux density* and is represented by the letter *B*. The unit of flux density is the *gauss*, which is equal to one maxwell per square centimeter.

Permeability

The amount by which the flux density (*B*) exceeds the field intensity (*H*) in a coil or solenoid depends upon the type of core. The ratio between these two quantities (*B/H*), is called the *permeability* of the core. For example, soft iron has a permeability of about 2000. Earlier in the discussion of magnetism, permeability was defined as the ease with which a substance conducts magnetic lines of force. Permeability is represented by the Greek letter mu (μ). In any ferromagnetic substance permeability is not a constant quantity, but a quantity that depends largely upon the intensity of the field.

For example, magnetization curves which indicate the relation of flux density *B* to field intensity *H*, show that in the case of silicon steel (Figure 2–18), the flux density increases almost in direct proportion to *H* at low field intensities. At large field intensities, however, the steel becomes saturated with flux and a large change of *H* is required to increase *B* further. At length, increasing *H* produces no appreciable change in *B*.

You can find permeability from the values of *H* and *B* in the diagram by solving the equation,

$$\mu = \frac{B}{H}$$

Example

Assume H = 70 and B = 1600. Find μ.

$$\mu = \frac{B}{H}$$

$$\mu = \frac{1600}{70} = 22.86$$

Likewise, find μ when B = 10000 and H = 3.

$$\mu = \frac{10000}{3}$$

$$\mu = 3333$$

(Note the difference in permeability at the different points on the curve.)

Earth's Magnetic Field

Magnetic lines of force surround the earth much in the same way that lines of force surround a bar magnet. The earth has two magnetic poles, one near the geographic north pole and the other near the geographic south pole. These two facts are responsible for man's most important navigational instrument — the magnetic compass. Dating back to the middle ages, the magnetic compass is still the most widely used navigational instrument. Basically, it is an artificial magnet mounted in such a way that it aligns itself with the earth's magnetic lines of force, with one end of the magnet always pointing toward magnetic north.

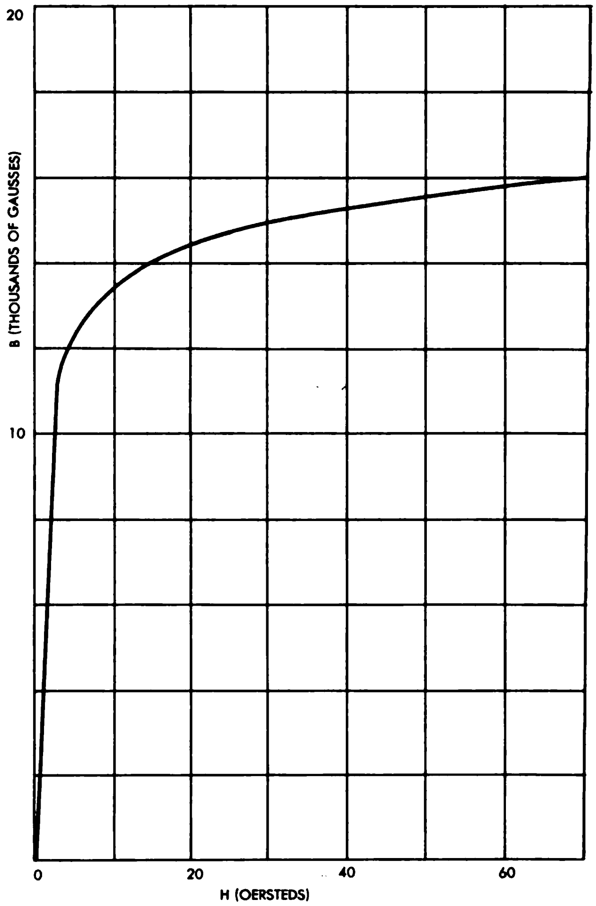


Figure 2–18. Magnetization Curve of Steel

Modern Magnetic Theory

In experiments consisting of breaking a magnet in two and then breaking each of the parts in two, and so on as far as physically possible, it was found that the parts were magnets themselves. It was presumed that if the breaking process were continued until the parts were the size of molecules, each part would still be a magnet with north and south poles.

Modern magnetic theory is based on two fundamental concepts, the spinning electron and critical atomic distances. Additional ideas which make the explanation of magnetic phenomena easier to understand are the group action of masses of molecules, or *domains*, and the crystal arrangement in metals. While electrons may not actually spin, the use of this idea makes it easier to explain ferromagnetism.

You have been told that atoms consist of a nucleus with a positive charge, surrounded by electrons moving in orbits around the nucleus, somewhat like our solar system. In addition, consider that the electrons, like the earth, are spinning about axes through their centers, some in one direction, and some in the opposite. Since these negative charges are moving, they set up a magnetic field, which, however, is very small. Since each electron's field is so very minute, this would not make the material magnetic. The electrons are grouped in *shells* about the nucleus. The electron spins in all the shells are balanced except in the next to the outermost shell. In ferromagnetic materials, there is an unbalance of spins in this shell. That is, there are more electrons spinning in one direction than in the opposite.

This excess of spins in one direction in a single atom cannot produce the over-all effect of magnetism if adjacent atoms act in opposite directions. Spins in adjacent atoms must aid each other. Chrome, manganese, iron, cobalt, and nickel all have excess spins, but manganese, for instance, is not considered a magnetic material. Another factor also must be considered, the distance between atoms.

There is a force between atoms that tends to align the electron spins in the same direction. In widely spaced atoms this force approaches zero. As atoms are drawn closer together, there are strong positive forces for the alignment of electron spins. These forces become zero at what is known as the critical atomic distance. If the atoms become still closer together, there again is a strong force, but now it produces electron spins that are parallel but in opposite directions. Although they have excess spins, manganese atoms are too closely spaced to be ferromagnetic. However, a slight displacement is produced by combining certain metals into alloys, such as silver-manganese-aluminum, a permanent-magnet material called Silmanel.

The forces between adjacent atoms which align the electron spins do not extend over the whole piece of material. The parallel alignment of electron spins takes place over very small volumes (not visible unless magnified). In ferromagnetic materials there are thousands of these units, called *domains*. Each of them has parallel electron spins and is magnetized to saturation in some direction. The over-all effect of these different directions of magnetization is zero; there is no external magnetic flux. The material is said to be unmagnetized.

Ferromagnetic metals have a crystal structure. In each crystal the atoms are arranged in a pattern which repeats itself. Due to the motion of its electrons, each atom is a miniature magnet. The atoms affect each other just as if they were tiny bar magnets. Because of the crystal structure, these magnetic forces align themselves more easily in certain directions than in others. Thus, the magnetization in the domains is not in just any direction, but is parallel to one of the directions of easy magnetization, equally divided among them so that the over-all effect is zero. When an external magnetizing force is applied to such material in certain directions, the electron spins are aligned more easily than when the same force is applied in another direction.

When a small magnetizing force is applied to an unmagnetized material, those domains whose magnetization direction is closest to that of the external mmf become slightly larger. (This occurs from 0 to Point *a* on the curve in Figure 2–19.) At this point, as the magnetizing force is increased, whole domains realign their "atomic magnets" into the direction of easy magnetization that is closest to the direction of the applied mmf. What happens is that the electron spins change from one stable direction to another. From Point *a* to Point *b* the electron spins change, domain by domain. Although the curve looks perfectly smooth, the change is actually in steps, caused by the sudden realignment of domains, which is called the *Barkhausen effect*.

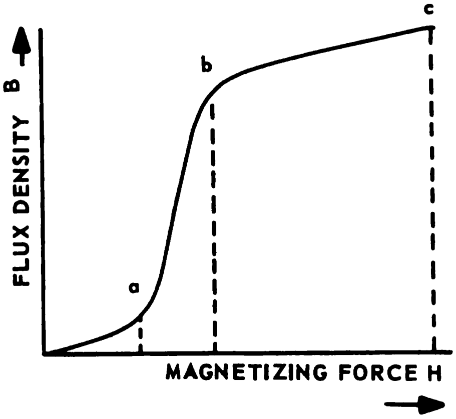


Figure 2–19. Magnetization Curve

From Point *b* to Point *c* the stronger magnetizing force gradually changes the direction of magnetization of the domains from one of the easy magnetization directions of the crystal to the direction of the applied field. At saturation all are parallel to the applied field.

ELECTROMAGNETISM

Over a century ago, Oersted, a Danish physicist, discovered that a current-carrying conductor is surrounded by a magnetic field. He also discovered that at any point the direction of this field is tangent to a circle about the conductor, that its strength diminishes as the distance from the concentric circles increases, and that lines of force form about the conductor.

The lines of force about a current-carrying conductor travel in either a clockwise or a counterclockwise direction, depending upon the direction of electron flow. At A in Figure 2–20, the electrons are moving into the page. In this case, the direction of the field about the conductor is counterclockwise. At B the electrons are moving out of the page. In this case, the direction of the field is clockwise.

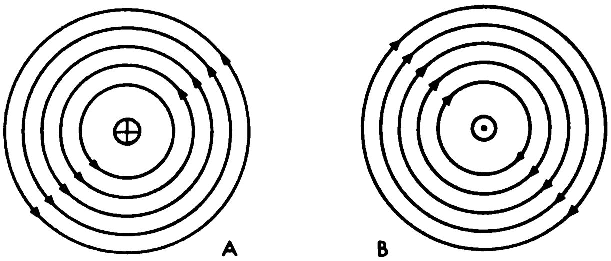


Figure 2–20. Magnetic Fields about Conducting Wires

To determine the direction of the lines of force about any conductor use the left-hand rule, which states that if you grasp a conductor with your left hand so that your thumb points in the direction of electron flow, your fingers will indicate the direction of the lines of force.

To find the strength of the field at any point about a long, straight, current-carrying conductor, use the equation,

$$H = \frac{2I}{10r}$$

H is the strength of the field in oersteds, I the curren in amperes, and r, the distance from the conducto in centimeters.

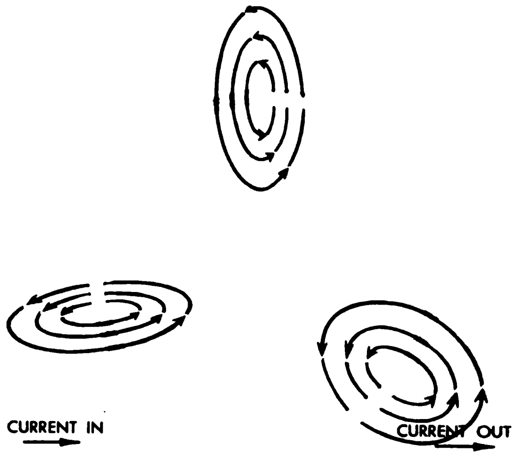


Figure 2–21. Effect of Loop on Fields

Magnetic Field about a Loop

If you bend the straight conductor into single turn loop as is shown in Figure 2–21, th lines of force will concentrate within the loo This concentration occurs because all lines force enter the loop from one side and leave the other. To find the intensity of the field the center of the loop, use the equation,

$$H = \frac{2\pi I}{10r}$$

I is the current in amperes, r the radius of the loop centimeters, and H the field intensity in oersteds.

Field in a Solenoid

If you wind several turns of wire close t gether into the form of a coil, the magnetic fiel about each turn will all have the same directio When the length of the coil is small compare to its radius, you can find the strength of t field at the center by the equation,

$$H = \frac{2\pi NI}{10r}$$

N equals the number of turns, r the radius in cen meters, and I the current in amperes.

When the length of the coil is longer than t radius, the coil is called a *solenoid.*

When current flows through it, the coil (solenoid) is surrounded by a magnetic field li

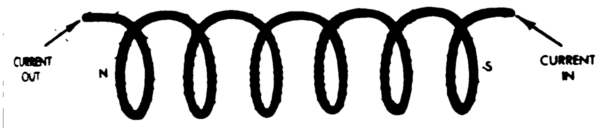


Figure 2–22. Magnetic Field Surrounding a Solenoid

that shown in Figure 2–22. One end of the coil is called the north magnetic pole, and the other end the south magnetic pole. To determine the polarity of a coil, use this left-hand rule. Grasp the coil with the left hand so that the fingers point in the direction of electron flow. The thumb points in the direction of the north pole. To obtain the intensity of the field at the axis and near the middle of a coil, use the equation,

$$H = \frac{4\pi NI}{10l}$$

I is the current in amperes, N the number of turns, l the length of the coil in centimeters, and H the intensity in oersteds.

Let $n = \frac{N}{l}$ (number of turns per unit length). The equation then becomes $H = 0.4\ \pi\ nI$. You can see that the strength of the field about a solenoid depends only upon the magnitude of current and the number of turns per unit length.

Inserting a soft iron core into a solenoid greatly increases the number of magnetic lines of force. This increase in magnetic lines is not from an increase in the intensity of the field, which depends only on current and turns per unit length, but from additional lines produced by the magnetization of the iron core.

Magnetic Circuits

Because magnetic flux or lines of force form closed loops, the path that the flux loops follow is called the magnetic circuit. Electrical circuits and magnetic circuits have many points of similarity. The force which produces a flow of electrons in an electrical circuit is the electromotive force. In the magnetic circuit the force which produces the flux is called the magnetomotive force (*mmf*). Similarly, just as the resistance opposes the flow of current in an electrical circuit, so *reluctance* opposes the magnetic flux in a magnetic circuit. Similarly, just as *conductance* indicates the ease with which electrical current flows, so *permeability* indicates the ease with which magnetic lines of force flow in a magnetic circuit.

No name has been given to the unit of reluctance in a magnet circuit. Quantitatively, one unit of reluctance is the reluctance of a magnetic circuit one centimeter long and one square centimeter in cross section with unit permeability. Mathematically, the unit of reluctance is expressed by the equation,

$$\mathcal{R} = \frac{l}{\mu A}$$

where l represents length in centimeters, A the cross-sectional area in square centimeters, and μ the permeability.

In much the same manner that Ohm's law expresses the relation between current, voltage and resistance, the expression,

$$\phi = \frac{\text{mmf}}{\mathcal{R}}$$

gives the relation between magnetomotive force, flux and reluctance where ϕ is flux in maxwells, *mmf* the magnetomotive force in *gilberts* (a gilbert is the mmf required to establish a flux of one maxwell in a magnetic circuit in which the reluctance is one unit), and $\mathcal{R}$ the reluctance.

Force of Magnetic Fields on Conductors

If a current-carrying conductor is placed in a uniform magnetic field (a field that is of the same strength everywhere in the vicinity of the conductor), the conductor will move. The direction of the movement will be at right angles to the magnetic field. The cause of the movement is the force due to the interaction of the field about the conductor (due to the electron flow through the conductor), and the magnetic field in which the conductor is placed. Figure 2–23 shows a conductor placed in a uniform magnetic field. The electron flow in the conductor is into the page, as is indicated by the symbol + at the center. The direction of the magnetic lines of force about the conductor, therefore, is counterclockwise.

Below the conductor the lines of force produced by the conductor are in the same direction as those produced by the two poles and tend to reinforce them. Above the conductor the lines

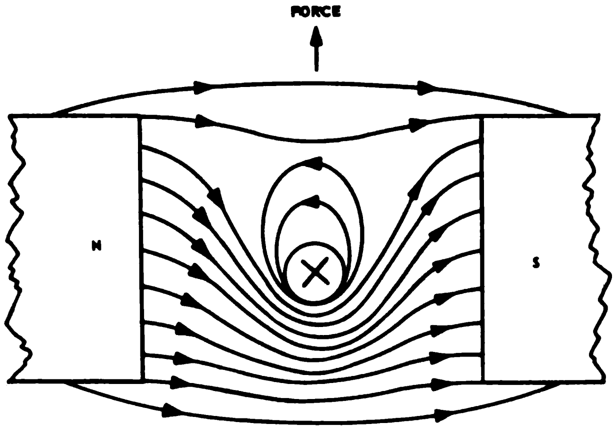


Figure 2–23. Composite Magnetic Field of Magnets and Conductor

of force of the two fields are in opposite directions and tend to counteract each other. The resultant field below the conductor is strong, whereas the resultant field above the conductor is weak. The repulsive action between lines of force in the same direction will force the conductor upward toward the weaker field.

By experimentation it can be shown that this force is proportional to the flux density, the current, and the length of the conductor perpendicular to the magnetic field. This is expressed mathematically by a formula known as Ampere's Law, which is

$$F = \frac{BIl}{10}$$

where F is the force in dynes, B is the strength of the field in gauss, I is the current in the conductor in amperes, and l is the length of the conductor in centimeters.

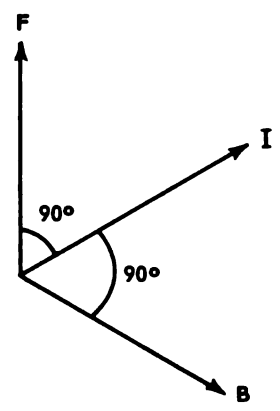


Figure 2–24. Direction of Force on a Conductor

Notice that the force, the flux, and the current flow in Figure 2–23 are mutually perpendicular. This is shown by the vectors in Figure 2–24.

If the conductor makes an angle ϕ with the flux, as shown in Figure 2–25, the effective length of the conductor is x, which is numerically equal to $l \sin \phi$. The force exerted on any current-carrying conductor in a magnetic field may be expressed as

$$F = \frac{BIl}{10} \sin \phi$$

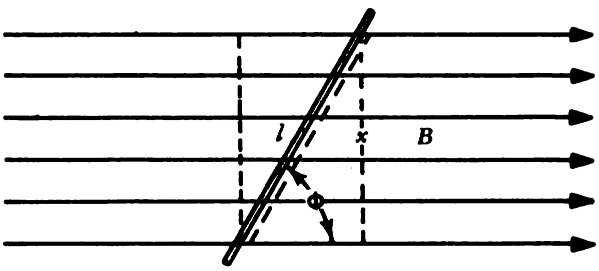
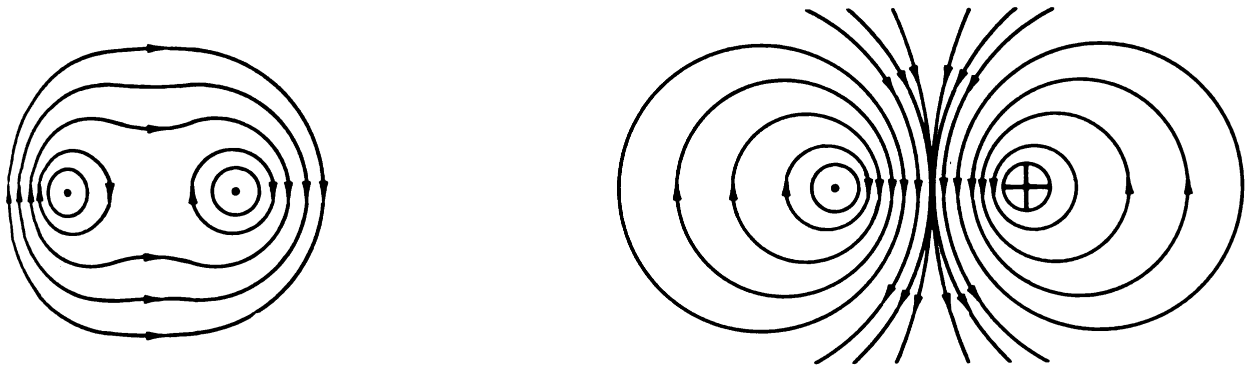


Figure 2–25. Conductor at an Angle with Flux

Whenever two current-carrying conductors are placed near each other, the magnetic fields about the conductors produce a force which reacts on each conductor. Figure 2–26A shows the fields about two conductors carrying current flow in the same direction (out of the page). The magnetic lines of force about each conductor are in a clockwise direction. Between the two conductors the lines of force are in opposite directions and tend to counteract each other. This weakens the field between the conductors and they move *toward* each other. Figure 2–26B shows two conductors carrying current in opposite directions. The magnetic lines of force about the left-hand conductor are clockwise, while those about the right-hand conductor are counterclockwise. The resultant field between the conductors is strengthened since the lines of force are in the same direction, and the two conductors are forced apart.

Applications of Electromagnets

Among the most common applications of electromagnetism are relays, electrical measuring devices, electric motors, electric generators, loudspeakers and transformers.


CONDUCTION IN SAME DIRECTION

CONDUCTION IN OPPOSITE DIRECTIONS

Figure 2–26. Field Surrounding Two Adjacent Conductors

In the most common type of relay, a current change in an electromagnet in one circuit produces an armature movement that opens or closes contacts. These contacts act as switches in another circuit. In Figure 2–27 when the switch in the low-voltage circuit is closed, current flowing through coil *L* sets up a magnetic field. This magnetic field attracts the soft iron armature *A*, which moves down and makes contact at *C*, closing the switch in the high-voltage circuit. When the switch in the low-voltage circuit is opened, the field collapses and the tension of spring *S* pulls the armature up, breaking the contact at *C* and opening the high-voltage circuit. This type of relay is a single-pole, single-throw relay.

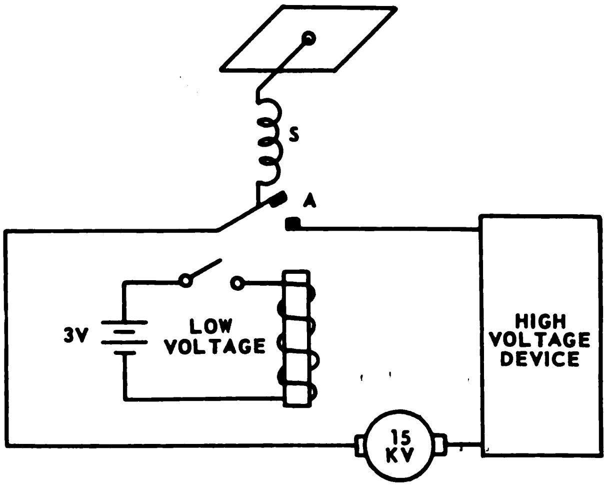


Figure 2–27. Relay Circuit

Hysteresis

When a piece of iron is magnetized, considerable energy is expended in lining up the molecular magnets in the iron in a definite direction. What happens when the magnetizing force is reversed? Figure 2–28 explains this. For a given iron core, the flux density increases along the curve *OA* as the magnetizing force is increased. But when the magnetizing force is decreased back to zero, the flux density does not decrease along the curve *AO* to zero. Instead the flux density decreases more slowly along the curve *AC*.

When the magnetizing force reaches zero, a certain value of flux density remains (*OC*). This value is caused by the residual magnetism. The iron continues to be a magnet even after the magnetizing force is removed. If, at this point, the magnetizing force is reversed, the field intensity is in the opposite direction, but the flux density has the same polarity as before, due to residual magnetism. Increasing the magnetizing force will gradually reduce the flux density (along the curve *CD*) to zero at point *D*. It requires a magnetizing force of *OD* to reduce the flux density to zero. The force *OD* is called the *coercive force.*

As the magnetizing force continues to increase, the iron core becomes magnetized along *DE*, in a direction opposite to its original magnetization. At point *E* the magnetizing force is again decreased to zero. The flux density decreases more slowly along the line *EF*. By the time the magnetizing force reaches zero, the flux density has a value of *OF*, which is equal to *OC*. This is due to the residual magnetism in this direction. As the magnetizing force is again increased in the

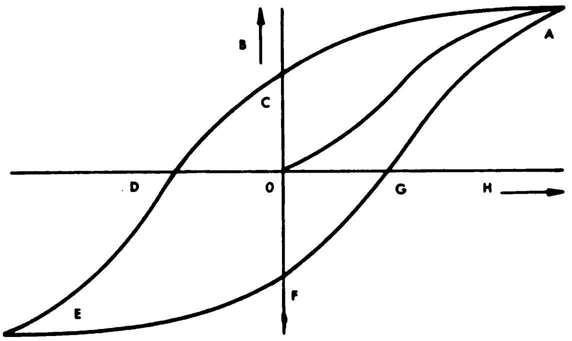


Figure 2–28. Hysteresis Loop

original direction, the flux density decreases along the curve *FG* to zero. If the magnetizing force continues to increase, the flux density will follow the path *GA*.

Notice that the flux density of the iron core does not become zero each time the magnetizing force does. A coercive force is necessary to reduce it to zero. This lagging of the magnetic flux behind the magnetizing force which produces it is *hysteresis*, which is a word meaning "a lagging behind." The entire curve ACDEFGA is known as the hysteresis loop.

When the magnetism of an electromagnet is reversed rapidly, as in an alternating current electromagnet, the iron core becomes heated, and considerable energy is wasted in producing this heat. This loss is called hysteresis loss and is explained as a kind of molecular friction caused by the reversal in the position of the molecular magnets which iron and other magnetic materials are thought to possess.

Experiments have shown that the energy loss due to hysteresis is proportional to the area of the hysteresis loop. For this reason material for the cores of electromagnets and transformers used in alternating current circuits should have a hysteresis loop with a very small area. The distance *DG* (Figure 2–28) must be relatively small. Perminvar, an iron-nickel-cobalt alloy has a hysteresis loop which is nearly a straight line for magnetic fields of less than 3 oersteds.

INDUCTION

Michael Faraday's discovery that a voltage may be developed by changing the direction of the flux or the amount of flux that links with a conductor or coil is considered one of the greatest scientific discoveries. At the time of his discov ery, the crude and inefficient battery invented b Volta in 1801 was the only source of electricit For 11 years Faraday had been working to fin a more efficient source.

On August 31, 1831 Faraday wound two sep arate coils of wire upon an iron ring. The end of one coil were connected to a battery an switch. The ends of the other coil were joine by a long wire which passed over a compa needle. When the switch was closed completin the battery circuit, the compass needle move then oscillated back and forth, and finall stopped in its original position. When the switc was opened, the needle again moved, in the o posite direction, oscillated, and came to res This momentary flow of current was complete unexpected, and Faraday had difficulty explai ing it. Later he showed that magnetic and ele trical actions were reciprocal because *it is nece sary for electricity to move before a magnet effect is produced, and it is necessary for ma netism to move (or change) before an electric effect is produced.*

In this experiment, current flowed in the se ond coil when the magnetism in the iron co changed as a result of the closing or opening the electric circuit of the first coil. This is know as *the principle of electromagnetic inducti* The same results would occur had a permane magnet been moved in and out of a coil of wi Electrons flow through the coil of wire wh there is a relative motion between the magne field of the magnet and the coil; an emf is be induced in the coil.

Faraday's Law

Later experimentation showed that *the m nitude of the generated voltage is directly p portional to the rate at which a conductor c magnetic lines of force.* This statement is kn as Faraday's law. The rate at which a condu cuts the magnetic lines of force depends u the number of turns in the coil, the strengt the magnetic field, and the speed with which coil, or field, is moved. When a wire is m parallel to the magnetic field, it cuts no line force and no emf is induced. Cutting acro field at an angle less than 90° will result lower voltage than if the wire were moved a same speed perpendicularly to the field.

When a conductor moves at a constant speed across a magnetic field of uniform flux density, *1 volt is generated for every 100,000,000 (10^8) lines cut per second.* The amount of voltage generated this way can be put in the form of a formula as

$$e = \frac{\phi}{t \times 10^8} \text{ volts}$$

where e = voltage generated in the conductor
ϕ = total flux cut
t = time, in seconds, during which the cutting takes place.

When the conductor consists of several turns in the form of a coil and when the magnetic field is not uniform, or is not cut at a uniform rate, the formula becomes

$$e_{av} = \frac{-N\Delta\phi}{\Delta t \times 10^8} = \frac{-N\Delta\phi}{\Delta t} \times 10^{-8} \text{ volts}$$

where e_{av} = average induced emf
N = number of turns in the coil
$\Delta\phi$ = change of flux in maxwells during
Δt = time increment in seconds

The negative sign in the equation indicates that the polarity of the induced voltage is in opposition to the effect that is producing it. The Greek letter delta (Δ) is used to indicate a small change.

To illustrate this, take a wire coil wound in the form of a solenoid and place it so that its turns are in a horizontal plane. Across the coil connect a galvanometer to complete the circuit so that there will be current flow when a voltage is induced. When the north end of the magnet is thrust into the center of the coil, a voltage is induced. As shown in Figure 2–29, the current which arises from this voltage will flow in the direction indicated by the arrow labeled *i.* So far as the external circuit is concerned, the polarity of the induced voltage is as shown, the top lead being positive and the bottom lead negative. This causes the top end of the coil to appear as a north pole, which opposes the downward direction of the magnet.

The induced polarity of the coil must be the same as that of the approaching magnetic pole. Otherwise, it would be necessary only to start a magnetic pole into the coil and, upon its inducing an opposite pole in the coil, the magnet would be drawn in, inducing additional emf without work. This would provide an unlimited source of energy without doing any work, a condition which is contrary to the law of the conservation of energy.

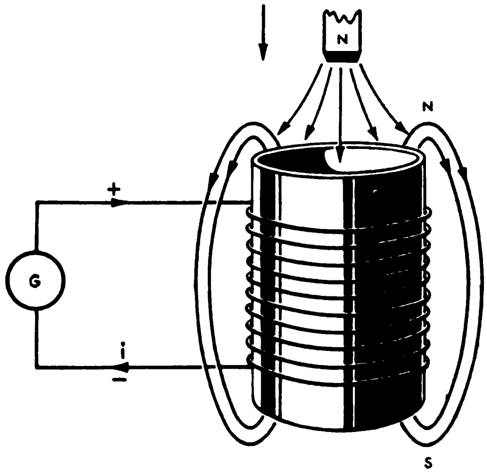


Figure 2–29. Inducing Voltage in Solenoid by Moving Magnet

Next, consider a wire moving downward with its length perpendicular to a magnetic field which runs from the left to the right, as shown in Figure 2–30. Think of the lines as being distorted as though they were rubber bands and were wrapping themselves about the wire. These lines of force, then, would encircle the wire in a counterclockwise direction. Cutting of the lines of force induces an emf along the wire so that, if current could flow in the example of Figure 2–30, it would flow into the page.

A simple rule determines the *direction of electron flow* through the wire. Grasp the wire with the left hand with the fingers pointing in the direction the distorted lines of force go around the wire; the thumb, extended along the wire, points in the direction of the electron flow. With

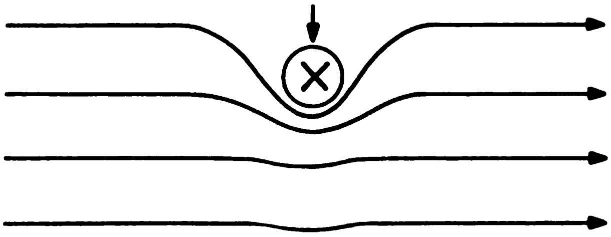


Figure 2–30. Conductor Cutting a Magnetic Field

the current flowing into the page in the conductor of Figure 2–30, the magnetic field about the wire due to the electron flow would be counterclockwise. These lines of force would be aiding the magnetic field below the wire and canceling above the wire, tending to force the wire upwards against the motion which is producing the emf.

Lenz's Law

Heinrich Lenz in 1834 summed up induction in the form of a law which is used to predict the direction of induced voltages and currents. Lenz's law says: *The electrodynamic action of an induced current opposes equally the mechanical action inducing it.* That is, the current set up by an induced voltage always tends to create flux in such a direction as to oppose the effect which produced the induced voltage. This law is based on the law of the conservation of energy. It took mechanical energy to push the magnet into the coil in opposition to the field developed by the current in the coil; it also would take energy to pull the magnet out of the coil against the field developed by the current induced as the magnet is pulled forth.

Mutual Induction

In an electrical circuit a change of current is always accompanied by a change in the magnetic field surrounding the circuit. If the current is increasing, the field is said to be *expanding*, that is, its intensity at any particular point is increasing. On the other hand, if the current is decreasing, the field is said to be *collapsing*, or decreasing in intensity. When a conductor is placed within the magnetic field of a circuit i which the expanding or collapsing lines of forc cut the conductor, a voltage will be induced in i

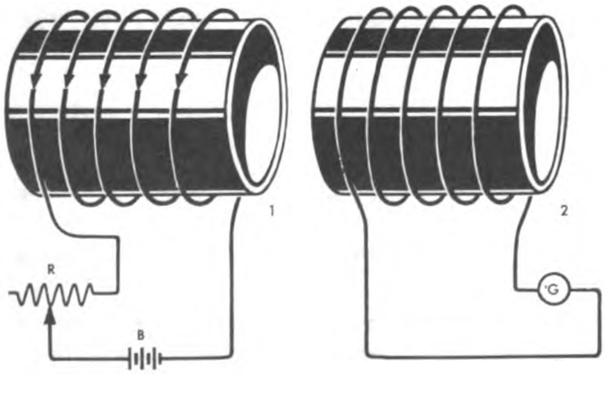


Figure 2–31. Mutual Inductance

In this connection, consider coils 1 and 2 c Figure 2–31, placed side by side. Assume tha battery B is connected to coil 1, and that ga vanometer G (an instrument for indicating th amount and direction of current flow) is cor nected to coil 2. As long as the current flowin in coil 1 is steady, the magnetic field (ϕ_1) likewis will be steady, and there will be no voltage ir duced in coil 2. However, if you change variab resistance R, there will be a change in the currer in coil 1. As a result, the flux through coil will change, and the galvanometer will show momentary deflection. This deflection is due the current resulting from the voltage induce in coil 2. The action in which a change in cu rent in one coil induces a voltage in another cc is called *mutual induction.*

If, in a short period of time, the current in cc 1 in Figure 2–31 changes and causes a chan in the flux linking coil 2, the voltage induced coil 2 is an emf of mutual induction. Its avera value is

$$e_{av} = N_2 \frac{\Delta\phi_2}{\Delta t} 10^{-8} \text{ volts}$$

where N_2 is the number of turns in coil 2, $\Delta\phi_2$ the char in flux from coil 1 which links coil 2, and Δt the ti in seconds required to change the flux by $\Delta\phi_2$ lines.

Even though coils 1 and 2 are close togeth not all of the flux produced by coil 1 (ϕ_1) lin coil 2. The proportion that does link coil 2 is dicated by the symbol k, called the *coefficie of coupling*, which is less than unity. The equ tion for the induced voltage can then be writ

$$e_{av} = N_2 \frac{k\Delta\phi_1}{\Delta t} 10^{-8} \text{ volts.}$$

The flux in coil 1 may be assumed to be p portional to the current flow (i_1) in that c The change in flux, then, is proportional to change in current (Δi_1). Since k and N_2 constants for any given set of coils, the indu voltage in coil 2 is equal to

$$e_{av} = M \frac{\Delta i_1}{\Delta t} \text{ volts}$$

where M is the *mutual inductance*, or coeffici of mutual inductance between coil 1 and coi M depends on the magnetic coupling between two coils, and is measured in henrys. *The he* is *the inductance in a circuit in which an emf of one volt is induced by a current changing at the rate of one ampere per second.* All of the preceding equations may be written with a minus sign to indicate that the induced voltage opposes the force that created it.

The formula indicates that the more rapid the change of current in coil 1, the greater the induced voltage. In tests, it can be shown that if coils 1 and 2 are interchanged, the voltage induced in coil 1 by a change of current in coil 2 is equal to,

$$e_1 = M \frac{\Delta i_2}{\Delta t} \text{ volts.}$$

When a soft iron core is inserted between two coils, the mutual inductance between them is greatly increased. Inductance is proportional to the number of lines of force, and since soft iron can conduct many lines of force, the inductance of the coil is greatly increased. However, the inductance can only be increased to an amount represented by the permeability of the soft iron. Since the permeability of iron depends on the field which, in turn, depends on the current in the coil, the inductance of an iron core coil is not constant over large changes of current.

The direction of the current resulting from the voltage induced in a coil can be determined by Lenz's law. Refer back to Figure 2–31 for a moment. An increase in the current in coil 1 is equivalent to moving it toward coil 2 and would induce a voltage causing the current in coil 2 to be opposite to the current in coil 1. Decreasing the current in coil 1 will cause current in coil 2 to be in the same direction as in coil 1.

Self-Inductance

You have just seen that a change in the current flowing in one coil induces a voltage in a second coil nearby. When the current flowing in the coil changed, the flux through that coil also changed. When the flux linking a coil changes, a voltage is induced in that coil. You have seen that the voltage induced by a change in the flux linking a coil is

$$e_{av} = -N \frac{\Delta\phi}{\Delta t} 10^{-8} \text{ volts.}$$

By definition, *self-inductance*, or just inductance, of a circuit with constant magnetic reluctance is the flux linkages per ampere of current producing the flux. This can be written as

$$L = \frac{N\phi}{I} \text{ henrys}$$

L is the symbol for inductance, and ϕ is measured in maxwells. The product $N\phi$ is referred to as flux linkages, since the flux loops ϕ are linked by *N* turns.

If the flux and the current are changing, ϕ/I is the rate of change of flux with respect to current. If small changes of flux and current are taken ($\Delta\phi$ *and* Δi), the equation for *L* may be written

$$L = \frac{N\Delta\phi\ 10^{-8}}{\Delta i} \text{ or } \Delta iL = N\Delta\phi\ 10^{-8}$$

Substituting for $N\Delta\phi 10^{-8}$ in the equation for the voltage

$$e = -L \frac{\Delta\phi}{\Delta i} \text{ volts.}$$

The minus sign in the formula indicates that the induced voltage, which is called the *counter emf* or back emf, is of such a polarity as to oppose the change of current that caused it.

The inductance of a coil of many turns is much greater than the inductance of a coil of a single turn, since the induced voltage is dependent not only on the change in flux, but also on the number of turns through which the flux passes. Inserting an iron core in a coil greatly increases the inductance. However, the increase is not constant over a wide range of current flows.

When two coils are placed close together, the relation between the mutual inductance, *M*, of two coils and their individual inductances, L_1 and L_2, is equal to

$$M = k\sqrt{L_1 \cdot L_2}$$

GENERATORS

Whenever a conductor moves in a magnetic field in such a way as to cut across lines of force, a voltage is induced in the conductor. The direction of the current arising from this voltage depends upon the direction of the magnetic field and the direction of the movement of the conductor. If the conductor moves first in one direction and then in the opposite, the induced voltage will be alternately of one polarity and then of the opposite. The current will flow first one way and then the other. If the direction of

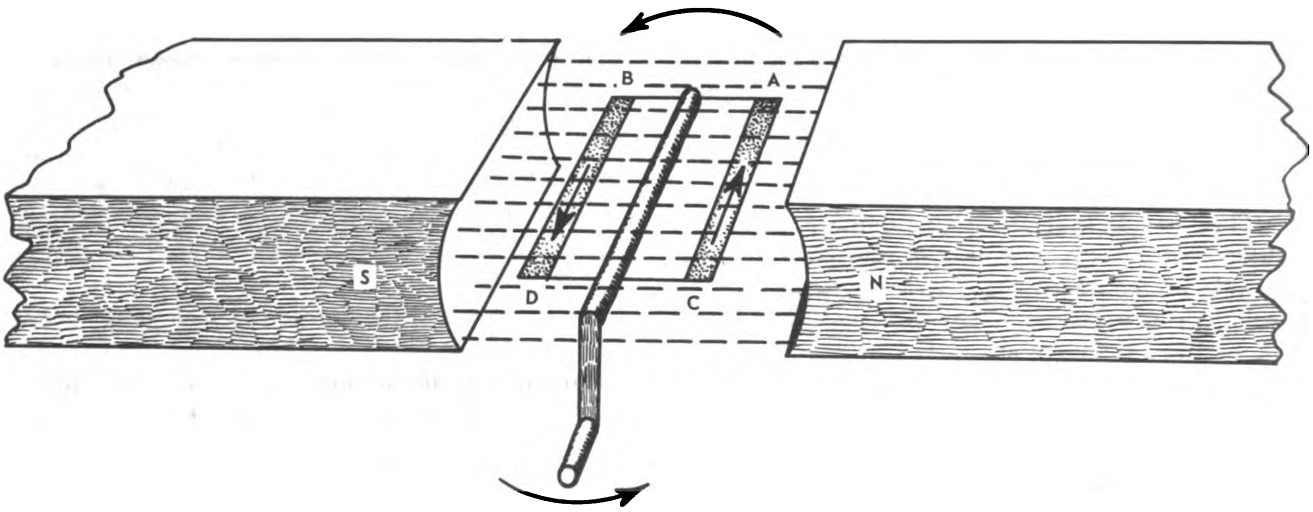


Figure 2–32. Elementary Generator

the motion of the conductor continually reverses, the current will reverse direction periodically. A current which periodically reverses direction is called alternating current (AC). On the other hand, a current which flows continuously in the same direction is called direct current (DC).

If mechanical force is used to cause a conductor to cut magnetic lines of force, an electromotive force will appear across the conductor. This type of machine is known as a generator. A generator which produces an alternating voltage and current is called an AC generator. One which produces direct voltage and current is known as a DC generator. Essentially, the two types are alike, their principal difference being in the method in which the generated energy is taken from the generator.

Simple Alternating Current Generator

The elementary generator of Figure 2–32 consists of a conductor formed into the loop BACD located in the magnetic field between the poles of a magnet. The conductor is rotating in the direction shown. As side BD moves down across the lines of force, a voltage which would cause current to flow from B to D is induced in the

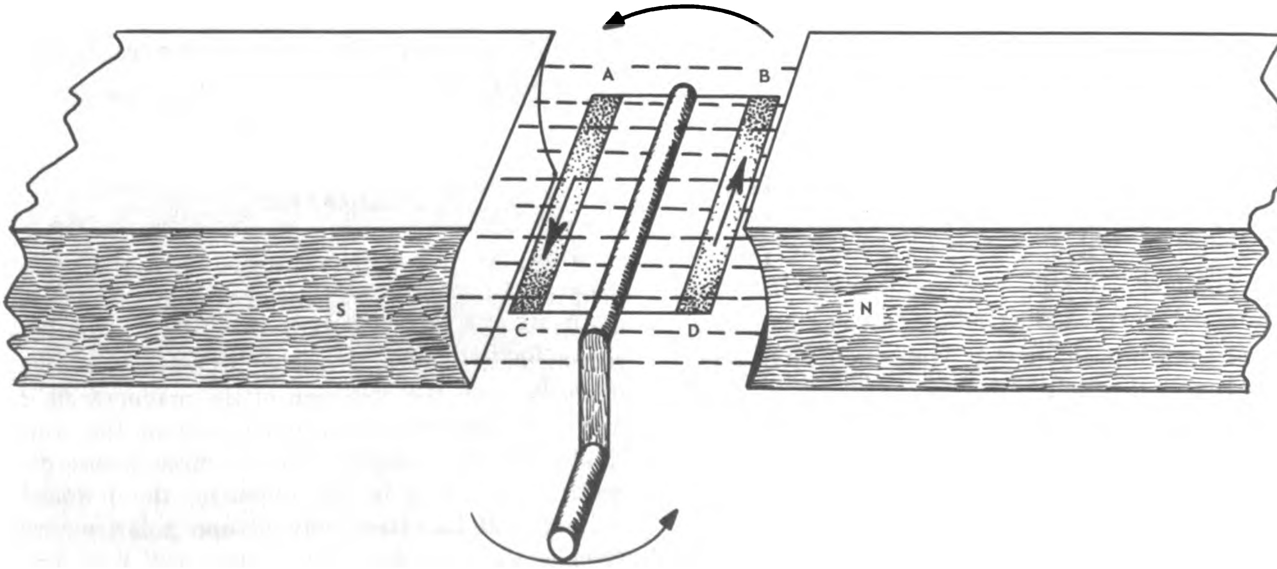


Figure 2–33. Elementary Generator

conductor. At the same time, the opposite side of the loop is cutting the same lines of force in the opposite direction. The induced voltage in side AC will cause current to flow from C to A. Thus a current will flow around in the loop in the direction BDCA as marked in Figure 2–32.

When the loop reaches a vertical position, the sides of the loop no longer are cutting the lines of force; they are moving parallel to them, and no voltage is induced into the loop. In Figure 2–33 the loop position is the reverse of that of Figure 2–32. Side AC is cutting downward and side BD is cutting upward. The voltage induced in side AC will cause current flow from A to C, and the voltage induced in side BD will cause current flow from D to B. Current flow in the loop at this time, then, is in the direction ACDB, the reverse of the current flow when the loop was in the position shown in Figure 2–32. During one half of the revolution, the current in a closed loop rotating in a magnetic field flows in one direction; during the other half of the revolution, current flows in the reverse direction.

In order to make use of the voltage induced in the rotating coil, the two ends of the coil are connected to two slip rings. Each ring is continuous and is insulated from the shaft and the other ring. Two metal or carbon brushes (X and Y in Figure 2–34) which are connected to the load (resistor R) rest on the rings. As the coil is rotated, the brushes make sliding contact with the slip rings, completing the circuit at all times.

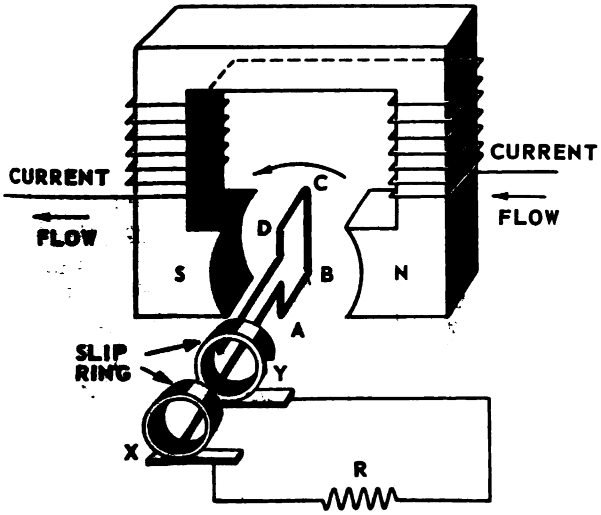


Figure 2–34. Elementary AC Generator

The amplitude of the induced emf depends upon the number of magnetic lines of force cut per unit time. The number of lines cut depends upon the strength of the magnetic field and the rate at which the conductor is moved through the field. In Figure 2–34, if the strength of the electromagnet is increased, the speed of rotation being unchanged, the induced voltage will increase. Similarly, if the field strength is unchanged but the coil is rotated at a higher speed, more lines will be cut per unit time, and a greater voltage will be induced. Another way of increasing the number of lines cut is to increase the number of conductors.

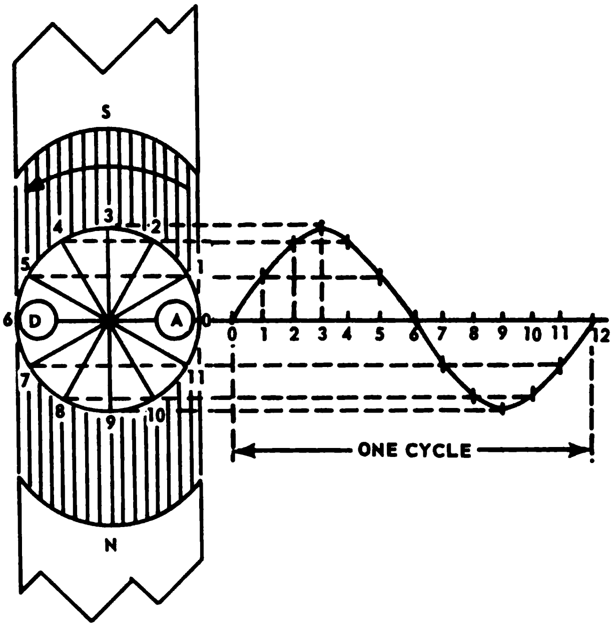


Figure 2–35. Graph of Induced EMF

To investigate how the emf varies from moment to moment, consider the cross section of coil shown in Figure 2–35. *A* and *D* are the ends of the two conductors facing you. The coil is rotating at a constant linear velocity *V* in a uniform magnetic field. As has been explained, when conductor *A* is in position 0, the voltage induced in the coil is zero.

As the coil rotates, *A* moves toward position 1 and a voltage is induced in the coil in such a direction that current will flow out at *A*. We will call this direction positive. The linear velocity (*V*), which is no longer parallel to the lines of force, can be resolved into two components, one of which is parallel to the lines of force and the other perpendicular to the lines of force. The component parallel to the lines of force cannot induce an emf, but the component perpendicular to the lines does produce an emf. To resolve the linear velocity, represent it as the vector $V\angle\alpha$. Then the horizontal component (which is perpendicular to the lines of force) is

$$V_h = V \sin \alpha$$

When *A* is at position 3, the angle is 90°. The induced voltage, as previously shown, is maximum (E_{max}).

$$E_{max} = V \sin 90^\circ = V(1) = V$$

Substituting in the general equation, the instantaneous voltage induced is

$$e = E_{max} \sin \alpha$$

One complete rotation of the coil is 360° or 2 π radians. If the circle described by conductor *A* in Figure 2–35 and the time base are divided into 12 equal parts, then the instantaneous voltages induced at each position of the coil can be plotted against time. Knowing the value of α (30° at point 1, 60° at point 2, etc.), you can find and plot the induced voltages for each of the 12 times indicated along the time base, giving the sine wave shown in Figure 2–35.

If *t* is the time it takes the coil to rotate through the angle α, then the angular velocity ω of the coil is equal to the angular distance divided by time, or

$$\omega = \alpha/t$$

which can be solved for

$$\alpha = \omega t$$

The equation for the induced voltage can now be written

$$e = E_{max} \sin \omega t$$

Direct Current Generator

Since the emf induced in the rotating coil is alternating, slip rings cannot be used to conduct current from the coil to the external load in a DC generator. Instead a single, split ring (or *commutator*) is substituted for the two rings. Each half of the ring is connected to one end of the generator coil. At the instant the induced voltage in the loop of Figure 2–36A reaches zero, the brushes cross the split in the ring. Induced voltage in the coil at the instant shown in Figure 2–36A is such that current flow is from side C1 to ring S1, where it is picked up by brush B1. Current flows through the load to brush B2, which is in contact with ring S2, and into side C2 of the coil.

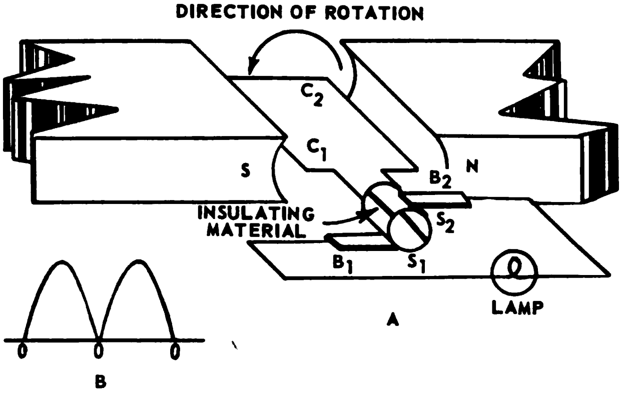


Figure 2–36. Elementary DC Generator

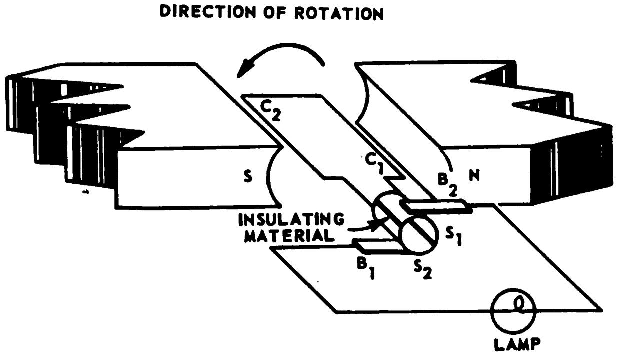
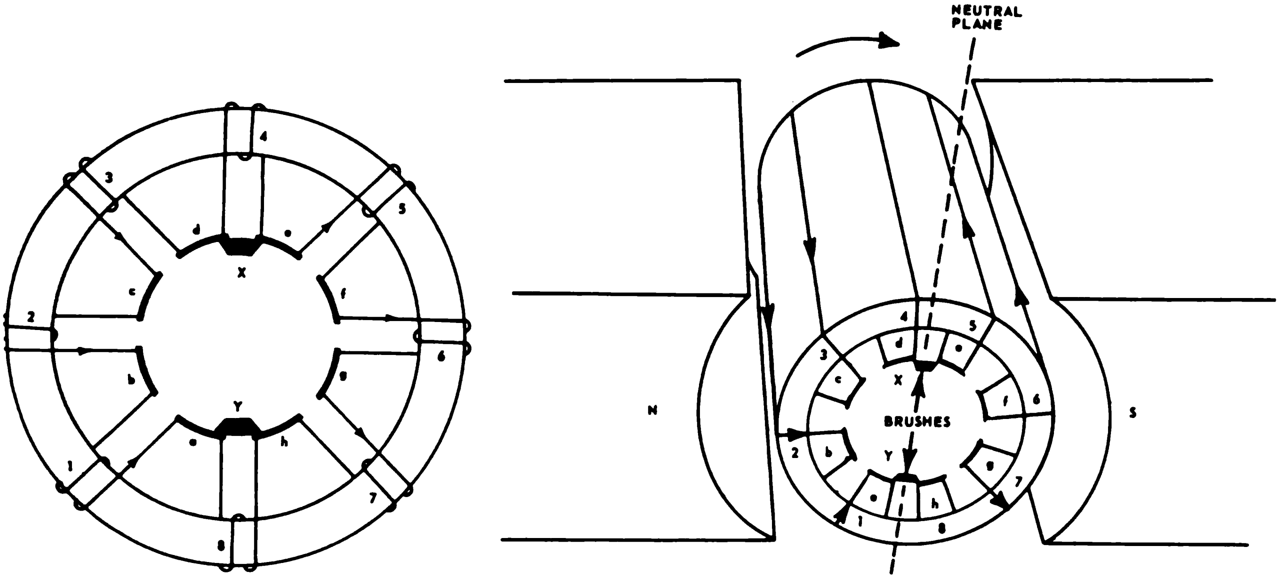


Figure 2–37. Elementary DC Generator

When the coil reaches the position shown in Figure 2–37, side C2 is cutting down through the magnetic field. The induced voltage now causes current to flow from side C2 of the coil to S2. But brush B1 is now in contact with S2, and current continues to flow through the load from B1 to B2 and back through S1 to side C1 of the coil. Although the induced emf in the coil reversed direction, the brushes were in contact with the opposite ends of the coil, and current in the external circuit continued to flow in the same direction.

At points 0 and 6 in Figure 2–35 no voltage is induced in the coil. These are the points marked 0–0–0 in Figure 2–36B. A comparison of the waveshapes of Figure 2–35 and Figure 2–36B



A COMMUTATOR CONNECTIONS

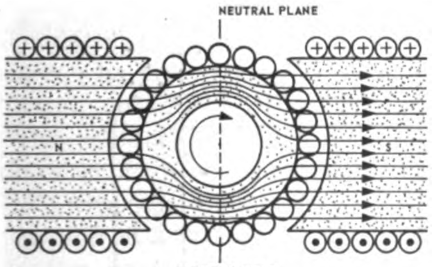
B NEUTRAL PLANE

Figure 2–38. DC Generator with Eight Coils

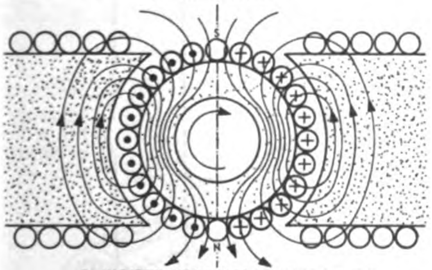
shows that the negative half of the sine wave has been reversed and made positive. Notice that in Figure 2–36A and Figure 2–37 brush B1 is always negative and brush B2 is always positive. The current flow in the external circuit varies from zero to a maximum, but it is always in the same direction. This is known as *pulsating DC.*

A generator using a one-turn loop for the armature produces only a small voltage output. To increase the output and at the same time make it steady instead of pulsating, several coils are equally spaced and connected in series around the armature. The armature is a cylindrical core of iron *laminations* (thin sheets of the metal). The commutator is divided into sections called commutator *segments.* The number of commutator segments is equal to the number of coils. Each coil is connected to two commutator segments, as shown in Figure 2–38A. One end of coil 1 goes to segment *a*; the other end, to segment *b*. One end of coil 2 goes to segment *b*; the other end, to segment *c*, and so on around the armature with the other end of coil 8 connected to segment *a*. The two brushes are located so that the coil they short out has no voltage induced in it at that instant. The plane of the brushes is perpendicular to the magnetic lines of force and is called the *neutral plane.*

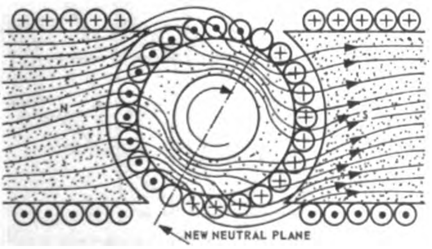
The magnetic field due to the electromagnet is shown in Figure 2–39A. The magnetic field due



A MAGNETIC FIELD



B MAGNETIC FIELD DUE TO ARMATURE CURRENT



C RESULTANT MAGNETIC FIELD

Figure 2–39. Effect of Armature Current on Generator Field

to the current flow in the armature winding is downward through the armature and perpendicular to the magnet's field, as shown in Figure 2–39B. The resultant field is displaced, due to the armature current, in the direction of rotation of the armature. The neutral plane, instead of being perpendicular to the axis of the magnets, is at an angle, as indicated by the dashed line in Figure 2–38B and Figure 2–39C. No voltage is induced in the conductors in the neutral plane since they are moving parallel to the lines of force. At the instant shown no voltage is being induced in coils 4 and 8.

Coils 1, 2, and 3 in Figure 2–38B are moving up through the magnetic field. The induced voltages in these conductors cause current to flow from segment *d* to *c* to *b* to *a*. Since these all tend to produce current flow in the same direction, the voltages are additive, and the amplitude of the resulting voltage is increased. At the same time coils 5, 6, and 7 are cutting down through the magnetic field. The induced voltage in these conductors causes current to flow from commutator segment *e* to *f* to *g* to *h*.

As the armature rotates, different commutator segments pass under the brushes, but at all times current flows toward brush Y and away from brush X. The net result is that there are two parallel paths between the positive brush and the negative brush at all times. The voltage between the positive brush and the negative brush is the sum of the induced voltages in the coils which are in series between the brushes. Assume that the voltage induced in each coil is a sine wave. Then the output voltage is a rectified sine wave (Figure 2–36B). The emf of each coil is plotted separately in Figure 2–40. The voltages do not reach zero nor maximum at the same time because they are spaced differently.

Since the voltage between the brushes is the sum of the individual voltages in the coils, the total output is that shown by the waveshape at

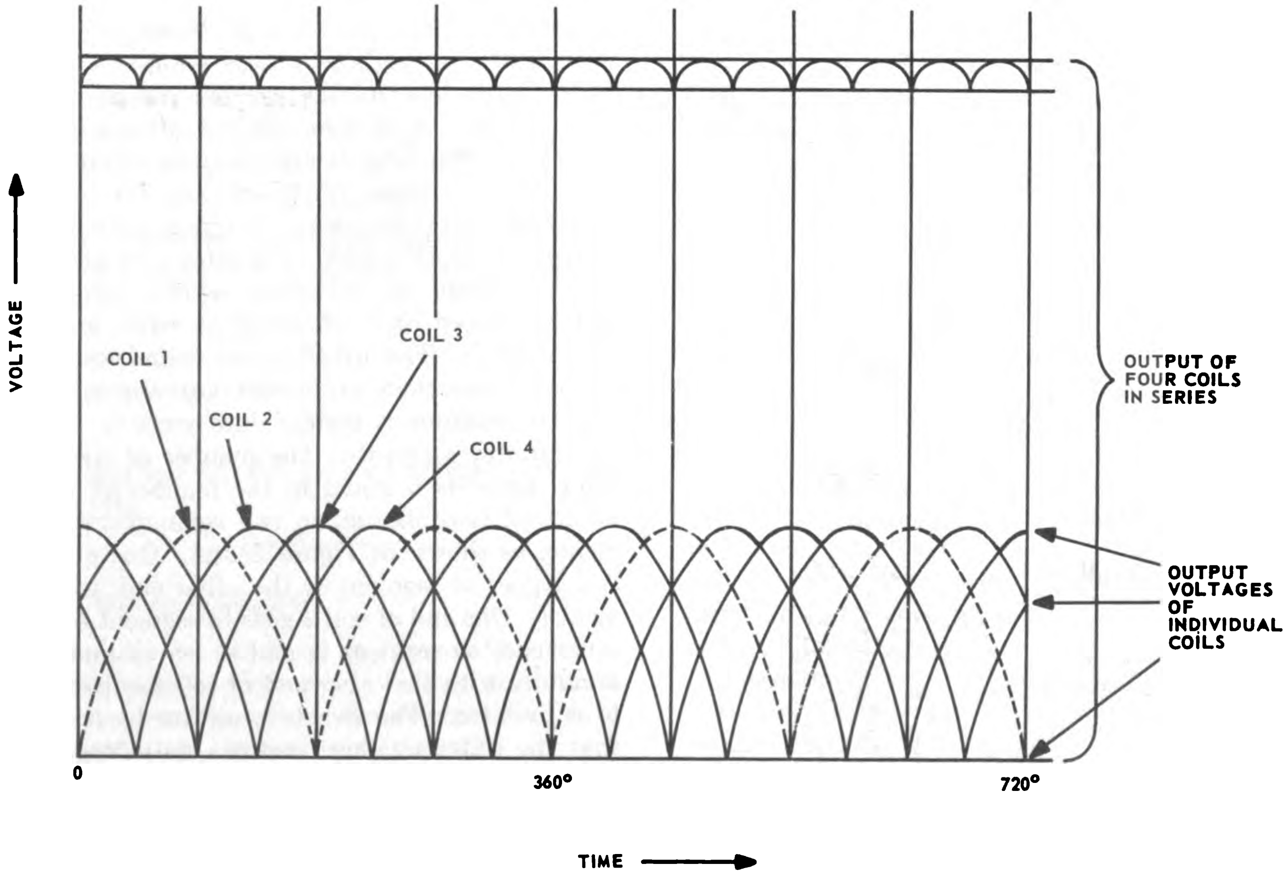


Figure 2–40. Effect of Coils in Series on Output Voltage

the top of the illustration. The dashed wave is the output of coil 1. If the armature is rotated at the rate of 2 revolutions per second, there will be four alternations per second. This variation in the output of a DC generator is called *ripple.* The frequency of the ripple depends upon the number of coils, the number of poles, and the speed of rotation. The greater the number of coils per pole, the smaller the amplitude of the ripple voltage. The output of all four coils (1, 2, 3, and 4) in series has a ripple frequency of 16 (four times the ripple frequency for one coil alone). This is much closer to a steady DC.

In the simple generator shown in Figure 2–38A, the armature is the ring type. This type is no longer used because the part of the windings on the inside of the ring do not cut flux and consequently have no emf induced in them. In the drum type winding, the conductors are all placed in slots in the surface of the armature and are connected to each other by front and back connectors, or coil ends (*ad* and *bc* in Figure 2–41 are coil ends). The advantages of the drum winding are that all of the armature copper (except the coil ends) is active in generating an emf, and the coils can be preformed and insulated before being placed on the armature, unlike the ring type. The conductors are connected so that when one side of the coil is under the center of a north pole, the other side is under a south pole. The operation of a drum-wound armature is essentially the same as that of a ring-wound armature.

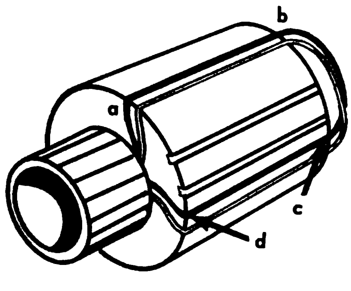
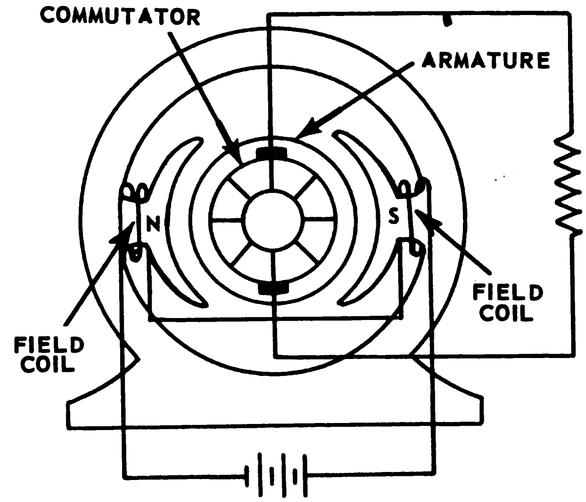
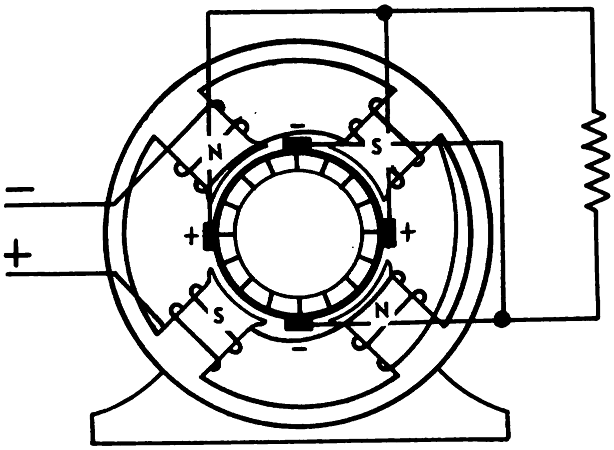


Figure 2–41. Two Coils in Place on a Four-Pole Drum-Wound Armature



A TWO-POLE GENERATOR



B FOUR-POLE GENERATOR

Figure 2–42. Generators

Modern generators do not use permanent magnets to produce the magnetic field. Since electromagnets can be designed which are much stronger than any permanent magnets, these are used to provide the strong magnetic field in which the armature rotates (Figure 2–42). The coils of the electromagnet are called field coils since the current flowing through them produces the magnetic field. The field coils are wound on a soft iron core, which is laminated to prevent eddy currents. Figure 2–43 shows a four-pole field structure with the resultant magnetic field. By varying the voltage applied to the field coils, it is possible to vary the strength of the magnetic field and thereby control the output of the generator.

Usually, the number of brushes in a generator is equal to the number of poles. Each brush is located in the neutral plane between each pair of poles. The positive brushes are connected together to form the positive output, and the negative brushes are connected together to form the negative output. This arrangement in a four-pole machine provides four parallel paths through which the armature current can flow. This makes the internal resistance of the generator low. The voltage generated is the voltage of any one of the parallel paths, and the current capacity is equal to the sum of the currents in the four paths.

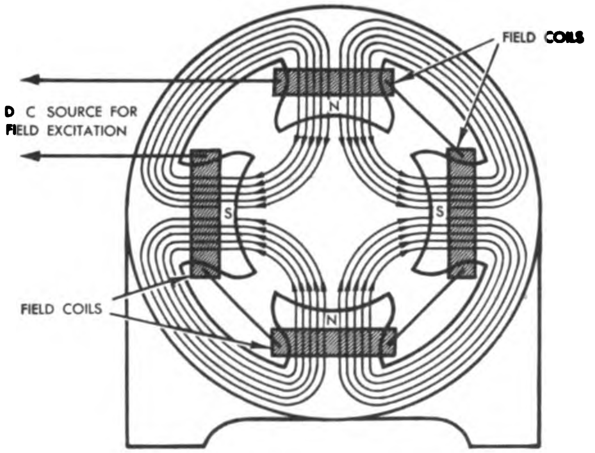


Figure 2–43. Field Structure of Four-Pole Generator

The voltage of a generator can be computed by multiplying the average voltage induced in a single conductor by the number of conductors in any one of the parallel paths. If the armature is driven at a rate of *N* revolutions per second and there are *P* poles with a flux of ϕ per pole, each conductor cuts ϕPN lines per second, and the voltage induced in it is $\phi PN \times 10^{-8}$ volts. If *z* is the total number of conductors and there are *p* paths, the number of conductors per path is *z/p*. Thus the average output of the generator in volts is

$$E = \frac{\phi PNZ}{p \times 10^{8}} \text{ volts}$$

In any given generator the quantities *P*, *Z* and *p* are fixed by the manufacturer. This leaves only ϕ and *N* as the variable quantities by which it is possible to control the output voltage. The usual practice is to keep *N* constant by driving the generator with a fixed-speed engine and to control the output voltage by controlling the flux. Usually the flux is controlled by controlling the current in the field coils of the generator.

Types of Generators

The field current of a generator can be produced by an external source (separately excited generator) or by the generator itself. When ar external source of voltage is used to excite th generator, it is usually another DC generato that is connected to the field windings. The fielc is constant whether the armature turns or not. A generator which supplies its own field current i called a *self-excited* generator. Self excitation i possible if the core in a generator has some residual magnetism when the armature is not rotating. As the armature is rotated, a small emf i induced which causes current to flow throug the field, building it up.

Self-excited generators are classed accordin to the way in which the field is connected. Ther are three general types — *shunt*, *series*, and *compound*. In the shunt type the field coils are connected across the armature terminals, usually i series with a rheostat (Figure 2–44A). The shun field has comparatively high resistance so that won't draw too much of the generator curren The series generator has a winding of a few tur of low resistance connected in series with tl armature and the load (Figure 2–44B). Tl compound generator (Figure 2–44C) is similar the shunt but also has a winding connected eith in series with the armature or in series with tl load. Compound generators are further classifi as *cumulative compound* or *differential compound*, depending on whether the series field ai or opposes the shunt field.

Shunt Generator

The shunt field is connected across the armture terminals so that the generator output also the field voltage. At the start the induc voltage in the armature is zero. Due to the resual magnetism, there is a small induced emf the generator comes up to speed. This volta since it is applied to the field winding, causes current to flow. This current causes the manetic field to increase which, in turn, increases t induced voltage in the armature. An increase the induced voltage, in turn, produces a higl value of field current. Each increase in field crent produces an increase in induced emf. T increase in voltage, in turn, increases the fi current. The voltage continues to build up u the saturation point is reached. The genera

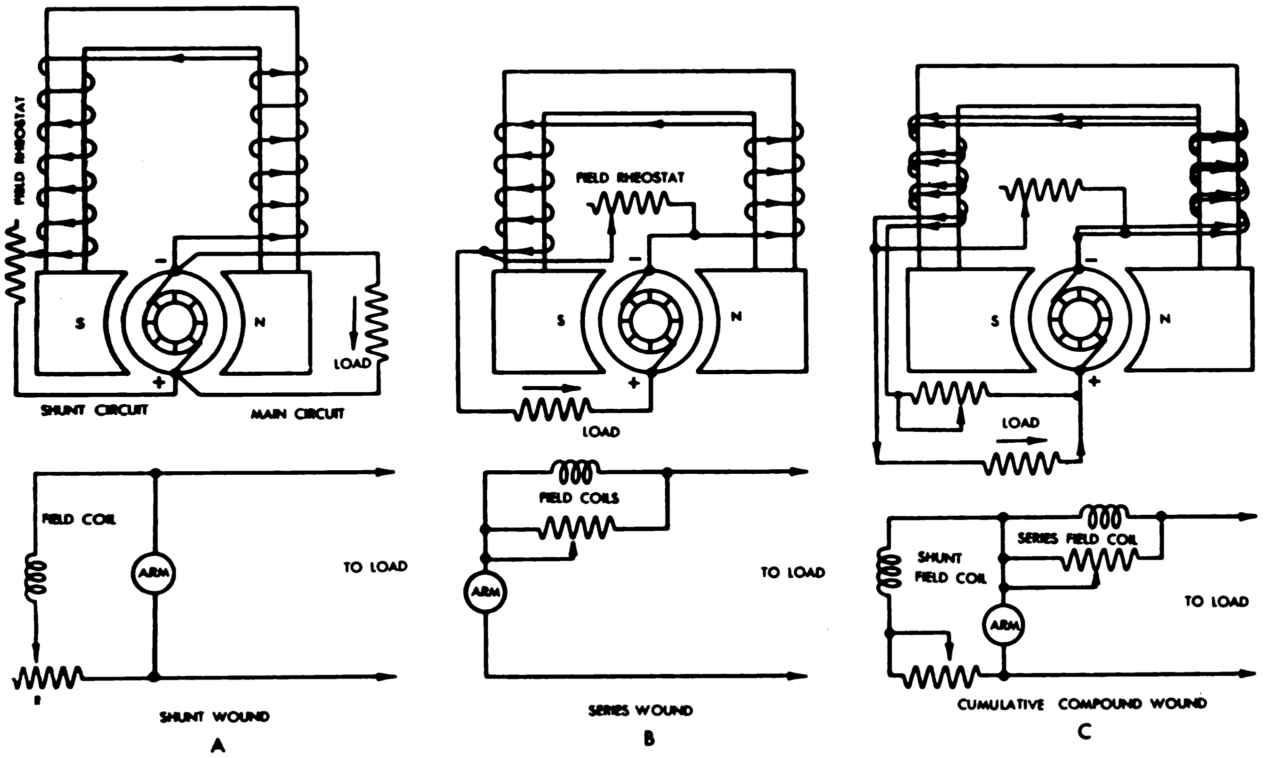


Figure 2–44. Types of Generators

voltage cannot increase beyond this point because here the field current produces an induced emf which is exactly the amount needed to cause that field current to flow.

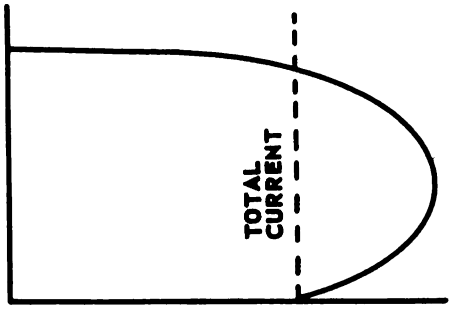
The terminal voltage of a shunt generator drops if, after building up to voltage, the generator is loaded (current is drawn). The drop in voltage increases with an increase in load (current). This is shown graphically in Figure 2–45A. There are three reasons for this voltage decrease. First, the increased load current flows through the armature resistance producing a larger IR drop. The terminal voltage is equal to the induced voltage less the IR drop across the armature. Second, the increased armature current produces a greater armature reaction (Figure 2–39). There is a component of the magnetic field due to the armature current that opposes the magnetism of the field coils. This weakened field reduces the induced emf. Third, the first and second causes result in a decreased field current which, in turn, results in a decrease in the induced emf.

When a generator is operating at or near rated voltage, a large decrease in terminal voltage results in a small decrease in flux, due to saturation and hysteresis. Therefore, loading a generator does not result in a cumulative decrease in output to zero volts.

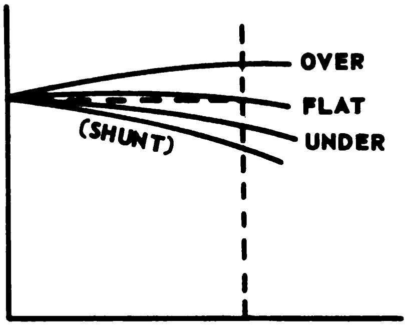
Compound Generators

Because the terminal voltage drops with load, a shunt generator is undesirable where there is a requirement for constant voltage. To provide a substantially constant output voltage, a few turns of wire connected in series with either the armature or the load are placed on the field core. In a cumulative compound generator these turns aid the shunt turns. As the load increases, the current through the series turns increases, increasing the field. This increase in flux increases the induced emf.

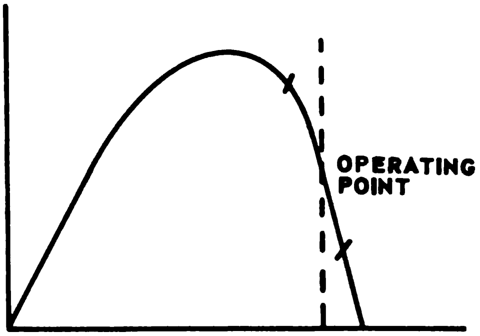
The ampere-turns of the series field can be adjusted so that the increase in induced emf balances the drop in voltage due to armature reaction and IR drop across the armature resistance (the first two causes of a decrease in terminal voltage of a shunt generator under load). Since the series field compensates for the first two causes of a decrease in voltage, there will be no drop in field current. In fact, the series turns can



A SHUNT



B COMPOUND



C SERIES

Figure 2-45. Load Characteristics of DC Generators

be adjusted to produce an increase in terminal voltage with an increase in load.

A generator is *flat-compounded* if the series field is such that the voltage is the same at rated load as at no load. The terminal voltage rises slightly from no load and then drops to the same voltage at rated load (Figure 2–45B). This curve is due to the saturation of the iron. If the terminal voltage is greater at rated load than at no load, the generator is *overcompounded*. This is desirable where the load is at a distance from the generator, in order to balance out the voltage drop in the line from the generator to the load. If the generator voltage rises enough to take care of this voltage loss, the voltage at the load remains constant. When the voltage at rated load is less than the voltage at no load, the generator is *undercompounded*. Most compound generators are wound overcompounded. Then the amount of compounding is regulated by shunting more or less current around the series field.

In a differential compound generator the two fields, due to the shunt and series windings, oppose each other. For any increase in load there is an increase in the strength of the series field. Because its direction is opposite to that of the shunt field, the total field strength is decreased. As a result, the output voltage decreases as the load increases. This type of generator is used in welding, where the output terminals of the generator are sometimes short-circuited.

Series Generator

In the series generator the field coils are connected in series with the armature and the external circuit. Of necessity, this type of machine uses very low resistance field coils consisting of a few turns of large wire. Generally the series generator is used for constant-current work, whereas the shunt generator is used for constant-potential work.

For the voltage to build up on a series generator, two things are necessary. The series field must be connected so that it will aid the residual magnetism, and the external resistance must be less than a certain critical value. Too high a resistance means that the field current will not be enough to develop sufficient magnetism. As more and more current is drawn by the load, the field strength and the induced voltage both increase. At some point, the terminal voltage becomes maximum. Beyond this point the terminal voltage drops rapidly to zero, because the armature reaction is so great.

Armature reaction is the interaction between the magnetic flux of the armature and that of the field in the generator. It causes a redistribution of flux (Figure 2–39) that must be taken into account when designing the generator. A component of the magnetic field due to the current through the armature opposes the field flux and, in the case of a series generator, becomes great enough to cancel most of it. Series generators are purposely designed to have very high armature reaction.

Figure 2–45C shows the characteristics of the series generator. The series generator is operated along portion *ab* of its characteristic. The current is practically constant and is not affected by a large change in the external resistance. The series field is shunted by a rheostat that is adjusted to maintain a constant current output from the generator as the output varies.

Effect of Variable Speed

In testing generators, it is assumed that the generator is operated constantly at its rated speed. If the speed of the prime mover, or driving motor, drops, the decrease in voltage is not a characteristic of the generator but is due to the change in speed of the prime mover. Practically, it is difficult to avoid a drop in the speed of the prime mover as the generator is loaded. In regulating the generator output, this voltage drop due to decreased speed must be taken into consideration.

Alternators

As previously mentioned, the principal difference between an AC generator and a DC generator is the method of connection to the external circuit; that is, the AC generator is connected to the external circuit by slip rings, while the DC generator is connected by a commutator. Another difference is that, since the output current of an AC generator sets up a magnetic field which reverses several times per second, it is not possible to use the generator output to excite the field coils. Therefore, an AC generator is separately excited, usually with a smaller DC generator that is located on, or geared to, the same shaft as the AC generator.

As pointed out, the induced emf in the armature of a generator is alternating and has a sinusoidal waveshape (Figure 2–35). When a conductor has passed one north and one south pole, one sine wave of voltage has been induced in it. In a two-pole alternating current generator, or alternator, when the conductor has completed one revolution of 360°, or 2π radians, one sine wave has been produced. Let s be the speed in revolutions per second, and f be the frequency of the emf wave in cycles per second. Since a sine wave is induced each time the conductor completes one revolution, f equals s for this alternator. At the end of t seconds the conductor will have completed st revolutions and produced ft sine waves, or cycles. The angle covered will be $2\pi st$ or $2\pi ft$ radians.

At constant speed or frequency $2\pi f$ is constant; therefore AC waves may be plotted against time as well as degrees, or radians. ω is the angular velocity in radians per second. Then $\omega = 2\pi f$, since $2\pi f$ is the number of radians per second through which the wave goes.

If the alternator has more than one pair of poles, a sine wave is induced in a conductor every time it passes a north and a south pole. That is, a cycle is completed every time a conductor passes one *pair* of poles. The frequency, then, is equal to the number of pairs of poles passed per second,

or $$f = \frac{P}{2} s,$$

where P is the number of poles and s, the revolutions per second.

Example

If the speed of an 8-pole alternator is 6000 rpm, what is the frequency of the output?

s is 6000/60, or 100 rps.

$$f = \frac{8}{2} 100 = 400 \quad \text{cycles per second.}$$

Since it is the *relative* motion between conductor and magnetic field that produces the emf in the armature, most alternators are designed with rotating field and stationary armature. In DC machines this is not practical. If the armature were stationary, the brushes would need to revolve with the field, due to the commutator. Since alternators use slip rings, it is not necessary that the armature rotate.

There are two important advantages to the stationary armature. The rotating armature requires two or more slip rings for conducting the current from the armature to the external circuit. They are difficult to insulate for the high voltages at which alternators are usually operated. Arc-overs and short circuits are frequent causes of troubles at the slip rings. With a stationary armature, the armature leads are continuously insulated conductors from the coils to the load. Also, it is easier to insulate the conductors in the armature if it is stationary.

Since the field voltage is rarely over 250 volts and little power is used, slip rings conducting current to the field cause little trouble.

There is a problem to get all the copper and insulation on a high-speed, high-voltage armature of small diameter. It is impossible to deepen the armature slots very much, and material cannot be cut away from the teeth without weakening the structure. But when the armature is stationary, deepening the slots does not cut away material from the teeth between the slots and there is plenty of space for the increased insulation and copper needed for the higher voltage.

With the stationary armature, alternators can be operated at high speeds and with a high output voltage very economically. Therefore, most generated electricity is alternating. Where DC is needed, it is generally cheaper to generate AC and then rectify it at the point where the DC is needed.

Polyphase Generators

Since the emf induced in the armature of a generator is alternating, the same sort of winding can be used on an alternator as on a DC generator. This type of alternator is known as a *single-phase* generator. Since the power delivered by a single-phase circuit is pulsating (the power is zero twice in each cycle when the power factor is unity), this type of circuit is objectionable in many applications.

A polyphase circuit is used for most applications instead of a single-phase circuit. If three coils are fastened 120° apart and are rotated in the magnetic field created by a pair of poles as shown in Figure 2–46A, the sine waves of voltage induced in the three coils will be 120° apart. (The shaded ends of the coils are 120° apart.) While the power of any one phase may be zero or negative, the total power is constant if the loads are balanced. This type of generator is known as a three-phase generator since it produces three sine wave voltages from the three coils.

In Figure 2–46A, the coils are rotating in a counterclockwise direction. In Figure 2–46B the turns of wire can be seen more clearly. The direction of induced voltage at the instant shown is indicated by arrows. The current is taken from each of the three coils by a pair of slip rings, terminals a_1 and a being connected to slip rings a_1 and a, terminals b_1 and b, to slip rings b_1 and b, and terminals c_1 and c, to slip rings c_1 and c.

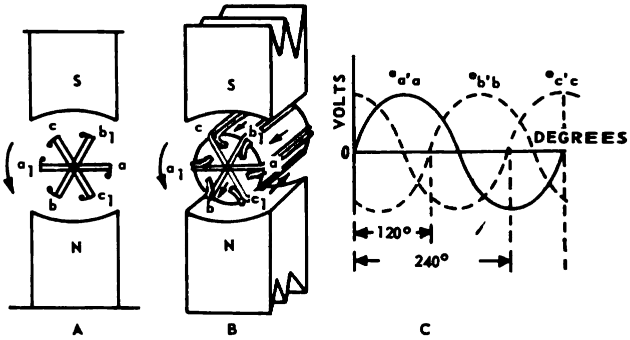


Figure 2–46. Three-Phase Generator

At the instant shown, the voltage induced in coil a_1a is zero; the induced voltage will be going positive as the coil moves on. The voltage induced in coil b_1b is approaching its maximum negative value (current is leaving the generator at terminal b_1 and entering it at terminal b). The voltage induced in coil c_1c is past the maximum positive value (current is entering at terminal c_1). The waveshapes induced in each of the three coils in one complete revolution are shown in Figure 2–46C.

In the three-phase alternator of Figure 2–46 each of the coils can be connected to a single-phase circuit through its slip rings. Of course, if the alternator has a stationary armature and a rotating field, the slip rings would not be needed; the six leads would be taken directly from the armature. In practice, however, three-phase alternators are generally connected to give 3- or 4-wire, three-phase systems. They can be Y-connected (Figure 2–47A) or delta-connected (Figures 2–47C and 2–47D).

Figure 2–47A shows the three coils of Figure 2–46A connected in Y to give a three-phase, 4-wire system. The three coils are indicated 120° apart. The common connection is the *neutral* wire. When the load is balanced, there is no current flow in the neutral wire. The voltage across any individual phase winding with respect to neutral, or ground, is a single-phase voltage.

Figure 2–47C shows the same three coils arranged for a delta system. The ends of the coils that were connected to neutral in Figure 2–47A have been connected to the outer end of the next coil. Figure 2–47D shows the usual, simplified delta arrangement.

If the voltages in each of the three coils of Figure 2–47A is represented as a vector (Figure 2–47B), the voltage across any two phases can be shown to be 208 volts. It can be shown vectorially that in a balanced Y-system the line voltages are equal and differ in phase by 120°. Each line voltage differs in phase by 30° from one of its phase voltages. The line voltages are equal in magnitude to $\sqrt{3}$, or 1.732, times the phase voltage.

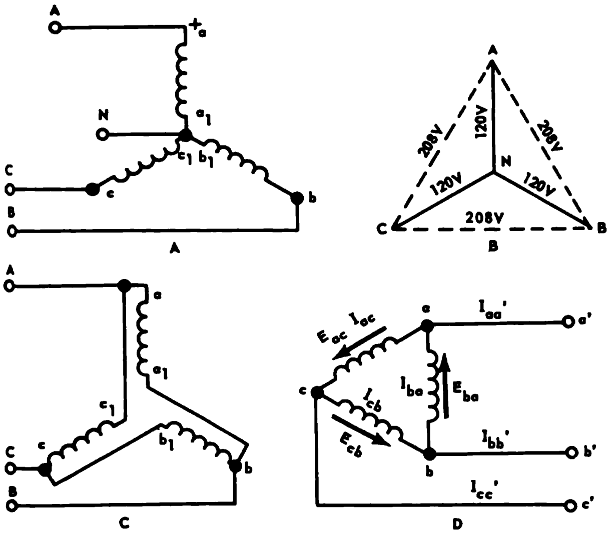


Figure 2–47. Y and Delta Connections for Three-Phase Generator

Figure 2–48 shows a Y-connected load made up of three equal resistors, *a′o′*, *b′o′*, *c′o′*, supplied by a Y-connected alternator. Since the loads are balanced, there is no current in the neutral wire. The coil currents, I_{oc}, I_{oa}, I_{ob}, are in series with the line currents, $I_{cc'}$, $I_{aa'}$, $I_{bb'}$; therefore they are equal. That is, the phase current *is* the line current.

In the delta system, the line voltage is equal to the coil voltage. In a balanced delta system the line current is $\sqrt{3}$ times the phase current.

DC MOTORS

Motor Action

A motor is a machine that changes electrical energy to mechanical energy. A motor operates because a force is exerted on a current-carrying conductor placed in a magnetic field. Figure 2–24 shows that this force is at right angles to the magnetic field (and to the conductor).

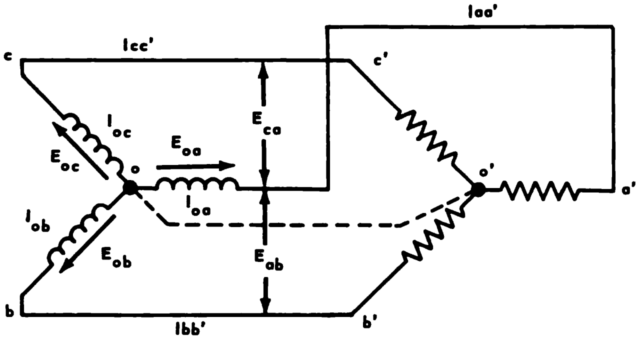


Figure 2–48. Y-Connected Load

Figure 2–49 shows the force acting on a single-turn coil in a magnetic field. Current is flowing out of the left-hand conductor and into the right-hand conductor. The force tends to move the right-hand conductor up and the left-hand conductor down (Figure 2–49A). Both forces acting on the coil produce a turning effect. This rotational force is called torque. When the coil

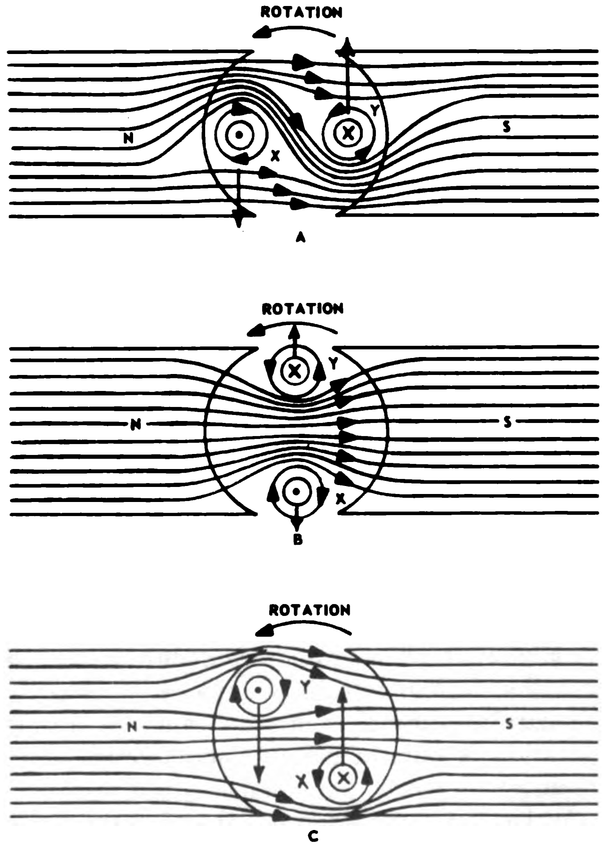


Figure 2–49. DC Motor Principle

reaches the position shown in Figure 2–49B, the forces tend to spread the coil apart and there is no torque. This position is called *dead center.* If the current in the coil reverses at this point and the coil is also carried beyond the dead center, forces will be developed as in Figure 2–49C. The torque tends to continue turning the coil in the same direction as before.

If the direction of current is always reversed at the proper time, a continuous rotating force is developed. A commutator is used to produce the change in current flow, just as in the DC generator.

A single coil motor, as shown in Figure 2–49, is impractical since it has dead centers and the torque is pulsating. A large number of coils wound on the armature will produce a much smoother torque and will eliminate the dead centers. The construction of a motor armature is the same as that of a generator armature. Additional pairs of poles, as in a generator, are also used.

The rotational force (torque) that turns the motor armature depends on two factors — armature current and the strength of the magnetic field. Mathematically, this relationship is expressed as

$$T = K_t I_a \phi$$

where K_t is a constant depending upon the physical construction of the motor, I_a is the armature current in amperes, ϕ is the flux from one N pole entering the armature, and T is the torque.

Counter Electromotive Force

The armature resistance of a small, 28-volt DC motor is extremely low — about 0.1 ohm. When the armature is connected across the 28-volt source, current through the armature will apparently be

$$I = \frac{E}{R} = \frac{28}{0.1} = 280 \text{ amperes.}$$

Such a high current is not reasonable, especially when the current drain during normal operation is found to be 4 amperes. Evidently the current through a motor armature during operation is determined by more factors than ohmic resistance.

As has been pointed out, the armature of a DC motor is like a DC generator armature. When the armature rotates in the magnetic field, the conductors cutting the field must have a voltage induced in them. Figure 2–50 shows a conductor on the armature of a DC motor moving up through the magnetic field. Applying the left-hand rule, you will find that the voltage induced in this conductor due to its upward motion through the magnetic field will cause current to flow in the conductor from right to left. This is in opposition to the current flowing in the wire due to the applied voltage. (The direction of this current is shown by the arrows in the wire in Figure 2–50.)

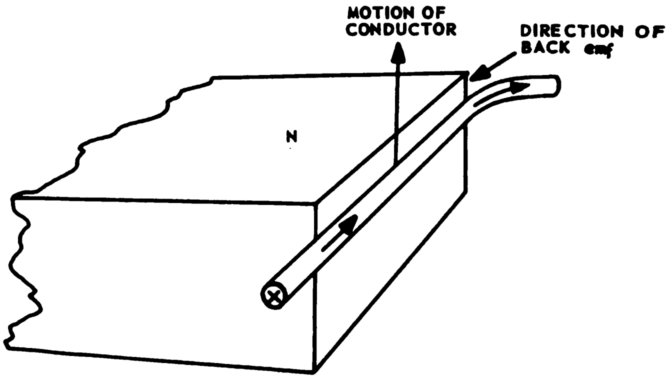


Figure 2–50. Direction of Current and Voltages in a Motor Conductor

If you determine the direction of the induced emf in all of the conductors on a motor armature, you will find that it always *opposes* the current due to the applied voltage. For this reason, the induced voltage is called the *counter electromotive force*, or *back emf.* Since the back emf opposes the current entering the armature, it must also oppose the applied voltage. The difference between the applied voltage and the back emf is the effective voltage in the armature.

The armature current, then, is

$$I_a = \frac{E_{app} - E_{cemf}}{R_a}$$

where E_{app} is the applied voltage, E_{cemf} is the back emf and R_a is the armature resistance. The back emf can be found by solving the preceding equation for E_{cemf}.

$$E_{cemf} = E_{app} - I_a R_a$$

The back emf is always less than the applied voltage.

For the 28-volt motor described,

$$E_{cemf} = 28\text{v} - 4 \times 0.1 = 27.6 \text{ volts}$$

The induced emf in a motor, like that in a generator, is determined by the number of poles,

$$E_{cemf} = \frac{\phi SPZ}{60\,P'\,10^8}$$

ϕ is flux from 1 N-pole
S is speed in rpm
P is number of poles
Z is number of conductors
P′ is number of parallel paths through the armature

P, *Z*, *P′* and 10^8 are constant for a specific motor; therefore, they can be replaced in the formula by K_1. The counter emf then becomes

$$E_{cemf} = K_1 \phi S$$

Solving for speed,

$$S = \frac{E_{cemf}}{K_1 \phi}$$

Letting $K = 1/K_1$, you can write the preceding formula as

$$S = K \frac{E_{cemf}}{\phi}$$

This formula shows that the speed of a motor is directly proportional to the counter emf and inversely proportional to the field flux.

If you substitute for E_{cemf} in this formula,

$$S = K \frac{E_{app} - I_a R_a}{\phi}$$

This formula shows that the speed of a motor varies with the load. The load on a motor can be considered as *applied* torque. The reaction of the motor must cause an increase in the *developed* torque for the motor to operate. Previously we showed that the torque of a motor depended upon the armature current and the flux. Will the torque of a motor increase with a stronger field? We have shown that an increase in flux decreases the speed of a motor. Because of inertia, the speed of the armature does not change instantaneously. Thus, the increased field strength induces a larger back emf. This increase in the back emf, while small, produces a large decrease in I_a. The decrease in I_a is greater than the increase in ϕ, and the torque decreases. The speed of the armature gradually decreases until the developed torque is that required by the armature under the changed conditions.

The circuits and characteristics of generators are also applicable to motors. A generator will run as a motor and convert electrical energy into mechanical energy, provided the proper voltage is applied to the output terminals. This voltage must be the same as that generated when the machine is rotated at the proper speed. DC motors, like DC generators, are classified as shunt, series and compound motors.

Shunt Motor

In a shunt motor the armature and the field coils, which have a fairly high resistance, are connected in parallel across the applied voltage. When a load is applied to a motor, the speed of the motor immediately tends to decrease. In a shunt motor, since the field coils are connected directly across the source, the flux remains constant. From the formula for counter emf you can see that a decrease in speed produces a corresponding decrease in E_{cemf}. A decrease in E_{cemf} causes an increase in I_a. Since torque depends upon armature current as well as field flux, this increase in armature current increases the torque developed by the motor. The back emf decreases and the armature current increases until the developed torque is approximately equal to the applied torque.

The speed of the shunt motor is

$$S = K \frac{E_{app} - I_a R_a}{\phi}$$

Since the applied voltage, armature resistance, and field strength are constant, the speed of the shunt motor is essentially dependent on the armature current. As we have shown, when the load on a motor is increased, the armature current increases. Because of armature reaction, ϕ will decrease slightly as the armature current rises. This slight decrease in ϕ, coupled with the rise in armature current, has the effect of maintaining the speed of the shunt motor almost constant.

Speed regulation of a direct current motor is the change in speed when the load is reduced from rated load to zero, expressed in percent of rated-load speed. The speed regulation is simply a measure of a motor's ability to maintain its speed when a load is applied. The shunt motor is considered to be a constant-speed motor, since its speed changes very little with load.

A rheostat is usually connected in series with the field of a shunt motor. Changing the amount of resistance in series with the field changes the strength of the field. In this way the speed of a shunt motor can be varied. An increase in field strength decreases motor speed.

A shunt motor is characterized by fairly high starting torque and current. Torque, as we have seen, is proportional to the product of the armature current and the flux. Since the applied voltage is always across the field coils, the flux is practically the same at all times in a shunt motor. However, the counter emf as the motor starts is very low. The armature current, therefore, is high. This means that the starting torque is equal to or even greater than the torque at full load.

Series Motor

A series motor is so-called because the field coils are connected in series with the armature. The field coils usually consist of a few turns of low resistance wire. Since the field and armature are in series, the strength of the field depends upon the armature current. For this reason, the torque of a series motor can be considered as proportional to I_a^2. Armature reaction, however, and saturation of the iron in the armature keep the torque from increasing as fast as the square of the current.

The formula for the speed of a series motor is slightly different from that of the shunt motor; since the armature current flows through the field coils, their resistance must be taken into consideration.

$$S = K\frac{E_{app} - I_a(R_a + R_f)}{\phi}$$

where K is a constant, E_{app} the applied voltage, I_a the motor current, R_a the armature resistance, ϕ the flux entering the armature from one pole, and R_f the resistance of the field coils. I_a and ϕ both vary with the load.

Let's see what happens when the load on the series motor is increased. The speed of the motor immediately tends to decrease, as we have seen before. This decrease in speed produces a decrease in the back emf, which allows more armature current to flow. The numerator of the fraction in the formula for the speed of a series motor becomes slightly smaller. The flux (ϕ) increases due to the increase in current flow. The increase in armature current increases the denominator and decreases the numerator of the fraction in the speed formula, resulting in a drop in the motor speed. The decrease in motor speed brings about an increase in armature current and field strength. The speed and current continue to change until the developed torque is balanced with the applied torque (and any losses).

When the load on a series motor is decreased, the speed tends to increase at that instant. An increase in speed results in an increase in the back emf. (Current and flux have not had a chance to change yet.) This increase in counter emf produces a decrease in the armature current and also in the field strength. As the current and the flux decrease, the developed torque decreases, and the speed also decreases. The speed and current, as before, continue to adjust until the torques are balanced.

If the load on a series motor should be removed, the speed would continue to increase, developing more and more back emf, thus cutting down the armature current and the field. (This increase in speed with decrease in load is shown graphically in Figure 2–51.) The flux becomes so small that the speed of the motor must become excessive in trying to reach a point where the counter emf is equal to the applied emf. (At this point the developed torque would balance the zero torque of no load.) With a motor where there is little residual magnetism in the field structure, the armature can reach such high speeds if the load is removed that the armature structure itself flies apart. For this reason, most large series motors are directly geared to their loads, rather than connected with belts.

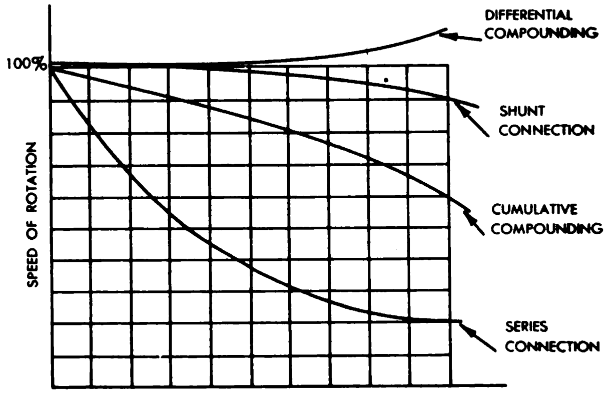


Figure 2–51. Load Characteristics of DC Motors

Series motors have a widely variable speed depending on the load, and a very high starting torque with moderately high starting current.

Series motors are used: (1) in work that requires a high starting torque, (2) where the load is constant so that variations in speed are not produced, or (3) where a change in speed with load is desirable to prevent excessive current flow with increased torque, as in a street car motor.

Compound Motor

Like the compound generator, the compound motor has both series and shunt field windings. The series winding may either aid the shunt winding (cumulative compound) or oppose the shunt winding (differential compound).

Cumulative Compound. The characteristics of the cumulative-compound motor lie somewhere between those of the series and those of the shunt motor. As the load is increased, the increase in current increases the flux due to the series winding. This increases the torque faster than the increase for a straight shunt motor. But this increase in flux decreases the speed more rapidly than the speed decrease in a shunt motor. The applied and developed torques are balanced with less speed decrease than in a series motor, but more than in a shunt motor, as shown in Figure 2–51.

Because of the series field, the cumulative-compound motor has a higher starting torque than a shunt motor. Cumulative-compound motors are used in driving machines that are subject to sudden changes in load. They are also used where a high starting torque is desired but a series motor cannot be used easily.

Differential Compound. Since the series field opposes the shunt field in this type of motor, an increase in load creates an increase in current and a decrease in total flux. These two tend to offset each other and result in a practically constant speed. However, since an increase in load tends to decrease the field strength, the speed characteristic becomes unstable. This type of motor is rarely used.

Control of Speed and Direction of Rotation

The most common method of speed variation is control of the main field flux. A rheostat is usually connected in series with the field windings. From the relationship between speed, flux and back emf, you can see that *decreasing* flux causes *increase* in speed, while *increasing* flux causes *decrease* in speed.

Reversing the polarity of the voltage applied either to the armature *or* to the field will reverse the direction of any of these motors. Figure 2–52A shows the force acting on the armature of a DC motor producing counterclockwise rotation. In Figure 2–52B the polarity of the voltage applied to the field coils has been reversed, reversing the direction of the field. Notice that the rotation is now clockwise. In Figure 2–52C both the field and the direction of current through the armature have been changed. As a result, the rotation is still counterclockwise, as it was in Figure 2–52A. Since this is true, it doesn't matter which side of the DC motor is connected to the positive and which to the negative side of the source voltage so far as direction of rotation is concerned.

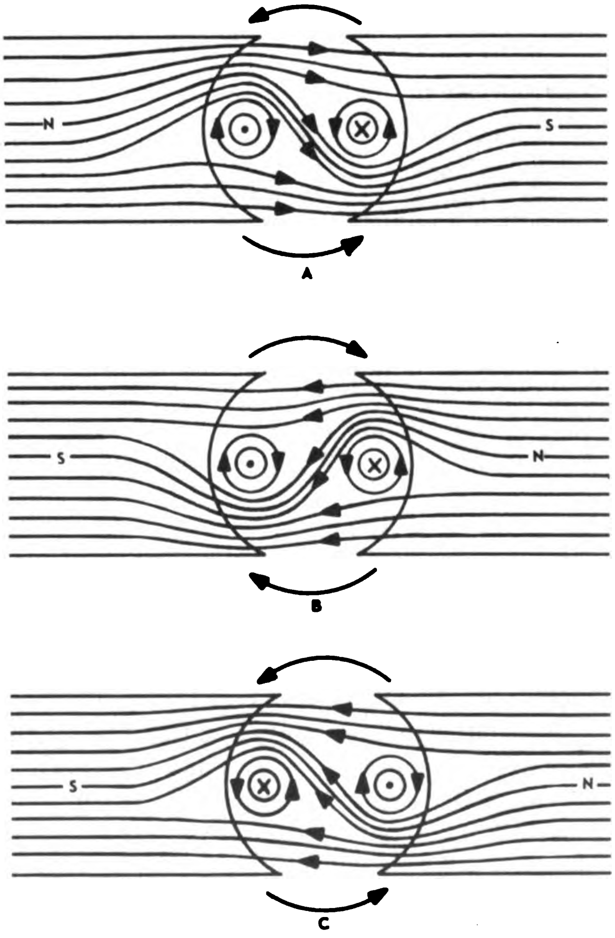


Figure 2–52. Direction of Rotation

AC MOTORS

Series Motor

Since changing the polarity of the voltage applied to both the armature and the field does not change the direction of rotation of a series motor, AC may be applied to the terminals, rather than DC. Since the field and armature are in series, the field and armature currents are in phase. Therefore, the flux is in phase with the armature current and the series motor develops about the same torque with AC applied as it does with DC. This is not true with the shunt motor, since the flux there is nearly 90° out of phase with the armature current.

The main differences between series DC motors and series AC motors are: (1) the field structure is laminated in the AC motor, (2) the number of turns per pole in the field coils is reduced to a minimum in the AC motor, and (3) the armature of the AC motor has more turns than that of the DC motor.

Induction Motor

Because of its ruggedness and simplicity, the induction motor is the most widely used AC motor. Figure 2–53 is an illustration of the basic principle of the induction motor. A horseshoe magnet is placed over a disk of metal so that its magnetic field passes through the disk. The disk is mounted on an axis so that it can rotate freely. The magnet is rotated in a clockwise direction above the disk (Figure 2–53A). As it moves, the magnetic flux cuts through the disk. This metal disk can be considered to be made up of many conductors side by side. Figure 2–53B is a view looking at the edge of the disk below the north pole. Since the results would be the same if the disk were moved in a counterclockwise direction (the relative motion between conductor and flux would be the same), the conductors are indicated as moving.

Eddy currents are induced in the disk by the cutting of the lines of force. They are shown in Figure 2–53B as coming out of the paper. The lines of force about the conductor are shown in Figure 2–53C. Remember that these lines of force are due to the induced current in the conductor. The magnetic field on the right side of the conductor in Figure 2–53C is weakened, while the field to the left is increased. The force acting on the conductor tends to move it to the left. This is the same direction we are moving the magnet above the disk.

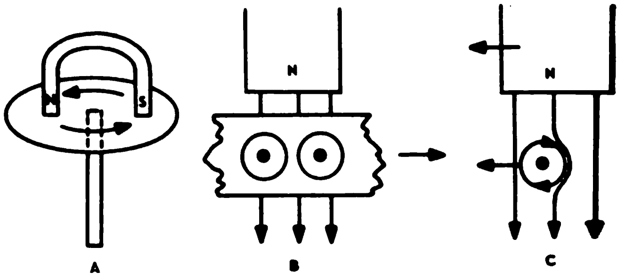


Figure 2–53. Basic Induction Motor Principle

The disk is rotating in the same direction as the magnet, but at a slightly slower rate. It can never rotate as fast as the magnet, because if it were to do so, there would be no relative motion. With no relative motion there would be no induced voltage and no magnetic field about the disk conductor. There would be, therefore, no reaction between magnetic fields, no torque, and a slowing down of the disk. Once the disk began to slow down, there would be relative motion between the magnet and the disk, resulting in induced currents in the disk with their accompanying magnetic fields. The disk once more would have a force tending to rotate it. The *difference* of speed between the magnet and the disk is called the *revolutions slip.*

Instead of a disk, a cylinder may be used. Instead of a horseshoe-shaped magnet, four poles may be rotated within the cylinder. The cylinder is more like the commercial induction motor and works on the same principle as the disk just described.

Rather than rotating a magnet, a rotating field is used in the practical induction motor. The rotating field is due to polyphase currents in polyphase windings. There is no mechanical rotation of the fields; they are produced by electrical means.

Two-Phase Rotating Field. Figure 2–54 illustrates the development of a two-phase rotating field. A, B, C and D are simplified sectional views of the two-phase, two-pole field (stator) winding. Figure 2–54E shows the vectors and the sine waves of current applied to windings *aa'* and *bb'* of the stator. The currents have a phase difference of 90°. In this explanation when we speak of current as being negative, we mean flowing

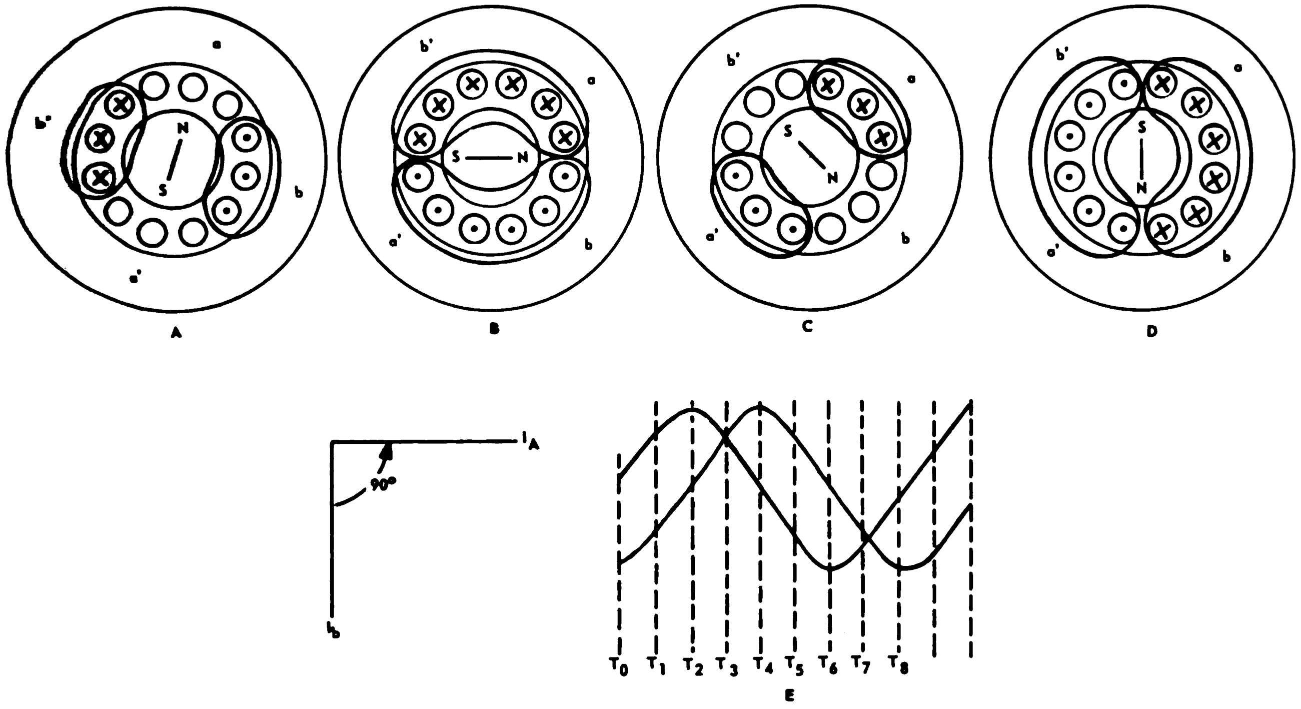


Figure 2–54. Rotating Field, Two-Phase Currents

in one direction; when we say current is positive, we mean it is flowing in the opposite direction. At times t_0 in Figure 2–54E, I_a is zero and I_b is maximum negative. There is no current flowing in winding *aa'*, but maximum current is flowing in the direction indicated by the + and the • in the winding *bb'*. This creates a magnetic field with the direction shown.

At time t_1, I_b has decreased, but is still negative. I_a is now positive and equal in amplitude to I_b. Current has decreased in winding *bb'*, but the field due to this current is in the same direction. Current is now flowing in winding aa' in the direction indicated by the + and • in the conductors in Figure 2–54B. The magnetic flux due to winding aa' is at right angles to that for bb'. The resultant field has the direction shown in Figure 2–54B. Notice that the north pole has moved 45° in a clockwise direction. At time t_2, I_a is maximum positive and I_b is zero. With no current flowing in winding *bb'* and maximum flowing in *aa'*, the field is as indicated in Figure 2–54C. The field, due only to the flux from winding *aa'*, has a direction that is 45° counterclockwise from that in Figure 2–54B. Each time the phase of the currents advanced 45°, the magnetic field rotated 45°.

At time t_3, both currents are positive and equal in amplitude. The direction of current in winding *bb'* is the reverse of that in A and B. The magnetic fields due to currents in the two windings combine to form a field that is vertical, as shown in D. An analysis for times t_4 through t_8 will show that the field continues moving in a clockwise direction until, at time t_8, it is back to the position shown in Figure 2–54A. In one cycle, the field has made one complete rotation. Thus, the rotating field for a two-phase, two-pole stator makes one rotation for each cycle. Its speed in revolutions per second is equal to the frequency of the supply voltage in cycles per second.

A four-pole stator, similar to the one just discussed, can be analyzed in the same way. Such an analysis will show that the rotating field advances only 22.5° for every 45° advance in current. Thus the speed of the four-pole rotating field will be one-half that of the two-pole motor.

Three-Phase Rotating Field. Figure 2–55 shows the sectional views of a four-pole, three-phase stator for an induction motor. For simplicity, only one conductor is shown for each section of the windings. Figure 2–55E shows the currents for the three windings and the vectors showing the phase relationships. An analysis

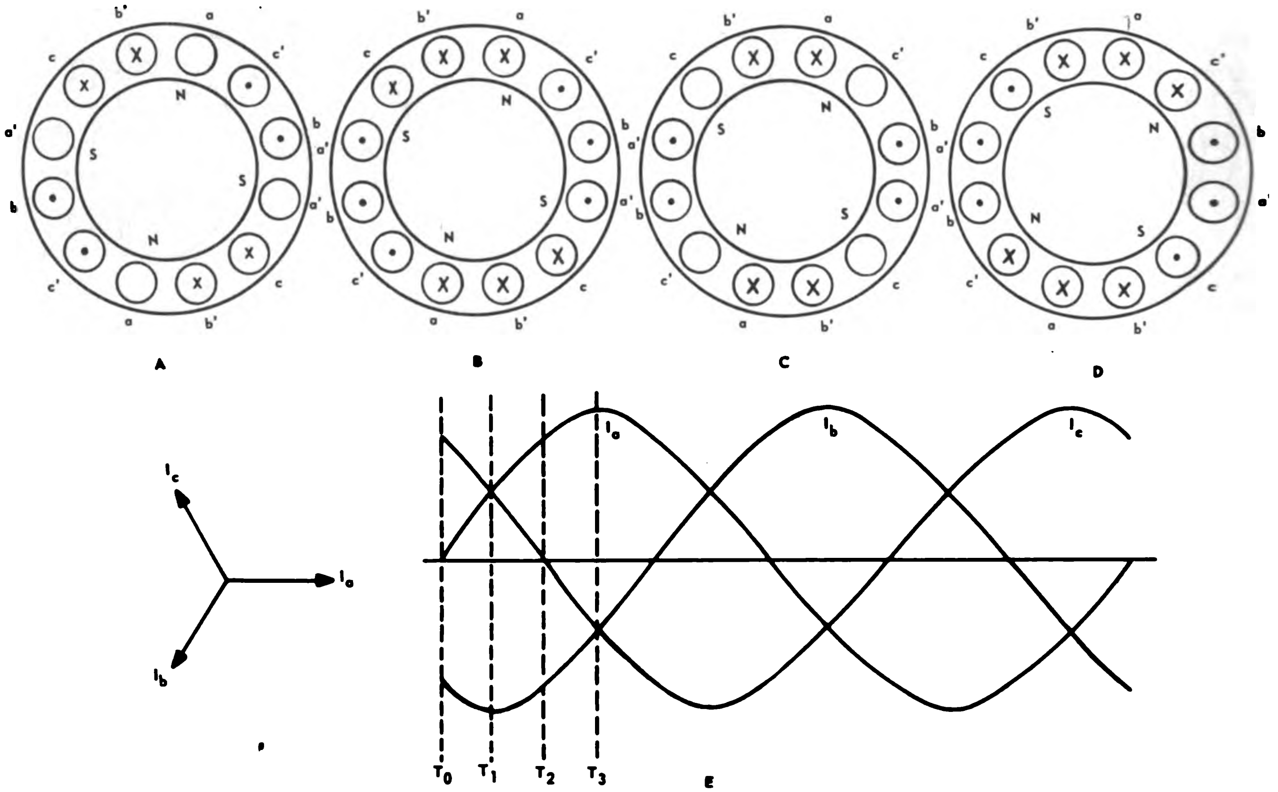


Figure 2–55. Rotating Field, Three-Phase Currents

similar to that for the two-phase, two-pole stator can be made of this circuit. The currents shown in Figure 2–55A are those at time t_0 in Figure 2–55E. Those in B are at time t_1, those in C are at time t_2, and those in D are at time t_3. Since I_a is zero at t_0, there is no current in windings *aa'*. I_b and I_c are equal and opposite. The magnetic field due to these currents is shown in Figure 2–55A.

At time t_1, I_b is maximum negative, while I_a and I_c are half their maximum positive values. The windings are so arranged that the currents are flowing into the page in the *a*, *b'* and *c* ends of the windings and out of the *a'* *b* and *c'* ends. The field has moved slightly in the clockwise direction. The time angle between t_0, t_1, t_2 and t_3 is 30°, but the field only advances 15° during each of these intervals.

As the current changes, the field gradually rotates. In Figure 2–55D the field has moved 45°, although at time t_3 the current has advanced 90°. The speed of the field rotation in revolutions per second is one-half the frequency of the current in cycles per second.

Since motor speeds are generally given in revolutions per minute, a formula for the speed of the rotating field can be derived.

$$S = \frac{f \times 60}{P/2}$$

$$= \frac{120f}{P} \text{ rev. per minute}$$

where f is the frequency in cycles per second and P is the number of poles.

From the illustrations and discussion on rotating fields, it is apparent that, to produce a two-pole rotating field, the angular space between windings (*a* and *b* in Figure 2–54; *a* and *b*, *b* and *c*, *c* and *a* in Figure 2–55) must be the same as the electrical degrees between the currents in the windings. With two-phase current, this is 90°; with three-phase current, it is 60°. To produce a four-pole rotating field, the angular space must be one-half the number of electrical degrees. However, the speed of the rotating field depends only on the number of poles. The number of phases has no effect on the speed of rotation of the field.

Operation. If an armature is placed in any of the stators just described, the rotating field will cut the armature conductors and induce currents in them. The magnetic field due to this current reacts with the rotating field and produces a torque. This torque tends to turn the armature in the direction the field is rotating. (This is the same principle that produced the rotation in the simple disk described earlier.)

If you wish to reverse the direction of rotation of a two-phase rotating field, reverse the leads of either phase. To reverse the direction of rotation of a three-phase rotating field, you may interchange any two leads.

You will recall that in the simple induction motor we explained that the rotor cannot turn as fast as the rotating field. The speed of the rotating field is called the *synchronous* speed of the motor. The difference in speed between that of the rotating field and the speed of the rotor is called the revolutions slip. Usually the slip is given as a fraction of the synchronous speed. That is, a motor whose synchronous speed is 3600 rpm and whose rotor speed is 3528 rpm has a slip of 2%.

$$(3600 - 3528) \div 3600 = 2\%$$

Squirrel-Cage Motor

This simple induction motor is so-called because the armature, or rotor, consists of bars with the ends connected together by *end rings*. It looks very much like the cage that spins when the squirrel inside it starts running.

As the rotating field moves, currents induced in the rotor bars set up magnetic fields. The interaction of the two fields causes the rotor to turn in the direction of the rotating field. This motor is considered a constant speed motor, like the DC shunt motor. At no load the slip of the squirrel-cage motor is very small, but as the load is increased, the slip increases. In order to produce the torque required by the load, more current must flow. More lines of flux must be cut per unit time to induce this current. Since the field cannot be changed, the rotor must have a greater relative motion with respect to the rotating field, or a greater slip. The slip increases until the proper torque is produced.

Split-Phase Motor

When it is necessary to operate an AC motor with single-phase current, a two-phase motor may be used. The single-phase supply is effectively "split" into two phases that are approximately 90° apart. One such system is shown in Figure 2–56A. A capacitor is used in series with one phase winding to give the 90° phase shift required. In the vector diagram E_{app} is the applied voltage. The current through winding $b (I_b)$ leads E_{app} by an angle as shown. Since current and voltage across a capacitor are 90° out of phase, the voltage across the capacitor (E_c) lags the current (I_b) by 90°. The current through winding a (I_a) lags the applied voltage by an angle less than 90°. Notice that I_a and I_b are almost 90° apart. The voltage across winding b (E_b) leads the current (I_b) by an angle of less than 90°. Because I_a and I_b are approximately 90° out of phase, the motor will operate as a two-phase motor does.

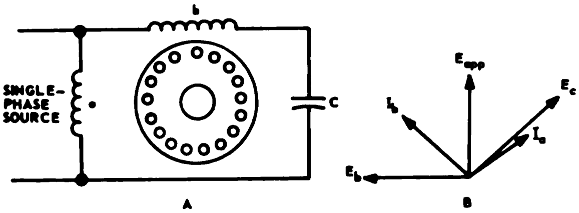


Figure 2–56. Split-Phase Motor

Unlike other motors we have studied, the currents in the induction motor armature are induced currents, rather than being conducted into the armature from an outside source of power.

Synchronous Motor

The design of a synchronous motor is almost the same as an alternator., Figure 2–57A shows the poles of a synchronous motor field and two of the conductors in the armature. The field is constant, since it is due to direct current flow. The current through the armature is alternating. At the instant shown, the torque on the two conductors tends to move them to the left. Since this is alternating current, the next half-cycle will find the current through the armature reversed, and the torque reversed if the wires remain in the same relative position. If the armature moved far enough so that wire *2* were beneath the N pole and wire *1* below an S pole, the torque would be in the same direction, keeping the armature turning in the same direction as during the first half-cycle.

If each half-cycle sees the armature conductors moved the distance of one pole, the torque will

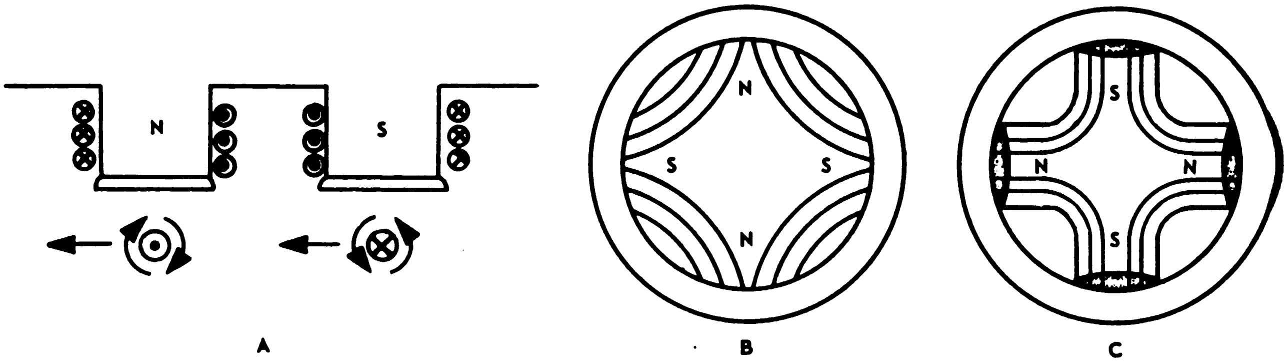


Figure 2–57. Synchronous Motor Principle

always be in the same direction, resulting in continuous motion. Thus, the synchronous motor must operate at constant speed, if the frequency is constant. If the average speed differs from synchronous speed, the average torque becomes zero and the motor stops.

Figure 2–57B shows a rotating field for a four-pole synchronous motor. This field is produced in the same way as the one in Figure 2–55. At the instant shown the two N poles are in a vertical position, and the two S poles in a horizontal position. Now let us insert an X-shaped rotor in this field. The flux from the field will move the rotor so that the flux is maximum, and there is very little air gap at each pole. The pole pieces of the rotor are locked in with the poles produced by the rotating field in the stator. Now assume that the current flowing in the armature (rotor) produces poles that rotate at the same speed as the poles due to the stator winding. They will lock in, just as though everything were at a standstill (Figure 2–57C). The rotating poles of the rotating field of the stator pull the rotor around with them at the same speed.

A synchronous motor is not self-starting. First it must be brought up to synchronous speed. This may be done by means of a direct current exciter or a small induction motor geared to the synchronous motor shaft. However, where the power factor is very low, as in clock motors, they will operate without DC excitation.

TERMS USED IN AC

Previously you learned that the output voltage of an AC generator can be expressed in two ways. One is graphically by a sine wave, as shown in Figure 2–35. The other is by the equation,

$$e = E_m \sin \omega t$$

where e equals the voltage, E_m the maximum value of generated voltage, and ωt the angular velocity multiplied by the time.

When a generator produces an AC voltage, the current arising from it varies in step with the voltage. Like the voltage, the current can be represented graphically by a sine wave and algebraically by the following equation,

$$i = I_m \sin \omega t$$

where i equals the current, I_m the maximum value of generated current, and ωt the angular velocity multiplied by time.

Frequency and Cycle

During the time the coil in a generator rotates through 360°, that is, one complete revolution, the output voltage goes through one complete *cycle.* During one cycle, the voltage increases from 0 to positive E_m in one direction, decreases to 0, increases in the opposite direction to negative E_m, and then decreases to 0. The first 180° (one-half of the voltage cycle) is called the *positive* alternation and the last 180° (from 180° to 360°) the *negative* alternation. The value of the E_m voltage at 90° is called the *maximum amplitude*, or *peak* voltage value. The time required for a positive and a negative alternation is called the *period*, and the number of complete cycles per second is called the *frequency* of the sine wave. When the angular velocity ω at which the coil rotates is expressed in radians per second, the mathematical relation between ω and f is given by the equation

$$\omega = 2\pi f$$

Effective Value of AC

Since a sine wave of alternating current (or voltage) varies continually between zero and maximum (or peak) values, first in one direction, then in the other, the question of numerical values arises. The value of AC most commonly used is *effective value.* The effective value of alternating current is the amount of alternating current which produces the same heating effect as an equal amount of direct current. In other words, one ampere (effective value) of AC will produce the same amount of heat in a given conductor in a given time as one ampere of DC. Since the heating effect of a quantity of current is proportional to the square of the current, it is possible to calculate the effective value of alternating current by squaring all the ordinates in a sine wave, taking the average of these values, and then extracting the square root. The effective value, being the root of the mean (average) square of the instantaneous currents, is also known as the *root-mean-square* or *rms* value.

To understand the meaning of effective current, study the diagram in Figure 2–58. In it the instantaneous values of i are plotted in the upper curve and the corresponding values of i^2 are plotted in the lower curve. The i^2 curve is also a sine curve. It has twice the frequency of the i curve and varies above and below a new axis. The ordinate of this axis is the average of the i^2 values, and the square root of this ordinate is the *rms*, or effective, value of current. Since the i^2 curve varies uniformly from 0 to I_m^2, its average value is $\frac{1}{2} I_m^2$. Thus the *rms* value is $\sqrt{\frac{1}{2} I_m^2}$ or $\frac{I_m\sqrt{2}}{2}$ which simplified is equal to $0.707\ I_m$. The rms, or effective value, is commonly represented by I. Hence, the mathematical relation between effective and maximum (or peak values) of current can be expressed as $I = 0.707\ I_m$.

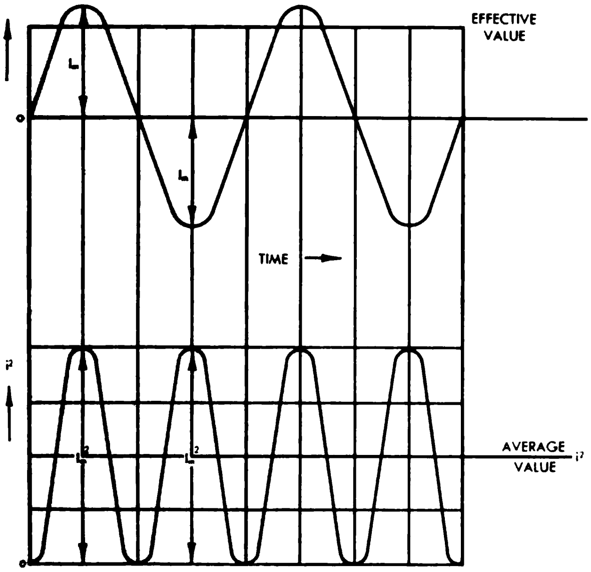


Figure 2–58. Determining Effective Value of Current

The relation between maximum (or peak) values of emf and effective emf is the same as the relation between peak and effective current. Thus,

$$E = 0.707\ E_m$$

The reciprocal of 0.707 is 1.414. Therefore,

$$E_m = 1.414\ E \text{ and}$$
$$I_m = 1.414\ I$$

Average Value of AC

Another value in a sine wave sometimes used is the average value of the ordinates during the positive alternation. Average value is not as widely used as the *rms* value, but in some instances is a more descriptive value of current or voltage. However, you can always assume that when an AC voltage is described by a value but is not designated *rms*, peak or average, *rms* value is meant. For example, the 110-volt AC which is used for lighting and other purposes in your home means 110 volts *rms*. The mathematical relation between I_{av}, I_m, and I is given by the formula $I_{av} = 0.636\ I_m = 0.9I$. Similarly the relation between E_{av}, E_m, and E is given by the formula $E_{av} = 0.636E_m = 0.9E$.

Phase

Phase is the fraction of a cycle that has elapsed since a voltage or a current has passed through a given value. Usually, this value is the zero value. By referring to Figure 2–35 showing the sine wave of voltage and taking point 0 as the starting point or zero phase, you can see that the phase at 1 is 30°, at point 2 it is 60°, at point 3 it is 90°, and so on throughout one complete cycle until point 12 where the phase is 360° or 0°.

A term more commonly used than phase is *phase difference.* Phase difference can be used to describe two voltages having the same frequency that pass through zero values at different

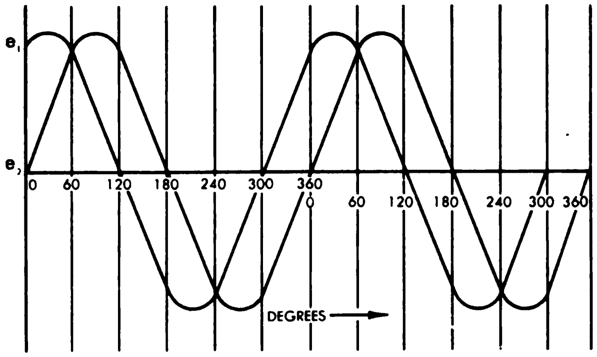


Figure 2–59. Phase Relations

instants. In Figure 2–59 the degrees along the axis indicate the phases of the voltages e_1 and e_2 at any instant. At the 0° position, e_2 equals 0. On the other hand, e_1 passes through the zero value at 120° and 300°, both of which are 60° *ahead* of e_2 in time. Thus the voltage e_1 is said to *lead* e_2 by 60 electrical degrees. Another way of saying this is that e_2 *lags* e_1 by 60 electrical degrees.

The concept of phase difference is also used to compare two different currents or a current and a voltage. The expression *in phase* means that the phase difference between two currents or a current and a voltage is zero degrees, and *out of phase* means that their phase difference is some amount other than zero.

One fact to remember is that the phase angle of a voltage is not necessarily the same as the angle of rotation of the armature in a generator. If the generator has more than one pair of poles, the output voltage may go through a corresponding number of cycles for each revolution of the armature, depending on the number and placement of the brushes. This gives rise to the term *electrical degrees,* a term that is used to distinguish between phase angle and the angle of rotation of an armature.

AC CIRCUIT COMPONENTS

Resistance

Resistance, you recall from the previous discussion of DC, is the property by which a conductor opposes the flow of current in it. In alternating current circuits the resistance of a conductor opposes alternating current in the same way that it opposes direct current. At lower frequencies, the DC resistance of a given conductor is essentially the same as its AC resistance. However, at ultra-high frequencies current has a tendency to flow only in the surface of a conductor. This tendency is known as *skin effect* and causes the resistance of the conductor to increase. The higher the frequency, the more pronounced the skin effect becomes. Because of it, many conductors of high-frequency current in radar equipment are silver plated on the outside to reduce the resistance of the surface layer.

Inductance

In the discussion of induction you learned that a coil opposes any change in the current that flows through it by building up a counter emf. This counter emf is an induced voltage, which is

$$e_i = -L \frac{\Delta i}{\Delta t}$$

where e_i is the counter emf, L the inductance in henrys, Δi the change in current, and Δt the change in time.

The term $\frac{\Delta i}{\Delta t}$ is the rate of change in current with respect to time.

In alternating current the instantaneous value of i is equal to $I_m \sin \omega t$. The instantaneous value of e_i is equal to $-\omega L I_m \cos \omega t$.

This is also the equation of a cosine curve, a curve which has the same shape as a sine wave curve but differs in phase from it by 90°. That this phase difference exists is apparent, if you recall that it is the *rate of change* of current that determines the value of the counter emf. The counter emf therefore becomes maximum, not at the time of maximum current but at the time the current is changing most rapidly, that is, at the time when i is zero. By Lenz's law the counter emf is in such a direction as to oppose the change in current. Hence, if i is increasing, the counter emf will be in the opposite direction to the current. When i is decreasing, the direction of the emf is the same as that of the current. The counter emf (e_c) lags the current (i) by 90°. Figure 2–60 shows these relationships.

In an AC circuit with both inductance and resistance, you can resolve the emf applied to the circuit into two components — one that sends the current i through the resistance of the circuit (the IR drop) and the other that overcomes the opposition of the counter emf (equal to e_c but *opposite in phase*). The component which

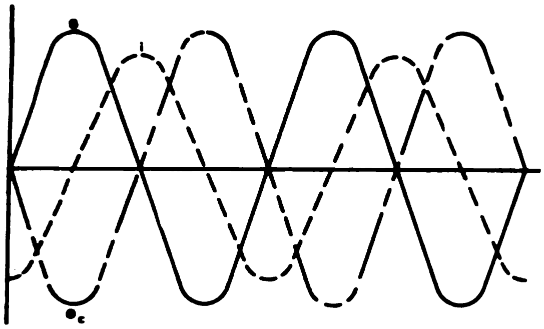


Figure 2–60. Voltage and Current Relations In an Inductance

overcomes e_c is represented by the sine wave *e*. This wave passes through zero (or maximum) *ahead* of the current and *e* is said to *lead i* by 90° (Figure 2–60).

Inductance in a circuit not only makes the current lag the voltage, but makes the magnitude of the current smaller. This choking down effect that results from the opposition to current flow caused by the inductance is known as *inductive reactance* and is given by the equation,

$$X_L = 2\pi fL$$

where X_L is the inductive reactance in ohms, f the frequency in cycles, and L the inductance in henrys.

When inductances are connected in series and are not close enough to be in each other's magnetic fields, the inductances as well as their inductive reactances add like resistances connected in series. Thus, in a series circuit the sum of the inductive reactances is expressed by the equation,

$$X_{Lt} = X_{L1}\ X_{L2} + X_{L3} + \ldots + X_{Ln}$$

and the sum of the inductances by the equation,

$$L_t = L_1 + L_2 + L_3 + \ldots + L_n$$

When inductances are connected in parallel, their inductances and the inductive reactances add by the sum of the reciprocals method, like resistances connected in parallel. Thus, in a parallel circuit, the sum of the inductive reactances is expressed by the equation,

$$X_{Lt} = \frac{1}{\frac{1}{X_{L1}} + \frac{1}{X_{L2}} + \frac{1}{X_{L3}} + \ldots + \frac{1}{X_{Ln}}}$$

and the sum of the inductances, by the equation,

$$L_t = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots + \frac{1}{L_n}}$$

When mutual coupling exists between two inductances, the equations become somewhat more involved. For example, in the case of two inductances L_1 and L_2 with a mutual inductance of *M*, the total reactance is equal to,

$$X_t = X_{L1} + X_{L2} \pm X_{2m}$$

The plus or minus sign preceding X_{2m} is a necessary part of the equation since the two coils may be located in such a way that the fields either aid or oppose each other. When the fields are aiding, X_{2m} is preceded by the plus sign; when they are opposing, it is preceded by the negative sign.

Capacitance

A capacitor is a circuit element in which charges of electricity can be stored. It is composed of two conductors, usually flat plates, separated by an insulator or dielectric. A capacitor can be charged by connecting it to a source of DC such as a battery. Due to the battery voltage, electrons will be removed from one plate of the capacitor and will flow through the external circuit to the other plate. The charge builds up as a result of the deficiency of electrons on one plate and the abundance of electrons on the other.

After a period of time, depending upon the capacity of the capacitor and the resistance of the conductors, the capacitor becomes fully charged; that is, the potential difference between the plates equals the voltage of the battery. If a battery of higher voltage were to be added, the capacitor would take on an additional charge. For any given capacitor, there is a definite ratio of the charge to the voltage. Mathematically, it is expressed,

$$C = \frac{Q}{V}$$

where C is the capacitance of the capacitor, V the voltage, and Q the charge in coulombs.

Capacitance is measured in farads. A capacitor has a capacitance of one farad when one coulomb of electricity produces a charge of one volt. For practical uses, the farad is much too large a unit. More commonly used are the microfarad (mf or μf), which is equal to one millionth of a farad, and the micromicrofarad (mmf or $\mu\mu$f), which is equal to one millionth-millionth of a farad.

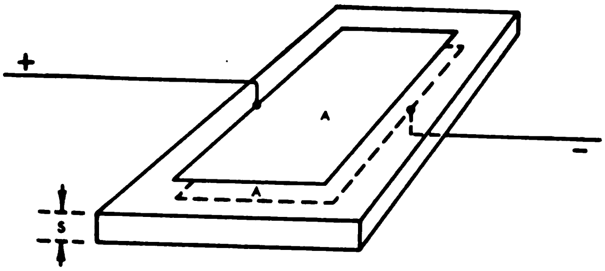


Figure 2–61. Parallel Plate Capacitor

In its simplest form, a capacitor consists of two parallel plates as shown in Figure 2–61. The value of the capacitance of a capacitor depends upon the area of the plates, the distance between them, and the dielectric. A given charging voltage will, if the area of the plates is increased, move an increased number of electrons to the negative plate before the voltage across the capacitor equals the charging voltage. In this condition the electrons spread over a wider area, and more are required to produce a certain voltage.

Whenever the distance between the plates is increased, the repelling force between the positive plate and the negative plate is less, and fewer electrons are forced away from the positive plate and fewer are deposited on the negative plate. Accordingly, since there is less movement of electrons, the capacitance is less. Finally, the dielectric determines the capacitance. The dielectric is rated according to its dielectric constant.

The dielectric constant of air is taken as the basis and is equal to one. In insulating materials the constant is usually much higher. The higher the dielectric constant of the dielectric, other factors being equal, the greater the capacitance of the capacitor. In the charging process of a capacitor, the electron orbits in the atoms of the dielectric become distorted. The energy required to distort the electron orbits is supplied by the charging source. When this source is removed, the capacitor starts to discharge, and the orbits return to their normal shape, releasing energy. Since this release of energy aids the normal discharge of the capacitor, it represents an additional source of stored energy.

The higher the dielectric constant, the greater the stored energy, for dielectrics with high constants are harder to distort than those with lower constants. Thus, capacitance varies with the dielectric. The relation between the capacitance, the area of the plates, the distance between the plates, and the dielectric constant is expressed mathematically, as

$$C = 0.2246\frac{KA}{S}$$

where C is the capacitance in micromicrofarads, K, the dielectric constant, A, the area of the plates in square inches, and S, the distance between the plates in inches.

When capacitors are connected in parallel, the total capacitance is the sum of the individual capacitances. Assuming each capacitor in Figure 2–62 has a capacitance of 0.001 mf, then the total capacitance is 0.003 mf. This is easy to understand if you keep in mind that when you connect capacitors in parallel, there is an increase in plate area. Let Q_1, Q_2, and Q_3 represent the charge on C_1 C_2, and C_3 respectively. Since the capacitors are connected in parallel, the same voltage E is across each.

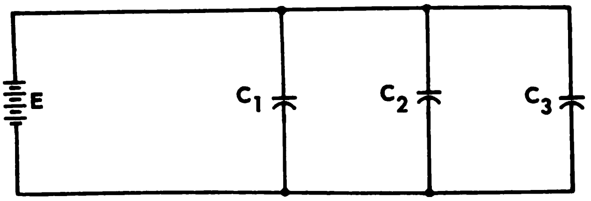


Figure 2–62. Capacitors in Parallel

Thus, if the charges of each capacitor are added,

$$Q_t = Q_1 + Q_2 + Q_3$$

The charge of a capacitor equals the charging voltage times the capacitance, or

$$Q = CE$$

Therefore, by substitution,

$$Q_t = C_1E + C_2E + C_3E$$

Similarly, $Q_t = C_tE$, hence

$$C_tE = C_1E + C_2E + C_3E$$

By dividing both sides by E, you get the formula for the capacitance of capacitors in parallel,

$$C_t = C_1 + C_2 + C_3$$

Sometimes capacitors are connected in series to lessen the voltage across each. In a series circuit the same current flows through all elements. Therefore, when capacitors are connected in series, the charges on each due to current flow are equal and the total charge can be expressed,

$$Q_t = Q_1 = Q_2 = Q_3$$

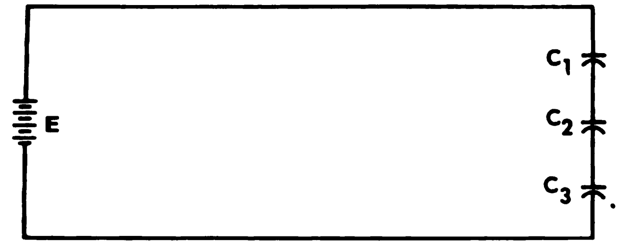


Figure 2–63. Capacitors in Series

Since the voltage drop around a series circuit equals the applied voltage, the voltages across the capacitors equal the applied voltage. Thus,

$$E_t = E_1 + E_2 + E_3$$

Capacitances in series combine by the reciprocal of the sum of reciprocals formula like resistances or inductances in parallel.

$$\text{Thus } \frac{1}{C_t} = \frac{E_t}{Q_t} = \frac{E_1 + E_2 + E_3}{Q_t} = \frac{E_1}{Q_1} + \frac{E_2}{Q_2} + \frac{E_3}{Q_3}$$

$$\text{But, } \frac{E_1}{Q_1} = \frac{1}{C_1}, \quad \frac{E_2}{Q_2} = \frac{1}{C_2}, \quad \frac{E_3}{Q_3} = \frac{1}{C_3}$$

$$\text{Therefore, } \frac{1}{C_t} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\text{From this, } C_t = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

The value of *i* at any instant during the period of charge of a capacitor is the rate of charge and is

$$i = \frac{\Delta Q}{\Delta t}$$

where ΔQ equals the change in charge and Δt the change in time.

The rate of charge is *C* times the rate of change of voltage, since *Q* equals *CV* and *C* is constant. Thus, the rate of charge is also equal to,

$$i = C\frac{\Delta v}{\Delta t}$$

Capacitive Reactance

When a capacitor is connected in a circuit to which a source of alternating voltage is applied, the capacitor will be charged first in one direction and then in the other. The electrons in the rest of the circuit will flow to and fro as an alternating current at the frequency of the applied alternating voltage. Although the current in the circuit appears to flow through the capacitor, it actually does not. Yet the effect is the same as if current were passing through the capacitor. When an AC ammeter is placed in the circuit, it shows a steady deflection just as if the capacitor were not in the circuit. However, the amount of current flow is less when the capacitor is in the circuit than when it is out of the circuit. This indicates that the capacitor offers opposition to the flow of current. This opposition, which is called *capacitive reactance,* is given by the equation,

$$X_c = \frac{1}{2\pi fC}$$

where X_c is the capacitive reactance in ohms, f the frequency in cycles, and C the capacitance in farads.

There is a difference in nature of inductive and capacitive reactances. Inductive reactance causes current to lag the voltage, while capacitive reactance causes current to lead the voltage. This is another way of saying that an inductance opposes a change in current, and a capacitance opposes a change in voltage. Still another difference is that inductive reactance is directly proportional to the inductance and frequency, while capacitive reactance is inversely proportional to capacitance and frequency.

PRACTICAL COILS AND CAPACITORS

All electrical networks may be reduced to three circuit components — resistors, inductors and capacitors. In practice none of the three is perfect. All coils possess a certain amount of resistance, even though the material used in them is of the best quality. Not only do coils have some resistance, but they also have a certain amount of capacitance between the turns of wire, a condition which must be taken into consideration, especially at the higher frequencies. Similarly, resistors cannot be perfect, for they inherently possess a certain amount of inductance. Even straight wires have inductance, another condition that must be taken into account at the higher radio frequencies.

In the discussion of DC circuits, neither coils nor capacitors were mentioned because a coil in a DC circuit is like a *resistor* and a capacitor like an *open* circuit, *after* the current in the circuit has reached a *steady state.* For example, if a capacitor, coil and resistor are connected in series and a steady source of voltage is suddenly impressed on the combination by closing a switch, there is a relatively short period, called the *transient* period, in which a current does flow before coming to a stop. After the current stops, the capacitor has the effect of an open circuit. Consequently there can be no further current flow.

In AC circuits all three types of circuit elements have one thing in common; they all *oppose* the flow of current. This does not mean that all elements act alike, for each opposes current flow in a different manner. But the *opposition* offered by each is measured in ohms and each can be treated graphically as a vector quantity.

Another factor to consider in connection with resistors, coils, and capacitors is that neither a perfect coil nor a perfect capacitor dissipates energy. A resistor is the only one of the three that does. It does this in the form of heat. A coil, to be sure, does store up energy in its magnetic field, but it gives this energy back to the circuit when the field collapses. The same is true of the capacitor, except that the energy is stored in the dielectric field between the capacitor plates. However, there is no such thing as a perfect coil or capacitor. *Practical coils and capacitors dissipate energy* because of the *resistance inherent in them.*

For some purposes the resistance of the coil or the capacitor in a circuit can be ignored. For other purposes, however, these resistances cannot be neglected. In circuit analysis, then, the resistance of a capacitor may be considered as a *small* resistance in *series* with a perfect (resistanceless) capacitor, or as a *large* resistance in *parallel* with a perfect capacitor. The resistance of the coil is separated from the coil in the same way. In addition, the capacitance between the turns of wire in the coil (distributed capacitance) is considered as a capacitor in parallel with the coil.

Figure 2–64A shows the equivalent circuit of a capacitor drawn as a perfect capacitor in series with a small resistance R_1. Figure 2–64B shows the equivalent circuit of a capacitor as a capacitor in parallel with a large resistance R_2. Figure 2–64C is the equivalent circuit of a coil drawn as an inductance in series with a small resistor R_3, both of which are in parallel with the distributed capacitance of the coil C_d. Figure 2–64D is the equivalent circuit of a coil drawn as an inductance in parallel with a large resistance R_4 and the distributed capacitance C_d.

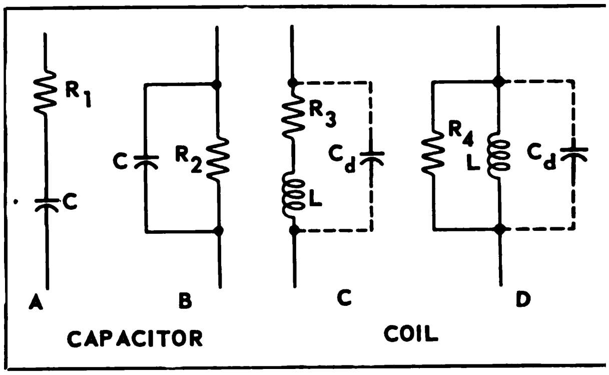


Figure 2–64. Equivalent Circuits

Dielectrics

Impregnated paper, mica, oil, oxides, ceramic and air are the dielectrics most frequently use in capacitors in radio and radar. Although paper dielectric has the advantage of low cost it is characterized by appreciable losses becaus of dielectric hysteresis when high frequency A is applied to it. These losses are similar to magnetic hysteresis and manifest themselves in th form of heat generated within the dielectri itself. They cause imperfect capacitor actio with a consequent lowered efficiency.

On the other hand, air is free from dielectri hysteresis. Losses in air capacitors are chiefl due to the solid dielectric material used in mounting the plates and to the charging current which cause a small I^2R loss in the plates.

The dielectric strength of practically all insulating materials diminishes with an increase i temperature so that the capacitor may brea down in continuous service under voltages tha could be withstood indefinitely in intermitten service. A capacitor with a wax-impregnated dielectric seems to undergo an aging process afte a period of months, so that a breakdown finall occurs. Oil-impregnated paper is considered superior to wax-impregnated paper, particularly a high voltages.

Variable Capacitors

Variable capacitors, which employ air dielectrics, are used to adjust the resonant frequenc of tuned circuits in radar and radio sets, wher a capacitance, continuously variable within limit and having small losses, is required. The angl of rotation of the shaft determines the capacitance of the capacitor. By using semicircula rotor and stator plates, it is possible to obtain a capacitance that is approximately proportional to the angle of rotation. Capacitors using this type of construction are called straight-line capacity (SLC) capacitors.

Straight-line capacity capacitors are rarely used in receivers designed for broadcast purposes since the various stations are separated by equal frequency intervals. Using SLC in broadcast receivers would result in crowding the higher frequency stations on the tuning dial. The shape of the plates may be modified so that the capacitance is proportional to the square of the angle of rotation. When this is done, the capacitors are called straight-line-wavelength (SLW) capacitors.

The ideal capacitor for broadcast reception is one in which the resonant frequency (with a given inductance) is proportional to the angle of rotation. This type of capacitor is called a straight-line-frequency (SLF) capacitor. In it the rotor plates are scimitar-shape, being rather long and pivoted near one end. Because of the shape of the rotor plates, the SLF capacitor requires more space than other types, which is considered a disadvantage.

Electrolytic Capacitor

The principle of the electrolytic capacitor is based upon the fact that when two aluminum plates are immersed in a suitable electrolyte (such as a solution of ammonium borate or sodium phosphate) and connected to a source of direct current, a thin insulative film of oxide forms on the positive plate. This film acts as a dielectric between the positive aluminum plate and the electrolyte, which form the capacitor.

The thickness of the film in the electrolytic capacitor depends on how large a voltage is used in the forming process, with higher voltages causing thicker films. The film gradually disintegrates when the forming voltage is removed and forms again when the voltage is reapplied. During the reforming of the film, a large leakage current flows, but it soon drops to a normal value. For example, for a capacitor with a working voltage of about 450 volts, the leakage current is about 200 microamperes per microfarad while the film is forming. Over a period of several hundred hours of continuous operation, the leakage current gradually drops to only a few microamperes per microfarad. When an electrolytic capacitor is operated for a considerable period of time at a voltage appreciably lower than the forming voltage, the thickness of the film decreases and the capacitance of the capacitor rises.

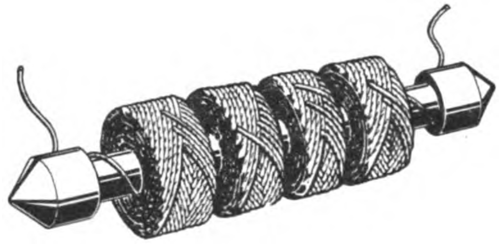
To prevent the film from completely disintegrating and ruining the capacitor, it is necessary to observe polarity when connecting an electrolytic capacitor into a circuit — that is, the positive plate of the capacitor must be connected to the positive side of the circuit. This is because electrolytic capacitors offer high resistance to current passing in one direction, but very low resistance to current in the other direction. This unidirectional-conduction characteristic makes them unusuable in alternating-current circuits. They are used chiefly in rectifier filter circuits where their comparatively large leakage losses are of little consequence.

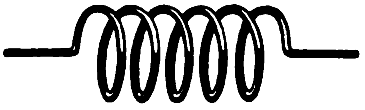
Because of the extreme thinness of the film (dielectric), it is possible to get very high capacitance in only a fraction of the space required by paper dielectric capacitors. Depending on the forming voltage, capacitances ranging from 1 to 1.5 mfd per square inch of area may be obtained with electrolytic capacitors.

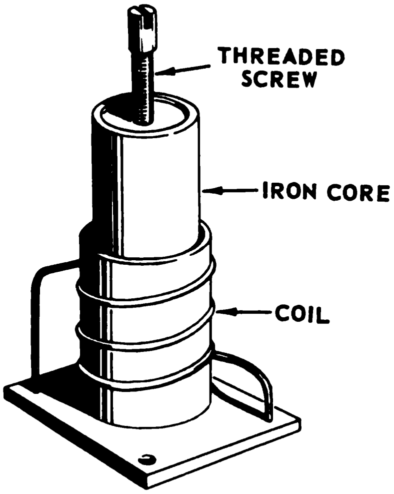
The maximum continuous working potential of electrolytic capacitors is about 450 volts. Voltages greatly in excess of this value will puncture the film. However, on reducing the voltage, the film is restored. Thus, the capacitor is said to be self-healing. Higher voltage operation is obtainable when two or more capacitors are connected in series. However, it usually is necessary to shunt each unit with a high resistance to insure that the voltage divides equally across the capacitors. Otherwise, the voltage will divide in accordance with the insulation resistance of the capacitors which, while high in value, may differ appreciably in each capacitor.

In recent years electrolytic capacitors have been developed which use a paste for the electrolyte. These capacitors are called dry electrolytic capacitors. In some types, a layer of gauze, or some other absorbent material, is saturated with the electrolyte and then rolled up between the two aluminum foils. Glycerin is usually mixed with the electrolyte because of its ability to absorb moisture.

Dry electrolytic capacitors are very compact and are available in very large values of capacitance for low voltage use. For example, one commercial dry electrolytic, which is rated at 6000


LOW FREQUENCY RF CHOKE COIL
A


HIGH FREQUENCY CHOKE COIL
B



PERMEABILITY TUNED COIL
C

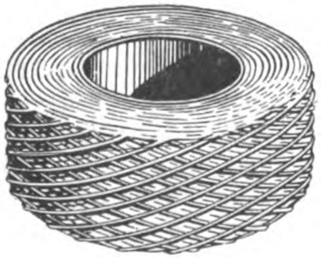

HONEYCOMB COIL
D

Figure 2–65. Various Types of Coils

mfd at a working voltage of 150 volts, has a volume of only 100 cubic inches. Those with working voltages of 450 volts have about a microfarad per cubic inch of volume, or about 15% of the space required by paper capacitors for the same voltage. The life of a dry electrolytic capacitor depends considerably upon the conditions of use.

Coils

Because of their property of inductance, coils of varied forms are used in radio and radar. Inductance, remember, is the property of a circuit that causes it to oppose any change in current flowing through it. Mathematically, it is equal to the number of lines of flux linkage per ampere divided by the current producing the flux that is,

$$L = \frac{N\phi}{I} \times 10^{-8} \text{ henrys}$$

where N is the number of turns in the coil, and ϕ is th magnetic flux linking the coil and produced by the current I. The term 10^{-8} converts the inductance int henrys.

This formula applies only to coils using ai cores. When an iron core, for example, is inserted into the coil (or in cases where the permeability of a core in a coil is changed), the inductance of the coil changes. The inductance o a coil is equal to

$$L = \mu L_0$$

where μ is the permeability of the core and L_0 is th inductance of the coil with an air core.

In radio, the use of iron-core coils is limited chiefly to power and audio frequency devices such as interstage transformers and choke coils Furthermore, their use is limited to frequencie below 150 kc unless especially prepared iron i used in the core. This special preparation consists of iron dust mixed with a suitable binder The binder serves to insulate the particles o iron from each other. These special cores (Figur 2–65C) are common in radar receiver circuits.

At high frequencies, the chief difficulty wit iron cores is increased eddy current losses. Edd current losses in iron cores are proportional t the square of the frequency and to the square o thickness of the laminations in the core. The become very great at high frequencies. Thes eddy currents in the core also induce currents o their own that act as a shield and prevent th main magnetic flux from penetrating very deepl into the laminations or particles of iron in the core. This causes the effective permeability of the iron to decrease with increased frequency.

Eddy currents are induced in a particle of iron through which alternating current flows. These currents flow around the outer periphery of the iron particle in a direction in opposition to the flux that produced them. Since the demagnetizing effect of eddy current is greatest along the center line of the iron particle, the flux density there is minimum and the flux at the surface is maximum. Since the permeability of iron is the ratio of the flux density to the magnetizing force, this constitutes a reduction in the permeability of the iron.

Eddy currents can be reduced by using sheet steel rolled thinner than .001 inch. Since it is very difficult to roll steel this thin, a still better method is to divide the core material (as explained previously) into fine particles and then mix the particles with an insulating compound. Since this effectively reduces the eddy current losses and, at the same time, increases flux penetration, the effective permeability of the iron in the core is increased.

As pointed out previously, in any coil there is a small amount of capacitance between each turn and all other turns, and from each turn to ground. These individual capacitances, which are called stray capacitances, are equivalent to small capacitors shunted across the terminals of the coil. Coils used in resonant circuits at the higher radio frequencies are nearly always single-layer coils, since they have low distributed capacitance at these frequencies. Multi-layer coils are seldom used at these frequencies, because of high distributed capacitance, unless they are *bank-* or *honeycomb-wound* (Figure 2–65).

In a multilayer coil wound in the ordinary fashion, the voltage between the first turn of one layer and the last turn of the next layer is quite high. This high voltage causes a large value of *displacement* current to flow through the capacitance between these turns and greatly increases the distributed capacitance of the coil. However, if the coil is bank-wound, then the distributed capacitance will be considerably below that of a similar layer-wound coil having an equal number of turns. In the honeycomb-wound coil, the layers are wound in such a way that there is a space between adjacent turns and the wires of one layer cross at an angle those of the layer beneath. This reduces the area of contacts and lowers the capacitance between the layers. The distributed capacitance is less than would be obtained with the ordinary form of layer winding of similar dimensions. Radio-frequency choke coils (Figure 2–65A) commonly use honeycomb windings.

SERIES AC CIRCUITS

Resistance Only

When an alternating voltage is applied to a series AC circuit containing resistance only, the current and voltage in the circuit are in phase. One method of showing the in-phase relationship is by sine waves. The current in the circuit is sinusoidal and will be maximum at the same time as the voltage is maximum. Another method of showing the same relationship is by a vector diagram. The usefulness of the vector method of representing phase of currents and voltages will become more apparent later in this manual. In the vector diagram in Figure 2–66, the current and voltage vectors coincide, showing that they are in phase, and differ only in amplitude. (Refer again to Chapter 1 for a discussion of vectors and their manipulation.)

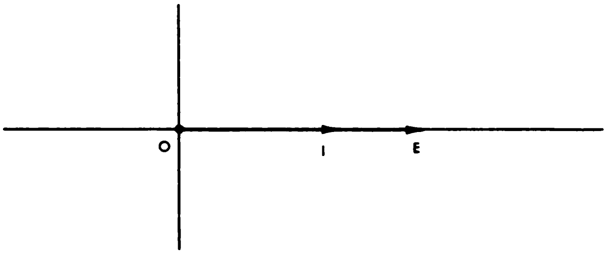


Figure 2–66. Vector Representation of Current and Voltage in a Resistive Circuit

Since a resistor presents the same opposition to the flow of alternating current as it does to direct current, current flow in an AC circuit containing resistance only is equal to the voltage divided by the *impedance*.

Impedance is a term embracing the total opposition to current flow in an AC circuit. In a circuit containing resistance only, resistance and impedance mean the same thing. Ohm's law for AC is

$$I = \frac{E}{Z}$$

where I equals the current in amperes, E the voltage in volts, and Z the impedance in ohms.

In some AC circuits, impedance includes not only resistance but also reactance, both capacitive and inductive. (The relation between Z, R and X_L or X_c is to be taken up shortly.)

The formula that gives the relation between current, voltage, and impedance in an AC circuit is called Ohm's law for alternating current. This formula is based on effective values, but is equally true for maximum, average, or instantaneous values of current and voltage.

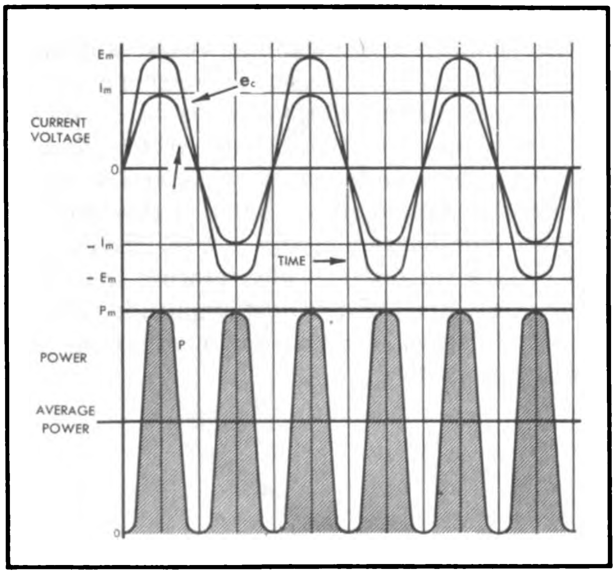


Figure 2–67. Current, Voltage and Power In a Resistive Circuit

Power in a resistive series AC circuit is equal to the product of the effective voltage and the effective current, and is equal to

$$P = EI$$

To understand how this formula is derived, refer to Figure 2–67 showing the relation between the current and voltage, and the instantaneous power resulting from them in a series resistive AC circuit. The power at any instant is the product of the current and voltage at that instant ($P = ei$). At the instant when $e = E_m$, i also is I_m and $P_m = E_m \times I_m$. When e is zero, i is zero and the power is zero. Then as e increases in a negative direction, i increases negatively and their product increases in a positive direction (the product of two negative numbers is positive). Thus P varies between $P_m = E_m \times I_m$ and *zero* at a frequency twice that of the voltage and current. The line drawn through the center of the power sine wave shows the average power. To determine the average power mathematically, it is necessary to perform the following operation:

$$P_{av} = \frac{P_m}{2} = \frac{E_m I_m}{2} = \frac{I_m}{\sqrt{2}} \times \frac{E_m}{\sqrt{2}}$$

The values represented by $\frac{E_m}{\sqrt{2}}$ and $\frac{I_m}{\sqrt{2}}$, are the effective values of voltage and current respectively. When multiplied together, they give the average power in a resistive circuit which, as stated previously, is,

$$P = EI$$

and represents the power being consumed in the form of heat.

Capacitance Only

When an alternating voltage is applied to a series AC circuit containing capacitance only, there is no opposition and maximum current flows at the first instant. There is no charge on the capacitor, so there is no voltage across it. As a charge builds up on the capacitor, the voltage across the capacitor rises. This voltage opposes the applied voltage and current decreases. When the charge on the capacitor is maximum, current is zero. At this point the applied voltage is reversing polarity. As shown by the sine waves in Figure 2–68, the current leads the voltage across the capacitor by 90°.

In the vectorial representation in Figure 2–68, the voltage E is the reference vector. I is drawn upward at 0, which in vector representations is called the leading position. It could have been drawn downward at 90°, but the practice is to draw the leading component in a counterclockwise direction. One point to notice here which has a bearing on power, as you will see later, is that the product of E and I is alternately positive and negative.

Current leads voltage in a capacitive circuit because the capacitor, as you saw earlier, opposes any change in voltage. The opposition that the capacitor offers to the flow of current is called capacitive reactance. It is indicated as X_c and, like resistance, is measured in ohms by the Ohm's law formula,

$$I = \frac{E}{Z} = \frac{E}{X_c}$$

where I is the current in amperes, E the emf in volts, and X_c the capacitive reactance in ohms.

Notice that the impedance in a capacitive circuit is represented by X_c, the capacitive reactance. So far you have seen that impedance in an AC circuit includes resistance and capacitive reactance.

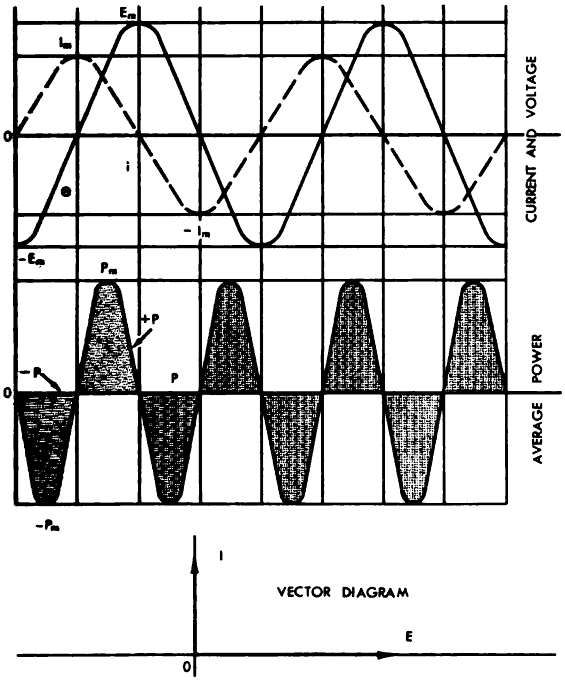


Figure 2–68. Current, Voltage and Power In a Capacitive Circuit

In resistive circuits you learned that the power consumed in the circuit is expressed by the equation, $P = EI$. This power is the true power in the circuit, and is the power in any circuit in which the current and voltage are in phase. For that matter, the formula is the same power formula you applied earlier in DC circuits. However, in AC circuits in which current and voltage are not in phase, as is the case when there is capacitance (or inductance) in the circuit, the true power is not equal to the product of E times I.

The reactive parts of the circuits, either the capacitance or the inductance, or both, return some energy to the circuit instead of converting it to heat. Therefore, power in a reactive circuit depends not only on the voltage and current, but also upon the reactive components of the circuit. In a theoretical circuit containing a pure capacitance, that is, one in which there is no resistance at all, no power is consumed.

To understand how capacitance affects power in a reactive circuit, refer to the power diagram in the illustration. When *P* in the power diagram is positive, it means that the capacitor is taking power from the circuit. When *P* is negative, the capacitor is giving power back to the circuit. Since the positive power lobes are symmetrical with the negative power lobes, the power returned to the circuit is equivalent to the power taken from it, and the average power is zero. In other words, a pure capacitance does not consume power.

Inductance Only

In a series AC circuit containing only inductance, the current and voltage, like those in a capacitive circuit, are not in phase. Because the inductance opposes any change in current, the voltage across the inductance leads the current by 90° — that is, the voltage reaches its maximum value 90° ahead of the current. The sine waves of voltage and current in Figure 2–68 should be interchanged for an inductive circuit.

By reversing *E* and *I* in the vector diagram in Figure 2–68, you will get the vector representation of the inductive circuit. Power consumption in a purely inductive circuit resulting from the out-of-phase relationship between the current and voltage, like that in a capacitive circuit, is also zero, since all power is returned to the circuit by the inductance in a circuit that has no resistance.

Circuit Containing Resistance and Inductance

In a series AC circuit that contains both resistance and inductance, the voltage across the inductance leads the current by 90°, and the voltage across the resistance is in phase with the current. Because of the phase difference caused by the inductive component, the total impedance in the circuit is not equal to the arithmetic sum of the ohmic values of the resistance and the inductive reactance, but is equal to their vector sum, and is expressed by the formula,

$$Z = \sqrt{R^2 + X_L^2}$$

Similarly, the total voltage drop in the circuit is the vector sum of the individual voltage drops, and is equal to

$$E = \sqrt{E_R^2 + E_L^2}$$

To understand how these formulas are derived, it is necessary to establish certain geometrical relationships by drawing a vector diagram involving the circuit components. Figure 2–69 shows a circuit with a resistance *R* in series with an inductive reactance. The vector diagram shows the voltage relationships of the circuit components. In the vector diagram, the current *I* is the reference vector or the 0° line, since this is a series circuit. In a series circuit the current is the same throughout the circuit.

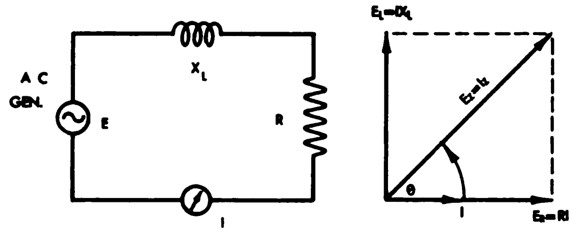


Figure 2–69. Circuit and Vector Diagram with Inductance and Resistance

Since the voltage across *R* is in phase with the current, it is drawn to coincide with *I*. Since the voltage across the inductance is equal to IX_L, and since it leads the current *I* by 90°, it is drawn perpendicular to E_R. The resultant of these two emf's in series, according to the parallelogram of forces method, is their vector sum. The diagonal E_Z of the parallelogram is the total voltage drop in the circuit. By Kirchhoff's laws, it is equal to the applied voltage *E*. The magnitude of *E* can be obtained from the right triangle whose two legs are E_R and E_L and whose hypotenuse E_Z is equal to *E*. Thus,

$$E = \sqrt{E_R^2 + E_L^2}$$

Since the voltage drop across E_R is equal to *IR*, and the drop across *L* is equal to IX_L, it is evident that the total voltage drop is given by the equation,

$$E = \sqrt{(IR)^2 + (IX_L)^2}$$

By factoring,

$$E = \sqrt{I^2 (R^2 + X_L^2)}$$

By taking the square root,

$$E = I\sqrt{R^2 + X_L^2}$$

But since $E = IZ$

the total impedance Z is

$$Z = \sqrt{R^2 + X_L^2}$$

In determining a vector quantity, it is necessary to consider phase as well as magnitude. Therefore, in addition to magnitude, the usual designation of voltage (or impedance) includes the phase angle θ between it and the quantity compared with it. One way of expressing the phase angle vectorially is in the polar form. By this method the applied voltage in a circuit is represented as $E\angle\theta°$ and the impedance as $Z\angle\theta°$.

A vector can also be expressed in rectangular form by giving it horizontal and vertical components. From Chapter 1, you recall that you designated the horizontal component as the real component, and the vertical component as the imaginary component. In order to indicate the 90° phase difference between them, you used the notation *j*. Thus, instead of the polar form $E\angle\theta°$, you can write the applied voltage in rectangular form as $E_R + jE_L$, and instead of the polar form $Z\angle\theta°$, you can write the total impedance in the rectangular form $R + jX_L$. These forms do not imply that the terms jE_L and jX_L are only imaginary, for they are as real as E_R and *R*. This system of notation is merely a convenient method for handling these quantities.

Figure 2–70 shows the current, voltage and power relations in an AC circuit that contains resistance and inductance connected in series. Since the voltage leads the current by θ degrees, there is a period of θ degrees during each power cycle where the product of *e* and *i* is negative — that is, when power is being returned to the circuit. Note that the negative power lobe is considerably smaller than the positive lobe. Hence, you see that the true power is neither *EI* nor zero. Since the negative power lobe is smaller than the positive lobe, the average of the two is not zero. True power is not equal to *EI*, for the product of these quantities is not equal to I^2R. This latter is the power dissipated by the resistor. Remember, the only element that actually consumes power in a circuit is resistance. In this circuit the product of *E* and *I* is the *apparent* power. The ratio of true power to apparent power is called the *power factor* and is expressed as,

$$\text{pf} = \frac{\text{P (true power)}}{\text{EI (apparent power)}}$$

Since $E = IZ$ and the true power P is equal to I^2R, the power factor ratio can also be written,

$$\text{pf} = \frac{I^2R}{I^2Z} = \frac{R}{Z}$$

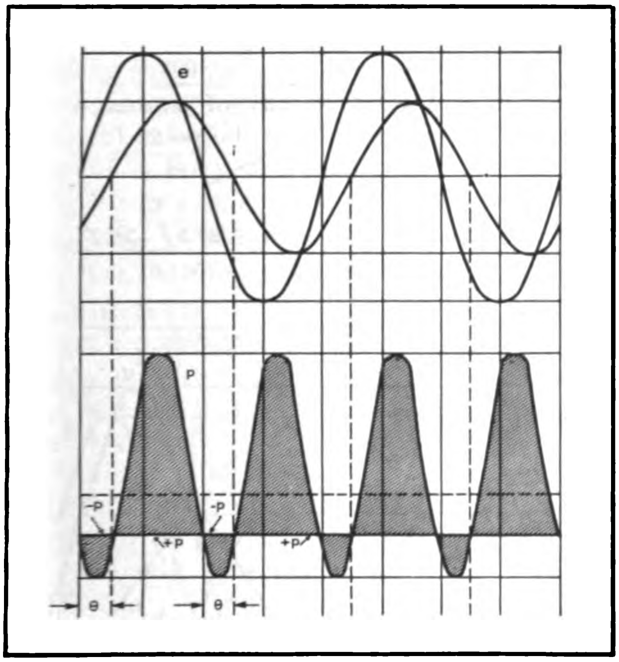


Figure 2–70. Current, Voltage and Power Relations in an RL Circuit

Earlier you saw that it was possible to express the relation of the quantities Z, X_L and R by the expression, $Z = R + jX_L$. In a graphical representation of this relationship, which is called an impedance triangle, you can see that $R = Z \cos \theta$. (See Figure 2–71.)

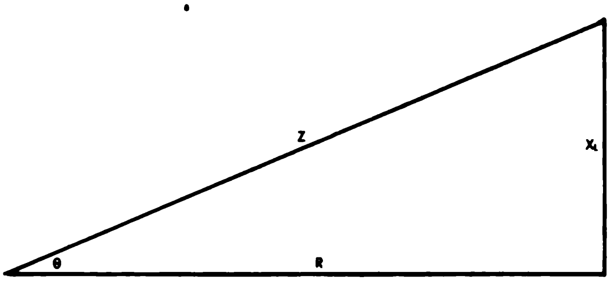


Figure 2–71. The Impedance Triangle

By substituting this value for R in the power factor formula, you obtain the formula

$$\text{pf} = \frac{Z \cos \theta}{Z} = \cos \theta$$

The power factor of the circuit is the cosine of the phase angle between the current and voltage in the circuit. Thus, the power factor is a number whose minimum value is zero and whose maximum value is one. In terms of this definition, the power in a reactive circuit, then, is equal to,

$$P = EI \cos \theta$$

Circuit Containing Resistance and Capacitance

In a circuit containing resistance and capacitance, the ohmic values of the resistance and the reactance, like the values of the resistance and the reactance in a resistive-inductive circuit, do not add arithmetically, but add vectorially. In a circuit containing resistance and capacitance connected in series, the total opposition to current flow is

$$Z = \sqrt{R^2 + X_c^2}$$

The effective voltage resulting from the out-of-phase relationship is

$$E = \sqrt{E_R^2 + E_c^2}.$$

Both of these formulas are easily proved by the impedance triangle and the parallelogram force method of vector representation. These methods are shown in Figure 2–72. The method

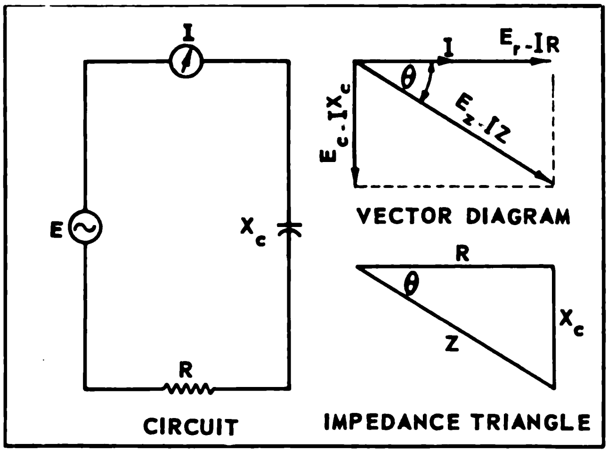


Figure 2–72. Resistance and Capacitance in Series AC Circuits

is similar to that employed in resistive-inductive circuits.

The vector quantities just illustrated can be expressed in polar or reactangular coordinates. The effective voltage in a resistive-capacitive circuit can be written

$$E_Z\underline{/-\theta^\circ} = E_R - jE_c$$

and the total impedance

$$X\underline{/-\theta^\circ} = R - jX_c.$$

For finding power in resistive-capacitive circuits, use the power formulas developed in the resistive-inductive circuits. This is possible since the cosine of a negative angle is equal to the cosine of the corresponding positive angle.

Examples

To understand the principles discussed in connection with AC circuits, study the solutions of the following circuit problems.

Problem 1

In the circuit in Figure 2–73, find the current flow and power dissipated across the resistor.

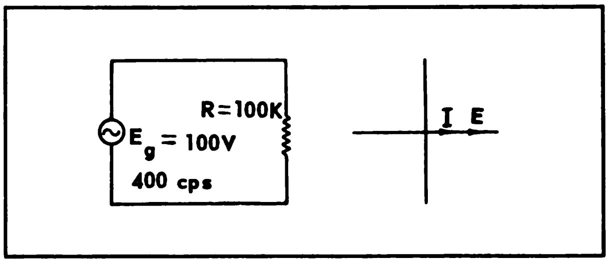


Figure 2–73. Series Resistive Circuit

Solution

Since the load in the circuit is a resistance, the current flow is in phase with the voltage output of the generator. By Ohm's law, the current is

$$I = \frac{E}{R}$$

$$I = \frac{100}{100 \times 10^3}$$

$$I = 1 \text{ ma}$$

To find the power dissipated across the resistor, substitute in the power formula,

$$P = EI \cos\theta$$

$$P = (100)\ (.001) \cos\theta$$

Since in a resistive circuit the phase angle equals 0°

$$\cos\theta = \cos 0^\circ$$

From tables

$$\cos 0^\circ = 1$$

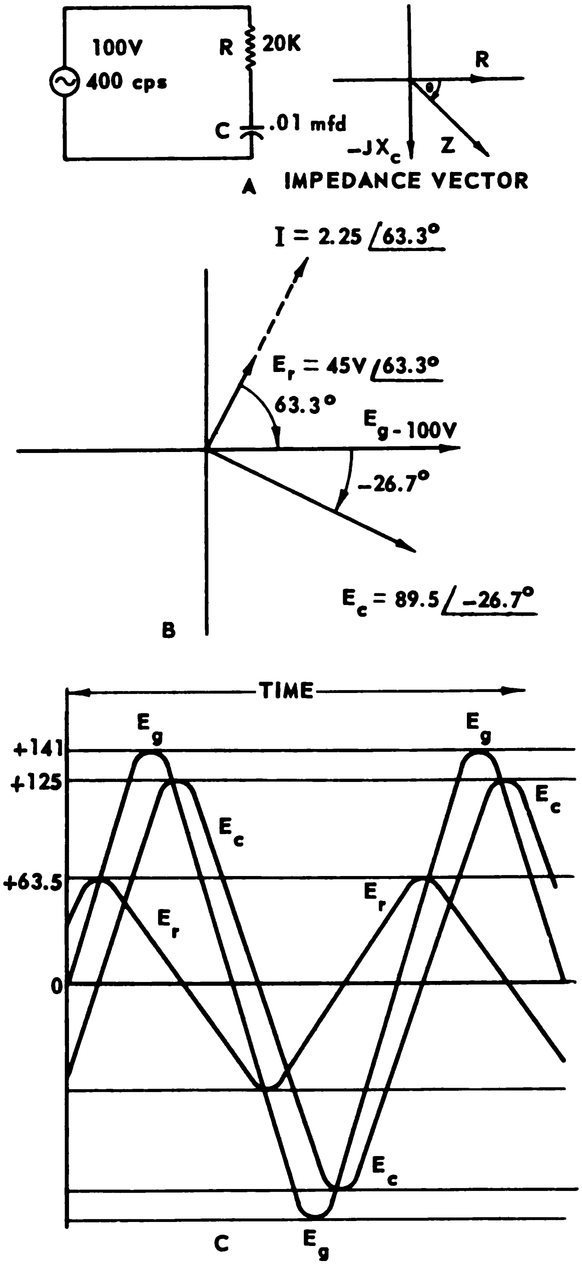


Figure 2–74. Series Resistive-Capacitive Circuit

Therefore, the power is,

$$P = (100)\ (.001)\ 1$$

$$P = 0.1 \text{ watt}$$

$$pf = \cos\theta = \cos 0^\circ = 1$$

Problem 2

When a capacitor or an inductor is placed in series with a resistor in an AC circuit, the current is not in phase with the applied voltage. In the circuit in Figure 2–74A, the reactance of the .01 capacitor at 400 cycles is $\frac{1}{2\pi fC}$ or 39.8K, and the impedance of the load Z is

20K − j39.8K (see the vector diagram in Figure 2–74). Find the current through and the voltage across the resistor (E_R) and the voltage across the capacitor (E_C).

Solution

You can solve for the current using either the rectangular form or the polar form for the impedance. Method 1 uses the rectangular form and Method 2 the polar form.

Method 1

$$I = E/Z$$

$$= \frac{100}{20{,}000 - j39{,}800}$$

$$= \frac{100\ (20{,}000 + j39{,}800)}{(20{,}000 - j39{,}800)\ (20{,}000 + j39{,}800)}$$

$$= \frac{(2 + j3.98)\ 10^6}{4\times10^8 + j7.95\times10^8 - j7.95\times10^8 - j^2 15.8\times10^8}$$

$$= \frac{(2 + j3.98)\ 10^6}{19.8 \times 10^8}$$

$$= \frac{(2 + j3.98)\ 10^{-2}}{19.8}$$

$$I = (0.1005 + j0.205)\ 10^{-2}$$

The current I is,

$$I = 2.28/63.9^\circ \text{ milliamperes}$$

Method 2

$$Z = 20K - j39.8K = 44.5K \angle -63.3^\circ$$

$$I = E/Z$$

$$I = \frac{100}{44.5K \angle -63.3^\circ}$$

The current I is,

$$I = 2.25 \angle 63.3^\circ \text{ milliamperes}$$

A comparison of the two methods shows that the solution by polar form is much simpler than the solution by rectangular form. For this reason, the polar form is used in this manual, except in cases where vectors are to be added or subtracted. The vector representation of the current is shown in Figure 2–74B.

From the value of current obtained, ($I = 2.25 \angle 63.3^\circ$), you can find the voltage across the resistor as follows:

$$I = 2.25 \angle 63.3^\circ \text{ milliamperes}$$

$$I = 2.25 \angle 63.3^\circ \times 10^{-3} \text{ amperes}$$

$$R = 20{,}000 = 20 \times 10^3 \text{ ohms}$$

$$E_R = IR = 2.25 \angle 63.3^\circ \times 10^{-3} \times 20 \times 10^3$$

Hence, the voltage across R is,

$$E_R = 45 \angle 63.3^\circ \text{ volts.}$$

The answer means that the voltage across the resistor is in phase with the current through the resistor, but leads the applied voltage by 63.3°. The sine wave representations of these voltages are shown in Figure 2–74C.

The find the voltage across the capacitor, use Ohm's law and solve as follows:

$$E_C = IX_C$$

$$= 2.25 \angle 63.3^\circ \text{ ma} \times 39.8 \times 10^3 \angle -90^\circ$$

The voltage across the capacitor is

$$E_C = 89.5 \angle -26.7^\circ \text{ volts } (I_C = 2.25 \angle 63.3^\circ)$$

Figure 2–74B shows the relationship of the circuit voltages and current vectorially, while Figure 2–74C shows them graphically. The generator voltage, contrary to usual practice, is used as reference here in order to show which voltage leads and which lags the generator voltage.

The apparent power delivered to the circuit is *I^2Z*, or 224 milliwatts, while the true power consumed is *I^2R* or 101 milliwatts. The power factor (true power/apparent power) is equal to 101/224 or 0.451. By the power formula *$pf = \cos\theta$*, it is equal to cos 63.3° or 0.449. The difference in results is due to the different mathematical approach.

Problem 3

In the circuit in Figure 2–75A, find the voltage across the inductor (E_L), and the voltage across the resistor (E_R).

Solution

The current which flows through a pure inductor lags the voltage across it by 90°. The reactance of the inductor, which is directly proportional to the frequency, is equal to $2\pi fL$. The values in the circuit make the reactance of the inductor $2\pi \times 0.159 \times 10{,}000$ or 10,000 ohms. The impedance of the circuit is equal to $10{,}000 + j10{,}000$ or $14.14 \times 10^3 \angle 45^\circ$. (See the vector diagram in Figure 2–75B.)

First find the current flow as follows:

$$I = E/Z$$

$$= \frac{100 \angle 0^\circ \text{ volts}}{14.14 \angle 45^\circ \text{ kilohms}}$$

$$= 7.07 \angle -45^\circ \text{ milliamperes}$$

By Ohm's law,

$$E_L = IX_L$$

$$= 7.07 \angle -45^\circ \times 10^{-3} \times 10 \angle 90^\circ \times 10^3$$

The voltage across L is,

$$E_L = 70.7 \angle 45^\circ \text{ volts}$$

$$E_R = IR = 7.07 \angle -45^\circ \times 10^{-3} \times 10 \times 10^3$$

The voltage across R is,

$$E_R = 70.7 \angle -45^\circ \text{ volts.}$$

Figure 2–75C shows these values vectorially. Figure 2–75D shows a graphical representation of the quantities just calculated.

The voltage across the inductor leads the generator voltage by 45°, and the voltage across the resistor lags the generator voltage by 45°. Notice also that the voltage across the resistor lags the inductor voltage by 90°, and the total current flow through the circuit lags the applied voltage by 45°.

In all cases of series AC circuits, the impedance into which the generator is working is equal to the vector sum of the reactances and resistances

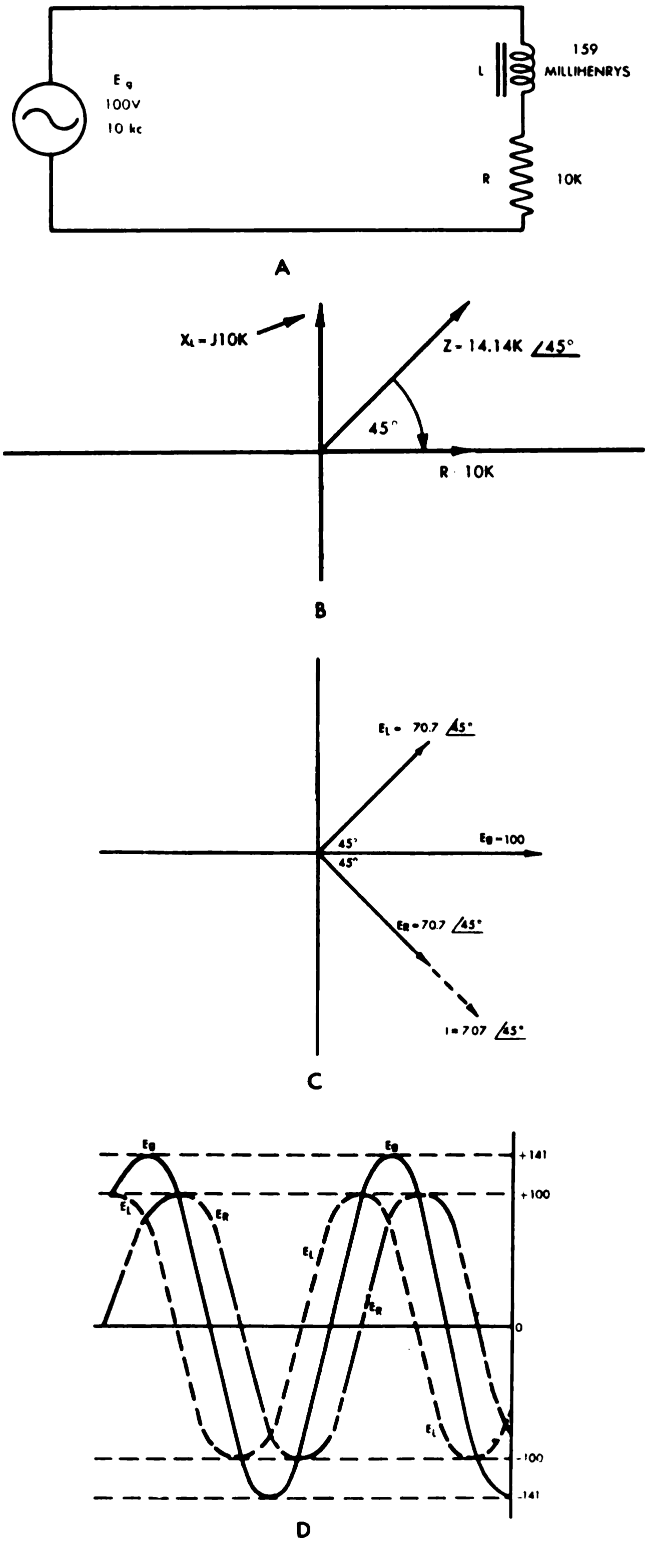


Figure 2–75. Series Resistive-Inductive Circuit

around the loop. To understand the mathematics involved, study the circuit and the impedance diagram in Figure 2–76.

Problem 4

A generator with an output of 100v at 10 kc is working into a load in the circuit; find the impedance into which the generator works.

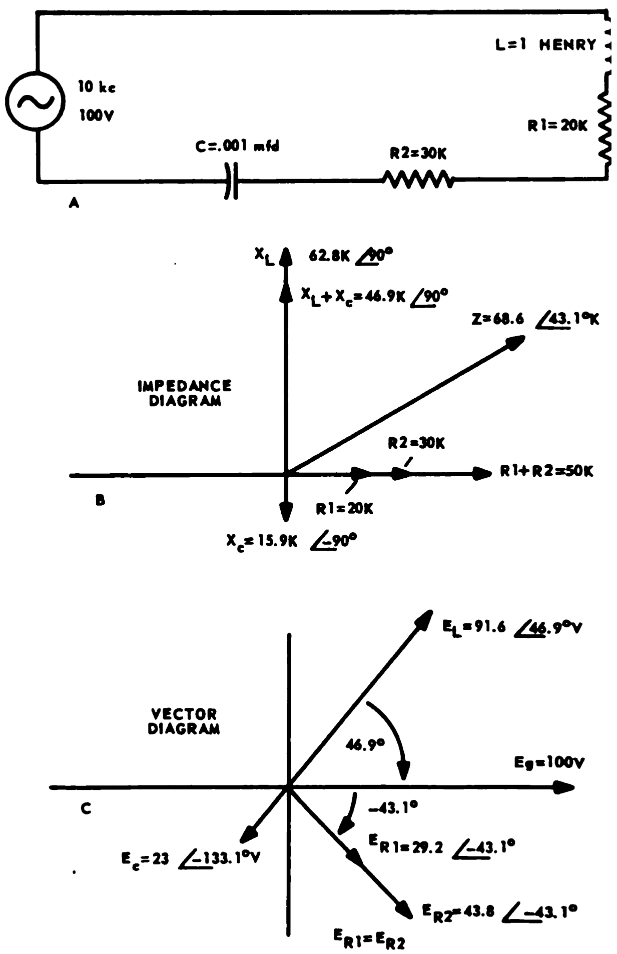


Figure 2–76. R, L, and C Series Circuit

Solution

$X_L = 62{,}800$ ohms

$X_C = 15{,}900$ ohms

The total impedance (Z) of the circuit is

$$Z = X_L + R_1 + R_2 + X_C$$

Substituting,

$$Z = [j62.8 + 20 + 30 + (-j15.9)] \times 10^3$$

The total impedance is

$$Z = (50 + j46.9) \times 10^3 \text{ ohms}$$

In polar form Z is

$$Z = 68.6 \times 10^3 \angle 43.1^\circ \text{ ohms.}$$

This circuit is said to be inductive, since th inductive reactance in it is greater than th capacitive reactance. In this type of circuit yo can expect to find the following conditions: *First,* the current will lag the generator voltage; *second,* the voltage across the capacitor will be 180° out of phase with the voltage across the inductor; and *third,* the voltage across the resistors will lag the generator voltage by the same angle as the current.

Find the current first, and then the voltages.

$$\text{The current, } I = \frac{100}{(68.6 \angle 43.1^\circ)\, 10^3} = 1.46 \angle -43.1^\circ \text{ ma}$$

$$E_c = 1.46 \times 10^{-3} \angle -43.1^\circ \times 15.9 \times 10^3 \angle -90^\circ$$
$$= 23.2 \angle -133.1^\circ \text{ volts}$$
$$E_L = 1.46 \times 10^{3} \angle -43.1^\circ \times 62.8 \times 10^3 \angle 90^\circ$$
$$= 91.6 \angle 46.9^\circ \text{ volts}$$
$$E_{R1} = 1.46 \times 10^{-3} \angle -43.1^\circ \times 20 \times 10^3 = 29.2 \angle -43.1^\circ \text{ volts}$$
$$E_{R2} = 1.46 \times 10^{-3} \angle -43.1^\circ \times 30 \times 10^3 = 43.8 \angle -43.1^\circ \text{ volts}$$

Figure 2–76C is a vector diagram of the voltages and currents in the circuit at A. They are drawn with the generator voltage (E_g) as reference.

Series Resonance

You have learned that the effective reactance in a series circuit containing both inductive and capacitive reactances is their difference. The circuit may act capacitively or inductively, depending upon which is the larger of the two. This fact expressed mathematically is $Z = R + j(X_L - X_c)$. Since X_c decreases and X_L increases with an increase in frequency, there is a frequency at which a given coil and capacitor will have equal reactances. This frequency is called the *resonant* frequency. The condition that exists in the series circuit when $X_c = X_L$ is known as series resonance. At series resonance the j term drops out of the impedance formula, and the impedance is the resistance only. Since there is zero reactance, the current is in phase with the voltage, and the power factor is one. Furthermore, since the only impedance is the resistance, the current becomes maximum at resonance.

In a series resonant circuit containing no resistance, the frequency at which the circuit is resonant is

$$f = \frac{1}{2\pi\sqrt{LC}}$$

f is the frequency in cycles
L is inductance in henrys
C is capacitance in farads.

The following is the derivation of the series resonance formula:

At series resonance,

$$X_L = X_c$$

But $$X_L = 2\pi fL \text{ and } X_c = \frac{1}{2\pi fC}$$

Therefore, $$2\pi fL = \frac{1}{2\pi fC}$$

or $$f^2 = \frac{1}{4\pi^2 LC}$$

Taking the square root,

$$f = \frac{1}{2\pi\sqrt{LC}}$$

At frequencies above resonance the inductive reactance in a circuit is greater than the capacitive reactance and the current lags the applied voltage. The circuit is said to be *inductive.* Below resonance, the capacitive reactance is greater than the inductive reactance and the current leads the voltage. The circuit is *capacitive.* Figure 2–77 shows a circuit that is resonant at 1000 kilocycles, and the graph of the current versus frequency of the applied voltage. Below 1000 kc the capacitive reactance increases and the current drops off rapidly. Above resonance, the current decreases due to the increase in inductive reactance. In practical circuits, the resistance R is low. Although it appears as a separate component in the circuit, it really represents the resistance of the inductor and the connector leads. At resonance, R is the only opposition to current flow; therefore, very high currents flow at resonance in a series resonant circuit. At frequencies that differ appreciably from the resonant frequency, current flow is practically independent of the resistance, being determined chiefly by the reactance.

In the series resonant circuit shown in Figure 2–77, the reactances X_L and X_c are each equal to 940 ohms and the current flow is approximately 0.133 ampere. Hence, the voltage across both E_L and E_c is equal to 125 volts. The drop across resistance R is only one volt, the applied voltage in the circuit. Voltages E_c and E_L are 180° out of phase and cancel each other when the voltages in the circuit are added. The ratio of E_L (or E_c) to E_R is 125; this is known as the Q of the circuit. Since R, the resistance in the circuit, is mostly the resistance of the coil, you will usually find the term Q applied to the coil in a circuit. The Q of a coil is the ratio of X_L to R at resonance.

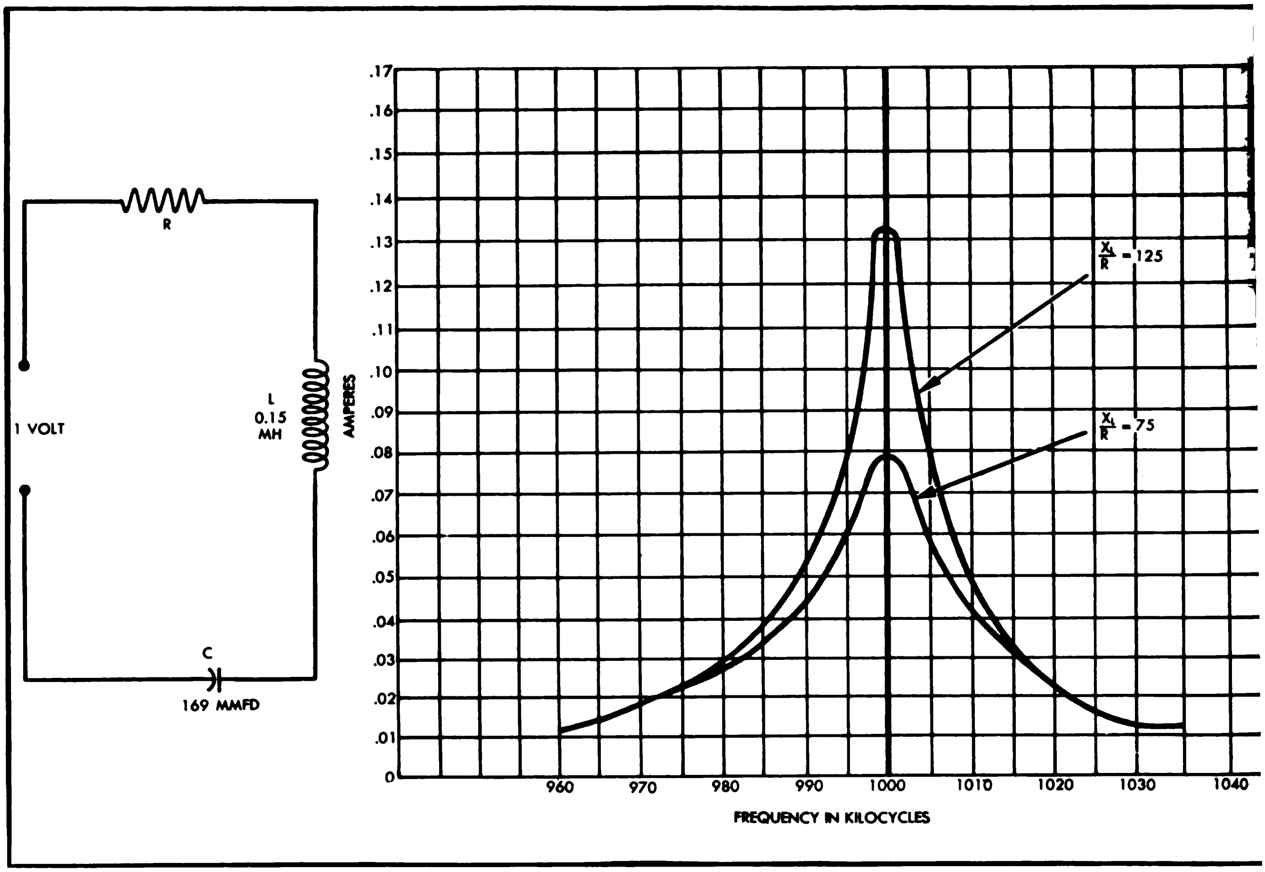


Figure 2–77. Series Resonance

The second curve shown in Figure 2–77 is for a lower *Q* circuit. The *Q* in it is 75. The lower *Q* of the second curve is due to a greater *R* in the circuit.

To understand the principles of series resonance, study the following examples.

Examples

Problem 1

In the circuit in Figure 2–78, E_g is equal to 100 volts. Using the values indicated, (a) find the resonant frequency (f_r), and I, E_L, E_c, and E_R, at resonance, and (b) show the magnitude of I, E_L, E_c, and E_R both graphically and vectorially.

Solution

a. Solving for the values required

1. Substituting the values given for this problem in the equation,

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$f_r = \frac{0.159}{\sqrt{28 \times .01 \times 10^{-11}}} = \frac{0.159}{\sqrt{28 \times 10^{-8}}} = \frac{0.159}{10^{-4}\sqrt{28}}$$

$$= \frac{0.159 \times 10^{4}}{\sqrt{28}} = \frac{1590}{5.3} = 300 \text{ cps}$$

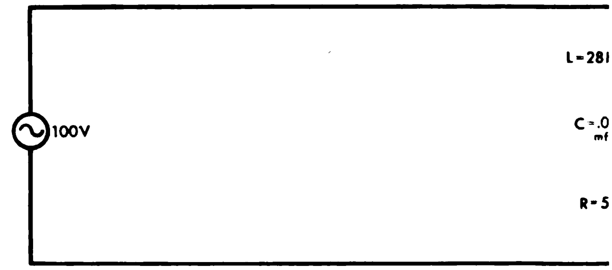


Figure 2–78. Series Resonant Circuit

2. By previous formulas,

$X_L = 53K$

$X_c = 53K$

$Z = 5K + j53K - j53K \text{ or } 5K$

$Z = 5K \angle 0°$

$$I = \frac{100}{5K \angle 0°} = 20 \text{ ma} \angle 0°$$

$E_c = (53 \times 10^{3} \angle -90°) \times 20 \times 10^{-3} \angle 0°$

$= 1060 \angle -90° \text{ volts}$

$E_L = (53 \times 10^{3} \angle 90°) \times 20 \times 10^{-3} \angle 0°$

$= 1060 \angle 90° \text{ volts}$

$E_R = 5 \times 10^{3} \times 20 \times 10^{-3} \angle 0° = 100 \angle 0° \text{ volts}$

b. **Showing graph and vector diagram.**

Draw the graph and vector diagram as shown in Figure 2–79. From the graph of the voltage across the circuit elements at resonance at A, you can see that since E_c and E_L are equal and 180° out of phase at any time, $E_L + E_c = 0$. Another point to notice is that E_c and E_L are much greater than the applied voltage. This is significant, since a capacitor with a peak voltage rating of 250 volts would break down if it were placed in this circuit even though the applied voltage is only 100 volts. A further consideration is that the voltage across the resistance in this circuit is equal to the applied voltage.

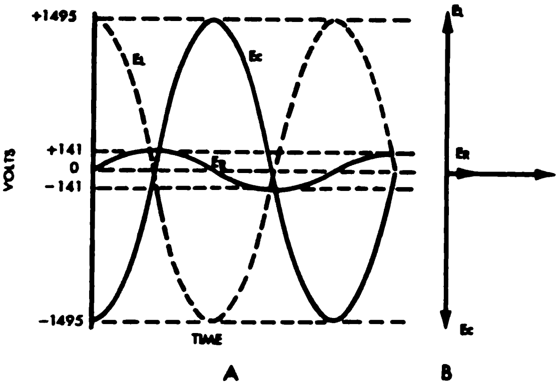


Figure 2–79. Graph and Vector Diagram for Problem 1

Problem 2

In the first problem it was assumed that the inductor had no resistance at all. This is not true, for the wire with which the inductor is wound always has some resistance. For accurate computations it is necessary to consider the ohmic resistance of the wire itself in an inductor. It is considered as in series with the inductor and is handled in the same manner as an external resistance. When this resistance is considered, it is impossible to measure the inductor voltage (E_L), since the voltage across the inductor is actually $E_{R-IN} + jE_L$, where E_{R-IN} is the voltage drop that results from the internal resistance of L.

In understanding more completely the effect of the internal resistance of a coil on circuit operation, solve for the values required in the previous problem when the coil is considered to have an internal resistance of 520 ohms as illustrated in Figure 2–80.

Solution

From the previous problem,

$$f_r = 300 \text{ cps}$$
$$X_L = 53\text{K ohms}$$
$$X_c = 53\text{K ohms}$$

Including the internal resistance of the coil, the impedance of this circuit is,

$$Z = j53K + 0.520K - j53K + 5K = 5.52K$$

Then finding the other required values at resonant circuit condition,

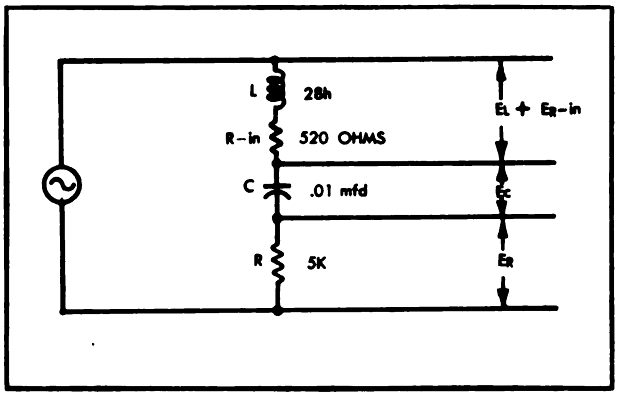


Figure 2–80. Circuit with R of Coil Included

$$I = \frac{100}{5.52K} = 18 \text{ ma}$$
$$E_L = 954 \underline{/90^\circ} \text{ volts}$$
$$E_c = 954 \underline{/-90^\circ} \text{ volts}$$
$$E_{R-IN} = 9.36 \text{ volts}$$
$$E_R = 90.64 \text{ volts}$$

In this circuit, the real voltage across L is equal to $E_{R-IN} + jE_L$ or $9.36 \underline{/0^\circ} + 954 \underline{/90^\circ}$ which is equal to $954.04 \underline{/89.4^\circ}$ or practically the same as E_L.

In comparing the results obtained with those in the previous problem, notice that the internal resistance actually lowers the voltage developed across $E_L + E_c$. Also notice that the current which was 20 ma in the first problem becomes 18 ma. You can see just how the internal resistance affects this circuit by studying the graph and vector diagram in Figure 2–81.

Continuing with the effect of the internal resistance, it is well to note that if this resistance were the total resistance in this series-resonant circuit, the current would be equal to 100/0.520K, or 192 ma. Further, if an inductor having the same inductance, but wound with

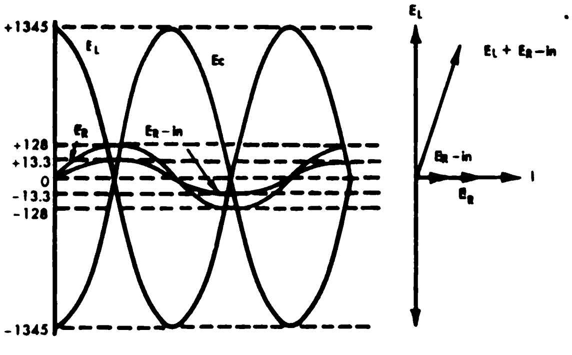


Figure 2–81. Graph and Vector Diagram for Problem 2

larger wire having less resistance were used, an even greater current would flow at resonance. In a circuit the ratio of inductive reactance of an inductor to its internal resistance, as you learned earlier, is the Q of the inductor. $Q = \frac{X_L}{R}$. The Q of the coil of the circuit in this problem is equal to 53,000/520, or 102.

High Q coils are preferred to low Q coils, especially at resonance. In resonant circuits, high Q coils permit high current to flow to the load, since there is very little power lost. This is due to the low resistance of the windings in the coil. Remember that the lower the internal resistance, the greater the Q.

Use of Series Resonant Circuits

One use of a series-resonant circuit is to filter out an unwanted frequency. Here is an example. In Figure 2–82, the generator is producing 24 volts DC with a 400-cps ripple. If C were tuned so that $X_L = X_C$ at 400 cps, the 400-cps ripple then would be effectively shorted out. This is because the impedance to the 400-cps voltage between points A and B consists only of the resistance of the inductor.

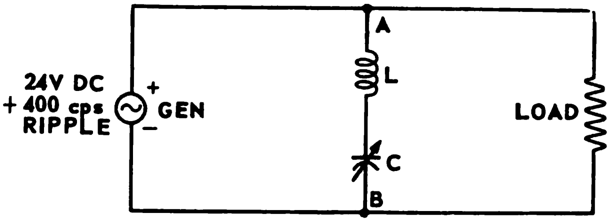


Figure 2–82. Series-Resonant Filter

PARALLEL AC CIRCUITS

Resistive Circuits

When resistors are connected in parallel across an AC voltage, the circuit behaves as a parallel DC circuit, and the circuit problems involved are essentially the same as those in parallel DC circuits. For example, in the circuit of Figure 2–83, using the values indicated, find the values of current and voltage in each of the three branches. Also find the total impedance.

In this problem, both Ohm's and Kirchhoff's laws are applicable. By using Kirchhoff's law which states that the voltages across elements connected in parallel are equal, it follows that since the voltage of the source in the problem is 12 volts, then the voltage across each resistor is also 12 volts.

Another one of Kirchhoff's laws states that the sum of the currents leaving a junction equals the current entering that junction. Therefore, the total current in this circuit equals the sum of the branch currents I_1, I_2, and I_3. It is necessary to find these currents first in order to find the total current. To find them, substitute the voltage for each branch and the resistance of each resistor indicated in the circuit in the Ohm's law formula, since R and Z are equivalent in a resistive circuit.

$$I = \frac{E}{Z}. \text{ Thus,}$$

In branch 1,

$$I_1 = \frac{E}{Z_1} = \frac{12}{6} = 2 \text{ amperes}$$

In branch 2,

$$I_2 = \frac{E}{Z_2} = \frac{12}{4} = 3 \text{ amperes}$$

In branch 3,

$$I_3 = \frac{E}{Z_3} = \frac{12}{3} = 4 \text{ amperes}$$

Then according to Kirchhoff's law, the total current I_t is $I_t = I_1 + I_2 + I_3$.

Thus,

$$I_t = 2 + 3 + 4 = 9 \text{ amperes}$$

Then by the Ohm's law formula, the total impedance Z_t is $Z_t = \frac{E}{I_t}$

Thus,

$$Z_t = \frac{12}{9} = 1.33 \text{ ohms}$$

Another way to find the total impedance and total current in this problem is to obtain the total resistance by the parallel resistance formula first, and then the total current directly by Ohm's law. Here is the solution:

$$\frac{1}{Z_t} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

Then by substituting the indicated values of resistances in this equation,

$$\frac{1}{Z_t} = \frac{1}{6} + \frac{1}{4} + \frac{1}{3} = \frac{2 + 3 + 4}{12} = \frac{9}{12}$$

$$Z_t = \frac{12}{9} = 1.33 \text{ ohms}$$

Now by substituting in the Ohm's law formula, $I_t = \frac{E}{Z_t}$

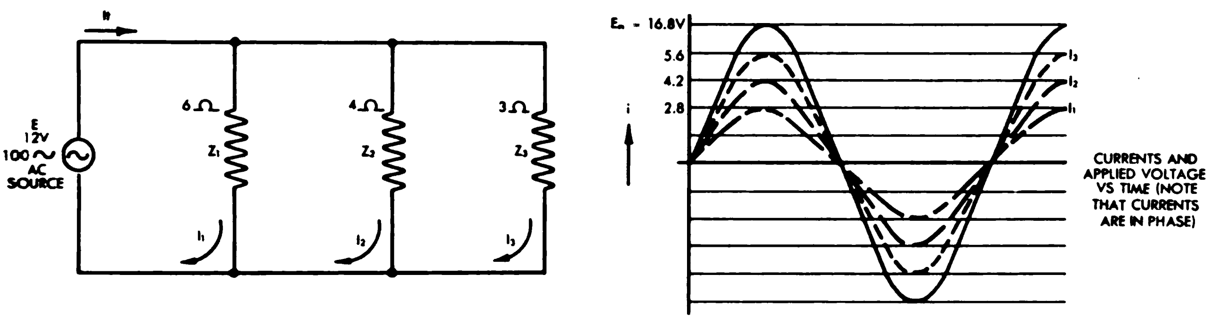

It
E
12V
100 ~
AC
SOURCE
6 Ω
Z1
I1
4 Ω
Z2
I2
3 Ω
Z3
I3
Em = 16.8V
5.6
4.2
2.8
i
I3
I2
I1
CURRENTS AND APPLIED VOLTAGE VS TIME (NOTE THAT CURRENTS ARE IN PHASE)

Figure 2–83. Parallel Resistive Circuit

$$I_t = \frac{12}{1.33} = 9 \text{ amperes}$$

Resistance and Capacitance

When a resistor and a capacitor are connected in parallel across an alternating voltage source as in Figure 2–84, the capacitor introduces capacitive reactance into the circuit as it does in a series AC circuit, but with a somewhat different action. In any parallel circuit, the voltages across the resistive branch and the capacitive branch are equal to each other and equal to the applied voltage. The current in the resistive branch is in phase with the applied voltage and also equal to the applied voltage divided by the resistance of the resistor. But the current and voltage through the capacitive branch are not in phase.

As you recall, current leads voltage in a purely capacitive circuit by 90°. The best way to understand how these currents and voltages act is by use of vector diagrams and sine waves. In this connection, first refer to the vector diagram in Figure 2–84. Notice first that the vector diagram differs from that in Figure 2–72 for the series resistive-capacitive circuit. The most important difference results from the fact that in a parallel circuit the voltage is the same in all parts of the circuit. *The voltage, therefore, is used as the reference vector and not the current, as in a series circuit.*

All other vectors are drawn with reference to the voltage vector. The voltage vector might be labeled E_a, E_R, or E_c, because all these voltages are equal and in phase. The current flowing in the resistor is in phase with the voltage applied to it. Thus, it is drawn to coincide with the voltage vector. But since the current through the capacitor leads the applied voltage by 90°, it is drawn perpendicularly upward from the voltage vector as shown. Now if you construct a parallelogram with these vectors, you can find

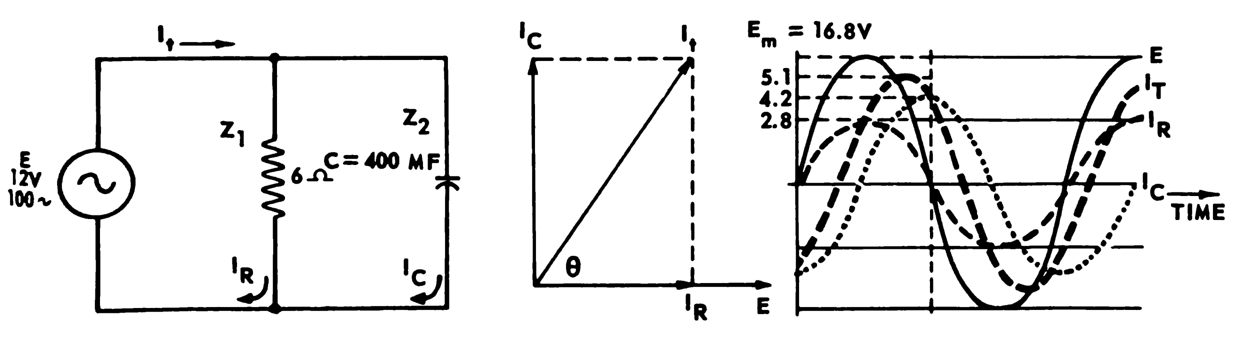


Figure 2–84. Parallel Resistive-Capacitive Circuit

the total current in the circuit. This is represented by the diagonal of the parallelogram. An important point to remember is that in a parallel AC circuit, the total current is not the algebraic sum but the *vector* sum of the branch currents.

The solid line is the voltage reference sine wave. The light dashed line represents the current through the resistor; the dotted line the current through the capacitor, and the dark dashed line the total, or *line,* current.

Example

Using the values given in the circuit in Figure 2–84, find the following quantities:

1. Branch Current

Solution

First, find the capacitive reactance of the capacitor.

$$X_c = \frac{1}{2\pi fC} = \frac{1}{6.28 \times 100 \times 400 \times 10^{-6}} = 4 \text{ ohms}$$

Now find the current in each branch by Ohm's law,

$$I = \frac{E}{Z}.$$

$$\text{In the } I_R \text{ branch, } I_R = \frac{E}{R} = \frac{12}{6} = 2 \text{ amperes}$$

$$\text{In the } I_c \text{ branch, } I_c = \frac{E}{X_c} = \frac{12}{4} = 3 \text{ amperes}$$

2. The Total Current

To find the total current, add the currents I_R and I_c vectorially. I_R is laid out along the E vector in the diagram in Figure 2–84 because current and voltage are in phase through a resistor. I_c is drawn at an angle of 90° counterclockwise from the E vector because current through a capacitor leads the voltage across it. The diagonal of the completed parallelogram represents the total current. The diagonal is also the hypotenuse of a right triangle. The hypotenuse of a right triangle is equal to the square root of the sum of the squares of the legs of the right triangle. For the diagram in Figure 2–84 this becomes

$$I_t = \sqrt{I_R^2 + I_c^2}$$

$$= \sqrt{2^2 + 3^2} = \sqrt{4 + 9} = \sqrt{13} = 3.6 \text{ amperes}$$

3. Phase Angle

Using trigonometry, we can obtain the phase angle. Any one of three trigonometric functions — sine, cosine, or tangent — can be used. The sine of an angle is the ratio of the opposite side to the hypotenuse; the cosine is the ratio of the adjacent side to the hypotenuse, and the tangent is the ratio of the opposite side to the adjacent side. However, we will use the cosine function.

In the vector triangle in Figure 2–84, I_R and I_t (one leg and the hypotenuse) include the phase angle θ. Using the definition of the cosine,

$$\cos\theta = \frac{I_R}{I_t}$$

Solving for θ

$$\theta = \cos^{-1}\frac{I_R}{I_t}$$

Substituting values for I_R and I_t

$$\theta = \cos^{-1}\frac{2}{3.6} = \cos^{-1} 0.555$$

From the trigonometric tables in the attachment,

$$\theta = 56^\circ$$

4. Total Impedance

In finding the total impedance, substitute the total current and applied voltage in the Ohm's law formula, $Z_t = \frac{E}{I_t}$.

$$\text{Thus,} \qquad Z_t = \frac{12}{3.6} = 3.33 \text{ ohms}$$

5. Power Factor

Since the current in this circuit is out of phase with the voltage, the power factor is not one. The resistor alone dissipates power. In an AC circuit that contains capacitance and resistance connected in parallel, the power factor equals the ratio of the true power (energy dissipated) to the apparent power delivered to the circuit. As shown before, the power factor is the cosine of the phase angle.

$$\text{pf} = \cos\theta = 0.555 \text{ or } 55.5\%$$

6. Power

As mentioned before, a resistor is the only element in a circuit that actually consumes power. Of course a capacitor takes power from the circuit during one alternation, but it returns it again to the circuit during the next alternation. Therefore, over a complete cycle, a capacitor dissipates no power.

In resistive-capacitive circuits there are several methods for determining power consumption. One method is to find the apparent power in the circuit, and then multiply it by the power factor. In this example, you have already found that the power factor is 0.555 or 55.5%. Therefore, it is only necessary to find the apparent power. Apparent power, if you recall, is the total current multiplied by the applied voltage — that is, $P_a = EI$. The total current in the circuit is 3.6 amperes. Thus, substituting in the formula, $P_a = EI$, you obtain,

$$P_a = 12 \times 3.6 = 43.2 \text{ watts}$$

Then multiplying this result by the power factor, the true power is,

$$P = 43.2 \times 0.555 = 24 \text{ watts.}$$

Here is another method. The true power dissipated in a circuit — that by the resistance — is equal to the square of the current through the resistor multiplied by the ohmic resistance of the resistor,

$$P = I_R^2 R$$

Therefore, substituting for I_R and for R (circuit in Figure 2–84),

$$P = 2^2 \times 6 = 4 \times 6 = 24 \text{ watts.}$$

Since the voltage across the resistor depends on the current through it and not on the total current, the power formula $P = E^2/R$ may be used. By substituting the voltage applied to the resistance and the ohmic value of the resistor, you likewise find that the true power is,

$$P = \frac{12^2}{6} = \frac{144}{6} = 24 \text{ watts.}$$

Solving this problem by using simple mathematics was easy enough. But you will often come across complicated problems where it is easier to use rectangular or polar notation. Here is how you can solve for some of the quantities in this problem by rectangular notation.

1. Total Impedance

When the resistor and capacitor are referred to rectangular coordinates, they are equal respectively to,

$$R = Z_1 = 6 + j0$$
$$X_c = Z_2 = 0 - j4$$

Substitute in the formula for impedances in parallel and simplify.

$$Z_t = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$Z_t = \frac{(6 + j0)\,(0 - j4)}{(6 + j0) + (0 - j4)} = \frac{-j24}{6 - j4}$$

Rationalize the denominator,

$$Z_t = \frac{(-j24)\,(6 + j4)}{(6 - j4)} = \frac{96 - j144}{52} = 1.84 - j2.77$$

This answer is of little value unless you intend to make additional computations. What you really want to know is the value of Z_t in ohms. This is what you do. First, notice that the answer has the form, $R - jX$, where R is the amount of resistance and X is the amount of reactance in an equivalent series circuit. The negative sign before the j indicates that the reactance is capacitive. This parallel circuit will have the same effect on current flow as the equivalent circuit in Figure 2–85.

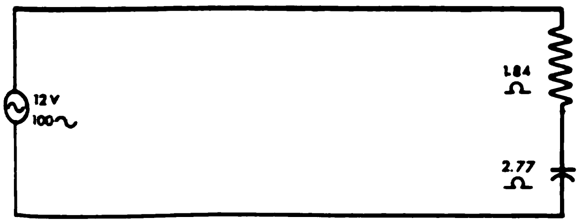


Figure 2–85. Equivalent Series Circuit

We have shown earlier that the impedance in a series resistive-capacitive circuit is

$$Z_t = \sqrt{R^2 + X_c^2}$$

Substitute the ohmic values in this formula and solve.

2. Phase Angle

Use the tangent function to find the phase angle.

$$\tan\theta = \frac{X}{R}$$

Substitute the values.

$$\tan\theta = \frac{2.77}{1.84}$$

Simplify.

$$\theta = \tan^{-1}\frac{2.77}{1.84} = \tan^{-1} 1.5$$

From tables, you find that

$$\theta = 56°$$

3. Power Factor

The power factor is equal to the cosine of θ.

$$\cos\theta = \frac{R}{Z_t}$$

$$= \frac{R}{\sqrt{R^2 + X_c^2}}$$

$$pf = \cos\theta = \frac{R}{\sqrt{R^2 + X_c^2}}$$

Substitute the values.

$$pf = \frac{1.84}{3.33} = 0.555$$

Circuits Containing Inductance and Resistance

Now consider an AC circuit in which inductance and resistance are connected in parallel. In studying this circuit, refer to the inductive-resistive circuit in Figure 2–86 and find the following circuit quantities.

1. Total Current

In constructing the vector diagram, it will be necessary to find the branch currents. Since the voltage across all circuit elements is the same, that is, the applied voltage $E_a = E_R = E_L$, the voltage becomes the reference vector. Since I_R, (the resistor current) is in phase with E_R and the applied voltage, it must be drawn to coincide with the voltage vector as shown. But since the current flowing through the inductance L lags the applied voltage by 90°, it must be drawn downward at a 90° angle as shown. Before you

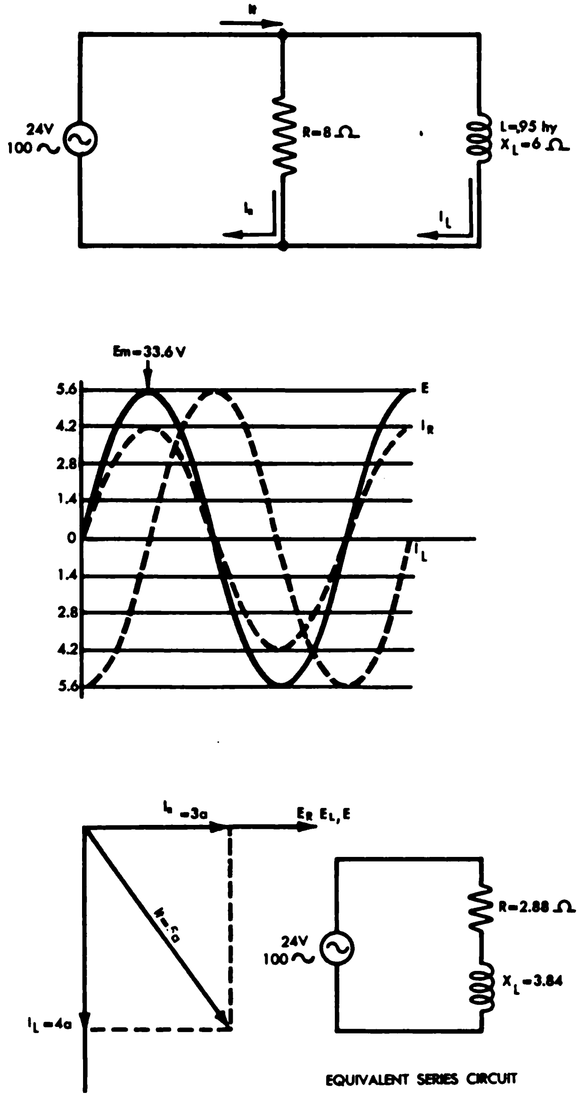


Figure 2–86. Inductive-Resistive Circuit

can actually draw these vectors, you must first determine their magnitudes. Find the current through each component,

$$I = \frac{E}{Z}$$

Substituting in this formula the values indicated in the circuit, the current in the resistive branch is

$$I_R = \frac{24}{8} = 3 \text{ amperes}$$

In finding the current through the inductive branch, it is first necessary to find the inductive reactance of that branch by use of the formula,

$$X_L = 2\pi fL$$

Substituting the values given in this formula

$$X_L = 6.28 \times 10^2 \times 0.95 = 6 \text{ ohms}$$

Then by Ohm's law

$$I_L = \frac{24}{6} = 4 \text{ amperes}$$

As illustrated, when the vectors are complete into a parallelogram, the diagonal is the vecto sum of the branch currents and also the hypotenuse of a right triangle. Since the legs o this triangle are I_R and I_L respectively, you ca find the hypotenuse from the equation for I_t.

$$I_t = \sqrt{I_R^2 + I_L^2}$$

Thus, in substituting the values for I_R and I_l

$$I_t = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \text{ amperes.}$$

2. *Total Impedance*

In finding the total impedance, substitute th values for I_t and applied voltage in Ohm's la for AC. Thus,

$$Z = \frac{E}{I_t}$$

$$Z = \frac{24}{5} = 4.8 \text{ ohms}$$

3. *Phase Angle*

In finding the phase angle, substitute th values for I_R and I_t in the formula

$$\cos\theta = \frac{I_R}{I_t}$$

Thus,

$$\cos\theta = \frac{3}{5}$$

$$\theta = \cos^{-1}\frac{3}{5}$$

$$\theta = \cos^{-1} 0.60$$

From tables,

$$\theta = 53^\circ$$

4. *Power*

In finding the true power (actual power co sumed), multiply the apparent power (EI_t) b the power factor. The power factor is equal *cos θ*. From the previous problem θ was foun to equal 53°. Thus, from trigonometric table *cos 53° = 0.6.*

Substitute for the values E, I_t and *cos θ* in t equation

$$P = EI_t \cos\theta$$

$$P = 24 \times 5 \times 0.6 = 72 \text{ watts}$$

Calculations for the circuit of Figure 2– can also be made by using rectangular notatio

The impedance of each branch is first written in rectangular notation, as in the problem for the resistive-capacitive circuit.

1. Total Impedance

In rectangular coordinates, the resistance and reactance are

$$R = 8 + j0$$
$$X_L = 0 + j6$$

Inductive reactance is always indicated with a + sign.

Substitute in the formula for impedances in parallel and simplify.

$$Z_t = \frac{Z_R Z_L}{Z_R + Z_L}$$

$$Z_t = \frac{(8 + j0)\ (0 + j6)}{(8 + j0) + (0 + j6)} = \frac{j48}{8 + j6}$$

Rationalize the denominator.

$$Z_t = \frac{j48\ (8 - j6)}{(8 + j6)\ (8 - j6)} = \frac{288 + j384}{100} = 2.88 + j3.84$$

Change from rectangular to the usual ohmic notation using the formula

$$Z_t = \sqrt{R^2 + X_c^2}$$
$$Z_t = \sqrt{2.88^2 + 3.84^2} = \sqrt{8.29 + 14.77} = \sqrt{23.06}$$
$$Z_t = 4.8 \text{ ohms}$$

2. Phase Angle

Use the tangent function to find the phase angle θ.

$$\tan\theta = \frac{X_L}{R}$$

$$\tan\theta = \frac{3.84}{2.88}$$

$$\theta = \tan^{-1}\frac{3.84}{2.88} = \tan^{-1} 1.33$$

From trigonometric tables,

$$\theta = 53°$$

The rectangular form of the total impedance, 2.88 + j3.84, is in the form $R + jX$. This form indicates that the series circuit equivalent to this circuit will contain a resistance of 2.88 ohms and a reactance equal to 3.84 ohms.

Circuits Containing Inductance and Capacitance

When both inductance and capacitance exist in combination with resistance in parallel circuits, the computations are somewhat lengthy, but not any more difficult than those in circuits which contain only resistance and capacitance. In the circuit illustrated in Figure 2–87, find the following quantities.

1. Total Current

In this circuit, the total current is the vector sum of the individual branch currents. The current in the resistive branch is in phase with the applied voltage, and thus is indicative of the true power consumption. The inductive current lags the applied voltage by 90°, and the capacitive current leads it by 90°. The effect of these currents is exactly opposite. In finding the branch currents, substitute the values, $R = 6$ ohms, $X_c = 24$ ohms, $X_L = 12$ ohms, in the Ohm's law formula, thus

$$I = \frac{E}{Z}.$$

$$I_R = \frac{48}{6} = 8 \text{ amperes}$$

$$I_c = \frac{48}{24} = 2 \text{ amperes}$$

$$I_L = \frac{48}{12} = 4 \text{ amperes.}$$

In this circuit, as in the previous circuit, the voltage vector is the reference. Therefore, in drawing the vector diagram, first lay out the voltage to scale as a horizontal vector to the right.

Since the current in the resistive branch is in phase with the voltage applied, lay out the resistor current (I_R) along the voltage vector. Since the current through a capacitor leads the impressed voltage by 90°, draw the capacitor current (I_c) perpendicularly upward from the starting point of the voltage vector. Next, construct the vector for the current flow through the inductance. As you recall, the current flowing in a pure inductance lags the voltage 90°. Therefore, draw this vector (I_L) perpendicularly downward from the starting point of the voltage vector. The resultant of these two vectors is shown by the open arrow in Figure 2–87.

Since the current flowing through the capacitor is 180° out of phase with the current flowing through the inductor, these two currents can be combined arithmetically. In this circuit, the inductor current is greater than the capacitor current. Thus, the resultant of these two currents ($I_L - I_c$) is a current of smaller value, but in a lagging position with reference to the applied voltage. The next step is to obtain the resultant between this current ($I_L - I_c$) and the vector of the resistor current (I_R). This is accomplished by completing the parallelogram between the

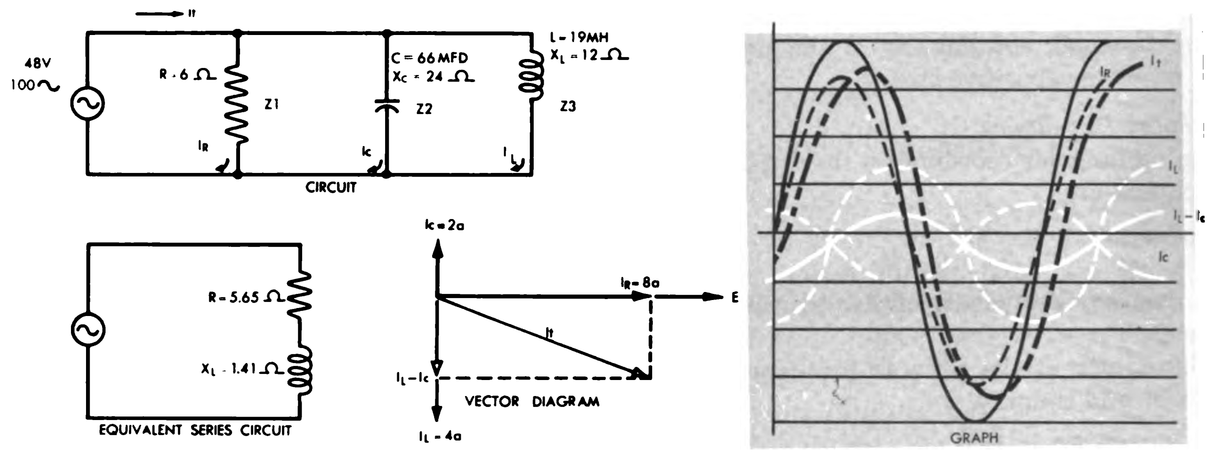


Figure 2–87. Inductive-Capacitive Circuit

($I_L - I_c$) vector and the I_R vector. The diagonal of this parallelogram represents I_t. Its scalar length is the value of the total current flowing in the circuit. The angular displacement between this current vector I_t and the voltage vector E is the phase angle. The cosine of this angle, $\frac{I_R}{I_t}$, is the power factor.

As you can see, the diagonal of this parallelogram is the hypotenuse of a right triangle of which the two sides are I_R and $I_L - I_c$. Therefore,

$$I_t = \sqrt{I_R^2 + (I_L - I_c)^2}$$

On substituting the previously found values for the currents in this formula, you obtain,

$$\begin{aligned} I_t &= \sqrt{8^2 + (4 - 2)^2} \\ &= \sqrt{64 + 4} \\ &= \sqrt{68} \\ I_t &= 8.25 \text{ amperes} \end{aligned}$$

Of course, there are circuits with resistance, capacitance, and inductance in which the current flowing through the capacitance is greater than the current flowing through the inductance. This shifts the parallelogram above the voltage vector. (In the circuit in Figure 2–87, notice that the parallelogram is below the voltage vector.) In determining the total current, it makes no difference which current — the inductive or the capacitive — is the larger, for the square of a negative number has a positive sign.

2. Phase Angle

In finding the phase angle, substitute the previously found values for I_R and I_t in the equation,

$$\text{Phase angle } \theta = \cos^{-1} \frac{I_R}{I_t}$$

$$\text{Thus,} \qquad \theta = \cos^{-1} \frac{8.00}{8.25} = \cos^{-1} 0.969$$

From trigonometric tables,

$$\theta = 14°$$

3. Power

You can find the true power consumed in t circuit by multiplying the product of the appli voltage and the total current by the power fact

$$P = EI \cos \theta$$

$$\text{True power} = 48 \times 8.25 \times 0.9697 = 384 \text{ watts.}$$

A simple method of checking this answer is multiply the voltage across the resistor by t current flow through it; that is, by performi the operations indicated in the formula,

$$P = EI_R.$$

Thus,

$$P \text{ (true)} = 48 \times 8 = 384 \text{ watts}$$

4. Total Impedance

In solving for the total impedance, substitu the calculated values in the Ohm's law formu

$$Z_t = \frac{E}{I_t}$$

$$Z_t = \frac{48}{8.25} = 5.81 \text{ ohms.}$$

The vector for the total impedance shows it be largely resistive, but since there is some ductive reactance, the current slightly lags t voltage.

Rectangular notation permits you to find t total impedance directly. In other words, it not necessary to find current and then to substitute in Ohm's law.

In the rectangular form the impedances of the branches, Z_1, Z_2, and Z_3 are 6, $-j24$, and $+j12$ respectively.

Substitute these values in the formula for impedances connected in parallel.

$$\frac{1}{Z_t} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

$$\frac{1}{Z_t} = \frac{1}{6} + \frac{1}{-j24} + \frac{1}{j12}$$

$$\frac{1}{Z_t} = \frac{4 + j1 - j2}{-j^2\,24} = \frac{4 - j1}{24}$$

$$Z_t = \frac{24}{4 - j1}$$

Rationalize.

$$Z_t = \frac{24}{4 - j1} \times \frac{4 + j1}{4 + j1}$$

$$= \frac{96 + j24}{17}$$

$$= 5.65 + j1.41$$

You can draw the equivalent series circuit from the rectangular form of Z_t as shown in Figure 2–87. It contains 5.65 ohms of resistance and 1.41 ohms of reactance. The sign in front of the reactive component ($j1.41$) is positive, indicating that the reactance is inductive.

Since 5.65 and 1.41 are the ohmic values of R and X in a series circuit,

$$Z_t = \sqrt{(5.65)^2 + (1.41)^2}$$

$$= \sqrt{31.9 + 2.0}$$

$$= \sqrt{33.9}$$

$$Z_t = 5.81 \text{ ohms}$$

Complex Circuit Combination

Another type of parallel circuit to consider is one containing an inductive-resistive branch and a capacitive-resistive branch. In studying this circuit, find the circuit quantities, Z_t, I_t, I_1, I_2, E_{R1}, E_{R2}, E_c and E_L, and draw the vector diagrams, using the circuit in Figure 2–88 and the following given values:

E = 100 volts	C = 63.3 mmfd
f = 500 kc	R_1 = 15K
L = 9.55 mh	R_2 = 12K

To solve for Z_t, first find the reactances of L and C.

$$X_L = 6.28 \times 5 \times 10^5 \times 9.55 \times 10^{-3} = 30\text{K}$$

$$X_c = \frac{1}{6.28 \times 5 \times 10^5 \times 63.6 \times 10^{-12}} = 5\text{K}$$

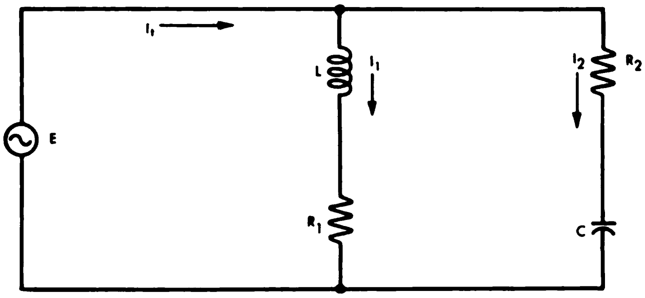


Figure 2–88. RCL Circuit

Now find the impedances of branches 1 and 2,

$$Z_1 = 15\text{K} + j30\text{K} = 33.6\text{K}\ \underline{/63.5^\circ}$$

$$Z_2 = 12\text{K} - j5\text{K} = 13\text{K}\ \underline{/-22.6^\circ}$$

Then, by substituting these values in the equation, find Z_t.

$$Z_t = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$Z_t = \frac{(33.6\text{K}\ \underline{/63.5^\circ})\ (13\text{K}\ \underline{/-22.6^\circ})}{(15 + j30 + 12 - j5)\text{K}}$$

$$= \frac{436\text{K}\ \underline{/40.9^\circ}}{27 + j25} = \frac{436\text{K}\ \underline{/40.9^\circ}}{36.8\ \underline{/42.7^\circ}}$$

$$Z_t = 11.85\text{K}\ \underline{/-1.9^\circ}$$

Next, find the currents, I_t, I_1, and I_2.

$$I_t = \frac{E}{Z_t} = \frac{100\ \underline{/0^\circ}}{11.85\text{K}\ \underline{/-1.9^\circ}} = 8.43\ \underline{/1.9^\circ} \text{ ma}$$

$$I_1 = \frac{E}{Z_1} = \frac{100\ \underline{/0^\circ}}{33.6\text{K}\ \underline{/63.5^\circ}} = 2.98\ \underline{/-63.5^\circ} \text{ ma}$$

$$I_2 = \frac{E}{Z_2} = \frac{100\ \underline{/0^\circ}}{13\text{K}\ \underline{/-22.6^\circ}} = 7.8\ \underline{/22.6^\circ} \text{ ma}$$

Now the voltage drops, E_{R1}, E_L, E_{R2}, and E_c may be found.

$$E_{R1} = I_1 R_1 = 2.98\ \underline{/-63.5^\circ} \times 15\text{K}$$

$$= 44.6\ \underline{/-63.5^\circ} \text{ volts}$$

$$E_L = I_L X_L = 2.98\ \underline{/-63.5^\circ} \times 30\text{K}\ \underline{/90^\circ}$$

$$= 89.4\ \underline{/26.5^\circ} \text{ volts}$$

$$E_{R2} = I_2 R_2 = 7.8\ \underline{/22.6^\circ} \times 12\text{K}$$

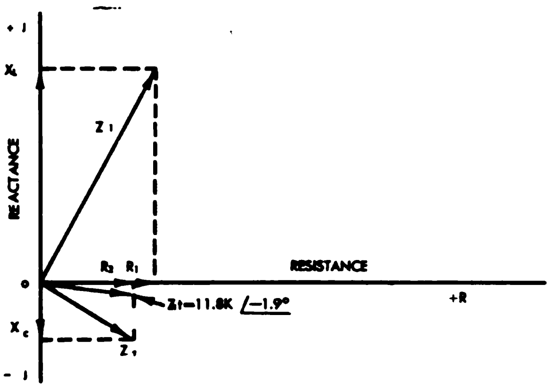
$$= 93.6\ \underline{/22.6^\circ} \text{ volts}$$

$$E_c = I_2 X_c = 7.8\ \underline{/22.6^\circ} \times 5\text{K}\ \underline{/-90^\circ}$$

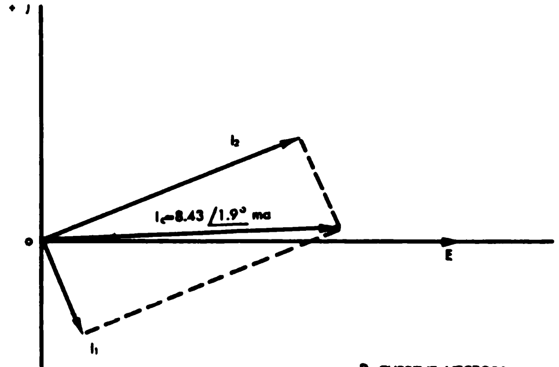
$$= 39\ \underline{/-67.4^\circ} \text{ volts}$$

Draw the vector diagrams as illustrated in Figure 2–89. In diagram A, notice that the total impedance vector lies slightly below the resistance axis. This indicates that there is a small amount of capacitive reactance associated with the total impedance. You therefore can expect the total current to lead the applied voltage by a small angle. If the applied voltage were plotted on the same graph, it would lie as shown in diagram B.

The voltage vector diagram is shown in Figure 2–90. The vector sum $E_L + E_{R1}$ and also



A IMPEDANCE VECTORS



B CURRENT VECTORS

Figure 2–89. Impedance and Current Vectors

$E_C + E_{R2}$ equals the applied voltage. E_L and E_{R1} are 90° out of phase. This same out-of-phase relationship also exists between E_C and E_{R2}.

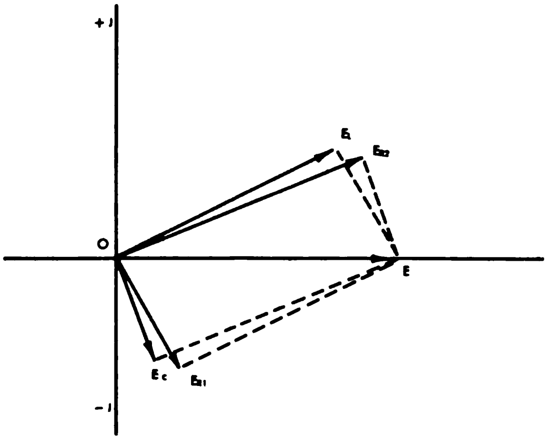


Figure 2–90. Voltage Vectors

Series-Parallel Circuits

The AC series-parallel circuit in Figure 2–9 is part of a Wien-bridge oscillator circuit. Th theory of operation of this circuit will be discussed later in this manual. With reference to i here, find the circuit quantities, Z_t, I_t, I_1, I_2, E and E_2, using the following given values:

$$E = 100 \text{ volts}$$
$$f = 2000 \text{ cps}$$
$$R_1 = R_2 = 90K$$
$$C_1 = C_2 = .001 \text{ mfd}$$

First, find X_{c1} and X_{c2}.

$$X_{c1} \text{ and} X_{c2} = \frac{1}{6.28 \times 2 \times 10^3 \times 10^{-3} \times 10^{-6}} = 79.5K$$

Then find Z_3 (impedance of R_1 and C_1 in series).

$$Z_3 = 90K - j79.5K = 120K \,\angle{-41.4^\circ}$$

Then find the total impedance, Z_t,

$$Z_t = 90K - j79.5K + \frac{(90K)\ (79.5K)\ \angle{-90^\circ}}{90K - j79.5K}$$
$$= 90K - j79.5K + \frac{7155K \,\angle{-90^\circ}}{120 \,\angle{-41.4^\circ}}$$
$$= 90K - j79.5K + 59.6K \,\angle{-48.6^\circ}$$
$$= 90K - j79.5K + 39.4K - j44.8K$$
$$Z_t = 129.4K - j124.3K$$

Thus,

$$Z_t = 179K \,\angle{-44^\circ}$$

Now I_t, I_1, and I_2 may be found.

$$I_t = \frac{E}{Z_t} = \frac{100 \,\angle{0^\circ}}{179K \,\angle{-44^\circ}} = 0.559 \,\angle{44^\circ} \text{ ma}$$
$$I_1 = \frac{I_t \times (-jX_{c2})}{R_2 - jX_{c2}} = \frac{0.559 \,\angle{44^\circ} \times 79.5K \,\angle{-90^\circ}}{90K - j79.5K}$$
$$= \frac{0.559 \,\angle{44^\circ} \times 79.5K \,\angle{90^\circ}}{120K \,\angle{-41.4^\circ}}$$
$$= 0.370 \,\angle{-4.6^\circ} \text{ ma}$$
$$I_2 = I_t \times \frac{R_2}{R_2 - jX_{c2}} = \frac{0.559 \,\angle{44^\circ} \times 90K}{120K \,\angle{-41.4^\circ}}$$
$$= 0.418 \,\angle{85.4^\circ} \text{ ma}$$

Finally, the voltage drops E_1 and E_2 may be found

$$E_1 = I_tZ_3 = 0.559 \,\angle{44^\circ} \times 120K \,\angle{-41.4^\circ}$$
$$E_1 = 67 \,\angle{2.6^\circ} \text{ volts}$$
$$E_2 = I_1R_2 = I_2X_{c2} = 0.370 \,\angle{-4.6^\circ} \times 90K$$
$$E_2 = 33.3 \,\angle{-4.6^\circ} \text{ volts}$$

The vector diagram in Figure 2–92 sho these circuit quantities. Notice that E_1 and are almost in phase with the applied voltage, At the same time, the line current is almost 4 out of phase with the generator voltage.

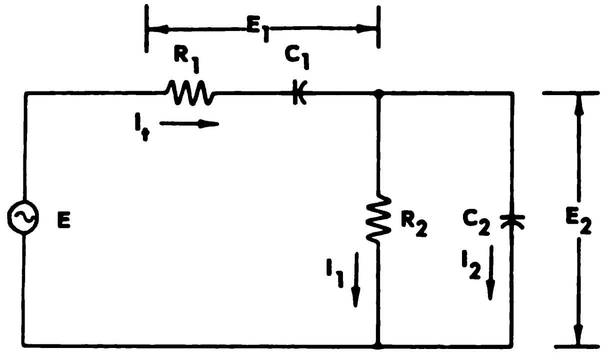


Figure 2–91. Series-Parallel Circuit

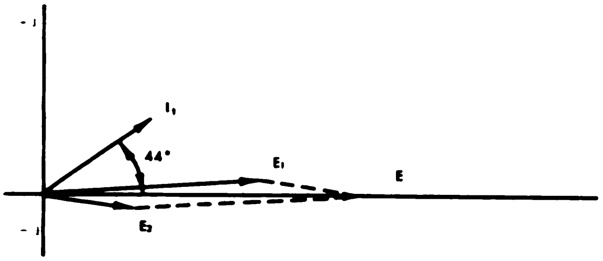


Figure 2–92. Vector Diagram

Parallel Resonance

A parallel-resonant circuit is one of the most widely used devices in electronic receivers, transmitters and frequency-measuring devices. Unlike a series-resonant circuit, a parallel-resonant circuit is characterized by high impedance and minimum current at resonance. The following discussion of parallel resonance is based on the parallel-resonant circuit shown in Figure 2–93.

In the circuit the following notation is used:

E_g = generator voltage.
R_g = generator resistance in ohms.
I_t = total current flow in amperes.
I_c = current through capacitor in amperes.
I_L = current through inductor in amperes.
R_L = internal resistance of inductor in ohms.
L = inductance of inductor in henrys.
C = capacitance of capacitor in farads.

An important consideration in the study of parallel resonance is the circuit condition which produces resonance. In this connection there are three definitions of the frequency at which a tank circuit is resonant. Based on the circuit illustrated in Figure 2–93, they are:

1. The frequency of E_g at which I_t is in phase with E_g. (This is the condition of unity power factor.)

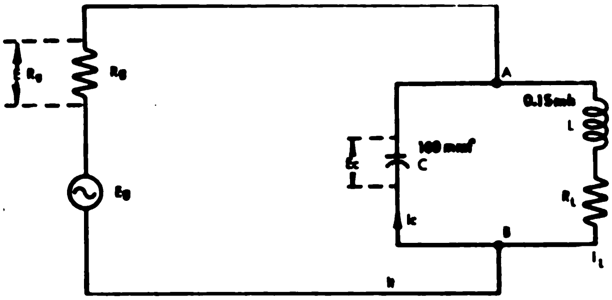


Figure 2–93. Parallel Resonant Circuit

2. The frequency of E_g at which X_L equals X_c.
3. The frequency of E_g at which the impedance of the tank is maximum.

These definitions lead to resonant frequencies that differ by as little as one percent when the circuit Q is large. For all practical purposes, the resonant frequency of a parallel circuit is the frequency that satisfies the second relation, $X_L = X_c$, or

$$\omega L = 1/(\omega C).$$

In simpler terms, the relation is written

$$\text{resonant frequency} = f_r = \frac{1}{2\pi\sqrt{LC}}$$

Another useful concept in the study of parallel resonance is the expression for the impedance of the tank,

$$Z_{AB} = \frac{-jX_c(R_L + jX_L)}{R_L + jX_L - jX_c} = \frac{X_LX_c - jX_cR_L}{R_L + jX_L - jX_c}$$

From the second definition of resonant frequency, you can see that when the frequency of E_g is such that $X_L = X_c$, then the expression for the impedance of the tank is,

$$Z_{AB} = \frac{X_L^2 - jX_cR_L}{R_L} = \frac{X_L^2}{R_L} - jX_c$$

From this expression, you can see that as R_L approaches zero, Z_{AB} approaches infinity. Since $X_L/R_L = Q$, you can write the relationship $Z_{AB} = (X_L^2/R_L) - jX_c$ as $QX_L - jX_c$. Thus, when the Q of the circuit is reasonably high, QX_L becomes much greater than $-jX_c$ and since arctan (X_c/QX_L), or arctan $(1/Q)$ is approximately equal to 90/0°, the impedance approaches pure resistance.

It should be apparent that the three definitions are identical when R_L is 0, and Q is infinitely large. Furthermore, for all practical purposes they are identical when the inductor L has a reasonably high Q.

Let us calculate the impedance (Z_{AB}) of the resonant circuit in Figure 2–93 using various values of R_L, for frequencies ranging from 960 kc through resonance (1000 kc) to 1030 kc. If we calculate the impedance between points A and B for R_L equal to zero, and plot a graph of impedance against frequency, we will get the dashed curve labeled $Q = \infty$ in Figure 2–94. Notice that we are not considering the phase angle in plotting this graph.

Next, calculate and plot the impedances for R_L equal to 7.5 ohms ($Q = 125$). This curve has a decidedly lower peak amplitude than the first curve we plotted. When $R_L = 9.4$ ohms, $Q = 100$; when $R_L = 12.5$ ohms, $Q = 75$. The other curves on this graph were obtained in the same way as were the first using the values of R_L that produce $Q = 100$ and $Q = 75$. Notice from the graphs in Figure 2–94 that as the Q decreases, the sharpness of the curve decreases. The lower the Q, the flatter the curve.

At the bottom of the graph in Figure 2–94 are curves of total current for these same values of Q, with degree of lead or lag plotted against frequency. Notice that at frequencies below resonance the current is lagging; that is, it is an inductive current. At frequencies above resonance, the current is leading; the circuit acts capacitively. This is easy to understand if you remember that at resonance $X_c = X_L$ and the branch currents would be equal and opposite ($R_L = 0$). As the frequency applied to the circuit decreases, X_c increases while X_L decreases. The branch currents are no longer equal and only part of the current through the inductive branch is canceled by the capacitive current. The line current (I_t), therefore, is inductive. Above resonance the opposite effect takes place. I_c exceeds I_L and the line current is capacitive.

Notice that, as Q decreases, the angle of lead or lag decreases for any one frequency, except that of resonance. For example, at 1005 kc, the circuit with a Q of 125 has an impedance of $70K \angle -50^\circ$. Similarly the circuit with a Q of 75 has an impedance of $53K \angle -40^\circ$.

As previously shown, the tank offers a pure resistance load at resonance to the generator of approximately QX_L or $Q\omega L$. The total impedance to the generator in Figure 2–93 is $R_g + QX_L$ when the frequency of E_g is such that the tank is resonant. Since the load for E_g is pure resistance, I_t is in phase with E_g.

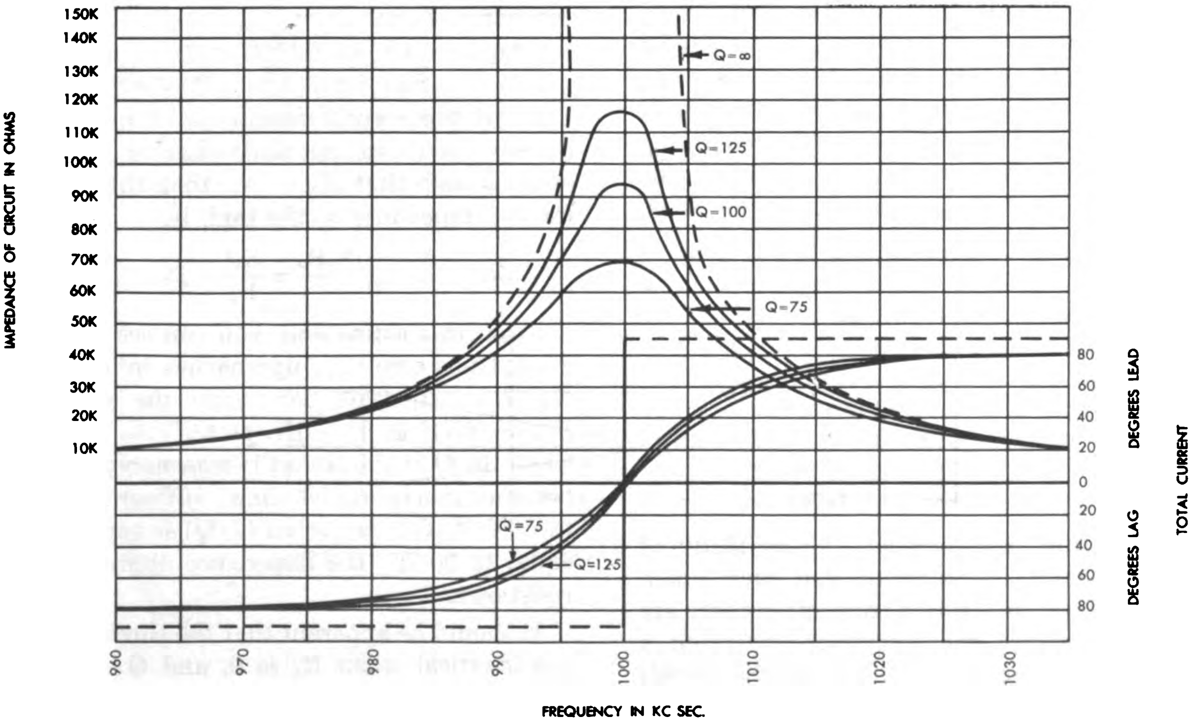


Figure 2–94. Magnitude and Phase Angle of Impedance as a Function of Frequency

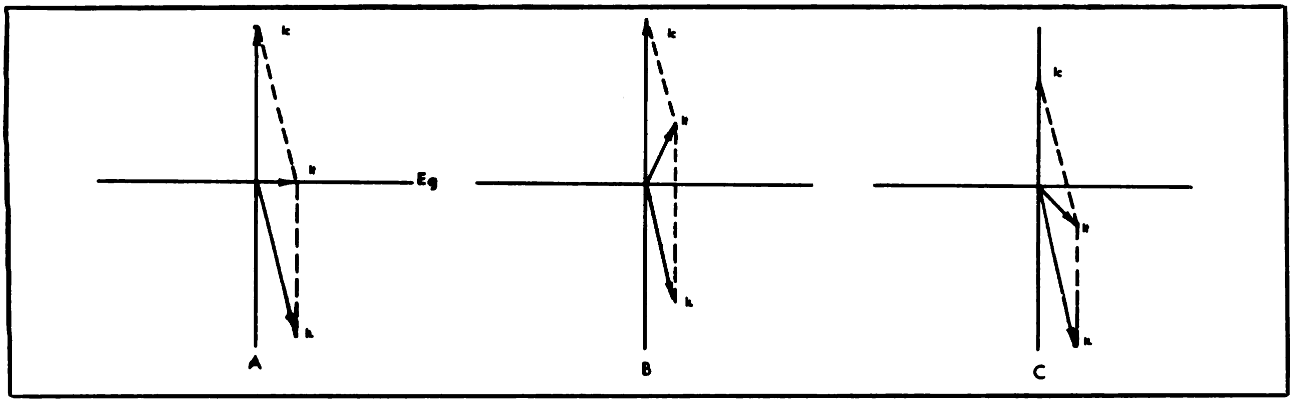


Figure 2–95. Currents at, above, and below Resonance

Since the tank acts as pure resistance, the voltage across the tank, E_c, is in phase with E_g, but the current through the capacitor leads the voltage E_c by 90°. The current through the inductor lags the voltage E_L by an angle whose tangent is QX_L/R_L. This angle approaches $-90°$ as R_L approaches zero.

The vector diagram in Figure 2–95A shows the phase and amplitude relationships of I_c, I_L, and I_t in a parallel-resonant circuit at resonance. From it you can see that I_c and I_L are much larger than I_t and are approximately 180° out of phase. As R_L approaches zero, I_t approaches zero.

At frequencies above resonance, the reactance X_c decreases and X_L increases. This causes an increase in current flow through C and a decrease in current through L. Therefore, the total current now leads the applied voltage, and the tank acts capacitively as shown vectorially in Figure 2–95B.

At frequencies below resonance, the reactance of C increases while that of L decreases. This causes a decrease in current flow through C and an increase in current through L. Since the total current then lags the applied voltage, the tank acts inductively as shown in Figure 2–95C.

If E_g in the parallel-resonant circuit of Figure 2–93 is constant and R_g equals 0, the voltage across the tank (E_c) is constant at all frequencies. But if the frequency of the generator is changed in steps from below resonance to above resonance, I_c will increase while I_L will decrease. As you can see in the graph in Figure 2–96, I_t will be minimum at resonance, since at resonance the impedance is maximum.

Also notice the graph in Figure 2–97 which shows the relationship of the quantities I_c, I_L, and E_c when R_g is 20,000 ohms.

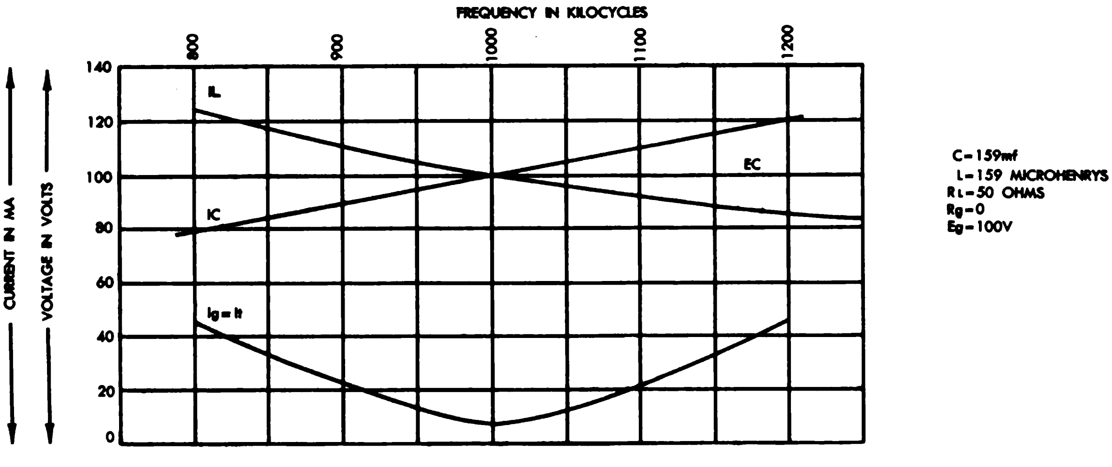


Figure 2–96. Currents and Voltages in a Circuit near Its Resonant Frequency

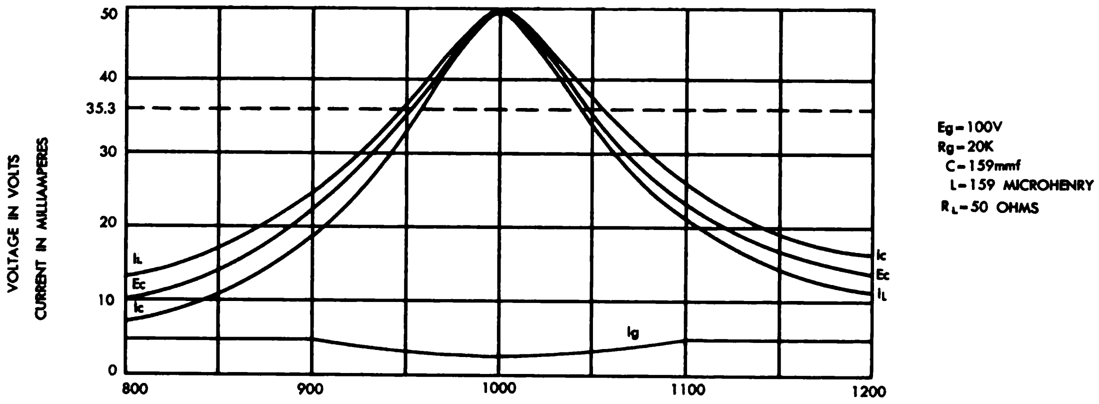


Figure 2–97. Effect of R_g on Currents and Voltages

Notice in this graph that the voltage across the tank becomes maximum at resonance. This is explained by the fact that, at resonance, the impedance of the tank is maximum. At frequencies above and below resonance, the impedance of the tank goes down, I_t increases, and a large portion of E_g is dropped across R_g.

Also notice in the same graph that, at any one frequency below resonance, I_L is greater than I_C while, at any one frequency above resonance, I_C is greater than I_L.

Bandwidth and Selectivity

One characteristic that makes a parallel resonant circuit very useful in radio circuits is its response to the resonant frequency and its discrimination against frequencies off resonance. In this connection, notice how the currents I_C and I_L and the voltage E_C in the graph in Figure 2–97 peak at the resonant frequency and drop off rapidly on either side of resonance. The rapidity at which the response drops off is the figure of merit of a resonant circuit and is called the *selectivity* or the *sharpness* of resonance.

An arbitrary standard has been set up to compare the selectivity of resonant circuits. This standard uses as reference points the two frequencies on each side of resonance at which the circulating tank current is 0.707 times its maximum value. The band of frequencies between these points is called the *bandwidth*. Since the reciprocal of the square root of two is 0.707, the power ($P = I^2R$) in the circuit is one-half maximum when the tank current is 0.707 times its maximum value. Therefore, the two frequencies are called the *half-power* points. From this it follows that selectivity is the ratio of the bandwidth to the resonant frequency ($\Delta f/f_r$).

In many radio circuits, the chief interest is not in current flow in the circuit, but in the voltage developed across the tank circuit. Near resonance the reactance of the capacitor in the tank circuit does not vary much; the graph of the capacitor voltage is practically the same as that of the capacitor current. Therefore, you can consider the bandwidth as a band of frequencies between the values of E_C that equal 0.707 times maximum value. Notice that the half-power points are 950 kc and 1050 kc and that the bandwidth is 100 kc in Figure 2–97.

To determine the factors which influence the selectivity of a parallel resonant circuit, convert the parallel combination of R_g, R_L, and X_L in Figure 2–93 into an equivalent series impedance, and find the Q of this combination when forming a series resonant circuit with C.

$$Q = \frac{R_gX_L - R_LX_L}{R_gR_L + X_L^2}$$

Since

$$\frac{\Delta f}{f_r} = \frac{1}{Q} \text{ and } R_LX_L \ll R_gX_L$$

Then,

$$\frac{\Delta f}{f_r} = \frac{R_gR_L + X_L^2}{R_gX_L} = \frac{R_L}{X_L} + \frac{X_L}{R_g} = \frac{1}{Q} + \frac{X_L}{R_g}$$

This expression shows that bandwidth and selectivity are primarily affected by the Q of the tank circuit and the generator resistance. Specifically, the greater the Q of the circuit, the sharper the resonance and the more selective the

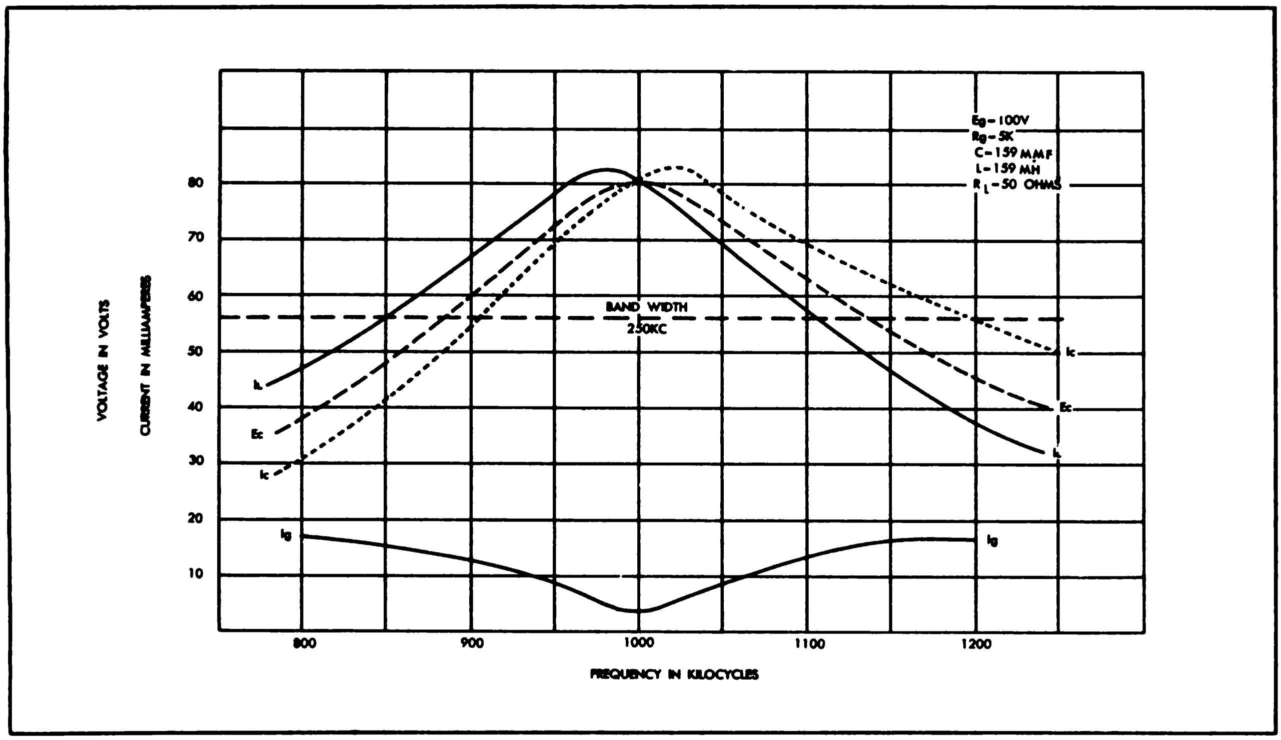


Figure 2–98. Effect of R_g on Bandwidth

circuit. R_g must be large in comparison to X_L if the circuit is to be selective. The greater the R_g, the greater the selectivity. To understand this point, notice the I_g, I_c, I_L, and E_c curves in Figure 2–98 for which R_g is 5K rather than 20K. In this case the bandwidth is 250 kc.

In an inductor without a shunt capacitor, there is one particular frequency at which the inductor itself resonates. This is due to the distributed capacitance between the windings of the inductor. In circuit operation, the distributed capacitance is effectively in parallel with the inductor, and thus forms a self-resonant circuit. Since such a resonant circuit is undesirable, various methods are employed in winding coils to minimize their distributed capacitance.

At microwave frequencies, it is necessary that inductances and capacitances in resonant circuits be very small. A resonant circuit in radar may consist of a horseshoe-shaped bar or a doughnut-shaped cavity instead of a capacitor and a coil. However, the principles of resonance explained before are applicable, no matter how high the frequencies or what physical form the resonant circuit may take.

AC NETWORK THEOREMS

While Ohm's law and Kirchhoff's law are the basic laws for solving network problems (combinations of circuit elements), there are other methods which are much more direct and shorter. Often Ohm's law or Kirchhoff's law leads to roundabout solutions that necessitate your finding quantities in which you are not directly interested. For example, in the Wien-bridge circuit previously solved in this chapter, it was necessary to find the total current before calculating the total impedance. In such circuits it would be much easier of you were able to find the impedance directly.

One method which makes it possible to find impedance directly uses T- and Pi-network transformations. These transformations enable you to find the impedance of the network by converting it to an equivalent impedance. Another method that permits direct calculation is Thevenin's theorem, which enables you to find current directly in a particular circuit. A third method, the maximum power transfer theorem, shows the relation that exists between the load impedance and the generator impedance when maximum power is delivered.

The only limitation to the validity of each of these methods is that the impedances of the network be *linear* and *bilateral.* A linear impedance is one that is independent of the amount of current flowing through it, and a bilateral impedance is one that conducts electricity well in both directions. This eliminates such elements as vacuum tubes, copper-oxide rectifier units, and saturable inductors.

Equivalence of T- and Pi-Networks

T- and Pi-networks get their names from their forms. In power systems, networks similar to these are called *Wye* and *Delta* connections.

A basic consideration in the equivalence of T- and Pi-networks is the theorem that states that in any network at any single frequency, a three-element T-structure can be interchanged with a three-element Pi-structure, provided certain relations are maintained. The following discusses these relations.

Suppose you are given the Pi-network shown in Figure 2–99, and it is necessary to find the equivalent T-network. The first consideration is the requirements for equivalence of these networks. In order for them to be equivalent, the impedance must be the same for both networks, looking into terminals a, b, c, and d in pairs. Equating the impedances, looking into terminals ab, ac, and cd in one network, to the corresponding impedances of the other, while all other terminals are disconnected, produces the following equations:

$$\text{Looking into ab, } Z_1 + Z_3 = \frac{Z_A(Z_B + Z_C)}{Z_A + Z_B + Z_C} \qquad (1)$$

$$\text{Looking into ac, } Z_1 + Z_2 = \frac{Z_B(Z_A + Z_C)}{Z_A + Z_B + Z_C} \qquad (2)$$

$$\text{Looking into cd, } Z_2 + Z_3 = \frac{Z_C(Z_A + Z_B)}{Z_A + Z_B + Z_C} \qquad (3)$$

You may solve these equations for Z_1, Z_2, and Z_3 in terms of Z_A, Z_B, and Z_C, or vice versa. Notice that only three equations are needed since there are three unknowns to find. To simplify your work, let $Z_A + Z_B + Z_C = S$. Then solve for Z_1, subtract equation (3) from equation (1), which leaves

$$Z_1 - Z_2 = \frac{Z_AZ_B + Z_AZ_C - Z_BZ_C - Z_AZ_C}{S}$$

$$= \frac{Z_AZ_B - Z_BZ_C}{S} \qquad (4)$$

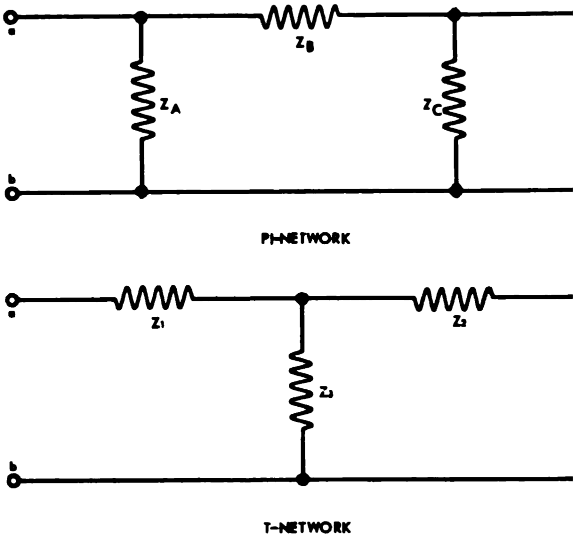


Figure 2–99. Pi- and T-Networks

Add equation (2) to equation (4).

$$2Z_1 = \frac{Z_AZ_B + Z_BZ_C + Z_AZ_B - Z_BZ_C}{S} = \frac{2Z_AZ_B}{S} \qquad (5)$$

$$Z_1 = \frac{Z_AZ_B}{S} = \frac{Z_AZ_B}{Z_A + Z_B + Z_C} \qquad (6)$$

Going through the same procedure for the othe combinations of equations (1), (2), and (3 gives,

$$Z_2 = \frac{Z_BZ_C}{Z_A + Z_B + Z_C} \qquad (7)$$

$$Z_3 = \frac{Z_AZ_C}{Z_A + Z_B + Z_C} \qquad (8)$$

In obtaining the equations for a T- to Pi transformation, multiply equations (6) and (7) equations (7) and (8), and equations (6) an (8) as follows:

$$Z_1Z_2 = \frac{Z_AZ_CZ_B^2}{S^2} \qquad (9)$$

$$Z_2Z_3 = \frac{Z_AZ_BZ_C^2}{S^2} \qquad (10)$$

$$Z_1Z_3 = \frac{Z_BZ_CZ_A^2}{S^2} \qquad (11)$$

Adding these three equations and factoring th right-hand side gives,

$$Z_1Z_2 + Z_2Z_3 + Z_1Z_3 = \frac{Z_AZ_CZ_B^2 + Z_AZ_BZ_C^2 + Z_BZ_CZ_A^2}{S^2}$$

$$= \frac{Z_AZ_BZ_C(Z_A + Z_B + Z_C)}{S^2}$$

$$= \frac{Z_AZ_BZ_C}{S} \qquad (12)$$

Substituting Z_2 for $\frac{Z_BZ_C}{S}$ in equation (12 produces the equation,

$$Z_1Z_2 + Z_2Z_3 + Z_1Z_3 = Z_2Z_A$$

Solving for Z_A,

$$Z_A = \frac{Z_1Z_2 + Z_2Z_3 + Z_1Z_3}{Z_2} \qquad (13)$$

In a like manner, by using equations (8) and (6), we can find Z_B and Z_C

$$Z_B = \frac{Z_1Z_2 + Z_2Z_3 + Z_1Z_3}{Z_3} \qquad (14)$$

$$Z_C = \frac{Z_1Z_2 + Z_2Z_3 + Z_1Z_3}{Z_1} \qquad (15)$$

From the preceding information, it is possible to make the transformation in either direction. You can prove that the networks on the preceding page are equivalent by connecting a generator across terminals ab, attaching a load Z_R across terminals cd and calculating current flow through the load in each case. This shows that the currents are equal and proves that the networks are equivalent.

The preceding transformation formulas are not easily remembered. Therefore, it is well to fix the processes of transformation in your mind by rules. For this purpose, the T- and Pi-networks are redrawn as shown in Figure 2–100. The rules are as follows:

Rule 1. Pi-to-T-Transformation. To find any of the elements of the T-network, multiply the *two adjacent impedances* of the Pi-network and divide their product by the *sum of the impedances* of the Pi-network. For example, suppose you want to find Z_2. Then divide the product of Z_BZ_C by $Z_A + Z_B + Z_C$. The result is Z_2. Use similar procedures for finding Z_1 and Z_3.

Rule 2. T-to-Pi Transformation. To find any of the elements of the Pi-network, find the *sum of the impedance products* of the T-elements two at a time and *divide* by the impedance element of the T-network which is *opposite* the desired Pi-element. For example, suppose you desire to find Z_C when Z_1, Z_2, and Z_3 are known. To do this, first form the sum $Z_1Z_2 + Z_2Z_3 + Z_1Z_3$ and then divide by Z_1, which is opposite Z_C. To obtain Z_A and Z_B follow a similar procedure.

Example

In the network of resistors in Figure 2–101, using the values indicated, find the impedance between points AB.

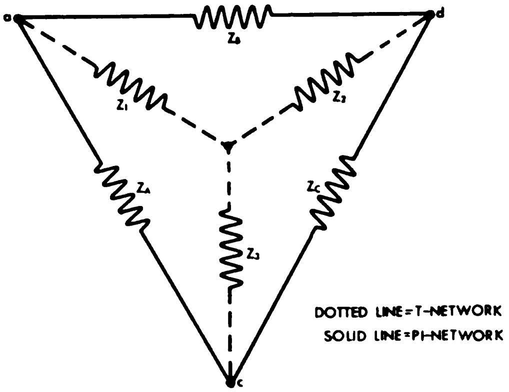


Figure 2–100. T- and Pi-Networks Redrawn

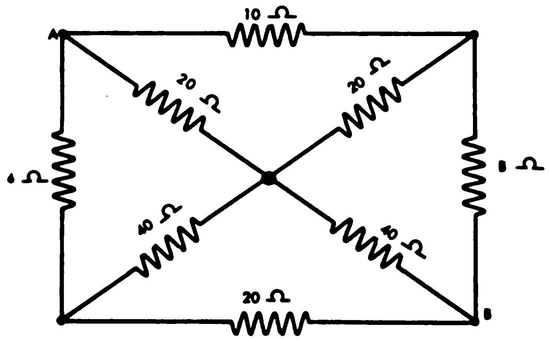


Figure 2–101. Circuit for Problem

Solution

As there are many methods for solving this problem, it is unimportant which three elements you transform first, or which kind of transformation you make. But, as you gain experience, you will find that by proper selection you can reduce the number of transformations necessary to reduce the circuit to series and parallel circuits. In this case, transform the Pi-network made up of the 20-, 40- and 8-ohm resistors to a T-network. Applying Rule 1 produces the following:

$$\frac{20 \times 8}{68} = 2.35 \text{ ohms}$$

$$\frac{40 \times 8}{68} = 4.7 \text{ ohms}$$

$$\frac{20 \times 40}{68} = 11.76 \text{ ohms}$$

After transformation the result is as illustrated by the equivalent circuit in Figure 2–102. But

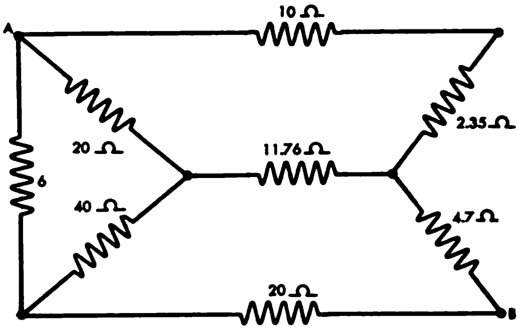


Figure 2–102. First Equivalent Circuit

it still does not lend itself to immediate calculation of the impedance between points AB. Therefore, transform the Pi-network consisting of the 6-, 40- and 20-ohm resistors into a T-network. To do this, perform the following calculations:

$$\frac{6 \times 20}{66} = 1.82 \text{ ohms}$$

$$\frac{6 \times 40}{66} = 3.64 \text{ ohms}$$

$$\frac{20 \times 40}{66} = 12.1 \text{ ohms}$$

The resulting equivalent circuit is as shown in solid lines in Figure 2–103. The 12.1 ohms has been combined with the 11.76 ohms from the circuit shown in Figure 2–102.

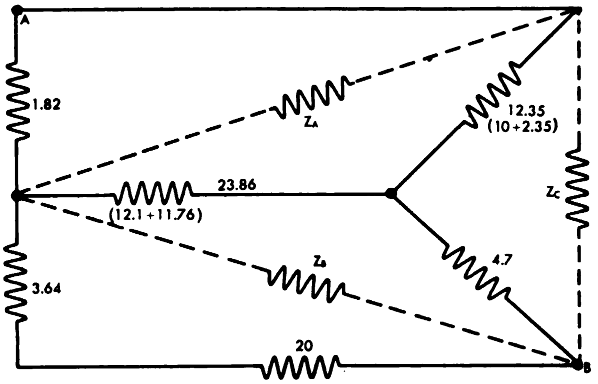


Figure 2–103. Second Equivalent Circuit

Transform the T-network composed of 23.86 ohms, 12.35 ohms, and 4.7 ohms into a Pi-network (shown in dotted lines) as follows:

$$Z_A = \frac{12.35(4.7) + 4.7(23.86) + 23.86(12.35)}{4.7}$$

$$= \frac{58.0 + 112.1 + 295.0}{4.7} = \frac{465.1}{4.7} = 99.0 \text{ ohms}$$

$$Z_B = \frac{465.1}{12.35} = 37.7 \text{ ohms}$$

$$Z_C = \frac{465.1}{23.86} = 19.5 \text{ ohms}$$

Combining 1.82 ohms, which is in parallel wit Z_A, yields 1.79 ohms. Z_B in parallel with 23.6 ohms gives 14.5 ohms. These two parallel circuits are in series with each other and are als in parallel with Z_C. Therefore, the impedanc between AB is 8.86 ohms.

Thevenin's Theorem

Thevenin's theorem states that the current i any impedance Z_R connected to two terminals o a network is the same as if Z_R were connected t a simple generator whose generated voltage is th open-circuited voltage at the terminals of th network, and whose impedance is the impedanc of the network looking back from the terminal with all generators replaced by impedances equa to the internal impedances of these generators.

In the diagrammatic representation of Thevenin's theorem in Figure 2–104A, E_{ab} is the voltage measured at terminals ab with Z_R remove Z_{ab} is the impedance measured back from th terminals ab. In measuring this impedance, assume that the generators are replaced by impedances equal to their internal impedance When the load, Z_R, is connected into the equivalent generator as in Figure 2–104B, the curren that flows through Z_R is the same as that whic flowed when Z_R was connected to the origina terminals ab. By Thevenin's theorem, the network shown at A can be reduced by T- and Pi transformations to a single T- or Pi-network an a single generator as shown in Figure 2–105A.

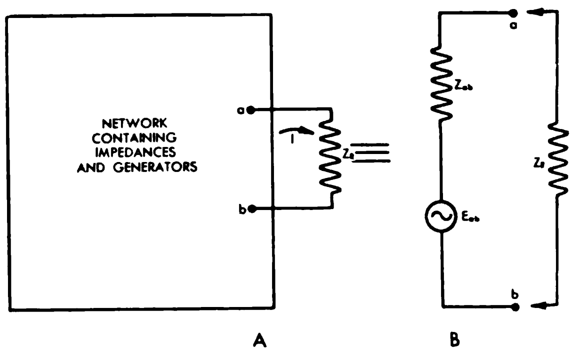


Figure 2–104. Thevenin's Theorem

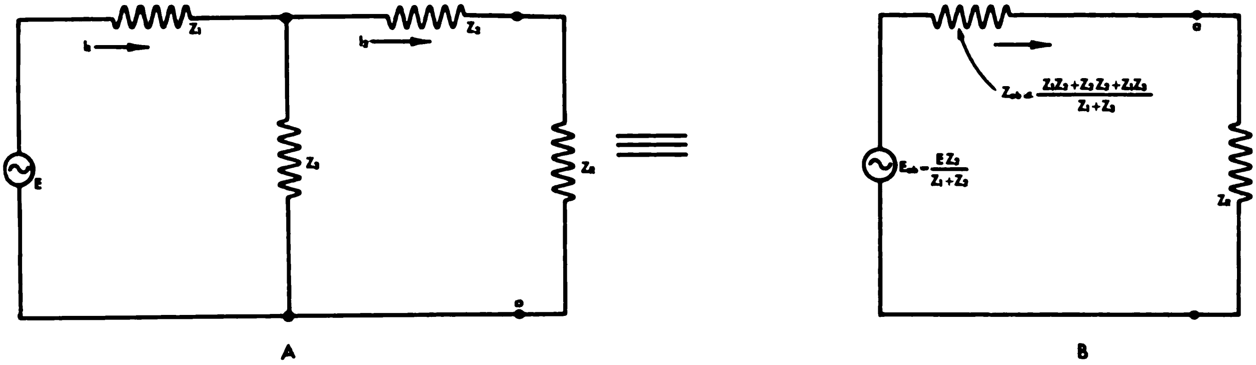


Figure 2–105. Equivalent Circuits Based on Thevenin

The proof of Thevenin's theorem is based on showing that the current flowing through Z_R in circuit B in Figure 2–105 is the same as the current which flows through Z_R when it is connected to the equivalent generator.

From the circuit at A in Figure 2–106, the current I_2 is,

$$I_2 = I_1 \frac{Z_3}{Z_3 + Z_2 + Z_R} \qquad (1)$$

The total current flow I_1 through the generator E is,

$$I_1 = \frac{E}{Z_1 + \dfrac{Z_3(Z_2 + Z_R)}{Z_3 + Z_2 + Z_R}} \qquad (2)$$

Substituting equation (2) in equation (1) gives I_2 in terms of the impedances and the generator voltage. Thus,

$$I_2 = \frac{EZ_3}{Z_1Z_2 + Z_2Z_3 + Z_1Z_3 + Z_R(Z_1 + Z_3)} \qquad (3)$$

Now find the equivalent generator according to Thevenin's theorem in the following manner: First, remove Z_R from the terminals ab. This makes the open-circuited voltage,

$$E_{ab} = E\frac{Z_3}{Z_1 + Z_3} \qquad (4)$$

This is the voltage of the equivalent generator. Now find the impedance of the equivalent generator by replacing the generator at A with its internal impedance (which is zero in this case) and measuring the impedance, looking in terminals ab. This impedance is

$$Z_{ab} = Z_2 + \frac{Z_1Z_3}{Z_1 + Z_3}$$

$$Z_{ab} = \frac{Z_1Z_2 + Z_2Z_3 + Z_1Z_3}{Z_1 + Z_3} \qquad (5)$$

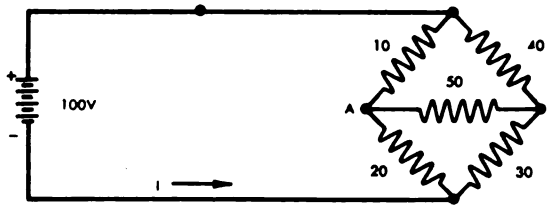


Figure 2–106. Bridge Circuit

After finding the equivalent generator by Thevenin's theorem, connect Z_R to this generator, and calculate the current flow through it by Ohm's law as follows:

$$I = \frac{\dfrac{EZ_3}{Z_1 + Z_3}}{\dfrac{Z_1Z_2 + Z_2Z_3 + Z_1Z_3 + Z_R}{Z_1 + Z_3}}$$

$$I = \frac{EZ_3}{Z_1Z_2 + Z_2Z_3 + Z_1Z_3 + Z_R(Z_1 + Z_3)} \qquad (6)$$

On comparing equations (3) and (6), you can see that they are equal, thus proving Thevenin's theorem.

Example

Using the values indicated in the bridge circuit in Figure 2–107, find the current flow through the 50-ohm resistor, assuming that the internal resistance of the battery is zero.

Solution

1. First, find the open-circuited voltage between points AB.

The drop across the 20-ohm resistor

$$= 100 \times \frac{20}{30} = 66.67 \text{ volts}$$

The drop across the 30-ohm resistor

$$= 100 \times \frac{30}{70} = 42.8 \text{ volts}$$

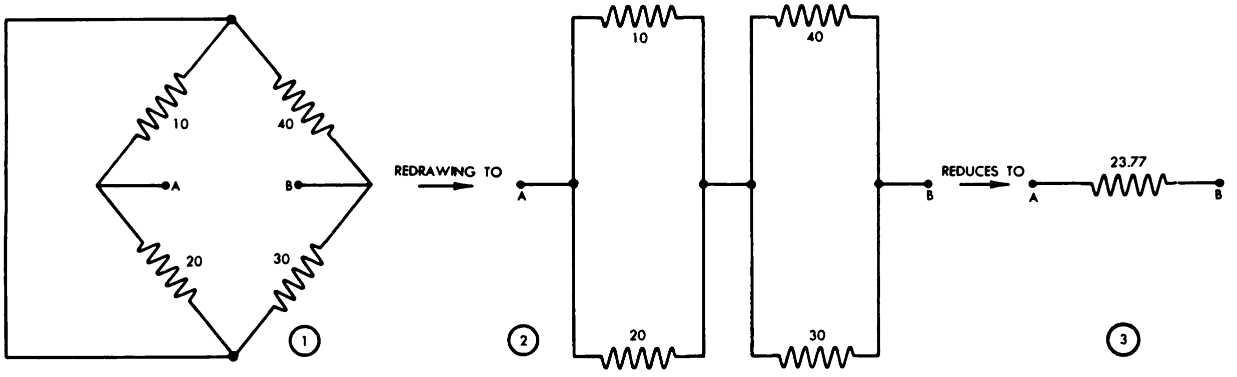


Figure 2–107. Equivalent Circuits of Bridge Circuit

The drop between AB

$$= 66.67 - 42.8 = 23.87 \text{ volts}$$

This also is the voltage of the equivalent generator.

2. Next, find the impedance looking into points AB (with the 50-ohm resistor disconnected as shown at 1 in Figure 2–107) with the generator replaced with its internal impedance (0 ohms). Redrawing the circuit of 1 produces the parallel combinations shown at 2 of which the series resistance equals 23.77 ohms (shown at 3).

3. Set up the equivalent generator circuit as in Figure 2–108. The voltage of this equivalent generator is the open-circuited voltage at points AB, and its internal impedance is the impedance looking in at points AB.

4. Lastly, connect the 50-ohm resistor to the terminals of the equivalent generator and calculate the current flow.

$$I = \frac{23.87}{23.77 + 50} = 0.324 \text{ amperes}$$

Thus, the current flow through a resistor can be calculated without setting up simultaneous equations whose solution, at best, is tedious. It is suggested that you study this example until you thoroughly understand each step. The four steps outlined are general and apply to all similar problems.

Maximum Power Transfer Theorem

The maximum power transfer theorem gives the conditions under which maximum power may be delivered to a load. A given emf source with a certain internal impedance does not furnish the same power or energy to different loads. Only when the impedance of the emf source is equal to that of the load is maximum energy delivered to the load. Even then the efficiency of transfer is only 50%. In communication circuits where the *total* power available is small, it is very important to match the impedance of the load to the source of power. Radio and radar, for example, deal with relatively weak signals; it is, therefore, important to pass as much of the signal as possible from one circuit to another.

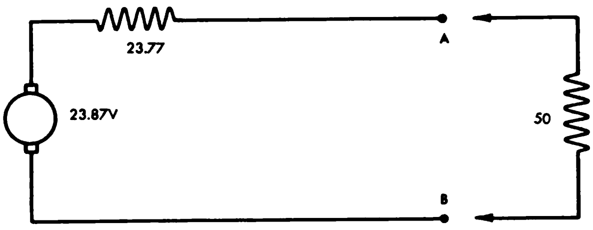


Figure 2–108. Equivalent Generator Circuit

For a generator that feeds through a network to supply maximum power to an impedance Z_R at the terminals (ab), Z_R must comply with the conditions required in the following cases:

1. When the phase angle of Z_R is unrestricted, Z_R must be the conjugate of the impedance Z_{ab} looking into the network at the terminals ab.

2. When the phase angle of Z_R is fixed, then the absolute values of Z_R and Z_{ab} must be equal.

Figure 2–109 indicates that any network consisting of two or more generators supplying power to another network may be reduced to an equivalent circuit containing only a single generator of certain internal impedance Z_{ab} feeding a load Z_R. This is an application of Thevenin's theorem.

The following summarizes the principal points of the maximum power transfer theorem:

1. When Z_{ab} is purely resistive, then Z_R must be entirely resistive and equal to Z_{ab} for maximum power transfer. Thus,

$$Z_{ab} = R_{ab} = Z_R = R_R$$

2. When Z_{ab} is complex,

$$\therefore Z_{ab} = R_{ab} - jX_{ab}$$

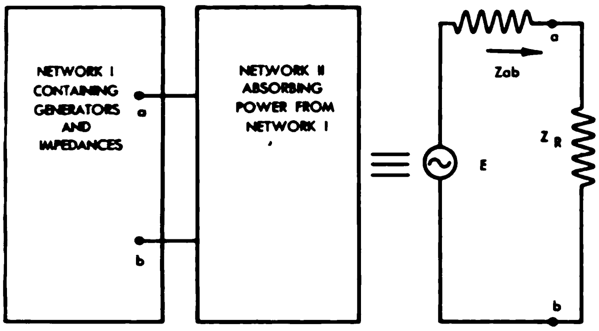

NETWORK I CONTAINING GENERATORS AND IMPEDANCES
a
b
NETWORK II ABSORBING POWER FROM NETWORK I
≡
E
Zab
a
Z_R
b

Figure 2–109. Maximum Power Transfer Theorem

Z_R must be complex and the conjugate of Z_{ab}. In other words,

$$Z_R = R_R + jX_R$$

where $R_R = R_{ab}$ and $X_{ab} = X_R$ for maximum power transfer.

3. When it is not possible to vary Z_R, both as to its resistive component or its reactive component, the transfer of power will be best when the absolute value of Z_{ab} and Z_R are equal. In the language of mathematical symbols, the best transfer of power takes place when

$$|Z_{ab}|=|Z_R|$$

The rigorous proof of maximum power transfer requires differential calculus, which is beyond the scope of this course. However, the following elementary presentation should be helpful. In this connection consider the condition in which Z_{ab} and Z_R are pure resistances. Then from the preceding illustration, the current through R_R equals

$$I = \frac{E}{R_{ab} + R_R} \qquad (1)$$

and the power in the load is

$$P = I^2R_R = \frac{E^2R_R}{(R_{ab} + R_R)^2} \qquad (2)$$

To determine for what value of R_R the power, as given by equation (2), is a maximum, it is necessary to make a graph of power versus R_R in terms of R_{ab} as shown in Figure 2–110.

From this graph you can see that the power to the load increases until the load resistance is equal to the internal resistance of the generator, and then begins to decrease gradually to zero.

Now, consider the case in which Z_{ab} and Z_R have reactance as well as resistance. In this case the absolute value of current is expressed by the equation,

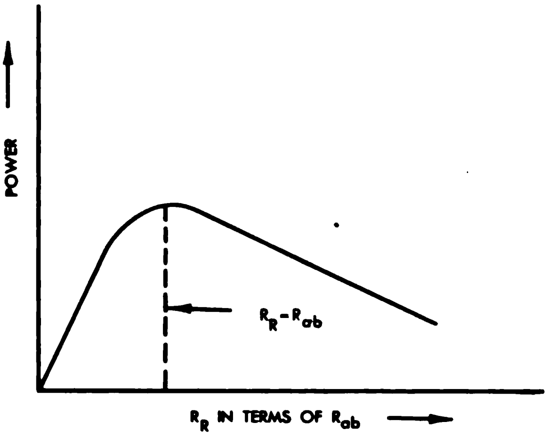


Figure 2–110. Power Delivered to R_R as R_R is Varied

$$|I|= \frac{E}{Z_{ab} + Z_R} = \frac{E}{(R_{ab} + R_R)^2 + (X_{ab} + X_R)^2} \qquad (3)$$

and the power delivered is equal to,

$$P = I^2R_R = \frac{E^2R_R}{(R_{ab} + R_R)^2 + (X_{ab} + X_R)^2} \qquad (4)$$

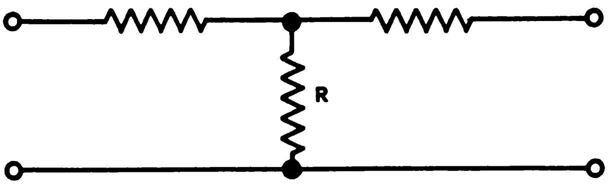
From inspection it is clear that, as far as X_R is concerned, the power in equation (4) is a maximum when $X_{ab} = X_R$. That is, when Z_{ab} is inductive, Z_R must be capacitive and vice versa. Now when this condition is fulfilled, equation (4) reduces to equation (2). Therefore, $R_R = R_{ab}$ is the value of R_R which makes the power a maximum. Thus, the circuit is series resonant, and the only opposition to the current is the total resistance of Z_R and Z_{ab}. At maximum power transfer, the total resistance is $2R_{ab}$.

Using the values indicated in the bridge circuit shown in Figure 2–106, find the value of the resistor across the points AB which will absorb the greatest amount of power from the 100-volt battery.

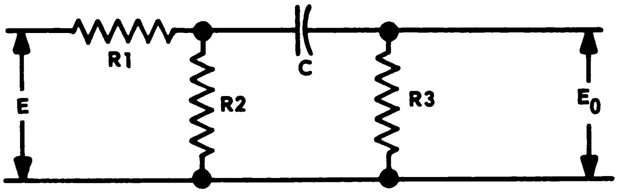
By applying Thevenin's theorem, you can find the equivalent generator circuit as shown in Figure 2–107. Since the internal impedance of the equivalent generator is 23.77 ohms, the resistor to be placed across AB must have the same value in order to absorb the greatest amount of power from the battery.

COUPLED CIRCUITS

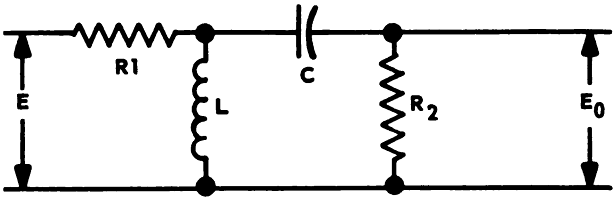
Interconnected circuits and nearby circuits that transfer electrical energy from one circuit to the other are called *coupled circuits.* There are four types of coupling illustrated in Figure 2–111.



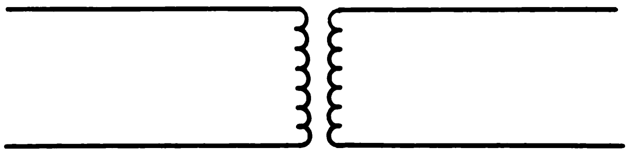
A DIRECT COUPLING



B RC COUPLING



C IMPEDANCE COUPLING



D TRANSFORMER COUPLING

Figure 2–111. Types of Coupling

Direct Coupling

Figure 2–111A shows two circuits directly connected, or coupled, together. The voltage developed across R is applied directly to the following stage.

RC Coupling

The circuit in Figure 2–111B is typical of networks used for connecting two amplifier stages in radio receivers and in other equipment using RC-coupled amplifiers. An important consideration in RC-coupled circuits is the comparison between the relative magnitude and phase of the output voltage, E_o (the voltage across R_3), and the impressed voltage, E. When you know the circuit constants, the frequency, and the magnitude of E, you can calculate the voltage, E_o, by means of complex algebra. However, before discussing the calculations involved, it is well to observe the following facts with reference to this circuit. The higher the frequency of the impressed voltage, the smaller the reactance of C and the more nearly the circuit is like a series-parallel resistive combination. On the other hand, the lower the frequency, the greater the reactance of C. This causes a smaller proportion of the voltage across R_2 to appear across R_3 as the output voltage, E_o, and a greater phase difference to exist between the voltages, E and E_o, with E_o leading.

You can see, therefore, that there is a wide variation in the magnitude of the output voltage and the phase relation between the input and output voltages. In practical circuits which use RC coupling, the range of variation is further widened by small distributed capacitances, called shunt capacitances, that appear across R_2 and R_3. (Shunt capacitances are considered in detail in a later chapter.) At the higher frequencies, these capacitances become increasingly important because they lower the output voltage and cause E_o to lag behind E.

In Figure 2–112, as the frequency increases from zero to mid-frequency range, gain increases and then levels off at a point at which capacitive reactance is negligible. The dotted portion of the curve from c to d represents the dropping off of gain. In RC-coupled amplifier circuits, this

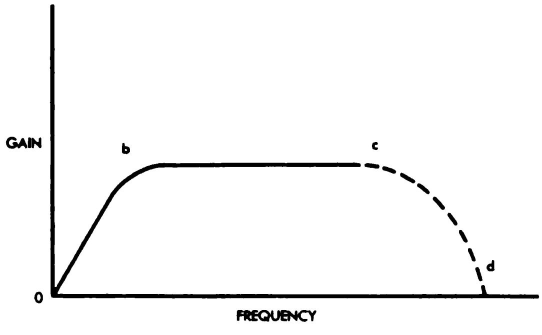


Figure 2–112. Approximate Frequency Response for RC-Coupled Circuits

drop is due to the shunting effect caused by distributed capacitances in the vacuum tubes and wires.

LC Coupling

Sometimes LC coupling is used to couple amplifier stages. With reference to Figure 2–111C, coil L may have either an iron core or an air core, depending on whether the signals being amplified are in the audio range (30 — 15,000 cps) or in the radio frequency range. In practical circuits, L not only has some resistance, but also is shunted by distributed capacitances in the coil windings and by the plate-to-cathode capacitance of the preceding tube.

An important fact to consider in LC coupling is the effect that high- and low-frequency voltage sources have on the output voltage. At high frequencies the reactance of a coil is high while that of a capacitor is low. For practical purposes you can consider that L and R_2 in the LC-coupled circuit are in parallel. The voltage output, then, will equal approximately the voltage across the coil. Thus, the higher the frequency, the greater the magnitude of E_o. Further, the voltage E_o will lead E. At lower frequencies, the coil has small reactance, but the capacitor has a large reactance. Therefore, the voltage across the coil will equal the voltage across the voltage divider, C and R_2. In this case, voltage E_o leads the input voltage E as before, but by a larger angle.

The shape of the response curve (Figure 2–113) depends on the relative values L, C, R_1 and R_2. In actual cases, the curve may have points of maxima and minima because of the resonance of the coil with its distributed capacitance.

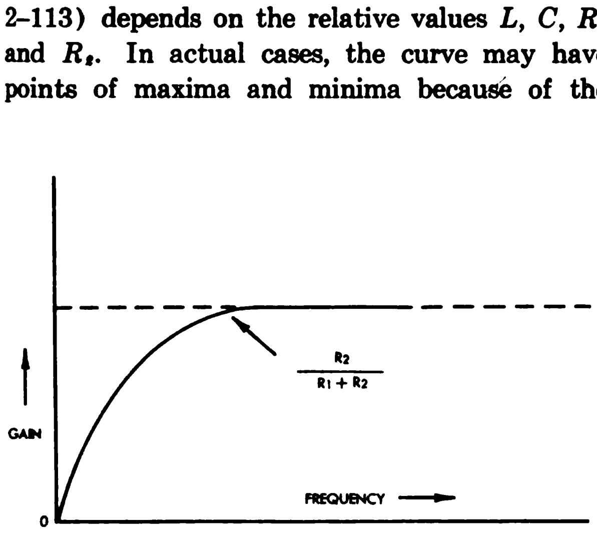


Figure 2–113. Approximate Frequency Response for LC-Coupled Circuits

Transformer Coupling

A transformer consists of two coils that couple energy from one to the other by mutual inductance. Depending on the frequency of operation and the particular piece of equipment in which it is installed, a transformer may use an iron, a magnetic alloy, or an air core. The coil that is connected to the AC source is called the *primary* winding, and the coil that is connected to the load is called the *secondary* winding. The relative number of turns of the windings depends on whether the voltage induced in the secondary is to be larger or smaller than the voltage applied to the primary; the actual number of primary turns depends on the magnetic properties of the core and the frequency.

In both radio and radar, transformers generally are classed as power, audio, and radio frequency transformers. Power and audio transformers use iron cores, or some alloy that has the desired magnetic properties. In ordinary broadcast receivers, the r-f transformers may or may not contain iron cores. In radar sets, powdered iron cores are commonly used in transformers.

The simple transformer shown in Figure 2–114 is a core type transformer in which the primary is wound on one leg of the core, and the secondary on the opposite leg. To reduce eddy current losses, the transformer core usually is constructed of laminated sheets instead of a solid piece.

When the primary current flows momentarily in the direction of the arrow labeled I_p, the magnetic flux ϕ flows in the direction indicated. To

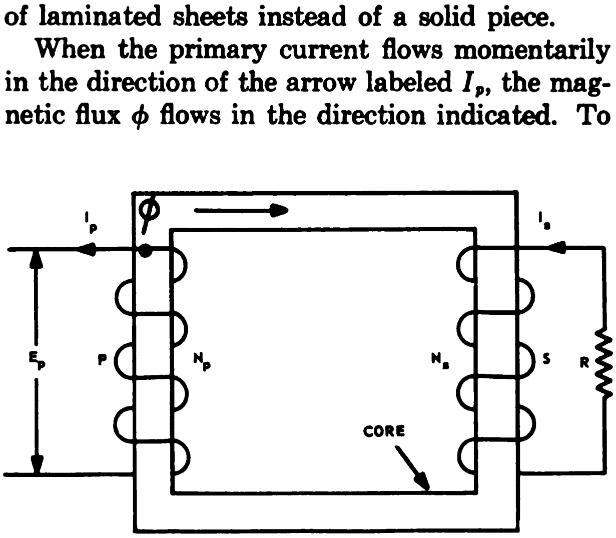


Figure 2–114. Simple Transformer

determine this direction, use the left-hand rule, which states that when the fingers of the left hand are placed so that they encircle the primary winding and point in the direction of electron flow, the thumb will point to the north pole of the electromagnet and the direction of flux flow.

According to Lenz's law, whenever the flux (ϕ) is varying and links a secondary winding in a transformer, a voltage is induced in the secondary. The polarity of the induced voltage is such that, when current flows in the secondary, the flux opposes the primary flux. The current in the secondary flows in the direction represented by the arrow labeled I_s in the illustration, and the flux set up by the secondary current opposes the initial flux produced by current flow in the primary winding.

Since a transformer requires a varying flux for operation, obviously it cannot be used with a DC source of voltage. When a steady voltage is suddenly connected to the primary, a voltage is induced in the secondary for an *instant*, but none at all after this transient period, for then current is not changing through the primary.

An iron-core transformer is a very efficient piece of electrical equipment. Any time power is supplied to the primary, a magnetic field is set up. It links the secondary windings and induces a voltage in it and causes current to flow in the load. In other words, power is transferred to the secondary. This transfer of power is greatest when all the flux set up by the primary links all the turns of the secondary.

Because of the high permeability of iron (and some of its alloys), its reluctance to the path of the magnetic field in a transformer is much less than that of air, and the amount of flux lost (leakage) to air, therefore, is small. This accounts for efficiencies as high as 99% in iron-core transformers. The effectiveness of the coupling between the primary and the secondary is expressed in terms of the coefficient of coupling K. The coefficient of coupling is defined as the ratio of flux that links the two coils to the flux created by the current flow through the primary.

The preceding paragraphs dealt with the theory and operation of transformers. Now it is well to study the following formulas for an ideal transformer (one assumed to have no flux leakage and no power loss in either the primary or the secondary).

For an ideal transformer, the formula for th instantaneous voltage which is induced acros the primary by the changing flux is,

$$e_p = -N_p \frac{\Delta\phi}{\Delta t} \times 10^{-8} \text{ volts} \qquad (1)$$

As the same flux links the secondary coil, th voltage induced in the secondary is,

$$e_s = N_s \frac{\Delta\phi}{\Delta t} \times 10^{-8} \text{ volts} \qquad (2)$$

Dividing equation (1) by equation (2) gives

$$\frac{e_p}{e_s} = \frac{N_p}{N_s} \qquad (3)$$

Since this relation is true for any instant, it i also true when the voltages are maximum. Thus

$$\frac{E_{pm}}{E_{sm}} = \frac{N_p}{N_s} \qquad (3a)$$

where E_{pm} is the maximum primary induce voltage and E_{sm} is the corresponding secondar voltage.

Since $E_{max}/\sqrt{2} = E_{eff}$, you can divide the numerator and the denominator of the left-han member of equation (3a) by $\sqrt{2}$.

This gives,

$$\frac{E_{pm}/\sqrt{2}}{E_{sm}/\sqrt{2}} = \frac{E_p}{E_s} = \frac{N_p}{N_s} \qquad (3b)$$

where E_p and E_s represent effective values of primary and secondary voltages respectively.

Equation (3b) is the expression of the relationship between the number of turns of the primar and secondary and the corresponding voltages i a transformer.

The power input at any instant in an idea transformer equals the power output at that instant, that is,

$$i_p e_p = i_s e_s \qquad (4)$$

where i_p and i_s represent the instantaneous value of the primary and secondary currents.

Since this relationship is true for all values, i follows that it is true for the maximum and th effective values of voltage and current. Thus,

$$\frac{E_p}{E_s} = \frac{I_s}{I_p} \qquad (5)$$

Then from equation (3b) and equation (5) it follows that,

$$\frac{I_s}{I_p} = \frac{N_p}{N_s} \qquad (6)$$

According to the maximum power transfer theorem discussed before in this chapter, it is necessary to match the impedance of two circuits to obtain the greatest transfer of energy. The simplest and most common circuit used for impedance matching is the transformer. There are a number of mathematical relationships between primary and secondary impedances and the corresponding turns in a transformer.

The impedance looking into the primary of a transformer is

$$Z_p = \frac{E_p}{I_p} \qquad (7)$$

The terms E_p and I_p are given by equations (3b) and (6) respectively. Substituting for these values in equation (7) and noting that E_s/I_s is Z_s, gives,

$$Z_p = \frac{E_p}{I_p} = \frac{E_sN_p/N_s}{I_sN_s/N_p} = Z_s\frac{N_p^2}{N_s^2} \qquad (8)$$

or,

$$\frac{Z_p}{Z_s} = \left(\frac{N_p}{N_s}\right)^2$$

From this equation, it is apparent that for a transformer with unity coupling, the impedance ratio is equal to the square of the turns ratio. For example, if R in the transformer shown in Figure 2–114 is 100 ohms, N_p is 100 turns, and N_s is 1000 turns, then the impedance looking into the primary is equal to 1 ohm. Another way of expressing this result is to say that the impedance *reflected* into the primary is 1 ohm.

The phase angle between the primary voltage and the primary current in a transformer is equal to the phase angle between the secondary voltage and the secondary current. This can be shown as follows:

The average power in the primary circuit is

$$P_p = E_pI_p \cos \theta_p$$

The average power in the secondary is

$$P_s = E_sI_s \cos \theta_s$$

where θ_p and θ_s are the phase angles of the primary and the secondary circuits.

Then, since $P_s = P_p$,

$$E_pI_p \cos \theta_p = E_sI_s \cos \theta_s$$

But from equation (5), $E_pI_p = E_sI_s$,

Therefore, $\cos \theta_p = \cos \theta_s$

$$\text{and } \theta_p = \theta_s \qquad (9)$$

So far, only an ideal transformer having unity coupling has been discussed.

Another important concept to consider is the general transformer circuit. In the following discussion, equations for calculating primary current, secondary current, input impedance, and reflected impedance are derived by Ohm's and Kirchhoff's laws. In addition, the concept of *mutual impedance* is discussed.

In the typical transformer circuit shown in Figure 2–115, the load (Z_2) is connected across the secondary and the impedance (Z_1) is in series with the primary. These impedances may be resistive, reactive, or any combination of these elements. Z_p is the impedance looking into the primary with the secondary open-circuited, and Z_s is the impedance of the secondary with the primary open. The mutual impedance is defined as $Z_m = j\omega M$ where M is the mutual inductance between the primary and the secondary.

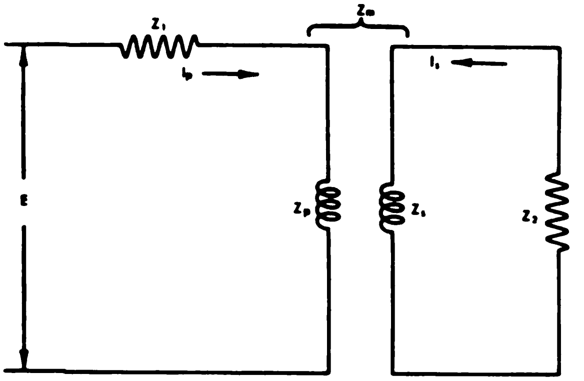


Figure 2–115. Typical Transformer Circuit

Applying Kirchhoff's laws first to the primary and then to the secondary gives,

$$E = I_pZ_1 + I_pZ_p + I_sZ_m \qquad (10)$$

$$0 = I_sZ_2 + I_sZ_s + I_pZ_m \qquad (11)$$

The term I_sZ_m is the voltage drop in the primary due to current flow in the secondary and the mutual impedance Z_m, while I_pZ_m is the voltage drop in the secondary due to I_p in the primary.

In equations (10) and (11), I_p and I_s are considered as unknowns. Solving for I_p and I_s simultaneously gives the equations,

$$I_p = \frac{E\,(Z_2 + Z_s)}{(Z_1 + Z_p)\,(Z_2 + Z_s) - Z_m^2} \qquad (12)$$

$$I_s = \frac{-EZ_m}{(Z_1 + Z_p)\,(Z_2 + Z_s) - Z_m^2} \qquad (13)$$

An important fact to understand is that these equations are *vector equations* and it therefore is necessary to use complex expressions to represent the various impedances. The negative sign of equation (13) indicates that the secondary current is 180° out of phase with the primary current.

To find the impedance looking into the primary circuit from the source of applied voltage E, divide E by I_p. Thus,

$$Z = \frac{E}{I_p} = \frac{(Z_1 + Z_p)(Z_2 + Z_s) - Z_m^2}{Z_2 + Z_s} \qquad (14)$$

$$= Z_1 + Z_p - \frac{Z_m^2}{Z_2 + Z_s}$$

In this equation the term $\frac{Z_m^2}{Z_2 + Z_s}$ is the impedance reflected into the primary by the secondary. In generalized impedances, the sign preceding this term must always be negative and the denominator is usually written $R \pm jX$. The latter is expressed this way so that the reflected impedance will contain (in addition to a resistive term of positive sign) a reactive term whose sign depends upon the nature of the total reactance in the secondary circuit. The sign of this reflected reactance is always opposite to that of the total secondary reactance. In other words, when $Z_2 + Z_s$ is inductive, the reflected term is of the nature of $-jX$ and reduces the inductive reactance looking into the primary. With perfect coupling and with Z_s as a pure resistance, this reflected term is just large enough to cancel completely the inductive reactance of the primary. The impedance looking into the primary, therefore, is in the nature of a pure resistance.

The preceding equations are applicable to all transformers and are frequently used in electronics. In power transformers, the coefficient of coupling is nearly equal to unity and produces a ratio of transformation that is very nearly equal to the turns ratio. In radio and radar circuits, particularly where the primary and secondary are tuned to resonance, the coefficient of coupling may be very small. Because leakage losses are often 95% or more, the effective ratio of transformation differs greatly from the ratio of turns on the transformer. Therefore, it is better to analyze loosely coupled circuits in radio and radar in terms of mutual inductance rather than turns ratio.

With equation (14) it is possible to construct the equivalent circuit of the typical transformer as illustrated in Figure 2–116. In this case, the impedance looking into the primary is the sum of three quantities, Z_1, Z_p, and the reflected term $-Z_m^2/(Z_2 + Z_s)$.

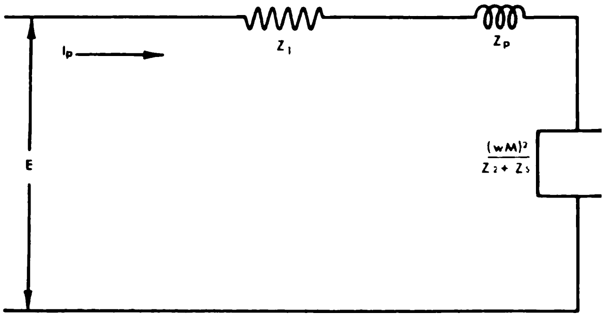


Figure 2–116. Equivalent Circuit Referred to the Primary

Voltage and Current Phase Relationship in Transformers

Among the uses of a transformer is that o obtaining a required phase shift between circuits By using vectors, we can understand how this can be accomplished. While transformer losses wil have an effect on the magnitude and direction o a vector, it is small and can be neglected withou serious error.

IRON-CORE TRANSFORMERS WITH NO LOAD. The primary and secondary windings of the trans former under consideration are in the same direc tion. In Figure 2–117A, the primary voltage E is shown as the base vector. Since there is no load on the secondary, the only current flowing in the primary is the magnetizing current I_m The primary can be considered to contain only inductance, if the resistance of the winding and core losses are disregarded. I_m can be plotted a lagging E by 90°. The magnetic flux ϕ is alway proportional to the magnetizing current and in phase with it (Figure 2–117B). This flux pro duces in the primary winding a counter emf (E_c that is dependent on the rate of change of flux This voltage is equal in magnitude and opposit in direction to E. It is maximum when I_m and the flux are passing through zero and producing a maximum rate of change (Figure 2–117B).

Since the flux also links the secondary, a volt age E_i is induced in the secondary. This voltag is also dependent on the rate of change of flux and therefore is in phase with E_c and 180° out of phase with E. The completed vector diagram in Figure 2–117A shows this 180° phase shift between the primary and secondary voltages.

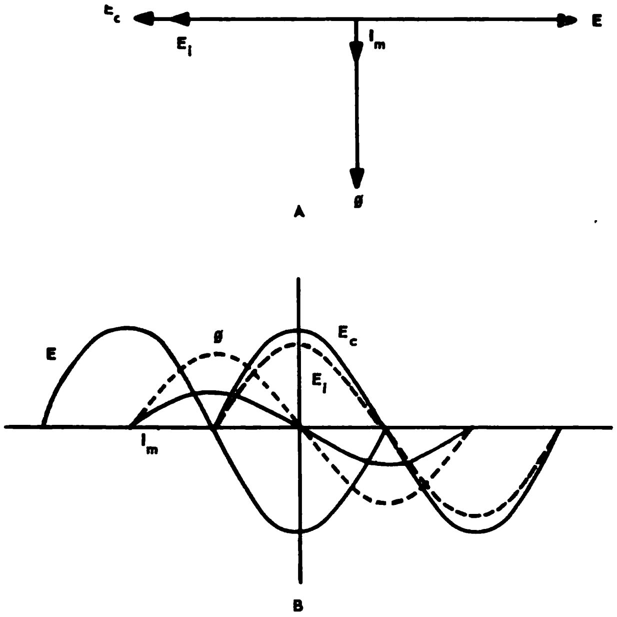


Figure 2–117. Vectors and Waveforms for an Iron-Core Transformer with Open Secondary

Iron-Core Transformers with Resistive Loads. The vectors in Figure 2–118B show the operation of the transformer with a resistive load across the secondary shown in A. As with the unloaded transformer, winding resistances and core losses are negligible and have been disregarded. Current flowing in the secondary tends to magnetize the iron core in a direction opposing the magnetizing action of the primary current. This causes the primary current to increase. The amount of increase (I_p) is such that the added magnetizing ampere turns balance the demagnetizing ampere turns of the secondary.

The current flowing in the primary is the vector sum of the magnetizing current and the current flowing to supply power to the secondary. The magnetizing current and its associated flux always lag the primary voltage by 90°. Since the primary circuit consists of the resistance reflected from the secondary plus the inductance of the primary winding, the primary current lags the voltage by some angle that is dependent on the relative values of resistance and inductance.

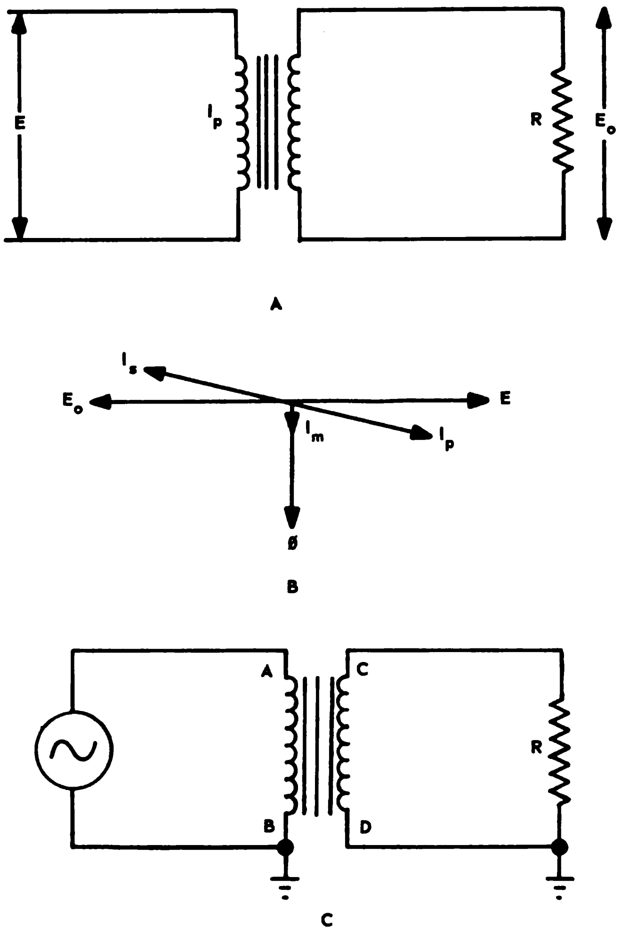


Figure 2–118. Iron-Core Transformer with Resistive Load

Because the same flux that produced the counter emf produces the voltage induced in the secondary, these two voltages are in phase with each other and 180° out of phase with the primary voltage. The secondary winding and the resistive load form a series RL circuit. The secondary current, then, lags the output voltage E_o by some angle dependent on the relative values of resistance and inductance. Note that the secondary current is also 180° out of phase with the primary current.

In the cases cited, the transformers produced a 180° phase shift. In practice, the phase of the voltage applied to the load depends upon the way in which the secondary is wound with respect to the primary and the way the load is connected to the transformer. Assume in Figure 2–118C that the voltage from A to B is 180° out of phase with the voltage from C to D. Then the voltage across the load is 180° out of phase with the primary voltage. If the leads connecting load R to the secondary are reversed (point C is grounded instead of point D), the voltage applied to the load will be in phase with the primary voltage. Thus, the voltage applied to the load may be either in phase with or 180° out of phase with the primary voltage, depending upon the way in which the load is connected to the secondary.

Air-Core Transformers with Secondary Open. In the circuit of an air-core transformer with no load connected to the secondary, assume that a sinusoidal current is flowing in the primary coil. There is little coupling between the primary and the secondary, since all of the flux emanating from the primary is not cutting the secondary winding. This coupling is sufficient, however, to induce a small voltage in the secondary. Since the secondary is open, no current will flow in it. There is no reflection of impedance into the primary. As with the iron-core transformer, the windings have some resistance and the transformer has losses. These are small and are disregarded in the preparation of the vector diagrams.

The current flowing in the vector diagram shown in Figure 2–119B lags the primary voltage by 90°. This is because the primary circuit contains only inductance. The voltage induced in the secondary is maximum when the primary current is passing through zero and producing a maximum rate of change of flux. Thus, the voltage induced in the secondary lags the primary current by 90° and lags the primary voltage by 180°. The phase relationships here are the same as with the iron-core transformer.

Air-Core Transformers with Secondary Tuned. In actual practice, the secondary of the air-core transformer is usually tuned with a capacitor, as shown in Figure 2–120A. The current flowing in the secondary winding is the same as that in the capacitor, so the circuit operation is that of a series LC circuit. In an air-core transformer, the coupling is so small that current flowing in the secondary circuit has negligible effect on the primary circuit. Thus, it can be seen that the primary is a simple inductive circuit in which the current lags the voltage by 90°. This is shown by the vectors in Figure 2–120B. The induced voltage lags I_p by 90°, as shown by the vector E_i.

If the secondary circuit is tuned to resonate at the frequency of the primary voltage, the reactance of the capacitor cancels the reactance of the secondary winding (series resonant circuit), leaving only the resistance of the circuit. Therefore, the secondary current is in phase with the

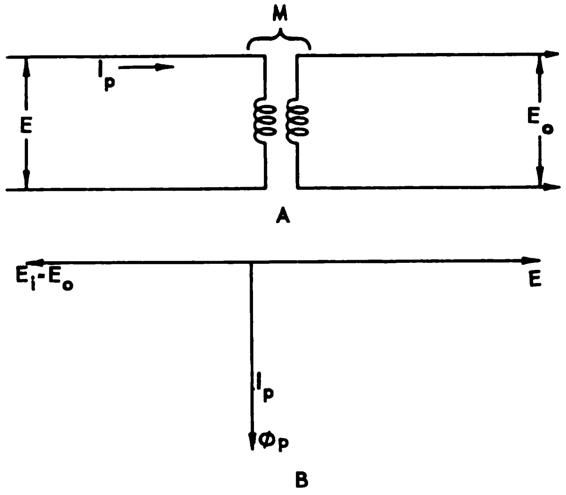


Figure 2–119. Air-Core Transformer with Open Secondary

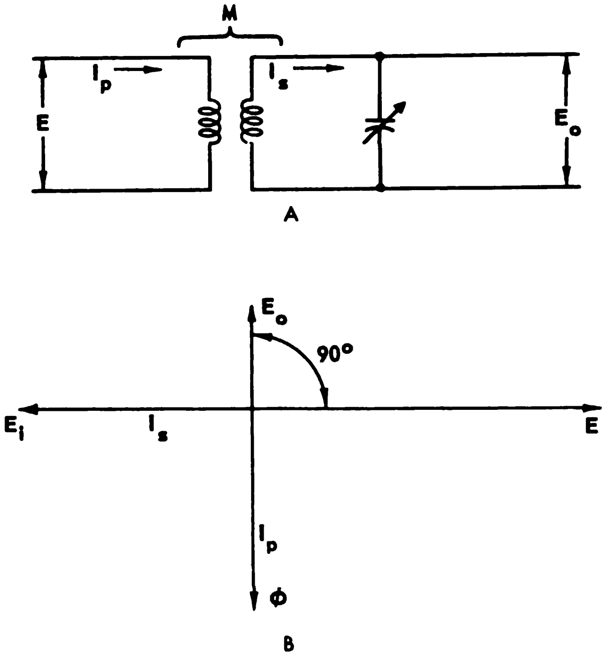


Figure 2–120. Air-Core Transformer with Secondary Tuned

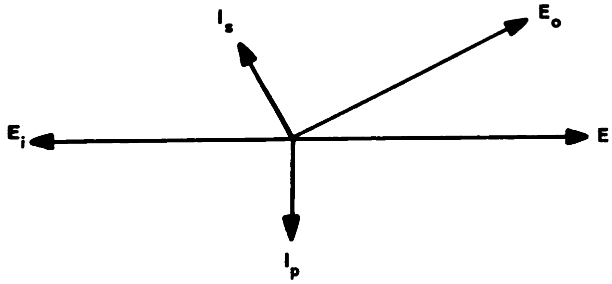


Figure 2–121. Applied Frequency Higher than Tuned Secondary Frequency

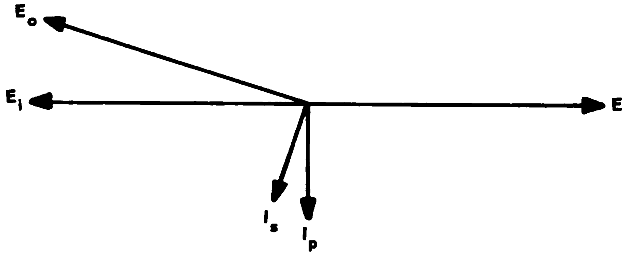


Figure 2–122. Applied Frequency Lower than Tuned Secondary Frequency

induced voltage. The output voltage is obtained across the capacitor. Since the voltage across a capacitor lags the current by 90°, E_o lags E_i by 90° and leads the primary voltage by 90°.

We can determine what to expect if a higher frequency is applied to the primary of this circuit, leaving the secondary tuned to the original frequency. The E, I_p and E_i vectors for this circuit are the same as in Figure 2–120B. They are shown in Figure 2–121. In the secondary, however, since the applied frequency is higher than the resonant frequency of the circuit, X_L exceeds X_c, and the circuit acts inductively (series resonance). This being the case, I_s lags E_i. As the applied frequency is increased, the angle by which I_s lags E_i increases. At some frequency, the circuit becomes predominantly inductive, and I_s lags E_i by approximately 90°. Since the output is taken across the capacitor, E_o lags the current by 90°. Thus, as the frequency rises above the resonant frequency of the secondary, the inductive effect of the secondary circuit causes the output voltage to lag the primary voltage by some value from about 0° to nearly 90°.

Changing the inductance or capacitance of the secondary, with the input frequency kept constant, will also produce a change in the phase shift. As the resonant frequency of the tuned circuit becomes lower than the input frequency (the input frequency is above the resonant frequency of the tuned circuit), the phase angle between the output voltage and the input voltage varies from nearly 90° to about 0°.

At a frequency below the resonant frequency, the vector diagram will appear as in Figure 2–122. Again, the E, I_p and E_i vectors are the same as in Figure 2–120B. In the secondary, however, since the input frequency is lower than the resonant frequency, X_c exceeds X_L, and the circuit acts capacitively. In this case, I_s leads E_i. As the frequency is decreased, the angle of lead increases. At some frequency, the circuit becomes predominately capacitive, and I_s leads E_i by nearly 90°. The output, taken across the capacitor, lags the current by 90°. Thus, variations in frequency below the resonant frequency of the tuned circuit cause the output voltage to lead the primary voltage by some value between 90° and 180°.

Again changing the inductance or capacitance of the secondary with the input frequency kept constant will produce a change in the phase shift. As the resonant frequency of the secondary becomes higher than the input frequency (the input frequency is below the resonant frequency of the tuned circuit), the phase angle between the output voltage and the input voltage varies from 90° to 180°.

The degree of phase shift between input and output voltages across a tuned transformer can be set anywhere from practically 0° to nearly 180° by proper adjustment of the inductance or capacitance in the circuit.

Power Losses in Transformers

There are two kinds of power losses in transformers — *copper loss* and *iron loss*. Copper loss is due to resistance in the primary and secondary windings. Mathematically, it is equal to,

$$P_c = I_p^2R_p + I_s^2R_s$$

Iron loss is due to hysteresis and eddy currents. Hysteresis, as you recall, is the lagging of the magnetic flux (produced in a core) behind the magnetizing force. Energy is needed to overcome the frictional opposition that the molecules composing the magnetic material display to the orientation of the small elemental magnets twice during each cycle of alternating current or voltage. This energy must be supplied by the source of voltage and therefore constitutes a loss in power.

In advanced texts it is proved that hysteresis loss equals

$$P_h = kB^n fV10^{-7} \text{ watts}$$

where k and n are constants, depending on the kind and treatment of the core material; B is the maximum value of the flux density used; f is the cycles per second, and V is volume of the core in cubic centimeters.

Eddy currents are produced by the changing flux density inside the core material. The changing flux density induces an emf not only in the windings but in the core itself. The induced emf's, in turn, produce extraneous currents that are called eddy currents. High eddy currents cause excessive heating of the core and effectively shield a winding from the inductive influence of the changing flux. Therefore, they represent a power loss which, in the interest of efficiency, should be reduced to a minimum.

Eddy current loss is equal to,

$$P_e = 1.6 \times 10^{-11} (fLB)^2 V \text{ watts}$$

where *L* is the thickness of the core plates. (For the meaning of the other terms refer to the equation for hysteresis loss.)

Note in the equation that eddy-current losses are proportional to the square of the thickness of the core plates. This means that the thinner the laminations, the smaller the eddy-current loss. It explains why transformer cores are laminated.

The efficiency of a transformer depends on the amount of copper loss (P_c), hysteresis loss (P_h), and eddy-current loss (P_e). The smaller the sum of these losses, the more efficient the transformer. There are lower resistance materials usable for transformer windings, but copper has the least resistance commensurate with cost. Loss due to hysteresis is reduced by the use of core material having a small hysteresis loop. The most practical and effective way to reduce eddy-current loss, as pointed out before, is to laminate the core material.

Types of Transformers

POWER TRANSFORMERS. The theory of operation and the formulas for power transformers already have been discussed to some extent. The following are some of the facts about the physical construction, voltages employed, and uses of power transformers.

The physical size of a transformer depends on the amount of power that it is to handle, the number of secondary windings that are needed, the frequency at which it is to operate, and so on. Typical secondary voltages in transformers in conventional power supplies, such as those used in home broadcast receivers and radar receivers, are 350, 700, 6.3, and 5 volts. The higher voltages are rectified by vacuum tubes, filtered and used as plate voltages for the various amplifier stages, while the small voltages are used directly to supply heater and filament power to vacuum tubes. In radar transmitters, transformers step pulse voltages up to over 200 kilovolts.

Physically, the size of these transformers is not as large as you might expect, since they are not required to handle power continuously. In practice, a radar transmitter is operated only for extremely short periods of time; for example, 0.3 microsecond at intervals of several hundred microseconds. In home receivers, the usual input voltage to transformers ranges from 110 to 120 volts at 60 cycles. For airborne radar equipment, AC voltages vary from about 80 to 110v at frequencies ranging from 400 to 1600 cps. Ground equipment uses the available commercial power — usually 220v, 60 cps three-phase.

AUDIO TRANSFORMERS. Audio transformers transfer voltages over a wide range of audio frequencies, rather than at a single frequency like the power transformer. They have cores of iron or alloy, and are designed to carry a limited amount of direct current in the primary winding without affecting the audio frequency response.

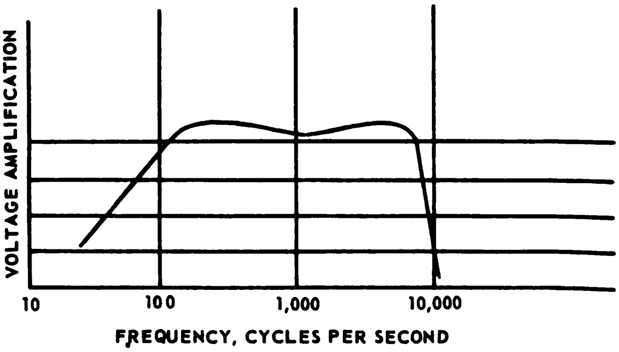


Figure 2–123. Typical Response Curve for Audio Transformers

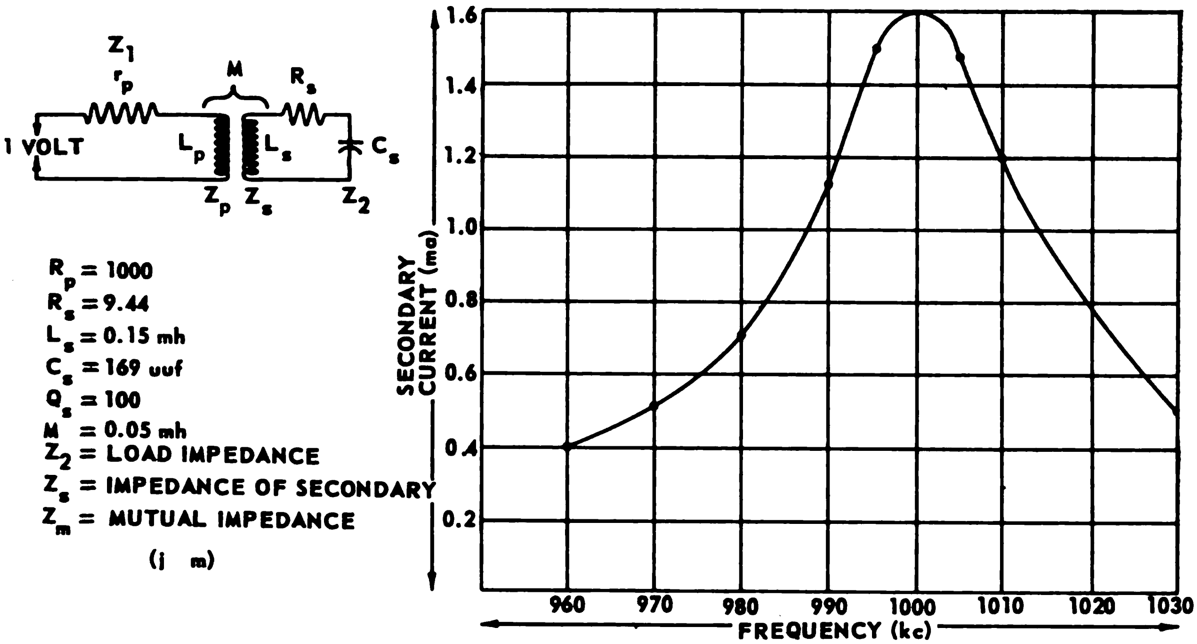


Figure 2–124. Response Curve for Transformer with Tuned Secondary

Since the ideal audio transformer has a flat frequency response, it can step up or step down voltages having different frequencies by the same amount. Practical, inexpensive audio transformers are designed with a nearly flat response for the mid-frequency range (100–4,000 cps). At lower and higher frequencies their output voltages begin to fall off. Above 4 or 5 kc, a resonant peak appears because of distributed capacitance in the transformer windings and the input capacitance of vacuum tubes to which it is connected.

Audio-frequency (a-f) transformers are constructed more or less like power transformers. However, greater care is exercised in the choice of core material and special precautions are taken to reduce capacitances between the primary and secondary windings as well as between the turns of each winding.

Other characteristics of a-f transformers are coefficients of couplings ranging from 0.99 to about 0.999, a primary inductance of 50 to 100 henrys, and a secondary inductance that is greater than the primary inductance by the square of the turns ratio. (The typical turns ratio is 3:1.) The formulas for power transformers are applicable to audio transformers over the flat portion of the frequency range.

Audio-frequency transformers are used in public address systems, and in the audio stages of broadcast and communications receivers for matching the output of the last stage of this equipment to the loudspeakers.

TUNED TRANSFORMERS. A tuned transformer is a radio-frequency transformer in which a capacitor is placed across either the primary or the secondary, or both. Transformers are tuned whenever they are required to display a frequency discriminator effect, that is, to pass a certain frequency or a band of frequencies. The type of response curve desired determines whether both the primary and secondary are tuned.

Figure 2–124 has a diagram of a transformer that has an untuned primary and a tuned secondary. It is typical of the transformers used in transformer-coupled radio-frequency amplifiers. Usually Z_1 is the plate resistance of a vacuum tube and is very large in comparison to the inductive reactance of the primary. The curve shown is the graph of secondary current versus frequency of the voltage applied to the primary. This curve has the same shape as an ordinary resonance curve, but has a lower Q than the secondary circuit curve alone.

The term for the impedance reflected into the untuned primary by the tuned secondary of a transformer is

$$\frac{Z_m^2}{Z_p + Z_s} \text{ or } \frac{(\omega M)^2}{R_s + j\left(\omega L_s - \frac{1}{\omega C_s}\right)}.$$

This term is subtracted from the primary impedance, since I_s is 180° out of phase with I_p.

Within a limited frequency range, the numerator $(\omega M)^2$ in the equation does not vary appreciably. The denominator, $R_s + j\left(\omega L_s - \frac{1}{\omega C_s}\right)$ represents the series impedance of the secondary. This is the form of the impedance of a parallel resonant circuit $\left(Z_t = \frac{(\omega L)^2}{Z_s}\right)$. Thus, the reflected impedance of a tuned secondary circuit varies with frequency according to the same general law as does the parallel impedance of the secondary circuit, but the magnitude of the curve depends upon the mutual inductance M.

When both the primary and the secondary of a transformer are tuned, the behavior of the transformer depends on the coefficient of coupling between the primary and the secondary. Notice in Figure 2–125 the circuit with a transformer having a tuned primary and secondary circuit, and the curves showing the primary and secondary current for three values of coupling. When the coefficient of coupling is small ($k = 0.002$), the primary current curve is practically the same curve that would be obtained if the tuned circuit were considered alone, and the secondary current is small and more peaked than if the secondary were considered alone.

As the coefficient of coupling is increased, the curve of primary current becomes broader. This is caused by the reduction in primary current at resonance and the increase in current at frequencies slightly off resonance. Meanwhile, the secondary current peak becomes larger and somewhat broader. The broadening continues until the resistance which is coupled from the secondary into the primary is equal to the primary resistance. This is the condition of critical coupling and causes the secondary current to reach its maximum value as shown by the curve labeled $k = 0.01$. The curve of secondary current is now somewhat broader than the resonance curve of the secondary alone.

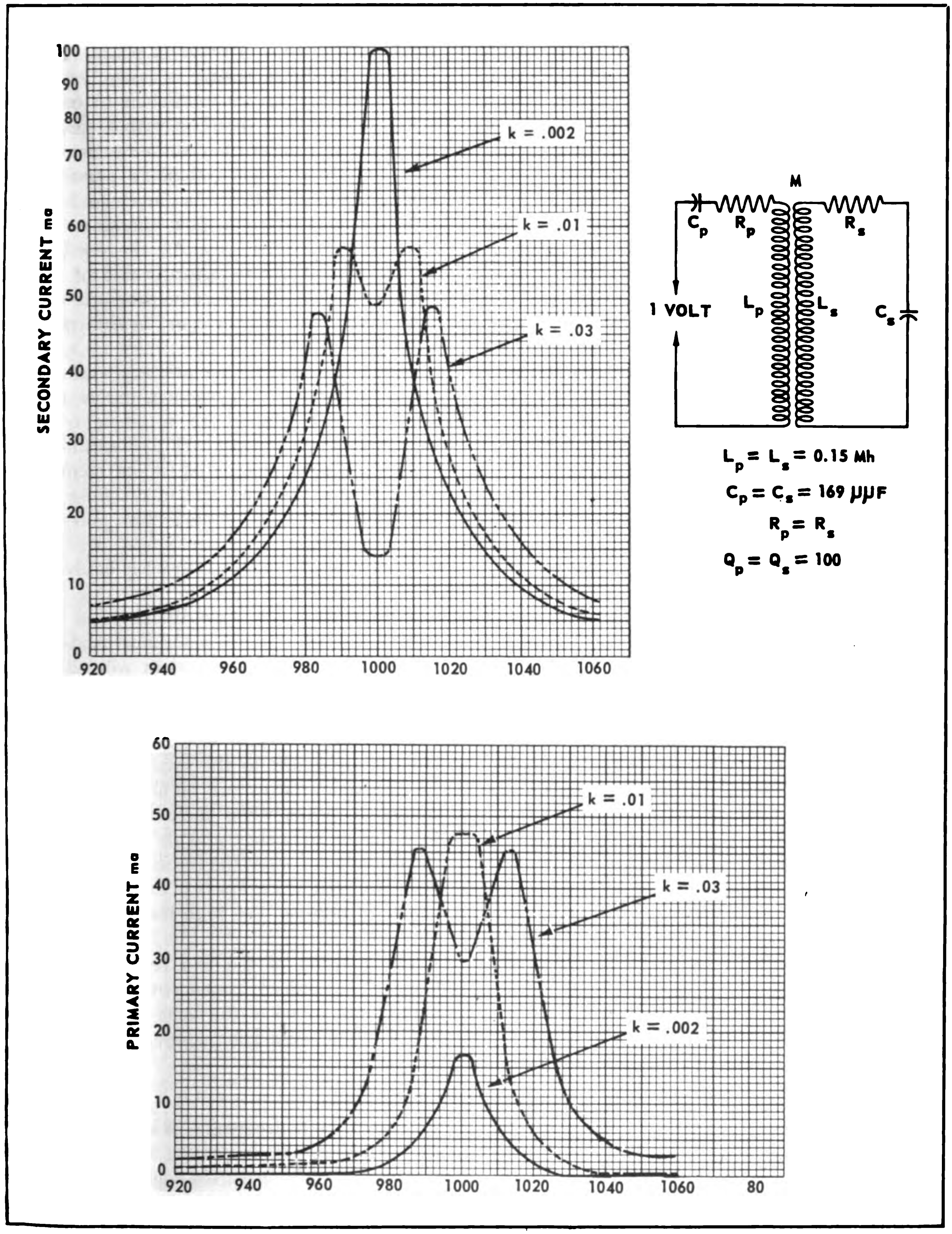
At the same time, the primary current curve has two peaks, since it is a little larger just below and above resonance than at resonance. When the coupling becomes greater than critical, for example, when $k = 0.03$, the double peaks of primary current become more prominent and farther off resonance. Besides, the secondary current begins to show double peaks. Notice that the peaks of primary current are decreased, but that the peaks of secondary current are almost the same as for critical coupling.

The double humps of the current curves are explained by the nature of coupled impedance. In understanding the explanation, first consider the total primary impedance. It consists of the actual impedance of the primary and the impedance coupled from the secondary. In the case of a tuned secondary and untuned primary, coupled impedance was proved to be essentially that of a parallel resonance curve. Therefore at resonance it is inductive; and above resonance it is capacitive. When the coupled impedance is added to the self-impedance of the primary, the effective primary resistance at resonance increases above the value it would have if there had been no secondary. This results in less primary current. Furthermore, the greater the degree of coupling, the greater the reflected and effective resistance, and the lower the current.

At frequencies below resonance the self-impedance of the primary is capacitive and the reflected impedance is inductive. The effective series impedance, therefore, is decreased, and the current reaches maximum value at a frequency below resonance where greatest cancellation of reactance occurs. Above resonance a similar situation exists, except that the self impedance is inductive and the coupled impedance is capacitive.

The amount of current in the secondary depends on the secondary impedance and the voltage induced in the secondary by the primary current. The induced *voltage* curve has approximately the same form as the primary current curve. This is because it is equal to $\omega M I_p$, and ωM does not vary appreciably over the small range of frequencies. Thus, the secondary current curve is almost exactly the same as the product of the shape of the primary current curve and the shape of the resonance curve (current) of the secondary when each is considered separately. Since the latter curve is sharply peaked, the secondary current curve is more peaked than the primary current curve.

The value of M in these circuits is usually equal to, or slightly greater than, the critical value. Besides having a flatter top, the sides of the curve fall off more rapidly than with a single tuned circuit. This makes the discriminator effect between the pass frequency and other frequencies much better than with a single tuned



$L_p = L_s = 0.15$ Mh

$C_p = C_s = 169\ \mu\mu F$

$R_p = R_s$

$Q_p = Q_s = 100$

Figure 2–125. Response Curve with Primary and Secondary Tuned

circuit. Such a circuit, therefore, has good selectivity. Tuned transformers are used in i-f amplifier stages in radio and radar receivers.

FACTORS AFFECTING BANDWIDTH OF TUNED TRANSFORMERS. The shape of a selectivity curve depends not only on the degree of coupling between primary and secondary, but also on the *Q's* and the resonant frequency of the primary and the secondary. It was pointed out before that for any given double-tuned circuit, optimum (or critical) coupling produces the greatest amount of current flow in the secondary and the greatest voltage output. The shape of this curve, as well as the magnitude of the current flow in the secondary, depends on the primary and secondary being tuned to the same resonant frequency and the *Q* of each circuit.

The higher the *Q*, the greater the selectivity; that is, the steeper the slope of the sides of the curve, or in other words, the narrower the bandpass. The *Q* of the primary or the secondary depends, among other things, on the resistance of the wire with which it is wound and whether or not a resistor is connected in parallel with the tuning capacitor. Circuits with resistors across transformer windings are said to be *loaded.* These resistors reflect a certain amount of resistance in series with the capacitor across the other winding. This tends to lower the effective *Q* of the circuit. In some applications, where a wide band of frequencies needs to be passed, resistors are connected across the transformer windings.

It can be proved that with critical coupling between the primary and secondary, $\omega M = \sqrt{R_1 R_2}$, where R_1 and R_2 are the resistances of the primary and secondary, respectively.

The expression for the coefficient at critical coupling is

$$k = \frac{1}{\sqrt{Q_1 Q_2}}$$

where Q_1 and Q_2 represent the *Q's* of the primary and secondary.

When the primary and secondary resonate to the same frequency and are coupled loosely, the primary current vs. frequency curve closely approximates the series-resonant curve for the primary when it is considered alone. As coupling is increased, the primary current curve broadens and peak current values decrease. With the passing of the point of critical coupling, double peaks and further broadening occur.

So far as the secondary is concerned, current flow is small when the coefficient of coupling is small, and current variation with respect to frequency closely approximates the resonance curves for the primary and secondary circuits, when each is taken separately. As coupling is increased, the secondary current flow increases and the current curve broadens. As the coefficient of coupling exceeds the critical value of the current curve, the peaks become progressively more pronounced and farther apart as the coupling is increased.

When the *Q's* of the primary and the secondary are equal, the response curves are symmetrical with respect to a mean frequency. However, when the primary and secondary are tuned to frequencies that differ slightly, the effect is as if the coefficient of coupling were increased and the two circuits were tuned to the same frequency. In other words, detuning the circuits when the *Q's* are equal is equivalent, so far as shape is concerned, to increasing the effective coupling. The only difference is that detuning causes the absolute magnitude of the response curve to be less than when the same shape is obtained without detuning.

When the *Q's* differ, the curves are not symmetrical, regardless of whether or not the two circuits are tuned to the same frequency. The low-frequency peaks will be less than the high-frequency peaks when the secondary is tuned to a *higher* frequency and its *Q* is *higher* than the primary, or when the secondary is tuned to a *lower* frequency and its *Q* is *lower* than the primary. Otherwise, the high-frequency peak will be depressed.

The position of the two resonant peaks for two tightly-coupled, tuned circuits of relatively high *Q* and tuned to the same frequency can be found by the equation,

$$f = \frac{f_o}{\sqrt{1 \pm k}}$$

Where f_o is the frequency to which the primary and secondary are tuned, and *k* is the coefficient of coupling. The negative sign in the denominator is used for finding the position of the upper resonant peak; the positive sign for the lower. This formula becomes inaccurate as *k* approaches critical coupling.

CHAPTER 3

Measuring Instruments

The solution of many problems involving circuit quantities such as voltage, current, and resistance was taken up in the preceding chapters. However, you did not learn how these quantities were measured, either for obtaining the data for problems or for checking the results of the computations. For measuring electrical quantities there are many useful instruments, among which the most common are the ammeter, the voltmeter, and the ohmmeter. You will find others with specialized applications such as the synchroscope.

This chapter deals with the principles, construction, and application of instruments commonly used for making DC and AC measurements, instruments for measuring resistances and reactances, combination meters, the vacuum tube voltmeter, and the oscilloscope (an instrument for observing and studying alternating current waveforms). Not only will you learn how measuring instruments perform their functions, but also what precautions you must observe in using them.

DC MEASURING INSTRUMENTS

Basic Principles

The principal DC measuring instruments are the ammeter (for current), the voltmeter (for voltage), and the ohmmeter (for resistance). Basically, all three are current measuring instruments, their principal difference in construction being only in how an external resistance is connected with respect to the moving element. In the ammeter the resistance is connected *across* the moving coil; in the voltmeter it is connected *in series* with the moving coil. In the ohmmeter the resistance is connected in series with the coil, but, in addition, this instrument supplies its own source of power.

Ammeters are connected in series in the circuit in which the current is being measured; voltmeters are connected in parallel with the circuit element across which the voltage is being measured. The power in a circuit is always *removed* before measuring resistances with an ohmmeter.

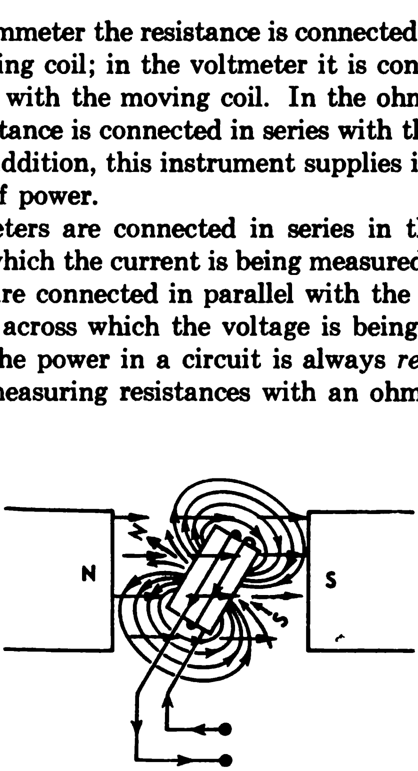


Figure 3–1. Turning Force on a Coil Carrying Current

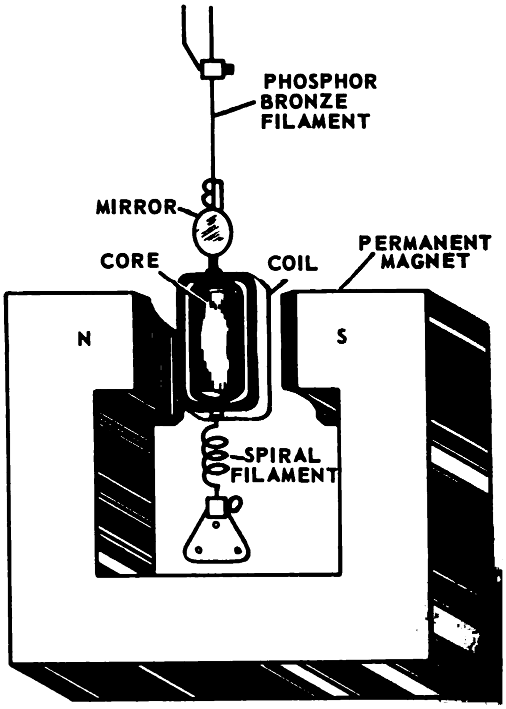
The DC ammeter is an instrument designed to measure the amount of direct current flowing in an electrical circuit. It operates upon the principle that a force is exerted upon a current-carrying conductor suspended in a magnetic field. Figure 3–1 shows a coil mounted between the poles of a magnet so that it is free to turn in the magnetic field. When the coil is connected in a circuit, current flows through the coil in the direction indicated by the arrows and sets up a magnetic field around the coil. This field has the same polarity as the adjacent poles of the permanent magnet. The interaction between the two fields produces forces which cause the coil to rotate (clockwise in Figure 3–1) until it reaches a position where its flux is in the same direction as that of the permanent-magnet field. The north pole face of the coil is facing the south pole of the magnet.

D'Arsonval Galvanometer

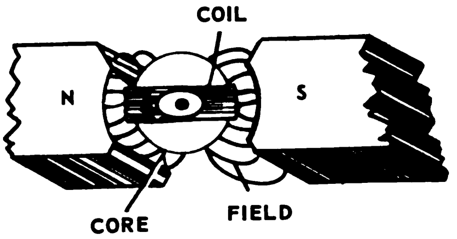
The D'Arsonval galvanometer consists of a coil of many turns of fine wire suspended between the poles of a permanent magnet (Figure 3–2A). Often there is a fixed cylindrical iron core inside the coil and the pole faces are shaped to provide a radial field (Figure 3–2B). The iron core insures that the magnetic field is uniform in strength and constant in direction. The air gaps between the coil and the pole faces and between the coil and the iron core are about $\frac{1}{16}$-inch in length. The coil is suspended by a single fine strip of phosphor-bronze, which serves as one lead to the coil. The other lead is a loose spiral of fine wire leading down from the bottom of the coil.

The coil's turning produces in the suspension strip a torsion which opposes the turning of the coil. When the turning force due to the current flow in the coil is equal to the restraining force of the suspension strip, the coil stops turning. Since the flux between the pole faces and the coil is practically radial, the deflection of the galvanometer is proportional to the current for all practical purposes. One system of reading the deflection of the D'Arsonval galvanometer is with a mirror fastened to the suspension wire. A beam of light is reflected from the mirror onto a scale at a distance away. As the mirror turns, the beam of light travels across the scale.

If the galvanometer coil starts to swing back and forth, as in a pendulum coming to rest, it will oscillate a long time before it comes to rest with the forces equalized, unless some means of damping, or slowing down, the oscillations is provided. In electrodynamic damping, the most common type used, the coil is wound on an aluminum bobbin. When the bobbin is moved through the magnetic field, currents are induced in the bobbin in such a direction as to produ a field opposing the motion of the coil. Wi the proper amount of damping, the coil will con to rest after only one or two oscillations.



A PRINCIPLE OF D'ARSONVAL MOVEMENT



B RADIAL FIELD

Figure 3–2. D'Arsonval Galvanometer

Weston-Type Instrument

Current and voltage measurements usually ı quire rugged and portable instruments. F measuring direct current and voltages, the W(ton-type instrument, based on the principle the D'Arsonval galvanometer, is used almost uı versally. It is constructed so that it is portal and is provided with a pointer and scale for i dicating the deflections of the moving coil. Tl permanent-magnet moving-coil element is often called the "D'Arsonval movement." It may be used as either ammeter or voltmeter.

The essential parts of the Weston-type instrument are shown in Figure 3–3. The permanent magnet is made in horseshoe form, usually of chrome or tungsten steel. Two soft-iron pole pieces are fitted to the poles of the magnet. A bracket of nonmagnetic material holds the iron cylinder which fits inside the coil. The length of the air gap is very much shorter than with the D'Arsonval galvanometers. The coil is wound on an aluminum bobbin which supports the coil mechanically and supplies the damping.

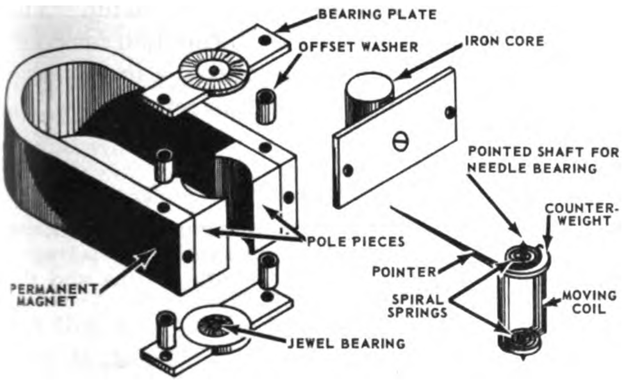


Figure 3–3. Weston-Type Instrument

Instead of hanging the coil, it is supported at both top and bottom by hardened steel pivots turning in cup-shaped jewels, making the moving coil almost frictionless. This method of suspension, unlike the D'Arsonval galvanometer, makes the instrument portable. Two phosphor-bronze, flat spiral springs, one at the top and the other at the bottom, serve as leads to the moving coil and also control the turning. Any tendency of the coil to turn is opposed by these two springs which are coiled in opposite directions to compensate for temperature changes which would cause the spiral spring to coil or uncoil.

A pointer made of aluminum is attached to the moving element. This pointer is carefully balanced by counterweights so that the whole moving element stays zeroed even if the instrument is not level. Since the deflection of the moving coil is practically proportional to the current in the coil, the graduations on the scale are nearly uniform.

Since the moving coil can handle only a very small current, the Weston-type instrument or D'Arsonval movement is generally used with a shunt when employed as an ammeter, and with a high series resistance when used as a voltmeter. Ammeter shunts are low resistances connected in parallel with the meter movement so that only a small fraction of the current flows through the moving coil.

The shunt, generally made of manganin, has a very low temperature coefficient, while the copper coil has a much higher temperature coefficient. This means that an increase in temperature will change the proportion of current going through the coil and cause the meter to read low. To prevent this happening, a *swamping resistor* of approximately zero temperature coefficient and having a resistance four or five times that of the coil is connected in series with the coil. Then a change in coil resistance due to a change in temperature has small effect on the over-all resistance of the coil circuit.

In order to have accurate current readings, the addition of an ammeter to a circuit should have no effect on the current flowing in the circuit. This means that the instrument movement designed for use in an ammeter must have a coil wound for low resistance so that the voltage drop across the instrument is low. Then the ammeter will not add any great resistance when connected in the circuit. The cross section of the wire in the coil is large and the number of turns small. On the other hand, the moving coil of an instrument movement to be used in a voltmeter is usually wound of finer wire and consists of many turns so that it has a higher resistance.

The principal advantages of permanent-magnet moving-coil instruments are: (1) lower power consumption, (2) high torque-to-weight ratio, (3) uniformity of the scale and the possibility of a very long (about 300°) scale, (4) a single instrument, with shunts and series resistances, covers a large range of both current and voltage, (5) no hysteresis errors and very little error due to stray magnetic fields, (6) perfect damping by the eddy currents induced in the metal bobbin on which the moving coil is wound.

Errors due to friction and heating, the same as in other instruments, will be found. Then, too, the weakening of the permanent magnet in time will cause errors.

Ammeters

The sensitivity of a meter movement is inversely proportional to the amount of current that causes the indicator to deflect full scale. The smaller the current required for full-scale deflection, the more sensitive the meter movement. For measuring current in electronic equipment, ammeters with a sensitivity of 0.1 ampere or even 1 milliampere are used. Not too uncommon are meters with a sensitivity of 100 microamperes.

To measure larger amounts of current than the coil itself can safely carry, a resistance is connected in parallel with the coil. The current being measured divides between the coil and the resistor, a small portion flowing through the coil, the remainder through the parallel resistor, called the *meter shunt*. The shunt may be built into the meter or it may be mounted externally. Meters designed to measure several ranges of current employ a shunt for each range. The shunts are mounted on a common terminal board and are connected to a multipole switch. Setting the switch to the desired range connects the proper shunt resistor into the meter circuit. Shunt resistors usually contain only a fraction of an ohm of resistance and consist of a few inches of a metal alloy having a low temperature coefficient. The alloy is drawn into a wire and is wound around a piece of mica or fiber and mounted on a terminal board.

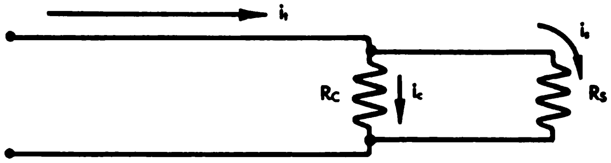


Figure 3–4. Shunt Resistor(R_S) Increases Current Range

Since current through the two parallel branches divides in a ratio inversely proportional to the resistance of each branch, it is possible to calculate the current flowing through the coil as well as the total current flow in a circuit in which current is being measured. In the circuit of Figure 3–4, for example, if $R_s = 5R_c$ and $I_c = 0.5$ ma, find the current flowing in the shunt (I_s) and the total current flowing in the circuit. Since the resistance of the coil is five times that of the shunt, and since the current divides in inverse proportions to the resistance, five times as much current will flow through the shunt as through the coil. Therefore, the current through the shunt, $I_s = 5(0.5 \text{ ma})$ or 2.5 ma. The total current in the circuit is equal to $I_c + I_s$. Thus, the total current, $I_t = 0.5 + 2.5$ or 3 ma.

Calculations such as those involved in this example will enable you to determine the size of the shunt required to extend the range of an ammeter. Although in actual practice it is seldom necessary to perform these calculations, it is well to know how meters having the same movement are constructed with different ranges, and how a range switch on the same meter changes the current range of the meter. The following example illustrates the method for computing values of shunts required in a multirange meter having a single meter movement.

Example

Problem

Assume that you desire to construct the ammeter shown in Figure 3–5 with the ranges 0-10 ma, 0-0.1 amp, and 0-1.0 amp. If the meter sensitivity is 1 ma and the coil resistance is 75 ohms, find R_{s1}, R_{s2}, and R_{s3}.

Solution

1. Range 0-10ma.

Since a full scale reading on the meter will indicate 10 ma, then 9 ma must flow through the shunt and 1 ma through the coil.

Since

$$\frac{R_{s1}}{R_c} = \frac{I_c}{I_s}$$

Then

$$\frac{R_{s1}}{75} = \frac{1}{9}$$

$$R_{s1} = 8.33 \text{ ohms}$$

2. Range 0-0.1 amp

R_{s2} must carry 99 ma (0.1 amp − 1 ma)

Since

$$\frac{R_{s2}}{R_c} = \frac{I_c}{I_s}$$

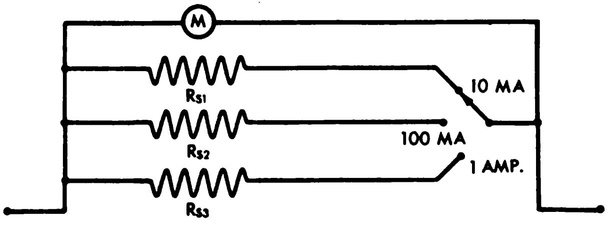


Figure 3–5. Computing Values of Shunts

Then

$$\frac{R_{s2}}{75} = \frac{1}{99}$$

$$R_{s2} = \frac{75}{99} = 0.758 \text{ ohm}$$

3. Range 0-1.0 amp

R_{s3} must carry 999 ma (1.0 amp — 1 ma)

Since

$$\frac{R_{s3}}{R_c} = \frac{I_c}{I_s}$$

Then

$$\frac{R_{s3}}{75} = \frac{1}{999}$$

$$R_{s3} = \frac{75}{999} = 0.751 \text{ ohm}$$

When using an ammeter, observe the following precautions:

1. Always connect an ammeter in series with the element through which the current flow is to be measured.

2. Never connect an ammeter across a source of voltage, such as a battery or generator. Remember that the resistance of an ammeter, particularly on the higher ranges, is extremely low and that any voltage, even a volt or so, may cause very high current to flow through the meter causing damage to it.

3. Use a range large enough to keep the deflection less than full scale. Before measuring a current, form some idea of its magnitude. Then switch to a large enough range or start with the highest range and work down until you reach the appropriate one. Most accurate readings are obtained at approximately half-scale deflection. Many a milliammeter has been ruined by attempting to measure amperes. Therefore, be sure to read the lettering either on the dial or on the switch positions, and choose the proper range before connecting the instrument in the circuit.

4. Observe proper polarity in connecting the meter in the circuit. Current must flow through the coil in a definite direction in order to move the indicator needle up-scale. Current reversal because of incorrect connection in the circuit results in a reversed meter deflection and frequently causes bending of the meter needle. You can avoid improper meter connections by observing the polarity markings on the meter and by remembering that the *black* meter leads are the *negative* leads and the *red* the *positive* leads.

The Voltmeter

Previously it was stated that the Weston-type, or D'Arsonval, movement can be used either as an ammeter or as a voltmeter. Thus, you can measure voltage with the ammeter just described by placing a resistance in series with the coil and measuring current flowing through it (Figure 3–6). In other words, a voltmeter is a current-measuring instrument designed to indicate voltage by measurement of the current flow through a resistance of known value. Various voltage ranges may be obtained by adding various resistors called *multipliers* in series with the coil.

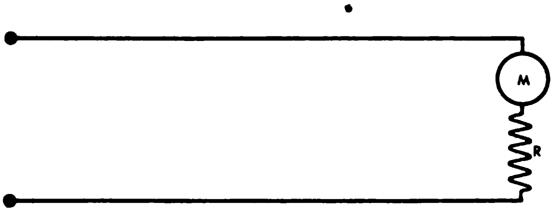


Figure 3–6. Voltmeter Circuit

The following example illustrates how you can calculate the values of multipliers needed to extend the range of a voltmeter.

Example

Problem

What size multiplier does a voltmeter having a 0-1 ma meter movement with 75 ohms resistance require for ranges of 0-10v, 0-100v, and 0-500v?

Solution

1. Multiplier R_1

Since 1 ma causes a full-scale deflection, the resistance $R_1 + 75$ must be such that the voltage drop across it is 10v when 1 ma flows.

Thus:

$$R_1 + 75 = \frac{10}{.001} = 10{,}000 \text{ ohms}$$

$$R_1 = 9925 \text{ ohms}$$

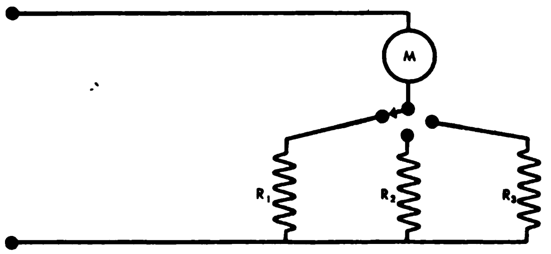


Figure 3–7. Computing Values of Multipliers

2. Multiplier R_2
Similarly,

$$R_2 + 75 = \frac{100}{.001} = 100{,}000 \text{ ohms}$$

$$R_2 = 99{,}925 \text{ ohms}$$

3. Multiplier R_3
Similarly,

$$R_3 + 75 = \frac{500}{.001} = 500{,}000 \text{ ohms}$$

$$R_3 = 499{,}925 \text{ ohms.}$$

The sensitivity of a voltmeter is measured in *ohms-per-volt* and is determined by dividing the *resistance* of the meter and the multiplier by the full-scale reading in *volts.* It is just another way of stating what current will cause full-scale deflection. A voltmeter should have very high resistance so that it will draw very little current and affect the circuit as little as possible during voltage measurements. Sensitivity, therefore, is an indication of the measuring quality of a voltmeter. Generally a meter with a 1,000 ohms-per-volt sensitivity is satisfactory for most uses. For radar or radio circuits with high resistances, however, a meter with a sensitivity of 20,000 ohms-per-volt or higher is required for accuracy.

When using a voltmeter, observe the following precautions:

1. Always connect a voltmeter in parallel across the portion of the circuit in which voltage is being measured, as shown in Figure 3–8.

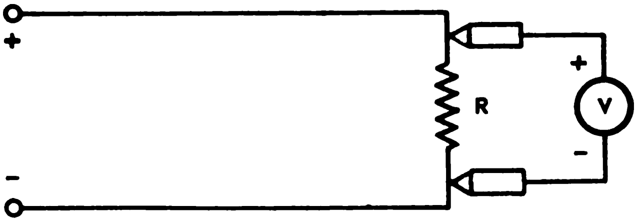


Figure 3–8. Voltmeter in Parallel

2. Use a range large enough to insure less than full-scale deflection.
3. Observe the proper polarity in connecting the voltmeter across the circuit.

The accuracy of a reading indicated by a voltmeter depends upon the sensitivity of the meter. As previously mentioned, when a voltmeter having a low resistance — that is, one with low sensitivity — is connected into a high resistance circuit, the voltage indicated by the meter is somewhat in error depending on the meter sensitivity. This error in reading is due to the *loading* effect which the meter has on the circuit in which voltage is being measured.

To understand just how a meter affects, or loads, a circuit, refer to Figure 3–9A. According to the values indicated, the total current flow in the circuit is equal to $\frac{150}{100{,}000 + 50{,}000}$, or 0.001 ampere. The applied voltage divides across the two resistors — 0.001 × 100,000, or 100 volts, across the 100,000-ohm resistor, and 0.001 × 50,000, or 50 volts, across the 50,000-ohm resistor.

When you connect a voltmeter across the 100,000-ohm resistor, you naturally expect the meter to read 100 volts. However, depending upon the sensitivity of the meter, it may give readings other than the 100 volts you expect. For example, suppose you are using a voltmeter

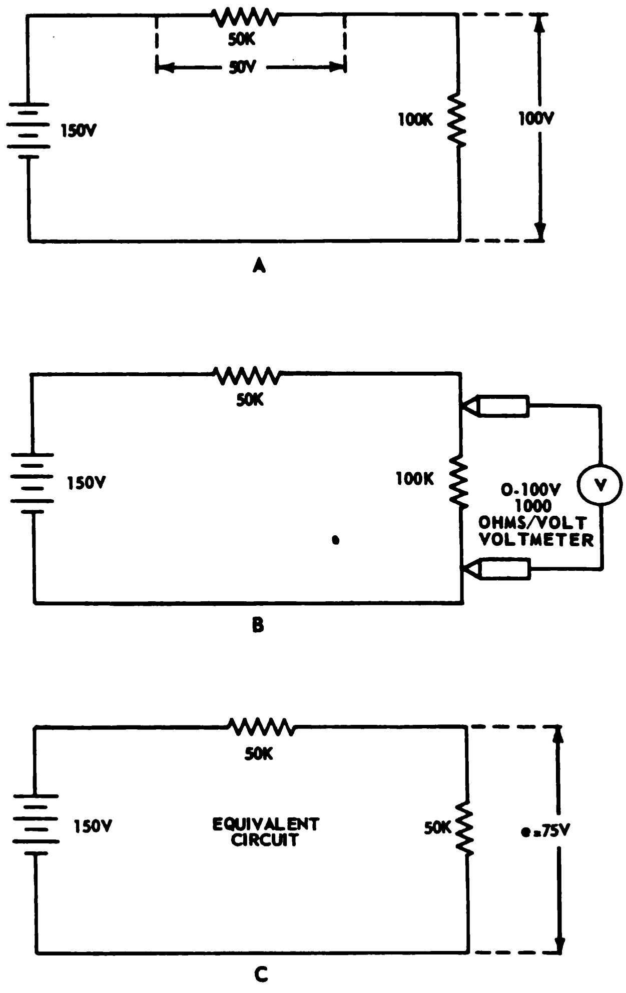


Figure 3–9. Loading Circuit with Voltmeter

with a sensitivity rating of 1,000 ohms-per-volt on the 0–100-volt range. The resistance of the meter on this range equals 1,000 ohms × 100, or 100,000 ohms. (To find the resistance of a voltmeter, multiply the sensitivity rating by the voltage.)

When you connect this voltmeter across the 100,000-ohm resistor in the circuit, you put the meter in parallel with this resistor, and the circuit effectively becomes as shown in the equivalent circuit (Figure 3–9C). As you can see, the meter is in parallel with the 100,000-ohm resistor, and since the meter resistance is equal to the resistance of the circuit resistor, the effective resistance is 50,000 ohms. In other words, the 100,000-ohm circuit resistor is effectively replaced in the circuit by an equivalent 50,000-ohm resistor.

The total circuit resistance is no longer 150,000 ohms, but 100,000 ohms, and the total current flow is no longer 0.001 ampere but $\frac{150}{100{,}000}$ or 0.0015 ampere. The voltage across the meter and the circuit resistor now equals the product of the total current and the 50,000 ohms (which represents the equivalent resistance of the meter and the 50,000-ohm resistor). Therefore, the meter reads only .0015 × 50,000, or 75 volts. Thus, the loading effect of the meter on the circuit has reduced the voltage across the resistor from 100 volts to 75 volts.

Obviously, this error in the reading is due to the loading effect the meter has on the circuit. When the meter is removed, the voltage across the 100,000-ohm resistor again becomes 100 volts.

You can avoid loading a circuit by using a voltmeter in which the resistance is large compared to the circuit element across which voltage is being measured. If a voltmeter with such high sensitivity is not available, you can improve accuracy by using a higher range on the voltmeter you have. Thus, for example, if you had used the 0–500-volt range on the meter, the voltage across the resistor would have read approximately 91 volts.

On the 0–500-volt range, the resistance of the meter equals 1,000 × 500 or 500,000 ohms. When the meter is put across the circuit, the parallel resistance of the circuit resistor and the meter equals $\frac{100{,}000 \times 500{,}000}{100{,}000 + 500{,}000}$, or about 83,000 ohms. The total resistance in the circuit is then 83,000 plus 50,000, or 133,000 ohms. The total current flow is $\frac{150}{133{,}000}$ or approximately .0011 ampere. Thus, the voltage across the meter and the circuit resistor is equal to .0011 × 83,000 or approximately 91 volts, which is much closer to accuracy than the reading obtained with the 0–100-volt range.

The loading effect of a voltmeter explains why the same meter gives different readings on different ranges when it is placed across the same circuit element. You saw this proved in the example where the 0–500-volt range gave a much more accurate reading than the 0–100-volt range on the same meter.

Although the highest range of a voltmeter gives readings more nearly equal to the actual voltage, it is not always desirable to use it. You will find it much more difficult to read higher-range scales accurately than the lower-range scales where the numbers are less crowded.

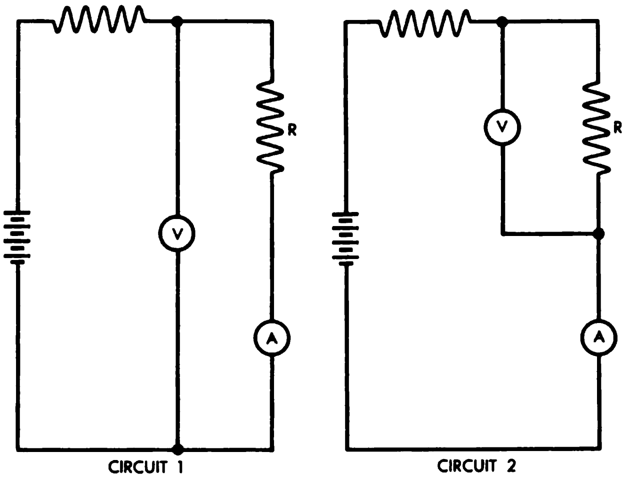


Figure 3–10. Measuring Current and Voltage Simultaneously

If you wish to use a voltmeter and an ammeter in the circuit at the same time, there are two possible ways to connect them, but each produces an error. In the first case, shown in Figure 3–10, the voltmeter reads not only the voltage drop across the resistor R but also that across the ammeter. In the second case the ammeter reads not only the current flowing through the resistor R, but that through the voltmeter as well. Both methods further emphasize the fact that the resistance of an ammeter must be very low to keep the voltage drop across it small, and that the resistance of a voltmeter must be great to keep the current flowing through it small.

The correct connection of the circuit is the one leading to the least error. It depends on the relative values of resistance. If the resistance of the circuit is small, approaching the resistance of the ammeter, use the second circuit as it leads to the least error. If the resistance of the circuit is large compared to the voltmeter resistance, use the first circuit. For intermediate values of circuit resistance, either circuit connection is satisfactory.

The Ohmmeter

You can determine the resistance of a circuit element by first measuring the current flowing through it with an ammeter, then the voltage drop across it with a voltmeter, and finally applying Ohm's law. It is much more practical, however, to use an ohmmeter with which you can read resistance directly from a scale.

Ohmmeters possess a number of features not found in ammeters and voltmeters. An ohmmeter must supply its own power. Usually this is supplied by a battery of known voltage. In an ohmmeter there are two resistors, one fixed and one variable. The fixed resistor, marked R in Figure 3–11, is of such a value that when the points A and B are shorted, the meter will read full scale. The variable resistor is connected in parallel with the meter and is called the *zero adjustment*. It serves to compensate for changes in the battery voltage. Usually it is adjusted by a control knob on the meter panel.

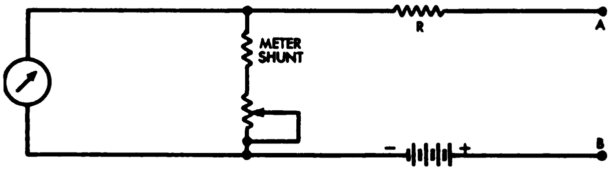


Figure 3–11. Ohmmeter Circuit

If zero resistance is connected between points A and B in the ohmmeter, the meter pointer will deflect full scale. If a resistance equal to R is connected between these points, the deflection will be half scale, and if a resistance equal to 2R is connected, the deflection will be one-third scale. The difference in deflection indicates that the upper end of the scale reads low resistance.

The scales on ohmmeters (Figure 3–12) are not linear but are similar to a scale of reciprocals. At the lower end (high resistance end), the calibrations are crowded, making accurate readings difficult. For this reason, it is best to select a range in which the indicator will fall in the upper portion of the scale.

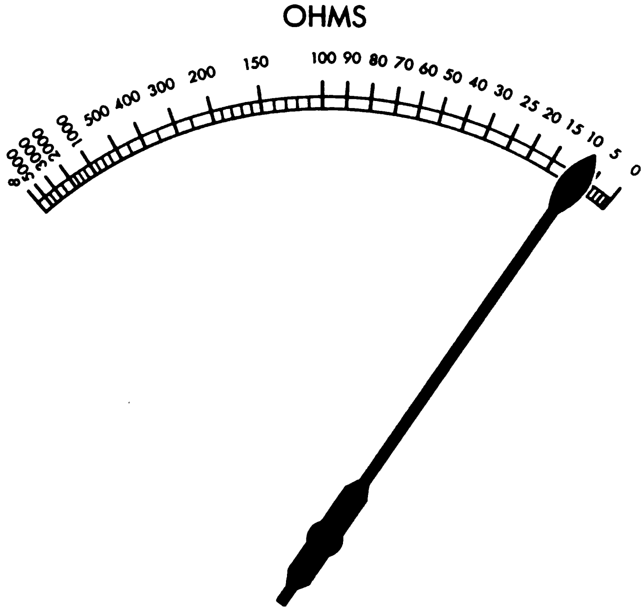


Figure 3–12. Typical Ohmmeter Scale

A typical ohmmeter has more than one range. Additional ranges are made possible by the use of various values of R and battery voltages. On some meters there is a special range for reading very low resistances. This is made possible by connecting the unknown resistance *in parallel with* a known resistance. Readings of resistance in this case appear on a special scale called *low ohms*. On this scale, zero is at the left.

The ohmmeter is not as accurate a measuring device as the ammeter or the voltmeter because of the circuit associated with it. Therefore, do not expect to read resistance with greater than five to ten percent accuracy. While there are instruments which read the resistance of an element with very great accuracy, they usually involve some sort of bridge network and are much more involved in use. For the work you will do, the ohmmeter is accurate enough.

In addition to measuring resistance, the ohmmeter is a very useful instrument for checking continuity in a circuit. Often when troubleshooting electronic circuits or wiring a circuit, you cannot readily make visual inspections of all parts of the current path. Therefore, it is not always apparent whether a circuit is complete, or whether current may be flowing in the wrong part of the circuit because of contact with adjacent circuits. The best method of checking a circuit under these conditions is to send a current through it.

If the conductor makes a complete circuit, current will flow through the circuit. The ohmmeter is the ideal instrument for checking circuits in this manner. It provides the power to cause the current to flow and the meter to indicate the relative amount that is flowing. To make the check, first study the circuit diagram, and then check the corresponding parts of the circuit itself with the ohmmeter. It will indicate perfect conduction, partial conduction (resistance), or no conduction at all.

When using an ohmmeter, proceed as follows:

1. Choose a range which you think will contain the resistance of the element that you are measuring. In general, use a range in which the reading will fall in the upper half of the scale (near full-scale deflection). Reading the scale is easier there and gives greater accuracy.
2. Short the leads together and zero the meter, using the zero adjustment. If you change ranges at any time, remember to readjust to zero ohms.
3. Never attempt to measure resistance in a circuit while it is connected to a source of voltage.
4. Connect the unknown resistance between the test leads and read its resistance from the scale. When applicable, disconnect at least one end of the element being measured to avoid reading the resistance of parallel paths.

AC MEASURING INSTRUMENTS

The Weston-type, or D'Arsonval movement, instrument is not suitable for measuring alternating current, since the torque acting on the coil would be alternating. This would cause the moving element to oscillate about the zero position. If the current through the coil is of a very low frequency, the pointer can be seen to move, but if the current alternates rapidly, the moving element cannot follow the alternations and there is no deflection to be observed. If, however, a Weston-type instrument is modified to include a device (called a rectifier) for changing AC into a unidirectional current, it can be used to measure AC.

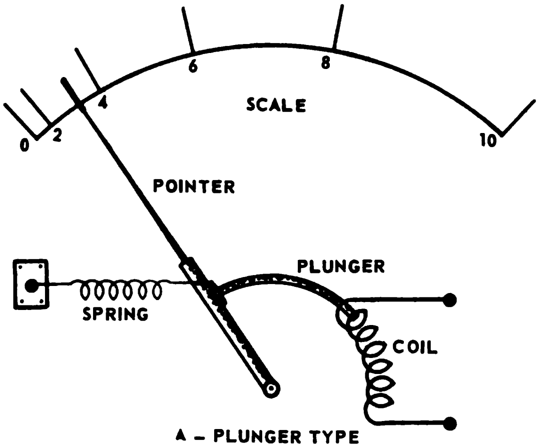
There are several types of instruments in common use which can be used to measure AC as well as DC. Among these are (1) moving iron, (2) electrodynamometer, (3) hot wire, (4) permanent-magnet moving coil in conjunction with rectifier or thermocouple.

Moving Iron

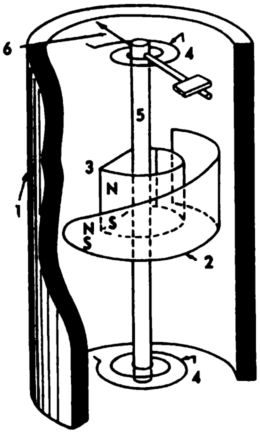
There are two general types of moving iron instruments, the attraction and the repulsion. Figure 3–13A shows the plunger-type moving iron instrument. A small rod of soft iron is mounted on a shaft which is free to pivot. Current flowing through the coil creates a magnetic field which attracts the iron and moves the pointer across the scale. Whether the current is direct or alternating, the magnetic field will attract the iron rod. This type of instrument is practically obsolete.

Figure 3–13B shows an iron-vane instrument of the repulsion type. The shapes of the irons vary in different makes of instruments. In this make, a small strip of soft iron bent into a cylinder (3) is mounted on a spindle (5) which is free to turn. A similar piece (2), tongue or wedge-shaped, with a larger radius is fixed inside a cylindrical coil (1) wound with many turns of wire. The current being measured flows through this coil. This current magnetizes both strips of iron. The upper edges of the two strips will always have the same polarity and the lower edges of the two strips will always have polarity opposite to the upper edges, no matter which direction current is flowing. As shown in Figure 3–13B, the upper edges are north poles and the lower edges south poles.

Since like poles repel, there will always be a force of repulsion between the upper and the lower edges. Because of the shape of the fixed strip, the repulsion causes the movable vane to rotate about its axis (5). The spiral springs (4) resist the rotation and cause it to be proportional to the torque, which is dependent on the square of the current flowing in the coil. The pointer (6) connected to the movable vane moves across a scale indicating the amount of current flow in the coil.



A – PLUNGER TYPE



B – IRON VANE TYPE

Figure 3–13. Moving Iron Instrument

Since the movement of the needle is proportional to the *square* of the current flow, the distances between the scale markings are not uniform. A *linear* scale of a meter is shown in Figure 3–14A. The distances between 1 and 2, between 2 and 3, between 3 and 4, etc., are all equal. If the current is doubled, the pointer will move twice the distance; if the current is tripled, the pointer will move three times the distance, and so on. In an instrument where the turning of the movable part is proportional to the square of the current, when the current is doubled, the pointer moves *four* times as far; when the current is tripled, the pointer moves nine times as far. Figure 3–14B shows a square law scale. Notice how difficult it is to read at the left end of the scale.

Errors in moving-iron instruments are caused by hysteresis and stray magnetic fields. In addition, AC voltmeters have errors due to the change in impedance of the coil with changes in frequency. However, the iron-vane ammeter is practically the only kind used for commercial instruments.

Hot-Wire Instruments

In hot-wire instruments, the heating effect of an electric current is used. When a metal is heated, it expands; when it is cooled, it contracts. If the resistance and the coefficient of expansion of the wire are constant, the heating and the expansion of the wire are both directly proportional to the square of the current. With a large enough expansion, a pointer can be deflected across a scale to indicate the amount of current.

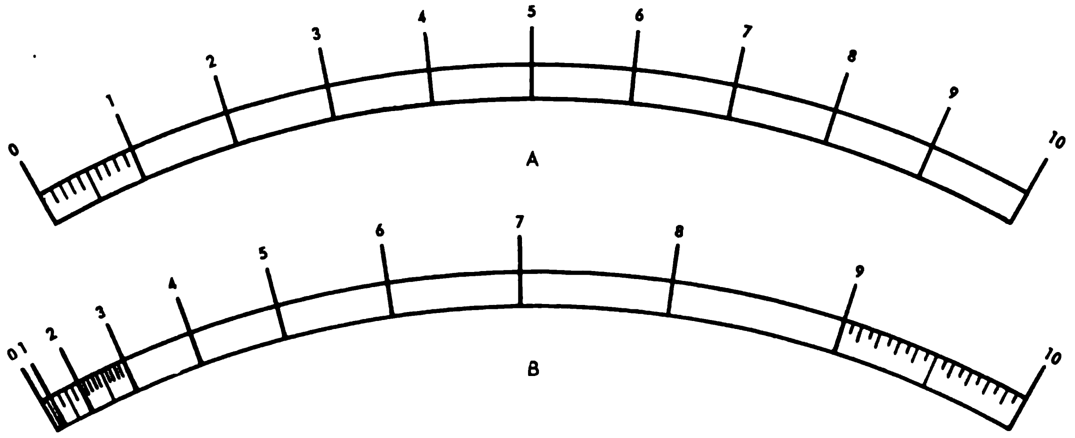


Figure 3–14. Linear and Square Law Scale

Figure 3–15 shows the principle of operation of this type instrument. When the meter is properly connected in the circuit, current flows through wire AB. A phosphor-bronze wire is fastened at one end to the center of wire AB, passes around a pulley supporting the pointer, and is connected at the other end to a tension spring.

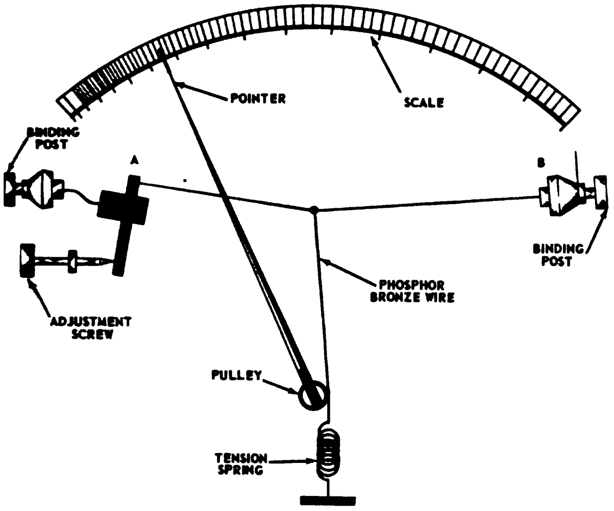


Figure 3–15. Hot-Wire Meter Movement

When wire AB becomes heated by the current flowing through it, the wire expands, increasing its length. As the wire sags, the spring pulls the phosphor-bronze wire through the pulley, which turns and moves the pointer across the scale. The movement of the pointer is proportional to the lengthening of the wire, which is proportional to the square of the current flowing through wire AB. The wire must be made as thin as possible so that it will reach a steady temperature quickly when current flows through it, yet it must have enough strength to withstand the stress placed on it by the tension of the spring.

Hot-wire instruments can be used to measure both DC and AC. Since the deflection of the pointer depends upon the heating effect of the alternating current, hot-wire instruments measure the rms value of the current, regardless of the waveform or frequency. This is the main advantage of the instrument. The calibration is the same for AC measurements as for DC measurements. In addition, since magnetic fields are not used to operate the instrument, stray magnetic fields will not affect the operation.

The disadvantages of the hot-wire instrument are many. (1) It takes time for the wire to heat. (2) While the base of the instrument is usually made of materials with the same coefficient of expansion as the wire so that changes in temperature outside the instrument will not affect the readings, the wire changes temperature quickly while the temperature of the base takes time to change. This difference in temperature of the working wire and the base causes shifting of the zero point and requires frequent zero adjustment. (3) The instrument cannot stand overloads. Since the hot wire is so fine, it fuses quickly if the current exceeds the working value by an appreciable amount. Fuses will not give adequate protection as the wire may melt before the fuse. (4) The instrument is fragile. (5) It has high power consumption. (6) Since the deflection of the pointer is proportional to the square of the current, the scale is difficult to read at the left-hand end. (See Figure 3–14B.)

Dynamometer Instruments

The dynamometer instrument is an electromagnet, moving-coil instrument. The permanent magnet of the Weston-type instrument is replaced by either one or two fixed coils which carry the current to be measured (or a current proportional to the voltage to be measured). Usually air-core coils are used to avoid introducing hysteresis and eddy current errors when the instrument is used to measure AC. Figure 3–16A shows the principle of the electrodynamometer. The fixed coil is split into two electrically continuous parts. The moving coil is mounted between the two fixed coils. It is mounted on a vertical spindle with a hardened steel pivot at each end. These turn in jeweled bearings. There are two spiral springs similar to those used with the Weston-type instrument. They oppose the turning of the coil and conduct the current to the moving coil. As the springs can conduct only a very small current, the movable coil is wound with fine wire.

Assume that at one instant the direction of the field of the fixed coils in Figure 3–16A is from the bottom to the top. At the same time, the current in the moving coil produces a field whose direction is indicated by ϕ_m. The reaction of the two fields tends to turn the moving coil in a clockwise direction so that the fields can align themselves and the number of flux linkages can become a maximum. The torque developed is proportional to the two fields and the sine of the angle between the moving and fixed coils. Since the flux in each case is proportional to the current through the coils, the torque is proportional to the product of the two currents, or I^2.

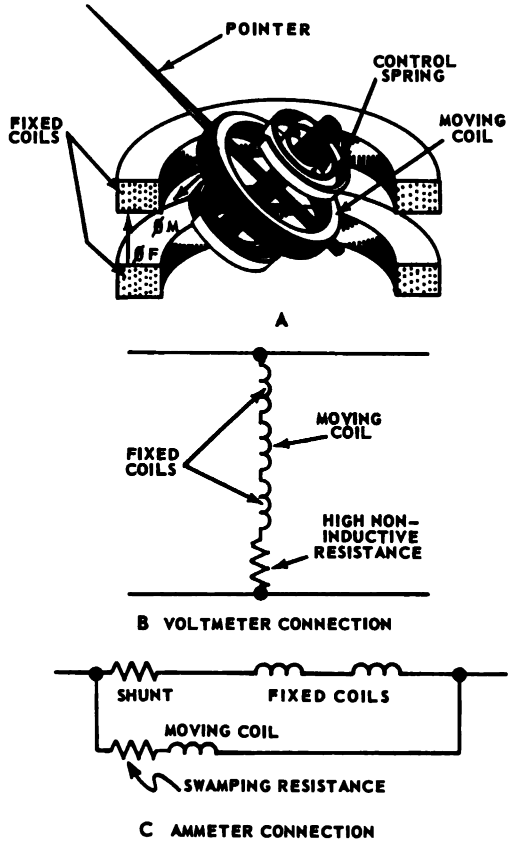


Figure 3–16. Electrodynamometer

Figure 3–16B illustrates the connections for a dynamometer voltmeter. The resistance limits the current through the fixed coils, which are wound with fine wire and connected in series with the moving coil. The current in the dynamometer is proportional to the voltage being measured. Therefore, the deflection of the needle is proportional to the square of the voltage. The scale that must be used is similar to Figure 3–14B, a square-law scale.

The dynamometer type of voltmeter requires about five times as much current as the permanent-magnet moving-coil DC voltmeter of the same rating. It uses, therefore, an appreciable amount of power. Unless the instrument is shielded, stray magnetic fields, wires carrying current, inductive apparatus, and even iron alone, if brought too near, may cause large errors in the indications.

The current that can be measured without the use of a shunt is small because of the heavy moving coil that would be required if it were to have high current-carrying capacity. Designing a shunt is not simple, since the division of current between coil and shunt depends upon the impedances and they depend upon the frequency of the current. A shunt would be accurate at one frequency only, and then only if the waveform were a pure sine wave. Figure 3–16C shows the usual connection for an ammeter. With this connection, the time constants of the two parallel branches can more easily be made equal so that the division of current between them will be independent of frequency. Figure 3–17 illustrates another type of dynamometer ammeter.

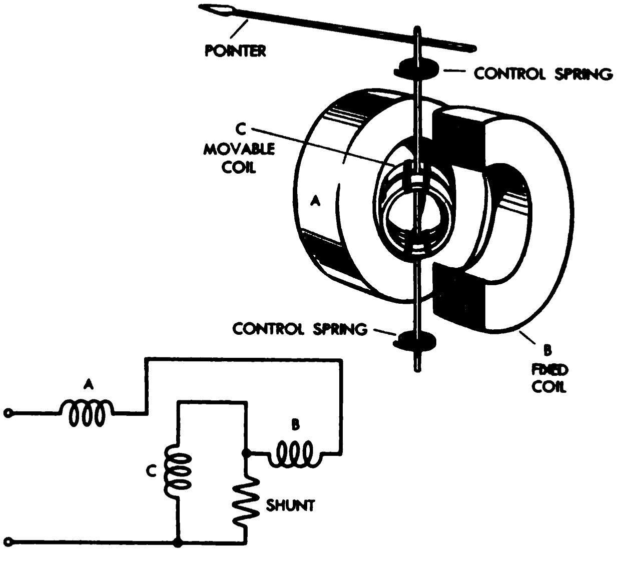


Figure 3–17. Dynamometer Type Ammeter

The dynamometer type instrument is not nearly as useful for measuring DC as the permanent-magnet type. It must have a heavier moving system so the torque-weight ratio is small. Friction tends to cause errors. Since the deflecting torque varies with the square of the current, a non-linear scale must be used. In addition, dynamometer instruments are more expensive than the permanent-magnet type. For these reasons dynamometer ammeters and voltmeters are rarely, if ever, used in DC circuits. The iron-vane ammeter is more commonly used in AC circuits than the shunted dynamometer ammeter.

Thermocouple Instruments

The thermocouple, in itself, is not a meter movement; however, it is commonly used with DC meter movements to measure alternating currents or voltages. The thermocouple is a very simple source of electricity. When the junction between two unlike metals is heated, an emf is generated between the unheated ends. While all metals and alloys will produce an emf in this way, certain combinations, such as antimony and bismuth, copper and constantan, iron and copper-nickel alloy, produce a greater voltage for the same amount of temperature difference.

Figure 3–18A illustrates a typical thermocouple ammeter. The heating element (ab), through which the current to be measured flows, is a wire, or strip of resistance alloy, having practically zero temperature coefficient of resistance (resistance does not change with temperature change) connected between two large copper blocks. The hot junction (c) of a thermocouple is hard-soldered or welded to the center of the heater. The cold ends of the thermocouple (d and e) are soldered to the center points of two copper compensating strips that are connected to the leads to the meter movement. These strips are in thermal contact with the copper terminals of the heater, but are electrically insulated by thin mica strips. Since external temperature changes affect the cold ends of the thermocouple to a different degree from the hot junction, the large copper blocks and the copper compensating strips are used to prevent errors.

A vacuum thermocouple has a very small thermal junction in contact with a small heater (through which the measured current flows) all sealed within a small evacuated bulb. This reduces heat dissipation and prevents errors resulting from external temperature changes or air currents.

The scale is calibrated in terms of the current in the heater circuit. Since the heater temperature increases with the square of the current (I^2R), the usual permanent-magnet moving-coil type of movement would require a square-law scale. This would be undesirable in many cases, so a specially designed movement is used with the thermocouple. Figure 3–18B shows how the air gap is increased so that the flux density through which the coil turns decreases as the coil moves toward the high side of the scale. This decreases the sensitivity as the pointer moves up the scale and a nearly linear scale is obtained.

Because this instrument depends upon the heating effect of the current, it can be used to measure very high frequency currents and currents that are not sinusoidal. There are practically no capacitive or inductive effects in the heater circuit, so this thermocouple instrument works well in radio-frequency measurements.

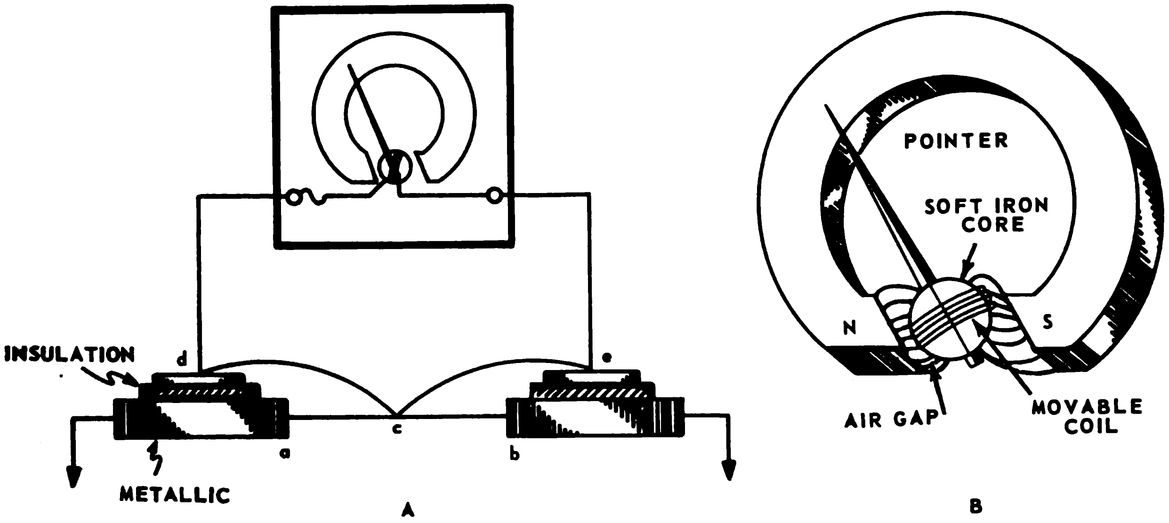


Figure 3–18. Thermocouple Instrument

The meter can be calibrated with direct current, because the indications depend on the heating effect of the current.

Meter movements used with thermocouples must be very sensitive, because the output voltage of the thermocouple may be only 15 millivolts and the internal resistance 5 ohms. The heater circuit is delicate and its rating is limited to very small currents. Thermocouple instruments are very delicate and should be handled with care.

By using a small heater wire and a high resistance in series, this instrument can be used as a voltmeter.

Rectifier-Type Instrument

A DC meter movement of the Weston, or D'Arsonval, type can be used for measuring alternating currents and voltages by rectifying the current before its flows through the moving coil of the meter movement. The rectifier is a circuit element designed to pass current in only one direction. Usually the rectifier is of the copper-oxide type, although some instruments use selenium rectifiers. (Solid state rectifiers are explained in Chapter 4.)

Figure 3–19 shows a bridge connection, which gives full-wave rectification. The solid arrows show the direction of current flow during the positive half-cycle of the input; the dashed arrows show the direction of current flow during the negative half-cycle of the input. Notice that the current flows through the meter movement in the same direction on each half cycle.

The deflection of the pointer is proportional to the *average* value of the rectified wave. The ratio of the rms value to the average value is 1.11 when the current is a sine wave. Assuming that the current is sinusoidal, then the scale of the instrument must be marked in values of 1.11 times the current actually measured to give rms values. If the instrument is used to measure nonsinusoidal waveforms, errors will result. Waveshapes having the same average value may have entirely different rms values. However, when measured with the rectifier-type instrument, they will all indicate the same rms value, 1.11 times the average value.

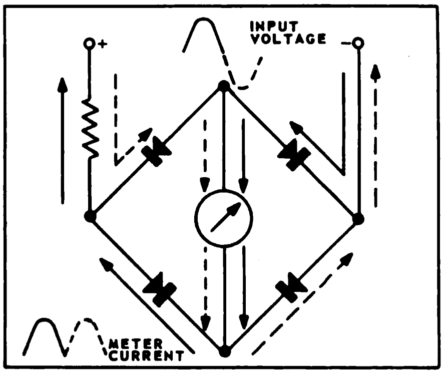


Figure 3–19. Bridge Rectifier Meter Circuit

The resistance of a copper-oxide rectifier in the forward direction of current flow varies with the amount of current over a wide range. This distorts the waveform of the alternating current flowing through the rectifier, effectively "peaking" it. Thus, the actual rms value of the current will be greater than that indicated by the rectifier instrument; that is, the instrument reads low. To take care of this, a swamping resistor is placed in series with the variable resistance of the rectifier itself so that the change in total resistance is small. As a result, the reading of the instrument will be nearly correct, provided the waveform is sinusoidal.

The resistance of the rectifier changes with age and use. The instrument is also affected by temperatures above 50°C. For these reasons the accuracy is usually given as ±5% of full scale. In addition, these instruments are subject to frequency errors that increase as the frequency increases. Nevertheless, these instruments are widely used in the measurement of small alternating currents and voltages in communication circuits, especially in the radio-frequency range, because of their sensitivity and uniform scale.

Observe the following precautions in the care and use of the rectifier meter:

1. Do not subject the rectifier to high temperatures or chemical vapors. Certain chemicals, such as sulphuric-acid vapors, condense on the rectifier, forming a path for current flow in both directions.
2. Keep in mind that the accuracy is usually not better than 5% of the full-scale reading..
3. Never remove the load from a bridge rectifier without first removing the applied voltage. Removing the load subjects the rectifier to the full applied voltage, and since rectifiers are not constructed to withstand more than a few volts, arcing will occur and puncture the oxide.

INSTRUMENT TYPE	ACCURACY CAPABILITY OF INSTRUMENT	EXPECTED ACCURACY RANGES OF AMMETER	EXPECTED ACCURACY RANGES OF VOLTMETER	ADVANTAGES
Permanent-magnet moving-coil (DC only) (Combined ammeter and voltmeter test set)	From 0.2% to 1%	Up to 500 amps.	Up to 750 volts	Most accurate for DC measurements
Moving iron	1% for DC and AC up to 60 cycles per second	0.5 to 10 amps.	Above 75 volts	Cheapest
Electrodynamic ammeters and voltmeters	0.3% to 0.5% for DC and AC up to 60 cycles per second	0.5 to 10 amps.	Above 75 volts	Measures either DC or AC of low frequency
	1%	Above 10 amps.	Above 300 volts (single range) or 600 volts (multirange)	
Hot-wire, (DC and AC up to 100 cps, shunted type; AC up to 1 mc, unshunted)	3.5%	Maximum current less than 3 amps.	DC and AC up to 5,000 cps	Calibration same for DC and AC. Readings independent of frequency, waveform, or stray magnetic fields.
Thermocouple (self-contained)	3.5%, DC and AC up to 1 mc	5 ma to 1 amp.	Rarely used	Gives extremely accurate readings at higher frequencies
Rectifier voltmeters and milliammeters	3.5%, 25 to 10,000 cps (Indications affected by waveform)	1 ma to 50 ma for full-scale deflection	Not less than 10 volts for full-scale deflection	For precision measurements

MEASUREMENT OF INDUCTANCE AND CAPACITANCE

A circuit similar to the ohmmeter circuit previously described can be arranged for measuring reactances if you replace the battery by a known source of AC voltage, and the resistor by a reactive circuit element such as a capacitor or an inductor. Current flow through the known and unknown reactances will give an indication of the reactance in the circuit being measured. If the frequency of the AC voltage source is constant, such as 60 cps, the scale can be graduated to read capacitance directly. An inductance is measured by using the meter in the same way as for measuring capacitance, except that a conversion chart or graph is used in conjunction with the meter. Inductance measurements are quite inaccurate in the case of inductors with large resistances.

An instrument of this type does not possess a high degree of accuracy as a measuring device. However, it is quite useful, providing you exercise some common sense in interpreting results. For making precise measurements of capacitance or inductance, a somewhat more elaborate circuit is required. Most precision-reactance measuring instruments employ some type of bridge arrangement, but these instruments are primarily for laboratory use and are not very practical for use as field-test equipment.

In measuring capacitance or inductance, first short the leads together and *zero* the meter as you do in measuring resistance. Then connect the capacitance or inductance element being measured between the leads and read the reactance from the calibrated dial. Make no measurements of circuit elements while voltage is applied. To obtain correct indications, be sure to disconnect one terminal from the circuit to avoid reading the reactance of parallel paths.

THE VACUUM TUBE VOLTMETER

It is desirable that voltmeters for use in servicing many electronic circuits consume practically no power, for the loss of even a small amount of power seriously disturbs the circuit being tested. The vacuum tube voltmeter meets this requirement and has a widespread use in the servicing of electronic equipment.

Essentially, this instrument consists of one or more vacuum tubes, a DC milliammeter, a voltage supply, and various circuit components such as resistors, capacitors, and switches. The chief advantages of the vacuum tube voltmeter are that little or no power is taken from the circuits being tested, deflections for small input voltage variations are large, and a high degree of accuracy in measuring circuits with high impedance and low power is obtained.

Either DC or AC voltages can be measured with the vacuum tube voltmeter. It operates because a change in grid voltage causes a change in plate current. When the meter is used as a DC measuring instrument, the vacuum tube in the meter acts as a DC amplifier; when it is used as an AC measuring instrument, a rectifier is used to furnish a DC voltage. In either case, the current flowing in the plate circuit is proportional to the voltage applied to the grid of the tube, and is indicated on the milliammeter, which is calibrated to read voltage.

TEST UNIT I–176

An instrument in wide use for making various electrical measurements is the Test Unit I–176. This unit is a combination meter comprising a single meter movement and a number of associated circuits. Switches on the meter connect various resistors, shunts, and other circuit components in different arrangements. Depending

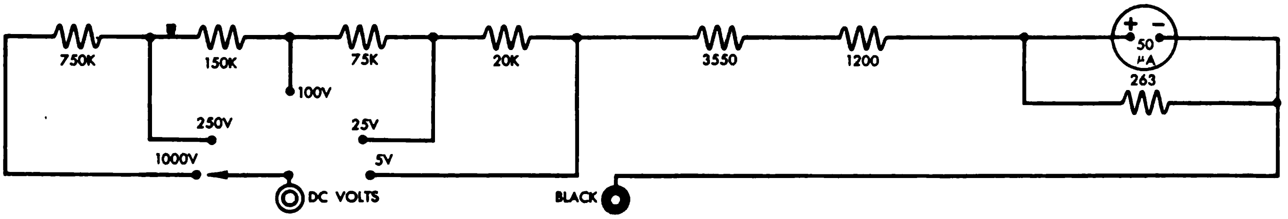


Figure 3–20. Simplified 1000 Ohms/Volt DC Voltmeter Circuit

upon the position of the switches, the I–176 can be used to measure AC voltage and current, DC voltage and current, and resistance. The following are types of measurements and ranges available in Test Unit I–176.

1. DC voltages: 0–5v, 0–25v, 0–100v, and 0–250v. These ranges are provided by the multipliers shown in the circuit of Figure 3–20.

The range selector switch connects the correct multiplier into the meter circuit. There is a selection between two sensitivity ratings — 1000 ohms-per-volt and 20,000 ohms-per-volt. The selection is made by the function switch. On the 20,000 ohms-per-volt position the 263-ohm meter shunt is out of the circuit.

An additional DC voltage range is available. This range is 0–5000v. It has a sensitivity rating of 20,000 ohms-per-volt and requires one special test lead which has a resistance of 50 megohms.

2. AC voltages: 0–5v, 0–25v, 0–100v, 0–250v, and 0–1000v. All these ranges have a sensitivity rating of 1000 ohms-per-volt. Except for the 1200-ohm resistor, this position of the switch connects the same multipliers as the DC voltage position (Figure 3–21A).

3. Direct current: 0–50 μa, 0–1 ma, 0–10 ma, 0–100 ma, 0–500 ma, 0–1 a, and 0–5 a. Various shunts are placed into the meter circuit by the function selector switch, the range selector switch, and by plugging the test leads into the separate test jacks.

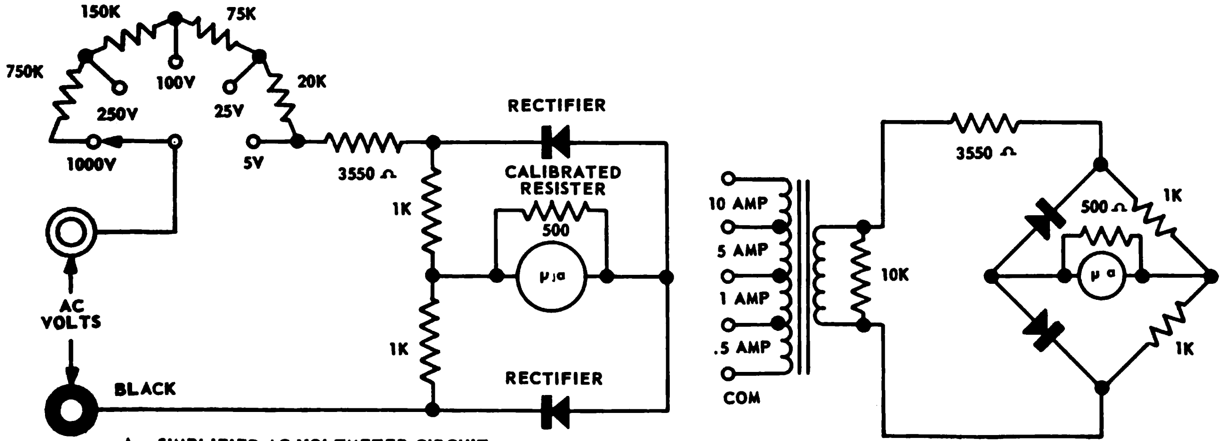
4. Alternating current: 0–500 ma, 0–1 a, 0–5 a, 0–10 a. Plugging the test leads into the jack corresponding to the desired range completes that range circuit. Each jack is connected to taps on the transformer primary. The secondary circuit of this transformer uses the same rectifier and meter used on the AC voltage position (Figure 3–21B).

5. Resistance: 0–1000 ohms, 0–100,000 ohms, and 0–10 megohms. The desired range is selected by the range selector switch and by plugging the test leads into the jack corresponding to that range. Power is supplied by either a 1.5v or a 22.5v battery, depending on the equipment.

THE OSCILLOSCOPE

The oscilloscope is an instrument consisting of a cathode-ray tube (CRT) and associated circuits (Figure 3–22) for use in viewing voltage waveshapes. The cathode-ray tube, which is discussed in detail later, consists of an electron gun for supplying a concentrated stream of electrons in the form of a beam, deflection plates (electrostatic CRT) for changing the direction of the electron beam, and a screen covered with a material that gives off light when struck by the stream of electrons from the electron gun.

The associated circuits in the oscilloscope include a time-base generator whose output is used to move the beam across the screen in a straight line and then quickly move it back to the starting point. The movement of the spot across the screen will appear as a solid line when its rate of moving exceeds the persistence of human vision or the persistence of the phosphorescent material forming the screen. This action is accomplished by applying a *sawtooth* voltage to the horizontal deflection plates. A sawtooth voltage is one that rises at a uniform rate to a certain



A SIMPLIFIED AC VOLTMETER CIRCUIT

B SIMPLIFIED AC AMMETER CIRCUIT

Figure 3–21. Simplified Meter Circuits

value and then quickly drops back to its starting value.

The sawtooth voltage wave is applied to the horizontal deflection plates, where it deflects the electron stream. Since negative voltages repel and positive voltages attract electrons, the gradual rise in voltage causes the left plate to become increasingly negative and the right plate increasingly positive, thereby causing the spot to move across the screen. The quick drop of the voltage back to its starting value returns the spot from right to left in a very short time, called th *flyback* time.

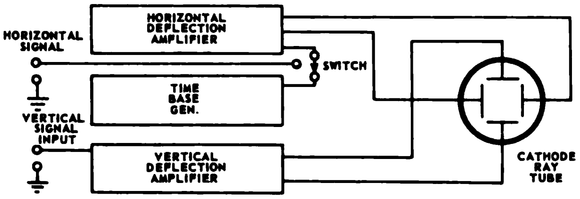


Figure 3–22. Block Diagram of Oscilloscope

The sawtooth voltage is produced by the tim base generator. This voltage or some other d sired external voltage for horizontal deflectio applied at the terminals marked *horizontal sign input*, is fed into the horizontal deflection ampl fier which increases the amplitude to that neede for a trace of the desired length. The voltage t be observed is applied to the *vertical signal* te minals and is amplified to the desired value b fore being applied to the vertical deflection plate With voltages thus applied simultaneously to t vertical and horizontal plates, the deflection the beam is the resultant of the two forces.

You can see how this works out by studyi Figure 3–23 which shows how a sine wave reproduced on the screen when a sawtooth vol age is applied to the horizontal plates and a si wave voltage to the vertical plates. The resulta is a single cycle of sine wave on the screen if t

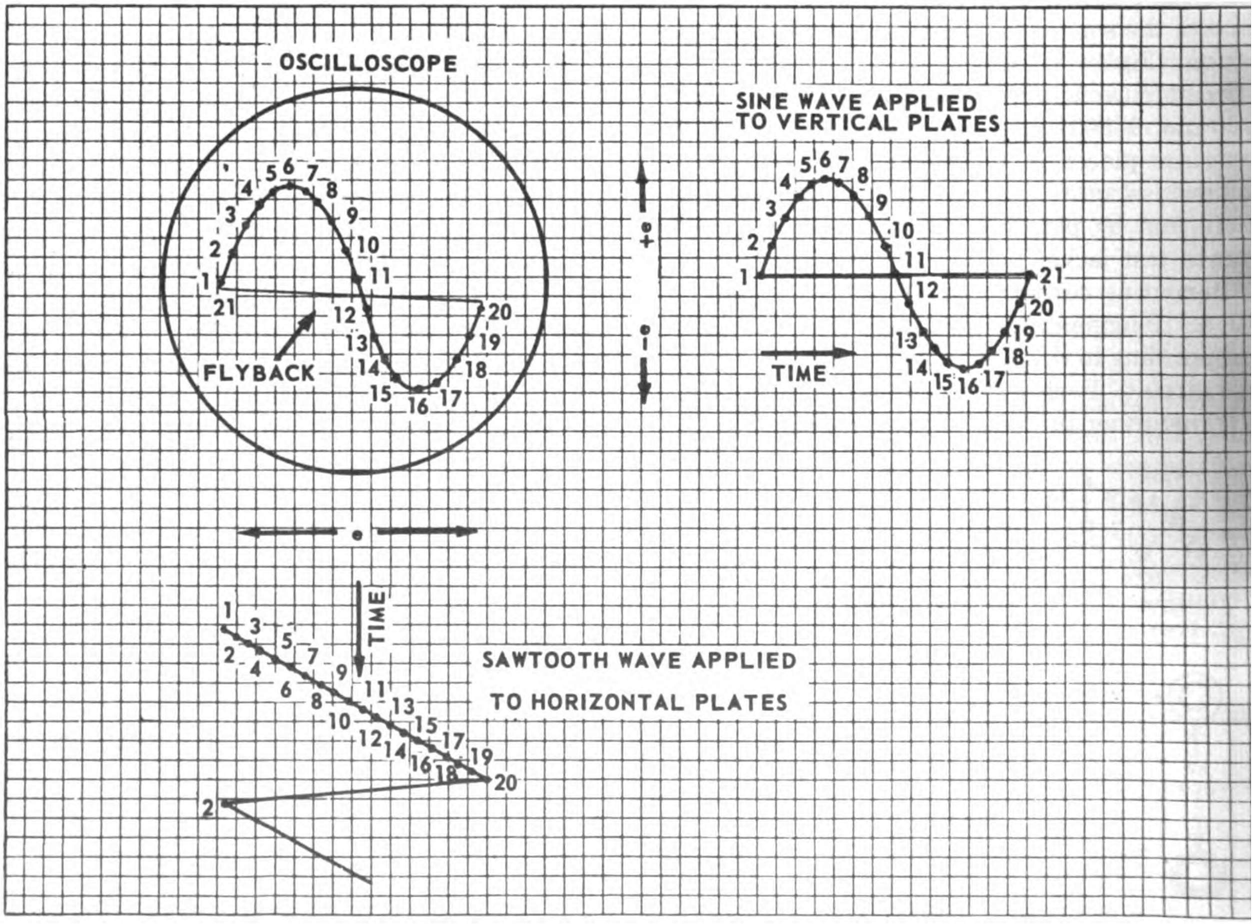


Figure 3–23. Development of Sine Wave on Face of Oscilloscope

duration of one cycle of the sawtooth voltage is the same as one cycle of the sine wave voltage.

Getting two cycles of the sine wave on the screen requires that the period of the sawtooth voltage be twice that of the sine wave; that is, its frequency must be one-half of the sine wave frequency. Thus, you can see that the sweep generator must be variable in frequency to permit observation of other than one cycle or, for that matter, waves of any frequency other than that of the sawtooth voltage.

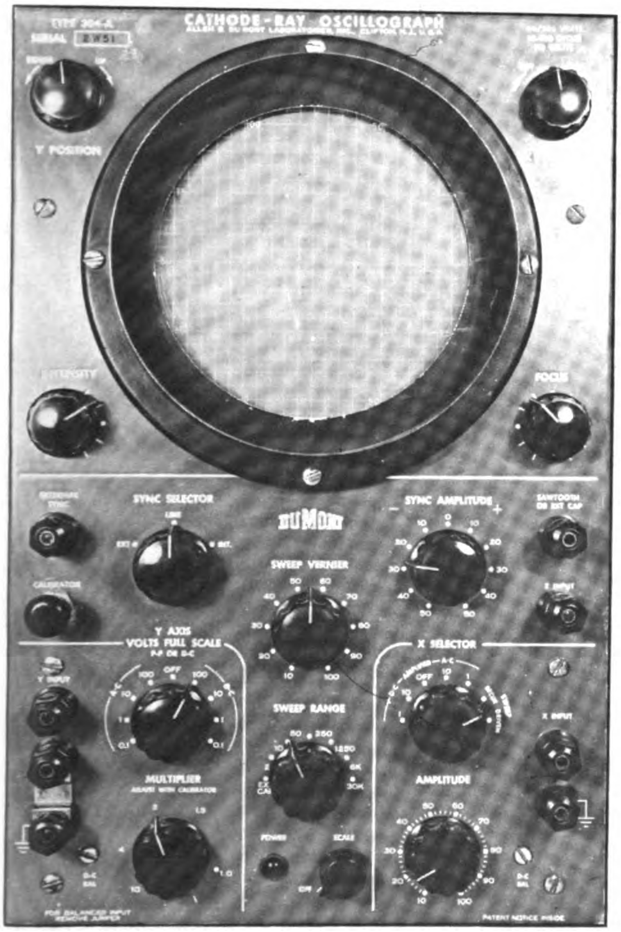


Figure 3–24. DuMont 304–A Oscilloscope Controls

DuMont Type 304A Oscillograph

The DuMont 304A Oscillograph is a typical oscilloscope. There are others in common use, but this one serves to illustrate the kinds and purposes of the controls. In this instrument the *horizontal* axis is called the X axis, and the *vertical* axis, the Y axis. Figure 3–24 shows the physical location of the controls and Figure 3–25 the electrical location.

SCALE. This control varies the illumination intensity of the illuminated scale. The attached switch applies 115-volt, 50- to 400-cycle power to the oscilloscope. The pilot light goes on when power is applied.

INTENSITY CONTROL. This control determines the amount of beam current. Keep the intensity setting no higher than necessary for convenience of use, in order to conserve tube life. Do not permit a sharply focused spot or line to remain stationary on the screen at high intensity.

FOCUS CONTROL. This control sets the potential of the focusing electrode of the cathode-ray tube gun. Generally, there is a setting for optimum focus at each intensity level. This gives a clear-cut picture on the screen.

POSITION CONTROLS. The X POSITION and the Y POSITION controls permit adjustment of the position of the trace along the X axis and Y axis respectively. They serve to *center* the picture.

BALANCE CONTROLS. When properly adjusted, the DC BAL (Y AXIS) prevents change in the vertical position of the sweep trace when the MULTIPLIER control is varied. Proper adjustment of the DC BAL (X AXIS) prevents change in the horizontal position of the trace when the (X) AMPLITUDE control is varied.

Y AXIS VOLTS FULL SCALE. This switch allows you to make direct peak-to-peak reading of the input signal in steps of 0.1v, 1v, 10v and 100v for both AC and pulsating DC inputs. In the OFF position the input signal is removed from the attenuator.

Y INPUT. Three binding posts provide for the connection of external signals to the Y amplifier. A jumper is provided for conventional single-ended inputs. The jumper is removed for balanced input (no attenuation). The third post is the ground connection.

MULTIPLIER. This is a potentiometer that provides continuously variable amplitude control of the vertical deflection.

CALIBRATOR. This switch, when pushed in, provides calibrating voltage to check the setting of the MULTIPLIER control.

X-SELECTOR. This switch selects internal or external signals for the horizontal deflection. The internal signal is a linear sawtooth sweep, either recurrent or driven. On external signals, either DC or AC coupling with attenuation ratios of 1 or 10 is provided. In the OFF position the grid of the input cathode follower is grounded.

(X) Amplitude. This control varies the overall gain (amplification) of the X amplifier. The range of this control is such that a signal causing 5 inches deflection at maximum AMPLITUDE control setting will be cut down to between 0.1 and 0.5 inch at minimum setting.

Sweep Range. This switch varies the sweep frequency in steps from 2 to 30,000 cycles per second by connecting in different size capacitors. The ranges which are marked on the front panel, however, serve as a guide only. They are not exact frequency calibrations.

Sweep Vernier. Used in conjunction with SWEEP RANGE control, this control provides continuous variation of the sweep frequency by controlling the rate at which the sweep capacitor is charged.

Sawtooth or EXT CAP. This binding post makes available a sawtooth output when X SELECTOR is set at SWEEP. When an external capacitor is connected between this terminal post and ground, the SWEEP RANGE switch being set on EXT CAP, a very low-frequency time base may be obtained. The larger the capacitor used, the lower the frequency of the sweep produced.

X Input. These binding posts provide for connection of external signals to the X amplifier. (The lower post is the ground connection.)

Sync Selector. This switch determines the source of the signal with which the sweep frequency is synchronized. Internal, line-frequency or external signals may be used. In the INT position the signal is obtained from the Y amplifier. In the EXT position the synchronizing voltage is obtained from a signal connected between ground and the EXTERNAL SYNC binding

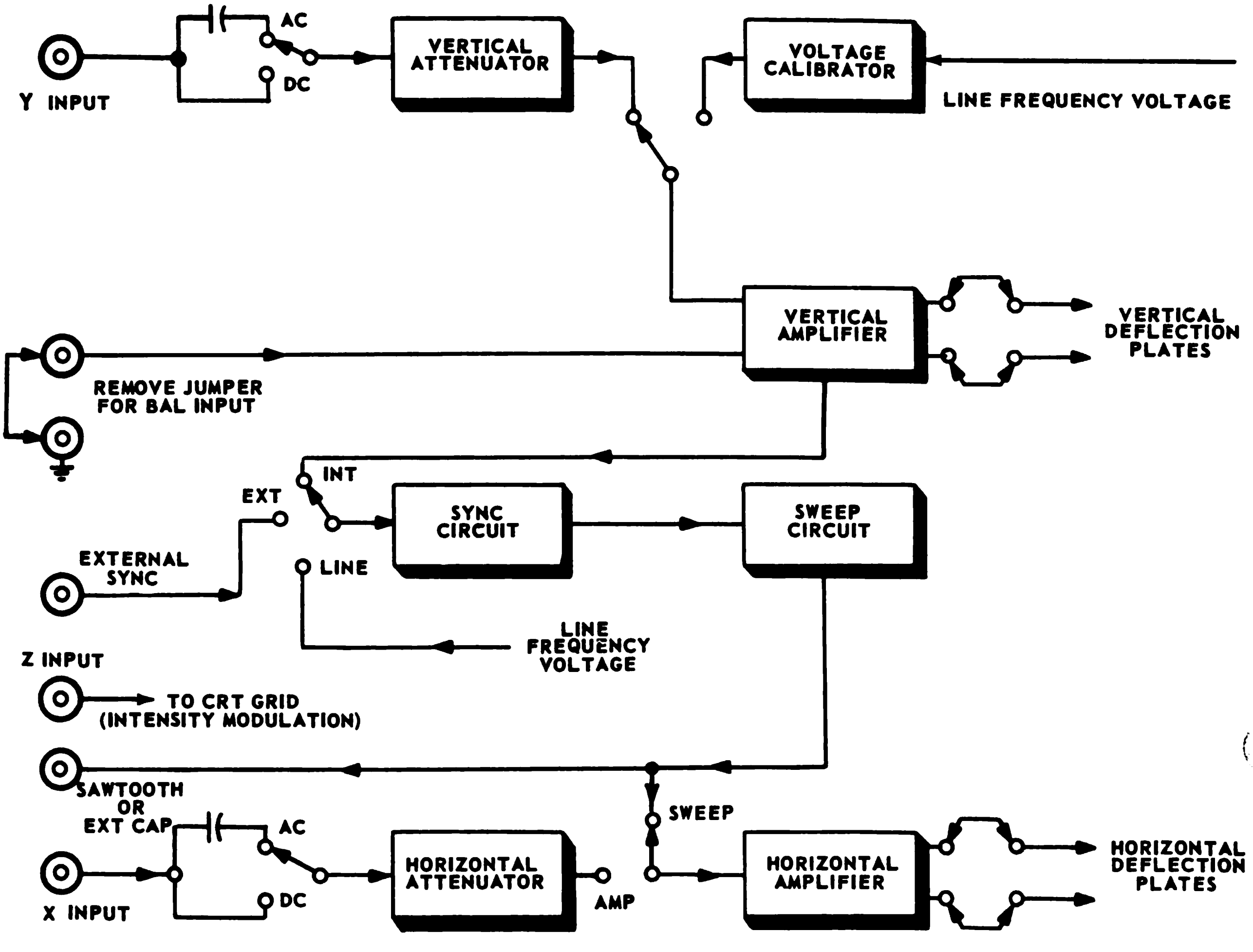


Figure 3–25. Functional Block Diagram of DuMont Oscilloscope

post. On LINE position voltage from the line supplying the power to the oscilloscope is used for synchronization.

External Sync. This is the binding post provided for connection of external synchronizing signals.

Sync Amplitude. This control selects the polarity and varies the amplitude of the synchronizing voltage. The minimum setting of this control should be used, since too large a synchronizing signal may distort the output of the sweep circuit, producing non-linearity or a shift in frequency. The pattern should be stabilized as far as possible with the SWEEP VERNIER control before advancing the SYNC AMPLITUDE control.

Z Input. Through this binding post external signals can be used to intensity modulate the trace.

Rear Terminal Posts. The terminal posts on the back of the oscilloscope may be used to apply signals directly to the deflection plates. (There are capacitors between these terminals and the plates because of the high DC voltage on the deflection plates.) Figure 3–26A shows the rear panel of the DuMont 304–A oscilloscope

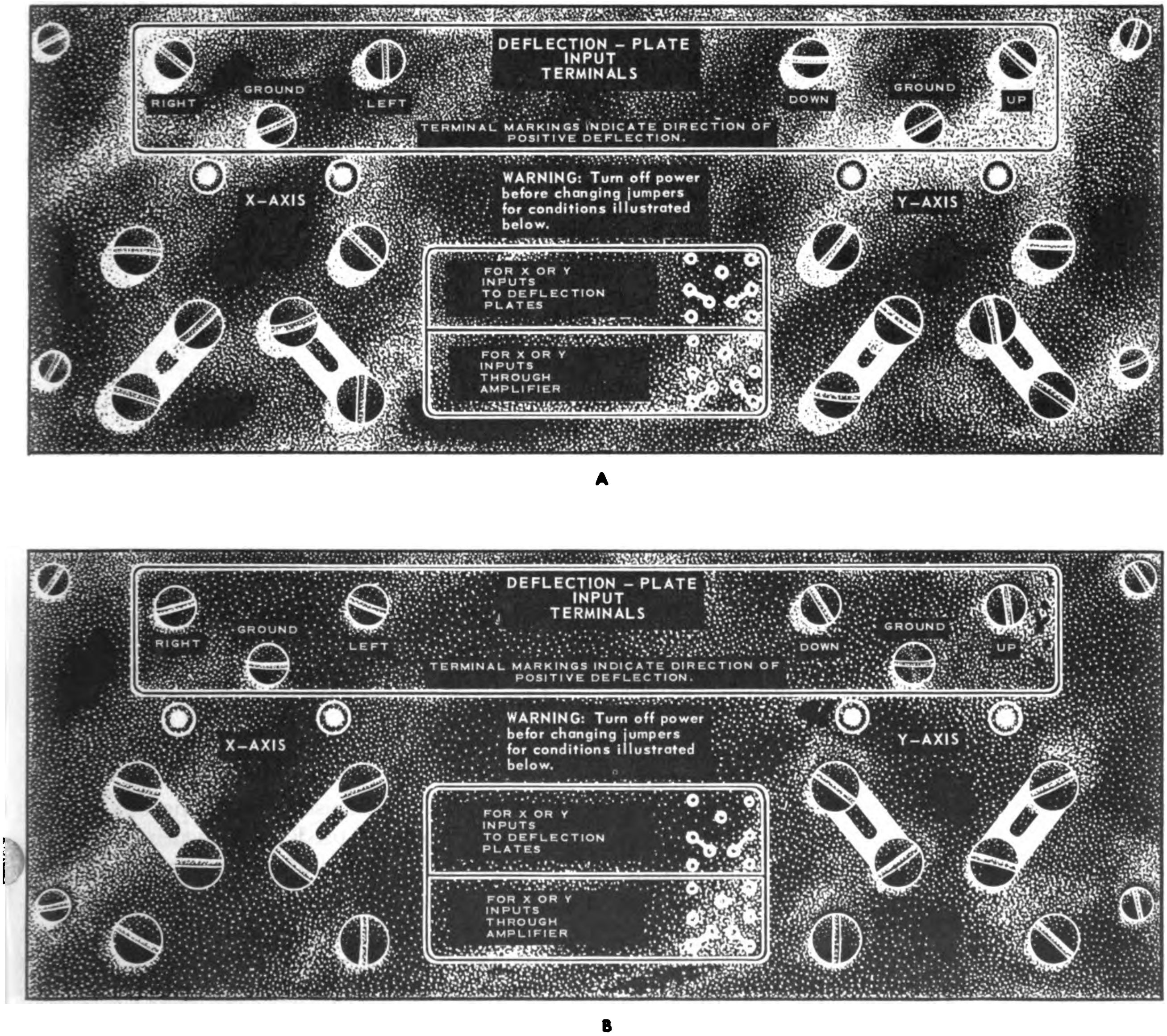
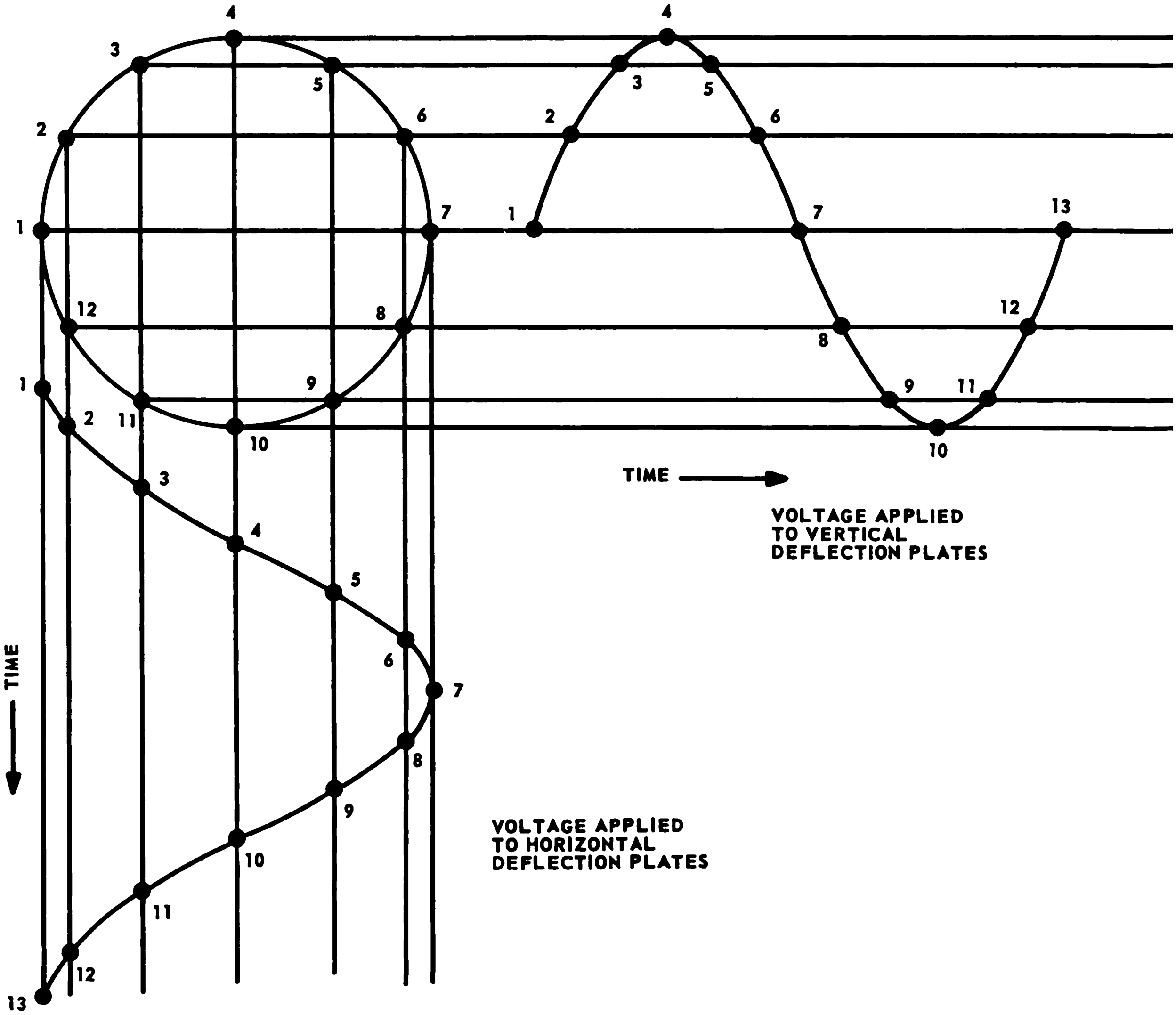

DEFLECTION - PLATE INPUT TERMINALS
RIGHT
GROUND
LEFT
DOWN
GROUND
UP
TERMINAL MARKINGS INDICATE DIRECTION OF POSITIVE DEFLECTION.
X-AXIS
Y-AXIS
WARNING: Turn off power before changing jumpers for conditions illustrated below.
FOR X OR Y INPUTS TO DEFLECTION PLATES
FOR X OR Y INPUTS THROUGH AMPLIFIER
A
DEFLECTION - PLATE INPUT TERMINALS
RIGHT
GROUND
LEFT
DOWN
GROUND
UP
TERMINAL MARKINGS INDICATE DIRECTION OF POSITIVE DEFLECTION.
X-AXIS
Y-AXIS
WARNING: Turn off power befor changing jumpers for conditions illustrated below.
FOR X OR Y INPUTS TO DEFLECTION PLATES
FOR X OR Y INPUTS THROUGH AMPLIFIER
B

Figure 3–26. Rear Panel of DuMont 304–A


4
3
5
2
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1
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12
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1
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2
11
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4
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6
7
8
9
10
11
12
13
TIME
4
3
5
2
6
1
7
13
12
8
9
11
10
TIME
VOLTAGE APPLIED TO VERTICAL DEFLECTION PLATES
VOLTAGE APPLIED TO HORIZONTAL DEFLECTION PLATES

Figure 3–27. Generation of Circular Pattern by two Sine Waves 90° Out of Phase

with the jumpers connected in the "through amplifier" position. Figure 3–26B shows the jumpers in the "deflection plates" position. In this position signals can be connected to the appropriate UP, DOWN, LEFT or RIGHT terminals.

These designations denote for each terminal the direction of deflection resulting from the application to that terminal of a positive-going signal. With direct connections the highest frequency signal that can be observed is dependent upon the input capacity to the deflection plates, and the transit time of the electrons between the deflection plates. There is no amplification or attenuation provided with direct connections, and it takes about 15 rms volts to produce one i of beam deflection.

Uses

In addition to its use in observing waveshap the oscilloscope is used for measuring voltag currents, frequency, and phase relations.

Voltage and Current Measurement. A v age is measured by comparing the deflection produces with the deflection produced by known voltage. Do not change the vertical g control while comparing the deflections. Si deflection is proportional to voltage, this is a v good method for measuring peak voltage of

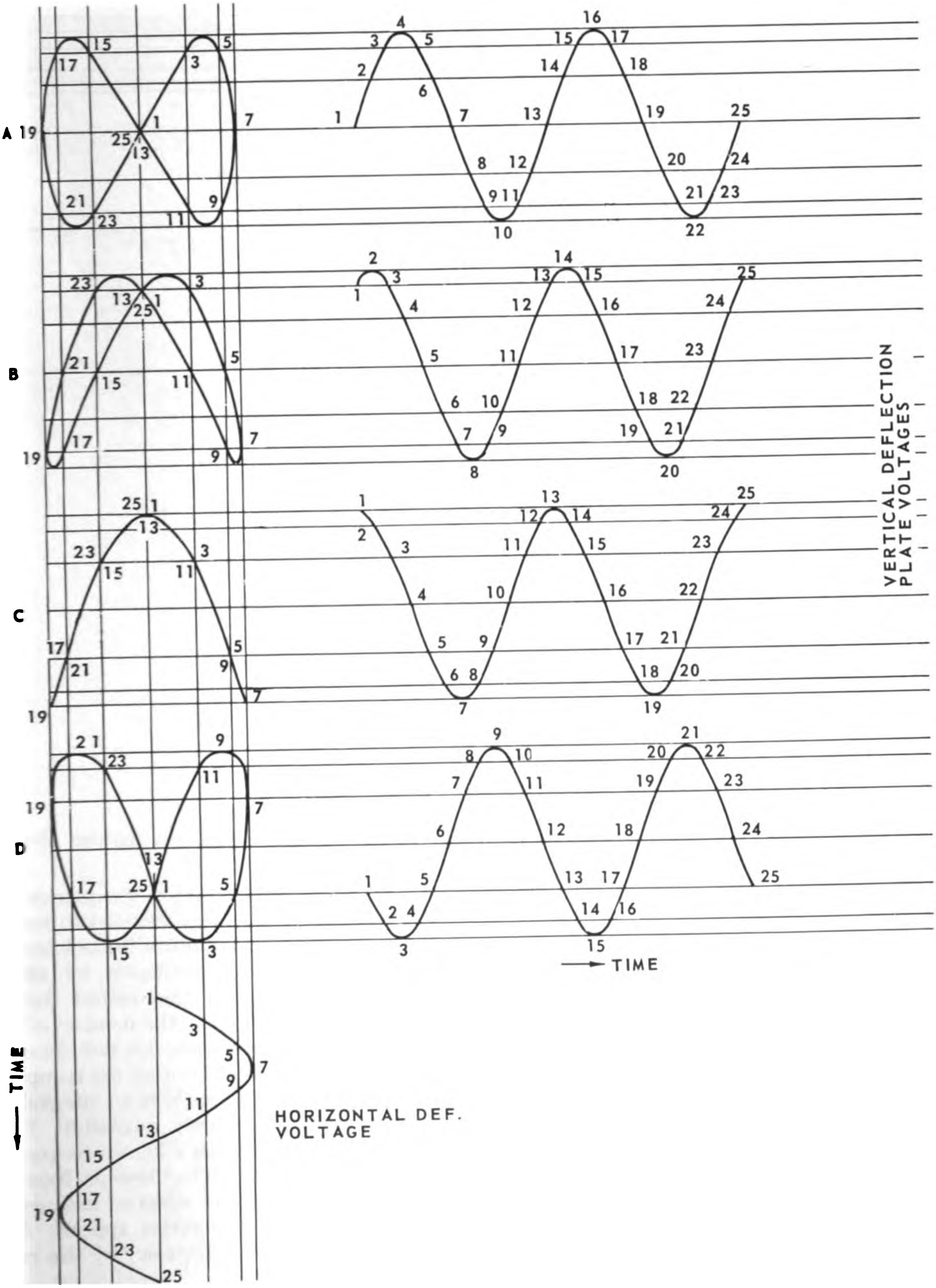


Figure 3–28. Lissajous Figures for 1:2 Frequency Ratio

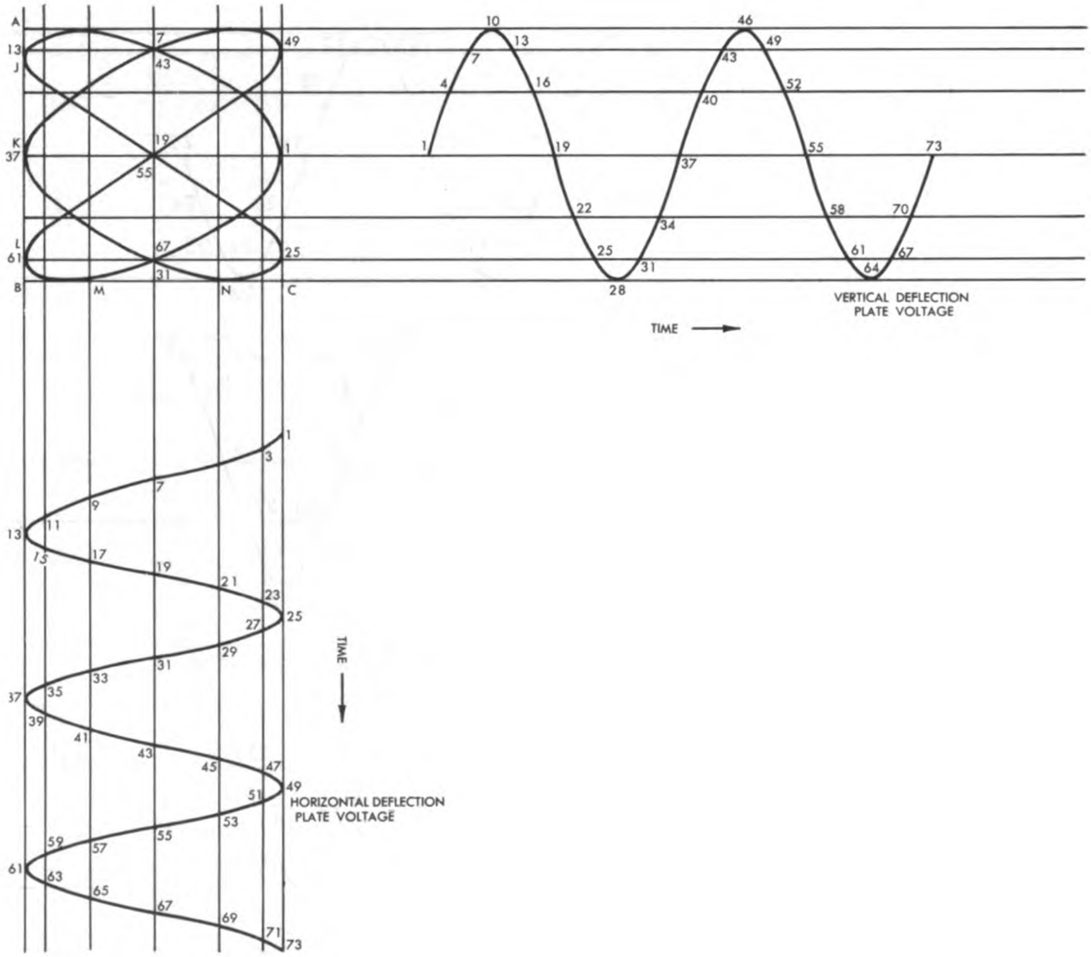


Figure 3–29. Lissajous Figures for 2:3 Frequency Ratio

waveshapes (other than sine waves). First, apply the known voltage and then adjust the vertical gain control to give as much amplitude as possible without letting the image extend beyond the flat surface of the screen. Measure the peak amplitude accurately. Do this with a ruler and dividers or by counting squares on the shield in front of the screen. Apply the unknown voltages and take a similar measurement.

By comparing deflections, you get the ratio of the peak voltages. If both are sine wave voltages, the rms values have the same ratio. If either is nonsinusoidal, the only comparison possible is the peak values. The meters you have encountered thus far do not read peak values, but you know that the peak value of a sine wave is 1.414 times the rms value. However, that is not true for other waveshapes.

Measure currents by passing the current through a known resistance, and measuring the resulting voltage drop in the manner described for measuring voltages.

FREQUENCY MEASUREMENT. Frequency measurement may be made by either of two methods. The first method is to compare the unknown frequency with a known frequency by applying them one at a time to the vertical deflection plates and then comparing the number of cycles which appear on the screen. Do not change the sweep frequency controls during the comparison. Unless the two frequencies have an integral ratio, this method is not entirely successful. For example, suppose that when a known frequency of 1000 cps is applied to the oscilloscope, four cycles appear on the screen, and when an unknown frequency is applied, five cycles appear. If you calculate the unknown frequency by the relation

$\frac{f}{1000} = \frac{4}{5}$, you get $f = 1250$ cycles per second.

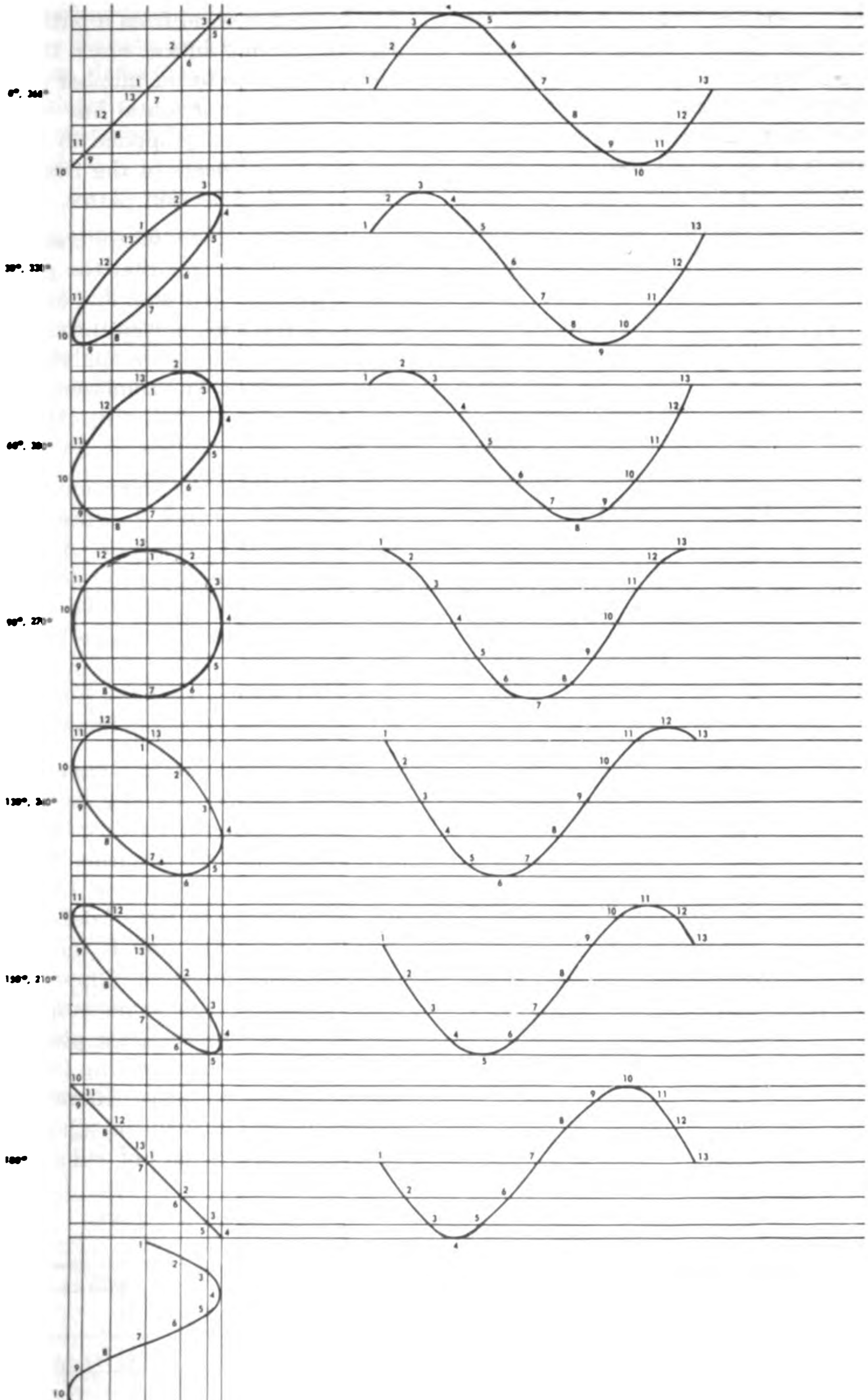


Figure 3–30. Lissajous Figures Indicating Phase Difference between Sine Waves of Same Frequency and Amplitude

Suppose the unknown frequency were some figure such as 1223. In this case, you can only approximate its frequency since changing the sweep frequency slightly will only tell you if the unknown frequency is above or below 1250. If you have to increase the sweep frequency to get five cycles, then the unknown frequency is above 1250 cps. In using this method of approximating frequency, place the SYNC SIGNAL AMPLITUDE control in its minimum position for best results.

A second method for determining the frequency of a sine wave is to apply the unknown frequency to the vertical deflection plates and a known frequency sine wave to the horizontal deflection plates. The resulting figure on the screen is called a *Lissajous figure*, and from it you can determine the unknown frequency. The simplest type of Lissajous figure is a circle. A circle can be produced by two voltages differing 90° in phase having the same frequency and amplitude. Figure 3–27 shows the generation of a circular pattern by two sine waves 90° out of phase.

Figures resulting from the application of one frequency to the vertical deflection plates, and another frequency half the value of the first to the horizontal plates are shown in Figure 3–28. It makes no difference what the actual frequencies are as long as they have a ratio of 1:2. Note that the phase angle difference alters the pictures of the waveshapes. Two voltages in phase appear on the screen as shown at A. If the phase angle is 90°, the loops are closed as shown at C. Phase angles greater than 180° appear inverted as shown at D.

Another common Lissajous figure is the one showing the result of applying voltages having frequencies in a ratio of 2:3 to the horizontal and vertical deflection plates respectively (Figure 3–29).

As you can see, it is possible to draw a great number of pictures for different frequency ratios, but that would be quite a lengthy process. Instead, examine Figures 3–28 and 3–29 and see whether any general rule will work for determining the frequency ratio from a pattern. In this connection note in Figure 3–29 that the pattern touches the horizontal line BC at the two points M and N, and the vertical line AB at the three points J, K, L. From these observations you can form the following general rule for determining the frequency ratio from a pattern: The ratio of the number of points at which the pattern touches the vertical line to the number of points at which it touches the horizontal line is the same as the ratio the frequency applied to the horizontal deflection plates bears to the frequency applied to the vertical deflection plates.

In applying this rule to Figure 3–28C, don't be confused by the fact that the pattern appears to touch the top horizontal line only once. Remember that the spot moves through point 1 twice. Another fact to know in applying the rule is that it holds good for any combination of vertical and horizontal lines, providing they intersect the pattern.

For accurate determination of frequencies by the Lissajous figures, you need a calibrated variable frequency generator. Apply the output of this generator to one set of deflection plates and the frequency being measured to the other deflection plates. Then determine the frequency ratio from the relationship given in the preceding rule. Read the frequency of the generator and use the ratio to determine the unknown frequency.

DETERMINING PHASE RELATIONS OF VOLTAGES. Lissajous figures may also be used to determine the phase relations of two voltages. Figure 3–30 shows the patterns resulting when two voltages of the *same frequency and the same amplitude* are applied to both sets of deflection plates. The pattern is a circle only if the two voltages are 90° (or 270°) apart in phase, of the same frequency and of the same amplitude. You can determine the approximate phase angle by comparing the patterns resulting from unknown voltages to these patterns. Remember that if one voltage is greater than the other, the pattern will never become circular but will always be elliptical.

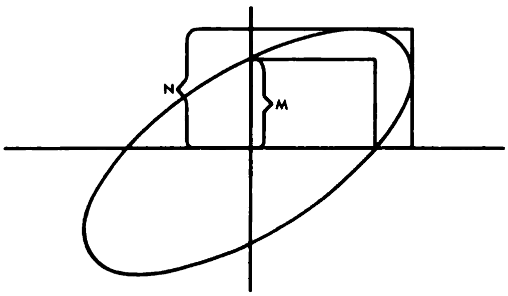


Figure 3–31. Phase Angle by m and n

A more exact method for determining the phase angle is measuring the intercept *m* on the Y-axis, and the maximum deflection *n* on the Y-axis (Figure 3–31). Exercise care both in centering the figure, and in measuring the quantities *m* and *n*. The phase angle is arc sin (m/n).

SYNCHROSCOPE

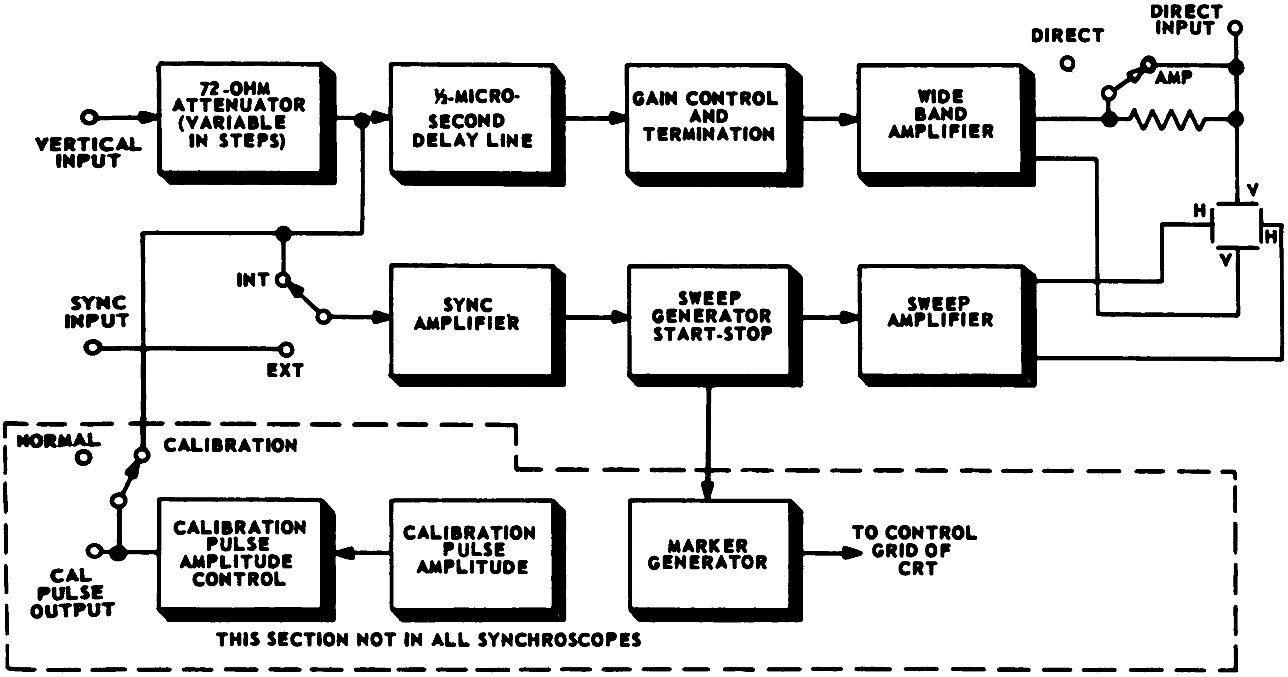
The synchroscope is a special type of oscilloscope. The trace is produced only when the sweep circuit is triggered, as contrasted with the continuous sawtooth sweep furnished by the oscilloscope. The synchroscope can be used to observe recurrent pulses or waveforms; the sweep is usually very fast to permit display of short pulses.

Figure 3–32 is a block diagram of the signal and sweep channel circuits of a typical synchroscope, which differ from those of an oscilloscope. The input circuit of the signal channel is usually an adjustable attenuator. This provides approximately the same input level to the amplifier for different amplitude input signals. The artificial delay line is used to delay the signal to be observed until the sweep circuit has been triggered by the undelayed signal.

Without the delay line the beginning portion of the waveform would not appear on the trace, since time is needed for the input signal voltage to reach the amplitude required to trigger the sweep circuit. The pulse as it would appear without the delay line in the circuit is shown in Figure 3–33B. With the delay circuit the signal reaches the amplifier one-half microsecond after the start of the trace and the entire pulse is seen, as shown in Figure 3–33C. In addition, the delay line is used to furnish a series of pulses spaced one microsecond apart for calibration of short time intervals.

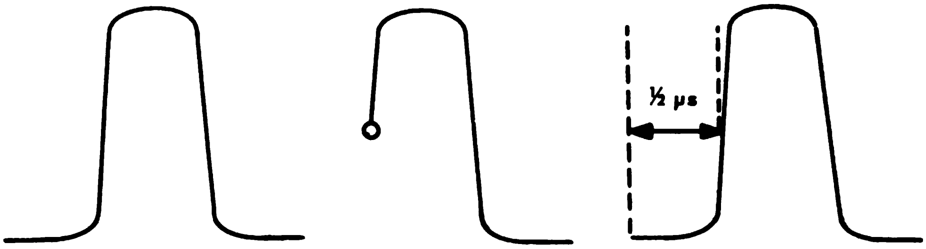
The gain control varies the amplitude of the input to a wide band, or video, amplifier. The output of this amplifier is fed to the vertical deflection plates, which have an external connection as well.

The sync switch in the sweep circuit allows for either internal or external synchronizing voltages to be fed to the sync amplifier. The start-stop sweep generator does not develop a sweep voltage until a trigger voltage of sufficient amplitude is fed into the circuit. The sweep speed is adjustable, its duration varying from a very few microseconds to about 250 microseconds.



DIRECT INPUT
DIRECT
AMP
72-OHM ATTENUATOR (VARIABLE IN STEPS)
½-MICRO-SECOND DELAY LINE
GAIN CONTROL AND TERMINATION
WIDE BAND AMPLIFIER
VERTICAL INPUT
V
H
H
V
INT
SYNC INPUT
SYNC AMPLIFIER
SWEEP GENERATOR START-STOP
SWEEP AMPLIFIER
EXT
NORMAL
CALIBRATION
CALIBRATION PULSE AMPLITUDE CONTROL
CALIBRATION PULSE AMPLITUDE
MARKER GENERATOR
TO CONTROL GRID OF CRT
CAL PULSE OUTPUT
THIS SECTION NOT IN ALL SYNCHROSCOPES

Figure 3–32. Block Diagram of a Typical Synchroscope



A – INPUT PULSE B – WITHOUT DELAY C – WITH DELAY

Figure 3–33. Synchroscope Trace With and Without Delay

Some synchroscopes have provisions for the calibration of input voltages and sweep times. In calibrating, the calibration pulse is adjusted until it is the same amplitude as the unknown voltage; this value is read from the calibration pulse control. Marker pulses are used to calibrate sweep time. These marker pulses appear as a series of bright dots spaced at regular intervals. The time duration of the interval is selected by the operator.

CHAPTER 4

Electron Tubes and Power Supplies

The most important single device in electronics is the vacuum tube. Facts have established that without it, the present day knowledge of this science would have been impossible. The first radio receivers used no vacuum or electron tubes. They used a crystal semiconductor as a rectifier (detector) in conjunction with a coil and a variable capacitor for maximum signal. However, modern radio, radar, and electronic computer equipment employ a wide variety of vacuum tubes. The widespread use of the tube is evident in today's world of radio, television, phonographs, movies, and so forth. Most equipment today is multitube, employing thousands of tubes in some cases.

This chapter discusses the various types of vacuum tubes commonly used in radio and electronic circuits. You will learn about tubes with only two elements, tubes with three or more elements, and specially designed tubes, called special purpose tubes. In addition, you will learn about power supplies which furnish energy for the operation of these tubes. What you learn about vacuum tubes, transistors, and power supplies will largely determine how much you will be able to understand about radio and radar circuits.

EMISSION OF ELECTRONS

In 1873 F. Guthrie performed one of the earliest experiments toward vacuum tube development. His experiment used a metal ball heated to a dull red and a negatively charged electroscope. Elster and Geitel, two German scientists, later in 1882 and 1889 made further studies of the conductivity of a gas near heated solids. Edison, not knowing what Elster and Guthrie were doing, made an important discovery in 1883, when he found that a hot-spot developed on the carbon filament of incandescent lamps. In experimenting with an evacuated tube containing a heated filament near a metal plate, Edison discovered that current flowed between the filament and the metal plate.

This occurred when the plate was more positive than the filament, but not when the plate was negative with respect to the filament. *Thermal emission* is the name given to his discovery of electron flow from the filament to the inserted plate. This is the process by which vacuum tubes obtain their supply of electrons from the cathode. The property of emission was studied by other scientists; a third electrode was introduced by Lee De Forest, who is given most of the credit for developing the vacuum tube.

Thermal Emission

This process of thermal emission, which is known as the *Edison effect,* may be explained by the electron theory of matter, which assumes that the outer electrons in the atoms of metals are very loosely bound to the nucleus. When a voltage is applied to the filament in an evacuated tube, the free electrons move along the filament and collide with other free electrons. The collisions between the electrons generate heat. They also give some of the moving electrons enough energy to overcome the attractive force within the wire and break out from the surface of the wire. These electrons are called *emitted electrons.* The filament which emits the electrons is called the *cathode*.

When a plate placed near the emitter has a voltage applied to it making it more positive than the emitter, the plate attracts the negative electrons, causing a current to flow from the filament to the plate. This can be proved by connecting a milliammeter in the plate circuit. If the plate is connected to the negative side of the voltage source making the plate more negative than the filament, there will be no current flow. The electrons emitted from the heated filament will be repelled by the negative plate.

The escape of electrons from the surface of a metal is analogous to the escape of molecules from a liquid during evaporation. In liquids the molecules evaporated are the molecules that possess enough energy to overcome the force at the surface which tends to keep them within the liquid. In the case of heated metals, the process is similar and, in a sense, can be considered the evaporation of electrons from a heated surface.

The laws governing the emission of electrons from hot surfaces are based upon rather complex mathematical processes. However, certain basic concepts of emission can be understood without an extensive study of these laws.

Early vacuum tubes used pure tungsten filaments as emitters. Operated at 2300°C, a tungsten filament furnishes approximately 8 ma of current per watt of heating power. Higher temperatures give increased emission and shortened life, while lower temperatures have the opposite effect. A search for more efficient emitters led to the discovery that certain oxides used as coatings for tungsten gave increased emission. From this discovery came a filament consisting of a nickel ribbon coated with a mixture of barium and strontium oxides.

The operation of this emitter at 1000°C produces approximately 100 ma of current per watt of heating power. The temperature of this emitter must be maintained within closer limits, both above and below the rated amount, to prevent shortening its life. A third type of emitter was developed by coating tungsten with thorium. Operating this emitter at 1700°C produces a current of approximately 70 ma per watt. Like the oxide-coated emitter, its useful life is shortened by its being operated above or below its rated temperature.

Figure 4–1 shows the electron emission for three different types of emitters. Note that the emission current increases exponentially as the temperature is increased. This increase will continue until the filament burns out from overheating. Rather than exceed an optimum temperature when increased current requirements exist, manufacturers increase the emitter area o the tube.

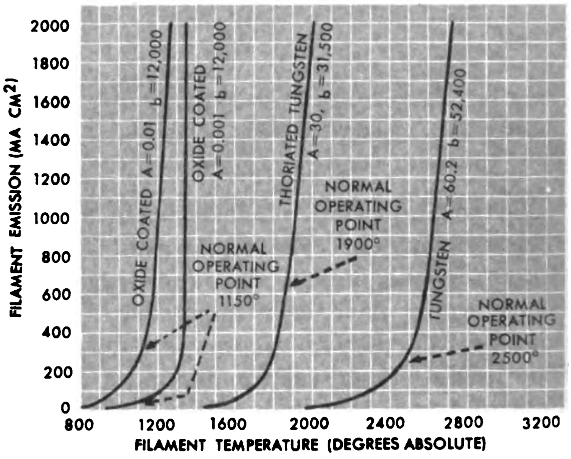


Figure 4–1. Electron Emission of Different Emitters

The emitter may be heated directly by passing a current through it. The emitter in a directly heated tube is called the filament (or filamentary cathode). The use of an alternating current to heat the filaments of amplifier tubes has an undesirable effect. The current through these tubes varies at the frequency (usually 60 cycles) o the filament voltage, and an undesirable hum appears in the output. This problem can be solved by balancing the filament voltage with respect to ground. The use of a DC filament voltage also solves this problem.

A tube which has a cylinder-shaped emitter containing an insulated heater is called an indirectly heated tube. In this type of tube, the emitter is referred to as the cathode. This cathode often consists of an oxide-coated, nicke cylinder which contains a heater wire. The use of an AC heater voltage does not vary the current through the tube, and undesirable hum voltages are not ordinarily a problem where indirectly heated tubes are used. Figure 4–2 illustrates the construction of directly and indirectly heated tubes.

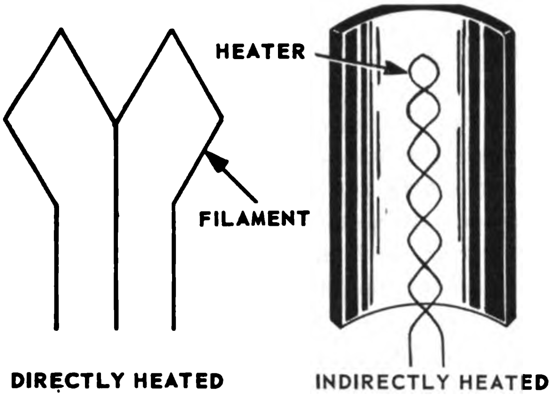


Figure 4–2. Emitter Heating

Filament and Heater Voltages

The voltage and current required for heating the emitter to the proper temperature vary considerably in different tubes. Tube manuals give the filament or heater voltage and current ratings of particular tubes. Current ratings vary considerably for different tubes, even with the same voltage ratings. These variations are related directly to the emission capabilities of the tubes.

Common filament voltages for directly heated tubes are 1.4, 2.8, and 5.0 volts; other values are used for transmitter power amplifier tubes, and range up to 33 volts. Indirectly heated tubes have heater voltages of 6.3, 12.6, 25, 50, 70, and 117 volts; the first two mentioned are most common, and are generally referred to as 6- and 12-volt tubes, respectively. A 1.4-volt tube is designed for use where voltage is supplied by a dry-cell battery. A 117-volt tube can be heated directly from a 117-volt AC line.

Connecting Heaters

Heaters may be connected in parallel when they have the same voltage rating. This is a common practice in receivers using a power transformer for stepping down the line voltage to suitable voltages for operation of the receiver. For example, a power transformer which receives voltage from a 117-volt AC power source may have three secondary windings: one to supply the voltage for the plates of the tubes; one to supply voltage for the rectifier filament; and one to supply 6.3 volts for the heaters of the other tubes. All of these 6.3-volt tubes can be connected in parallel across the 6.3-volt secondary winding. The total current drain through this winding is equal to the sum of the current ratings of the individual tubes connected across it.

Receivers which operate from either AC or DC do not use power transformers, but obtain power directly from the line. Heaters in these receivers are connected in series across the power source (line). However, for this connection to work satisfactorily, the combined voltage ratings of all the tubes must equal the line voltage. If this is not the case, then the applied voltage is dropped to the combined voltage ratings of all the tubes by connecting a resistor in series with the power source. Furthermore, when heaters are connected in series, all tubes must have equal current ratings. If any tube has a rating lower than the others, a resistor is shunted across this tube. This resistor carries additional current, and effectively makes the current rating of this tube equivalent to that of the other tubes.

The following examples illustrate typical conditions that you will encounter in connecting heaters of vacuum tubes:

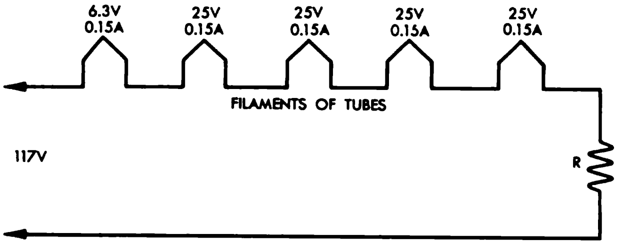


Figure 4–3. Series Heaters

Example 1

Problem

In Figure 4–3, five heaters are connected in series. Find the resistance and the power dissipated by the resistor, R, using the values indicated.

Solution

The sum of the voltage ratings of the five tubes equals 106.3 volts. Subtracting this voltage from the voltage of the power source, 117 volts, gives 10.7 volts, which is the voltage drop across resistor R. Then according to Ohm's law,

$$R = \frac{E}{I},$$

$$R = \frac{10.7}{0.15} = 71 \text{ ohms.}$$

Using the power formula,

$$P = EI,$$

$$P = 10.7 \times 0.15$$

$$P = 1.605 \text{ watts}$$

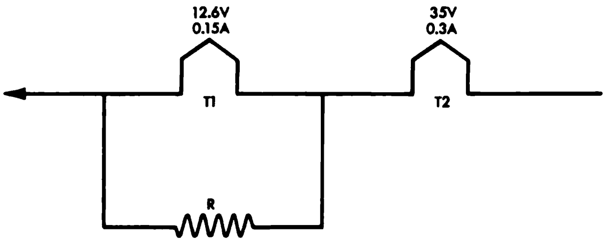


Figure 4–4. Series Heaters with Different Current Rating

Example 2

Problem

In Figure 4–4, two heaters are connected in series. V2 requires 0.3 ampere, but V1 needs only 0.15 ampere. Find the value of R and its power dissipation.

Solution

If 0.3 ampere is required for V2, and V1 needs only 0.15 ampere, the other 0.15 ampere must flow through R. By Ohm's law,

$$R = \frac{E}{I} = \frac{12.6}{0.15} = 84 \text{ ohms}$$

Finding power dissipation of R,

$$P = 12.6 \times 0.15 = 1.89 \text{ watts.}$$

DIODES

The simplest form of vacuum tube contains two electrodes — a cathode and a plate. This type of tube is called a *diode*. Edison discovered what may be called the earliest diode, his special lamp. *Edison effect* was the name given to his discovery. Fleming, in 1905, patented a device known as the *Fleming Valve*, which was used as a detector of radio currents. The modern diode differs little in principle of operation from these early models. It is much improved, however, in higher vacuum, longer life, and increased ruggedness.

A diode can be used to change alternating current to pulsating direct current, since it permits current flow in only one direction. When a positive voltage is applied to the diode plate, electrons will flow from the cathode to the plate and return to the cathode through an external circuit. When a negative voltage is applied to the plate, the free electrons in the space surrounding the cathode will be forced back to the cathode and no plate current will flow. When an alternating voltage is applied to the plate, the plate will alternately become positive and negative. Plate current will flow only during the time that the plate is positive, and since it flows through the tube in only one direction, it is said to be *rectified.*

Diode rectifiers are used in AC operated receivers and transmitters to convert AC into DC for supplying the control grid bias, and the plate and screen voltages for all other tubes. They are also used as detectors, or demodulators, in receivers. Rectifiers and detectors are discussed in detail later.

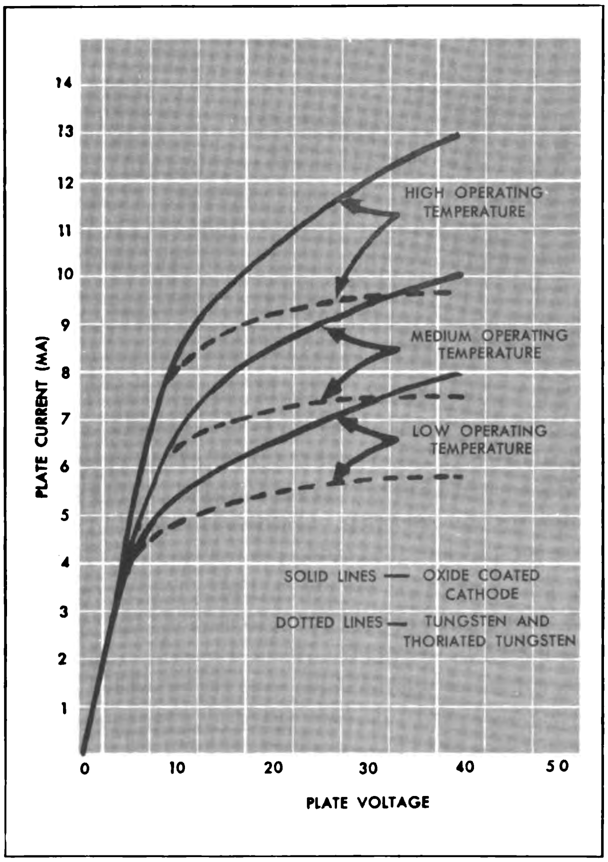


Figure 4–5. Diode E_p-I_p Characteristics

The relation between plate current (the number of electrons attracted to the plate) and plate voltage in a diode is given in Figure 4–5. It shows the results obtained in a typical diode operated at several cathode temperatures. Note that, at low voltage, plate current depends entirely on plate voltage and is independent of cathode temperature. At high plate voltages, plate current is determined largely by cathode temperature and is practically independent of plate voltage.

At low plate voltages, electrons are emitted by the cathode faster than they are attracted to the plate. Electrons, therefore, tend to remain in the space between the cathode and plate, and produce a negative charge (called *space charge*) which exerts a repelling force upon other electrons being emitted from the cathode. At any instant, the number of electrons traveling between the cathode and plate cannot be greater than the number required to neutralize the attraction of the plate voltage. All electrons in excess of this number are repelled back to the cathode. Thus, plate current is independent of cathode emission as long as the negative space charge exists.

At higher plate voltages the plate attracts more of the electrons emitted from the cathode and reduces the tendency for electrons to remain in the space charge region and to repel others. The higher the plate voltage, the greater the proportion of electrons attracted and the smaller the space charge.

Tube current levels off at high plate voltages because the plate attracts electrons from the cathode as rapidly as they are emitted. Under these conditions, increasing the plate voltage makes no difference because the plate current is already equal to the total emission of the cathode. The tube is said to be operating at *emission saturation.* The point at which no additional current flows when the plate voltage is increased is called the *saturation point.* When the cathode temperature is raised, however, the cathode emits electrons at a faster rate, emission saturation occurs at a higher plate voltage, and a higher current flows.

TRIODES

A triode is a three-element vacuum tube containing a grid, a cathode, and a plate. Figure 4–6 depicts typical triode structures. The grid is usually an open coil or mesh of fine wire and is placed between the cathode and plate. The turns of wire in the grid are spaced far enough apart so that the passage of electrons from the cathode to the plate is virtually unobstructed. The electrons that pass from the cathode to the plate constitute plate current flow in the tube.

If the grid is made more negative than the cathode, some of the electrons emitted by the cathode will be repelled by the grid and therefore will not reach the plate; plate current, accordingly, will be decreased. It follows that as the grid voltage is made increasingly negative, fewer and fewer electrons will reach the plate.

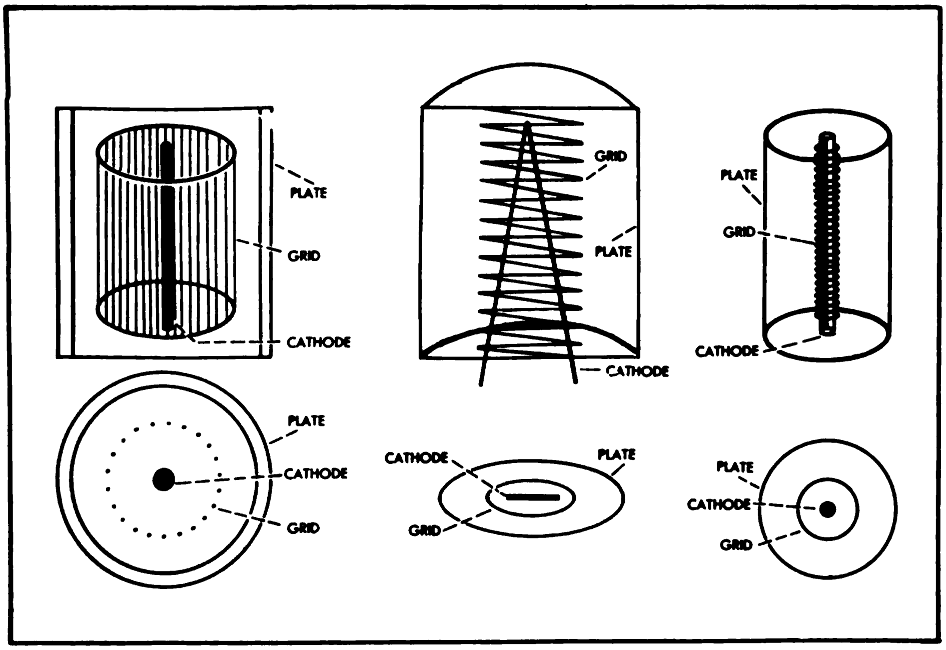


Figure 4–6. Typical Triode Structures

Eventually, when the grid is made negative enough with respect to the cathode, no electrons will reach the plate, and current will be cut off entirely. The point at which plate current ceases to flow is called *cutoff*, and the negative grid voltage required to stop the plate current is called *cutoff voltage.*

Plate current increases as the grid is made less negative. When the grid is at the same potential as the cathode, it will have no effect on the passage of electrons to the plate. If the grid is made positive with respect to the cathode, it will aid in drawing electrons from the space charge between the plate and grid, and there will be an increase in plate current. Not all of these electrons will go to the plate, however. The grid itself will act as an anode (plate) and a current, called *grid current*, will flow through it. If the grid is made positive enough, it will attract the majority of the total number of electrons emitted by the cathode, since it is closer to the cathode (and space charge) than the plate.

Because the grid is so close to the cathode, a comparatively small change in the voltage on the grid in a triode has the same effect on plate current flow as a much larger change in the plate voltage. It is this feature that gives the triode its chief property — *amplification.*

Here is how amplification is achieved. A small alternating voltage applied to the grid causes comparatively large variations in plate current at the frequency of the applied voltage. If a resistor, called the *load resistor*, is placed in series with the plate circuit, the varying plate current flowing through this resistor will develop a voltage which has a frequency equal to that of the AC voltage applied to the grid. The magnitude of the voltage across the load resistor, however, will be much greater than that of the voltage applied to the grid. Since the voltage appearing in the output is greater than the voltage applied to the grid, it is said to be *amplified.*

There are many possible applications of the triode, as with the diode, but all applications are founded on one basic function. The function of the diode is to convert AC power to DC power. The basic function of the triode is that of an amplifier of current, voltage, or electrical power.

Characteristic Curves of Triodes

The behavior of a triode is best described with curves, called *characteristic curves.* These curves are obtained by keeping one of the three quantities (grid voltage, plate voltage, or plate current) constant and observing the relationship between the other two. Since any of the three quantities may be kept constant, there are three families of curves.

In the three families of curves shown in Figures 4–7, 4–8, and 4–9, each curve is obtained by applying specific voltages to the grid and plate

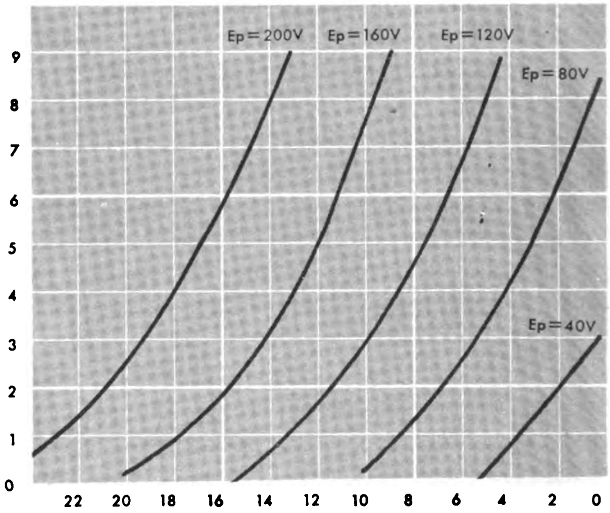


Figure 4–7. E_g-I_p Curves of a Triode

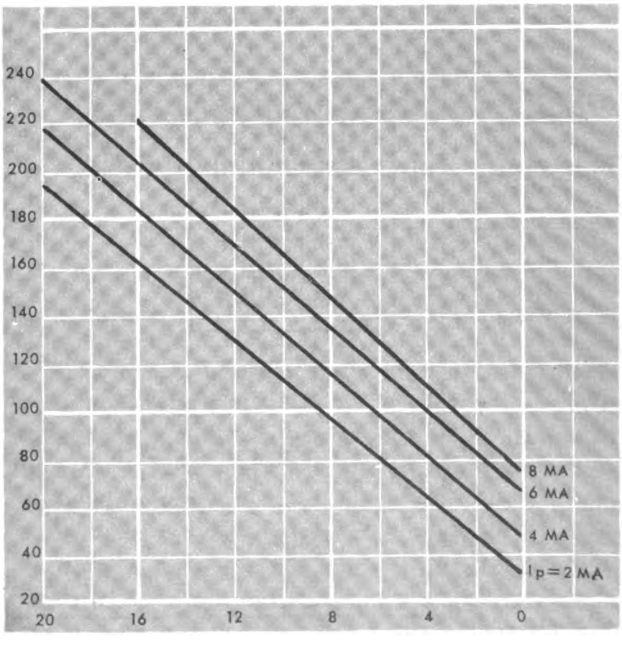


Figure 4–8. E_p-E_g Curves of a Triode

and by observing the plate current. The E_g–I_p curves are obtained by keeping the plate voltage (E_p) constant and plotting the plate current (I_p) against the grid voltage (E_g). The E_p–I_p curves are obtained by holding the grid voltage (E_g) constant and plotting the plate voltage (E_p) against the plate current (I_p). The third family, the E_p–E_g curves, results from keeping the plate current constant, and plotting the grid voltage (E_g) against the plate voltage. The most commonly used curves are the E_p–I_p curves. You can find these curves in any tube manual.

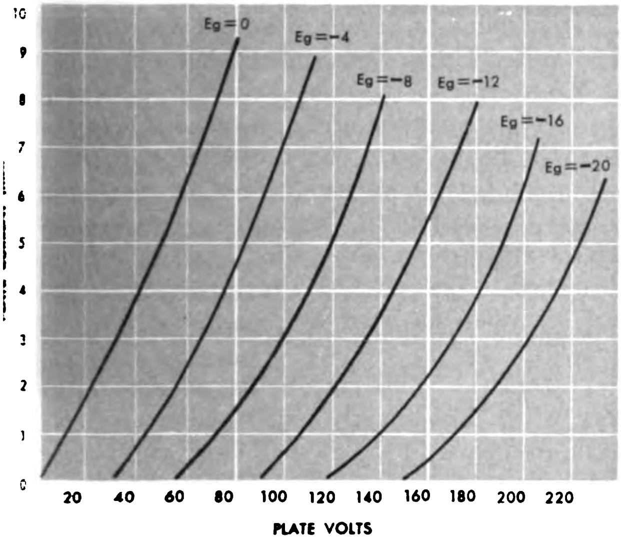


Figure 4–9. E_p-I_p Curves of a Triode

Curves of this type are called *static curves*, since they are obtained by using DC voltages under nonoperating conditions. Characteristic curves obtained under operating conditions are called *dynamic curves*. Dynamic curves are useful only when circuit elements are operated under the same conditions that the curves were obtained. Static curves, although obtained under nonoperating conditions, will give you considerable information relative to the tube even under operating (dynamic) conditions. For this reason they are the ones usually given.

Amplification Factor. The ratio of the plate voltage change required for a given change in plate current to the change in grid voltage which will produce the same change in plate current is called the *amplification factor* of a tube. It is commonly identified by the Greek letter μ (mu) and expressed mathematically,

$$\mu = -\frac{\Delta e_p}{\Delta e_g} \text{ (When } I_p \text{ is constant)}$$

where Δe_p is change in E_p, and Δe_g is change in E_g.

In triodes, the amplification factor varies from about 3 to about 100.

AC Plate Resistance. The ratio of a small change in plate voltage to the change in plate current it produces when the grid voltage is constant is called the dynamic or AC plate resistance of a tube and is represented by r_p. The formula for plate resistance is,

$$r_p = \frac{\Delta e_p}{\Delta i_p} \text{ (When } E_g \text{ is constant)}$$

Δi_p is change in I_p.

Plate resistance is important in determining the performance of a tube in a given circuit, since it forms a voltage divider with the load resistance. This divides the voltage applied across the tube in a ratio depending on the value of the plate and load resistances. The larger the load resistance with respect to the plate resistance, the smaller the ratio and the greater the gain of the circuit. Any decrease in this ratio results in an increase in gain; any increase results in a decrease in the gain.

Transconductance. Another important tube constant is called transconductance and is represented by g_m. (When the term transconductance is used without qualification, grid to plate transconductance is always meant.) Transconductance tells how much plate current change is caused by a small change in grid voltage. The formula for this constant is,

$$g_m = \frac{\Delta i_p}{\Delta e_g} \text{ (When } E_p \text{ is constant)}$$

The unit of transconductance is the mho.

Transconductance bears a mathematical relation to the amplification factor and plate resistance of a tube and is expressed in terms of these constants as,

$$\mu = g_m r_p.$$

Tube Constants and Characteristic Curves. The tube constants μ, r_p, and g_m may be determined with reasonable accuracy from E_p–I_p curves. To understand how they can be determined in this way, refer to the E_p–I_p curves shown in Figure 4–9. On these curves, note that when

E_g changes from −4 to −8 volts with I_p remaining at 5 ma, E_p changes from 88 volts to 125 volts. Thus,

$$\mu = -\frac{88 - 125}{(-4) - (-8)}$$
$$= -\frac{-37}{4}$$
$$= 9\frac{1}{4}$$

According to a tube manual, the amplification factor for the tube for which the curves are drawn is 9. Thus, for practical purposes, the curves provide values which give results that are reasonably correct. In general, the answer you obtain from the curves will vary somewhat, depending upon the values of E_g and I_p.

By assuming a value for E_g and then comparing the change in E_p with the change in I_p, you can approximate r_p. For example, assume E_g is equal to −8. When I_p changes from 4 ma to 5 ma, E_p changes from 112 volts to 125 volts. Hence,

$$r_p = \frac{125 - 112}{.005 - .004}$$
$$= \frac{13}{.001}$$
$$= 13{,}000 \text{ ohms}$$

Mathematically, the ratio expressed by the plate resistance is the slope of the curve at a particular point. On the same curve or on different curves, it has different values at different points. In other words, plate resistance depends, to some extent, on the conditions under which the circuit is operated.

When finding the g_m, using the same E_p–I_p curves illustrated previously, assume that E_p is equal to 100 volts. Thus, when E_g changes from −4 volts to −8 volts, I_p changes from 6.4 ma to 2.8 ma (according to the curves). Therefore, by substituting in the equation for transconductance,

$$g_m = \frac{.0064 - .0028}{(-4) - (-8)}$$
$$= \frac{.0036}{4}$$
$$= .0009 \text{ mhos, or } 900 \text{ micromhos.}$$

Bias

The application of an alternating voltage between the grid and cathode of a triode tube will cause variations in the plate current. It will also cause current to flow from cathode to grid during the positive half-cycles of the voltage. This grid current consumes power and tends to introduce distortion in the plate current waveform. It is desirable that the output voltage (which appears across the plate load resistor) be, except for an increase in amplitude, a faithful reproduction of the input (grid) voltage. The flow of grid current can be avoided by using a DC voltage between grid and cathode so that the grid is always more negative than the cathode. This voltage is called *bias*.

For example, if the alternating voltage has an effective value of 3 volts, its peak value will be 4.24 volts. Under these conditions, the placing of 7.5 volts DC in series with the alternating voltage (between the grid and cathode) will prevent the flow of grid current. The grid voltage will vary from negative 3.26 volts (−7.5v + 4.24v) to negative 11.74 volts (−7.5v −4.24v). This voltage difference, called bias, is generally furnished in one of three different ways. These are identified as *fixed bias*, *cathode bias*, and *grid-leak bias*.

FIXED BIAS. This may be supplied by a battery, which is commonly called a C battery. This term goes back to the days of early radio before receivers were operated from 110-volt AC sources. The filament voltage was supplied by the A battery; the plate supply voltage was furnished by the B battery, and the bias was obtained from a C battery. Troublesome C battery replacements were eliminated in later sets by a voltage divider circuit across the plate power supply that furnished negative DC voltages for the grids of the tubes.

CATHODE BIAS. A simple solution to the problem of furnishing a negative DC voltage to the grid of the tube was found in the insertion of a resistor between the cathode of the tube and ground. The flow of plate current back to the cathode develops an IR drop across this resistor, making the cathode more positive than ground. With the grid of the tube connected to ground through either a resistor or the secondary winding of a transformer, no DC voltage can exist between grid and ground as long as no grid current is allowed to flow.

If the voltage developed across the cathode resistor is 10 volts, the cathode is therefore a positive 10 volts with respect to ground potential.

Looking at it in another way, the ground is 10 volts negative with respect to the cathode. Since the control grid is at the same DC potential as ground, the grid is a negative 10 volts with respect to the cathode. It also follows that unless an alternating voltage with a peak value of more than 10 volts is applied between grid and cathode, no grid current will flow in the circuit.

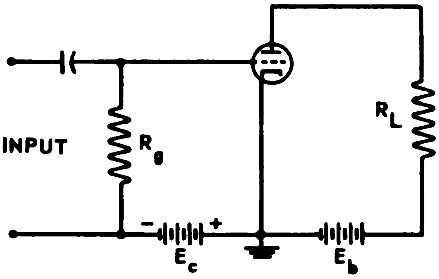
With an alternating voltage on the grid (signal voltage) of the tube, however, the voltage developed across the cathode resistor will vary in accordance with this voltage. Thus, the cathode voltage will increase in a positive direction at the same time that the alternating voltage on the grid increases in a positive direction. This effectively reduces the positive increase on the grid. When the alternating voltage on the grid goes in a negative direction, the cathode voltage decreases proportionately. This effectively decreases the negative increase in signal voltage on the grid. The net result of these variations is to decrease the amplitude of the signal voltage and thereby reduce the output of the amplifier.

A capacitor is usually connected across the resistor to keep the AC component of cathode current from varying the IR drop across the resistor. This is effectively a filter circuit used to maintain a constant voltage across the cathode resistor. Ohm's law is used to calculate the value of the cathode resistor. For example, if the desired bias is 10 volts and the cathode current is 30 ma, then

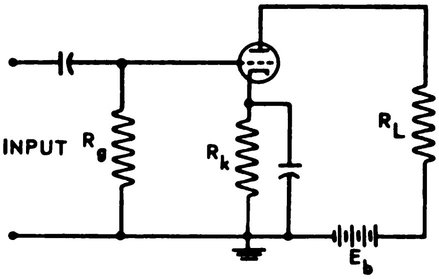
$$
\begin{aligned}
R_k &= \frac{E_g}{I_k} \\
&= \frac{10}{.03} \\
&= 333 \text{ ohms.}
\end{aligned}
$$

The reactance of the capacitor should be not more than one-tenth the value of R_k at the lowest frequency to be amplified. If this frequency is 100 cycles, then X_c should not exceed 33 ohms. To find C,

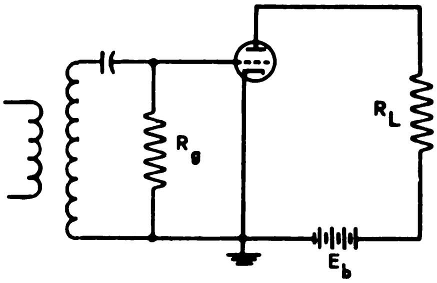
$$
\begin{aligned}
X_c &= \frac{1}{2\pi fC}\text{, or} \\
C &= \frac{1}{2\pi fX_c} \\
&= \frac{0.159}{100 \times 33} \\
&= 4.82 \times 10^{-5} \text{ farads} \\
&= 48.2\ \mu f.
\end{aligned}
$$



A FIXED BIAS



B CATHODE BIAS



C GRID LEAK BIAS

Figure 4–10. Types of Bias

Grid-Leak Bias. Grid-leak bias, which depends upon a flow of grid current on positive peaks of the signal voltage, is used where input power to the grid (called driving power) and some distortion of the output voltage is allowable. A typical circuit that uses this type of bias is illustrated in Figure 4–10C. When the grid is more positive than the cathode, there is a low resistance to current flow between cathode and grid, and the grid-leak capacitor charges rapidly through the tube. The charge on this capacitor tends to "leak" off, or neutralize itself, by flowing through resistor R_g (called the grid-leak resistor) and the secondary of the transformer. But before much of the charge can be neutralized, the alternating voltage reaches its positive peak again, and the flow of grid current recharges the capacitor.

This action is easy to understand when you picture the capacitor as charging through a low resistance, and discharging through a much higher resistance. The average charge that is maintained on the capacitor functions as DC voltage; it can be measured by connecting a DC voltmeter (which will respond to the average current flowing through it) between the grid and ground. It is significant to note that the presence of grid-leak bias is dependent upon the application of a signal voltage which will drive the grid positive with respect to the cathode. In some amplifiers, the loss of signal voltage can cause an excessive flow of plate current.

Load Lines

You can apply characteristic curves in analyzing an amplifier circuit as shown in Figure 4–11. The straight line drawn across the E_p–I_p curves is called the *load line*. It is the result of plotting the values of plate current and plate voltage i the triode circuit and connecting all of the plotte points by a straight line. One extremity of th load line is the point where the plate current i zero and the voltage drop across the load re sistor is zero. The other extremity is the poin where the plate current is absolutely unlimite by the tube — that is, the tube acts as a shor circuit. Current flow at this point is limited onl by the load resistor and is obtainable by Ohm' law. When the tube is shorted, plate voltage i zero. The slope of the load line is determine by the load resistor, hence the name *load lin* The greater the load resistance, the less the slop The load line and the E_p–I_p curves in Figur 4–11 are based on the triode circuit shown.

Here is how the line is located. If the plat current drops to zero, the voltage on the plat will be 240 volts. Thus one end of the load lin is located at a point where $I_p = 0$ and $E_p = 24($ When the tube is shorted, the plate voltage i zero and the current is the supply voltage divide

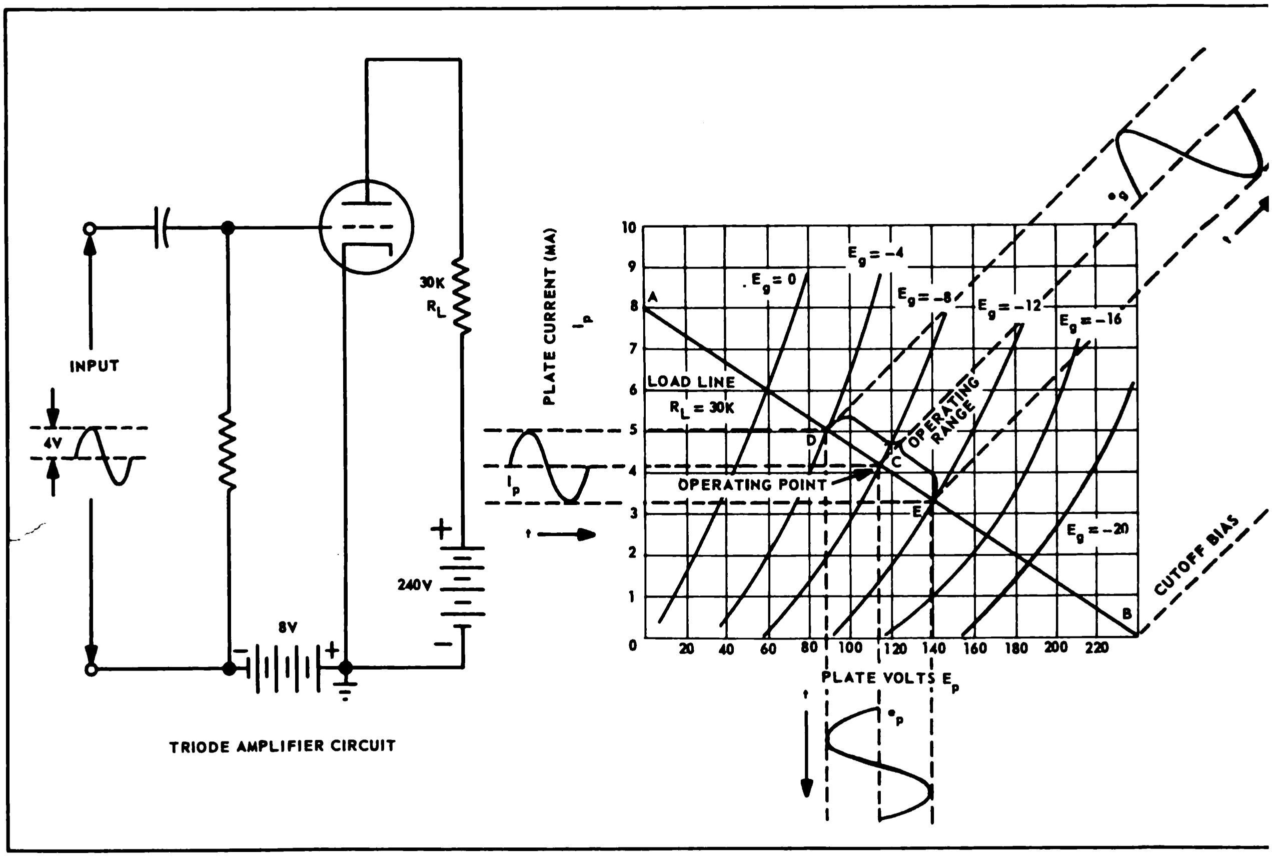


Figure 4–11. Analyzing the Operation of a Triode

by the load resistance $\frac{240}{30K}$ or 8 ma. At this point $E_p = 0$, and $I_p = 8$ ma. This locates the other end of the load line. If one of the quantities E_p, E_g, or I_p is given, you can determine the value of the other two with the load line.

In the circuit of the triode, the battery bias is 8 volts. With no signal applied to the grid, you can determine the static E_p and I_p from the intersection (C) of the —8-volt curve and the load line. The point C, called the operating point, corresponds to $I_p = 4.2$ ma and $E_p = 114$ volts. Suppose now you apply an AC voltage of 4 volts peak amplitude to the grid. Since the grid is biased at —8 volts, this 4-volt signal will cause the grid voltage to vary between —4 volts and —12 volts. The two curves $E_g = -4$ and $E_g = -12$ intersect the load line at points D and E, which correspond to $E_p = 87$ volts and 141 volts and $I_p = 5.1$ ma and 3.3 ma respectively. The portion of the load line between D and E is called the operating range, since the normal voltage variations during operation stay within these limits.

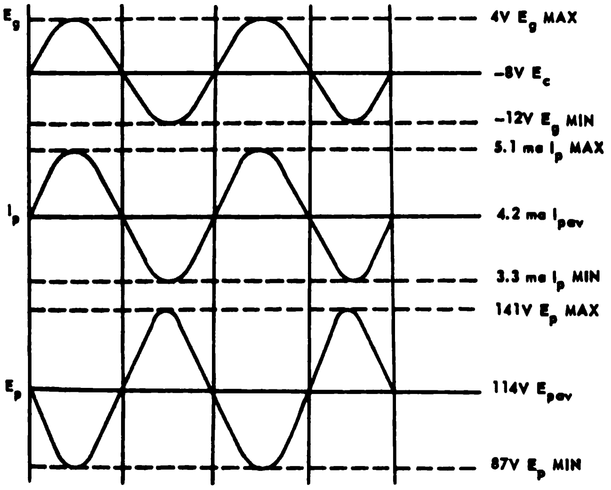


Figure 4–12. Phase Relation of E_g, I_p, and E_p

The waveshapes of the plate voltage and current, and the grid voltage of the triode curves are reproduced in Figure 4–12 in time reference to each other. They point out that the plate voltage variation is 180° out of phase with the variation in grid voltage. The grid voltage variation, e_g, is the applied AC signal. It has a peak amplitude of 4 volts. The variation in plate voltage, e_p, is the AC component of E_p and has a peak amplitude of 27 volts. The gain of the stage is 27 volts divided by 4 volts, or 6.75. The reason that the amplification of the stage is less than the amplification factor of the tube (9 in this tube) is that a voltage divider is formed by r_p and R_L, and only the voltage across R_L is usable as an output voltage. This is discussed in more detail in a later chapter.

The following problems illustrate further uses of load lines:

Problem 1

Find the value of R_k that will give 8 volts cathode bias in the circuit of Figure 4–13.

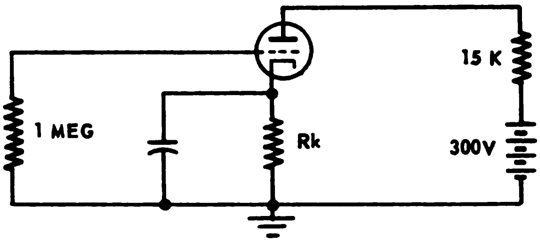


Figure 4–13. Cathode Bias

Solution

Using the family of E_p-I_p curves in Figure 4–14, locate the intersection of the E_p-I_p curve marked —8 volts and the load line. This gives point P, which is projected on the vertical axis (point S) to determine I_p, which is 5.5 ma. Determine the value of R_k by Ohm's law. Thus,

$$\begin{aligned} R_k &= E_k/I_p \\ &= 8/5.5 \times 10^{-3} \\ &= 1450 \text{ ohms.} \end{aligned}$$

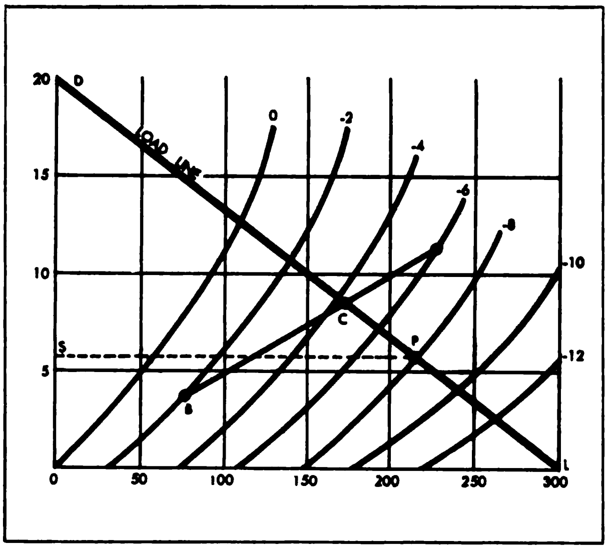


Figure 4–14. I_p-E_p Curves of Triode in Circuit

Problem 2

Using the same circuit, find the cathode bias if $R_k = 500$ ohms.

Solution

Assume that $I_p = 4$ ma; then the drop across $R_k = 500 \times 4 \times 10^{-3} = 2$ volts. Locate the point on the $E_g = -2$ curve which is opposite 4 ma on the vertical axis. This gives point B. Assume the $I_p = 12$ ma, and locate a corresponding point in a similar manner (point A). Connect these two points. This line will intersect the load line in some point C. The point C lies between the E_p-I_p curves marked -4 and -6. The approximate value is determined by proportional distance from the two curves. In this case, the value is about -4.3 volts. This means the cathode is 4.3 volts more positive than the grid.

Interelectrode Capacitance

You are familiar with the fact that two conductors separated by a dielectric form a capacitor whose capacitance depends upon the size of the conductors, the distance between them, and the dielectric. In a vacuum tube the capacitance between the electrodes, called interelectrode capacitance, connects the electrodes together capacitively, as shown in the illustration, Figure 4–15.

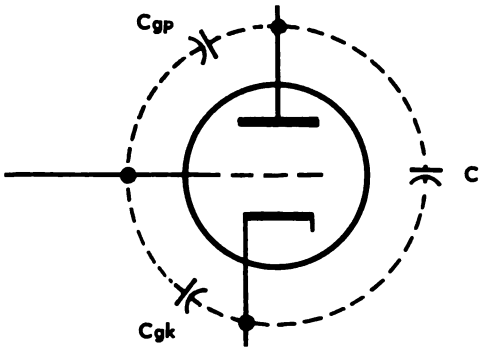


Figure 4–15. Interelectrode Capacitance of Triode

The largest capacitance usually exists between the plate and cathode because of the large area involved. C_{gk} is the grid-to-cathode capacitance, and C_{gp} is the grid-to-plate capacitance. Though these capacitances are small, being measured in micromicrofarads, they couple the elements of the tube. At low frequencies the coupling is negligible, but with an increase in frequency it becomes increasingly objectionable. The grid-to-plate capacitance has the most disturbing effect on the tube operation since it couples energy from grid circuit to plate circuit and vice versa.

TETRODES

A tetrode is a four-electrode vacuum tube. Essentially, it is a triode with the addition of a screen grid located between the control grid and the plate. Figure 4–16 gives the schematic representation of a tetrode tube. The addition of the screen grid materially reduces the grid-to-plate capacitance, making the tetrode more useful for higher frequency work than the triode. The positive voltage applied to the screen grid, usually somewhat less positive than that at the plate, changes the electrostatic field within the tube.

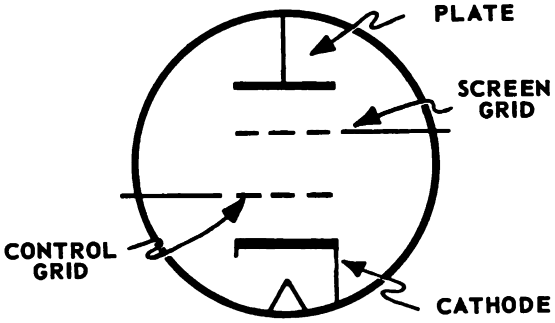


Figure 4–16. Schematic Representation of the Tetrode

As a result, the screen grid exerts a greater influence than the plate on the number of electrons drawn from the space charge. Furthermore, since the screen voltage is held constant, the varying voltage at the plate varies the electrostatic field between the screen and the plate but does not affect the electrons near the control grid. In general, increasing screen grid voltage increases plate current. While the screen grid, being positive, shares electrons with the plate, the construction of the screen grid is such that the majority of the electrons pass through and on to the plate.

The division of the electrons depends upon the voltages applied to the electrodes. As the voltage applied to the screen grid is increased, both the plate and the screen current increase, because the higher screen voltage causes more electrons to leave the space charge. The higher the screen voltage, the greater the proportion of electrons the screen takes. When the maximum number of electrons from the space charge is being obtained, the total current does not increase with higher screen voltages. The plate and the screen grid must then divide the available electrons between them. At this point, higher screen voltages increase the screen current at the expense of the plate current, and plate current decreases with an increase of screen voltage.

Secondary Emission

An effect called *secondary emission* is produced when electrons strike the plate or screen grid with sufficient force to knock other electrons from them. These electrons, thus freed, travel to the electrode exerting the greater force on them, that is, the electrode with the more positive potential, and produce a flow of current from one electrode to the other. In a diode or triode these electrons are attracted back into the plate again and have no effect on the operation of the tube. But in a tetrode, if the positive screen grid close to the source of the secondary-emission electrons has a higher potential than the plate, the electrons will tend to flow to the screen grid, causing a flow of electrons away from the plate greater than the flow of primary electrons to the plate. As the plate potential rises to the screen grid potential, the secondary-emission electrons are attracted back to the plate.

Characteristics of Tetrodes

At very low plate voltages the number of secondary electrons produced at the plate is small, and there is a tendency for space charge electrons to accumulate in a cloud in front of the plate and to turn back some of the arriving electrons. This space charge is just like the one at the cathode, but here the electrons turned back are attracted to the screen grid. Under these conditions, the plate current depends upon the plate voltage and is much less than the total space current. Figure 4–17 shows the effect of increasing plate voltage on the plate current.

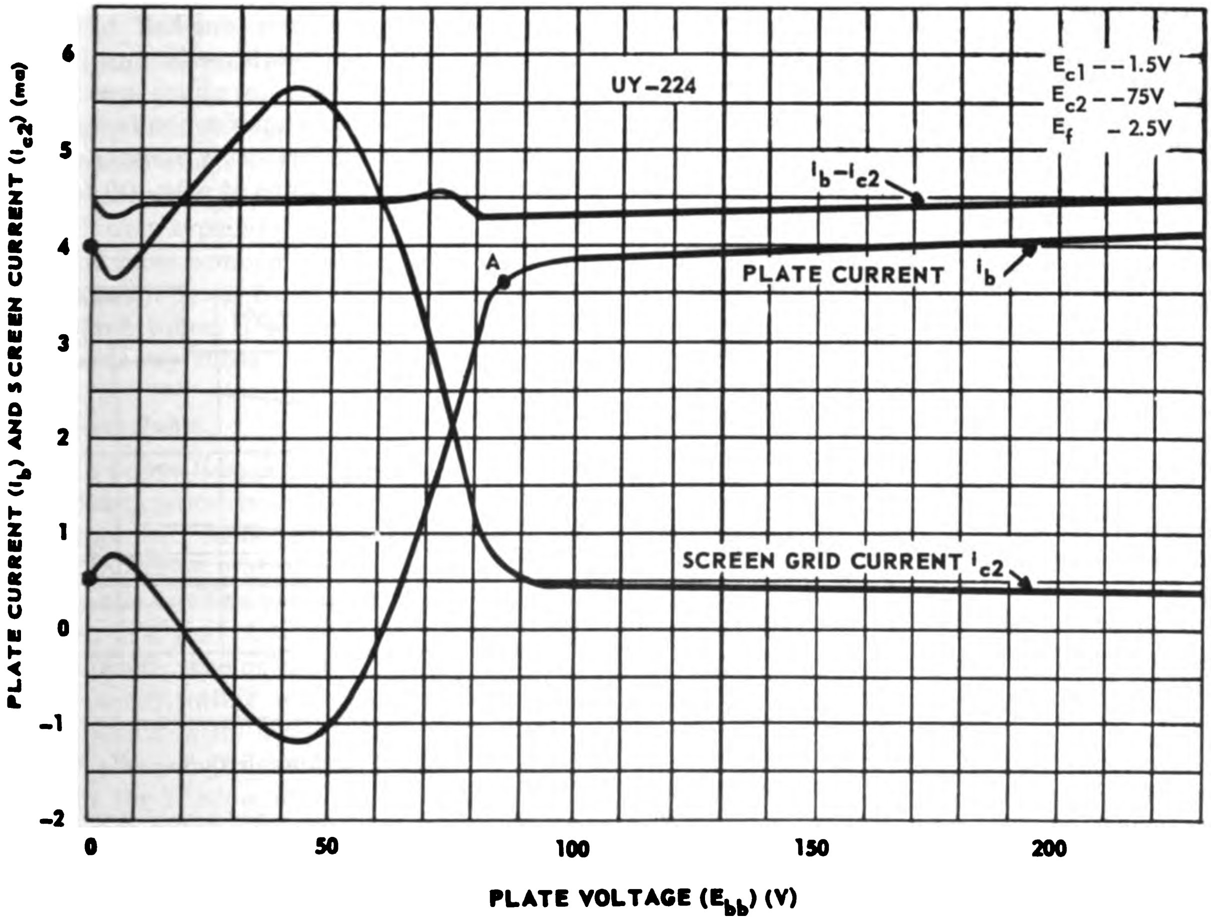


Figure 4–17. Typical I_p-E_p Tetrode Characteristics

While the number of primary electrons that the plate receives is independent of the plate voltage (dependent upon screen grid and control grid potentials), the number of electrons lost to the screen through secondary emission increases as the plate voltage increases. There is a transition between this region of decreasing plate current with increasing plate voltage and the region where the plate current becomes practically independent of the plate voltage. It occurs when the screen and plate are at approximately the same voltage.

When the plate voltage is appreciably more positive than the screen grid, the plate retains the primary electrons it receives. In addition, it receives some secondary electrons from the screen grid, although the number of such electrons is small — never enough to produce a negative screen-grid current in commercial tubes. This results in the plate current being very nearly equal to the total space current and substantially independent of the plate voltage, since the total space current depends primarily upon the potentials of the screen grid and the control grid.

To overcome the effects of secondary emission, plate voltage is not allowed to swing below a certain potential where the plate current begins to dip, except where this phenomenon is specifically being used (negative resistance or dynatron oscillator). With high plate voltage the tetrode has high plate resistance and amplification factors ranging from 500 to 800. The transconductance is about the same as that of a triode operating with similar plate current. The plate-to-grid capacitance, however, is lower than in a triode.

The high amplification factor and lower plate-to-grid capacitance make the tetrode particularly useful as an amplifier of radio frequency voltages.

PENTODES

To overcome the effects of secondary emission, there are tubes in which a third grid, called the *suppressor grid*, is added between the screen grid and the plate. This type of tube is known as a *pentode*, or five-electrode tube. The suppressor grid is usually connected directly to the cathode or to ground. In addition to repelling secondary electrons to the plate, it provides additional shielding within the tube. Note that secondary emission is not eliminated, but is merely prevented from reaching any point where it will interfere with the operation of the tube. The suppressor grid does not materially affect the flow of primary electrons to the plate since their flow depends almost entirely upon the screen grid and control grid voltages.

Although the screen grid in either the tetrode or pentode greatly reduces the effect of the plate voltage upon plate current flow, the control grid, because of its closeness to the cathode in the space charge region, still is able to control the total space current flow and, consequently, the plate current. For this reason the transconductance of a pentode (or a tetrode) is of approximately the same value as that of a triode of similar structure.

On the other hand, since the plate voltage has relatively little effect upon the plate current flow in a pentode, the values of r_p and μ are very high. This is obvious if you remember the definitions of these tube constants. The amplification factor of pentodes varies from 100 to 5200, and the plate resistance from one-half to over one megohm. The high plate resistance makes it difficult for a pentode to utilize much of the advantage of its high amplification factor. Therefore, pentode amplifiers using resistance-capacitance coupling have gains of only 100 to 200.

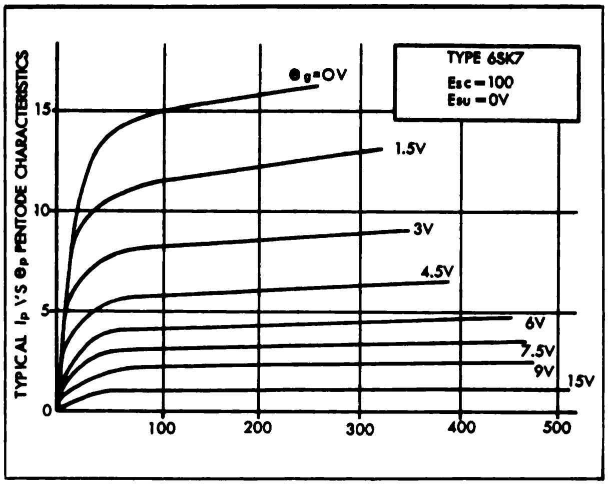


Figure 4–18. Characteristic Curves of a Pentode

Pentode Characteristics

A typical set of characteristic curves is shown in Figure 4–18. The fact that the plate voltage is relatively ineffective in controlling the plate current is shown by the small slope of the curves beyond the point where the plate voltage is high enough to keep the electrons in the space between screen grid and plate from bing attracted back to the screen grid. The plate voltage at which this occurs is less than the screen grid potential, since the electrons enter the space with sufficient velocity to cause them to travel on to the plate in spite of the higher screen grid voltage.

Uses

Pentodes are used mostly as radio frequency voltage amplifiers. They are also suitable as power amplifiers, having greater power sensitivity than triodes. Power sensitivity is the ratio of output power to AC grid voltage. For a triode to develop the same power output as a pentode, the amplification factor of the triode has to be quite low and, because of this low amplification factor, a larger input voltage is required to obtain power output comparable to that of a pentode.

SPECIAL PURPOSE TUBES

In addition to the types of tubes discussed thus far, there are tubes of special design intended to give optimum performance in a particular application or to combine in one envelope functions normally requiring two or more tubes. Tubes of this type are called *special purpose tubes* The most commonly used special purpose tubes are beam power tubes, remote cutoff tubes, multipurpose tubes, UHF tubes, gas-filled tubes, and cathode-ray tubes.

Beam Power Tube

A beam power tube is a special type of tetrode with characteristics similar to those of a pentode power amplifier. Although a beam power tube has no suppressor grid, the effect of one is obtained by the creation of a space charge between the screen grid and plate. This is accomplished by appropriate spacing of the screen and plate and by concentrating electron flow into well-defined beams with beam-forming plates and specially aligned grid wires. Thus, the screen grid is in the shadow of the control grid, and electrons can pass between them without changing direction. This creates a space charge that causes secondary electrons from the plate to be repelled to it without adversely affecting the normal flow of electrons from the cathode.

The characteristic curves for beam power tubes differ somewhat from pentode curves. The transition between the region where the plate current is practically independent of the plate voltage and the one where the plate current is controlled by plate voltage is more abrupt than in the pentode, and occurs at a lower value of plate voltage.

Remote Cutoff Tube

A remote cutoff tube is a pentode voltage amplifier tube constructed in such a way that space current decreases gradually at highly negative control grid voltages rather than quickly at a well defined cutoff point. (See Figure 4–19.)

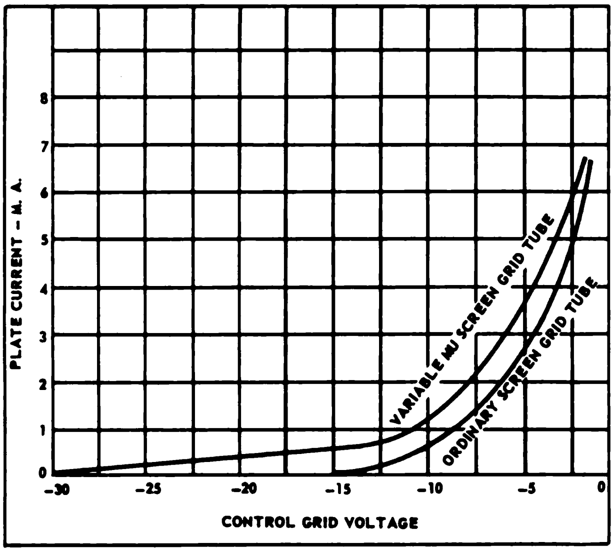


Figure 4–19. Variable–Mu Tube vs. Ordinary Tube

Other names for remote cutoff tubes are variable-mu and super-control tubes. The control grid structure is not spaced uniformly; the windings are close together at the ends and more widely spaced at the center. This gives different amplification factors and cutoff voltages at different parts of the grid structure.

This means that when the grid goes negative enough to cause current through part of the grid to cease, it will still flow through some other portion which requires a more negative voltage to cut off the tube. For this reason, the plate current flow tapers off slowly and requires a much more negative voltage than an ordinary tube for the plate current to reach zero. Remote cutoff tubes are used in circuits with automatic volume control and in circuits in which the gain is controlled by varying the bias on one or more tubes.

Multipurpose Tubes

A multipurpose tube is one which contains two or more tubes in the same envelope. Frequently you will find two diodes or two triodes located in the same envelope. Such tubes are known as duplex diodes or twin triodes. Some other multipurpose tubes in common use are duplex-diode-triodes, twin pentodes, and diode-heptode tubes.

UHF Tubes

UHF (ultra high frequency) tubes are especially constructed for operation at higher frequencies, where it is necessary to reduce interelectrode capacitances and to cut down transit time (the time electrons require to travel from the cathode to the plate). This is done by making the electrodes quite small and spacing them very closely together. Because of these construction features, the power handling ability of this type of tube is somewhat less than that of tubes used at lower frequencies.

In UHF tubes there is frequently no tube base. Connections to the electrodes are made through pins which protrude through the envelope in a way that keeps the leads short and minimizes capacitance between them. Three special types of UHF tubes, the *acorn tube*, the *doorknob tube*, and the *lighthouse tube*, are so named because of their shapes and sizes. Acorn and doorknob tubes are available in diodes, triodes, and pentodes. The lighthouse tube is designed to fit directly into the end of the concentric tubing used to form the tuned or tank circuit in UHF circuits. By directly connecting the tube in this manner, losses due to connecting wires are eliminated.

Gas-Filled Tubes

Although the presence of gas in a high vacuum tube is objectionable, there are special purpose tubes which are gas filled. Common gases for this purpose are neon, nitrogen, argon, and mercury vapor. The gas-filled tube is characterized by the fact that the voltages applied to the electrodes have little effect on the current flow once they reach a certain minimum value. This is because the gas becomes *ionized* — that is, the molecules of gas lose electrons through collision with other electrons in the interelectrode space and become positively charged ions which drift toward the cathode and attract more electrons out of the cathode, thus increasing the current flow.

There is a certain minimum voltage difference between plate and cathode that will cause the electrons to move with sufficient velocity to produce ionization. This is called the ionization potential or firing voltage. Once the tube is ionized, the plate voltage can be reduced somewhat without affecting the flow of electrons. There is a minimum value of plate voltage necessary to maintain ionization. This voltage is called the extinguishing or de-ionizing voltage. Refer to Figure 4–20, which shows the characteristics of a VR 105–30.

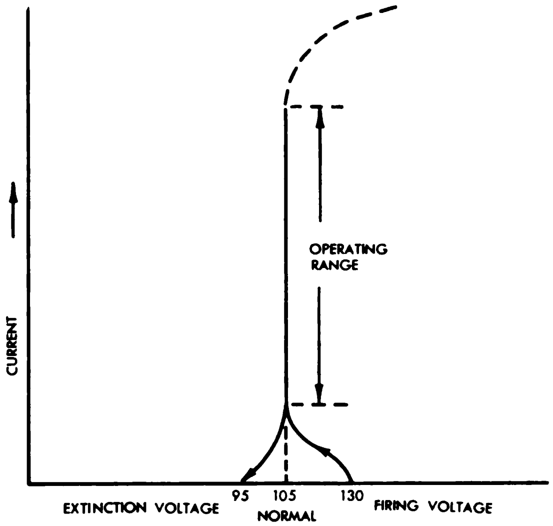


Figure 4–20. Characteristics of Gas-Filled Diode VR 105–30

When a gas-filled tube conducts current, it behaves like a low resistance that varies in value in such a way that the voltage developed across it remains practically constant over a moderately wide range of current. This characteristic makes gas-filled tubes useful as voltage regulator tubes, mercury vapor rectifiers, protective devices, and thyratrons.

A *voltage regulator* tube is a tube especially designed to keep the voltage at a practically constant value. An example of a voltage regulator tube is VR 105–30. The first number in the tube designation indicates the value of the constant voltage that the tube maintains; the second number is the current limit for desired regulation. Between 5 and 30 ma, the output will vary no more than one volt. If the current exceeds 40 ma, the tube will no longer regulate and is likely to be damaged. The use of the voltage regulator tube is described later in this chapter in voltage regulator circuits.

Another type of gas-filled tube in wide use is the *mercury vapor* tube. This tube contains a small amount of mercury which vaporizes when the cathode is heated and which ionizes when plate voltage is applied. The plate-to-cathode voltage drop in the mercury vapor tube is practically constant at 15 volts regardless of the amount of plate current flow. The tube can withstand reasonably high inverse voltages. Mercury vapor diodes are widely used as rectifiers in power supplies where considerable current is drawn.

A *thyratron* is a gas-filled triode or tetrode in which the grid controls the ionization voltage. A sufficient negative voltage applied to the grid, with plate voltage removed, will prevent electron flow after plate voltage is applied. If grid voltage is gradually made less negative with plate voltage applied, a point will be reached where electron flow will suddenly begin. The value of the grid potential is called *starting voltage.* Tube 884, a triode, is a widely used thyratron. Figure 4–21 shows its characteristics.

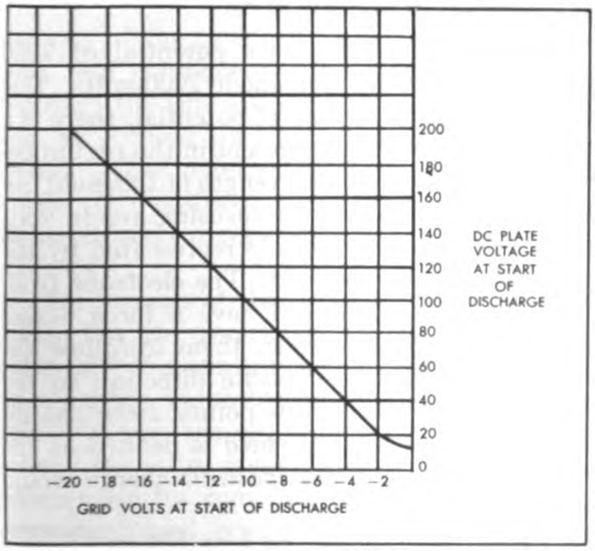


Figure 4–21. Characteristics of a Thyratron

Note the plate voltages necessary to produce ionization for various values of grid voltage. The difference in the nature of the grid control indicates that the fields of use of the high vacuum tube and the thyratron do not, in the main, overlap. In industry this tube has many applications. It is essentially an electronic switch with an internal impedance that is very high (switch open) or very low (switch closed), although it cannot be opened by the same method by which it was closed (the grid). In radar its principal use is as a sawtooth generator to provide sweep voltages for cathode-ray tubes.

Gas-filled diodes are also used in radar equipment as protective devices to keep receivers from being overloaded during transmission. The protective tube and circuit in radar equipment is called the T–R box.

Cathode-Ray Tube

The *cathode-ray tube* (CRT) is a tube especially designed to measure or display electrical phenomena which cannot be studied by other means. Not only is the CRT the heart of the oscilloscope, but it is widely used in radar and air defense equipment to display visually information obtained from a transmitter or a receiver.

The CRT consists of four essential parts: an evacuated glass envelope, an electron gun for producing a stream of electrons, a means of deflecting the electron stream, and a screen which transforms the electrical energy of the electron stream into light. The working parts of the CRT are enclosed in a glass envelope having a high vacuum in order to permit the beam to be sharp and to preserve the heater.

Commercial tubes are of various sizes, having screens varying from one inch in diameter, for use in radio servicing, to giant tubes of 20 or more inches in diameter. CRT's for most airborne radar equipment are 3, 4, or 5 inches in diameter. Ground equipment has CRT's 12 inches in diameter. Construction of the glass envelope is influenced by the effects of atmospheric pressure. On the face of a five-inch tube the pressure is almost 300 pounds. In most equipment there is a transparent shield between the operator and the face of the CRT. The screen serves to protect the operator and also to provide a place for cutting calibration marks.

The electron gun, which virtually shoots electrons at the screen in the CRT, consists of a

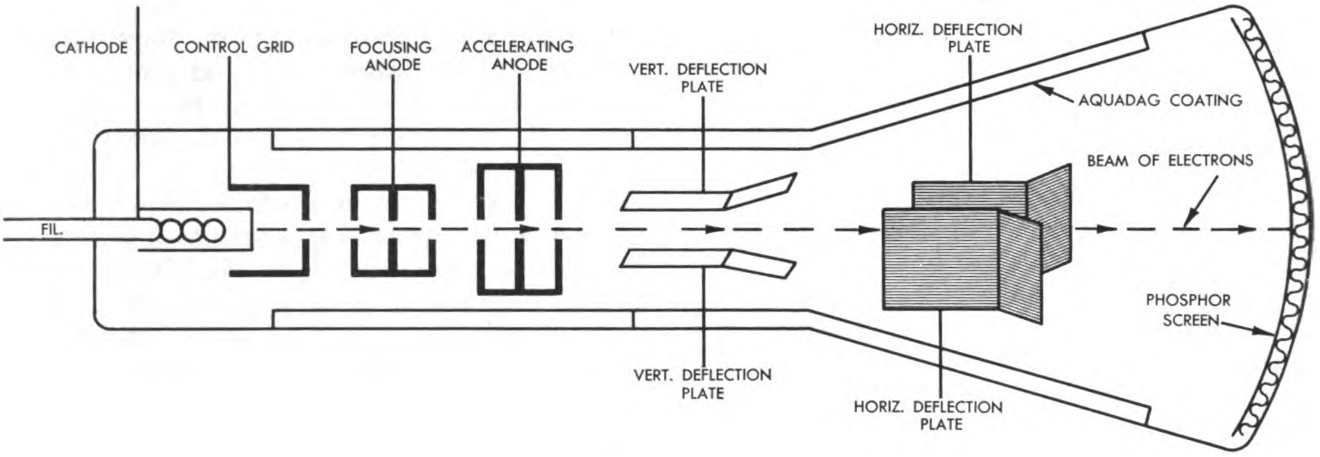


Figure 4–22. Construction of Cathode-Ray Tube

heater, a cathode, a control grid, a focusing anode, and an accelerating anode. In Figure 4–22, note these parts as well as their relative positions within the tube. Connections to the various electrodes are brought out through pins in the base of the tube rather than as shown. In many tubes the cathode is connected internally to the heater. The heater wire is heated by an alternating current furnished by a separate transformer. The cathode is a nickel cylinder, the end of which is coated with oxides of barium and strontium so that it emits electrons freely in the desired direction.

The accelerating anode is a cylinder. Inside it is a diaphragm which has a small opening in its center. Since the accelerating anode is highly positive, it will attract the electrons emitted by the cathode. The voltages applied to the accelerating anode range from 250 volts to as high as 10,000 volts. Three- to five-inch tubes commonly use 1500 to 2000 volts. This high voltage causes the electrons to travel at a high velocity, which is proportional to the square root of the voltage difference and may be of the order of 10,000 miles per second for a 1000-volt potential difference. While the major portion of the electrons in the beam are attracted to and captured by the accelerating anode, many pass through the opening in the diaphragm and proceed on to the screen.

The grid is cylindrical in shape and partially surrounds the cathode. It serves to control the space current flowing from the cathode to the other electrodes, as in other vacuum tubes. The control grid is operated at a more negative potential than the cathode. If made negative enough, it will cut off the flow of electrons. As stated earlier in the discussion of the oscilloscope in the preceding chapter, the grid voltage control is called the *intensity control.* In radar equipment it has the same name.

You can see that if some means is not provided for focusing, the electrons leaving the cathode will be emitted in a cone and, because of mutual repulsion, will be still further diffused and strike the screen in a scattered mass having a fuzzy or feathered appearance. The focusing and accelerating anodes make up an electronic *lens,* which concentrates the electron beam and makes the image on the screen clear cut. To understand how this focusing action takes place, refer to Figure 4–23.

The focusing anode has a potential of 1200 volts and the accelerating anode 2000 volts. Due to the 800 volts difference of potential, there is a strong electrostatic field present in the region between these anodes. The strength of this field can be varied by changing the focusing anode voltage. The electrostatic field is represented by the curved lines in Figure 4–23. The electrons passing through this field will have a force acting upon them tending to cause them to follow the lines of force (in the opposite direction to the one indicated by the arrow points, since the direction of an electrostatic field is defined as the direction in which a unit *positive* charge would be acted upon).

An electron entering the lens has two forces acting upon it, one force being from the acceleration given it by the attraction of the accelerating

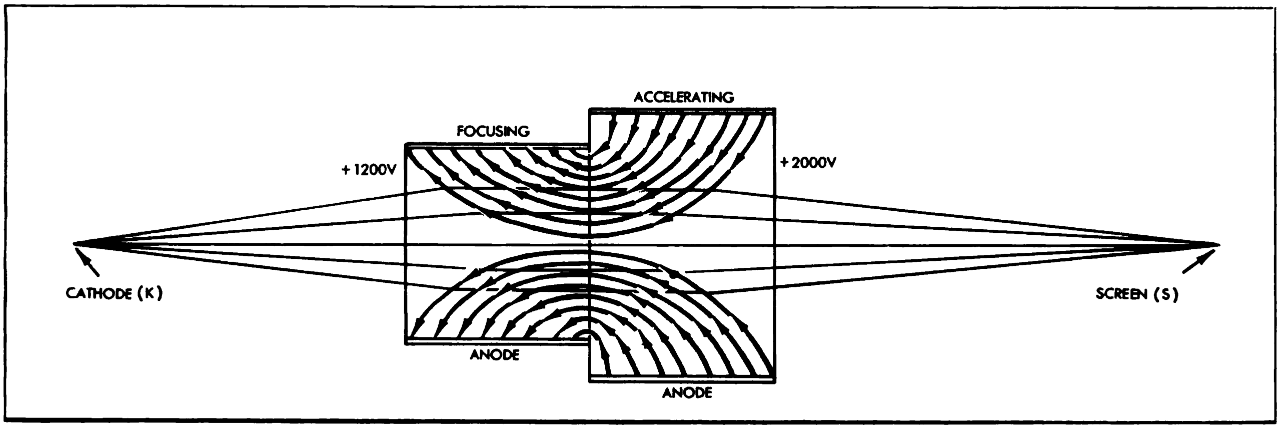


Figure 4–23. Focusing Action

anode, and the other force being created by the electrostatic field between the two anodes. The tendency is for the electron to be deflected and, instead of traveling in a straight line, to move in a direction tangent to the lines of force. The amount of deflection from its original path depends upon its velocity and the relative curvature of the lines of force. This curvature, in turn, depends upon the difference in potential between the anodes.

All electrons passing through the lens at a given angle tend to come together at a point called the focal point. This focus is comparable to the focus of a camera lens. You can see now why it is desirable to use small openings in diaphragms in the cylindrical elements. This insures that only those electrons which leave the cathode (K) at a small angle from the longitudinal axis of the tube will go through these openings, and consequently the stream of electrons reaching the lens will not be widely diffused and can be focused into a well defined beam. By changing the potential applied to the focusing anode, the position of the focal point can be changed, and correct adjustment will occur at the screen (S).

The purpose of the screen in the CRT is to make the electron beam visible. The screen, composed of a semitransparent substance known as *phosphor*, is located on the inside face of the tube. When the electron stream strikes the screen, the latter gives off light, whose color depends upon the composition of the phosphor. A commonly used screen coating for commercial oscilloscopes is Willemite zinc silicate, which gives off a green light. Other screen coatings are phosphorescent zinc sulphide, which produces yellow light, and calcium sulphide, which produces a white light shaded with green.

An important consideration in the choice of phosphors is its persistency. Persistency determines the length of time that the screen will continue to glow after the beam of electrons has been removed or moved to a new location. If the screen has a short, or low, persistency, this period of time is 0.1-second or less; if the persistency is long, or high, the period is 1 second or longer. Between these two limits the persistency is medium. The Willemite screen has low persistency. Those used in radar may be any of the types given, depending upon the use of the equipment.

You can readily see that some means must be provided for removing the electrons from the screen; otherwise the negative charge on the screen would build up to such a point that no more electrons could reach it. The usual method for removing the electrons is to place a conducting coating of aquadag, a graphite coating, covering all of the inside of the tube except the face, and to connect the coating to the cathode. Secondary emission from the screen, which is sufficient to prevent serious effects from a negative voltage there, is then collected by the aquadag coating and returned to the cathode.

The remaining part of the CRT is a means of deflecting the electron beam. The two types of deflection are *electrostatic* and *electromagnetic*. The type of tube shown in Figure 4–22 is designed for electrostatic deflection. Two pairs of deflection plates set at right angles to each other are used, and the electron beam passes between them. The plates are called the horizontal and vertical deflection plates according to the direction in which they deflect the beam, and *not* according to their positions in the tube.

Suppose the upper vertical deflection plate has a positive DC potential applied to it, and that the lower vertical plate has a negative DC potential applied to it. The electrons in the beam will then be attracted by the upper plate and repelled by the lower plate, with the result that the electron beam is deflected upward. If the lower plate were made positive and the upper plate negative, the beam would be deflected downward. If an AC voltage were applied to the plates, the beam would be deflected up and down alternately at the frequency of this voltage. This would cause a vertical line to appear on the face of the scope unless the frequency were very low. The amount of deflection depends upon the amplitudes of the voltages applied to the plates.

The deflection sensitivity of a CRT is the amount of deflection of the spot on the screen for a given voltage on the deflection plate. Deflection sensitivity is measured in millimeters per volt. Deflection factor, a more practical value, is measured in volts per millimeter. The deflection sensitivity is commonly about 50 volts per inch, which means that a positive voltage of 50 volts applied to the upper plate will cause the spot to be deflected upward one inch, or that an AC voltage which has a peak amplitude of 50 volts will, if applied to the vertical deflection plates, cause a deflection of two inches (one inch up and one down).

The second set of plates, called the horizontal deflection plates, functions in the same way as the vertical plates. A positive voltage applied to the right horizontal plate will move the beam to the right; one applied to the left plate will move it to the left, and so on. You have already seen in the preceding chapter how a sawtooth voltage applied to the horizontal plates gives a horizontal trace which can be used as a linear time base.

The deflection sensitivity of a tube is affected by several factors. First, the distance between the plates and the screen determines the swing of the beam for a given deflection angle. When the screen is farthest from the plates, the arc subtended by the deflection angle is greatest and the tube is most sensitive. When the vertical plates are farther from the screen than the horizontal plates, the sensitivity is greater in the vertical direction.

A second factor affecting the deflection sensitivity is the voltage applied to the final anode. The speed of the electrons in the beam is determined by the anode voltage. The faster the electrons travel, the greater the force required to deflect them and the shorter the time they will need to pass between the plates. Hence, higher voltages reduce the sensitivity.

Since both of the factors mentioned here are more or less fixed in any given tube, it is possible to state the deflection sensitivity in both directions at the time of manufacture.

Electromagnetic deflection is used in many cases where it is impossible to get a suitable electrostatic deflecting voltage. In electromagnetic deflection, the deflecting force is due to the magnetic field set up within the tube by the set of coils arranged around the neck of the tube. One of the big advantages of this kind of deflection is simpler tube construction, as there are no deflection plates within the tube. The sensitivity

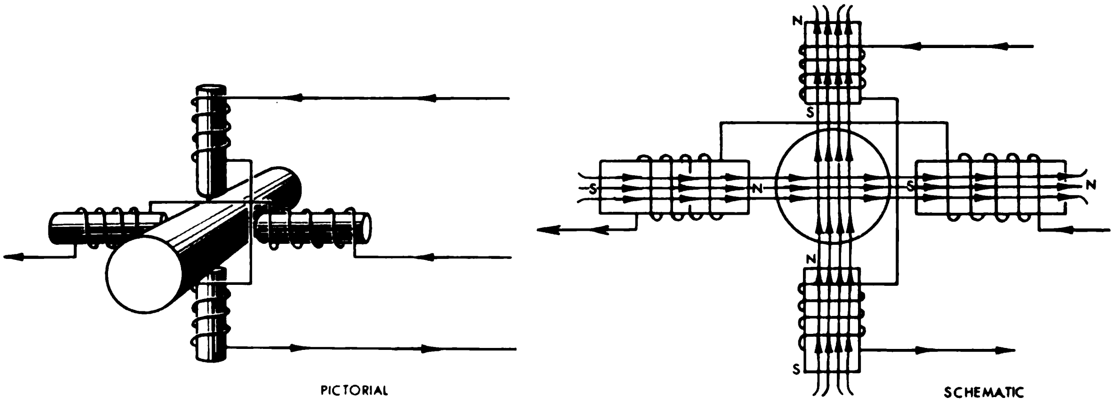


Figure 4–24. Electromagnetic Deflection

is also less affected by an increase in anode voltage. Focusing can also be done electromagnetically, which still further simplifies the tube construction.

If the two sets of coils are placed at right angles (see Figure 4–24), the two magnetic fields are perpendicular to each other. The strength of each field is proportional to the current flowing through the set of coils producing it. You are familiar with the fact that a current-carrying conductor located in a magnetic field has a force exerted upon it which is at right angles to the field and also at right angles to the direction of electron flow.

The stream of electrons is affected in the same way; as the electron stream enters the field, it is deflected in a direction perpendicular to the field in an amount proportional to the field strength. Deflection in a vertical direction is produced by the coils located on either side of the tube, and deflection in a horizontal direction, by the coils above and below the neck of the tube. Deflection sensitivity is usually expressed in inches per ampere turns.

Electromagnetic focusing is obtained by winding a coil around the neck of the tube in such a way that the magnetic field is lengthwise with the tube. This means that electrons traveling down the axis of the tube are unaffected, but those traveling at a small angle have a force applied to them perpendicular to the magnetic field, tending to bring the electrons back into a line of travel that focuses the beam at the screen. Electromagnetic deflection and focusing are widely used in radar equipment, but only occasionally in oscilloscopes.

Magnetic deflection has a disadvantage in that positive ions, which travel slower than electrons, are not fully affected by the deflecting field and will always strike the center portion of the screen. This tends to damage the fluorescent material and cause a dark spot. In addition, satisfactory deflection is not produced at higher frequencies because of insufficient field intensity.

Typical circuits used in conjunction with a CRT are shown in Figure 4–25. Although there are more complicated applications, all use essentially the same principle. Voltages for operation of the CRT illustrated are obtained from two rectifier circuits, one called the low-voltage rectifier and the other the high-voltage rectifier. The output voltages of these rectifiers are approximately 420 volts positive and 780 volts negative, respectively. The cathode voltage is fixed at negative 720 volts. The grid voltage is varied between negative 720 volts and negative 780 volts by a 50 K potentiometer that acts as an *intensity control.*

The position of the movable tap on the 250-kilohm potentiometer, labeled *focus*, controls the

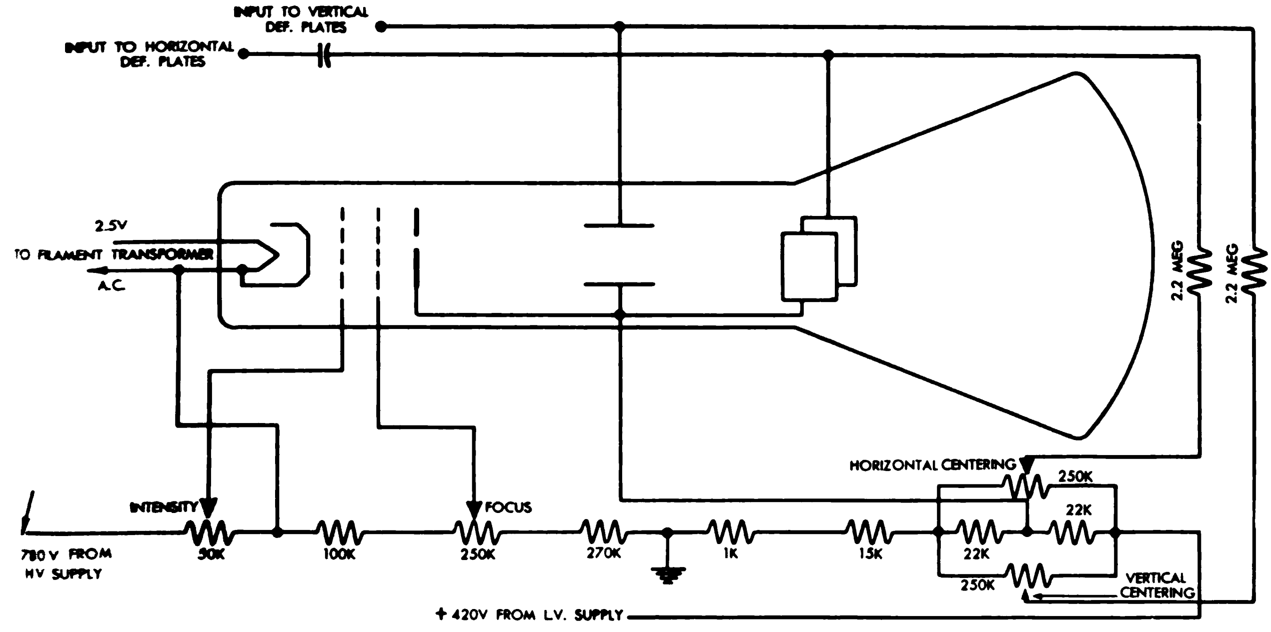


Figure 4–25. Typical CRT Circuits

focusing anode voltage. The focus control varies the anode voltage between 320 volts negative and 590 volts negative. The accelerating anode is kept at a fixed value of approximately 275 volts. The voltage between the accelerating anode and the focusing anode ranges from 595 volts to 865 volts, giving ample control over the focusing action.

The lower vertical deflection plate and the left-hand horizontal deflection plate are connected internally to the accelerating anode and are maintained, therefore, at a voltage of 275 volts positive. The movable tap on the 250 K potentiometer labeled *vertical centering* gives voltages between 120 volts and 415 volts. This range is from 155 volts below to 140 volts above the voltage of the lower plate. This makes it possible to center the beam vertically at any spot on the face of the CRT. The right-hand horizontal deflection plate is connected to a similar potentiometer marked *horizontal centering* which functions in a like manner for positioning the spot horizontally.

The sweep voltage is applied to the right-hand horizontal plate through the coupling capacitor (at the top of the diagram). This voltage may come from the sawtooth generator in the oscilloscope through the horizontal amplifier or from an external sweep circuit either through the horizontal amplifier or directly to the plate. The signal input to the upper vertical plate is applied through the coupling capacitors and may come through the vertical amplifier or be applied directly to the plate. The two 2.2-megohm resistors serve as decoupling resistors to keep the signals from being attenuated in the voltage divider system.

POWER SOURCES FOR OPERATING VACUUM TUBES

As previously pointed out, vacuum tubes require voltages of various values for their heaters screens, and plate circuits. Except for the heaters, which can be heated by either DC or AC these voltages are DC. The most convenien source of DC voltage for vacuum tubes is a rectifier. A rectifier is a device which changes AC into DC by permitting current either to flow i only one direction or to flow more readily in on direction than in the opposite. An ideal rectifie is one that offers zero impedance to the flow o current in one direction and infinite impedanc to the flow of current in the opposite direction Devices commonly used for changing AC int DC for use in radio and radar circuits are electron tube, metallic-oxide, and crystal rectifiers which will be explained later.

Electron Tube Rectifier

The rectifier tube is a diode type tube. Rectifier tubes with only one plate and one cathod are called *half-wave rectifiers* since current flow only during one-half of the AC cycle. When tw plates and one or more cathodes are used in th same tube, current can flow on both halves o the AC cycle. Tubes of this type are called *full wave rectifiers.*

HALF-WAVE RECTIFIER. A half-wave rectifie (Figure 4–26) conducts only during the half o the input cycle that makes the plate more positive than the cathode. The output voltage is reproduction of the positive alternation of th voltage across the secondary of the input trans

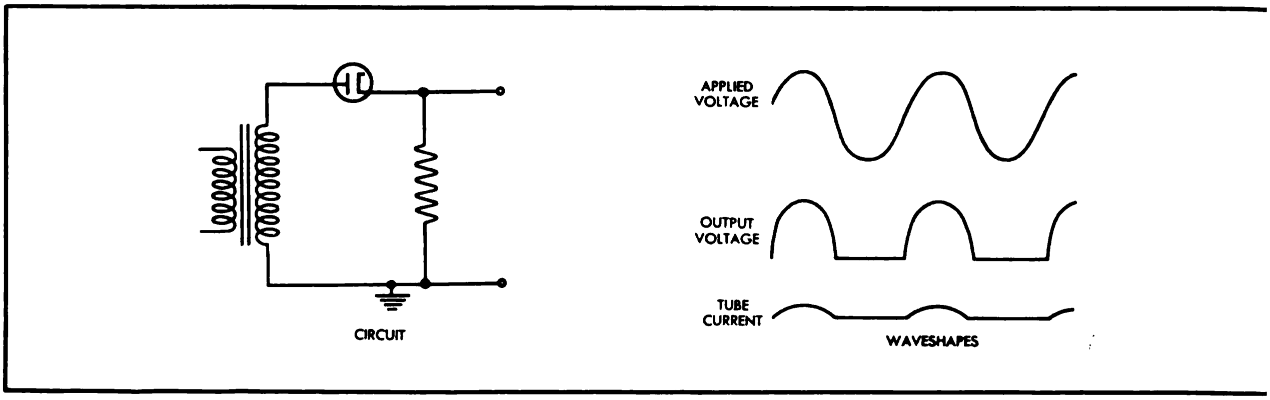


Figure 4–26. Half-Wave Rectifier

former except for a low voltage drop across the tube, which acts as a low resistance when conducting. The output voltage is a pulsating positive voltage that has an average value equal to one-half of the average value of the secondary voltage, or 0.318 of the peak value of the secondary voltage.

The efficiency of a half-wave rectifier is low. The voltage drops to a low value if a load requiring much current is connected to the tube. Hence, its use is limited to places where voltage is the primary concern and current drain is low. This makes it a suitable power supply circuit for anode (plate) voltages in the oscilloscope. The half-wave rectifier has a serious defect when used in circuits requiring appreciable current. Current through the secondary is in one direction only. This tends to cause saturation of the core with residual magnetism.

Full-Wave Rectifier. A full-wave rectifier conducts on both halves of the input voltage in such a way that current always flows through the load resistor in the same direction. Note in the illustration of the full-wave rectifier in Figure 4–27 that the secondary is grounded at the center, and that the opposite ends are connected to the plates of two diodes. These two diodes conduct alternately, since at any given instant one plate is positive and the other negative, while a half-cycle later the voltages are reversed. As a result, both halves of the input are reproduced in the output. The output voltage has an average value of twice that of a half-wave rectifier with the same input. This makes the full-wave rectifier a more efficient power source than a half-wave rectifier. It is capable of standing a higher current drain without excessive drop in voltage than a half-wave.

The tube used must be capable of withstanding a voltage twice the peak of the secondary voltage applied to each tube. This is the voltage applied between plate and cathode, with cathode more positive than the plate, when the tube is not conducting. When V_1 is conducting, its cathode is positive almost to the peak value of the secondary voltages and the cathode of V_2 is at the same potential. But the plate of V_2 is very negative at the same instant so that the cathode-to-plate potential is approximately twice the peak of the secondary voltage applied to each tube. This is called the inverse peak voltage and must be taken into consideration when designing a full-wave rectifier circuit. The peak inverse voltage rating is the maximum voltage that can be applied across a tube in a negative, or inverse, direction without damage.

Filter Circuits

The output voltages in the rectifier circuits just discussed are not suitable for supplying DC to most vacuum tubes because of their pulsating voltage output. The variations in the direct voltage are known as *ripple* voltage. Ripple voltage

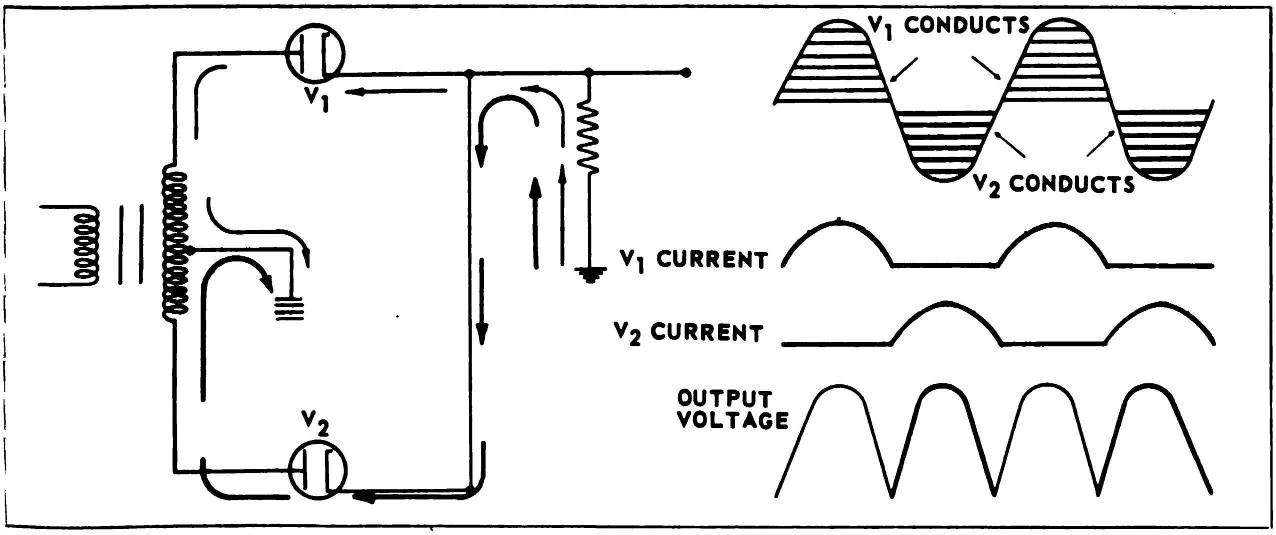


Figure 4–27. Full-Wave Rectifier

may be thought of as an alternating voltage superimposed upon direct voltage. Actually, the ripple voltage is composed of a fundamental frequency and a number of harmonic frequencies (harmonic frequencies are multiples of the fundamental frequency). The fundamental frequency is equal to that of the input voltage if the rectifier is half-wave, or twice the input frequency if the rectifier is full-wave. The amplitude (magnitude) of the fundamental component is greater than that of a harmonic. For power purposes the ripple voltage must be reduced to a low percent of the DC output voltage. A device for eliminating ripple is called a filter.

FILTER. A filter usually consists of capacitors connected in parallel and inductors connected in series with the load. A capacitor opposes any change in voltage across its terminals (across the load) by storing up energy in its electrostatic field whenever the voltage tends to rise, and returning the stored energy to the circuit whenever the load voltage tends to decrease. An inductor opposes a change in current through it (through the load) by storing up energy in its electromagnetic field when the current tends to increase and by releasing energy from the magnetic field to maintain current flow when it tends to decrease.

Another way of looking at the action of a filter circuit is to consider that the capacitor forms a low impedance path to ground for the ripple voltage and offers a very high impedance to the DC, whereas the inductor offers a low impedance to DC and a very high impedance to the ripple voltage. Either way you look at it, it leads to the conclusion that there is a reduction of the amplitude of the ripple voltage without seriously affecting the DC voltage.

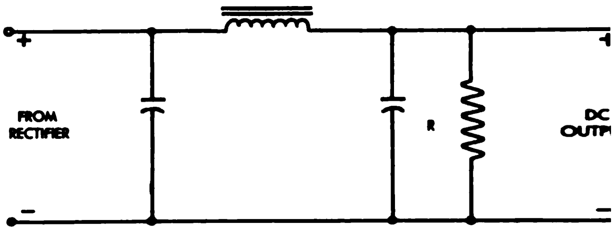
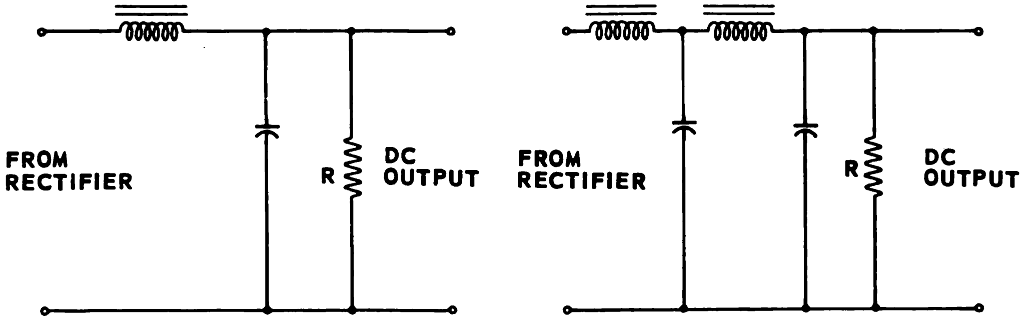


Figure 4–28. Capacitive Input Filter

CAPACITIVE INPUT FILTER. One type of filte Figure 4–28, is the capacitive-input, Pi-type filte A capacitive-input filter is characterized by hig output voltage at low current drain, the voltag being almost equal to the peak value of the sec ondary voltage applied to each tube. Howeve the voltage falls rather rapidly as the curren drain increases, reaching as low as 90% of th rms value when the full rated current is drawn This fall of voltage under increased load con ditions is referred to as poor regulation. A capaci tive-input filter gives poor regulation, because a the load increases the capacitor is able to dis charge more rapidly between cycles. In additio to decreasing the output voltage, this tends t increase the amplitude of the ripple voltage.

An important feature of this filter is the bleede resistor, which is used chiefly to discharge th capacitors when the equipment is turned off. A bleeder resistor should draw 10% or less of th rated current output of the power supply.

INDUCTIVE INPUT FILTER. Another type o filter, Figure 4–29, is the inductive-input (choke input) filter. The inductive-input filter is use wherever we must draw a heavy load, or whereve good regulation is demanded. With the same AC input voltage, its output voltage is lower than that of the capacitive-input filter. When the tub



SINGLE SECTION

TWO SECTION

Figure 4–29. Inductive Input Filter

draws no current, an inductive-input filter holds the voltage at practically the same value as a capacitive-input filter. When the tube draws a small amount of current, the voltage drops rapidly to a value that remains fairly constant over a wide range of current drains. If the bleeder resistor is small enough to insure drawing sufficient current to operate at the beginning of the linear portion of the voltage-current curve, this type of filter provides very satisfactory results under changing load conditions.

There are many other types of filters, but the action of each is very similar. Sometimes the inductor is replaced by a less expensive resistor, but a resistor has the disadvantage of offering as much impedance to DC as to the ripple voltage. In some applications, resonant circuits are used as filters. A series resonant circuit in parallel with the load and a parallel resonant circuit in series with the load form a low impedance path to ground for the ripple frequency, and a high impedance path to block its passage through the load.

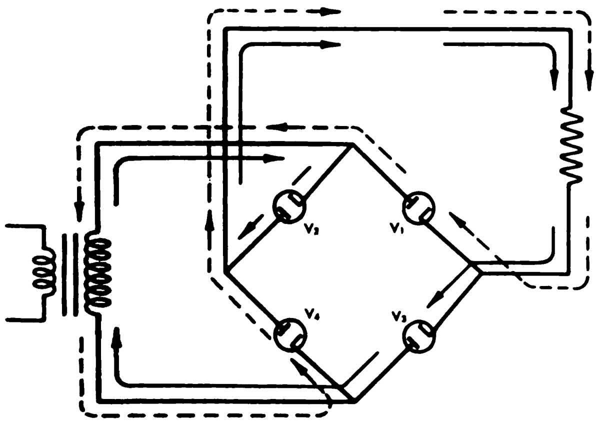


Figure 4–30. Bridge Rectifier

Bridge Rectifier

The bridge-type rectifier, shown in Figure 4–30, is a full-wave rectifier capable of withstanding a higher inverse voltage than a two-tube full-wave rectifier. Because the inverse voltage appears across two tubes in series, it is therefore only half as great per tube. By tracing current flow you can see that V_1 and V_4 are in series, and V_2 and V_3 are in series. At the same time, the series arrangement in this type of circuit offers some disadvantages. Instead of the drop across one tube, there are two drops to be deducted from the output voltage. When directly heated tubes are used (Figure 4–30B), another disadvantage is that the circuit requires three filament transformer windings since only V_1 and V_3 have the same DC voltage on the filaments. The main point in favor of the use of the bridge circuit is that the secondary of the transformer does not have to be center tapped, and the output voltage is twice as great as it would be if the same transformer were used in a conventional full-wave rectifier.

Metallic Rectifiers

The principal metallic rectifiers are the copper oxide and the selenium rectifiers. Both are rectifiers because they permit current to flow more readily in one direction than in the opposite. A copper oxide rectifier consists of a thin film of copper oxide on a copper plate. A selenium rectifier consists of a prepared film of selenium on a metallic substance such as iron. A selenium rectifier has a somewhat lower resistance than the copper oxide rectifier and is therefore more efficient, since it can pass higher current. This type of rectifier cannot stand high voltages, since the film is very thin. When several such sections are built up in series to form a stack, the breakdown voltage is increased. To reduce the danger of breakdown from inverse voltage, these rectifiers are usually arranged in a bridge circuit as shown in Figure 4–31.

Solid State Rectifiers

The early metallic rectifiers still in general use are the copper oxide and selenium types. Later, improved crystal rectifiers of silicon and germanium were put into use, and large area-type germanium rectifiers were put into use in the early 1950's. Metallic rectifiers today are of two general types, point-contact and extended-surface contact. As their names suggest, the first type conducts only in spots, while the second conducts over the whole surface with a thin barrier layer or junction between. (These rectifiers usually are made up of a semiconductor and a good conductor.) Electron flow is greater from the good conductor to the semiconductor.

The ohmic resistance of the rectifier depends not only on the direction of the applied voltage, but also on its amplitude. The copper oxide rectifier is made up of layers of copper oxide

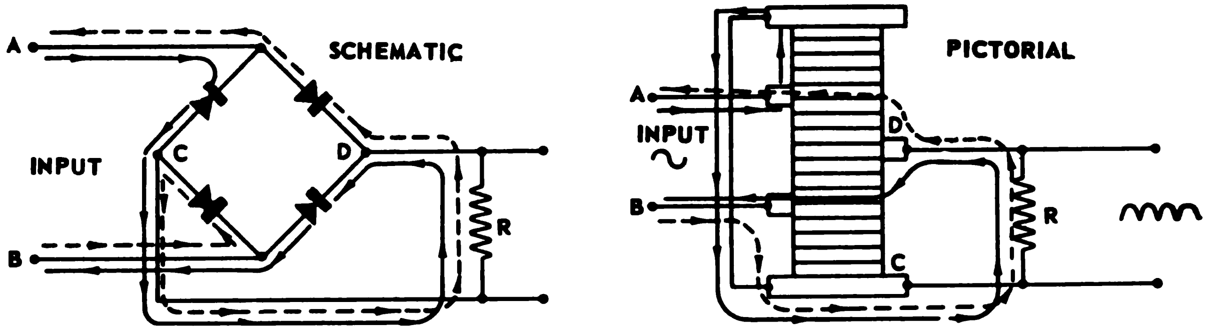


Figure 4–31. Selenium Bridge Rectifier

(semiconductor) and copper. Electrons move more easily from the copper to the oxide. In the selenium rectifier the selenium is the semiconductor and the conductivity is greater from the sprayed alloy layer to the selenium.

The germanium rectifier, unlike the copper oxide and selenium rectifiers, has *two* semiconductors in contact separated by a so-called P/N junction. Electrons move from the N type to the P type. This is explained in Chapter 8.

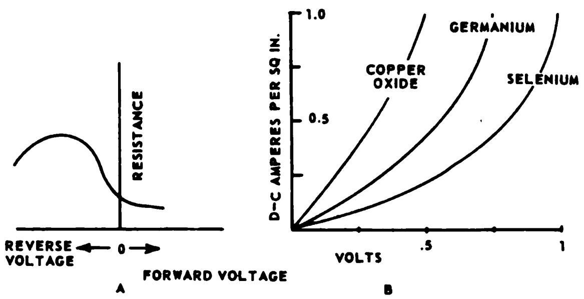


Figure 4–32. Current Flow, Normal Direction

Current-voltage curves for metallic rectifiers show that the relationship is not linear. Figure 4–32B shows the current per square inch for three types of rectifiers; in each case, the applied voltage caused current to flow in the normal direction. In order to include germanium on the same graph, current readings for it were multiplied by 100. Figure 4–33 shows the currents in the same rectifiers when the applied voltage was reversed.

Current in the reverse direction depends on the length of time the reverse voltage is applied as well as on its amplitude.

When metallic rectifiers are used in the operation of magnetic amplifiers, resistance in both directions must be considered. The ratio of fo ward current to reverse current at rated volta influences the amplifier performance.

Typical ratios of forward current to rever current are:

Copper oxide at 6 volts and ½ amp/sq in.	175:1
Copper oxide at 12 volts and ½ amp/sq in.	100:1
Selenium at 15 volts and ⅓ amp/sq in.	110:1
Selenium at 26 volts and ⅓ amp/sq in.	66:1
Dot germanium at 65 volts and 400 ma.	133:1
Large-area germanium at 65 volts and 75 amp.	750:1

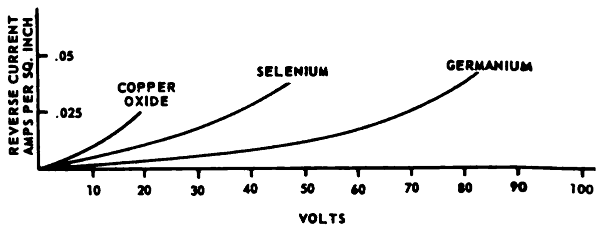


Figure 4–33. Current Flow, Reverse Direction

Several of the disadvantages of reverse currer flow are: the gain of magnetic amplifiers of th amplistat type is reduced; the DC output reduced since a portion of the direct current by passes the load, and the rectifier heats up be cause of the power loss due to the reverse curren

Metallic rectifiers are affected by temperatur atmospheric conditions, and aging (in coppe oxide and selenium types). At present, ge manium rectifiers have not been used long enoug to discover any aging.

The structure of these rectifiers resembles tha of capacitors and there is a certain amount capacitance between the rectifier terminals. Th amount depends upon the frequency and the ap plied voltage, the past history, and the metho of manufacture. Since silicon and germaniu point-type rectifiers have very low capacitance, they can be used in high-frequency work.

Polyphase Rectifiers

When large amounts of power are needed, polyphase (generally three-phase) circuits are used in preference to single-phase. There are three reasons for this: most commercial users and radar sets require three-phase power; the unfiltered output voltage of a polyphase rectifier is normally smoother than that of a single-phase rectifier; and supply transformers and associated equipment give better performance with certain polyphase circuits than with single-phase circuits.

A three-phase generator is mounted with three different sets of armature coils 120° apart. A magnetic field rotates within these coils. In this manner three equal single-phase voltages are produced that have a phase angle between them of 120°. The three-phase current flows from the generator to a transformer with a Delta-connected primary. The turns ratio is such that stepped-up voltages are induced into the Wye-wound secondary of the transformer. These voltages are rectified by the three-phase, bridge-type rectifier. Figure 4–34 shows the primary and secondary transformer windings.

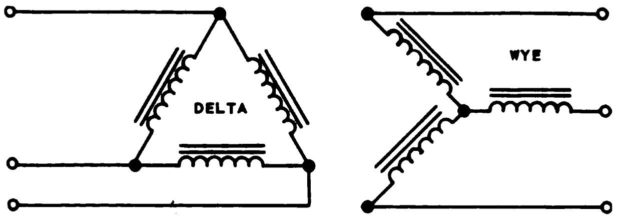


Figure 4–34. Delta and Wye Transformer Windings

Before making a detailed analysis of this circuit, a study of the three-phase bridge-type rectifier is necessary. Figure 4–35A depicts the simplified version of the rectifier, and Figure 4–35B shows the voltage waveforms.

Three-phase voltages and currents have a phase displacement of 120 electrical degrees. In respect to time, this displacement is one-third the period of the frequency of operation. The voltage waveforms, Figure 4–35B, show the three-phase voltages present at A, B, and C. Note that Phase A starts at zero reference time, while Phase B lags Phase A by 120°; Phase C leads Phase A by 120° and lags Phase B by 120°. In the simplified three-phase bridge circuit, the current flow through the load resistance produces a positive polarity output. A negative voltage is produced simply by reversing the cathode and plate connections to each tube.

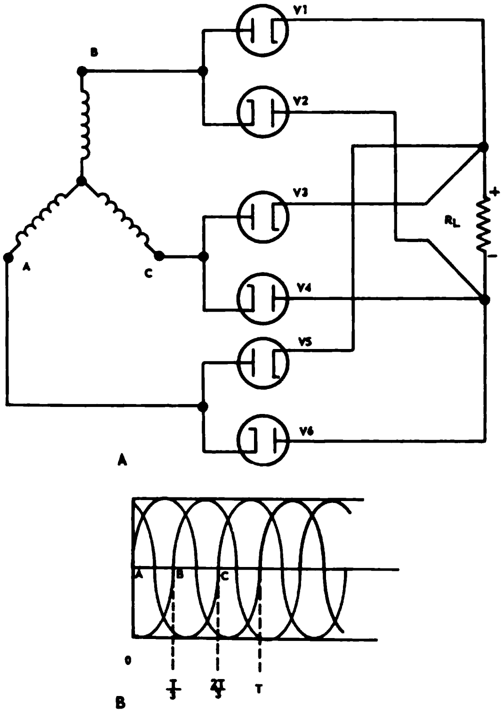


Figure 4–35. Simplified Three-Phase Bridge Type Rectifier

Disregarding a third-phase leg of the transformer, let us see how the input voltage affects the remaining two legs and the resulting output from the full-wave rectifier. Transformer legs A and B are part of the Wye-wound secondary as illustrated in Figure 4–34. Figure 4–36 illustrates the theory of operation for the three phases; the accompanying waveshapes aid in the analysis.

Phase A and Phase B form a series circuit (Figure 4–36A). On the positive alternation of Phase A voltage, assume that point A is positive with respect to point B. Under this condition, current will flow from point B through V2, R_L, V5, and through transformer leg A. On the negative alternation of the cycle, current will flow from point A through V6, R_L (in the same direction as before), V1, and through transformer leg B. The polarity of the voltage across

R_L is negative at the bottom and positive at the top for both positive and negative alternations. Full-wave rectification has been accomplished.

Again disregarding the third legs of the transformer, we see a series circuit in legs B and C (Figure 4–36B). On the positive alternation of the input cycle, assume that point B is positive with respect to point C. Current will flow through V4, R_L, V1, and through transformer leg B. On the negative alternation of the cycle, current will flow from point B through V2, R_L, V3, and through transformer leg C. Again the voltage across R_L is negative at the bottom and positive at the top for both alternations of the cycle.

Transformer leg B is disregarded this time, and we have a series circuit with legs A and C (Figure 4–36C). On the positive alternation assume that point C is positive with respect to point A. Current will flow from point A through V6, R_L, V3, and through leg C to complete the circuit. On the negative alternation, current flows from point C through V4, R_L, V5, and through leg A to complete the circuit. Once again we find that R_L has the same outpu polarity.

Figure 4–36G illustrates the resulting wave forms that appear across R_L, combining all thre phases of rectification. The ripple frequency i six times the input frequency. With a 60-cycl input, the output ripple frequency is 360 cycles The amplitude of ripple in the output is ap proximately 5.7% of the amplitude of the outpu voltage. With this low amplitude ripple fre quency, very little filtering is necessary in orde to have a smooth DC output.

Voltage Doubler

A voltage doubler is a circuit arrangemen capable of delivering a no load DC voltage tha is equal to twice the peak voltage of the AC lin from which the circuit receives its energy. I consists of two rectifier tubes and a circuit tha connects the outputs of the two tubes in series While the full-wave rectifier uses a double tub that has a common cathode, the voltage double tubes must have separate cathodes. It is ver similar to the bridge circuit, except that two o

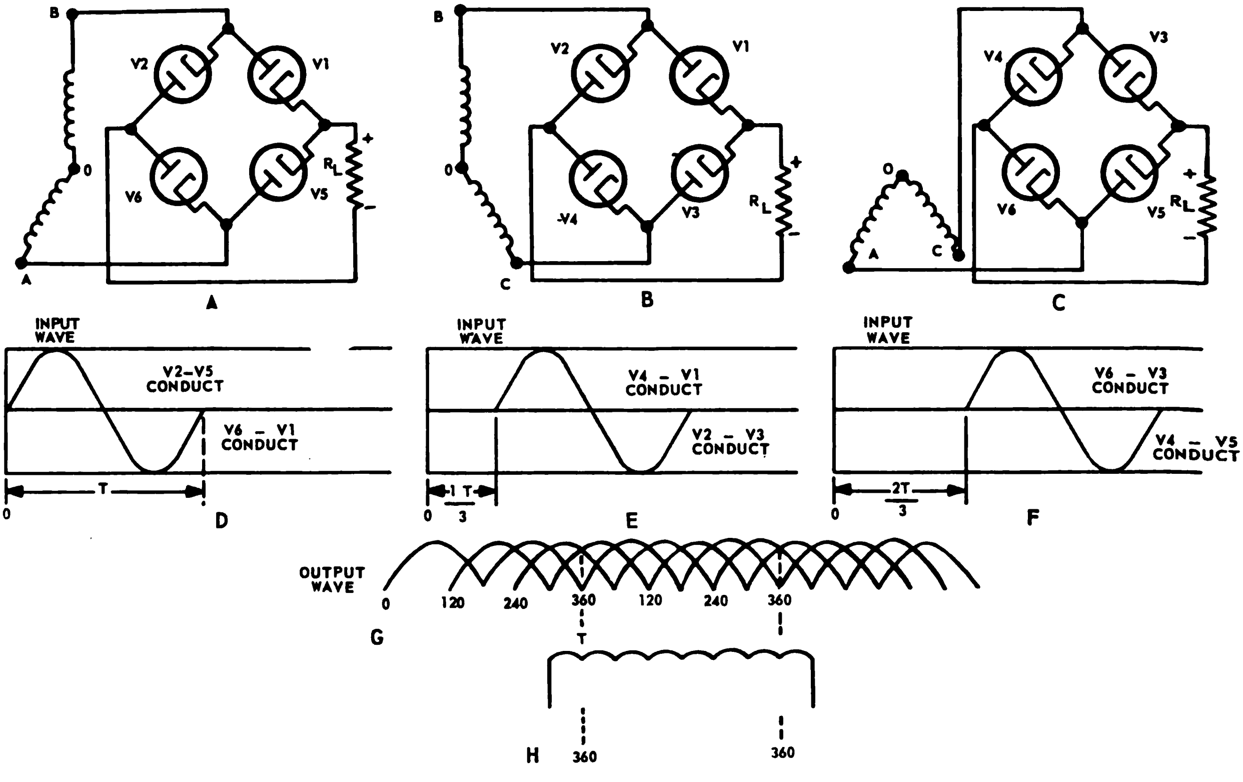


Figure 4–36. Three-Phase Rectification and Resulting Waveforms

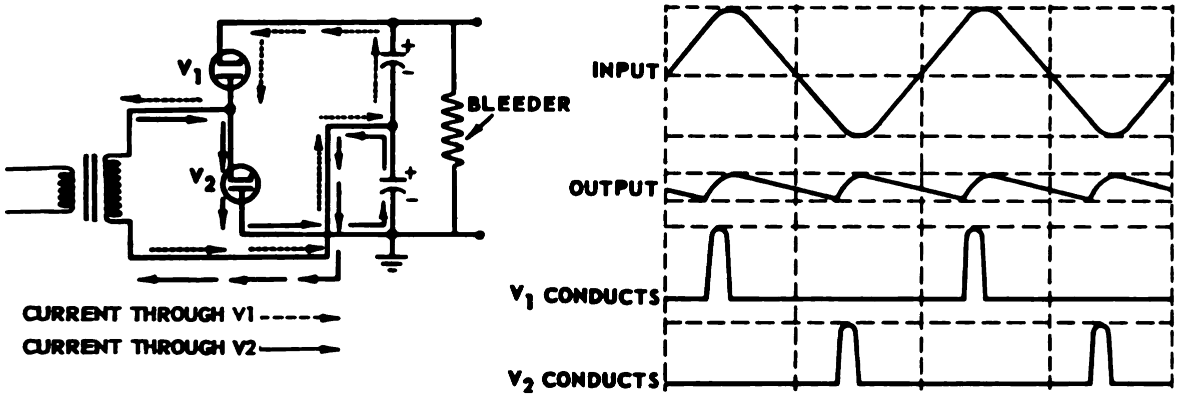


Figure 4–37. Voltage Doubler

the tubes have been replaced by capacitors. The voltage doubler is a good source of power in places where high voltage with low current drain is required.

An elementary circuit using two half-wave rectifiers is shown in Figure 4–37. To learn its operation, first consider the upper half. When the plate of V_1 is positive, current flows through the tube and charges the upper capacitor. Next, consider the lower half. When the plate of V_2 is positive, current flows through the tube and charges the lower capacitor. Since the tubes are connected opposite to each other across the applied AC, the two capacitors charge in the same direction. If no current is taken from them by the load, each will charge to the peak value of the voltage across the secondary. Since the capacitors are effectively in series, the voltage across both capacitors (the output voltage of the doubler) is approximately twice the peak value or 2.82 times the rms value of the secondary voltage.

To discharge the capacitors when the equipment is off, a bleeder resistor is connected across the output. It is made large to keep the capacitors from discharging appreciably between charging alternations. Due to the discharge of the capacitors through the bleeder, there is a small amount of ripple. Each tube conducts each time its plate is more positive than the capacitor voltage, and therefore conduction is both small and short in duration. Since current drawn from a voltage doubler power source depends upon the discharge of the capacitors, the amount of current that can be drawn is limited. During discharge the voltage drops rather rapidly and the ripple becomes appreciable.

Voltage Regulator Circuits

As operation of radar and computer circuits requires very stable voltages for optimum results, it is well to consider some of the circuits designed to maintain the output voltage of a power source constant at a predetermined level, in spite of varying input voltage or varying load conditions. There are other electronic circuits and mechanical devices in use for this purpose, but the ones discussed here are typical of the types you will probably work with.

SIMPLE REGULATOR CIRCUIT. The circuit in Figure 4–38 shows a simple voltage regulator circuit. Note that it uses a VR tube, which was discussed earlier in this chapter, and a limiting resistor R. Remember, too, that the VR tube is characterized by the fact that over a wide range of current flow there is little change in the voltage drop across it. In operation, a VR 150–30 tube draws current up to 30 ma and maintains the output voltage 150 volts within two volts. If the input voltage to the regulator were to rise, more current would flow through the VR tube and there would be a greater voltage drop across the resistor R. This voltage drop would subtract from the rising voltage and tend to maintain the

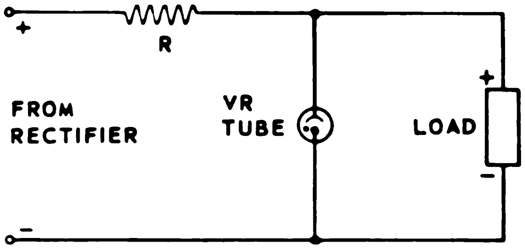


Figure 4–38. Simple Voltage Regulator

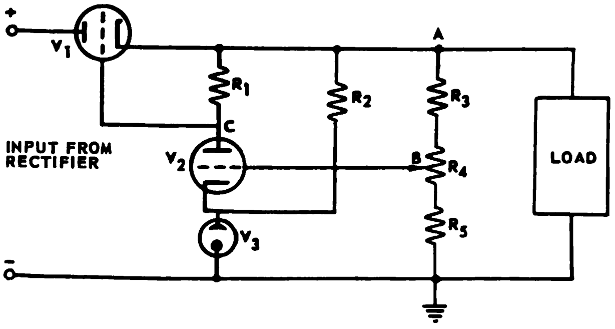


Figure 4–39. Electronic Voltage Regulator

voltage across the load and across the VR tube constant.

On the other hand, if the input voltage falls, the VR tube will conduct less current, the drop across R will be less, and, as before, the voltage across the load will remain at the desired level. When the load draws more current, the voltage tends to drop because of voltage drop across R. The VR tube compensates for this by conducting less, decreasing the total current, and, as a result, maintaining the load voltage constant. Regulation with this circuit is satisfactory as long as the current remains between 5 and 30 milliamperes.

ELECTRONIC VOLTAGE REGULATOR CIRCUIT. The electronic voltage regulator shown in Figure 4–39 is capable of very close regulation at a level which may be set by varying the potentiometer setting. This circuit contains a vacuum tube (V_1) in series with the load. The voltage across the load is regulated by controlling the conduction of V_1 — that is, making V_1 act as a variable resistor that automatically adjusts itself to the correct value. The function of V_2 is to control the bias on V_1 according to the output voltage. V_3 is a VR tube whose purpose is to maintain the cathode of V_2 at a fixed positive potential.

The voltage divider system, composed of R_3, R_4, and R_5, is arranged so that the variable arm of R_4 can be adjusted to a positive voltage sufficiently low to bias V_2 for operation in the linear portion of the Eg–Ip curve. R_1 is the load resistor of V_2; it is connected in series with the VR tube, V_3. The IR drop across R_1 is applied between the grid and cathode of V_1 to control the conduction of this tube.

If the output voltage (point A) tends to rise because of an increase in input voltage or to a decrease in current drawn by the load, the voltage at point B will rise by an amount dependent upon the resistances from A to ground and from B to ground. The voltage at B is the voltage applied to the grid of V_2 and determines how much V_2 will conduct. When V_2 conducts more, there is an increase in current through R_1 and, consequently, an increase in the IR drop across it. A greater negative voltage is applied to the grid of V_1, which causes it to conduct less, counteracting the tendency of the rise in output voltage which set off this action.

If the output voltage should tend to fall, the voltage at B drops, causing a decrease in the voltage across R_1 which produces a rise in voltage at C. This causes V_1 to conduct more, resulting in a greater output voltage. It has already been pointed out that V_2 must be biased to operate in the linear portion of the characteristic curve. This is also true of V_1, since accurate regulation will be proportional to the linearity of these tubes.

The setting of B determines the bias of V_2 and the consequent current flow. The current flow through R_1 determines the bias on V_1, and the current flow through V_1 determines the load voltage. Hence, the output voltage is adjustable within limits by setting the movable tap, point B, of potentiometer R_4. This circuit, with some refinements, is used in the power sources of several radar equipments. Note that the total rectifier current flows through V_1.

The great amount of current flow required in some sets may result in finding two or more tubes functioning in parallel to serve the purpose of V_1. You can see that the rectifier and filter circuit must be designed for greater current capacity than the load is expected to draw since the regulator also draws current. To illustrate, there is current flow through R_1, V_2, and V_3; a second path consists of R_2 and V_3; and a third path includes R_3, R_4, and R_5.

This extra current drain is the toll exacted for the regulation of the output voltage within very close limits. As the regulator tends to maintain a constant voltage, it will substantially reduce any ripple that may remain in the output of the filter of the rectifier that supplies the regulator. Thus, regulated power supplies not only provide a stable voltage, but also furnish one which is more thoroughly filtered than that of unregulated power supplies.

CHAPTER 5

Principles of Radar

Radar circuits are, for the most part, just new combinations of circuits that have been used for many years in other instruments. Up to this point, the manual has described these standard circuits. The rest of the manual, however, deals with these same circuits in new combinations — combinations called radar circuits.

The purpose of this chapter is to prepare you for the discussion of the radar circuits in the following chapters. It discusses the basic principles of radar, the components of radar sets, and in general gives you a basis for understanding individual radar circuits in terms of a complete radar set.

DEFINITION

The word *Radar* is an acronym of the expression *RA*dio *D*etection *A*nd *R*anging. The italicized capitalized letters, as you can see, form the word *Radar*. Radar is thus an application of radio principles to detect objects that cannot be observed visually, and to determine their direction, range, and elevation.

PRINCIPLES OF OPERATION

You know from experience that when a loud noise is made, such as a shout or a gunshot, you often hear an echo of that noise. Sometimes you hear several echoes from the same noise, some of them apparently coming from different directions. Without looking you know that a building, hill, grove of trees, or some other object is nearby in the direction of the first echo, and another similar object still farther away in the direction of the second echo. Obviously, the loud noise consisted of sound waves which began at your location and, on striking the object, were reflected. As these reflected sound waves move by you, you hear them.

According to the science of physics, sound travels at a constant rate. Therefore, it took a certain amount of time for the sound waves to go to the object and return to you as an echo. Still more time would have been required had the object been farther away. Furthermore, you are able to tell the direction of the object that caused the echo because of your ability to hear. With this faculty, your brain tells you that the object creating the echo is to the right, left, or in front of you.

In the same way, radar depends on creating and picking up an echo — a radio echo. Radar sets send out short, strong bursts of radio energy. Many of these bursts of energy strike objects in the vicinity and are reflected back to the site of the radar set. When the time required for the energy to go to the object and to return is carefully measured and translated into distance in miles or feet, it is possible to determine the distance of the object causing the echo.

In early radar sets, two antennas were used to compare the strength of the reflected energy. From this comparison it was possible to determine the direction from which the echo came. For the same purpose, modern sets employ various types of directional antennas. Thus radar, like the method you use in determining the direction and distance of an object causing a sound echo, determines the distance and direction of an object creating a radio echo.

In radar terminology, the reflected energy is called the *radar echo* or simply the echo. The object, principally due to military influence, is called the *target*.

Modern radar technique is highly developed. Sets are unbelievably accurate; in many cases they are almost completely automatic. Yet, regardless of their stage of development, all depend upon this simple principle of creating and detecting a radio echo.

RADAR SYSTEMS

Essentially, a radar system consists of a transmitter which sends out radio signals, a receiver which is located at the same site, and an indicator which usually gives a visual indication of echoes returned by a target.

When a radio signal which has a constant frequency is emitted by a radio transmitter, radio waves, in a manner similar to light and sound waves, travel out in all directions and are reflected by any object that they strike. On striking the object, components of the wave are reflected and likewise travel in all directions. Some of the reflected waves return to the site of the transmitter originating them, where they are picked up by the receiver, providing it is tuned to the correct frequency.

This is a simple explanation. Naturally, there are some problems which must be solved. First, there is the problem of having a powerful transmitter near the receiver and operating on the same frequency. In order for the receiver to detect the reflected signal, and thus indicate the presence of the target, the transmitter signal must be prevented from affecting the receiver.

The second problem is measuring distance. If a continuous constant frequency signal were transmitted, it would be impossible for the receiver to distinguish between various echoes at different distances, for they would all be alike. Thus, some means must be provided to eliminate the problems involved in using constant frequency signals.

There are several systems that satisfactorily solve these problems. They are the frequency modulation system, the frequency-shift system, and the pulse modulation system.

Frequency Modulation System

Because each cycle of a frequency modulate wave differs in frequency from the others o that wave, a frequency modulation system make it possible to identify each cycle of a wave transmitted and to recognize it from all others whe it returns to the receiver. By designing a transmitter which produces a signal that regularl changes over a known range of frequencies, it i possible to identify any particular reflected signal cycle.

An example of a frequency modulated signa is plotted against time in Figure 5–1. As shown the 420-mc frequency increases linearly to 46 mc and then quickly drops to 420 mc again When the frequency drops to 420 mc, the frequency cycle starts over again.

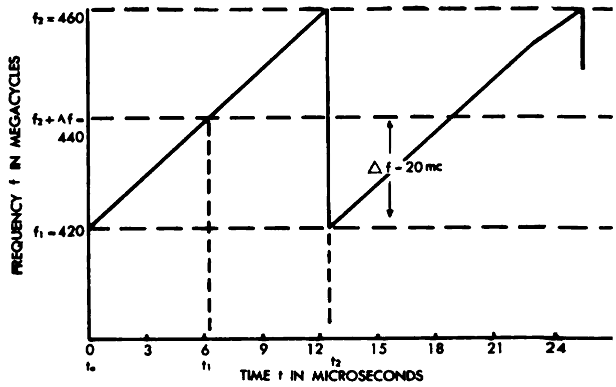


Figure 5–1. Frequency Modulation Chart

Since the frequency regularly changes 40 m with respect to time, you can use its value a any time during its cycle as the basis for computing the time elapsed after the start of the frequency cycle. For example, at time t_0 the transmitter sends a 420-mc signal toward a object. It strikes the object and returns to the receiver at time t_1 when the transmitter is sending out a new frequency of 440 mc. Thus, since f changes from 420 mc to 460 mc in 12.4 microseconds, the time required for the 420-mc signal to change from 420 to 440 mc is $\frac{20}{40}$ of 12.4 o 6.2 microseconds. Radar waves travel to and return from an object one mile away in 12.4 microseconds. Therefore, the distance of the object in the example is $\frac{6.2}{12.4}$ or $\frac{1}{2}$ mile.

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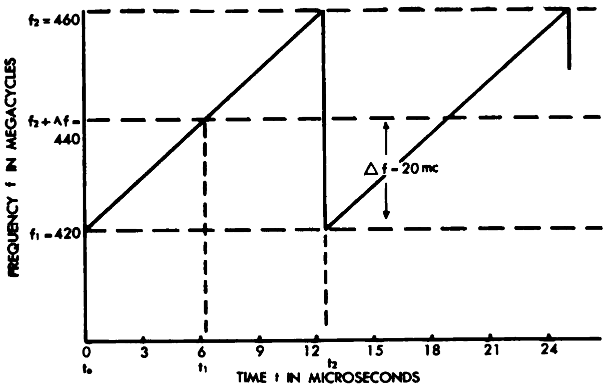

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In a frequency modulation system, two separate signals are fed to the receiver at the same time. For example, at t_1, the 440-mc transmitted signal and the 420-mc reflected signal reach the receiver simultaneously. When these two signals are mixed in the receiver, a beat note results. The frequency of the beat note varies directly with the distance to the object, increasing as the distance increases. A device that measures frequency can be calibrated to indicate range (distance to object).

Frequency-Shift System

Another system for making the received echo change continuously in frequency in order to locate remote objects is the frequency-shift system. This system is based on the *Doppler effect*, a familiar example of which is the change in pitch of an automobile horn as the car approaches. Radio waves act in the same way. If the source of radio energy (in the case of radar, an object from which radio waves are reflected) is moving rapidly, the frequency of the radiated energy (the echo) changes.

In radar, the use of the Doppler effect to locate objects is based on the transmission of continuous or unmodulated radio waves toward the object. At the receiver the frequency of the waves reflected by the object is changed, provided the object moves toward or away from the receiving point. Crosswise movement (which would describe a circle around the receiving point) would not change the frequency. The amount of change is proportional to the speed at which the object is moving toward or away from the receiving point.

Since the receiver is located near the transmitter, it receives a signal from both the transmitter and the remote object. When the object is moving toward or away from it, the receiver gets a frequency from the remote object that is different from the transmitter frequency. The detector in the receiver responds to the difference in frequency. If the object is not moving or if it is moving crosswise to a radius drawn through it, the returning frequency is the same as the transmitter frequency, and the detector response is zero. It is therefore impossible by the frequency-shift method to detect objects which are not approaching or moving away from the receiver.

The frequency-shift principle is sometimes used in conjunction with a pulsed radar set to eliminate echoes from stationary objects. For example, at air bases in mountainous areas, the mountains cause radar echoes so strong that the weaker echoes from aircraft flying in the vicinity are obscured on the radar screen. By applying the frequency-shift principle, the moving objects can be differentiated from stationary objects. The echoes from the stationary objects can be eliminated, and the radar operator is able to see only the flying aircraft. In practice, the radar set itself uses the pulse system and the frequency-shift detector device is a supplementary component attached to the radar set. This device is called a *moving target indicator*.

Pulse System

The pulse system of detection is employed in almost all radar sets. In this system the transmitter is turned on for short periods and off for long periods. During the period when the transmitter is turned on, it transmits a short burst of energy called a *pulse*. When a pulse strikes any object, part of the reflected energy is returned to the receiver where it is displayed on the screen of a cathode-ray tube. Since the transmitter is turned off after each pulse, it does not interfere with the receiver as would be the case if a constant signal were used.

Complete location of an object in space by radar pulses depends upon two factors — the range or the distance of the target; and the direction, including both the azimuth and elevation directions of the target.

The successful use of a radar set depends primarily on its ability to measure distance in terms of time. When radio energy is radiated into space, it continues to travel with a constant velocity. Its velocity is that of light, or about 186,000 statute miles per second or 162,000 nautical miles per second. In more useful terms, radio waves travel a nautical mile in 6.2 microseconds. They travel a *radar mile*, that is, a go and return mile, in 12.4 microseconds.

This constant velocity is used in radar to determine the distance, or range, of a target by measuring the time required for a pulse to travel to a target and return. Suppose, for example, a pulse of radio energy is transmitted toward a target some distance away and the radar echo returns after 620 microseconds. Energy moves a mile and back, or a radar mile, in 12.4 microseconds. Therefore, the distance of the target is $\frac{620}{12.4}$ or 50 miles.

In order to use the time-range relationship, a time-measuring device must be used. The cathode-ray tube is useful for this purpose since it responds to changes as rapid as one microsecond apart. A time base is provided by using a linear sweep to produce a known rate of motion of an electron beam across the screen of the cathode-ray tube.

The formation of the time base is shown in Figure 5–2. In A, a radar pulse is leaving the aircraft. At the time the pulse is radiated, the spot on the screen of the cathode-ray tube is deflected vertically for a brief instant; then it continues across the screen to the right. In B and C the pulse is traveling toward the target and the spot is moving across the screen. When the pulse strikes the target, there is no deflection, since energy is at the target itself. In D the reflected pulse is returning. In E the reflected energy has returned to the receiver, and there is a second vertical deflection of the spot on the right side of the cathode-ray screen.

The distance between the two upward deflections serves as the basis for determining the range of the target from the radar antenna. Assume, for example, that the set is designed so the spot moves across the cathode-ray tube in 700 microseconds. The spot was almost to

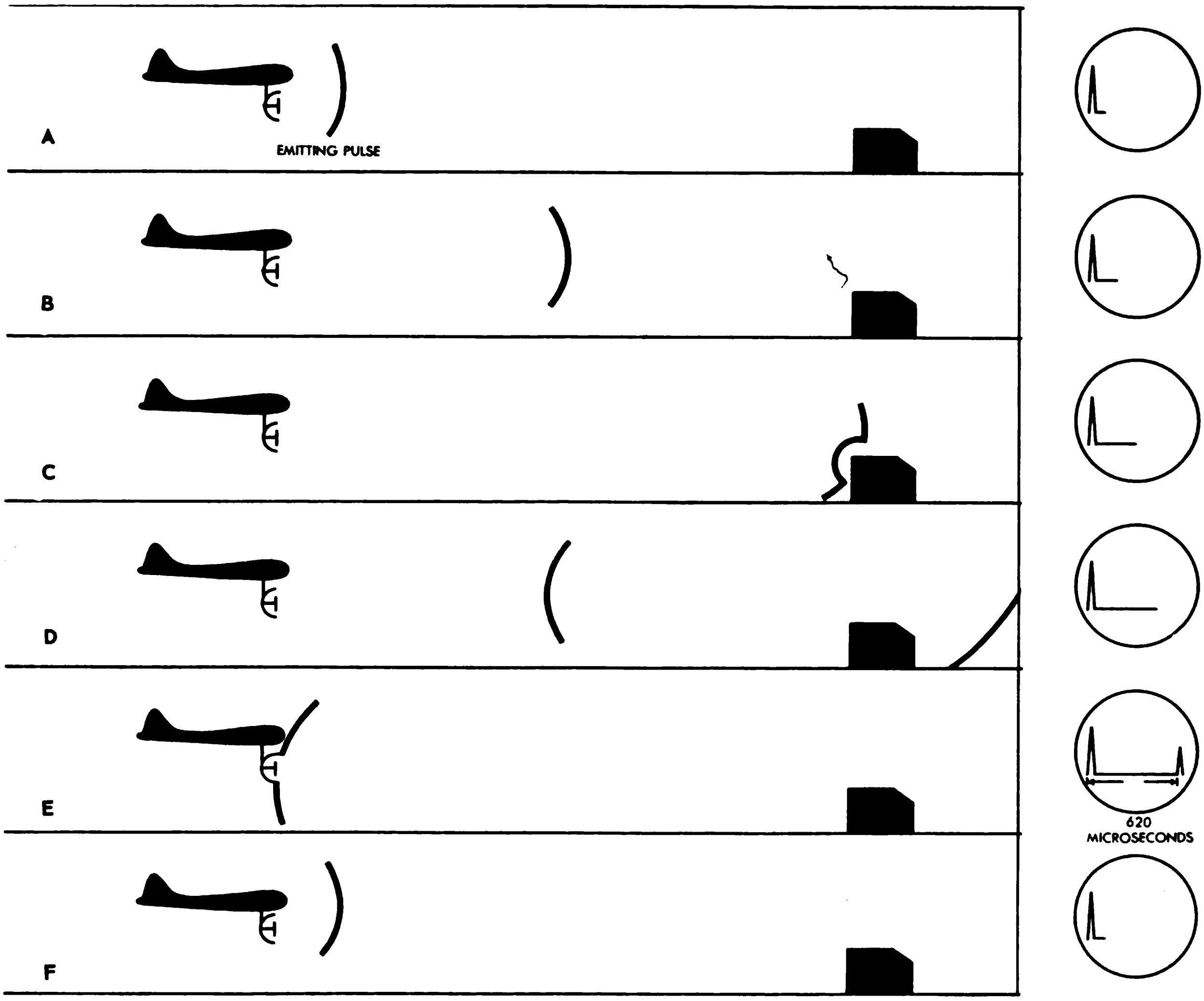


Figure 5–2. Formation of a Time Base

the end before the echo arrived, its position indicating that it took 620 microseconds to reach that point. Since radar waves travel one radar mile in 12.4 microseconds, the range of the target in the illustration is $\frac{620}{12.4}$ or 50 miles away. In F another pulse is being emitted, and a new time base is starting.

Two dimensions must be considered in determining the direction of a target. One is azimuth, which is the relative horizontal direction of the target with respect to some direction reference expressed in degrees. For example, this direction may be expressed with reference to true north if the radar set is a ground installation, or with reference to the heading of the aircraft if the set is airborne. The other dimension is elevation, which, like azimuth, can also be expressed in degrees. Elevation expresses the angular degrees that the target is above or below the radar set.

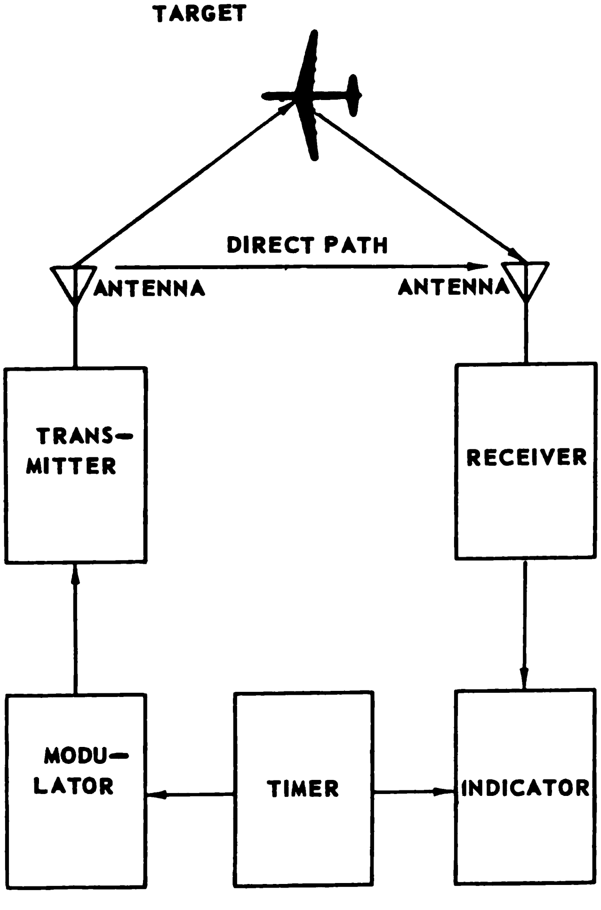


Figure 5–3. Elements in Pulse Radar System

The determination of azimuth and elevation depends upon the directional characteristics of antennas and antenna arrays. Antennas for performing these functions are discussed later in this manual.

BASIC ELEMENTS OF PULSE SYSTEMS OF RADAR

The basic elements in a typical pulse radar system are the timer, modulator, antenna, receiver, indicator, and transmitter, as shown in Figure 5–3.

Timer

The timer, or synchronizer, is the heart of all pulse radar systems. Its function is to insure that all circuits connected with the radar system operate in a definite time relationship with each other, and that the interval between pulses is of the proper length. The timer may be a separate unit by itself, or it may be included in the transmitter.

Modulator

The modulator is usually a source of power for the transmitter. It is controlled by the pulse from the timer. It sometimes is called the keyer.

Transmitter

The transmitter provides r-f energy at an extremely high power for a very short time. The frequency must be extremely high to get many cycles into the short pulse.

Antenna

The antenna is very directional in nature because it must obtain the angles of elevation and bearing of the target. To obtain this directivity at centimeter wavelengths, ordinary dipole antennas are used in conjunction with parabolic reflectors. Usually, in order to save space and weight, the same antenna is used for both transmitting and receiving. When this system is used, some kind of switching device is required for connecting it to the transmitter when a pulse is being radiated, and to the receiver during the interval between pulses. Since the antenna only "sees" in one direction, it is usually rotated or moved about to cover the area around the radar set. This is called *searching*. The presence of targets in the area is established by this searching.

Receiver

The receiver in radar equipment is primarily a super-heterodyne receiver. It is usually quite sensitive. When pulsed operation is employed, it must be capable of accepting signals in a bandwidth of one to ten megacycles.

Indicators

The indicators present visually all the necessary information to locate the target on the indicator screen. The method of presenting the data depends on the purpose of the radar set. Since the spot "scans" the indicator screen to present the data, the method of presentation is often referred to as the type of *scan.*

Figure 5–4 shows several of the common types of scan used in radar receivers.

In the type A scan the spot maintains a constant intensity, starting the instant a pulse of energy is radiated by the transmitter and traveling at a constant speed across the face of the indicator. When the spot reaches the right side of the indicator, it is blanked out, jumps quickly back to the left side, and repeats the process. Receiver echoes cause a vertical deflection of the spot, roughly proportional to the strength of the received signal. The horizontal distance between transmitted pulse and echo represents distance to the target. Although the principal function of this scan is to determine the distance of the object, it is possible to obtain a rough approximation of direction by rotating the antenna until the maximum echo is received.

The type B scan presents the pulse and the echo as bright spots (*blips*) on the time base line. In a system using this type of sweep a highly directional antenna system is rotated about a vertical axis. The time base itself moves across the screen synchronized with the antenna motion. The scanning spot moves from the bottom to the top of the screen, each vertical line being synchronized with a transmitter pulse for indication of range. At the same time, due to the antenna rotation, the sweep itself is slowly moved horizontally across the screen. This makes each trace slightly offset from the preceding so that the entire screen is covered. Instead of being a point, the origin is stretched out into a line at the bottom of the screen. This line represents the transmitted pulses and is the so-called "base line" of the pattern.

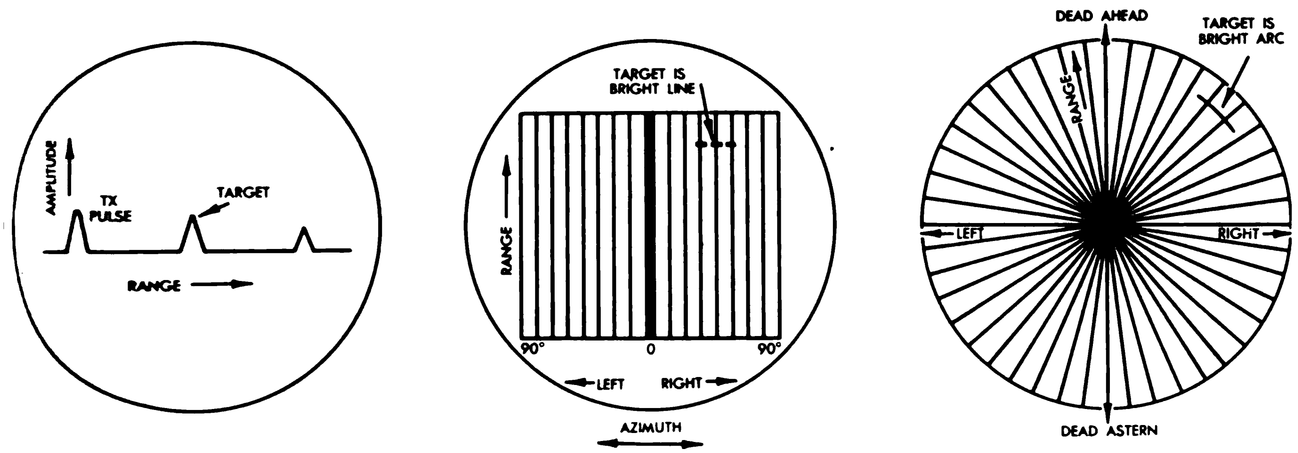
When an echo is received, the signal is impressed on the control grid of the indicator causing a bright spot to appear on the screen. The position of this spot to the right or left of the center line of the screen indicates the azimuth of the target. (Azimuth is the angle of the target, to the right or left of the radar set, in this case.) The height of the spot above the base line indicates the range, or distance, of the target. At short range the resolution (property of making visible the separate parts) in angle is much greater than in a PPI scan.

The type P scan is used as a Plan Position Indicator, hence the usual nomenclature, PPI. It plots range and azimuth in polar coordinates. The antenna generally is rotated uniformly about the vertical axis so that searching is accomplished in a horizontal plane. The beam is usually narrow in azimuth and broad in elevation, and large numbers of pulses are transmitted for each rotation of the antenna. As each pulse is transmitted, the unintensified spot starts from the center of the indicator and moves toward the edge along a radial line. Upon reaching the edge of the indicator, the spot quickly jumps back to the center and begins another trace as soon as the next pulse is transmitted.

As the antenna rotates, the path of the spot rotates around the center of the indicator screen so that the angle of the radial line, on which the spot appears, indicates the azimuth of the antenna beam, and distance out from the center of the indicator indicates the range. When an echo is received, the intensity of the spot is increased considerably, and a bright spot remains at that point on the screen, even after the scanning spot has passed it. Thus it is possible with this scan to produce a map of the territory surrounding the observing station on the indicator tube. This type of scan is useful when the radar set is used as an aid to navigation.

In the type C scan the echo appears as a bright spot with the azimuth angle as the horizontal coordinate and the elevation angle as the vertical coordinate. This type of scan has been used by night fighters to aid in following an enemy plane.

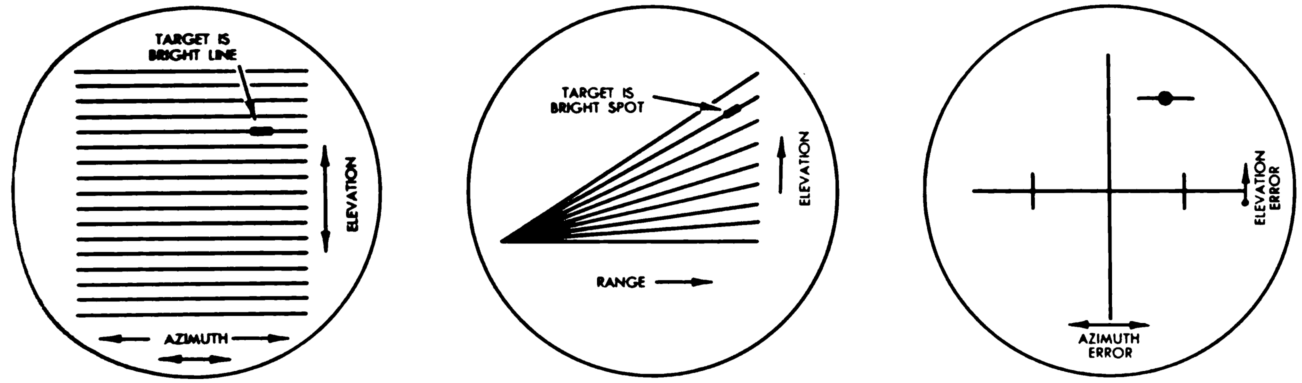
The type E scan is a modification of the B scan on which an echo appears as a bright spot



TYPE A SCAN

TYPE B SCAN

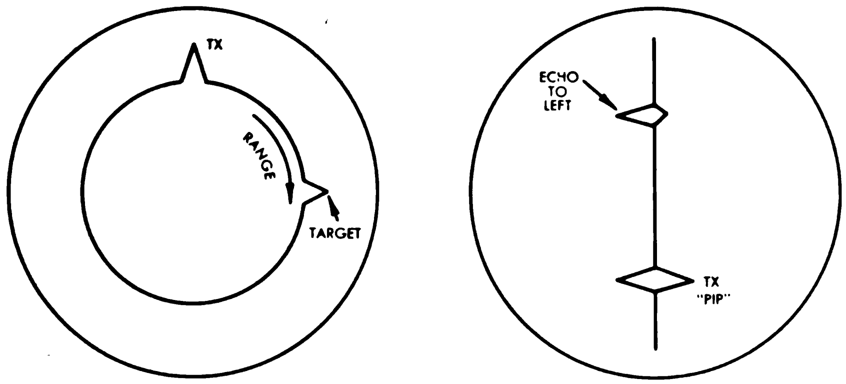
TYPE P (PPI)



TYPE C SCAN

TYPE E SCAN

TYPE G SCAN
SINGLE TARGET, WINGS INDICATE RANGE



TYPE J SCAN
SAME AS A, BUT TIME BASE IS CIRCULAR

TYPE L SCAN
RELATIVE STRENGTH OF "PIPS" ON EACH INDICATE AZIMUTH

Figure 5–4. Types of Scans

with the range as the horizontal coordinate and the elevation as the vertical coordinate. This type of scan is used in directing planes in making a blind landing. The approaching planes must follow a certain angular line to reach the touchdown point at the left of the screen.

The type G indicator, sometimes referred to as a "Spot Error Indicator", was commonly used with gun-laying equipment. It is a type C modified to give a rough indication of range. If the circuit has no target on which to lock, it will search for one, covering the range from zero (center) to maximum. When an echo returns, the faint dot and wing presentation will increase in brilliance and the set will lock on the signal. At this time the sweep voltage is removed and the signal appears as a dot on which the wings grow as the distance to the target is diminished. Its position on the screen gives error in both elevation and azimuth of the antenna. Centering the spot indicates correct aim. When the wings reach a prescribed length, the range is correct for firing the guns.

The type J scan is a modification of the A scan in which the spot rotates in a circle near the edge of the CRT face. An echo appears as a deflection from the circle. As the distance changes, the deflection moves around the circle like a pointer. Its chief use is as a radar altimeter.

The type L scan is another variation of the A scan which indicates azimuth by comparing signals from the left and right antennas. It is used in homing operations. The ratio of the signal amplitudes from the two antennas indicates the error in homing.

PULSE SHAPE, LENGTH, AND REPETITION FREQUENCY

The effectiveness of a radar set on a particular mission is increased by designing it so the transmitted pulse has the proper characteristics. The most important characteristics of a radar pulse are shape, duration, and repetition frequency (Figure 5–5).

Pulse Shape

The shape of the pulse determines range accuracy, minimum and maximum range, and resolution, or definition, of target.

Range accuracy depends to a large degree on the leading edge of the pulse. A pulse with a sharp leading edge provides a definite time from which to measure the start of the pulse and a better picture on the indicator scope than one produced by a pulse with a sloping leading edge.

Detection of objects at the minimum range of the set requires a pulse that has a sharp trailing edge. Sloping trailing edges increase the width of a pulse and consequently cause inaccuracy in range since the edge of the pulse may itself cover several range miles.

To achieve maximum range, the pulse must be flat topped and be sufficiently long in duration. A flat top in a pulse means that the energy in the pulse is constant for the duration of the pulse and all cycles reflected will have the greatest possible strength. Long pulses mean more cycles to strike the target and be reflected.

Range resolution, the ability to indicate very small differences in range where the targets are in the same direction, requires narrow pulses.

Pulse Width

The duration of the pulse (pulse width) determines the minimum range of the equipment. The transmitter must be shut off by the time the received echo returns from the target in order for the echo to show on the indicator as a separate echo and not be obscured by the transmitter pulse. Therefore, the shorter the duration of the transmitter pulse, the shorter the distance to the nearest target that can be observed. However, the transmitter pulse must contain a minimum of approximately 200 cycles of the radiated r-f to secure satisfactory reflection. Long pulses provide greater maximum range since a long pulse provides more energy for the target to reflect.

The wider the pulse, the narrower the bandwidth required in the radar receiver. The wider pulse width results in less noise and an extension of range. For these reasons the pulse width of a radar set depends, for the most part, on the purpose of the set. For example, in an altimeter or in air traffic control equipment, ability to detect and locate objects at very close range is essential, and narrowness of pulse is most desirable. In a bombing set, where high definition capability is important, a narrow pulse width is required. On the other hand, in early warning equipment, long range is a prime consideration, and wider pulses are used. Pulse widths used in radar equipment vary from as little as 0.18 microsecond to as much as 50 microseconds with the most common systems using pulse widths from 0.5 to 10 microseconds.

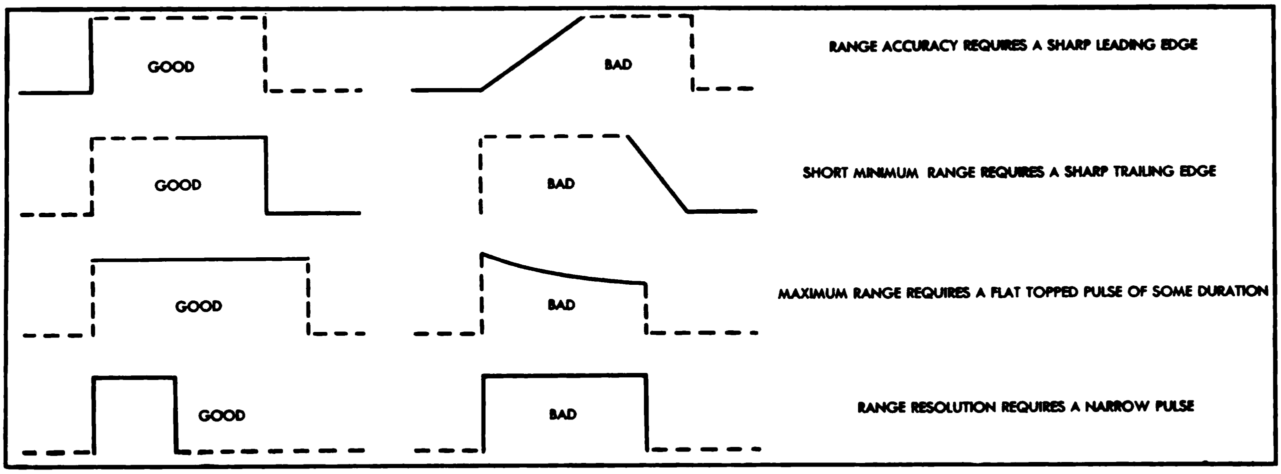

GOOD
BAD
RANGE ACCURACY REQUIRES A SHARP LEADING EDGE
GOOD
BAD
SHORT MINIMUM RANGE REQUIRES A SHARP TRAILING EDGE
GOOD
BAD
MAXIMUM RANGE REQUIRES A FLAT TOPPED PULSE OF SOME DURATION
GOOD
BAD
RANGE RESOLUTION REQUIRES A NARROW PULSE

Figure 5–5. Pulse Characteristics

Pulse Recurrence Time

The time from the beginning of one pulse to the start of the next is known as the pulse recurrence time (PRT). This includes both the transmitting time and the resting time.

Pulse Repetition Frequency

The rate at which bursts of energy are transmitted is known as the pulse repetition frequency (PRF). The maximum and minimum PRF for any one radar set depends upon the use that is to be made of the equipment. In order to reach great distances, it must be able to radiate plenty of power during the transmission of the pulse, and the rest cycle must be long enough to allow one transmission to travel well beyond the maximum range of the set and return.

The time from the beginning of one pulse to the start of the next pulse is the reciprocal of the pulse rate. That is, PRT = 1/PRF. Thus, for example, a pulse repetition frequency of 400 pulses per second has a pulse recurrence time of 1/400 second, or 2500 microseconds. A pulse, therefore, can travel to a target 200 miles away and return before the next transmitter pulse occurs. A pulse from a target 220 miles away would return to the receiver after the transmitter had emitted another pulse, appearing at about the 20-mile point on the indicator. Thus it is impossible to tell whether the observed signal is the echo from the pulse just transmitted or the echo of the preceding pulse. Such a condition is referred to as range ambiguity.

Although the pulse repetition rate must be kept low enough to realize the required maximum range, it must also be kept high enough to avoid some of the pitfalls a single pulse might encounter. If a single pulse were sent out by a transmitter, atmospheric conditions might attenuate it, the target might not reflect it properly, or moving parts — such as a propeller — might throw it out of phase or change its shape. When it did return, you might blink your eyes just when it would be displayed on the screen, and you would not see it at all. Thus, information derived from a single pulse would be highly unreliable. But by sending many pulses, one after another, many good ones will return.

The equipment will integrate, or sum up, all of the pulses and present you with a good clear reliable picture. Equipment is therefore designed in such a way that ten or more pusles are received from a single object. In this way effects of fading are somewhat reduced. At short ranges, such as those used in gun-laying sets, the repetition frequency is increased in order to insure accurate short-range measurements. Many echoes are received from one target, and the integrating effect is increased. This makes possible more accurate range and bearing determination, highly necessary in gun-laying equipment.

Another factor affecting the pulse repetition frequency is the rate of angular motion of the antenna in searching extensive regions. If the antenna is moved through too large an angle between pulses, not only will the number of pulses-per-target be too low, but there may even be areas in which targets may exist without their being detected. In this connection, still another factor to consider is the sharpness of the antenna beam. A sharp beam must be pulsed more often than a wide beam to avoid skipping over targets.

Tactical employment of a radar set determines, to a large degree, the pulse repetition frequency to be used. Long-range search sets, covering distances of at least 300 miles, require a pulse rate slow enough to allow echoes from targets at the maximum range to return to the receiver before the transmitter is again pulsed. Higher pulse rates are used in sets where the maximum range is around 10 miles.

Power Requirements in Pulse Systems

The maximum range which a radar set can attain depends largely on the power output of the set. Enough power must be radiated so that at the maximum range the received echo signal will have a power level at least equal to the electronic noise level at the receiver.

Radar systems have been developed to the degree where they transmit the largest peak power ever radiated by any type radio transmitting equipment. This peak power is sometimes in excess of 10 megawatts, and seldom less than 20 kilowatts.

In considering power requirements, you must distinguish between two kinds of power output measurements — *peak power*, which is the maximum power of the radar pulse of a radar transmitter, and *average power*, which is the power averaged over the whole pulse repetition period. Since the resting time of a radar transmitter is long in comparison with its operating time, the average power output is quite low in comparison with the peak power.

Both pulse width and pulse repetition frequency are determinants of average power. The designer first chooses a PRF which fits the range specifications. Next he computes the power requirements. Then the pulse width can be adjusted to allow for a proper level of average power. The power in the transmitter pulse is equal to the peak power multiplied by the pulse width. The average power is found by dividing this total pulse power by the time from one pulse to the next, or

$$P_{ave} = \frac{P_{peak} \times PW}{PRT}.$$

The ratio of the pulse width to the pulse recurrence time (PW/PRT) is known as the duty cycle. Substituting in the preceding formula we find average power is equal to the peak power multiplied by the duty cycle. Also, PRT and PRF are reciprocals of each other (1/PRT = PRF). Substituting this in the formula for average power, we get this equation:

$$P_{ave} = P_{peak} \times PW \times PRF$$

where power is in watts, PW in seconds and PRF in pulses per second.

The relationship of the various terms are shown in Figure 5–6.

Peak power is limited by the ability of circuit elements to withstand the stress of high voltage, and by the maximum range you wish to cover. The amount of power that a transmitter must radiate depends on the minimum power to which the receiver will respond, and on the amount of attenuation of the power to and from the target. (Moisture in the atmosphere, clouds, weatherproof covering of the antennas affect the attenuation.)

The effects of varying the different characteristics of the pulse are summarized in Figure 5–7.

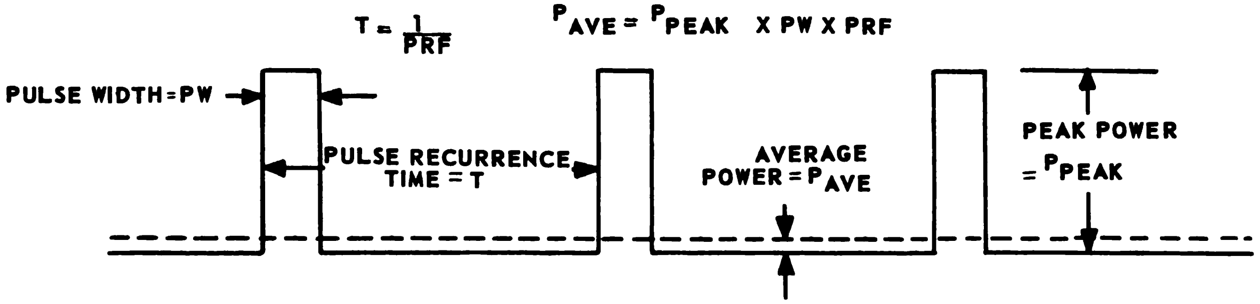
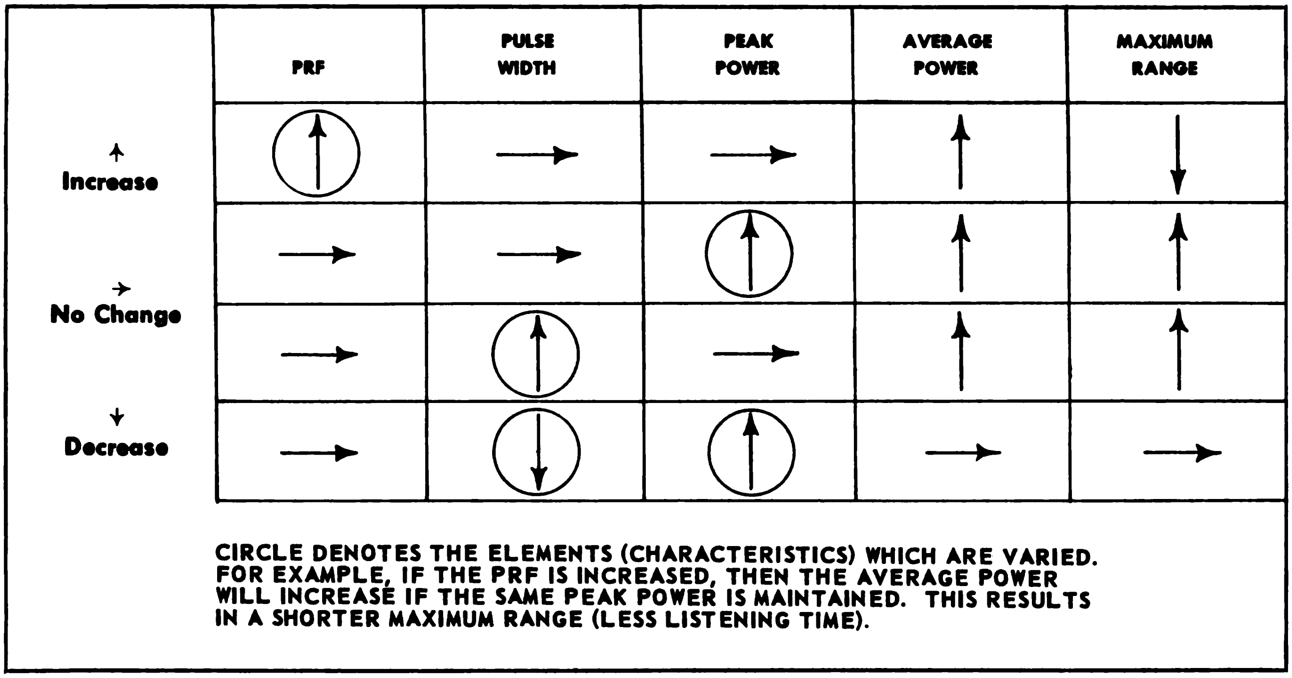

$T = \frac{1}{PRF}$
$P_{AVE} = P_{PEAK} \times PW \times PRF$
PULSE WIDTH=PW
PULSE RECURRENCE TIME = T
AVERAGE POWER = P_{AVE}
PEAK POWER = P_{PEAK}

Figure 5–6. Definition of Pulse Characteristics



↑ Increase
→ No Change
↓ Decrease

PRF	PULSE WIDTH	PEAK POWER	AVERAGE POWER	MAXIMUM RANGE
↑⃝	→	→	↑	↓
→	→	↑⃝	↑	↑
→	↑⃝	→	↑	↑
→	↓⃝	↑⃝	→	→

CIRCLE DENOTES THE ELEMENTS (CHARACTERISTICS) WHICH ARE VARIED. FOR EXAMPLE, IF THE PRF IS INCREASED, THEN THE AVERAGE POWER WILL INCREASE IF THE SAME PEAK POWER IS MAINTAINED. THIS RESULTS IN A SHORTER MAXIMUM RANGE (LESS LISTENING TIME).

Figure 5–7. Effect of Varying Pulse Characteristics

TYPES AND USES OF RADAR

1. *Air Aid to Intercept* (*AAI*). Airborne radar set used in interceptor aircraft to locate, track, and intercept hostile aircraft.

2. *Airborne Early Warning* (*AEW*). High-powered radar set installed in aircraft for the distant detection of approaching enemy aircraft.

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4. *Airborne Intercept* (*AI*). Short-range airborne radar used by fighter and interceptors to locate targets.

5. *Aircraft Control and Warning* (*AC&W*). Control and warning system established to control and report the movement of aircraft. It consists of radar and/or visual observation facilities, control centers, and/or filter centers, and the necessary communications.

6. *Airport Surface Detection.* Short-distance radar having high resolution and discrimination used by traffic control personnel. It provides, in graphic or pictorial form, plan position indication of the airport surface with the location of runways, taxiways, and ramps clearly defined; and presentation of all aircraft, vehicular, and other traffic, both moving and stationary.

7. *Airport Surveillance.* Radar located on or near an airport to provide an indication of the bearing and distance of each aircraft within the terminal area in which the airport is located. It is used to vector and give navigational guidance to arriving and departing aircraft, in the control of air traffic within the terminal area; it does not provide elevation data.

8. *Automatic Tracking* (*Auto-track*). Radar set which can continually and automatically correct its beam orientation to keep a selected target in its beam. Some tracking radars also supply range tracking information to computers for fire control systems.

9. *Beacon, or Navigational Aids.* Radar beams transmitted for the purpose of orienting aircraft. Provides bearing and distance to a known point.

10. *Electronic Countermeasures* (*ECM*). Various offensive or defensive tactics which use electronic and reflecting devices to reduce the effectiveness of enemy equipment, or tactics using electromagnetic radiations.

11. *Gap-Filler.* Short-range radar installations used in areas which are not adequately covered by long-range radar.

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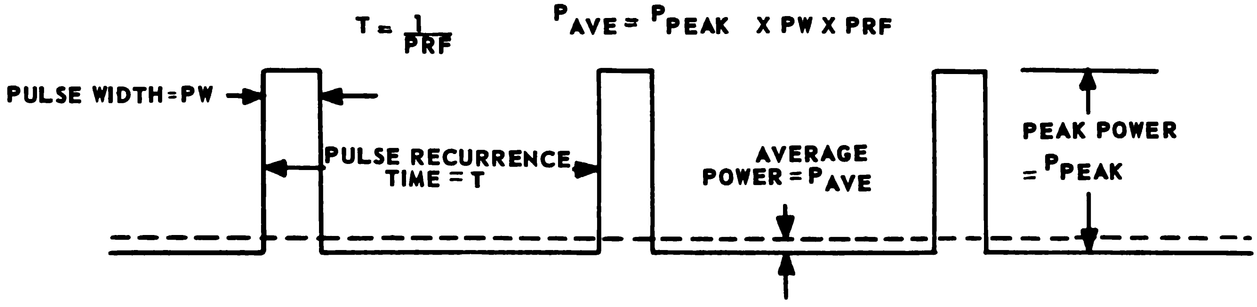

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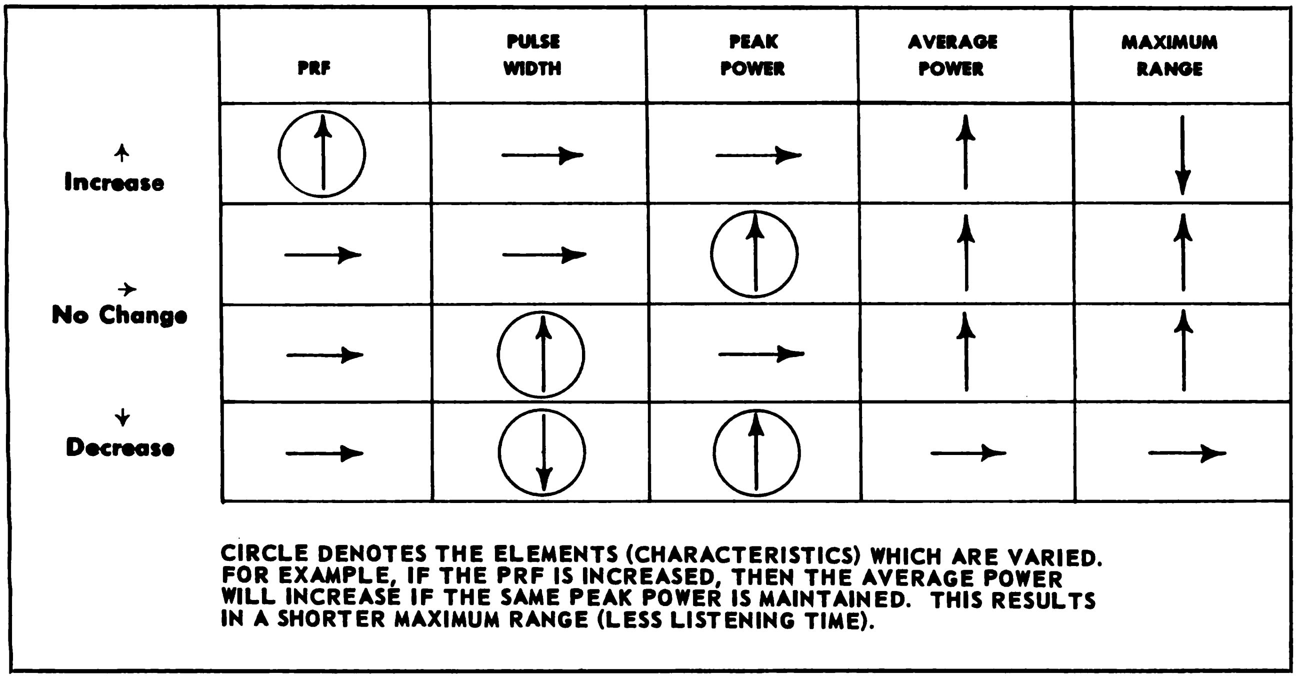

	PRF	PULSE WIDTH	PEAK POWER	AVERAGE POWER	MAXIMUM RANGE
↑ Increase	(↑)	→	→	↑	↓
→ No Change	→	→	(↑)	↑	↑
	→	(↑)	→	↑	↑
↓ Decrease	→	(↓)	(↑)	→	→

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11. *Gap-Filler.* Short-range radar installations used in areas which are not adequately covered by long-range radar.

12. *Ground Controlled Approach* (*GCA*). Radar system which provides both surveillance and

precision approach information by which aircraft approaches may be directed from the ground via radio communications. Two separate functions are performed by GCA. The first is vectoring and sequencing of all traffic within a terminal area. The second is the control of aircraft on final approach to a runway. This control is accomplished with ground-derived information, including distance from touchdown, deviation from a predetermined glide slope, and deviation from a predetermined course line.

13. *Ground Controlled Interception (GCI).* Systems employing radar techniques which will permit ground control of friendly aircraft or guided missiles for the purpose of effecting physical interception.

14. *Gun-Directing.* Radar used to direct antiaircraft artillery or similar fire.

15. *Identification, Friend or Foe (IFF).* System using radar transmissions to which equipment carried by friendly forces automatically responds, thereby distinguishing themselves from enemy forces.

16. *Precision Approach (PAR).* Rapid scanning radar system which is so located on an airport that aircraft on approach to the runway served by the system are presented on radar displays in terms of linear deviation from a desired glide path and in terms of distance to go to the touchdown point on the runway. PAR control personnel talk to the pilot over conventional air-ground communication circuits giving information necessary to direct the aircraft through the low approach phase of the landing operation.

17. *Radar Approach Control (RAPCON).* Use of radar for the direct control of aircraft within a given area. Usually two types of radar equipment are used. One is an air surveillance radar for control of the aircraft from the time it enters the control zone until it approaches the landing field. The other is a precision approach radar for control of the aircraft during its approach to the runway. The AN/FSA–4 control and monitoring equipment has been developed to provide control, coordination, and communication facilities to RAPCON.

18. *Tail-Warning.* Radar sets that warn of approach of aircraft from the rear.

CHAPTER 6

Timing Circuits

Control circuits in a radar set accomplish timing functions by producing a variety of voltage waveforms, such as square waves, sawtooth waves, trapezoidal waves, rectangular waves, brief rectangular pulses, and sharp peaks. In sound and in radio, tubes operate within the limits for which they are designed, but in radar timing circuits, they often are violently overdriven, frequently operating at points that range from well in the grid current region to far beyond cutoff. Although all these circuits are broadly classified as timing circuits, the specific function of any individual circuit might be timing, waveshaping, or wave generating. This chapter covers the principles of a variety of timing circuits, and gives you a foundation for understanding the operation of specific circuits in radar transmitters and receivers.

TRANSIENT AND NONSINUSOIDAL VOLTAGES

Most of the waveshapes in radar timing circuits are produced by combinations of vacuum tubes and resistors, or capacitors and inductors. During the past few years miniaturized circuits with miniature vacuum tubes have played an important role in the improvement of electronic circuits. Recently semiconductors (transistors), with their fast development, have been brought into use in the radar system of circuits. In the future we can expect to see the use of even smaller and lighter components in subminiaturization, and a new discovery called molecular electronics (see Chapter 8). Not only will these new developments be used in timing circuits, but also in other radar circuits.

The charging and discharging times of capacitors and inductors make these elements extremely useful in timing circuits because of their effects on transient and nonsinusoidal voltages, the voltages widely used in radar. A transient voltage is a brief voltage change, called a pulse, that occurs once and does not recur again for a comparatively long time. Such a voltage change is never sinusoidal but is usually periodic; that is, the change repeats itself at regular intervals. Examples of typical transient voltages are compared to a periodic sine wave in Figure 6–1.

Analysis

For the purpose of circuit design there are two ways of analyzing transient and nonsinusoidal voltages. One is to consider that the transient waveshape is a rapid change of voltage, which is followed after a certain interval by another and similar change. The other is to assume that the waveshape is the algebraic sum of a large number of sine waves having different frequencies and amplitudes. The second method of analysis is more useful in most cases. In the design of an amplifier, for example, this type of analysis is necessary because the amplifier cannot handle a transient without distortion unless it is capable of handling all the sine wave frequencies making up the transient.

Transient voltages have many different and complex shapes. The more commonly used transient waves in radar are the square wave, the sawtooth or triangular wave, and the peaked wave.

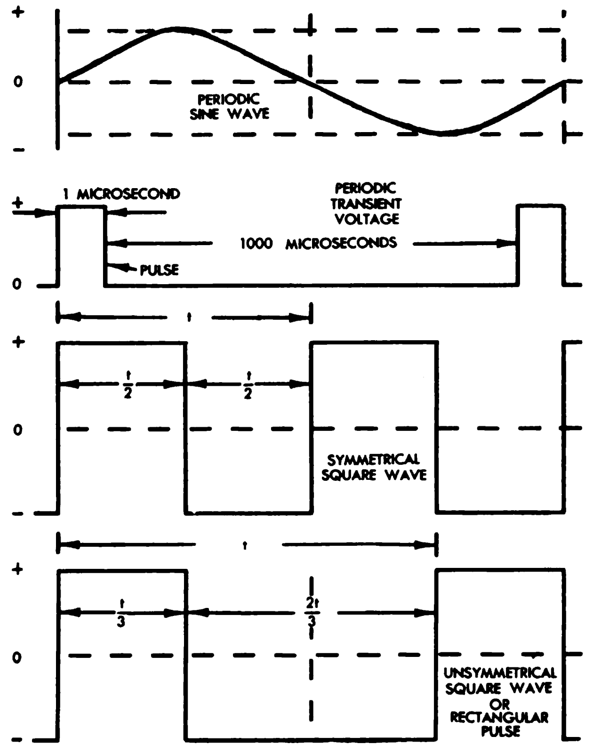


Figure 6–1. Typical Transient Voltages

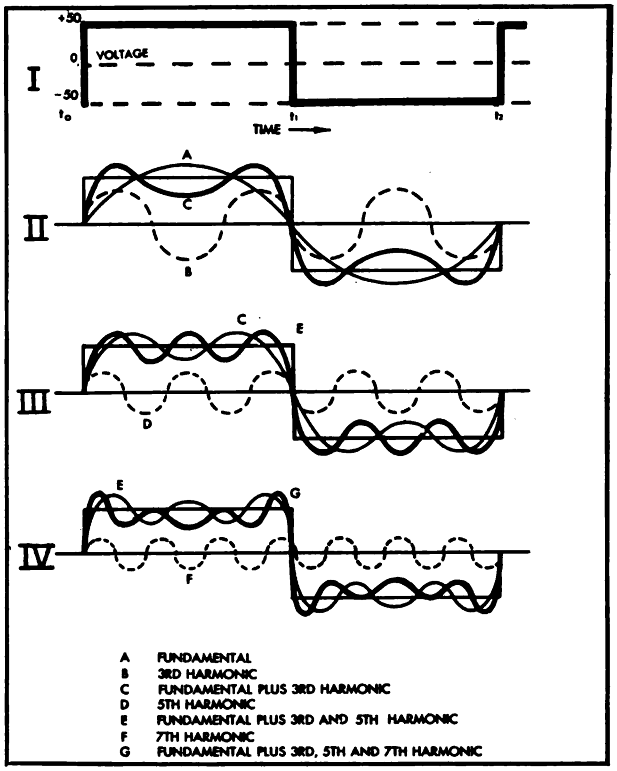

I
+50
0
-50
VOLTAGE
t₀
t₁
t₂
TIME
II
III
IV
A
B
C
D
E
F
G
A FUNDAMENTAL
B 3RD HARMONIC
C FUNDAMENTAL PLUS 3RD HARMONIC
D 5TH HARMONIC
E FUNDAMENTAL PLUS 3RD AND 5TH HARMONIC
F 7TH HARMONIC
G FUNDAMENTAL PLUS 3RD, 5TH AND 7TH HARMONIC

Figure 6–2. Composition of Symmetrical Square Wave

Square Waves

Figure 6–2 (I) shows a square wave in which voltage is plotted against time. This wave is called *symmetrical* because the alternations are identical.

In terms of the first method of analysis just described, you can consider the square wave in the illustration as a voltage that remains unchanged at plus 50 volts until time t_1, when it suddenly drops to minus 50 volts; it remains at this value until time t_2, then suddenly changes to plus 50 volts and remains at this value until time t_3, and so on.

In terms of the sine wave method, you can analyze the square wave by determining what sine waves are required to reproduce it. To reproduce a symmetrical square wave, you start with a sine wave, which is equal to the square wave repetition frequency, and add to it the odd harmonics of this frequency as shown graphically in the illustration at II, III, and IV. Diagram II shows the waveshape C formed by adding the fundamental frequency A and its third harmonic B, which has an amplitude 1/3 that of the fundamental. Notice that the resultant wave already slightly resembles a square wave as you can see by the square wave superimposed on the diagram. Diagram III shows the result of adding the 5th harmonic of the fundamental frequency to the resultant wave C. Notice in the resultant wave E that the corners are much sharper and the top somewhat flatter.

In diagram IV, the 7th harmonic at 1/7 amplitude is added to form the resultant curve G. This curve is fairly smooth across the top and fairly sharp at the corners. Adding the 9th harmonic at 1/9 amplitude, the 11th harmonic at 1/11 amplitude, and so forth, would further sharpen the corners and flatten the top of the wave.

The assumption that a square wave contains a number of frequencies, although it is actually just a change of voltage, is useful in studying the square wave in circuits. For example, according to the frequency method of analysis the higher harmonics are responsible for the sharpness in the corners of the square wave. Therefore, rounding off the corners of the square wave can be explained as discriminating against high frequencies by the circuit. Furthermore, since this

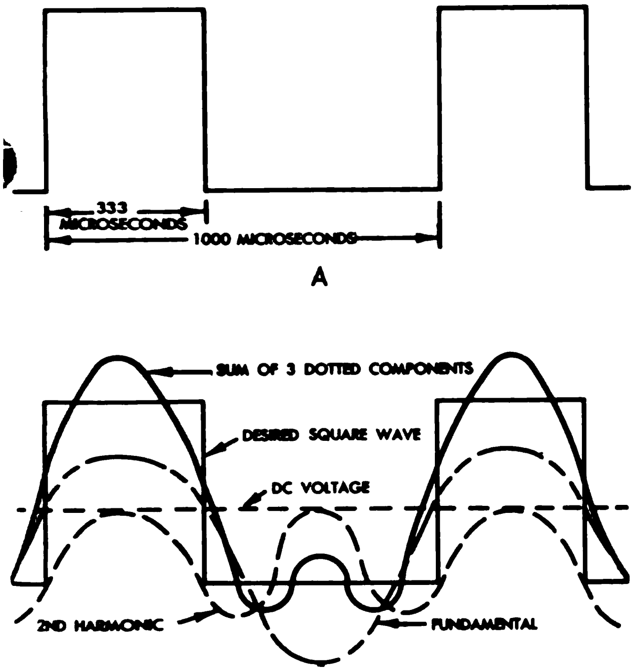


Figure 6–3. Composition of Square Wave with Short Duration Pulses

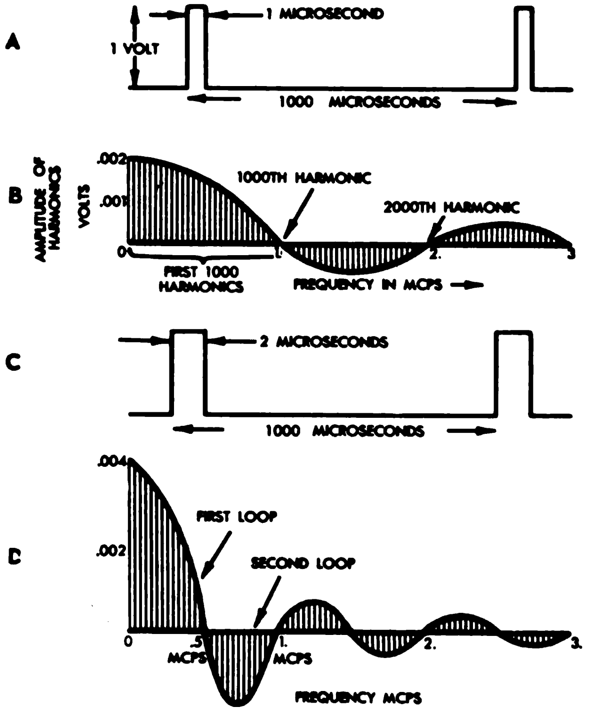


Figure 6–4. Harmonic Content of Short Square Pulses

method of analysis reduces a square wave to terms of sine waves of different frequencies, it is possible for engineers to design radar circuits by using conventional sine wave engineering practices.

Most square waves employed in radar circuits consist of short pulses separated by long time intervals, as shown in Figure 6–3A. Such pulses are constructed in the same way as square waves, but in addition to the sine waves of the fundamental pulse frequency and many harmonic frequencies of fractional amplitudes, a small DC voltage is required. In A, the duration of the pulse is one-third of the repetition time which, in this case, is 1000 microseconds.

To reconstruct this pulse, add a DC voltage equal to one-third of the desired pulse amplitude to a 1000 cps sine wave having an amplitude two-thirds of the pulse amplitude, a 2000 cps sine wave having an amplitude one-third of the pulse amplitude, etc. The solid curve in B represents the sum of all these voltages. Notice that there is a rough resemblance between the outline of the solid curve and the desired square wave. If you add all the harmonics up to about the 15th harmonic (15,000 cps), in the proper phase and amplitude, the resultant waveshape will be quite square.

An important fact to consider is the number of harmonics to include in forming the waveshape. The rule to follow is that the maximum number varies inversely with the pulse width. This means the wider the pulse, the fewer the harmonics; the narrower the pulse, the greater the number of harmonics. For example, in Figure 6–3, the width of the pulse is such that the first 15 harmonics can be included.

In Figure 6–4A a much narrower pulse is shown — a 1-microsecond square wave having a repeating frequency of 1000 pps. This pulse requires a great many more harmonics, as you can see from B. Here, the amplitude of each harmonic is plotted on the Y-axis, and its frequency is plotted on the X-axis. The tops of all the vertical lines are connected by a curve which forms a damped sine wave. It is impossible to show all the frequencies, since 1000 harmonics actually occur between zero and 1 megacycle.

When the pulse amplitude, shown at A, is 1 volt, the damped sine wave produced by connecting the amplitude of the harmonics is less than .002 volt. From the 1000th to the 2000th harmonic, it is 180° out of phase with the lower frequencies. The relative importance of the various frequencies depends upon the area under the sine wave curve, varying with the size of the area. Notice that the largest area occurs between zero and 1 mc. The significance of this is that any circuit which does not discriminate against any frequency from 1000 cps to 1 mc will pass this pulse with little distortion.

Figure 6–4D shows that doubling the pulse width to 2 microseconds (C) causes the amplitude of the harmonic frequencies to drop to zero at a half-megacycle. Thus, any circuit designed for operating with a 2-microsecond width will not discriminate against any frequency from 1000 cps to 500,000 cps.

All this leads to the conclusion that there is a certain relationship between the highest frequency that a circuit can handle and the width of the pulse. For example, in the case of the 1-microsecond pulse, the highest frequency was 1 mc, and in the case of the 2-microsecond pulse, 1/2 mc. We see that frequency varies inversely with pulse width. Mathematically this is expressed as,

$$F_h = \frac{1}{PW}$$

where F_h is the highest frequency for the circuit, and PW the pulse width in microseconds when the pulse width is much smaller than the pulse repetition period.

Note that the pulse repetition frequency is not considered in the equation, since it is quite evident by transient analysis that the pulse recurrence frequency has nothing to do with the harmonic frequencies in the pulse. For example, if the repetition frequency in Figure 6–4C changed to 500 cps, there would be twice as many harmonic frequencies in the first loop, but the loop still would pass through zero at 0.5 mc.

The lowest frequency that this circuit must be able to handle is the fundamental, or the pulse repetition, frequency. However, several of the low frequency harmonics can be removed from the curves in Figure 6–4B and D without noticeably reducing the areas. Thus a circuit may be designed for a 1-microsecond pulse and handle a frequency band as narrow as 10 kc to 1 mc.

Triangular and Sawtooth Waves

Triangular and sawtooth waves, like square waves, can be constructed with a series of sine waves. Circuits designed to handle the short sawtooth wave shown in Figure 6–5C must have about twice the bandwidth as a circuit required for a square pulse of the same dimensions.

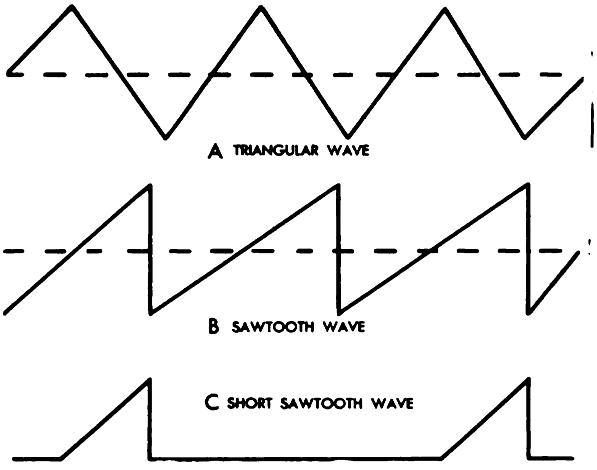


Figure 6–5. Triangular and Sawtooth Waves

A sawtooth wave contains both odd and even harmonics. The wave in Figure 6–6D, for example, contains all the harmonics up to the 7th. Notice how closely this wave resembles the sawtooth wave superimposed on it.

From the viewpoint of a transient voltage, you can consider the sawtooth voltage wave a slow linear voltage change from an initial value to a certain higher value, followed immediately by a sudden drop back to the initial value. The wave may repeat immediately or repeat after an interval of time has passed.

Peaked Wave

A peaked wave is another wave which can be constructed with a number of sine waves. Figure 6–7 shows how the addition of successive odd harmonics to a fundamental sine wave produces a peaked wave. In I, the 3rd harmonic, a wave having a lower amplitude, is added to the fundamental. During the one cycle of the fundamental, the 3rd harmonic is both in phase and out of phase with the fundamental, starting 180° out of phase. This produces a wave C with a higher peak and steeper sides than the fundamental. Next, the 5th harmonic is added to wave C. This produces a wave E in which the peak is further heightened and the sides further sloped. This process is carried further, as illustrated, with the addition of each harmonic producing a wave with higher and higher peaks and steeper and steeper

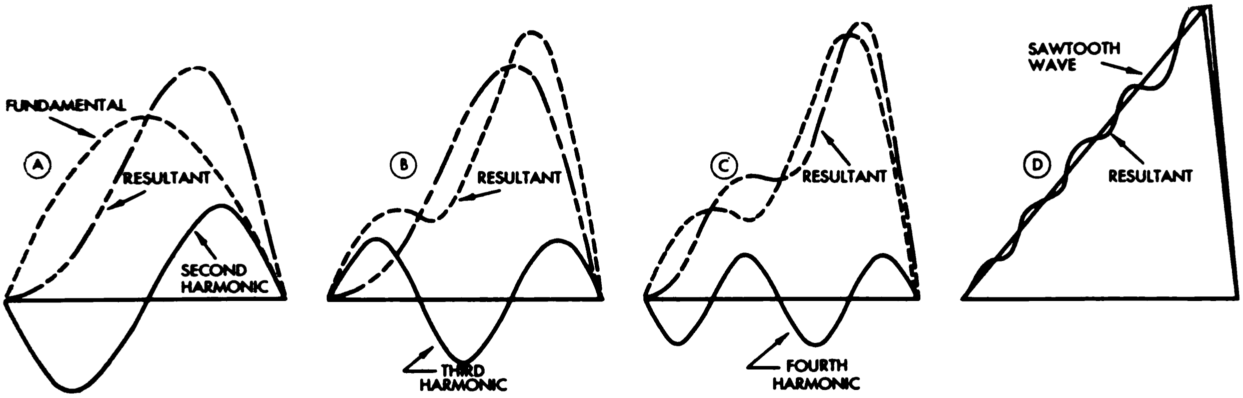


Figure 6–6. Composition of Sawtooth Wave

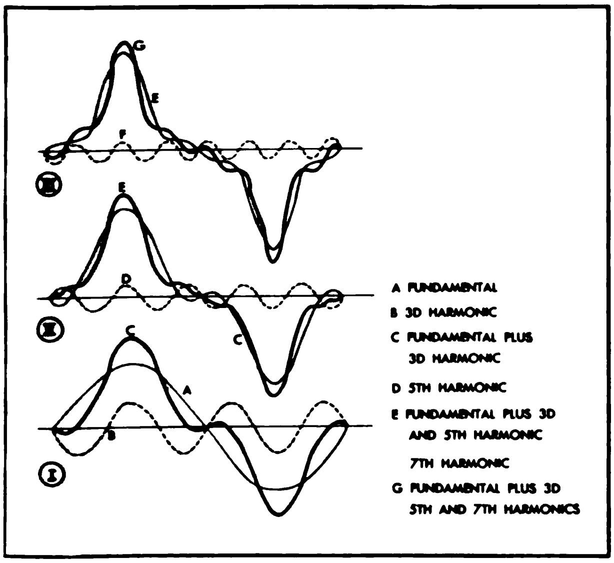

G
E
F
Ⅲ
E
D
Ⅱ
C
C
A
B
Ⅰ
A FUNDAMENTAL
B 3D HARMONIC
C FUNDAMENTAL PLUS 3D HARMONIC
D 5TH HARMONIC
E FUNDAMENTAL PLUS 3D AND 5TH HARMONIC
7TH HARMONIC
G FUNDAMENTAL PLUS 3D 5TH AND 7TH HARMONICS

Figure 6–7. Composition of Peaked Wave

sides. The resulting waveshape depends on the harmonics that are present, their relative amplitudes, and relative phase relationship.

TRANSIENT VOLTAGES IN RC CIRCUITS

The characteristics of the waveforms required by radar, such as slope, duration, or repetition frequency, are determined largely by controlling the variation of a voltage with respect to time. Since the charging or discharging of a capacitor through a resistor requires time, resistance-capacitance circuits are commonly used in radar.

To learn how a resistor-capacitor circuit affects a waveshape applied to it, it is well to investigate first the voltage and current changes that occur when a DC voltage is applied to the circuit. Most RC circuits and the voltages applied to them are reducible to a simple equivalent circuit containing a battery, a capacitor, and a resistor.

Figure 6–8A shows a simple series circuit consisting of a battery, a switch, a capacitor, and a resistor. According to capacitor theory, capacitor C offers infinite resistance to the direct current flow in the circuit, since it charges to a voltage equal and opposite to the battery voltage E_B when the switch is closed. When the capacitor is charged, the battery voltage is cancelled completely, leaving no voltage to cause a current to flow through the resistor. However, before the current does reach zero, time is consumed. This consumption of time provides the time element required in the formation of many radar waveshapes.

To understand how a capacitor affects the time delay in a circuit, first consider what happens in the capacitor itself and how it is affected by the rest of the circuit. Assume that, before you close switch Sw, the capacitor is completely discharged — that is, there are an equal number of electrons on each of its plates. When you close the switch, the battery voltage E_B immediately forces electrons into the capacitor plate connected to the negative terminal of the battery, placing a negative charge on it. This creates a stress on the dielectric that, in turn, forces electrons away from the other plate and makes its charge positive.

It might seem that if it were not for the limiting effect of the series resistor, the number of electrons that would accumulate on the capacitor plates would be infinitely high because, upon the closing of the switch, the current in the circuit

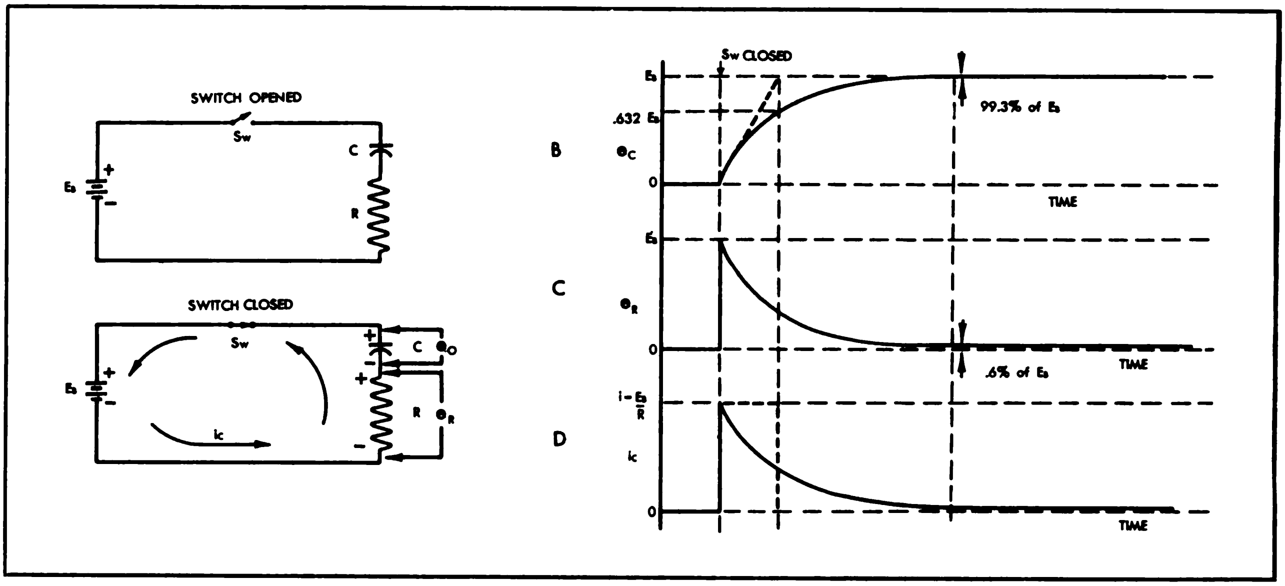


Figure 6–8. Current and Voltage During Capacitor Charge

would immediately jump from zero to a value determined only by the resistor. (That is, $i = \frac{E_B}{R}$ since the uncharged capacitor offers no opposition to current flow.) The fact is, however, that the capacitor itself offers opposition to the current. Just as soon as a few electrons accumulate on the capacitor plate, there is a like number forced away from the other plate. The net result is a small voltage difference across the capacitor, the plate with more electrons being the negative point of voltage, and the plate with less electrons being the positive point of voltage.

The polarity of the circuit then becomes as shown in Figure 6–8A. Notice here that the capacitor is charged to a voltage opposite in polarity to the battery voltage. Thus, in a short time after you close the switch, the movement of electrons — current flow — does not remain at its initial value, but drops to a lower value because the voltage across the capacitor subtracts from the charging voltage. Since the voltage applied to the capacitor is thus effectively reduced, it charges at a lower rate.

Now consider the action resulting during the further charge at a slow rate up to the point of so-called complete charge. (Theoretically the capacitor never completely charges to 100% charge.) When the switch was first closed as shown in Figure 6–8A, the capacitor charged rapidly to 63.2% of the applied voltage. At this point the current flow in the circuit was 36.8% of its initial value, and the resistor voltage 36.8% of the applied voltage. With the continuous increase in the voltage difference across the capacitor and its subtraction from the voltage applied to it, the charge *rate* tapers off, but the charge itself rises exponentially to the point where the capacitor is considered charged.

Notice the voltage and current curves shown at B, C, and D in Figure 6–8. Had the capacitor charged at its original rate, its charge curve would be the dotted line at B. The slope of this capacitor charge curve is expressed as E_B/RC.

The term RC in this ratio is called the time constant (TC) of an RC circuit. It is the time required to charge a capacitor to 63.2% of its final voltage. To find the time constant in seconds, multiply R, the resistance in ohms, by C, the capacitance in farads.

The capacitor voltage curve in Figure 6–8B and the instantaneous current curve in Figure 6–8D are exponential curves. An exponential curve is the graph of an equation in which a variable appears as an exponent.

The instantaneous resistor voltage curve in Figure 6–8C can be found from the instantaneous current values by using the Ohm's law formula, E = IR. Since R is constant, E, the voltage across the resistor, is directly proportional to the current through it. Therefore, its curve has

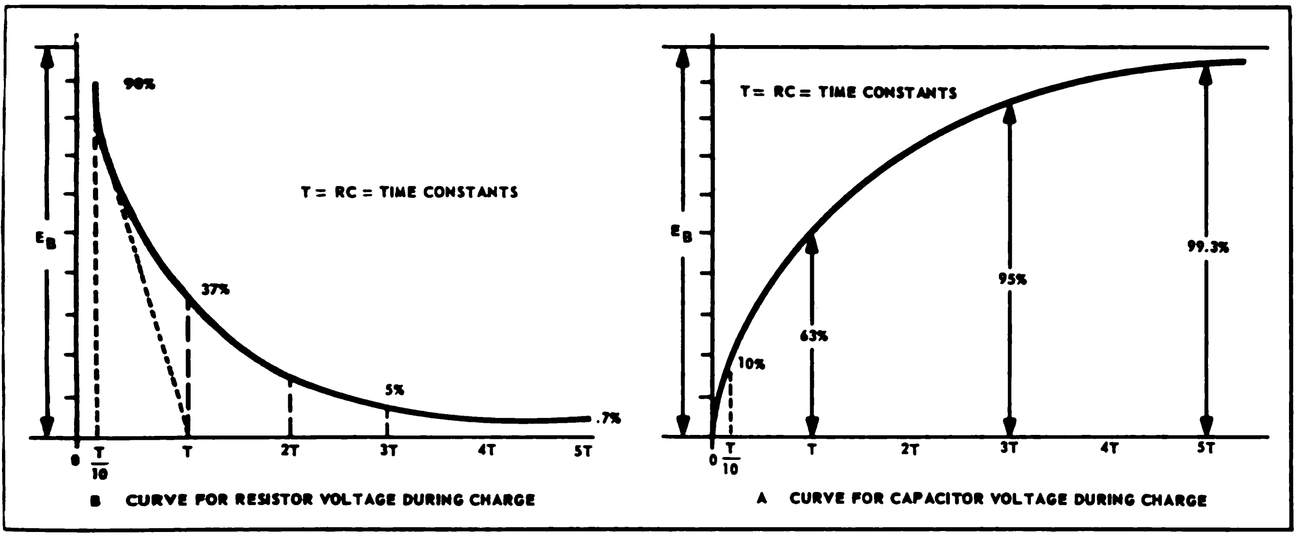


Figure 6–9. Exponential Curves for Rough Approximations

the same exponential slope as the instantaneous current curve.

Quantitative Values of Capacitor and Resistor Voltages

There are three methods of determining the instantaneous values of voltage across capacitors and resistors in RC circuits. The first is a rough estimate from the exponential curve with the values at four main points known. The second is a close approximation from a very accurate exponential curve on a universal time constant chart. The third involves solving the equation for the curve with the quantities for the RC problem inserted in the equation.

ROUGH APPROXIMATION CURVES. The voltage or current at any instant during the charging time of a capacitor can be approximated from the two curves shown in Figure 6–9. These curves employ only rough values, but it is well that you memorize them, since they occur frequently. They are also useful for approximating current at any instant during the charge of a capacitor. To find current, substitute the total current flow in the circuit for E_B on curve B.

With a known value of resistance and capacitance, it is simple mathematics to compute one time constant. After having found the time constant(s), extend up to the curve in the rough approximation chart in Figure 6–9A, and then across to the vertical column to find the percent of voltage or current. Roughly compute to find the instantaneous voltage using the percentage and known voltage. The same principle can be used in chart B for either resistor voltage or total current. The principle is reversed for time constants using instantaneous voltage or current. This is done by approximating the value on the vertical column, finding the corresponding point on the curve and extending down to the horizontal (time constant) line.

Example

Problem

Assume the RC is two time constants; find the instantaneous capacitor and resistor voltages. The applied voltage is 300 volts.

Solution

a. Extend a line from 2T on the horizontal line upward to connect with curve A and then across to the vertical column, which is in percentage; you will find it to be approximately 84%. Mental computation, multiplying 84% times 300 (84 × 3), gives 252 volts, the capacitor voltage.

b. Extend a line from 2T on the horizontal line upward to connect with curve B and then across to the vertical column, which is in percentage; you will find it to be approximately 13%. Multiplying 13% times 300 (13 × 3) gives 39 volts, the resistor voltage. Adding 252v (solution a) to 39v only gives 291v, whereas the total should be 300 volts, but remember, this is a rough approximation.

UNIVERSAL TIME CONSTANT CURVES. A universal time constant chart, Figure 6–10, provides a much more precise method than the method just described for determining the voltage and

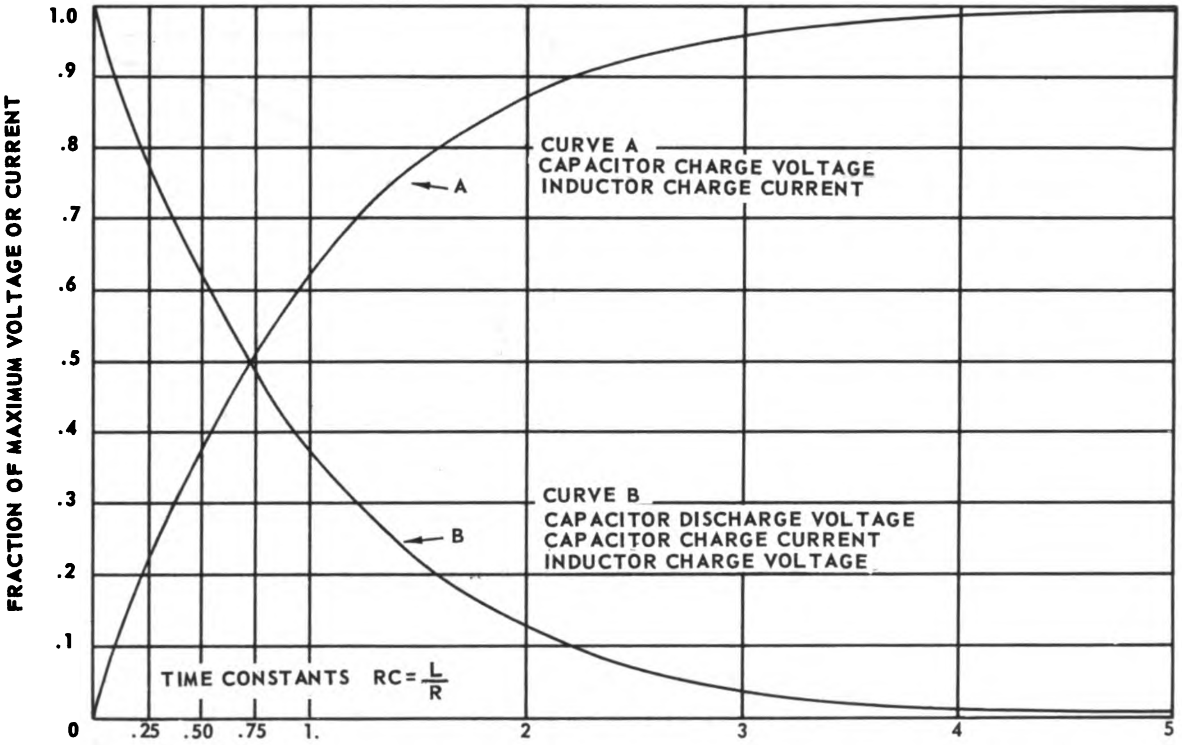


Figure 6–10. Universal Time Constant Chart

the current at any instant during the charge of a capacitor. The chart shows a number of time constants plotted against the fraction of maximum current or voltage. In learning to use this chart, study the following examples:

Examples

Problem 1

In the circuit in Figure 6–11, assume that C equals 0.001 mf and that R equals 100 K. Find the capacitor voltage 200 microseconds after the switch is closed.

Solution

a. Find the time constant.

$$\begin{aligned} TC &= RC \\ TC &= 100{,}000 \times .001 \\ &= 100 \text{ microseconds} \end{aligned}$$

b. Since 200 microseconds is equivalent to 2 time constants, enter the universal time constant chart at 2 along the bottom and proceed up to curve A. (This curve represents the capacitor charge voltage.) At curve A, proceed horizontally to the vertical scale and read 0.86. This represents the fraction of maximum voltage.

c. Thus, the capacitor voltage is about 86% of maximum. With 200 volts as the applied voltage, the capacitor voltage equals 200 × 0.86 or 172 volts.

Problem 2

In the same circuit, find the resistor voltage 200 microseconds after the switch is closed.

Solution 1

Using Kirchhoff's Law.

From the previous example, the capacitor voltage equals 172v. Since the circuit is a series circuit, the resistor voltage is the difference between the applied voltage and the capacitor voltage. Thus, it is 200—172, or 28 volts.

Solution 2

Using the universal time chart.

Knowing that the resistor voltage starts high and then drops off, you use curve B. Since there are two time constants involved, go up from 2 to curve B and then across to the vertical scale. At the intersection you find 0.14. Since the applied voltage is 200 volts, the resistor voltage must be 200 × 0.14 or 28 volts.

Solution 3

Using universal chart, find current.

By Ohm's law the maximum current is $\frac{200}{100K}$ or 2 ma.

At two time constants the current is 0.14 of maximum, according to curve B and the scale at the left. Therefore, it equals 0.14 × 2 ma or 0.28 ma. By Ohm's law, $E = IR$. The instantaneous voltage is 0.28 ma × 100 K or 28 volts.

Problem 3

In the same circuit, find how long it takes for the resistor voltage to drop to 120 volts.

Solution

120 volts is $\frac{120}{200}$ or 0.6 of 200 volts. Find 0.6 on the

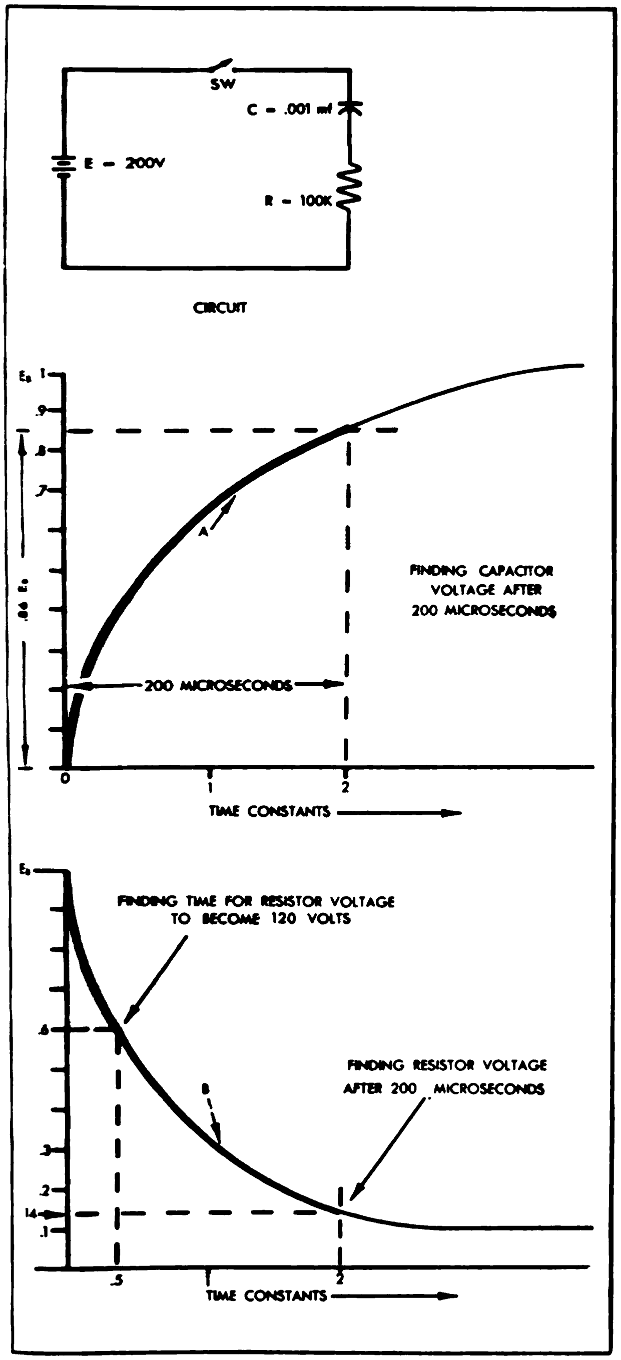


Figure 6–11. Using Universal Time Constant Chart

vertical scale, go to curve B, thence down to 0.5 time constants. If one time constant equals 100 microseconds, the time required for the resistor voltage to drop to 120 volts must equal 0.5 × 100 or 50 microseconds.

NOTE

Any value of voltage and any size of resistor or capacitor can be used with the universal time constant chart. Time can be long or short, providing all units have the same time constant. In Figure 6–11, after 5 time constants the capacitor is considered to be fully charged, and the resistor voltage and the circuit current are considered to be zero.

Exponential Equation. Another method of determining values of voltage across a capacitor and a resistor in a RC circuit is by use of the exponential equation for the instantaneous voltage curves. The exponential equation for the resistor voltage curve B in Figure 6–12 is,

$$e_R = E_B \epsilon^{\frac{-t}{RC}}$$

where e_R = instantaneous resistor voltage
E_B = the applied voltage
ϵ = the base of natural logarithms, 2.718
t = time in seconds
R = resistance in ohms
C = capacitance in farads
$\frac{t}{RC}$ = number of time constants

This equation leads to much more precise results than a universal time constant resistor voltage curve. For example, from the time constant curve you find that the resistor voltage reaches 37% of the applied voltage after one time constant. For purpose of comparison, determine the voltage across the resistor at this time by the exponential equation just given

$$\left(e_R = E_B \epsilon^{\frac{-t}{RC}}\right)$$

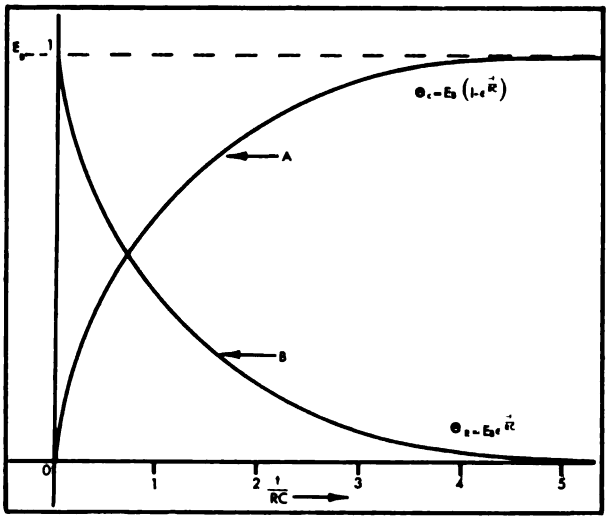


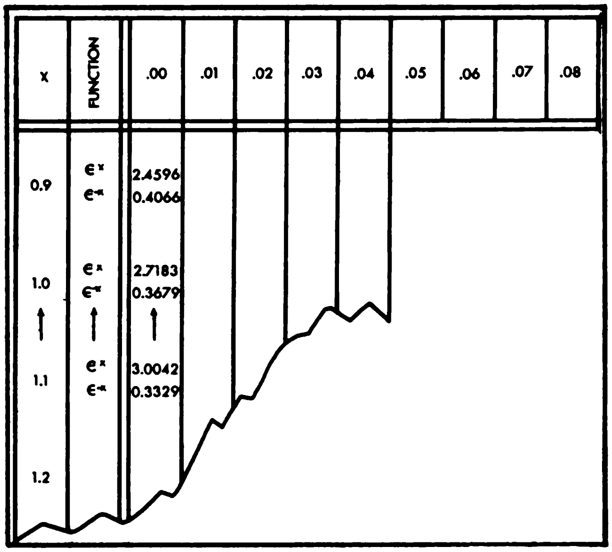
Figure 6–12. Exponential Equations of Voltage Curves

Since the number of time constants is 1, $e_R = E_B\epsilon^{-1}$.
From tables of hyperbolic functions (Figure 6–13),
ϵ^{-1} equals 0.3679
Therefore, substituting,

$$e_R = E_B \times 0.3679$$

NOTE

Finding the value of $\epsilon^{\frac{-t}{RC}}$ *requires the use of tables of hyperbolic functions of which an extract is shown in Figure 6–13. Thus, in finding the value of* $\epsilon^{\frac{-t}{RC}}$ *in the example, substitute 1, which is the number of time constants, for* $\frac{-t}{RC}$*. The expression then becomes* ϵ^{-1}*. Referring to the table, notice that when x (the exponent) equals 1, the function* ϵ^{-1} *equals 0.3679 as is indicated by the arrows.*



X	FUNCTION	.00	.01	.02	.03	.04	.05	.06	.07	.08
0.9	ϵ^{x} ϵ^{-x}	2.4596 0.4066								
1.0	ϵ^{x} ϵ^{-x}	2.7183 0.3679								
↑	↑	↑								
1.1	ϵ^{x} ϵ^{-x}	3.0042 0.3329								
1.2										

Figure 6–13. Extract from Tables of Hyperbolic Functions

According to the universal time constant chart curve, the voltage of e_R was indicated as approximately 37%. However, the equation and the table show it to be exactly 36.79% of the applied voltage at the end of one time constant. Therefore, for greatest accuracy, use the exponential equation method.

The equation for curve A, the instantaneous capacitor voltage charge curve is,

$$e_c = E_B\left(1 - \epsilon^{\frac{-t}{RC}}\right)$$

where e_c = capacitor voltage at any time, t.

Examples

To understand the use of these exponential equations, study the following examples and their solutions:

Problem 1

In the circuit in Figure 6–11, find the capacitor voltage 200 microseconds after the switch is closed.

Solution

$$e_c = E_B\,(1 - \epsilon^{-t/RC})$$

Substituting the values in the problem,

$$e_c = 200\left(1 - \epsilon^{\frac{-2 \times 10^{-4}}{10^{-9} \times 10^{5}}}\right)$$
$$= 200\,(1 - \epsilon^{-2})$$

From the tables, $\epsilon^{-2} = 0.1353$

Substituting,

$$e_c = 200\,(1 - 0.1353)$$
$$= 200 \times 0.8647 = 172.94 \text{ volts}$$

Problem 2

Using the same circuit and circuit values, find the resistor voltage 200 microseconds after the switch is closed.

Solution

$$e_R = E_B\epsilon^{-t/RC}$$

Substituting the values in this problem,

$$e_R = 200\,\epsilon^{\frac{-2 \times 10^{-4}}{10^{-9} \times 10^{5}}}$$
$$= 200\,\epsilon^{-2}$$

From the tables, $\epsilon^{-2} = 0.1353$

Substituting,

$$e_R = 200 \times 0.1353 = 27.06 \text{ volts}$$

Problem 3

Find how long it will take for the resistor voltage to drop to 120 volts. Assume that e_R = 120 volts.

Solution

$$e_R = E_B\epsilon^{-t/RC}$$

Transposing, $\frac{e_R}{E_B} = \epsilon^{-t/RC}$

Substituting, $\frac{120}{200} = \epsilon^{-t/RC}$, or $0.6 = \epsilon^{-t/RC}$

From the tables $\epsilon^{-x} = 0.6$ when x = 0.51 (x represents the exponent of ϵ)

Therefore,

$$\epsilon^{-t/RC} = \epsilon^{-0.31}$$
$$-t/RC = -0.51$$

Changing signs and transposing:

$$t = 0.51\,RC$$
$$= 0.51 \times 10^{5} \times 10^{-9}$$
$$\doteq 0.51 \times 10^{-4}$$
$$= 51 \text{ microseconds}$$

You can find the values for ϵ^{-x} on the more complete slide rules by placing the hairline on the ratio t/RC on the A scale, and reading the value $\epsilon^{-t/RC}$ on the LL00 or LL0 scales. The

LL00 scale reads values for $\epsilon^{-t/RC}$ ratios between 0.1 and 10. To familiarize yourself with the meaning of the slide rule values, it is a good idea at first to check the slide rule against the tables or the universal curve.

Capacitor Discharge Through Resistance

The discharge of a capacitor through a resistor follows the same exponential curve as the charge through a resistor as you can see in Figure 6–14. When the capacitor is charged to a voltage of E_c, there is an excess of electrons on one plate of the capacitor. This unbalanced condition creates a potential difference that exists as long as the switch is open (assuming the capacitor is perfect). The portions of the curves indicating open switch condition show that the capacitor voltage is E_c, the current in the circuit is zero, and the resistor voltage is also zero.

When the switch is closed, the electrons on the negative plate immediately surge through the circuit toward the other capacitor plate, building up a current that is limited to a value determined by Ohm's law — that is, a value equal to E_c/R. This is what happens: The initial current causes a voltage drop across the resistor equal to the capacitor voltage. But as soon as current flows, the number of excess electrons causing the current flow decreases. This causes a voltage decrease that, in turn, decreases the current. The decrease is not linear, however. The high initial current decreases the voltage rapidly, but subsequent lower voltages cause lower currents, which removes the charge at a lower rate. The discharge becomes slower and slower until at the end of five time constants, the current is less than 1% of the initial value. The voltage causing this low current is also extremely low because the capacitor is 99.4% discharged.

Theoretically, the time required to discharge the capacitor is infinitely long, but for practical purposes, five time constants may be considered as the time required for complete discharge.

To determine instantaneous values of current and voltage in RC circuits during capacitor discharge, use the same methods that you used previously in the charging circuit problems. One fact of significance in discharging circuits is that the voltages across the resistor and the capacitor follow the same curve inasmuch as they both start at high values and decrease exponentially to zero. Therefore, Curve B of the Universal Time Constant Chart, Figure 6–10, represents the instantaneous values of voltage both across the capacitor and the resistor during capacitor discharge. The only requirement for using it for both voltages is assigning e_R a polarity opposite that of e_c. Remember that the voltage across the resistor is not only equal to but opposite that across the capacitor when the capacitor is discharging. This is as it should be according to Kirchhoff's law, which states that the sum of the voltages around a series circuit is zero.

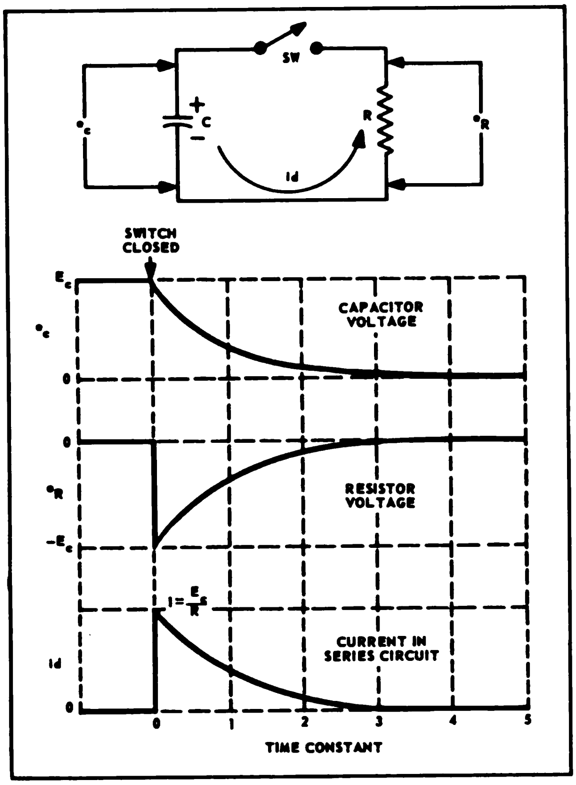


Figure 6–14. Current and Voltage During Capacitor Discharge

As just mentioned, you can use any of the three methods given earlier to solve for the voltage and current values — that is, you can use the curves which roughly approximate voltages and currents, the universal curves, or the exponential equations.

The following problems are solved by all three methods. They are first solved by the less precise approximation and universal time constant curves, and then by exponential equations. Note that the results obtained by the exponential

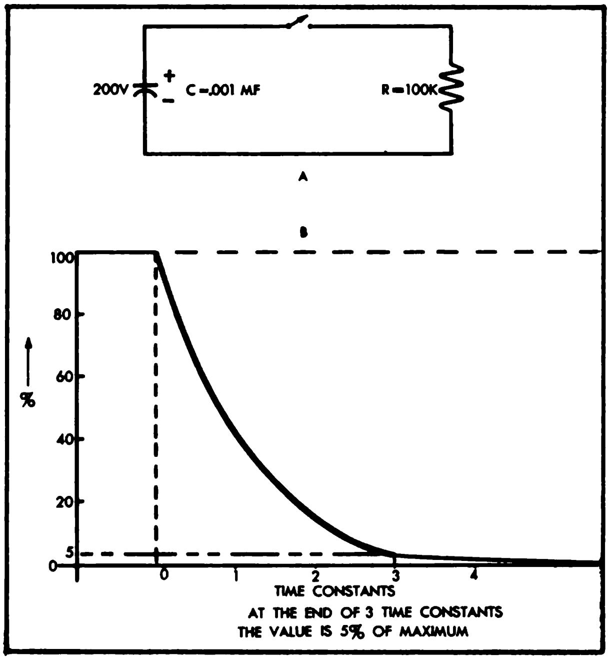


Figure 6–15. Capacitor Discharge

equations differ slightly from the answers obtained by use of the curves. The differences are due, of course, to the greater degree of accuracy obtainable by the equation method.

Examples

Problem 1

Assume that in the circuit in Figure 6–15, the capacitor is charged to 200 volts when the switch is closed. Find the capacitor voltage 300 microseconds after the switch is closed.

Solution

First find time constant by the equation, TC = RC. Substituting the values indicated in circuit,

$$TC = 100\ K \times .001\ mf$$
$$TC = 10^5 \times 10^{-9} = 10^{-4}$$

Therefore, TC = 100 microseconds

Since the time constant is 100 microseconds and since the problem requires 300 microseconds, the capacitor discharges for 3 time constants. Turning to Figure 6–15B, refer to the heavy lined curve. There you see that at 3 time constants the capacitor has discharged to 5% of E_c or .05 × 200, or 10 volts. Thus, after 300 microseconds, the capacitor voltage is 10 volts.

Problem 2

Find the current 200 microseconds after the switch is closed.

Solution

The current decreases from an initial value of 2 ma $\left(\text{calculated by Ohm's law, } \frac{E}{R} = \frac{200v}{100K}\right)$. The time of discharging is 200 microseconds, or 2 time constants. Refer to the universal time constant curve B, Figure 6–10; the value of current at 2 time constants is 14% of its maximum value. Since 14% of 2 ma is 0.28 ma, the current after 200 ms is 0.28 ma.

Problem 3

Find the resistor voltage after one time constant, using time constant curve B.

Solution

After 1 time constant, the voltage drops to 37% of its maximum value. Since the resistor voltage starts at 200 volts and drops to zero when the capacitor is completely discharged, it is 37% of 200 volts, or 74 volts, 1 time constant after discharge starts.

Notice here that curve B, which is the capacitor discharge voltage curve, also is used for voltage decrease across the resistor during discharge.

In problems concerning RC voltage values, the following exponential equations apply as indicated:

1. Instantaneous resistor voltage:

$$e_R = E_c \epsilon^{\frac{-t}{RC}}$$

2. Instantaneous capacitor voltage:

$$e_c = E_c \epsilon^{\frac{-t}{RC}}$$

3. Instantaneous discharge current:

$$i_d = \frac{E_c}{R} \epsilon^{\frac{-t}{RC}}$$

In each equation, E_c represents the initial capacitor charge voltage.

Examples

Problem 1

Find e_c when t equals 300 microseconds.

Solution

$$e_c = E_c \epsilon^{-t/RC}$$
$$e_c = 200\ \epsilon^{\frac{-3 \times 10^{-4}}{10^5 \times 10^{-9}}}$$
$$e_c = 200\ \epsilon^{-3}$$
$$\epsilon^{-3} = .0498 \text{ (from tables)}$$
$$e_c = 200 \times .0498$$
$$e_c = 9.960 \text{ volts.}$$

Problem 2

Find i_d when t = 200 microseconds.

Solution

$$i_d = \frac{E_c}{R} \epsilon^{-t/RC}$$
$$= \frac{200}{10^5} \epsilon^{\frac{-2 \times 10^{-4}}{10^5 \times 10^{-9}}}$$
$$= .002\ \epsilon^{-2}$$
$$\epsilon^{-2} = 0.135$$
$$i_d = .002 \times 0.135$$
$$= 0.27 \text{ ma}$$

Problem 3

Find e_R when $t = RC$ (time constant = 1)

Solution

$$e_R = E_b \epsilon^{-t/RC}$$

$$\text{if } t = RC, \text{ then } t/RC = 1$$

$$e_R = 200\,\epsilon^{-1}$$

$$\epsilon^{-1} = 0.368 \text{ (from table)}$$

$$e_R = 200 \times 0.368 = 73.6 \text{ volts}$$

Effect of Varying R and C

Thus far, the RC curves illustrated were drawn separately but in Figure 6–16 three curves are superimposed on one another for the purpose of showing what effect changing the time constant (RC) has on the waveshape produced by an RC circuit. Each curve represents an RC circuit having the same applied voltage but a different time constant. Naturally, in all RC circuits, values reach maximum (or zero) after 5 time constants. However, since the duration of each time constant is different, the curves differ in duration.

The curve labeled 20 microseconds, for example, is produced by a small RC, that is, a small capacitor or resistor. A small capacitor has the property of quick charging, and a small resistor connected to it permits a high charging current to flow, further facilitating rapid charging. Due to the small RC, this curve reaches its maximum limit quickly or, in approximately 100 microseconds. In comparison, the time required to charge the capacitor is much shorter than with the 100- or 200-microsecond curves.

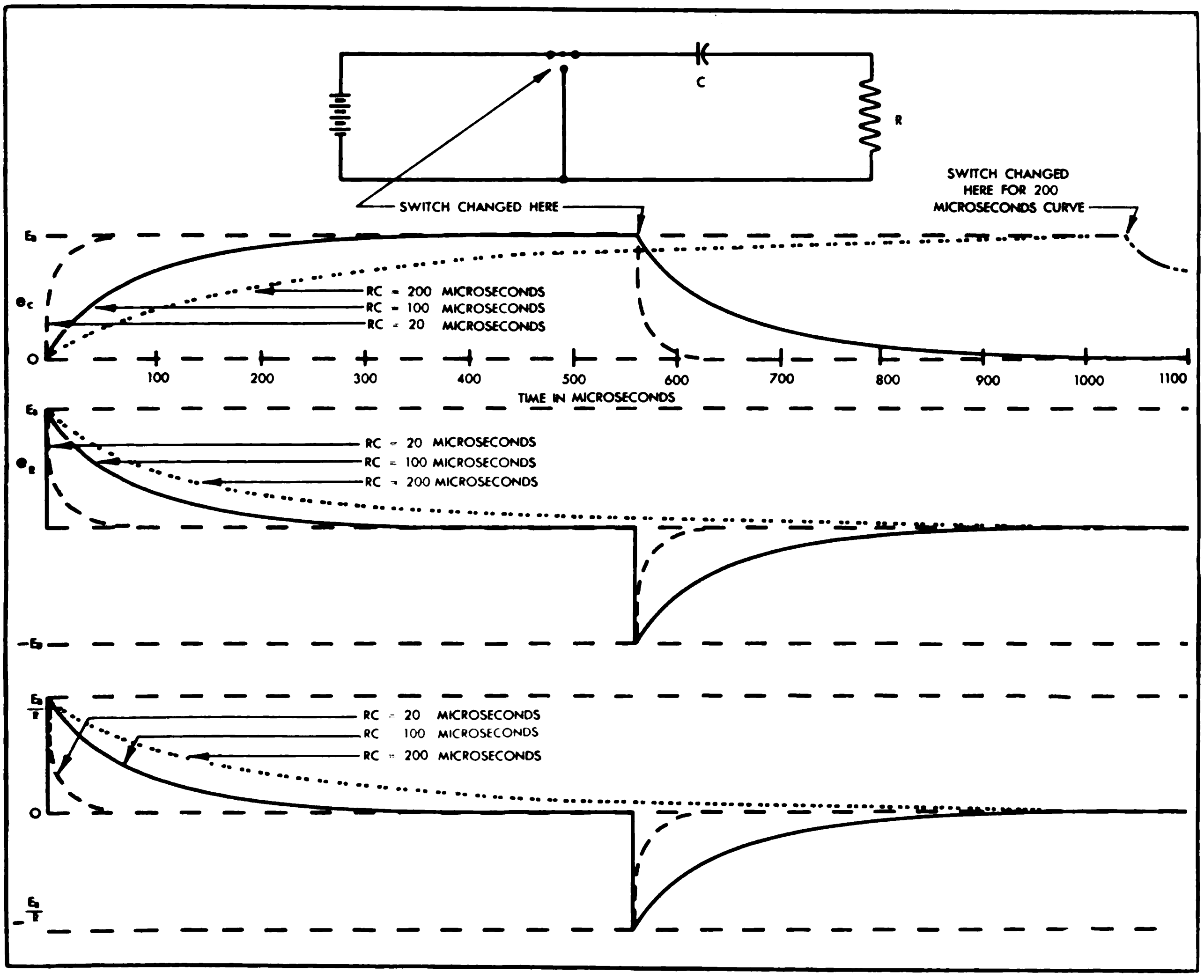


Figure 6–16. Effect of Varying R and C in RC Circuit

In Figure 6–16 a large RC value produces a time constant that is long in comparison to a small value of RC in the same circuit. A larger capacitor requires more current to charge it completely, while a larger resistor limits the current more and thereby increases the time required for charging. Thus, the curve labeled 200 microseconds has a long time constant when compared to the other two waveshapes. Notice that 1000 microseconds are required to fully charge the capacitor.

Notice that the e_R and i curves both follow the capacitor charge curve very closely. The first half of each of these curves represents the capacitor charge, and the last half the capacitor discharge. The difference between the two halves is due to the position of the switch.

Effect of a Medium Time Constant. Since most RC circuits in radar sets are associated with high speed timing devices, the voltages generally applied to an RC circuit are rapidly repeating square wave voltages rather than DC voltages. Figure 6–17 shows the capacitor and resistor voltages resulting from applying a square wave having a period of five time constants to the circuit at the top. For the purpose of analyzing the circuit, the square wave generator is redrawn into two equivalent circuits. They will make it easier for you to understand the action of the generator, providing that you regard the square wave first as a constant voltage lasting for a definite time, and then as a zero voltage lasting for an equal time.

Since the constant voltage equals the battery voltage during the first half of the square wave, the equivalent circuit for this time period (t_1) shows the battery voltage. During the second half of the square wave, or the zero voltage period, the capacitor discharge current flows through the generator. It is therefore not necessary to include the battery in the equivalent circuit for time t_2. The battery, however, must be replaced by whatever resistance the discharge current encounters in the generator, which in the case being considered is zero resistance.

The time constant used in Figure 6–17 is called a medium time constant. Actually any distinction between long, short, or medium time constants is arbitrary. Generally, the practice is to consider that a circuit has a long time constant when the RC product is equal to, or greater than, ten times the time of one alternation of the applied voltage; a short time constant when the RC product is equal to, or less than, one-tenth of the time of one alternation of the applied wave; and a medium time constant when the RC product lies between these two limits.

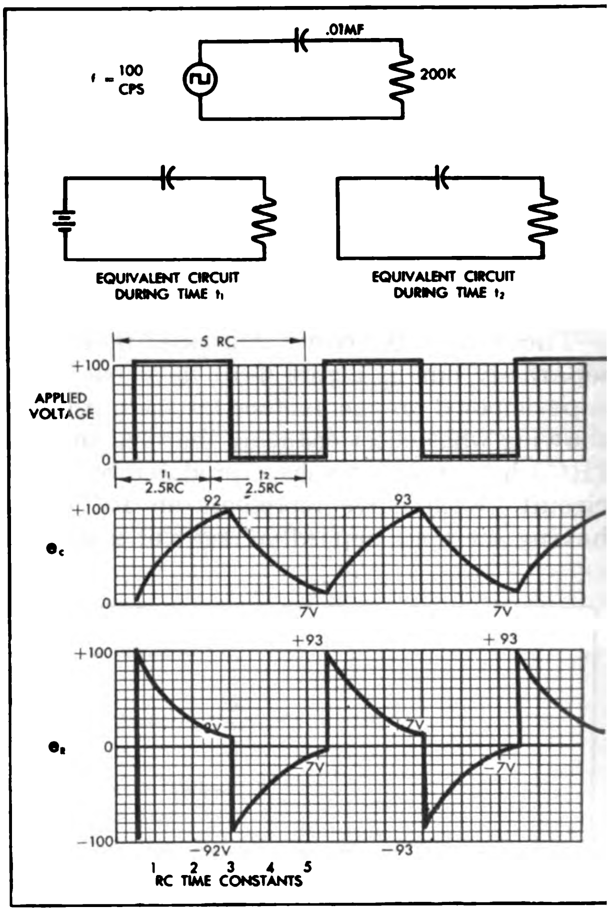


Figure 6–17. Effect of Medium Time Constant on Square Wave Input

Effect of a Short Time Constant. Consider now the effect of a short time constant on an RC circuit. The square wave shown in Figure 6–18 is applied to a resistance-capacitance combination with a time constant equal to .02 of a frequency cycle. Thus, in 20% of a half-cycle, five time constants will occur. If you assume as before that the square wave is a DC voltage applied for a short time and then shorted out, you can apply the rules you used previously. These stated that the charging rate for a capacitor with a short time constant is very high at first and then drops slowly to zero. Similarly, its discharge rate is very high to start with and then gradually decreases to zero.

The early rise of the capacitor voltage to the full applied voltage and its quick drop to zero cause the capacitor waveshape to resemble the square wave input. On the resistor, the short duration of the high charging and discharging currents has a quite different effect. The rapid rise and drop of the resistor voltage cause the resistor waveshape voltage to be peaked at each square wave changeover (change from maximum voltage to zero or from zero to maximum).

These sharp peaks are excellent for initiating action in another circuit, as, for example, in a radar transmitter that is designed to transmit a one-microsecond pulse whenever it is triggered by a positive voltage. The sharp positive pulses shown in Figure 6–18 may be used for this purpose since they occur at precisely the changeover time in the square wave. Referring again to Figure 6–18, notice that the universal exponential curve occurs over and over in the waveshapes. You can readily determine the time and amplitude of each curve by inserting the values for each curve from the universal time constant curve.

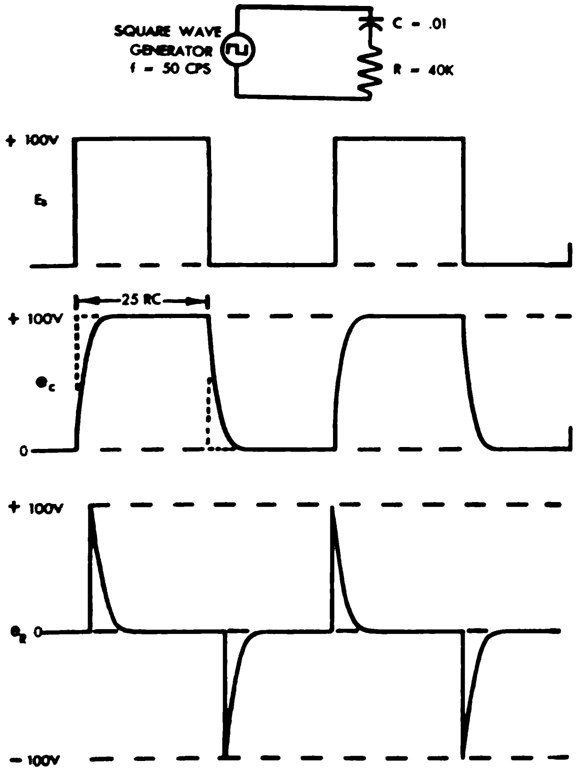


Figure 6–18. Waveshapes for Short Time Constant RC Circuit

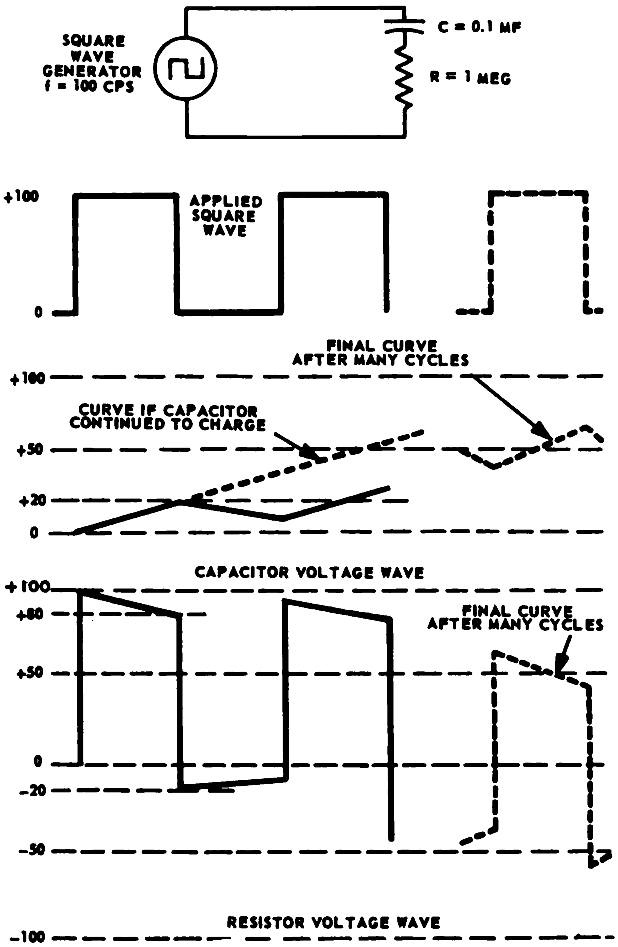

SQUARE WAVE GENERATOR f = 100 CPS
C = 0.1 MF
R = 1 MEG
+100
APPLIED SQUARE WAVE
0
FINAL CURVE AFTER MANY CYCLES
+100
CURVE IF CAPACITOR CONTINUED TO CHARGE
+50
+20
0
CAPACITOR VOLTAGE WAVE
+100
+80
FINAL CURVE AFTER MANY CYCLES
+50
0
-20
-50
RESISTOR VOLTAGE WAVE
-100

Figure 6–19. Waveshapes for Long Time Constant RC Circuit

Effect of a Long Time Constant. In Figure 6–19, which shows the waveshapes produced by an RC with a long time constant, the time constant is ten times as long as the period. On referring to the universal curve, you will see that the capacitor in the circuit will charge almost linearly for about 20% of a time constant. This causes the capacitor curve in the case illustrated here to be linear during the entire half-cycle. The dashed curve shows how the capacitor would have continued to charge if the applied voltage had not been removed. Note how the voltages in the circuit illustrated comply with Kirchhoff's law, which states that the sum of voltages around the circuit must equal the applied voltage.

During the first half-cycle, the applied voltage is 100 volts. Although the resistor and capacitor voltages are constantly changing, their sum at any one instant is also 100 volts. Thus, at the middle of the first half-cycle, for example, the capacitor voltage is 10 volts, and the resistor voltage is 90 volts, making a total of 100 volts and fulfilling Kirchhoff's law. During the other half-cycle, the applied voltage from the generator is zero. In this case the only voltage in the circuit is the capacitor voltage, which starts out at 20 volts and discharges 20%. As a result, the resistor voltage becomes minus 20 volts making all voltages around the circuit equal to zero and again fulfilling Kirchhoff's law.

The applied voltage in Figure 6–19 may either be a positive voltage or a voltage having a 50-volt DC component plus a great number of AC harmonics. In the latter case the action of the capacitor is to charge to the DC value after a number of cycles, this action being noticeable only during the first few cycles. Since a capacitor does not pass a DC voltage, the resistor voltage appears as pure AC as soon as the capacitor is completely charged. You can see this from the resistor voltage wave in Figure 6–19. Such a curve thus formed varies an equal amount on each side of the zero axis. Another important fact about this curve is that since each half-cycle is a replica of the exponential curve, you can determine instantaneous values of voltages either by the exponential equations or by the universal time constant chart, both of which were given earlier in this chapter.

Differentiation and Integration

The process of changing a square wave into a peaked wave is called *differentiation.* When the square wave is changed into a triangular wave, the process is called *integration.* Figure 6–20 shows both differentiated and integrated waveshapes, each of which may be expressed algebraically. When the algebraic expression representing the wave is differentiated, there results a new expression, which is the expression for the next lower curve shown. When the expression is integrated, the new expression is the same as the expression for the next higher curve. Notice that the top curve is not a sine wave but a series of hyperbolic curves which form a periodic wave shape. Differentiating or integrating a sine wave by an RC circuit produces another sine wave having a different amplitude and phase, but with the same sinusoidal waveshape.

In an RC integrating circuit (one that produces an output which is the integral of the input), the time constant must be very long and the output must be taken across the capacitor. In RC differentiating circuits, the time constant must be very short and the output must be taken across the resistance.

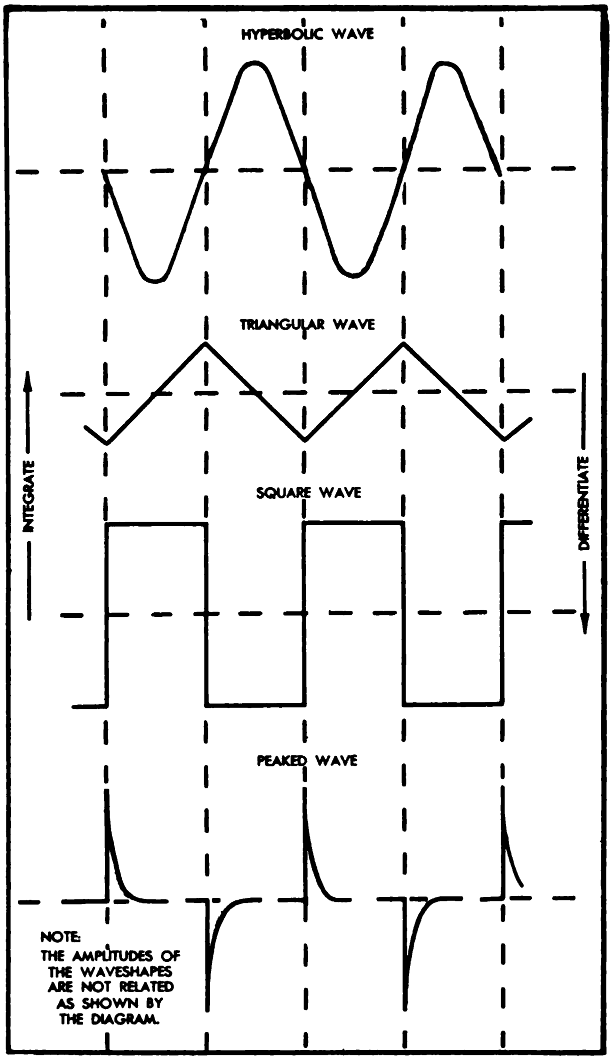


Figure 6–20. Differentiation and Integration

TRANSIENT VOLTAGES IN LR CIRCUITS

The operation of LR circuits is very similar to that of the RC circuits just discussed since the inductor current in an inductor-resistor circuit rises exactly like capacitor voltage in an RC circuit. With slight modifications, all the waveshapes obtainable in an RC circuit are also obtainable in an LR circuit.

LR Charging Circuits

Figure 6–21 shows typical current and voltage curves produced in LR circuits. In the LR circuit shown, inductance L is assumed to be perfect, that is, it offers no resistance to DC. When the switch is open, there is no voltage across the circuit and, because of this, the current and voltage are zero. When you close the switch, the battery voltage E_B is applied across the resistor R and the inductance L. Current attempts to flow, but the inductor opposes this change in current by building up a back emf that, at the initial instant, exactly equals the input voltage E_B. Since no current can flow under this condition, there is no voltage across the resistor R. The two bottom curves thus show that at the instant the switch is closed in the circuit, all the voltage is impressed across L and no voltage across R.

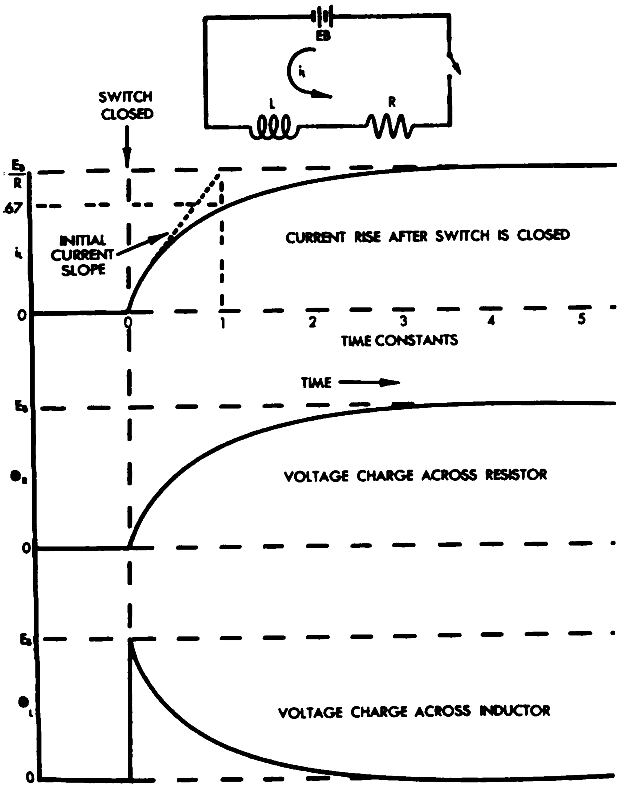


Figure 6–21. Current and Voltage Curves in LR Charging Circuit

As current starts to flow, the voltage e_R appears across R, and the voltage e_L across L is reduced by the amount of e_R at that instant. A reduced voltage across the inductor means a less rapid increase in i_L and thus a less rapid increase in the resistor voltage. The e_L curve shows that e_L finally becomes zero when the current stops increasing while the e_R curve shows that e_R builds up gradually to the input voltage as the charging current rises. Under a state of steady conditions, the resistor is the only factor that limits the magnitude of the current.

Suppose you examine the three curves in Figure 6–21 closely. Notice in the i_L (instantaneous current) curve that the current starts from zero and increases rapidly at first and then tapers off to practically a zero rate of increase at the end of 5 time constants. According to Ohm's law, the instantaneous values of the voltage across the resistor are directly proportional to the current flowing through it. Therefore, the e_R curve is similar to the i_L curve.

Since the sum of voltages around a series circuit (the circuit illustrated is a series LR circuit) equals zero, and since the applied voltage is constant, the inductor voltage e_L is always equal to the difference between E_B and e_R. The inductor voltage equals E_B when the resistor voltage e_R equals zero, and equals zero when e_R is at maximum value. If you were to reverse the polarity of the e_L curve, you might also consider it as representing the back emf in the inductor. (The polarity of back emf is always opposite to that of the applied voltage.)

At first the back emf is high since the current tends to jump to a value of E_B/R. This large back emf opposes the applied voltage and keeps the current small. In order for the back emf to be sustained at its initial high value, the current must continue to increase at the same rate as initially (dotted line in i_L curve). This is not possible, however, for as the current increases, the resistor voltage increases in proportion. The increase in resistor voltage effectively reduces the inductor voltage. Hence, the back emf decreases while current and resistor voltage increase. All three curves change along exponential curves as shown in Figure 6–21.

LR Time Constant

The meaning of the time constant in an LR circuit is analogous to the meaning of the time constant in an RC circuit. An LR time constant is defined as the time required for the current through an inductor to increase to 63.2% of its maximum value. The formula for the LR time constant in seconds is

$$TC = \frac{L}{R}$$

where L represents the inductance in henrys, and R the resistance in ohms.

The ratio $\frac{L}{R}$ also represents the time required for the resistor voltage to equal 63.2% of the applied voltage and the time required for the inductor voltage to equal 36.8% of the applied voltage.

LR Charging Problems

The shapes of the curves for current and voltage in the LR circuits are like those in the RC circuit. Therefore, you can use the rough approximation curves, the universal time constant curves, and the exponential equations for determining instantaneous values of current and voltage.

Study the following sample problems to learn how to find instantaneous values of current and voltages in LR circuits. These problems are solved first by the curve chart method and then by exponential equations.

Examples

Using curves.

Problem 1

Using the values indicated in Figure 6–22, find the amount of current 200 microseconds after the switch is closed.

Solution

Find the time constant by the formula, $TC = \frac{L}{R}$

$$TC = \frac{.01 \text{ henry}}{100 \text{ ohms}}$$

$$= \frac{10^{-2}}{10^{+2}}$$

$$= 10^{-2} \times 10^{-2}$$

$$= 10^{-4}$$

$$TC = 100 \text{ microseconds}$$

Since the time constant is 100 microseconds and the time involved is 200 microseconds, the charging action lasts 2 time constants. From the rough approximation curves in Figure 6–9 or the universal time constant chart in Figure 6–10, you find that the current in the inductor is 87% of maximum at 2 time constants. Calculating the maximum current by the formula $I = \frac{E_B}{R}$,

$$I = \frac{50}{100} = 0.5 \text{ ampere.}$$

Since the maximum current is 0.5 ampere, it will be 0.87 of 0.5 or 0.435 ampere at the end of 200 microseconds.

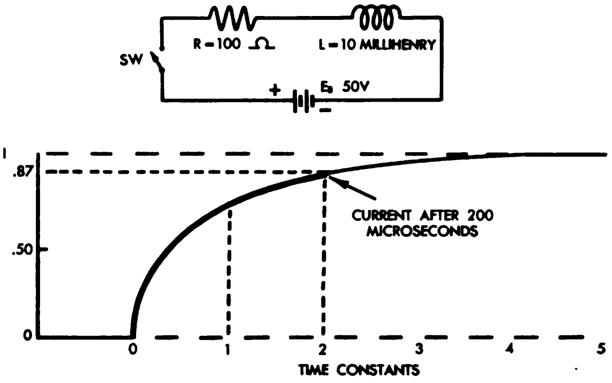


Figure 6–22. Inductor Resistor Circuit

Problem 2

Using the same circuit repeated in Figure 6–23 and the same time constant, find e_R and e_L, 50 microseconds after the switch is closed.

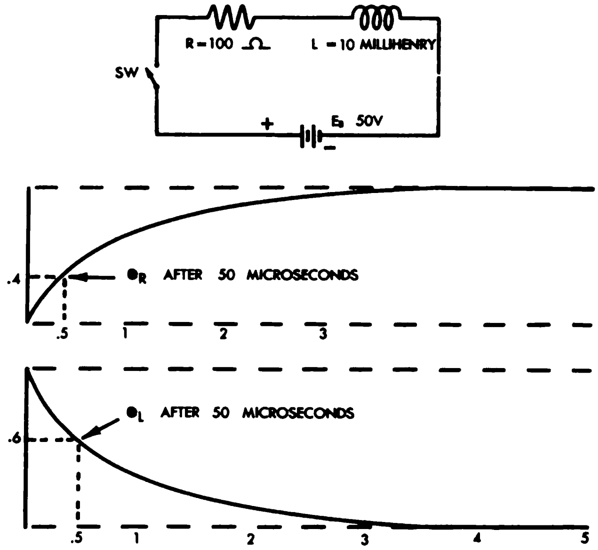


Figure 6–23. LR Circuit and Voltage Curves

Solution

The rough approximation curves do not contain a 50-microsecond time. Therefore, refer to Figure 6–10, the universal time constant chart. (Curve A is the resistor voltage curve; B the inductor voltage during charge.)

From curves A and B

$e_R = 0.4$ of maximum voltage

$e_L = 0.6$ of maximum voltage

Therefore, $e_R = 0.4 \times 50$ volts

$= 20$ volts

$e_L = 0.6 \times 50$ volts

$= 30$ volts

The following are the LR charging circuit exponential equations:

1. Instantaneous current: $i_L = \frac{E_B}{R}\left(1 - \epsilon^{\frac{-tR}{L}}\right)$
2. Instantaneous resistor voltage: $e_R = E_B\left(1 - \epsilon^{\frac{-tR}{L}}\right)$
3. Instantaneous inductor voltage: $e_L = E_B \times \epsilon^{\frac{-tR}{L}}$

Examples

Using exponential equations.

Problem 1

Using the same circuit, find i_L when $t = 200$ microseconds.

Solution

$$i_L = \frac{E_B}{R}\left(1 - \epsilon^{\frac{-tR}{L}}\right)$$

$$i_L = \frac{50}{100}\left(1 - \epsilon^{\frac{-200 \times 10^{-6} \times 10^{2}}{10^{-2}}}\right)$$

$$i_L = 0.5\ (1 - \epsilon^{-2})$$

From tables, $\epsilon^{-2} = 0.135$

Therefore, $i_L = 0.5\ (1 - 0.135)$

$i_L = 0.5\ (0.865)$

$i_L = 0.4325$ ampere

Problem 2

Find e_R when $t = 50$ microseconds.

Solution

$$e_R = E_B\left(1 - \epsilon^{\frac{-tR}{L}}\right)$$

$$e_R = 50\left(1 - \epsilon^{\frac{-50 \times 10^{-6} \times 10^{2}}{10^{-2}}}\right)$$

$$e_R = 50\ (1 - \epsilon^{-.5})$$

$\epsilon^{-.5} = 0.607$ (from tables)

$e_R = 50\ (1 - 0.607)$

$e_R = 19.6$ volts

Problem 3

Find e_L when $t = 50$ microseconds.

Solution

$E_B = e_R + e_L$

$e_L = E_B - e_R$

$= 50 - 19.6$

$= 30.4$ volts

LR Discharging

The curves shown in Figure 6–24 are formed when the source of energy is removed from a series LR circuit, and the energy stored in the magnetic field is returned to the circuit by the collapse of the field. At the initial instant when you open the switch, the resistor voltage e_R attempts to decrease to zero, but the inductance of the inductor resists any change in current and tends to maintain the current flow. This action of the inductor occurs because opening the switch removes the applied voltage, causes the current flow in the circuit to tend to cease, and starts the collapse of the magnetic field about the inductor. This collapse induces a voltage in the inductor, which causes the current to flow in the same direction that it did when the circuit was charged. Now the self-induced voltage is the source voltage in the circuit and its polarity is opposite to that of the original source voltage. As the discharge current i_d begins to decrease, the voltage across R decreases proportionately.

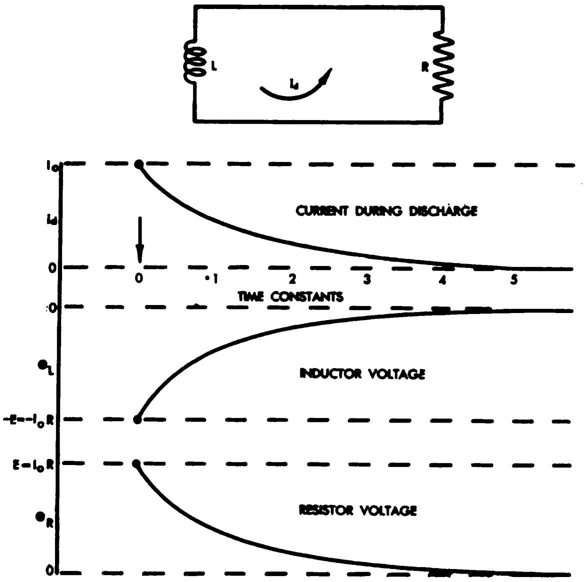


Figure 6–24. Voltage and Current Curves During Collapse of Inductor Field

Since $e_R + e_L = 0$, as previously mentioned in LR charging, the value of e_L also decreases. This change continues as shown in the e_R and e_L curves in Figure 6–24 until the e_R and e_L voltages are both zero and no current is flowing. The discharge current at the end of one time constant is 36.8% of maximum.

LR Discharging Problems

Study the following sample problems to learn how to find the instantaneous values of current and voltage in LR discharging circuits. The methods employed are like those used previously in RC discharge circuits. The formula for the LR time constant previously discussed applies here too.

Examples

Using curves.

Problem 1

Find the current 150 microseconds after the current has started to decrease in Figure 6–25.

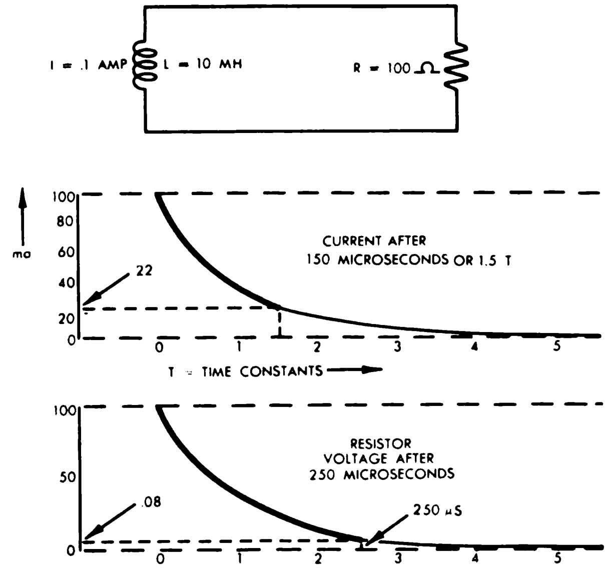


Figure 6–25. LR Discharge Circuit

Solution

Find the time constant,

$$TC = \frac{L}{R} = \frac{.01}{100} = 10^{-4} = 100 \text{ microseconds}$$

Since the time involved is 150 miscroseconds, the number of time constants is 1.5. Then, since the current starts high and decreases to zero, refer to curve B, Figure 6–10. This curve shows that the current drops to 22% of its initial value after 1.5 time constants. The initial current is 100 ma; therefore, the current after 150 microseconds is equal to 100 × 0.22, or 22 ma.

Problem 2

Using the same circuit, find the resistor and induct voltages 250 miscroseconds after the current starts d creasing.

Solution

The time given, 250 microseconds, constitutes 2.5 tin constants. Figure 6–10 shows that after 2.5 time co stants, the voltage drops to 8% of its initial valu Therefore, since the maximum voltage, E_R = IR = C ma × 100 ohms, or 10 volts, the voltage e_R, after 2 time constants, equals 10 × .08, or 0.8 volt. In findi the inductor voltage, utilize the fact that the induct voltage is the source voltage in an LR discharg circuit and, at all times, is equal to the voltage acro the resistor. Therefore, since the resistor voltage, e is 0.8 volt after 2.5 time constants, the inductor vol age, e_L, equals 0.8 volt.

Using exponential equations.

Use the following exponential equations for solvi discharge LR circuit problems:

1. Instantaneous current: $i_d = I_o \epsilon^{\frac{-tR}{L}}$
2. Instantaneous inductor voltage: $e_L = I_oR\epsilon^{\frac{-tR}{L}}$
3. Instantaneous resistor voltage: $e_R = I_oR\epsilon^{\frac{-tR}{L}}$

The symbols in these equations stand for the follov ing:

I_d = Current at any time t during discharge.
i_o = Initial current at start of discharge.
e_L = Instantaneous voltage across inductor.
e_R = Instantaneous voltage across resistor.

The other symbols have the same meaning as i exponential equations previously given.

Problem 1

Using Figure 6–25, the circuit shown with the prece ing sample problems, find e_R when t is 250 microsecond

Solution

$$e_R = I_oR\epsilon^{-tR/L}$$

$$e_R = 10^{-1} \times 10^{2}\epsilon^{\frac{-250 \times 10^{-6} \times 10^{2}}{10^{-2}}}$$

$$e_R = 10^{-1} \times 10^{2} \times \epsilon^{-2.5}$$

$$e_R = 10\,\epsilon^{-2.5}$$

$$\epsilon^{-2.5} = 0.0821 \text{ (from table)}$$

$$e_R = 10 \times .0821$$

$$e_R = 0.821 \text{ volt.}$$

Problem 2

Using the same circuit, find out when the voltag across the inductor will equal 3 volts.

Solution

When working a problem of this type, first make rough estimate of the result in order to check th answer you obtain by the equation. For example, i this sample problem, the initial voltage e_L is I_oR, o 0.1 × 100, or 10 volts. In one time constant, the indu tor voltage in an RL discharge circuit drops to 37

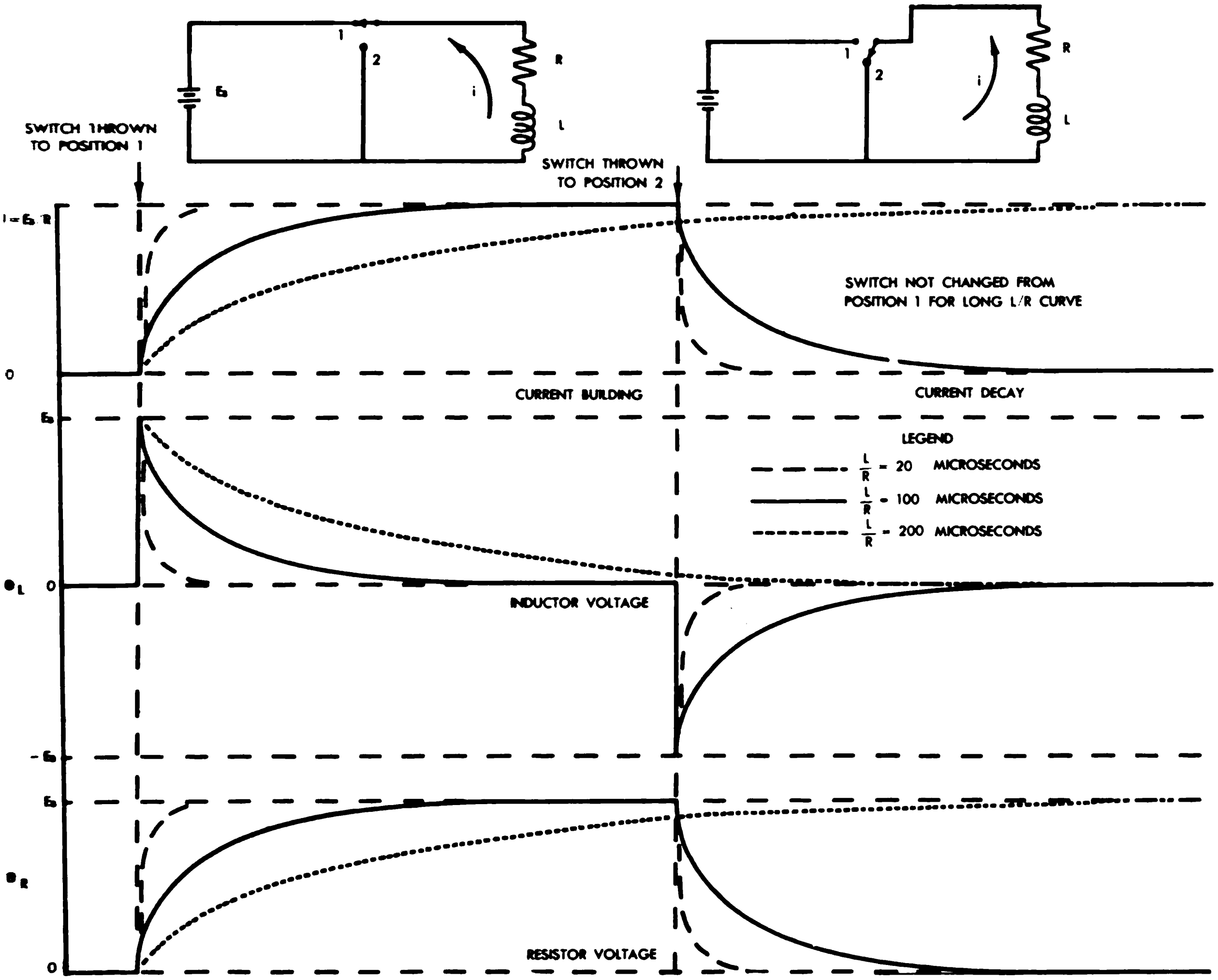

SWITCH THROWN TO POSITION 1
1
2
E_b
i
R
L
SWITCH THROWN TO POSITION 2
1
2
i
R
L
$I = E_b/R$
0
E_b
e_L 0
$-E_b$
E_b
e_R
0
SWITCH NOT CHANGED FROM POSITION 1 FOR LONG L/R CURVE
CURRENT BUILDING
CURRENT DECAY
LEGEND
$\frac{L}{R}$ = 20 MICROSECONDS
$\frac{L}{R}$ = 100 MICROSECONDS
$\frac{L}{R}$ = 200 MICROSECONDS
INDUCTOR VOLTAGE
RESISTOR VOLTAGE

Figure 6–26. Effect of Varying R and L

of its initial value. Three volts is 30% of 10 volts and one time constant equals 100 microseconds. Therefore, the voltage in this problem must drop to 3 volts in slightly more than one time constant, that is, 100 microseconds.

Use the equation, $e_L = I_oR\,\epsilon^{\frac{-tR}{L}}$

Dividing by I_o R gives: $\frac{e_L}{I_oR} = \epsilon^{\frac{-tR}{L}}$

In this equation, substituting the values given for R and L in the circuit

$$\frac{3}{10^{-1} \times 10^2} = \epsilon^{\frac{-t \times 100}{10^{-2}}}$$

$$\frac{3}{10} = \epsilon^{\frac{-t \times 100}{10^{-2}}}$$

$$0.3 = \epsilon^{\frac{-t \times 10^2}{10^{-2}}}$$

$$0.3 = \epsilon^{-t \times 10^4}$$

In the tables, $\epsilon^{-x} = 0.3$ when $-x = -1.2$; therefore,

$$\epsilon^{-t \times 10^4} = \epsilon^{-1.2}$$

$$-t \times 10^4 = -1.2$$

$$t = 1.2 \times 10^4$$

$$t = 120 \text{ microseconds}$$

Since one time constant equals 100 microseconds, 1.2 × 100 or 120 microseconds, are required for the inductor voltage to drop from 10 volts to 3 volts.

Effect of Varying R and L

As shown in Figure 6–26, varying the size of R and L in an LR circuit changes the charge and discharge curves. The changes result from the fact that any change of either R or L changes the LR time constant. The solid-line curves shown represent the instantaneous values of current and voltage during charge and discharge when the time constant is 100 microseconds.

Decreasing the time constant sharpens the slope of both the charge and discharge curves. Notice, for example, the dashed-line curve. It represents the effect of reducing the time constant in the circuit to 20 microseconds. A time constant may be reduced either by decreasing L or by increasing R. Decreasing L decreases the opposition to current flow change and permits current to build up and decay quickly. Increasing R does not affect the inductance, but it does reduce the maximum current flow. Since the maximum value is lower, the current reache its maximum value more quickly.

Because the current builds up very rapidly i a circuit with a short time constant, the voltag across the resistor (e_R), which keeps in step wit the current, similarly builds up rapidly. Th inductor voltage (e_L), which drops just as fa as the resistor voltage increases, likewise for a sharp peak. When current is decaying, th resistor voltage and the inductor voltage similarl decay much more rapidly than in a circuit wit a longer time constant.

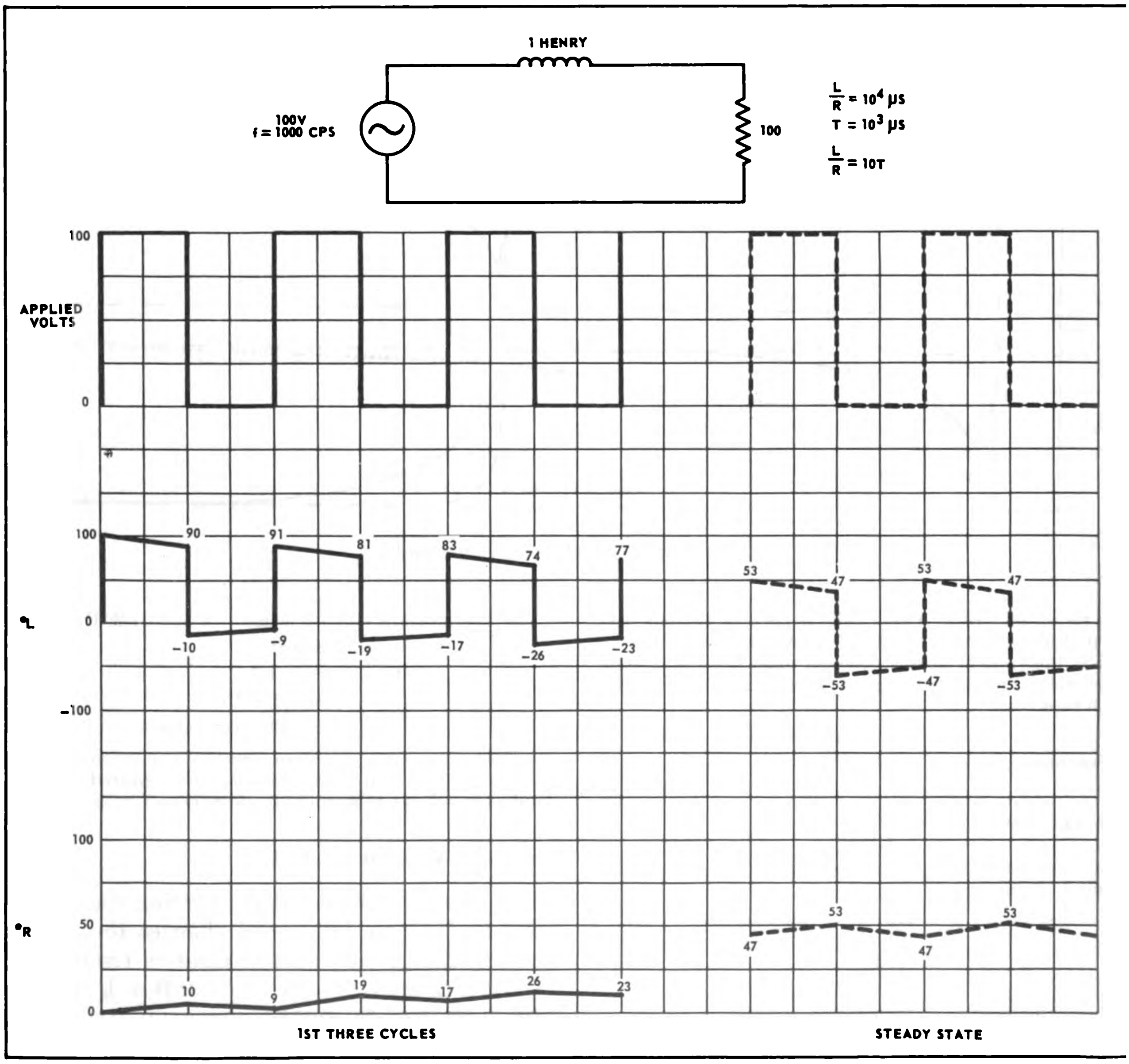


Figure 6–27. Effect of Long LR Time Constant

Increasing a time constant produces curves with gradual slopes. For example, notice the dotted-line curve. It represents the curve formed by a 200-microsecond time constant. The rise of current is slower than before because the inductor furnishes more opposition to the flow of current. The effect is noticeable also in the other 200-microsecond curves, where the fall of voltage across the inductor is very slow, and the rise of voltage across the resistor requires appreciably more time than with a 100-microsecond time constant.

Similarly, the decay of current takes longer since a large inductor stores more energy than a small one and consumes more time in dissipating it through the resistor. If the time constant were lengthened by decreasing R, the same relationships would hold true, because the smaller resistor would dissipate the energy stored in the inductor more slowly. You can see why by analyzing the basic concepts, energy dissipated is power, and power is I^2R. Therefore, if the current (I) is constant and if the resistance (R) is decreased, the power dissipation is less at any single instant. This means that if the rate of dissipation is decreased by using a smaller resistor, a longer time is required to dissipate the energy stored in the inductor.

Long LR Time Constant. The waveshapes illustrated in Figure 6–27 are those produced when a square wave is applied to an LR circuit with a long time constant. The time constant used is ten times the period. One important fact to observe here, other than the waveshapes, is how the inductor voltages change polarities at the time that the inductor acts as the source of voltage in the circuit. During current build-up, the end of the inductor that current enters may be called the negative end, for during this time the inductor acts just like any resistance or, for that matter, like any other load. However, when the source of voltage is removed, the magnetic field collapses, inducing a voltage that causes the current to continue flowing in the same direction. The inductor becomes a source, and, like a battery, the current leaves the negative end and enters the positive end. Since current flows in the same direction during either build-up or decay, the polarity of voltage actually reverses at each half-cycle.

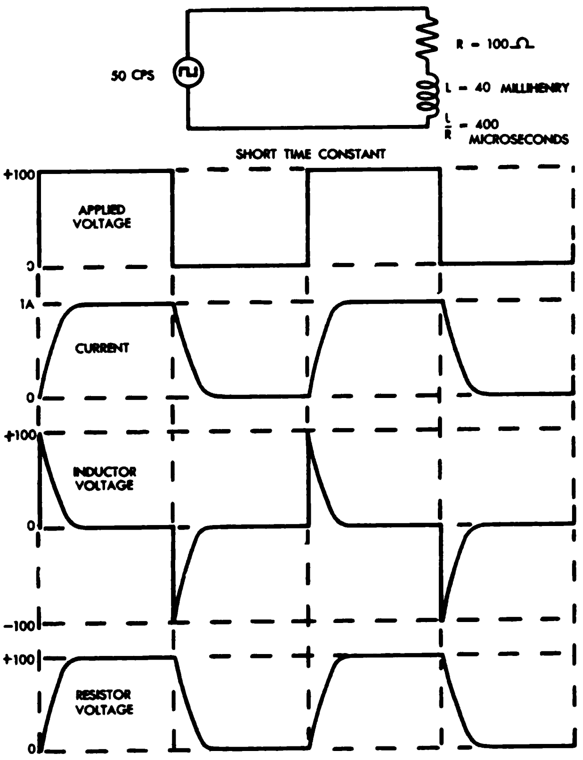


Figure 6–28. Effect of Short LR Time Constant

Short LR Time Constant. The waveshapes in Figure 6–28 are due largely to using a smaller inductor. The small inductor allows the current to reach a maximum value very quickly and thus causes the resistor voltage to be nearly a replica of the input voltage. Another result is that the inductor voltage is differentiated into a sharp peaked wave.

Medium LR Time Constant. The waveshapes produced by applying a square wave to a medium LR time constant circuit with the values indicated are shown in Figure 6–29. The time constant employed is equal to the time duration of one-half cycle of the input square wave. Therefore, the current reaches 0.63 ampere, or 63% of its maximum charge value during the first half-cycle. During the second half-cycle, when the applied voltage is zero and the inductor furnishes the current, the current drops to 0.23 ampere or 37% of 0.63 ampere. Since the applied voltage varies between 0 and 100 volts, there exists a 50-volt DC component in the waveshape. This DC component causes the current during charge and discharge to vary about an average value of 50 volts divided by 100 ohms, or 0.5 ampere.

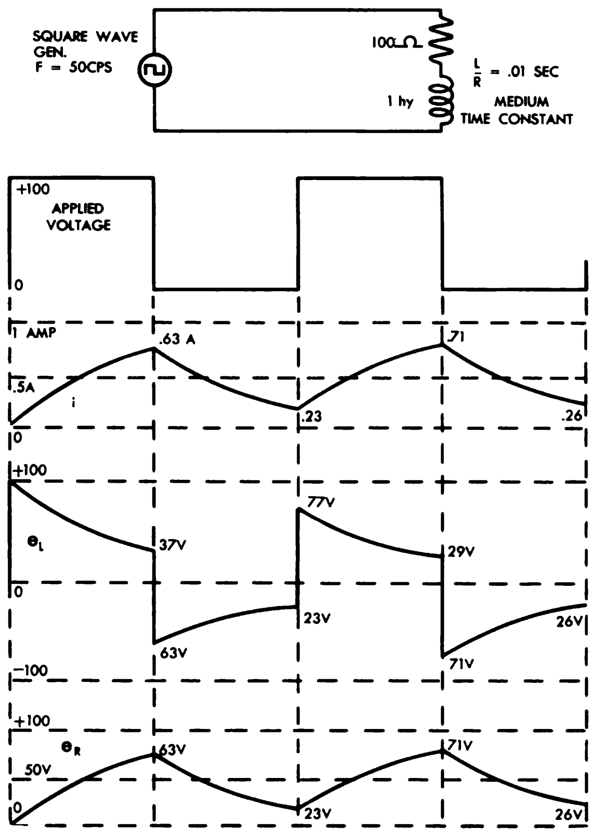


Figure 6–29. Effect of Medium LR Time Constant

However, because of the inductance in the circuit, the current does not reach this average value immediately but approaches it gradually. It does this by starting each current half-cycle a little higher.

For example, the average value in the second half-cycle is 0.42 ampere; the current range is 0.63 ampere to 0.23 ampere. In the third half-cycle it is 0.47, the range being 0.23 to 0.71, and in the fourth, the average is about 0.49 with a range of 0.71 to 0.26 ampere. During the first positive half-cycle, the inductor voltages are entirely positive. With succeeding cycles of the input voltage, these voltages work down until they center around the zero voltage line; that is, about as much of the inductor voltage is above zero as is below. The e_R voltage wave increases in value with each half-cycle of the applied voltage, thus indicating that a DC voltage component exists in the square wave. Notice that at the end of the first half-cycle the resistor voltage is 63 volts, and at the beginning of the second half-cycle it is 71 volts. After 5 half-cycles of the input voltage, the waveshape repeats at the values in the last half-cycle shown.

Frequently, the applied voltage does not contain a DC component at all, but is a pure square wave. In that case waves like those shown in Figure 6–30 are produced. The current and voltage waves produced when the pure square wave is applied to the LR circuit shown are identical to those produced when the square wave has a DC component, but the resistor voltages are 50 volts less than their original value.

Quantitative Values of Current and Voltage

An easy means of determining the voltage or current any time a square wave is applied to an LR circuit is to reduce the circuit to a simple DC circuit. For example, in Figure 6–31, the LR circuit can be represented by two equivalent circuits — one (t_1) representing the .01 second of the square wave when 100 volts is applied, and the other (t_2), the .01 second when zero voltage is applied.

During t_1 the circuit may be reduced to an LR circuit in which the generator is replaced by a battery with a voltage equal to the generator voltage. This is proper since the current rises from zero when a battery is used just as it does when the square wave voltage is applied. Therefore, if you know the time the voltage is applied and the values of L and R, you can compute the value of current at any time.

During time t_2 you can reduce the LR circuits shown to a circuit containing a resistor and an inductor in which current is flowing.

To understand how to apply these equivalent circuits in finding either current or voltage at a fixed time, study the following example:

Example

In the circuit of Figure 6–32, find the current at the end of each of the first two time constants.

Solution

When the square wave generator is producing the positive half-cycle, it applies a constant 100 volts to the circuit at A. (Diagram B shows the equivalent circuit.)

During this time current rises toward its maximum $\frac{E}{R}$ value of 1 ampere. Since the time constant is .01 second, and since the time of a half-cycle is also .01 second, the 100 volts is applied for one time constant. Therefore, at the end of one time constant, the current equals 63% of maximum, or 0.63 ampere.

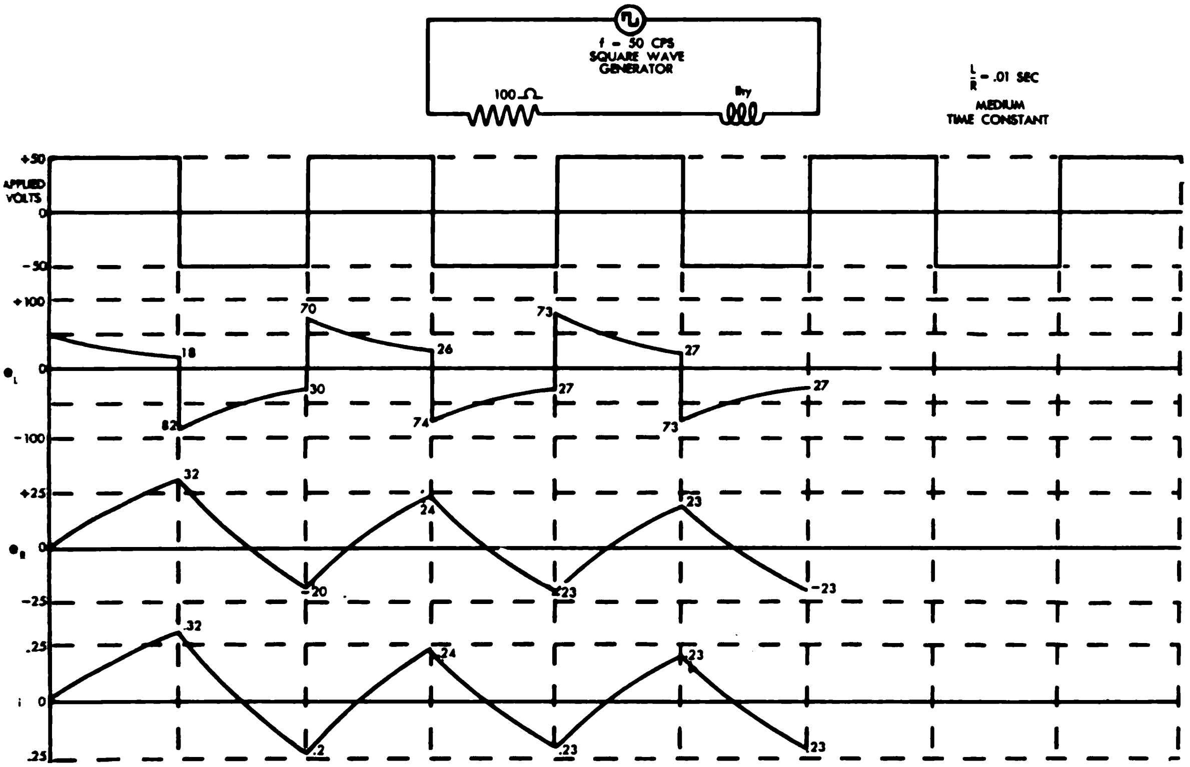


Figure 6–30. LR Waves when Input Contains no DC Component

Diagram C shows the circuit condition when the applied voltage is zero. Initially the current is 0.63 ampere, as just determined. The time of this half-cycle, or of one time constant, is also .01 second. Since current in an LR circuit drops to 37% of its initial value in one time constant, it here equals 37% of 0.63 ampere or 0.233 ampere at the end of the second time constant.

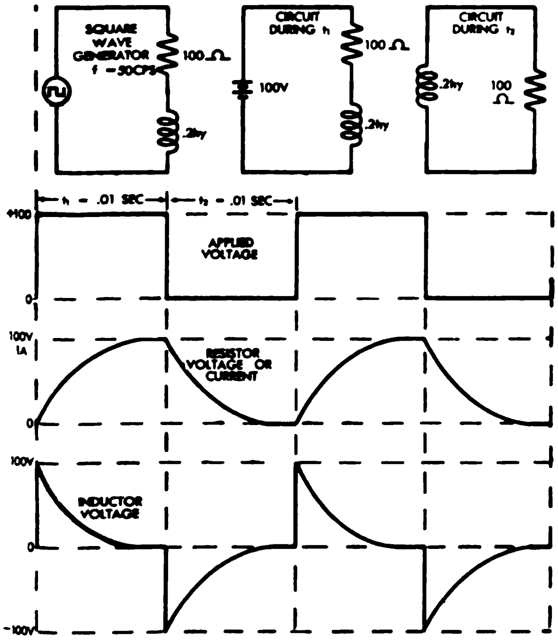


Figure 6–31. Equivalence in LR Circuit

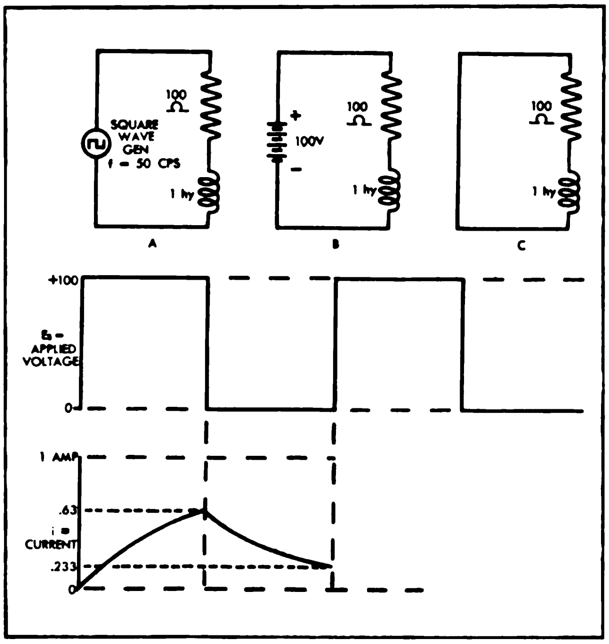


Figure 6–32. Circuit with Voltage and Current Curves

TRIANGULAR WAVES IN RC AND LR CIRCUITS

All the voltages thus far described in connection with RC and LR circuits were voltages of sudden changes, such as a voltage that changes to full magnitude when a switch is closed or such as the voltages of the steep wavefront of a square wave. However, not all voltages applied to RC and LR circuits are sudden change voltages. For example, triangular, or sawtooth waveshape voltages, such as those shown in Figure 6–33A, are commonly used in radar circuits. It is important that you know the effect that these waveshapes have on long and short time constant RC and LR circuits.

Response to Triangular Wave Input

For ease of understanding the effect of triangular waves in LR and RC circuits, first examine the first rise in voltage when this wave is applied to the RC circuit in Figure 6–33B. Figure 6–33C shows the voltage wave enlarged several times for easier study. Notice that the voltage starts at zero and keeps on increasing at a constant rate. The triangular waveshape is thus different from a square wave, in which the voltage rises quickly, then levels off. Likewise, the effect of a triangular wave on the capacitor differs from that of a square wave. With a square wave the capacitor charges exponentially to the value at which the square wave levels off, but with a triangular wave the voltage increases as the capacitor charges toward it (note the triangular shape of the wave), and the capacitor must continually charge toward a new higher value.

Initially when the voltage e_1 is applied to the circuit at B, the current rises with the voltage because the capacitor is initially uncharged. This current flow charges the capacitor with a voltage that opposes the applied voltage. This should decrease the current, but since the input voltage is continually increasing, the opposing voltage is more than overcome by the applied voltage, and the current actually increases, but less rapidly than before. As the current increase continues, the capacitor becomes charged to a higher voltage and the rate of current increase falls off some more until finally the current is high enough to raise the capacitor voltage at the same rate the input voltage waveshape increases. At this point, the current becomes constant and the capacitor voltage slope is essentially the same as the input voltage slope. At this time the constant current through the resistor causes a constant resistor voltage.

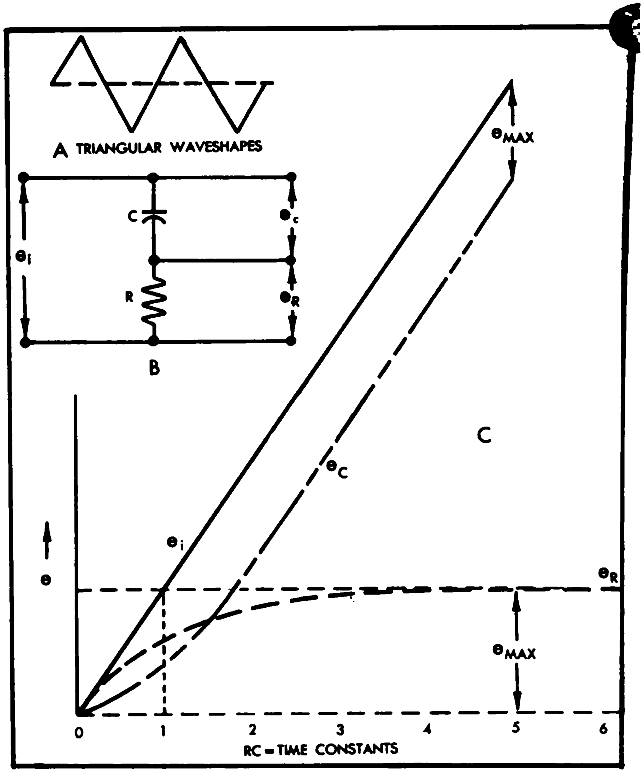


Figure 6–33. Response of RC Circuit to Triangular Wave Input

Since current increases from zero to a constant value in an RC circuit, its rate of increase is exponential. You can use the curve for e_r as the current curve. Since both the current and voltage are exponential, at the end of one time constant e_r will be 63.2% of the voltage applied, and the current will be 63.2% of its final value. After 5 time constants, the current will level off to a value equal to i_{max} (which is the value e_{max} shown in Figure 6–33).

The maximum value of resistor voltage (e_{max}) is a function of the time constant and the rate of rise. It is expressed by the equation,

$$e_{max} = \frac{\Delta e_I}{\Delta t} RC$$

In this equation, e_{max} = Maximum resistor voltage
R = Resistance in ohms
C = Capacitance in farads

$\frac{\Delta e_i}{\Delta t}$ = Rate of change of input voltage in volts per second. (The symbol Δ indicates a small change.)

Example

Problem

Assuming that the triangular waveshape in Figure 6–34 has a frequency of 50 cps and is applied to the circuit, find the maximum resistor voltage.

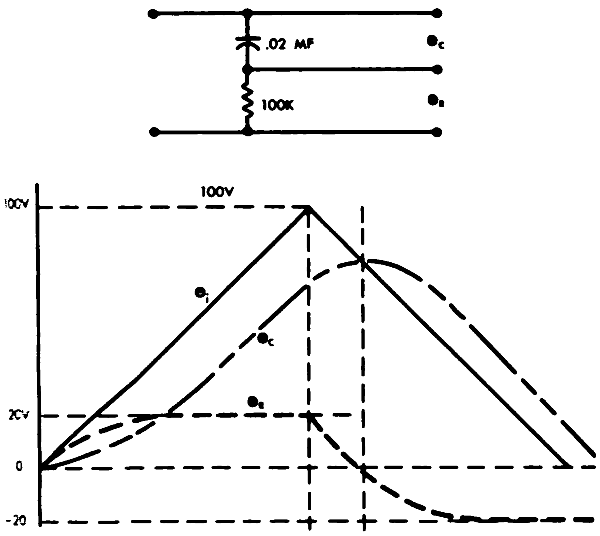


Figure 6–34. Capacitor and Resistor Voltages with Triangular Wave Input

Solution

Inspection shows that the change of the input voltage e_i is 100 volts during one half-cycle. You can represent this change in input voltage as Δe_i. Since the frequency is 50 cps, one cycle is $\frac{1}{50}$ or .02 second. Therefore the half-cycle is .01 second long. You can represent the change in time as Δt. Therefore, Δt, during the 100-volt rise, is .01 second.

Substituting in equation,

$$\begin{aligned} e_{max} &= \frac{\Delta e_i}{\Delta t} RC \\ &= \frac{100 \times 10^6 \times .02 \times 10^{-6}}{.01} \\ &= \frac{10^2 \times 10^6 \times 2 \times 10^{-8} \times 10^{-6}}{10^{-2}} \\ &= 2 \times 10 \\ &= 20 \text{ volts.} \end{aligned}$$

The capacitor continues to charge at the same rate until the rise of input voltage ceases. As the voltage abruptly starts decreasing during the second half of the cycle, the charging rate of the capacitor decreases until the voltage drops to the same value as the capacitor voltage. At this point, the current is zero. As the applied voltage continues to decrease, it falls below the capacitor voltage, and the capacitor starts discharging. This reverses the current, causing a negative voltage across the resistor. Again, the changeover cannot occur any faster than the capacitor can discharge, so the current, and consequently the resistor voltage change, is exponential. The decrease in voltage is at the same rate ($\Delta e_i/\Delta t$); so the negative voltage is also 20 volts.

Effect of Varying the Time Constant Ratio

The effect of varying the time constant is indicated in Figure 6–35. Diagram A shows the waveshape of the previous problem repeated over several cycles. As previously mentioned in connection with square waveshapes, the capacitor waveshape resembles the input when the RC is short, and the resistor waveshape resembles it when the RC is long. This is true for triangular waveshapes also. In diagram B, the RC is twice the time of the applied voltage; so the resistor waveshape very closely follows the input.

If the RC is shortened to equal the time the voltage is increasing, the resistor voltage reaches 63% of the applied voltage after one time constant. But, if the RC is shortened to one-fifth of the time the voltage is increasing, the capacitor charging rate quickly reaches the rate of applied voltage, after which the resistor voltage is constant. Notice in diagram A that this condition causes the resistor voltage to approach a square wave. This is differentiation, which was described earlier. When a triangular wave is integrated by use of a long time constant, the capacitor waveshapes will resemble a hyperbolic waveshape (Figure 6–35B).

An LR circuit has the same effect on triangular waveshapes as an RC circuit provided you observe the following changes:

First, the resistor waveshape of the RC circuit occurs across the inductor in the LR circuit while the capacitor waveshape occurs across the resistor. Second, the time constant is L/R instead of R×C. Third, the term $\Delta e_i/\Delta t$ must be replaced by $\Delta i/\Delta t$ to obtain the final voltage across the inductor.

When a triangular voltage waveshape is applied to an LR circuit, the current increases at a constant rate. This, in turn, causes the inductor

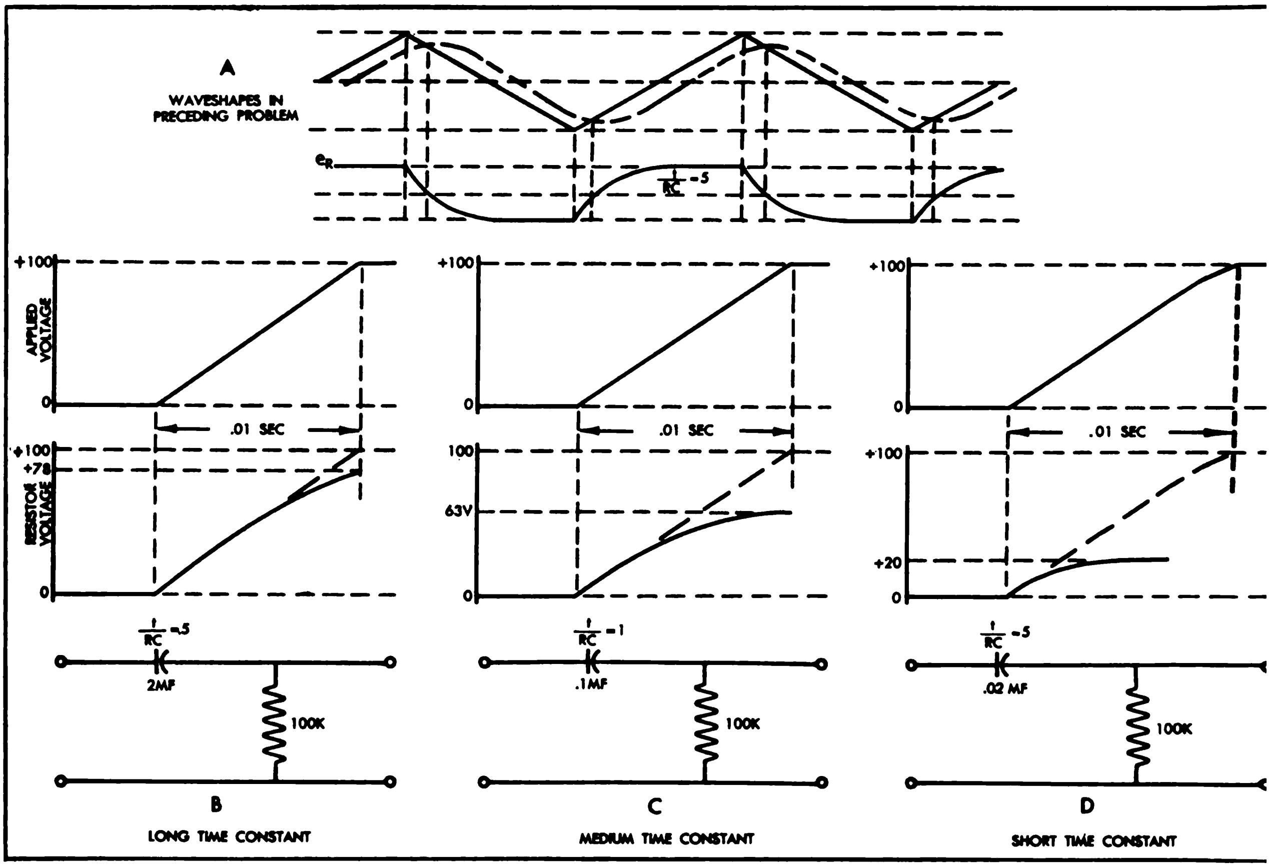


Figure 6–35. Varying the Time Constant with Triangular Wave Input

voltage to become constant after 5 time constants. You can express the maximum inductor voltage by the equation,

$$E_L = \frac{\Delta i}{\Delta t} \frac{L}{R}$$

where E_L is the maximum inductor voltage, $\frac{\Delta i}{\Delta t}$ the rate of change of current, L the inductance in henrys, and R the resistance in ohms.

LIMITING CIRCUITS

Limiting circuits are circuits that remove either one extremity or the other of an input wave. These stages are referred to as *limiters* or *clippers*.

Limiters are useful in a variety of ways. They are applicable in waveshaping circuits where it is desired to square off the extremities of the input signal. A sine wave can be converted into a rectangular wave by a limiter circuit. A peaked wave may be applied to a limiter to eliminate either the positive or the negative peaks from the output. Limiters are commonly used to prevent a voltage from swinging too far in eithe the positive or the negative direction.

Series Diode Limiters

Diodes are very useful for limiting since the conduct current only when the plate is positiv with respect to the cathode. Figure 6–36A show a series-connected diode (one in which the loa is connected in series with the tube) that is use to limit the positive input cycle of a sine wave The input voltage is e_i and the output is e When any input voltage, say 115 volts (163 volt peak value), is applied to the input terminals o the diode, the output follows the input only during the negative input cycle.

When the positive cycle is applied, the cathod becomes positive with respect to the plate an consequently the tube cannot conduct current Since there is no current flow, there is no voltag developed across the 1-megohm output resistor Therefore, the positive input cycle is limited t zero in the output.

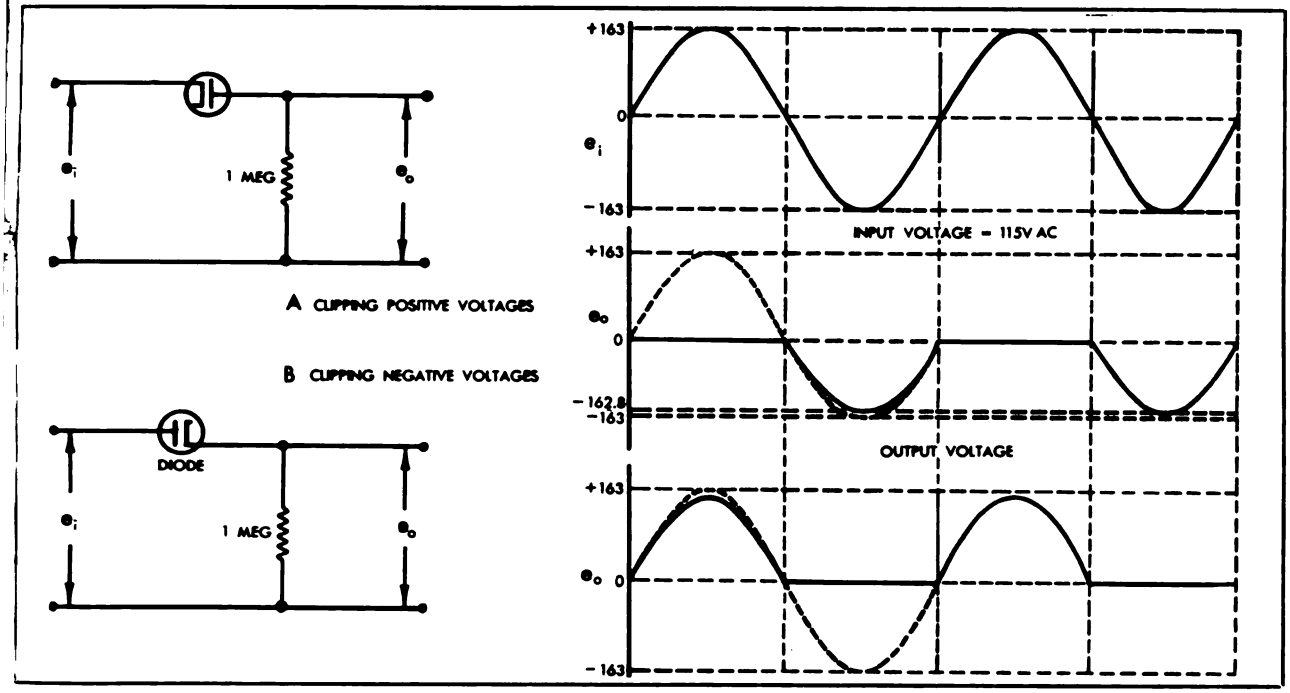


Figure 6–36. Series Diode Limiters

When the negative input cycle is applied, the plate becomes positive with respect to the cathode and the tube conducts current. This current flows through the circuit, developing a voltage across the internal resistance of the tube and the load resistor. These two resistances act as a voltage divider and divide the applied voltage, with $\frac{1}{1000}$ of it appearing across the tube (assuming the internal resistance of the tube is 1000 ohms), and with $\frac{999}{1000}$ of it across the load resistor. Since the applied voltage is 163 volts peak value, about 0.163 volt appears across the tube, and the remainder, or about 162.8 volts, across the load resistor. Thus, except for the small drop across the tube, the output is nearly equal to the positive input. Thus, the diode has limited, or clipped off, the positive input cycle, and has virtually reproduced the negative input cycle both in shape and magnitude.

When the input terminals are reversed as shown in Figure 6–36B, the diode limits the negative cycle of the input sine wave because during that cycle the plate is negative with respect to the cathode. During positive cycles, the tube can conduct because its plate is positive with respect to the cathode. The current develops across the resistor a voltage that follows the positive cycle of the input and is equal to it except for the small drop across the tube itself.

Shunt Diode Limiters

Another method of using a diode as a limiter is by connecting it in parallel with the load resistor. In Figure 6–37A, the diode is connected to limit the positive input cycle. When the positive cycle of the input voltage is applied to the circuit, the diode can conduct. Therefore, as in the case of the series diode limiter, the current flows through a voltage divider consisting of the one-megohm resistor and the internal plate resistance. This voltage divider divides the applied voltage at a ratio of one thousand to one million. One-thousandth of the applied voltage, or 0.163 volt appears across the tube. The remainder, about 162.8 volts, appears across the resistor. However, since the output terminals here are across the tube, the output is only 0.163 volt. Thus, during the time the input cycle is positive, the output is limited, or clipped, to practically zero voltage.

During the time the negative cycle of the input voltage is applied to the tube, the plate is negative with respect to the cathode and current does not flow. Therefore there is no voltage drop across the resistor. However, since

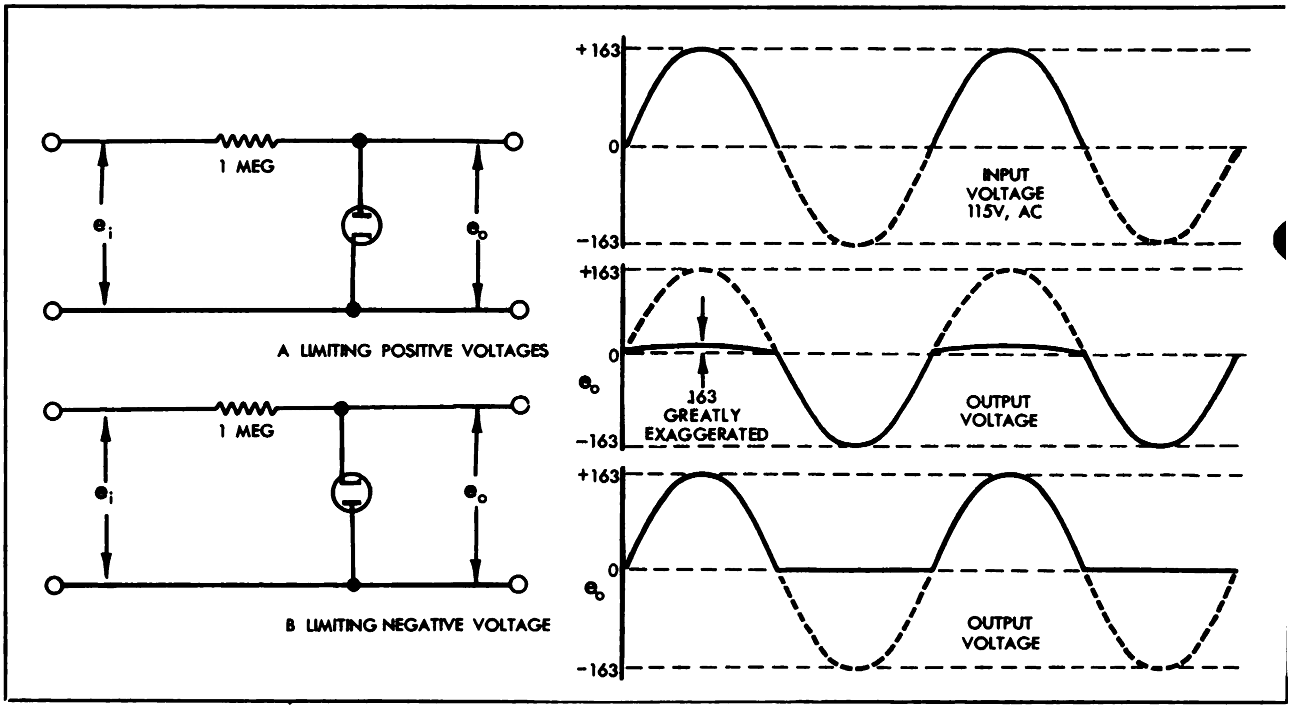


Figure 6–37. Shunt Diode Limiters

the tube appears as an open circuit, the applied voltage appears across its output terminals. Thus, the output voltage follows, and is equal to, the input voltage during the time the negative cycle is applied to the tube.

The negative input cycle may be removed by connecting the tube as shown in Figure 6–37B. When connected in this manner, the tube conducts when the cathode is negative and virtually all the applied voltage appears across the resistor; only a very small amount is across the tube. Therefore, since the output is taken across the tube, the negative input cycle has been practically reduced, or limited, to zero.

The input voltage can be limited to some value other than zero by maintaining the plate or cathode at that voltage with a battery or a biasing resistor. The two limiting circuits in Figure 6–38 — one for limiting the positive input cycle and the other for limiting the negative input cycle — both use a battery to supply the biasing voltage.

Circuit A, in Figure 6–38, is designed to limit the swing of the positive input cycle to +50 volts. The input voltage e_i is 115 volts effective value, or 163 volts peak value. The battery connected in the cathode circuit maintains the cathode 50 volts positive with respect to the plate when the tube is not conducting. As long as the input voltage is less than 50 volts, the tube does not conduct, but just as soon as the input exceeds this amount, current starts to flow and effectively connects the upper output terminal of the circuit to the positive terminal of the battery. Therefore, during the portion of the positive input cycle when the voltage exceeds the 50 volts on the cathode, the output voltage equals 50 volts, and the difference between the input voltage and this voltage (neglecting the drop across the internal resistance of the tube) appears across the 1-megohm resistor.

In Figure 6–38B, the battery E_B is so connected in the diode plate circuit as to make the plate 50 volts negative with respect to the cathode. As long as the input is positive or less negative than E_B the diode acts as an open circuit, and the voltage across the output is equal to the input. When the input becomes more negative than E_B, that is, when the cathode is negative with respect to the plate, the diode conducts current and connects the upper output terminal to the negative terminal of the battery. Therefore, during this part of the input cycle, −50 volts appears in the output and the rest of the 163 volts input, neglecting the small drop

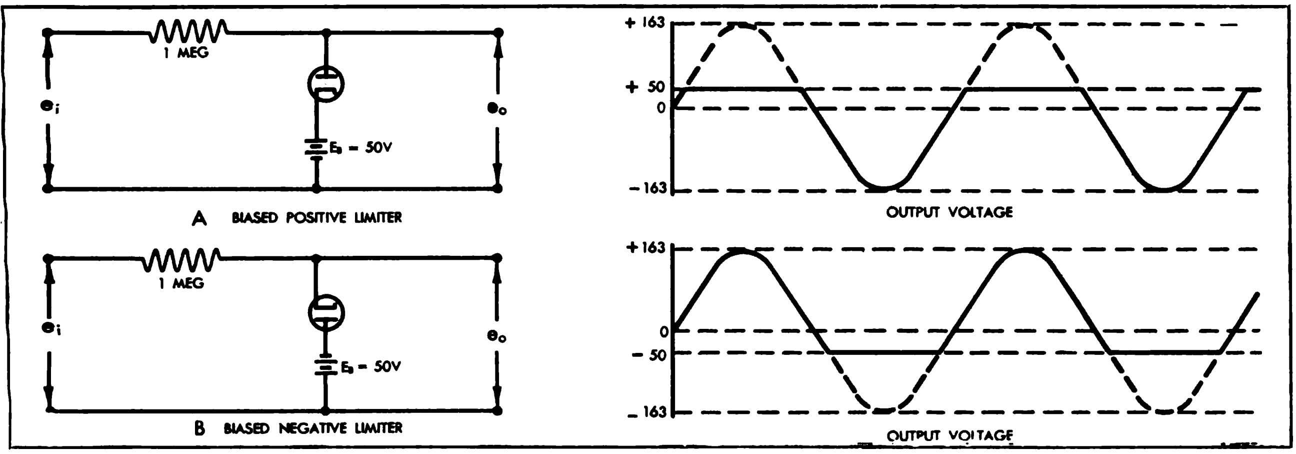


Figure 6–38. Limiting to Other Than Zero

across the tube, appears across the 1-megohm resistor.

Shunt diodes may be also used to limit the amount to which the input voltage can drop. In other words, only the peaks of waveforms are reproduced in the output. In Figure 6–39A, the diode conducts during the entire part of the input waveform that is below the positive voltage of the battery. The output voltage under this condition varies between the positive level of the battery voltage and the positive extremity of the input waveform.

In Figure 6–39B the entire portion of the input waveform above the negative potential of the battery causes the diode to conduct, thus producing an output voltage which varies between the negative level of E_B and the negative extremity of the input. In both circuits A and B the difference between the value of E_B and the applied voltage during the time the diode conducts is represented by the voltage drop across the 1-megohm resistor.

Crystal Diode Limiter

Although most radar circuits use diode vacuum tubes in limiter circuits, the crystal diode, or silicon crystal rectifier, is becoming increasingly popular for limiting. The crystal diode usually consists of a small piece of silicon and a pointed tungsten wire contactor. It limits or rectifies because it passes little current from the crystal to the wire, but permits current to flow readily from the wire to the crystal when an AC voltage is applied to its input terminals.

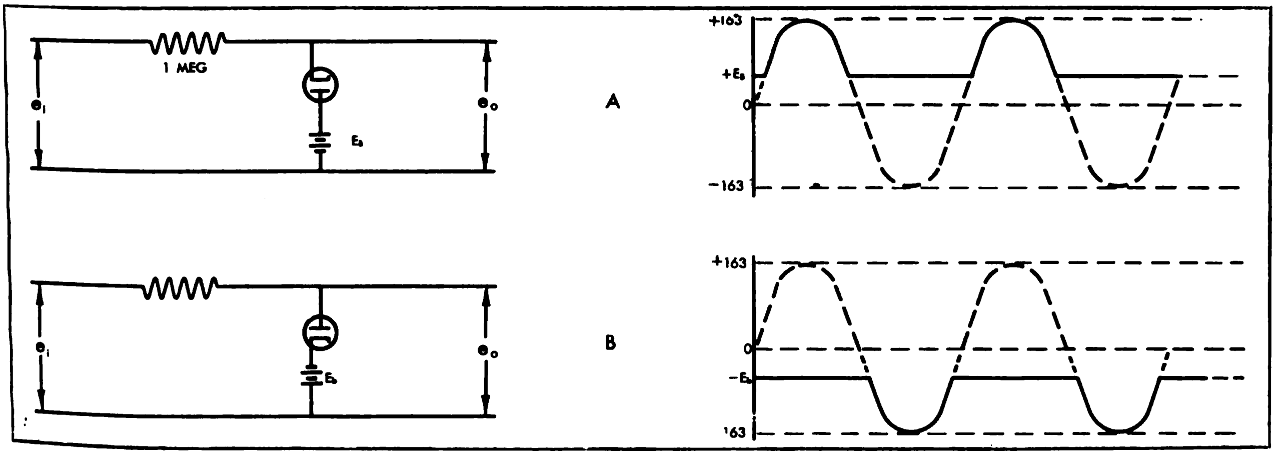


Figure 6–39. Maintaining Voltage Above a Certain Level

Since, like a diode vacuum tube, it conducts more current in one direction than in the other, the crystal assembly often is called a crystal diode. Not only is the crystal almost as efficient as the vacuum tube, but it requires no heater voltage and is entirely free of interference from the power frequency. It may be used in any of the diode limiter circuits previously described. The one illustrated in Figure 6–40 is for use in

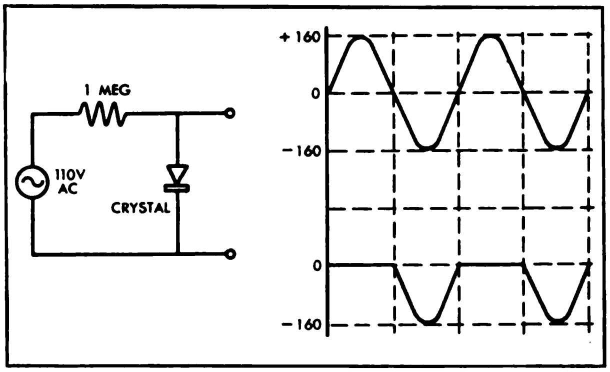


Figure 6–40. Positive Crystal Limiter

limiting, or clipping, the positive cycle of t
input waveform.

Grid Current Limiting

When a large limiting resistor is connected
series with the grid of a triode, tetrode, or pe
tode tube, the grid-cathode circuit may be us
as a limiter circuit in exactly the same manner
the plate-cathode circuit of the shunt-dio
limiter. What this means is that when the gr
is positive with respect to the cathode, the gr
will attract electrons just as does the plate
a diode when it is positive with respect to t
cathode. The limiter circuit in Figure 6–41
held normally at zero bias. During the positi
portion of the input signal, the grid tries to swi
positive.

Grid current flows through the resistor I
developing an IR drop across it with a polari
that opposes the positive input voltage. Sin
the input voltage must equal the sum of the dr
across R_g and the grid-to-cathode voltage, t
larger R_g is with respect to the cathode-grid r
sistance, the nearer the voltage on the grid

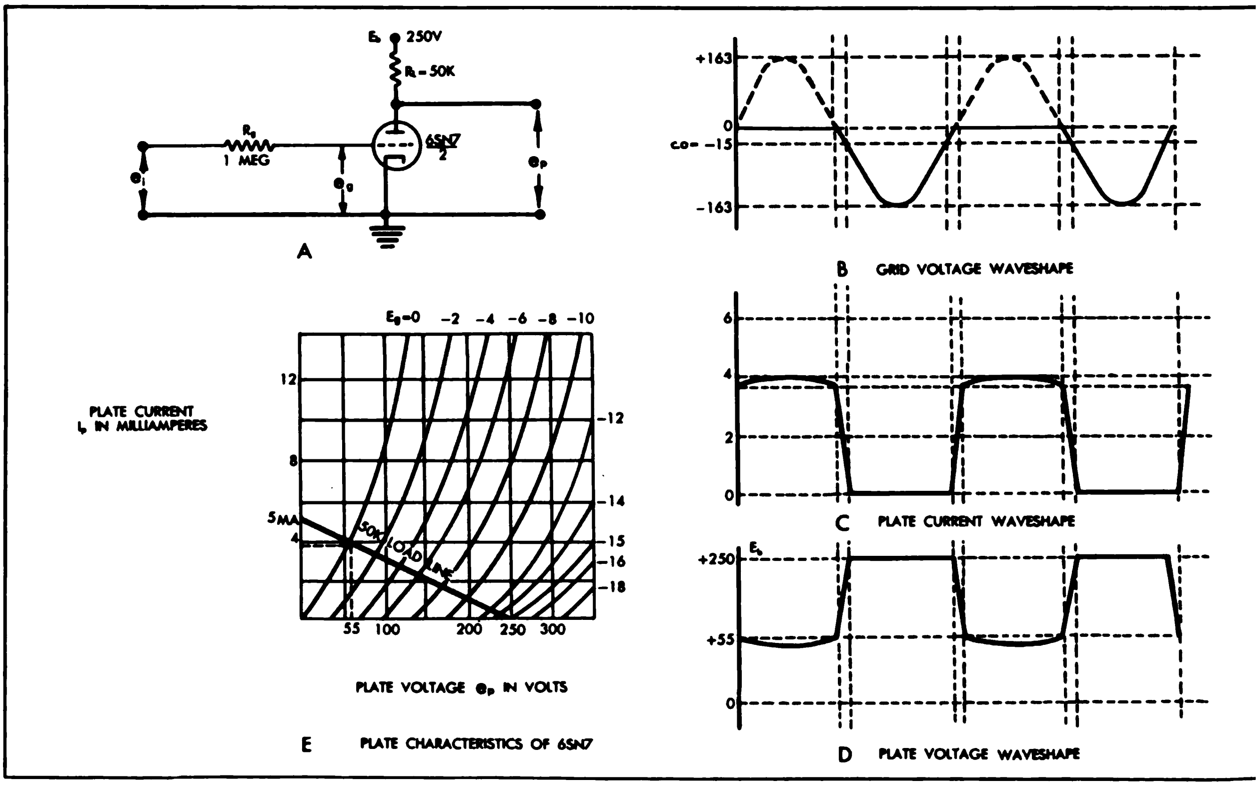


Figure 6–41. Grid Current and Grid Cutoff Limiting

limited to that of the cathode. In the circuit shown, the grid is never driven more than a fraction of a volt positive by the positive input cycle. This means that all voltages in the positive direction of the input signal are leveled off at zero.

If the negative peaks of the input voltage are more negative than the grid voltage required to cut off plate current, limiting occurs. The numerical values are given in Figure 6–41 to enable you to calculate the output. As before, the input voltage is a 115-volt sine wave and, as previously mentioned, the 1-megohm limiting resistor prevents the grid voltage from becoming more than a fraction of a volt positive. The grid voltage, however, can swing the full distance in the negative direction since no current flows from grid to cathode to reduce the applied voltage as when the positive signal is applied.

To observe the effect of the grid voltage variation, study the load line drawn on the characteristic curves in Figure 6–42. When the grid voltage is zero, the plate current is 4.0 ma and the instantaneous plate-to-cathode voltage e_p is 55 volts. As the voltage swings in the negative direction, the plate current decreases, and when the grid is —15 volts, the plate current is zero. The plate current cannot decrease below zero as the grid is made more negative, and therefore both the plate current and its resultant plate voltage become constant. This condition prevails until the grid voltage again becomes less than —15 volts.

With the applied waveshape indicated, the grid voltage exceeds —15 volts for most of the half-cycle. The plate voltage variation shows a rapid change to the maximum voltage of 250 volts, and remains at that value for most of the half-cycle. The plate voltage change is essentially a square wave except for a slight rounding that occurs because the grid draws current when it is driven positive a fraction of a volt. This voltage change is amplified in the plate circuit and rounds off the square wave.

The action just described is perhaps more correctly defined as squaring the waveshape since the plate waveshape rather than the input waveshape is limited.

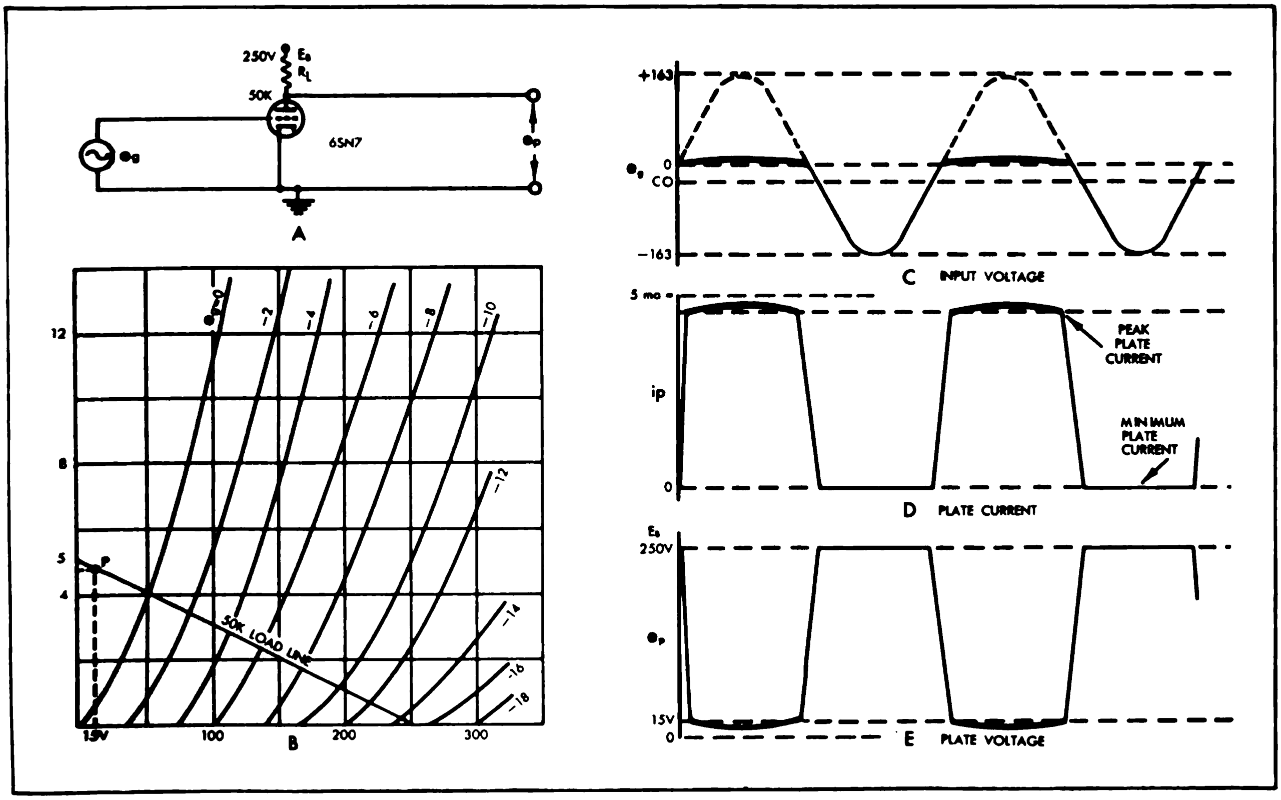


Figure 6–42. Saturation Limiting

Saturation Limiting

A grid limiting resistor is not absolutely necessary for limiting the positive half of an input cycle. If the grid is made positive by an input voltage, there is a saturation point beyond which the plate current will not increase. This point is indicated by a dot on the load line at P in Figure 6–42. That is the point at which the plate resistance of the tube is assumed to be zero. The maximum plate current obtainable then depends on the size of the plate load resistor.

Actually, there is always some resistance to electron flow in the tube, no matter how positive the grid becomes, and the current will stop increasing below the saturation value. Limiting occurs when plate current and plate voltages stop changing. This occurs at some positive value of grid voltage. Therefore saturation limiting occurs at a higher point on the grid voltage swing than when a limiting resistor is used. The curves also show limiting by plate current cutoff.

Double Diode Limiting

When a pair of biased diodes are connected as shown in Figure 6–43, both the negative and the positive half-cycles of the input voltage can be limited, leaving a waveshape that is fairly square. The voltage at which limiting occurs in each direction may be varied by varying the bias voltages on the diodes. In the circuit shown, tube V_1 limits the positive half-cycle and tube V_2 limits the negative half-cycle.

The Squarer and Peaker Circuit

The squarer and peaker circuit uses the principles discussed in the preceding limiter circuits. Each part of this circuit is used in several rad circuits, and the entire circuit is used in sor radar sets.

Notice the simple squarer and peaker circu in Figure 6–44A. The first tube is an overdriv amplifier that squares the input sine wavesha and amplifies it according to principles of trio limiting. The .002 mf capacitor and 500K r sistor form a short time constant that peaks t square wave. The grid of the second triode biased so that only a positive or a negative ampl fied peak appears in the output. Because of th limiting action this stage is called a limiter clipper.

Circuit Analysis

In analyzing the peaker circuit, first insp the circuit diagram. The first stage is a triod that looks like an unbiased amplifier stage. How ever, there is a 1-megohm grid limiting resist in the grid circuit. This stage applies its outpu through capacitive coupling to the second triod a stage that uses a 100K load resistor and fixe bias of 50 volts. Bias in this stage is provide by a 250K and 50K voltage divider across th positive 300-volt supply.

Now consider the operation of the peaker. A the input voltage goes from zero to a positiv voltage, the grid of the first triode is made posi tive with respect to its cathode. Immediately grid current flows through the cathode-to-gri resistance of the tube and the 1-megohm resisto Since the cathode-to-grid resistance is less tha 1000 ohms, the voltage developed by the gri current divides in a ratio of approximately 1 t 1000 across the tube and the grid limiting re sistor. At the positive peak of the input voltag

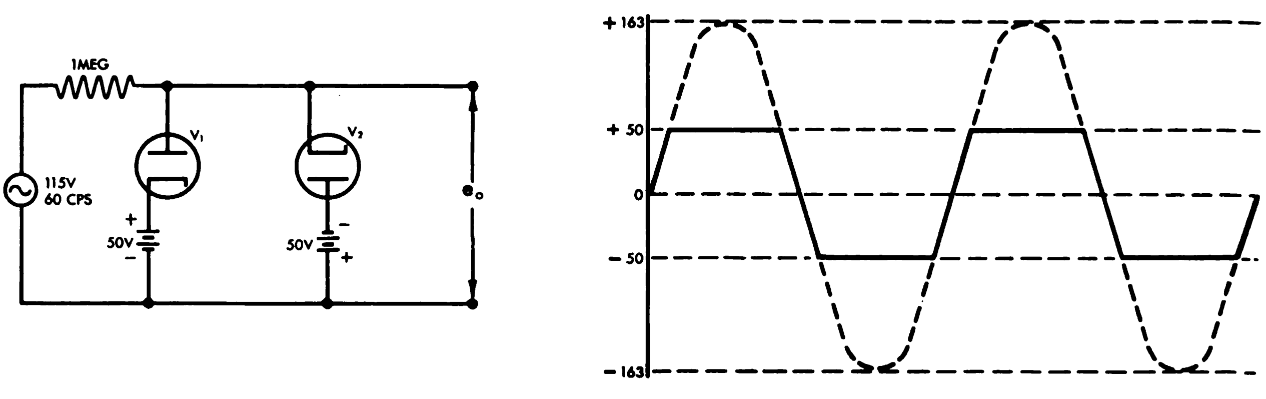


Figure 6–43. Double Diode Limiter

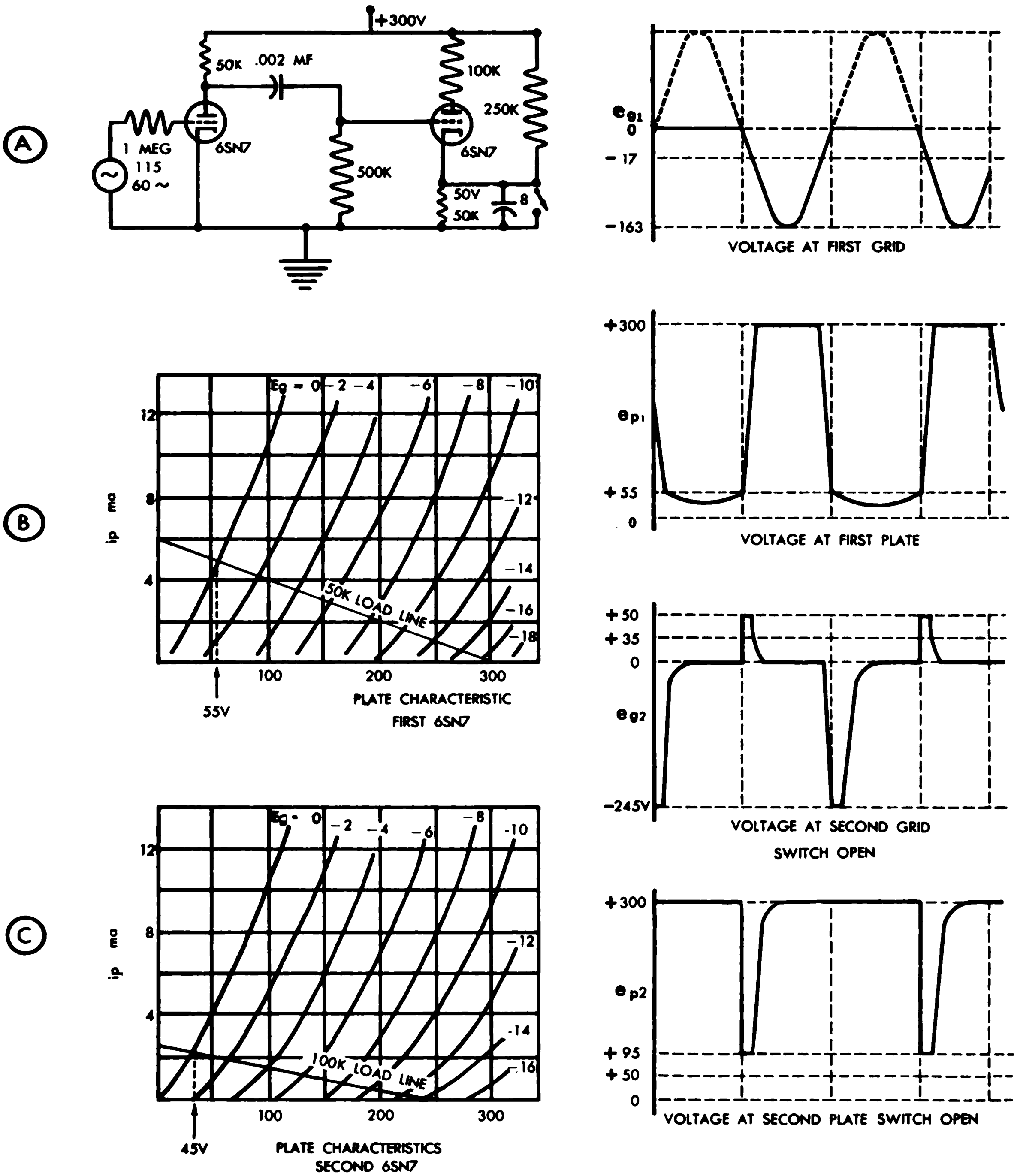


Figure 6–44. Squarer and Peaker Circuit

(peak 163 volts), the grid voltage is about 1/10 of a volt. The grid waveshape shows this voltage as a flat line at the zero level. (The dotted line is the applied voltage; the solid line, the actual grid voltage.)

The plate voltage of the first triode during this time is rather low because the plate current is unlimited by the grid. To determine its exact value, construct a load line on the characteristic curves as follows: First, calculate the maximum current by dividing the plate supply of 300 volts by the load resistance of 50K. Thus, the maximum current is 300/50K, or 6 ma. Draw a line across the curves from the 300-volt point on the bottom to the 6-ma point along the current scale. This line is the load line (see Figure 6–44B). The grid voltage is practically zero in the circuit. Therefore, finding the plate voltage for zero grid voltage is the first use of the load line.

At the intersection of the zero grid voltage curve and the load line, drop down to the plate voltage scale at the bottom. The point shows the plate voltage is 55 volts. During the positive swing of applied voltage, the grid actually goes slightly positive. This is shown by the amplified curve of the plate voltage change which is shown below the grid voltage curve. At the start of the half-cycle, the curve is at 55 volts, but drops below that slightly as the grid goes slightly positive, then returns to 55 volts at the end of the half-cycle.

Next, consider the input voltage during the negative half-cycle. First, however, remember that the grid does not draw current when it is negative. Therefore, when no current flows through the 1-megohm resistor, there will be no voltage drop across it. Thus, the full input voltage will appear at the grid during the entire negative half-cycle as is shown in the first grid voltage curve. Notice that the load line crosses the zero current axis at the end of the minus 17-volt grid curve. Thus, when the grid voltage reaches minus 17 volts, the current will have decreased to zero. The grid curves show that this value is reached very quickly during the negative half-cycle. The grid voltage continues to swing with the applied voltage through minus 163 and back to zero. All of this time the grid voltage is more than 17 volts negative, and the current remains at zero.

When the grid voltage starts to become negative, the decrease of current through the 50K load resistor causes the plate voltage to increase toward the 300-volt supply voltage. This change is very rapid because the grid voltage crosses the cutoff value in less than 1/100 of the cycle. Therefore, the plate voltage rise to +300 volts is almost vertical. The plate voltage remains at +300 volts during the entire time the grid voltage is below the cutoff value on the curve. As it comes above cutoff, the plate voltage again drops to 55 volts. Again, the grid voltage change from minus 17 to zero is very steep, and accordingly the plate voltage change is exceedingly fast. The long period of time that the grid is below cutoff and the rapid changes cause a square wave to be developed at the plate of the first triode.

The square wave output of the first stage is applied to an RC circuit consisting of the .002 mf capacitor and the 500K resistor in the grid of the second tube. The voltage across the resistor becomes the grid voltage. At the start of the first half-cycle the square wave is negative. Since the capacitor is initially uncharged, the application of this negative-going voltage to it immediately causes current to flow through the RC circuit. With no drop across the uncharged capacitor, the full voltage appears across the resistor. In Figure 6–44 the third curve, labeled e_{g2}, shows the entire voltage change occurring at the second grid.

Suppose for a moment you examine the conditions in the second tube when this grid voltage is applied to it. The cathode is 50 volts positive with respect to ground. The actual plate voltage is 300 volts minus the 50 volts on the cathode, or 250 volts. Notice the load line on the set of characteristic curves in Figure 6–44C. This line is drawn from the 250-volt supply voltage point to the 250/100K, or 2.5 ma, current point. Notice that the plate current becomes zero when the grid voltage is minus 15 volts. With 50 volts bias on this tube there will be no current until more than 35 volts is applied to the grid. Therefore, the negative swing of 245 volts applied to the grid will not change the plate current from its zero value. But also note that this waveshape is not like the applied square wave because the RC time constant is short.

The total time the voltage is applied is 1/120 or .00833 second, while the RC is .002 mf × 0.5 megohm or .001 second, a ratio of about 8 to 1. In other words, the capacitor is small and becomes charged during the very first portion of the square wave. As it becomes charged, the current through the 500K resistor gets lower and lower until it drops to zero after 5 time constants, or .005 second. The resistor voltage stays in step with this decreasing current so the voltage drops exponentially to zero. The grid voltage stays at zero for the duration of the half-cycle, or until the changeover of the square wave occurs.

At the end of the first half-cycle the voltage at the first triode changes 245 volts in the positive direction. Again this change immediately appears across the resistor. Nothing happens, however, until the voltage reaches 35 volts. Then current starts to flow in the tube. As the voltage increases from 35 volts to 50 volts, the actual grid-to-cathode voltage at the second tube rises from minus 15 volts to zero. Now the plate waveshape for this second tube can be examined. During the entire previous half-cycle, the tube was cut off and the plate voltage remained at +300 volts. But now current is flowing, and the voltage drops to that value of plate voltage for zero grid voltage.

This value can be determined from the load line again. Find the intersection of the zero grid voltage line with the load line and proceed downward to the plate voltage scale, which is 45 volts for this curve. This might seem inconsistent inasmuch as the plate voltage curve at C indicates that the plate voltage is 95 volts. But remember that the voltage is being indicated with respect to ground. Therefore, the 45 volts across the tube must be added to the bias voltage to get the total voltage from plate to ground. Now return to the second grid. As the square wave gets above 50 volts, the grid becomes positive and starts to draw current. So you see that the positive-going voltage on the second grid levels off at zero.

Only a small part of the remaining voltage rise of 245 volts ever appears between the grid and cathode. This is because the grid current flows through the .002 mf capacitor and 50K load resistor, and this combination of the 50K resistor and the high reactance of the small capacitor acts as a grid limiter to divide the total voltage. The small grid-to-cathode resistance of about 500 ohms will have a very small voltage drop across it. Remember, the 500K resistor is still in the circuit, but so much of the current flows through the grid itself that the 500K resistor can be practically disregarded during this time.

The low resistance and small capacitor form a very short RC circuit, so the capacitor will be charged in a hurry. The voltage at the grid then drops back to zero with respect to ground — which is 50 volts negative with respect to the cathode. So the triode conducts only during the brief period of time during which the grid is above cutoff. The resultant plate voltage drop is also very brief. The plate voltage stays at +300 volts most of the time. A negative pulse is thus generated for each cycle of the original input sine wave. Its amplitude is 300 — 95 or 205 volts.

When the switch is closed, this circuit will generate a positive pulse for each cycle of sine wave. As shown in Figure 6–45, the switch is closed in the circuit at A. When the switch is closed, it shorts out the 50K bias resistor and its parallel capacitor and connects the 300K resistor to ground. The three circuit elements become ineffective. An equivalent circuit may be drawn by omitting these elements as in the equivalent circuit at B.

Note that now there is no bias on the second triode, and that the full 300-volts supply voltage is across the tube and its load resistor. To evaluate the current and voltages, it is necessary to draw a new load line. This line extends from 300 volts at the bottom to 300/100K or 3 ma at the side (Figure 6–45C). With no bias, the grid voltage is zero, and the plate voltage under this condition is 40 volts.

On the negative half-cycles the square wave will be peaked in the same manner as before. But on the positive peaks, grid current flow occurs at zero instead of the plus 50 volts as before. Similarly, as in the previous case, the high impedance circuit of the .002 mf capacitor and 50K load resistor prevents the grid from getting more than a fraction of a volt positive during positive half-cycles.

The grid remains at zero except when the input voltage is negative-going. Therefore, at zero grid voltage where the plate current is high, the plate voltage, which varies inversely with the plate current, remains at a constant value of 40 volts. At the close of the first half-cycle, the voltage is negative-going. Here the grid voltage changes rapidly from zero to the cutoff value, and the plate voltage rises sharply from 40 volts to 300 volts. The grid voltage becomes negative by 245 volts, and the resulting plate voltage levels off at the supply voltage level of 300 volts. However, the rapid charge of the .002 mf capacitor causes the grid voltage to rise immediately to the cutoff value and beyond. As the grid voltage continues to rise to zero, the plate voltage drops back down to 40 volts.

If you study the plate voltage waveshape, you will notice the fairly constant low plate voltage with positive peaks with each cycle of input voltage. The positive peak voltage at the second grid only raises the grid a fraction of a volt above

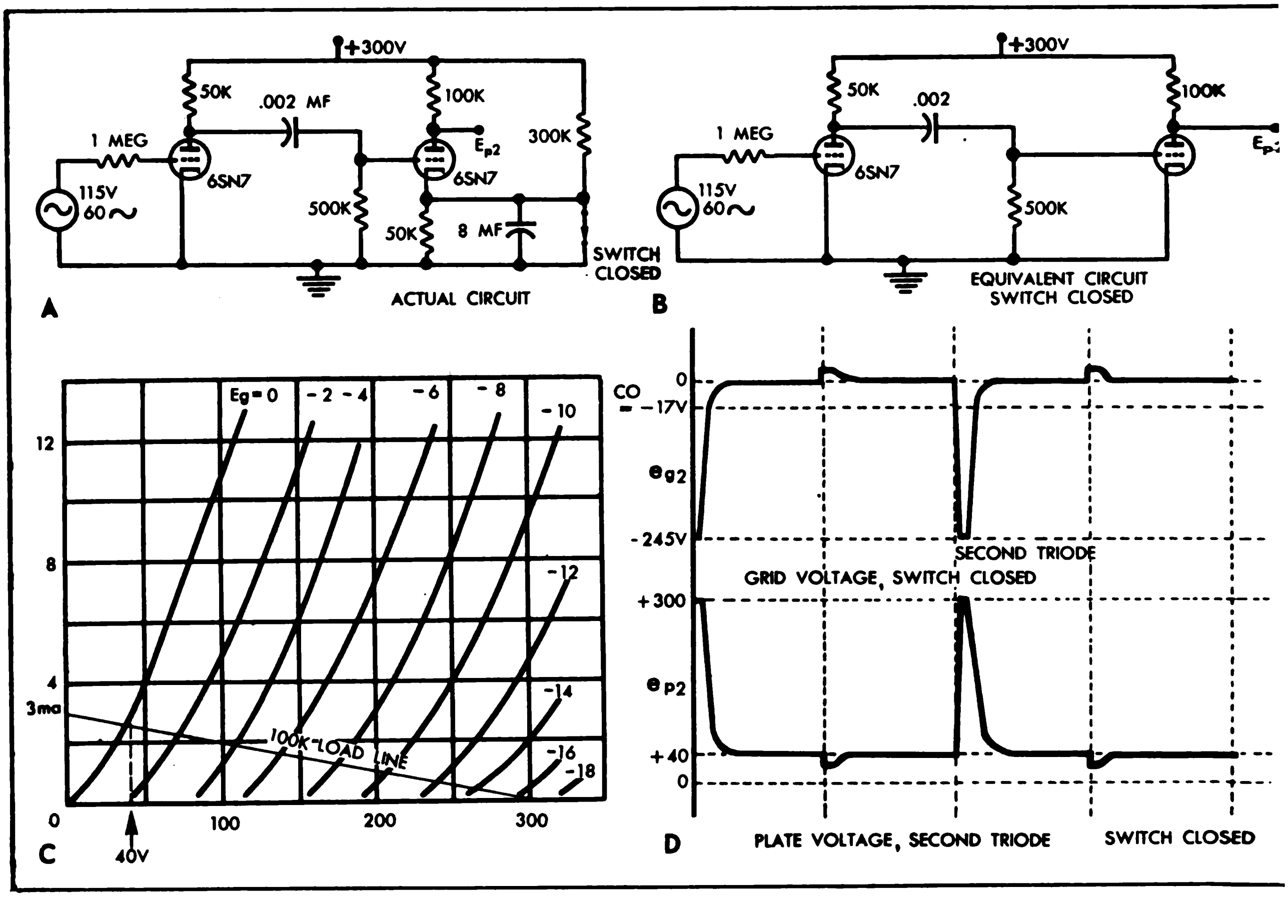


Figure 6–45. Squarer and Peaker with Switch to Form Positive Peak

zero, thus causing the plate voltage to dip slightly at each of these points. Thus you see, with this circuit it is possible to obtain either positive or negative peaked pulses by the flip of the switch.

CLAMPERS

A clamping circuit, or direct current restorer, as it is also called, is a circuit that shifts a waveshape so that it is all above or all below a certain voltage — often zero voltage. It is called a *clamping circuit* because it clamps the top or bottom of a voltage waveshape at a certain voltage, which may be zero, any positive voltage, or any negative voltage, depending on the design of the circuit.

Negative Clamping Circuit

A typical example of the need for a clamping circuit is the waveshape output produced by the squarer and peaker circuit (Figure 6–46) which shows a negative peak output. Although this voltage is called a negative peak voltage, it actually is all positive, the peak merely being less positive than most of the waveshape. To make it a truly negative peak voltage, all the positive voltage must be removed.

This is partly accomplished by a blocking capacitor, which removes all the DC component. With no DC component, part of the waveshape is positive and part is negative. For the waveshape to be all negative, a DC negative voltage must be added to it. Adding the negative DC voltage shifts the whole waveshape downward with respect to the zero axis, creating the desired waveshape as shown at the bottom in Figure 6–46. The action is accomplished by the clamping circuit which *restores* enough of the DC component to make the waveshape all negative.

Circuit Analysis. To analyze a simple clamping circuit in detail, study the elementary circuit illustrated in Figure 6–47A. Judging by its action, it actually consists of two time constants,

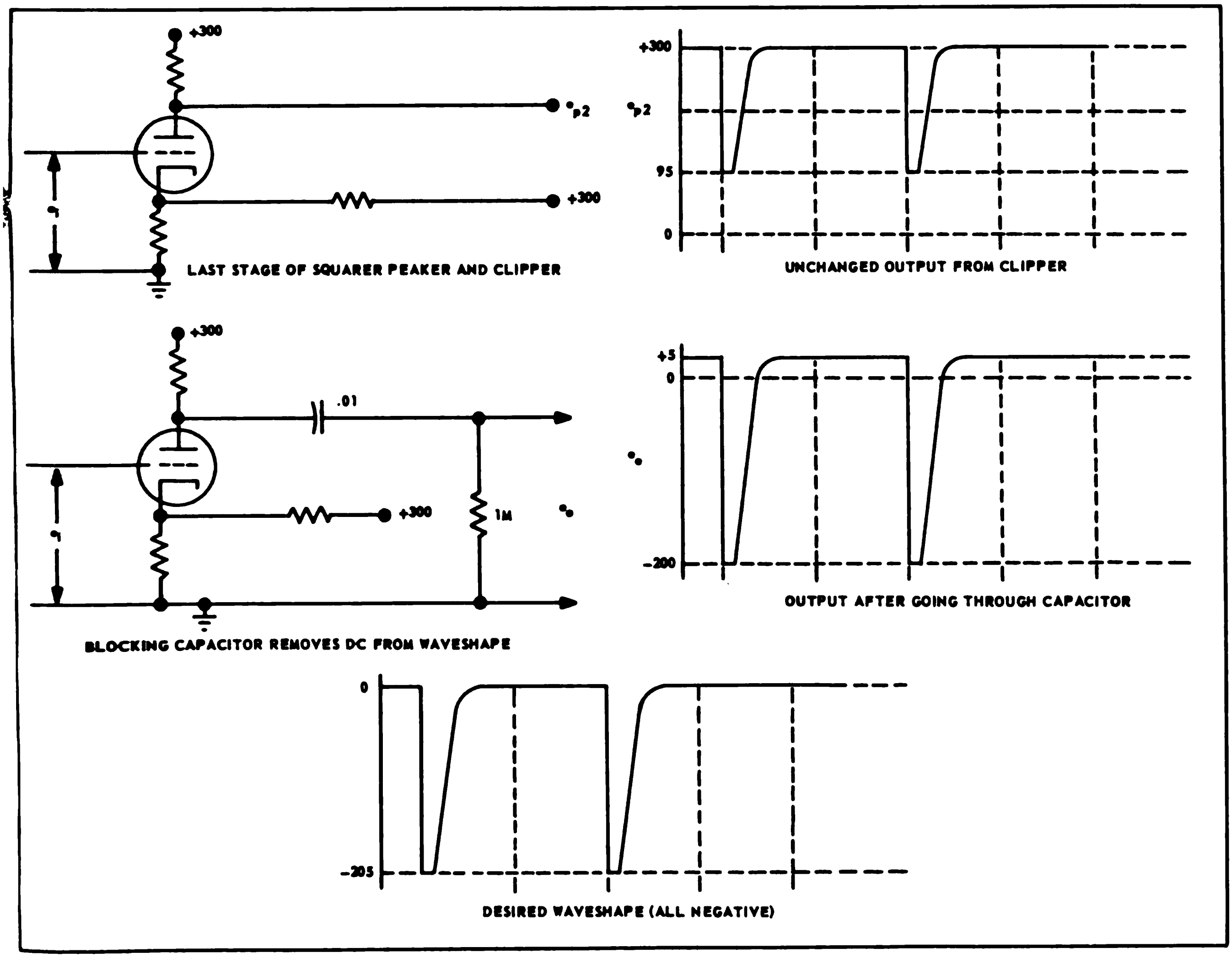

+300
e_{p2}
e_{p2}
+300
95
0
e_g
+300
LAST STAGE OF SQUARER PEAKER AND CLIPPER
UNCHANGED OUTPUT FROM CLIPPER
+300
.01
+5
0
e_o
+300
1M
e_o
e_g
-200
OUTPUT AFTER GOING THROUGH CAPACITOR
BLOCKING CAPACITOR REMOVES DC FROM WAVESHAPE
0
-205
DESIRED WAVESHAPE (ALL NEGATIVE)

Figure 6–46. Typical Need for Clamper

a short one during one half-cycle and a long one during the other half-cycle.

The applied voltage used in the circuit just mentioned is a square wave with a frequency of 500 cps and with a variation from plus 50 volts to minus 50 volts in amplitude, or 100 volts peak-to-peak. Starting from the beginning, assume that the capacitor is uncharged. When the first positive half-cycle is applied, the capacitor will charge toward 50 volts for 0.001 second. This positive voltage will cause the diode to conduct. Since the diode has an average internal impedance of 1000 ohms, the 50-volt potential will charge the capacitor through a resistance of 1000 ohms in parallel with 1-megohm, or 999 ohms. The time constant is 999 ohms × 1 mf or about 0.001 second. Due to these conditions, the capacitor will charge 63% during the half-cycle. or to about 31 volts. The equivalent circuit in Figure 6–47B for this half-cycle shows the action just described.

During the negative half-cycle a negative 50 volts is applied to the circuit. The capacitor is charged to 31 volts and both the applied voltage and the capacitor voltage cause current to flow in the resistor. During this cycle the diode plate is negative and does not conduct. Notice in the diagram that current from the discharge of the capacitor and current from the source both are in the same direction. Since currents in the same direction develop additive voltages, the voltage applied to the circuit equals 50 + 31 or 81 volts. R × C is 1-megohm × 1 mf or one second, which is 1000 times the time of a half-cycle. Therefore the capacitor will discharge only 0.1% during the half-cycle.

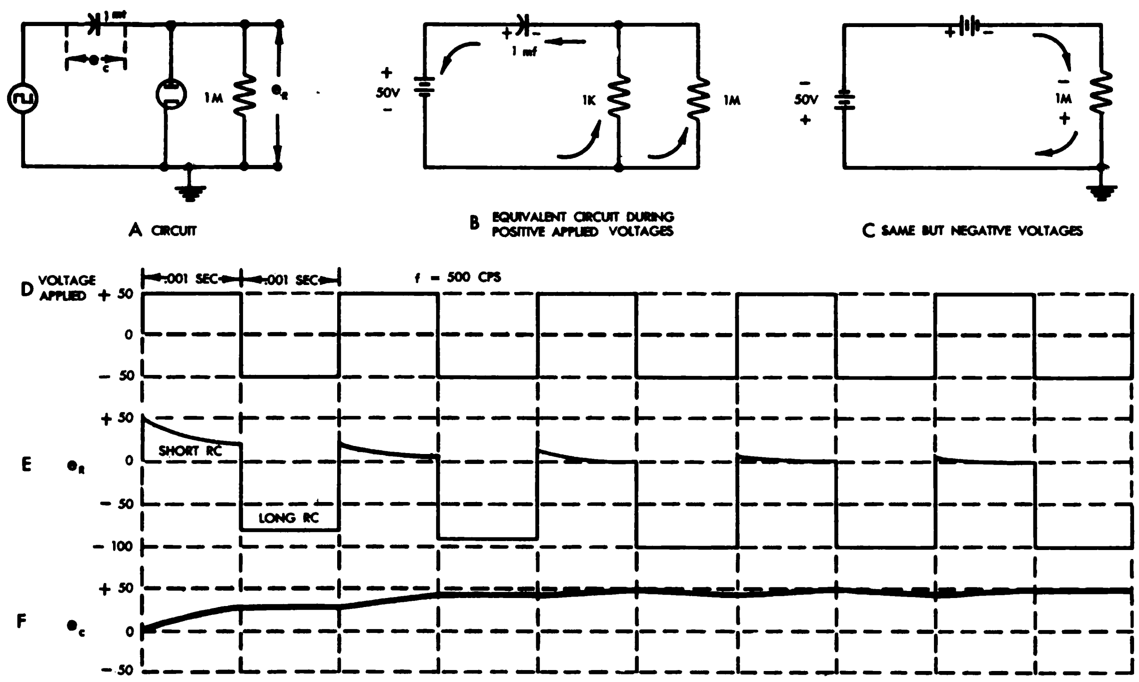


Figure 6–47. Clamping Circuit

On the next cycle the capacitor starts with almost a 31-volt charge. This 31 volts opposes the applied voltage and leaves only 50 − 31, or 19 volts to force current through the circuit. Since the time constant is short when the tube conducts, the capacitor will charge 63% of the 19 volts, or 12 volts. Adding this to the 31 volts already on the capacitor makes the charge at the start of the next half-cycle 43 volts. During the next half-cycle the discharge is slight, and the voltage at the source and the voltage across the capacitor cause 50 + 43, or 93 volts to appear across the resistor. The capacitor continues to be charged to 63% of the remaining voltage until its charge is practically 50 volts. At this time each positive half-cycle will cause practically zero voltage across the resistor while each negative half-cycle will cause 50 + 50 or 100 volts. In effect, the input voltage is half negative and half positive while the output (resistor) voltage is all negative.

Because of the preceding action the entire waveshape has shifted downward until the top is at the zero axis. Electrically you can say that a 50-volt DC component has been inserted. If the waveshape had been all negative at one time in the circuit, the circuit can be said to have *restored* it to negative.

Actually, there is a tiny bit of the waveshape left above the zero axis. This bit is the fraction of a volt required to recharge the capacitor after each half-cycle. With proper circuit design this can be made negligible.

The circuit described is a *self-adjusting* circuit, in that the top of the waveshape is clamped at zero regardless of the amplitude of the source voltage. When the amplitude is greater than shown, the capacitor charge becomes greater. When the amplitude decreases, the capacitor is not recharged as high during the charging time, and is allowed to discharge to the new lower voltage required.

VARYING THE R AND C. When the capacitor in a clamping circuit is made smaller, the charging rate is increased. However, the charging time is usually very short anyway and is of little consequence. But, on the other hand, the discharge time becomes faster. This is an advantage when the amplitude of the input waveshape is suddenly reduced, and a disadvantage when a constant input amplitude is present. As the capacitor discharges a lot with each cycle, a large part of the waveshape is shifted to the wrong side of the zero axis for recharging. A further result is that the recharging becomes a distortion of the original waveshape, a condition that usually is not desirable.

Increasing the resistor size increases the discharge time, causing less distortion during the cycle, but slowing the readjustment if a sudden drop in input amplitude occurs. The charge time (due to a sudden increase in input voltage) is unaffected because the diode resistance carries most of the current; the resistor current is too small to affect the charging time.

Circuit designs often incorporate a compromise between a short RC for following changes in amplitude and a long RC for least distortion. An important point to remember is that a clamping circuit will work with no resistor at all, even though most designs include a resistor in parallel with the diode.

The Positive Clamper

The clamping circuit just discussed is called a negative clamper. Figure 6–48A shows a simple negative clamping circuit with the input and output waveshapes. Although the input is a sine wave, the action is the same as that shown in Figure 6–47. Notice that the output waveshape is held below zero — it is a negative waveshape. To shift the waveshape in the positive direction, reverse the diode connections as shown in Figure 6–48B. The same input sine wave is used as in the negative clamper, but the output is all above zero. Therefore, the circuit is called a positive clamper.

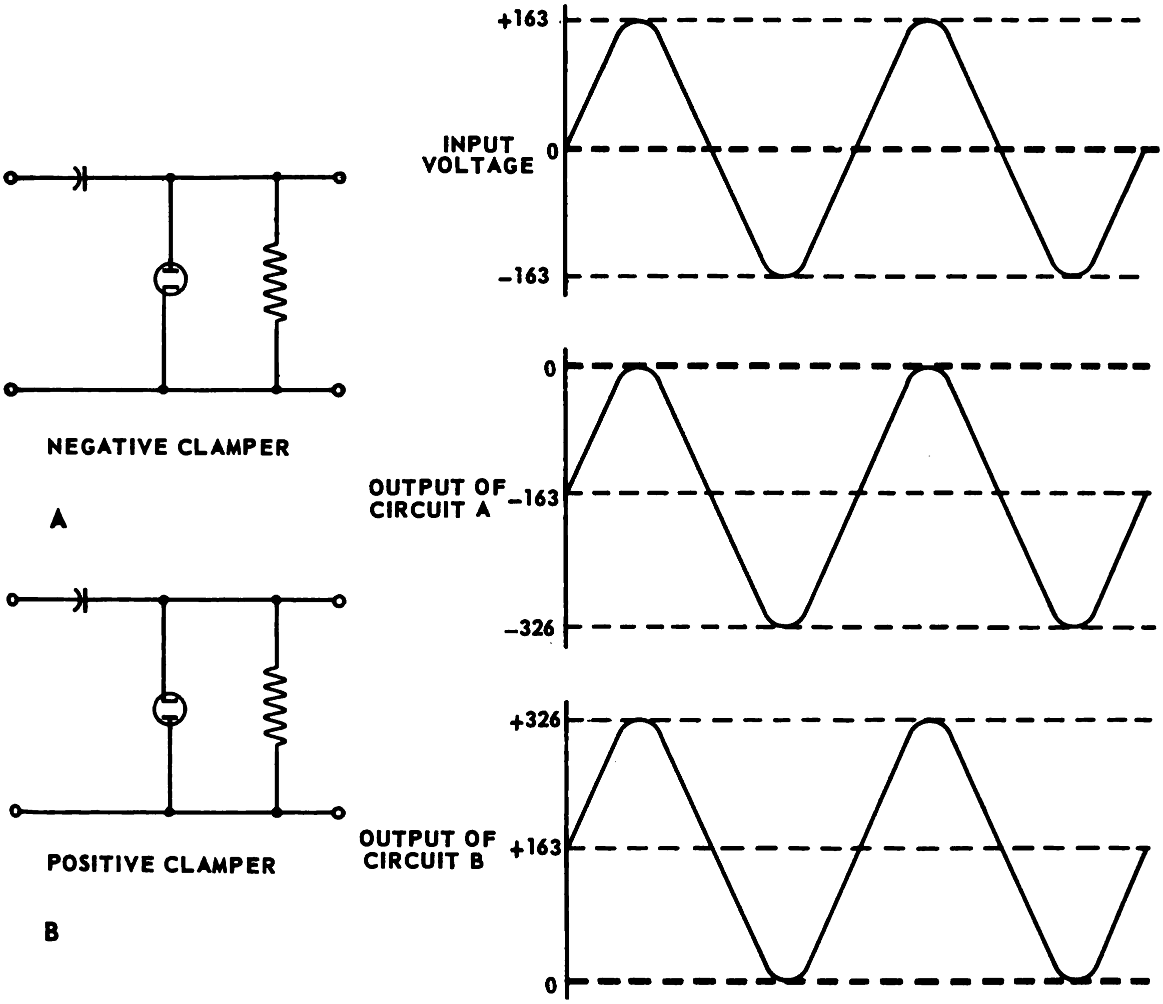
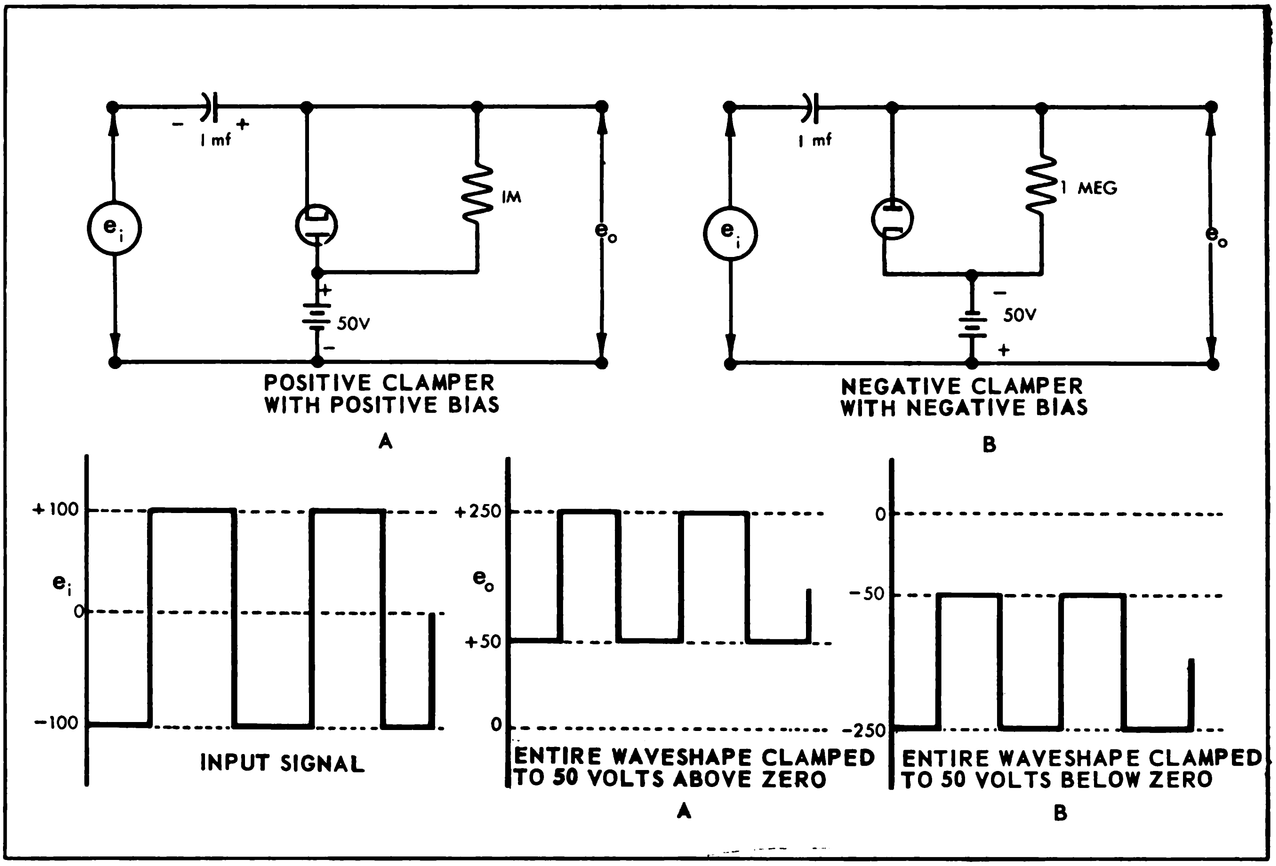


Figure 6–48. Positive Clamping Circuit



– + 1 mf
IM
ei
eo
+ 50V –
POSITIVE CLAMPER WITH POSITIVE BIAS
A
1 mf
1 MEG
ei
eo
– 50V +
NEGATIVE CLAMPER WITH NEGATIVE BIAS
B
+100
ei
0
–100
INPUT SIGNAL
+250
eo
+50
0
ENTIRE WAVESHAPE CLAMPED TO 50 VOLTS ABOVE ZERO
A
0
–50
–250
ENTIRE WAVESHAPE CLAMPED TO 50 VOLTS BELOW ZERO
B

Figure 6–49. Biased Clampers

Biased Clampers

A biased clamping circuit clamps a waveshape at an axis other than zero by inserting a DC bias of a voltage and polarity equal to the voltage at which clamping is desired. The only change in the circuit from the clampers just discussed is the insertion of a DC potential in series with the diode. The circuit in Figure 6–49A is a positive clamping circuit that shifts the entire waveshape to the positive side of zero. In operation, the DC source inserts an additional positive 50 volts, which shifts the entire waveshape to 50 volts above zero. Thus, the waveshape which originally varied from minus 100 volts to plus 100 volts, now varies from plus 50 volts to plus 250 volts.

Electronically, the shift is due to the DC source charging the capacitor to the DC potential, so that when the tube is nonconducting, the capacitor has an extra voltage on it in addition to that from the input voltage. For example, in this case, the square wave would charge the capacitor to half its peak-to-peak value, or 100 volts. This voltage is sufficient to clamp all the square wave in the output above the zero axis. Since the battery will add a 50-volt charge to the capacitor, the total charge will be 150 volts, and all the wave is clamped above +50 volts. The square wave input varies around this value in the output or between 150 + 100 and 150 − 100 which is 250 to 50 volts.

The circuit in Figure 6–49B is a negative clamping circuit that shifts the entire waveshape to the negative side of zero. In the circuit the DC source inserts in series with the tube and the 1-meg resistor an additional negative 50 volts, which shifts the entire waveshape to 50 volts below zero. The waveshape varies from minus 50 volts to minus 250 volts in contrast to the original waveshape, which varied from minus 100 volts to plus 100 volts. The operation of the negative biased clamper is similar to the positive biased clamper; however, the top of the wave is clamped to a negative 50-volt reference line.

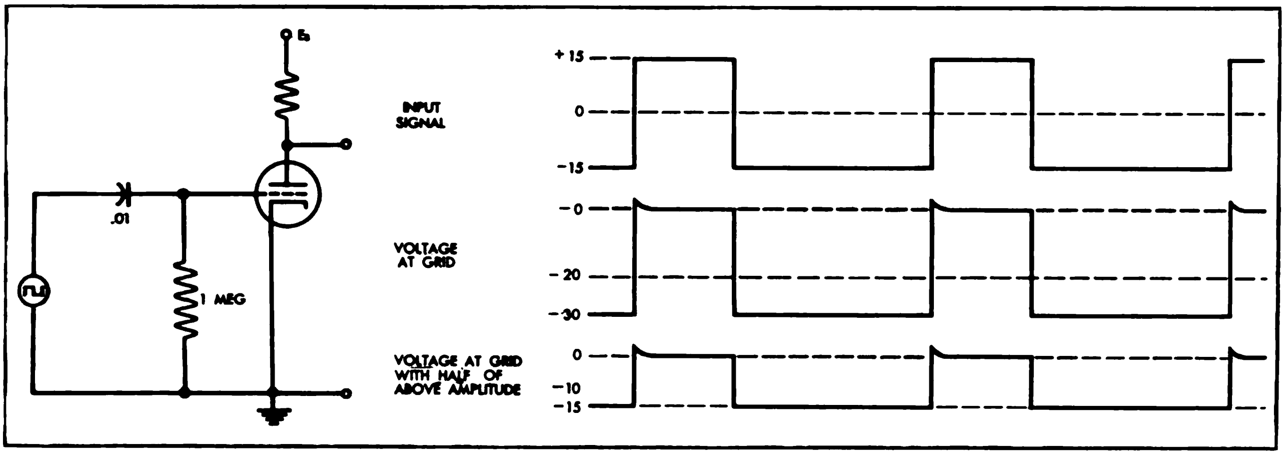


Figure 6–50. Clamping in the Grid Circuit

Clamping in a Grid Circuit

Clamping can occur in a grid circuit because the grid and cathode of a triode, in the absence of fixed bias, will form a diode and act as the clamping circuit. In the circuit in Figure 6–50, grid current flows during positive half-cycles, and charges the capacitor to a value which clamps the positive extremity of the input signal at zero. Except for the small positive voltage required to recharge the capacitor with each cycle, the signal at the grid will be all negative. The charge on the capacitor becomes the grid bias voltage.

This method for providing bias has the advantage of automatic adjustment to any amplitude of input signal. This means that, as in diode clampers, a larger signal will cause the capacitor to charge to a higher voltage, and reduction in signal amplitude will discharge the capacitor accordingly. In every case, the positive part of the signal is maintained at the zero axis. Note the input and grid waveshapes in Figure 6–50. The average bias (capacitor charge) for producing these waveshapes is minus 20 volts. The effect of a reduction in amplitude is shown in the last waveshape. The top is still clamped at zero. This clamping action does not change with frequency.

In radio circuits, grid clamping is called grid-leak bias, a type of bias that is generally used in Class C r-f amplifiers in communications transmitters, and in r-f oscillators.

TIME BASE GENERATORS

A time base generator circuit is a circuit that generates the voltage which causes the spot to move across the CRT screen at a constant rate. The distance along the trace formed by the moving spot is the basis of establishing time intervals. When recurrent voltage changes are superimposed on the trace, it is possible to measure both time and frequency of a radio signal, and in radar to measure the distance of targets. Commercial test oscilloscopes employ a thyratron tube in the time base generator circuit, because of the simple way in which the frequency can be changed. In radar sets the time base generator circuit usually employs a high-vacuum tube.

The Thyratron Sweep Generator

The movement of the spot in a CRT may be caused either by a voltage change on the deflection plates or by a current change in the deflection coils. The chief requirement is that the voltage (or current) must change linearly with time.

Although both RC and LR circuits produce a voltage change, the RC circuit is the one usually preferred as a means of generating a changing voltage. As previously mentioned, an RC circuit contains a DC voltage source, a series resistor and a capacitor. When those components are connected, the capacitor will start to charge but since the resistor limits the current, the charge will not be instantaneous, but at a rate that is exponential. The charging curve is quite linear in the very beginning, and the capacitor charging process is usually stopped before the slope of the charging curve changes appreciably. After the charging stops, the capacitor discharges, and the events just described repeat and the capacitor recharges.

Most time base generators use this very same charging and discharging process. Any circuit differences that exist result largely from the methods employed for high speed switching of the capacitor from charge to discharge. One device quick enough for this purpose is the electron tube. The circuit in Figure 6–51A shows a gas triode or thyratron, a special type of electron tube, used to provide the necessary switching operation, since this tube has certain special characteristics that make switching occur automatically. In the circuit the tube is in parallel with the capacitor.

When the circuit is turned on, there is no charge on the capacitor, and the tube acts as an open circuit due to the negative cutoff voltage on the control electrode, or grid. The capacitor charges along the first curve shown in Figure 6–51D. As the voltage increases, a potential is reached where the voltage across the tube is too high for the control electrode to keep the tube cut off and a small plate current starts to flow. This plate current ionizes the gas in the tube, and before the voltage can increase any further, a sudden high plate current flows. The plate voltage for the tube is provided by the charged capacitor.

Forcing a current through the tube causes discharge of the capacitor. Below a certain potential, the tube does not remain ionized. When the capacitor is discharged down to this potential, the gas in the tube deionizes, and the grid regains control of plate current. At this point the tube becomes an open circuit again and permits the capacitor to recharge. For further study refer to the equivalent circuits in Figure 6–51 — the charge of the capacitor at B and the circuit during discharge at C.

The curve for several charges and discharges at D shows that the voltage increases to the ionizing potential E_i, and then drops very abruptly to E_d, which is the deionizing potential. The slow rise is due to the size of R and C. The rapid drop in voltage forms an RC curve, but the resistance of the ionized tube is so small that the capacitor is discharged almost immediately, causing the curve to be practically vertical.

The voltage at which ionization occurs depends on the amount of voltage on the grid of the thyratron. If the grid is made less negative, a lower plate voltage will cause ionization as you see in the second curve at D. When ionization occurs earlier, the amplitude of the waveshape may decrease on one hand, but on the other, since the discharge time is the same, the wave will repeat more often; that is, its frequency will become higher. Another result is that the charging curve is much more linear; that is, its slope is more constant.

In thyratrons the deionization potential depends entirely on the gas pressure within the thyratron envelope. Deionization always occurs at the same potential regardless of the grid voltage or the applied plate potential. For example, the 884 tube, one customarily used in time base generator circuits, deionizes at about 20 volts.

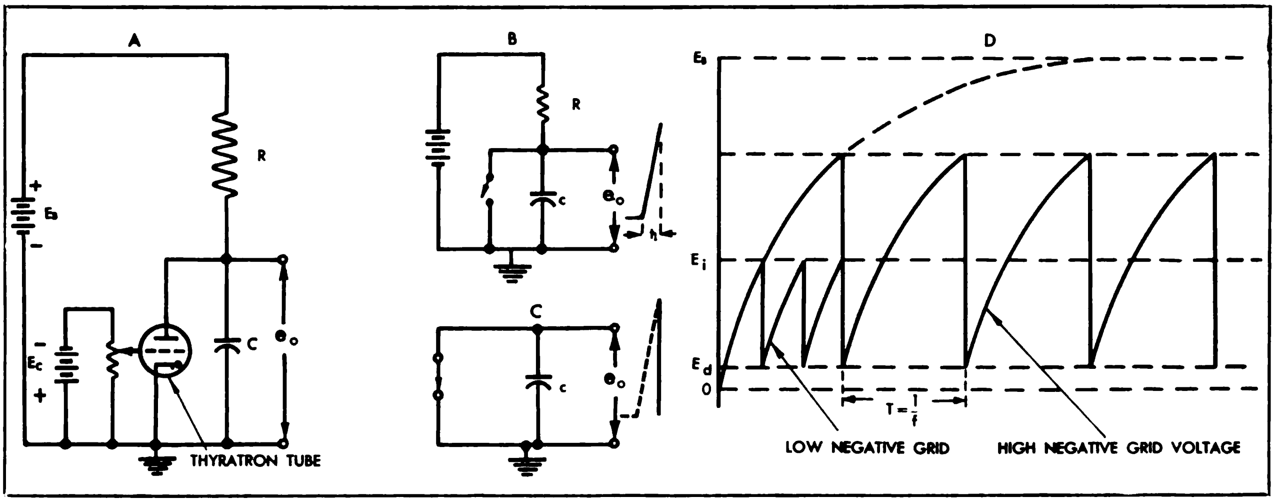


Figure 6–51. Thyratron Sweep Generator

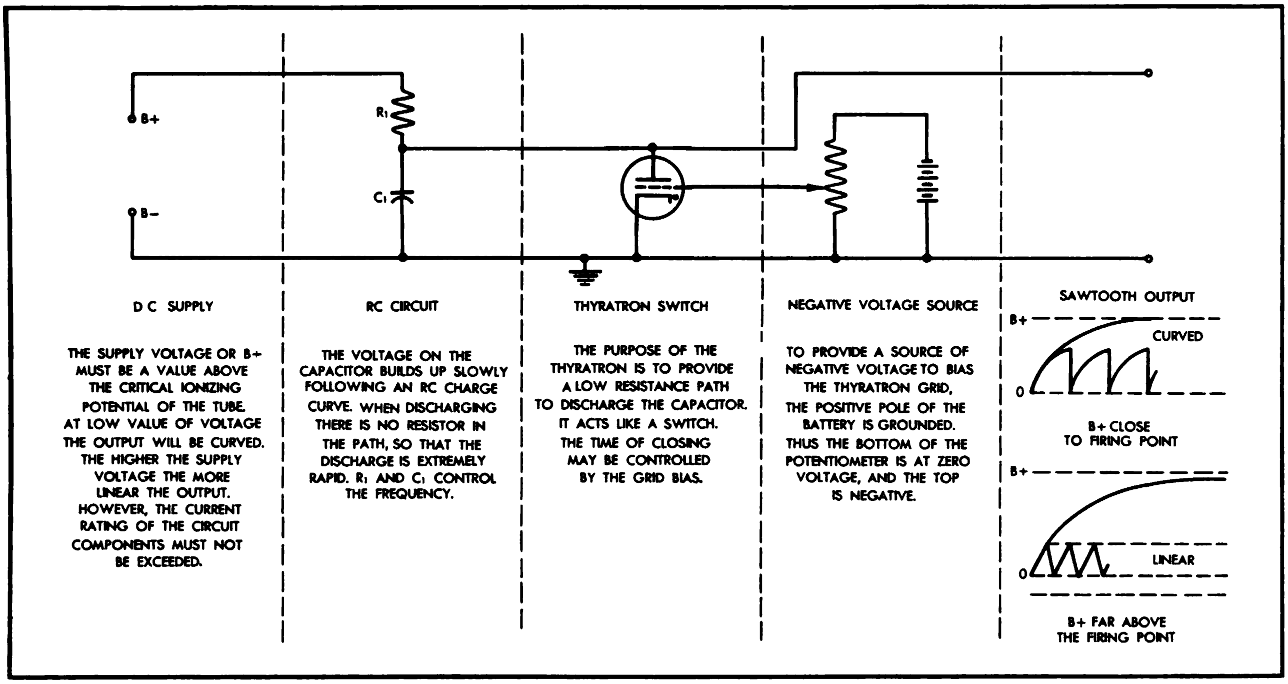

B+
B−
R₁
C₁
D C SUPPLY
THE SUPPLY VOLTAGE OR B+ MUST BE A VALUE ABOVE THE CRITICAL IONIZING POTENTIAL OF THE TUBE. AT LOW VALUE OF VOLTAGE THE OUTPUT WILL BE CURVED. THE HIGHER THE SUPPLY VOLTAGE THE MORE LINEAR THE OUTPUT. HOWEVER, THE CURRENT RATING OF THE CIRCUIT COMPONENTS MUST NOT BE EXCEEDED.
RC CIRCUIT
THE VOLTAGE ON THE CAPACITOR BUILDS UP SLOWLY FOLLOWING AN RC CHARGE CURVE. WHEN DISCHARGING THERE IS NO RESISTOR IN THE PATH, SO THAT THE DISCHARGE IS EXTREMELY RAPID. R₁ AND C₁ CONTROL THE FREQUENCY.
THYRATRON SWITCH
THE PURPOSE OF THE THYRATRON IS TO PROVIDE A LOW RESISTANCE PATH TO DISCHARGE THE CAPACITOR. IT ACTS LIKE A SWITCH. THE TIME OF CLOSING MAY BE CONTROLLED BY THE GRID BIAS.
NEGATIVE VOLTAGE SOURCE
TO PROVIDE A SOURCE OF NEGATIVE VOLTAGE TO BIAS THE THYRATRON GRID, THE POSITIVE POLE OF THE BATTERY IS GROUNDED. THUS THE BOTTOM OF THE POTENTIOMETER IS AT ZERO VOLTAGE, AND THE TOP IS NEGATIVE.
SAWTOOTH OUTPUT
B+
CURVED
0
B+ CLOSE TO FIRING POINT
B+
LINEAR
0
B+ FAR ABOVE THE FIRING POINT

Figure 6–52. Functions of Circuit Components

The circuit which was described is called a free-running circuit; when a DC potential is applied to it, it generates one time base after another indefinitely. Figure 6–52 explains further the function of the circuit components. There are other circuits which will be discussed later that must be driven. These generate only one time base and then wait for the driving voltage to start the charging process which generates the next time base.

Calculating Frequency of Oscillation. To calculate the frequency of oscillation of a thyratron sweep generator there are a few factors which you must know. Figure 6–53A shows a practical thyratron sweep generator circuit with an applied voltage of 250 volts. The 50K resistor in the plate circuit provides a minimum resistance in series with the tube. The 10K resistor in the grid circuit limits the grid current to a safe value. To show the details of this circuit's operation we

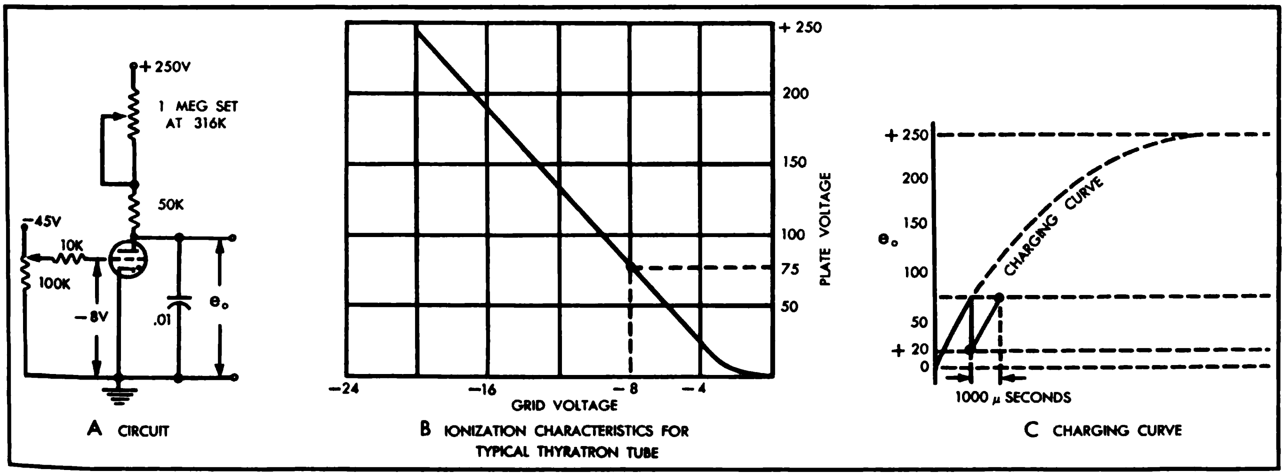


Figure 6–53. Practical Thyratron Sweep Generator Circuit

will use the values just given in a problem in which the bias is set at minus 8 volts and the variable resistance is set at 316K. Figure 6–53B shows the relation of the firing potential to the grid bias. Thus, when the grid voltage is minus 8 volts, the tube ionizes when the plate-to-cathode voltage reaches 75 volts. Using this information, you can plot a charging curve like that shown in Figure 6–53C.

This shows that during the first cycle, the capacitor voltage starts from zero, but on the second and succeeding cycles, it always starts charging from 20 volts, the deionization potential. This charge cancels 20 volts of the 250 volts plate potential. At 75 volts, the thyratron ionizes, stopping the capacitor charge. With 230 volts applied, therefore, one cycle has a duration equal to the time required to charge the capacitor from 20 to 75 volts, or 55 volts.

To determine what fraction of full charge is represented by 55 volts, use the universal time constant chart curve. From it you learn that this value is 55/230, or 0.239. Therefore, the capacitor is charged 23.9% before ionization. Using curve A in Figure 6–10, the Universal Time Constant Chart, find 0.239 of full charge; then move across to the curve, and down to 0.26 time constants. One time constant is RC, or 366K × .01, or .00366 second. Therefore, the time required for the charge is 0.26 × .00366, or .000952 second. Since the frequency is the reciprocal of the time of one cycle, F = 1/.000952, or 1050 cps.

For greater accuracy find t by the exponential equation, $e_c = E_B\ (1-\epsilon^{-t/RC})$. Since the capacitor starts charging from 20 volts toward 250 volts, the curve is the same as from zero to 230 volts. This makes the E_B in the equation equal to 230 volts. The capacitor charges from 20 to 75 volts, or to 55 volts of the 230 volts. Therefore, e_c is 55 volts. Substituting,

$$55 = 230\ (1 - \epsilon^{-t/3.66 \times 10^5 \times 10^{-8}})$$

$$\frac{55}{230} - 1 = -\epsilon^{\frac{-t}{3.66 \times 10^5 \times 10^{-8}}}$$

$$-0.761 = -\epsilon^{\frac{-t}{3.66 \times 10^5 \times 10^{-8}}}$$

From the table,

$$\frac{t}{3.66 \times 10^{-3}} = 0.273$$

$$t = 0.273 \times 3.66 \times 10^{-3}$$

$$t = 10^{-3} \text{ seconds}$$

$$F = \frac{1}{t} = \frac{1}{10^{-3}} = 1000 \text{ cps}$$

The time base voltage repeats 1000 times per second.

VARIATION OF CIRCUIT ELEMENTS. Any variation in the circuit constants affects frequency, linearity and amplitude. The effects of varying circuit elements in a thyratron sweep generator are summarized in Figure 6–54.

The use of a thyratron as a time base generator in a commercial oscilloscope requires a frequency that is variable. In these instruments coarse frequency changes are provided by switching the capacitors (Figure 6–56), and fine frequency adjustment, by varying the plate resistance and holding the grid bias constant. The usable frequency range extends from a few cycles to 50 kc in the commercial oscilloscope. The upper frequency limit is 50 kc because the discharge time at 50 kc is equal to the charging time, and half of any signal being investigated will be lost during the flyback (discharge) time.

SYNCHRONIZATION. In addition to all the factors just mentioned, the temperature of the gas, the slightly variable ionization potential, and the load on the circuit also affect frequency. Because of these factors the circuit is usually not stable enough to maintain an adequately constant frequency by itself and must be synchronized with an AC voltage that has a constant frequency. This synchronizing voltage may be any voltage that has a frequency near (usually a little above) the multiple, submultiple, or fundamental frequency of the thyratron generator. Due to the synchronization voltage the thyratron tube ionizes slightly ahead of normal time, and the capacitor discharges immediately and starts a new cycle.

Figure 6–55A shows a typical synchronization circuit in which a sine wave is used to synchronize the generator in the grid circuit. The sine wave varies the grid voltage and this, in turn, the ionization potential. The waveshape at B shows how the ionization potential varies at a slightly higher frequency than the natural free-running frequency of the generator. The dotted saw-tooth waveshape results from a constant ionization potential. At the time the capacitor voltage is about to reach the normal value, the ionization potential drops. When the capacitor voltage reaches the ionization potential, ionization occurs, starting a new cycle immediately.

This same action recurs with each cycle and causes the time base frequency to "lock-in" step

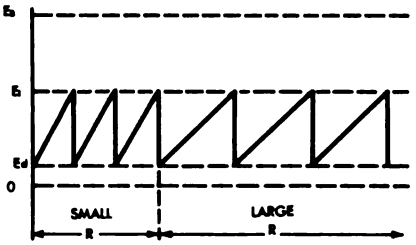
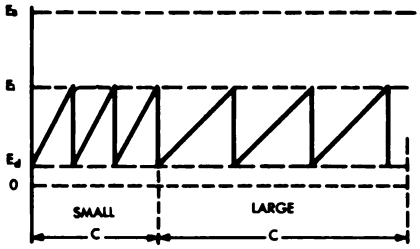
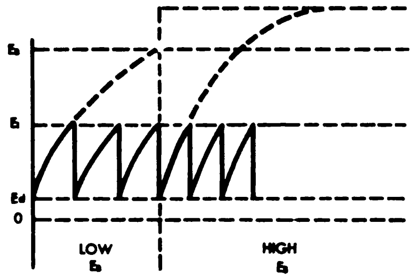
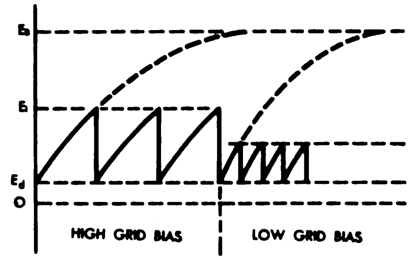
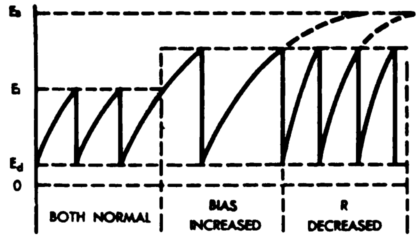
ELEMENT VARIED	DURATION OF ONE CYCLE	FREQUENCY	AMPLITUDE	LINEARITY	EFFECT ON WAVESHAPE
INCREASING R	LONGER	LOWER	UNCHANGED	UNCHANGED BECAUSE C STILL CHARGES TO SAME PERCENTAGE OF E_B	
INCREASING C	LONGER	LOWER	UNCHANGED	UNCHANGED	
INCREASING E_b	SHORTER	HIGHER	UNCHANGED	IMPROVED BECAUSE LESS OF CHARGING CURVE IS BEING USED	
DECREASING NEGATIVE VOLTAGE ON GRID	SHORTER	HIGHER	DECREASED BECAUSE IONIZATION POTENTIAL IS LOWER	IMPROVED BECAUSE A SMALLER PART OF THE START OF THE CHARGING CURVE IS USED	 E_b E_i E_d 0 HIGH GRID BIAS LOW GRID BIAS
INCREASING THE BIAS FOR GREATER AMPLITUDE AND DECREASING R TO RETAIN ORIGINAL FREQUENCY	UNCHANGED	UNCHANGED	INCREASED BECAUSE IONIZATION POTENTIAL IS HIGHER	DECREASED BECAUSE GREATER PART OF CHARGING CURVE IS USED	 E_b E_i E_d 0 BOTH NORMAL BIAS INCREASED R DECREASED

Figure 6–54. Effect of Varying Elements in Thyratron Sweep Generator

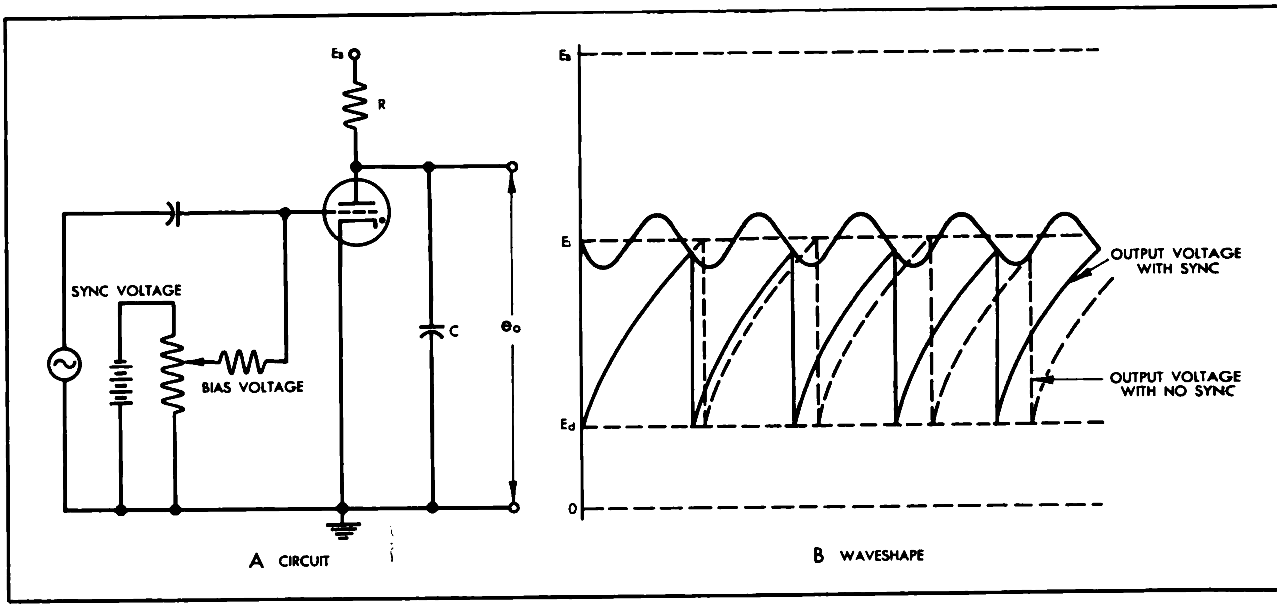


Figure 6–55. Synchronization with Sine Wave

with the synchronizing frequency. Often in the use of the commercial oscilloscope the waveshape being displayed on the CRT is used for synchronization. This is done so that the waveshape and time base will occur simultaneously and the pattern will be stationary. If there is no synchronization, a difference in frequency may exist and the pattern displayed on the screen may move forward or backward along the time base.

DuMont 208 Oscilloscope. A typical oscilloscope using a thyratron time base generator is the DuMont 208. Figure 6–56 shows the sweep-generator circuit. The tube is a type 884 thyratron. Fixed bias is provided in the cathode by a voltage divider from B+. The bias is held constant at 3.1 volts when the tube is nonconducting. Synchronization may be switched in from three sources — the signal being displayed, the 60-cycle power line frequency, or from a terminal post to which any other signal can be connected.

The 100K potentiometer is used to vary the amplitude of the synchronization voltage applied to the grid. One of the eight capacitors, when connected in the RC circuit, provides the coarse

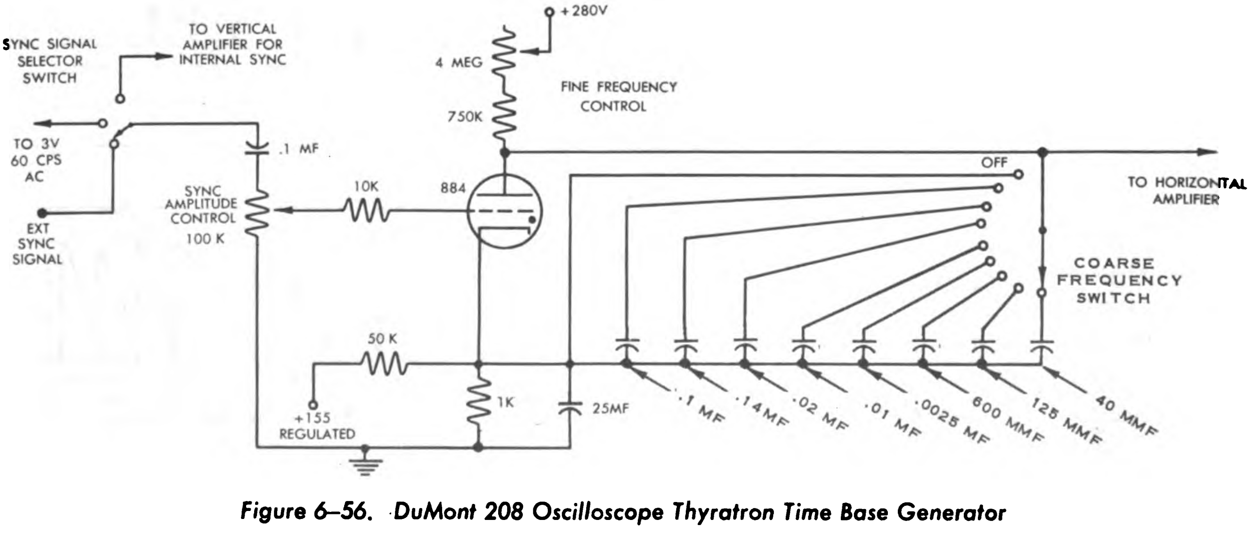


Figure 6–56. DuMont 208 Oscilloscope Thyratron Time Base Generator

frequency adjustment. The 4-megohm potentiometer provides fine tuning in that it can be used to set the frequency at exactly the desired value. Maximum linearity is obtained by keeping the amplitude of the output wave low. The desired amplitude is produced by the amplifiers. The output signal from this stage goes to a video amplifier.

The Hard Tube Sweep Generator

Because a thyratron is not readily adaptable for producing the time base with the spaced intervals required by radar equipment, the high vacuum, or hard tube, is used instead. A hard tube is one that is highly evacuated; that is, one from which as much gas as possible is removed. In contrast a *soft* tube is one with an incomplete vacuum; that is, one in which some gas remains in the envelope. A hard tube serves as a switch for discharging the capacitor, but it is not automatic and must be gated by an external circuit.

Figure 6–57A is the circuit of a hard tube sweep generator. In Figure 6–57B the top waveshape is the voltage that is usually used to operate the circuit. Normally, this voltage will hold the grid of the tube at zero. The tube in the circuit conducts a relatively large current. The load line in Figure 6–57C shows this current to be nearly 2 ma. According to the load line chart, the plate voltage due to this current is 25 volts.

A time base is formed whenever a radar pulse is transmitted. Simultaneously with the transmitted pulse the negative square wave input voltage drives the grid far below cutoff. As a result, the plate voltage tends to rise immediately to 250

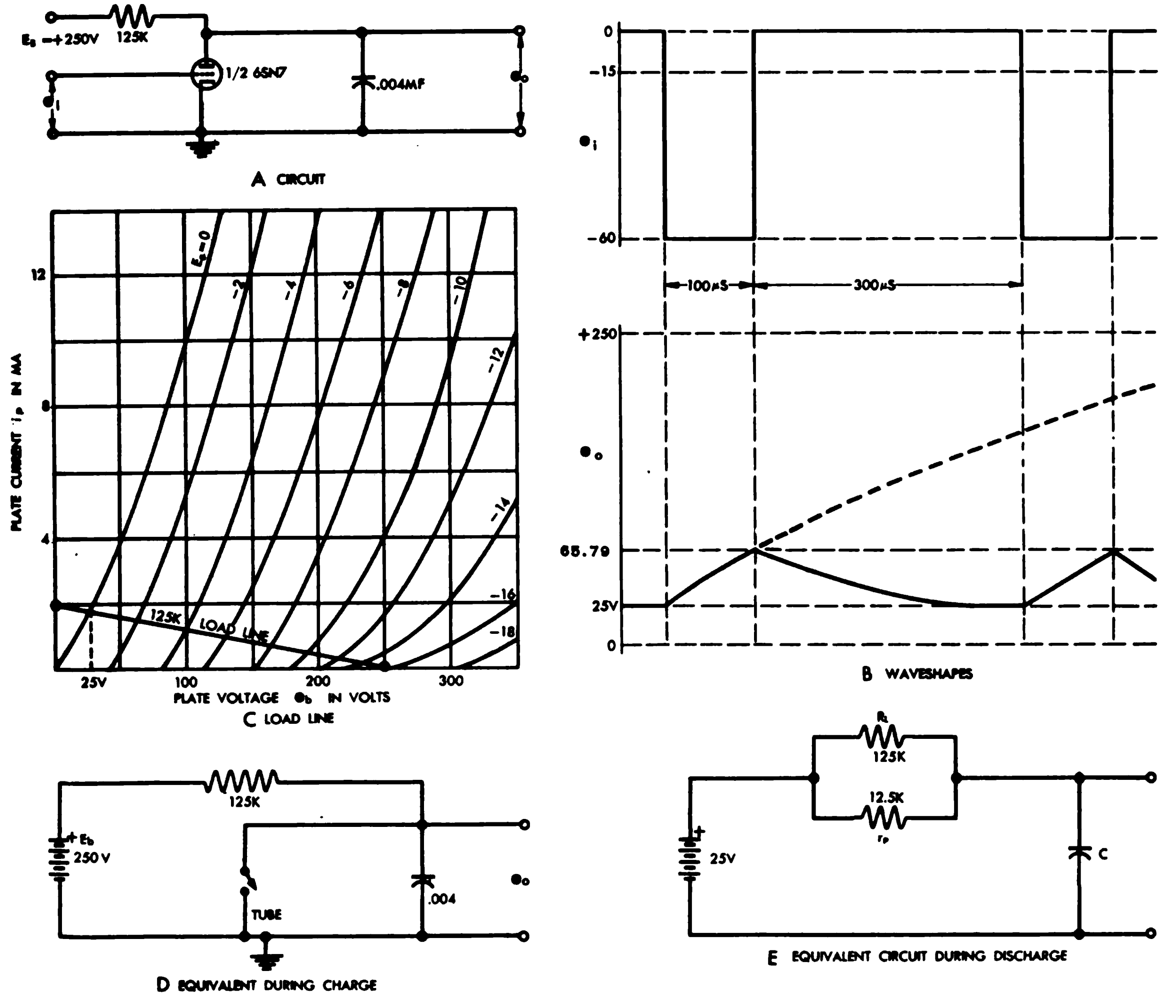


Figure 6–57. Hard Tube Sweep Generator

volts, but since it cannot rise faster than the capacitor can charge, its rise is exponential. If a long time constant is used, the capacitor can only reach a small percentage of full charge before the grid cutoff voltage is removed, and tube current discharges the capacitor back down to 25 volts (Figure 6–57B, lower waveshape). The long time interval between transmitter pulses and the short time constant formed by the low tube resistance insure capacitor discharge to 25 volts before the next cutoff voltage occurs.

You can readily calculate the amplitude and linearity of a time base such as the one formed in the preceding discussion in the following manner. First, draw the load line for the tube (Figure 6–57C). The load line shows that the voltage at the plate is 25 volts when the grid voltage is zero, and that cutoff for the tube is —16 volts when 250 volts is applied to the plate. Other factors to consider are that the —16 volts cutoff bias is easily exceeded by the —60 volts of the square pulse, which is 100 microseconds in dura-

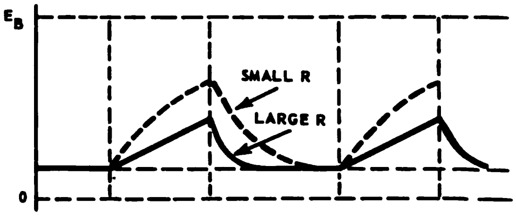
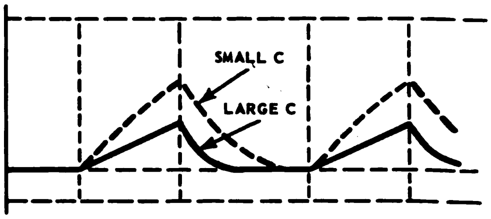
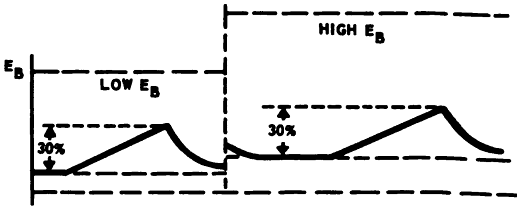
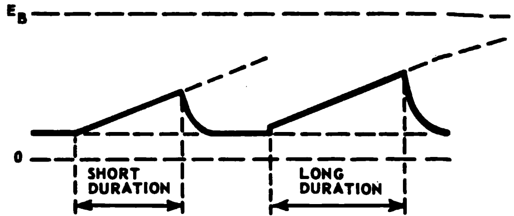
ELEMENT VARIED	EFFECT ON AMPLITUDE	EFFECT ON LINEARITY	EFFECT ON DURATION	WAVESHAPE
R IS INCREASED	DECREASED. BECAUSE CAPACITOR CHARGES SLOWER AND IS STOPPED BEFORE IT GETS AS HIGH	IMPROVED. BECAUSE LOWER AMPLITUDE MEANS LESS OF CHARGING CURVE IS USED	NO CHANGE. DURATION DETERMINED BY INPUT WAVESHAPE	
C IS INCREASED	DECREASED. CAPACITOR CHARGES SLOWER, AND IS STOPPED BEFORE IT GETS AS HIGH	IMPROVED. LOWER AMPLITUDE MEANS LESS OF CHARGING CURVE IS USED	NO CHANGE. DURATION DETERMINED BY INPUT WAVESHAPE	
E_B IS INCREASED	INCREASED. CAPACITOR CHARGES TO SAME PERCENTAGE WHICH IS A HIGHER VOLTAGE	NO CHANGE CAPACITOR CHARGES TO SAME PERCENTAGE AS BEFORE	NO CHANGE	
DURATION OF INPUT CUTOFF VOLTAGE INCREASED	INCREASED. CAPACITOR CHARGES LONGER AND SO TO A HIGHER VOLTAGE	DECREASED. MORE OF CHARGING CURVE IS USED	INCREASED. BECAUSE INPUT DURATION IS INCREASED	
AMPLITUDE OF INPUT SIGNAL INCREASED	NO CHANGE IF SIGNAL WAS ALREADY CUTTING OFF TUBE	NO CHANGE IF SIGNAL WAS ALREADY CUTTING TUBE OFF	NO CHANGE	NO CHANGE IN PLATE WAVESHAPE

Figure 6–58. Effect of Varying Elements in Hard Tube Sweep Generator

tion (represented as time t in the calculations), and that the applied voltage to the capacitor is 250 — 25 or 225 volts. To find the capacitor voltage (e_c) after 100 microseconds, perform the following operation:

$$e_c = 25 + E_B\,(1 - \epsilon^{-t/RC})$$

$$e_c = 25 + 225\,(1 - \epsilon^{-10^{-4}/1.25 \times 10^{5} \times 4 \times 10^{-9}})$$

$$e_c = 25 + 225 - 225\,\epsilon^{\frac{-1}{5}}$$

$$e_c = 250 - 225\,\epsilon^{-0.2}$$

$$e_c = 250 - 225\,(0.8187)$$

$$e_c = 250 - 184.21$$

$$e_c = 65.79 \text{ volts.}$$

The equivalent circuit during the charge is shown in Figure 6–57D. Here the tube acts as an open switch. If the rise continued at the initial rate, the curve would be perfectly linear. At the initial rate, the voltage would rise 225 volts during the first time constant. Since it is allowed to charge for 0.2 time constant, it would have risen 225 × 0.2 or 45 volts after this time. However, the actual rise is 40.79 volts (65.79 — 25). Thus, the actual curve varies from the linear curve by 4.21 volts.

When the grid voltage returns to zero, the equivalent circuit becomes as shown in Figure 6–57E, and the following action occurs. The capacitor discharges from 65.79 volts to 25 volts through the internal resistance of the conducting tube and the load resistor. At zero grid voltage, the plate voltage is 25 volts, and plate current approximately 2 ma. Although this value of plate current holds only when the grid voltage is zero, it is used here because the grid voltage remains constant at zero until the next cycle of negative voltage arrives. The resistance of the tube in parallel with 125K makes a total resistance of 11.35K. The time constant of this RC is 11.35 × .004 mf, or 45.5 microseconds. Because there are 300 microseconds between pulses, there is more than ample time for the capacitor to discharge to 25 volts. Figure 6–58 illustrates the effect of varying the elements.

As shown in Figure 6–59, the typical sweep generator circuit in a radar set has a negative square wave applied to the grid of the tube. The duration of the square wave is adjusted to exactly the time required for the radar wave to travel 10, 50, or 100 radar miles. When the range switch is on the 10-mile position, the input voltage is the 10-mile square wave, and the .003 mf capacitor is switched in the plate circuit. For the other ranges, larger capacitors are switched into the plate circuit, and the input square wave is longer. The 1-megohm potentiometers in the plate circuit vary the amplitude of the output sawtooth wave. They are adjusted so that the sweep line on the CRT screen will extend from the starting point to the edge of the screen.

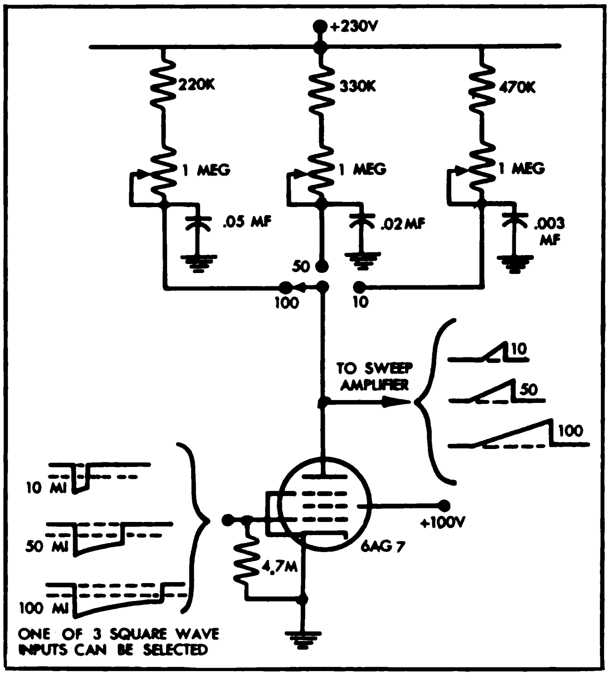


Figure 6–59. Typical Sawtooth Generator

Trapezoidal Waveshape Generator

The sawtooth sweep generator previously described is suitable for any electrostatically deflected CRT. However, you will find that the majority of the larger radar sets use magnetic deflection. In these cathode-ray tube circuits sweep voltages are applied to coils. In long range sweeps the sawtooth voltage is adequate for a linear sweep, but on short ranges, a special sweep voltage with a trapezoidal wave form must be applied to overcome the effects of the LR formed by the sweep coils.

The deflection of the spot in a magnetic CRT depends upon a change in magnetic field strength. Since this magnetic field strength varies directly with the current in the deflection coils, any li deflection of the spot requires a linear variation of the current through the deflection coil. A fairly linear current change occurs whenever a square wave is applied to an inductor if the first 10% of the time constant is used. As shown in Figure 6–60A, square wave e_a is applied to an LR circuit. (Remember that all deflection coil circuits are LR circuits because the coil wire and associated components introduce resistance in series with the actual inductance of the coil.) In this LR circuit the current change through the coil increases rapidly at first but then decreases when the voltage drop across the resistor decreases the voltage applied to the coil. During the first 10% of the rise, the increase in current is quite constant. (See Figure 6–60B.)

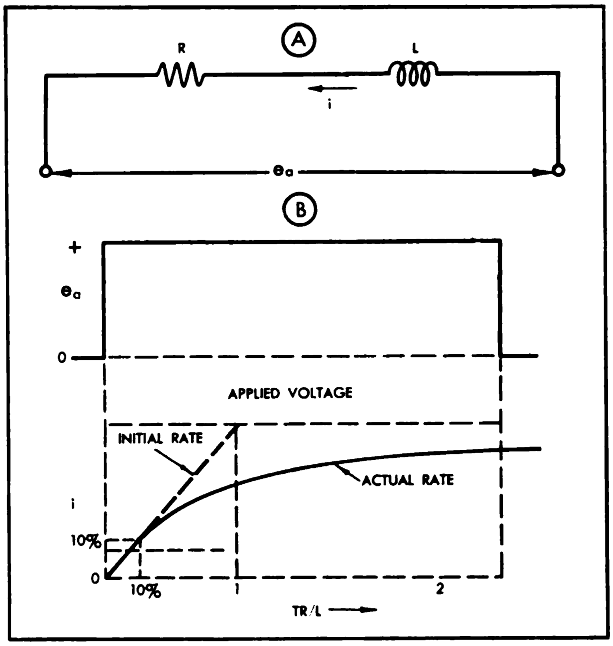


Figure 6–60. Square Wave Applied to LR Circuit

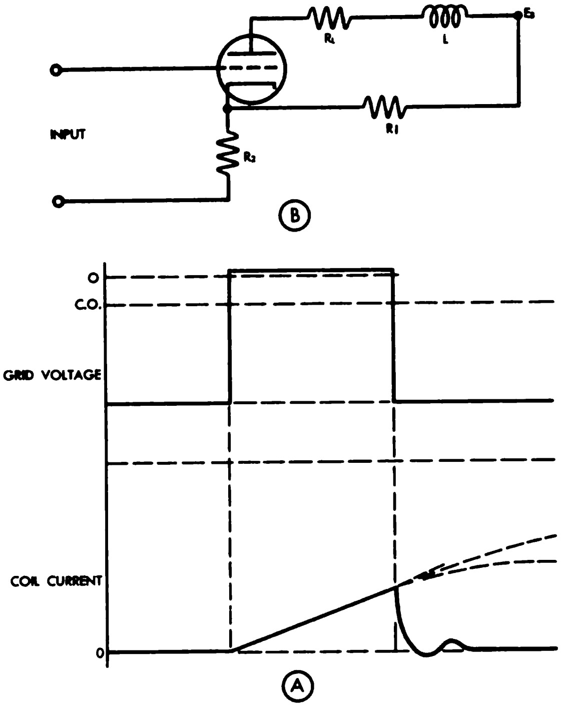


Figure 6–61. Linear Rise Due to Long Time Constant

Making the L/R much longer than the time of the applied voltage produces a reasonably linear sweep. Referring to the simple circuit in Figure 6–61B utilizing a square wave input, you see a tube that is normally cut off by fixed bias. When the sweep is ready to start, the grid voltage is brought up to zero with respect to the cathode by a positive square wave. Plate current flows, increasing at a rate determined by the inductance and resistance. This resistance is the plate resistance of the tube and the resistance of the coil. The time constant of the LR circuit formed is long, and therefore the current rise is slow. The pulse on the grid ends before the current becomes very high, thus keeping the current increase in the linear portion of the curve (Figure 6–61A). When the tube is cut off, the magnetic field collapses, causing an oscillatory transient. The transients can be prevented by a change in the circuit, or made harmless by blanking the spot during this time.

It is difficult to keep the series resistance small because the plate resistance of the vacuum tube involved is never small. Therefore, shorter time constants (larger R in equation L/R) are accepted, and the applied voltage is changed to compensate for the large R. Referring to Figure 6–60B, notice that if there were no resistance in this circuit, the current would increase indefinitely at the initial rate along the broken curve. This is because the fundamental relationship between the voltage and current in an inductor is $E = L\,di/dt$, where E is the applied voltage, L the inductance, di/dt the rate of change of the current. L and E are constant and time changes linearly; therefore the current *i* must change linearly to make the equation true. The rate decreases along the exponential curve because the voltage drop across the resistor, due to this same current through it, subtracts from the applied voltage to leave less voltage across the inductor. Less voltage causes the current to increase at a lower rate.

The effect of the resistor may be eliminated if the applied voltage is increased at the same rate that the resistor voltage increases. Under these conditions, the subtraction of the resistor voltage maintains a constant voltage across the inductor, and the current increases at a constant rate.

The applied voltage waveshape just described is called a trapezoidal wave. In the waveshape shown in Figure 6–62, the initial instantaneous rise is called the *jump* voltage. It is this voltage which starts the current flow in the inductor. The slow increase is called the *slope* voltage. Its action is to continuously compensate for the drop across the resistor.

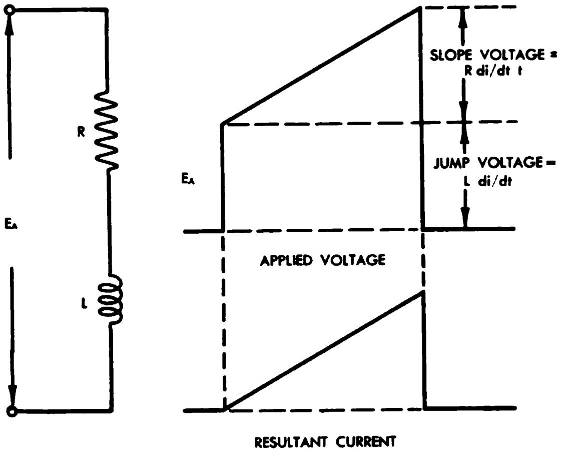


Figure 6–62. Trapezoidal Waveshape

A quantitative analysis shows that there is a direct relation between the L/R, the jump voltage, and the slope voltage. In addition, since the trapezoidal voltage is produced with an RC circuit, there is a direct relationship between the L/R of the sweep amplifier and the RC of the sweep generator.

The jump voltage is the voltage that would be applied to a pure inductance. It is a constant value equal to L di/dt, where L is the inductive reactance and di/dt is the current change in amperes per second. As the current change also occurs in the resistor, you can use the Ohm's law formula E = IR to determine the voltage across the resistor. The facts are that R is constant and I is the same continuously changing current that is flowing through the inductor. Thus, according to Ohm's law, the slope of the resistor voltage becomes R di/dt.

Remember, that in the determination of the waveshape required for any sweep coil, the ratio of the jump voltage to the change in resistor voltage is most important. This ratio can be expressed as

$$\frac{\text{jump}}{\text{slope}} = \frac{\text{L di/dt}}{\text{R di/dt}} = \frac{\text{L}}{\text{R}},$$

where di/dt is the rate of change of current. The ratio is equal to the time constant of the LR circuit.

To learn to use this relationship, study the following sample problems.

Examples

Problem 1

Assume the CRT and coils in the CRT assembly have been assembled and that as part of a radar set a linear time base for a 50-nautical mile range is required—that is, the spot must swing across the scope at a constant rate in 620 microseconds (the time of 50 radar miles of travel). Experiments with direct current indicate that a current of 100 ma causes the spot to move the required distance. Therefore, for a linear time base that covers the screen, the current in the coils must change at a constant rate from zero to 100 ma in 620 microseconds. The required voltage waveshape is shown in Figure 6–63C. Further measurements indicate that the total resistance in the coil windings is 400 ohms and the inductance is 100 millihenrys. The equivalent circuit of the coils is shown in Figure 6–63B. What jump and what slope voltages are required to cause a perfectly linear sweep on the CRT?

Solution

The voltage across the inductor E_L is L di/dt.

$$\text{Therefore, } E_L = 10^{-1} \times \frac{10^{-1}}{620 \times 10^{-6}} = \frac{10^{-1} \times 10^{-1}}{620 \times 10^{-6}}$$

$$E_L = 16.1 \text{ volts}$$

The resistor voltage change is R di/dt

$$\text{Therefore, it equals } 400 \times \frac{10^{-1}}{620 \times 10^{-6}} = \frac{400 \times 10^{-1}}{620 \times 10^{-6}},$$

or 6450 volts per second.

The change in voltage across the resistor is at a rate of 6450 volts per second. Since the time base only lasts 620 microseconds, the voltage change will be 620 millionths of this value, or $6450 \times 620 \times 10^{-6}$, or 40 volts.

You can easily check this result as follows: The current at the end of the time base is 100 ma and is the final resistor current. The resistance is 400 ohms; therefore $E_R = IR = 0.1 \times 400$ or 40 volts. The required input waveshape, drawn to scale, is shown in Figure 6–63C.

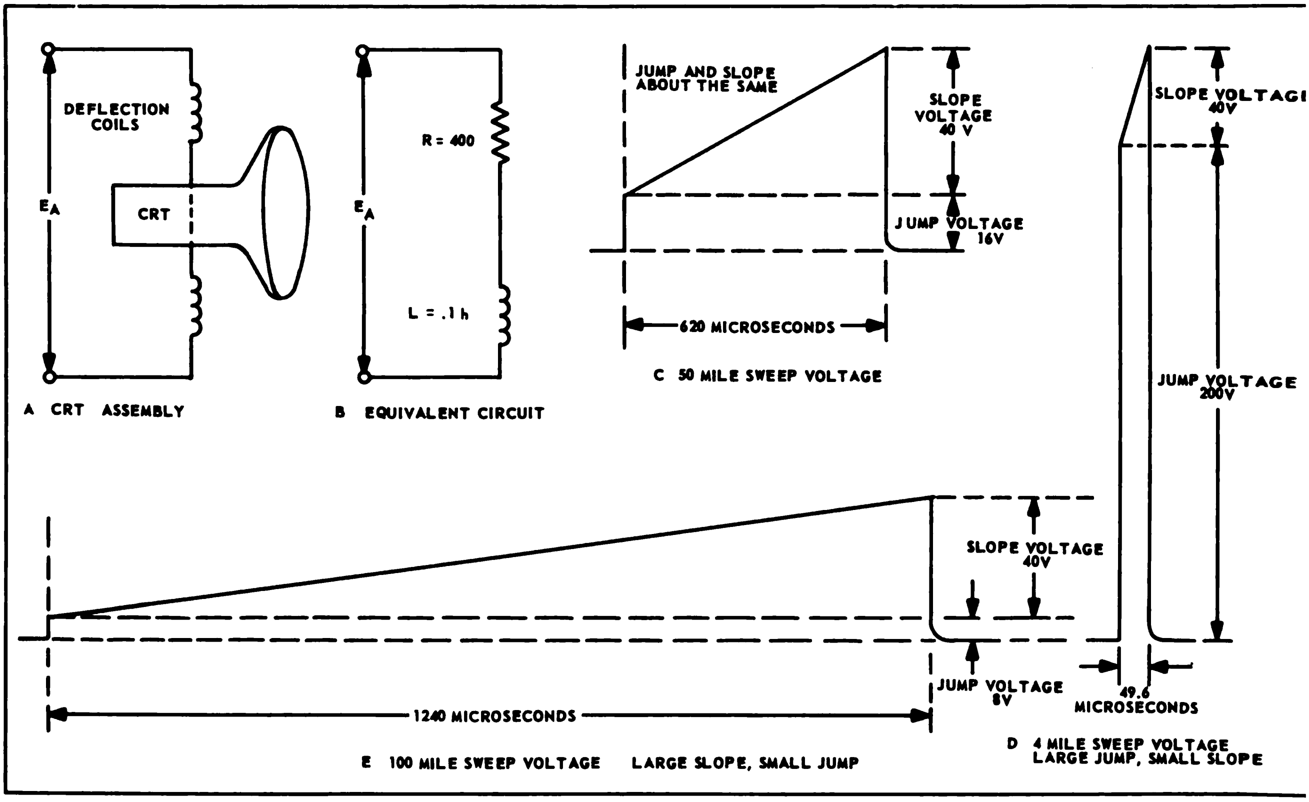


Figure 6–63. CRT Assembly and Waveshapes for 4-, 50-, and 100-Mile Ranges

Problem 2

Notice the difference in the waveshape when the same sweep coil is used for longer and shorter sweeps. In each case the final current is 100 ma for full scale deflection; therefore, the final resistor voltage is always 40 volts (100 ma through 400 ohms). Find the jump voltage for a 4-mile range.

Solution

The sweep time is 12.4 microseconds per mile. Therefore for 4 miles it is 4 times that, or 49.6 microseconds. Substituting in the equation $E_L = L\ di/dt$, you get

$$E_L = 10^{-1} \times \frac{10^{-1}}{49.6 \times 10^{-6}} = \frac{10^{-1} \times 10^{-1}}{49.6 \times 10^{-6}}, \text{ or } 200 \text{ volts.}$$

Since the 100 ma is a smaller percentage of the final current when 200 volts is applied than when 16 volts is applied, the rise is more linear and the slope voltage is a smaller proportion of the total voltage. This condition approaches the applied square wave shown in the illustration in Figure 6–60. When the percentage of slope voltage becomes somewhat smaller, it may be safely omitted without serious nonlinearity in current rise.

Problem 3

Calculate the applied waveshape for the 100-mile range and find the final slope voltage when the resistor voltage is still 40 volts across the 400-ohm resistor and the time is 1240 microseconds.

Solution

Find the jump voltage, E_L, by the equation $E_L = L\ di/dt$.

$$\text{Substituting, } E_L = 10^{-1} \times \frac{10^{-1}}{1240 \times 10^{-6}} = \frac{10^{-1} \times 10^{-1}}{1240 \times 10^{-6}}$$

$= 0.80645\text{v}$

Since the current rise is very slow, only a small voltage is applied to the inductor. In fact, the jump voltage is so small that it may be omitted altogether on this long range without appreciable change of the waveshape. The applied waveshape becomes a sawtooth waveshape similar to that used for the electrostatic CRT.

The waveshapes for the 4-, 50-, and 100-mile ranges are drawn to scale at C, D, and E, Figure 6–63. Notice that the small jump voltages go with the long ranges and the large jump voltages go with the very short ranges.

Trapezoidal Sweep Generator

Where it is necessary to supply a trapezoidal waveshape to a sweep coil, a vacuum tube and RC circuit may be used to produce the desired waveshape. Figure 6–64A shows a simple vacuum tube circuit that will produce this kind of waveshape, and Figure 6–64B is the equivalent of the producing circuit. The triode serves only as a high speed switch. When the tube is cut off by the driving pulse, the plate voltage rises toward the applied voltage at the rate the capacitor charges. However, some of the resistance of the RC circuit is between the plate and ground.

The high initial charging current flows through this resistor and the capacitor, raising the voltage immediately toward the applied voltage. The capacitor voltage and total resistor voltage curves are shown in Figure 6–64C. If R_2 is one-fourth the size of R_1, one-fourth of the voltage will be across it at all times. The curve for the R_2 voltage is a miniature replica of the total resistor voltage. The output voltage is the sum of the R_2 voltage and the capacitor voltage. This total voltage is the trapezoidal voltage required for the sweep coil circuit.

As before, the two elements — jump and slope — form the waveshape in the sweep coil. The ratio of the jump to the slope is L/R. To produce the same jump to slope ratio with an RC circuit, it is necessary that the RC circuit be designed so that R_2C is equal to the L/R of the sweep coil circuit. Mathematically, the relationship is expressed as $L/R = R_2C$.

This equation is derived as follows:

First, consider the jump voltage when the voltage E_B is applied to the circuit. The total voltage E_B appears across the resistors R_1 and R_2. The proportion of the voltage that appears across R_2 is expressed by the following equation:

$$E_{R2} = \frac{E_B R_2}{R_1 + R_2}$$

If R_2 is much smaller than R_1, you can simplify the equation and get an approximate answer by dropping R_2 from the denominator. The equation now reads,

$$E_{R2} = \frac{E_B R_2}{R_1}$$

Second, consider the slope, being sure not to confuse slope with the final slope voltage (Figure 6–65). Slope is a rate of so many volts per second while final slope voltage is so many volts at the end of the rise. The initial charging rate of the capacitor is such that it would charge the capacitor to the applied voltage in one time constant. If only a small part at the beginning of the curve is used, then the initial rate is the slope. Since the voltage increases at a rate of one E_B per time constant,

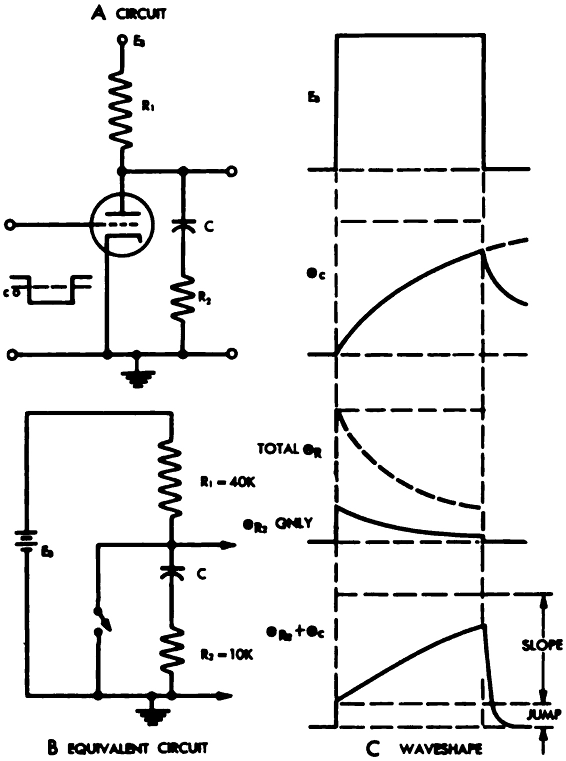


Figure 6–64. Trapezoidal Sweep Generator

$$\text{Slope} = \frac{E_B}{(R_1 + R_2)C}$$

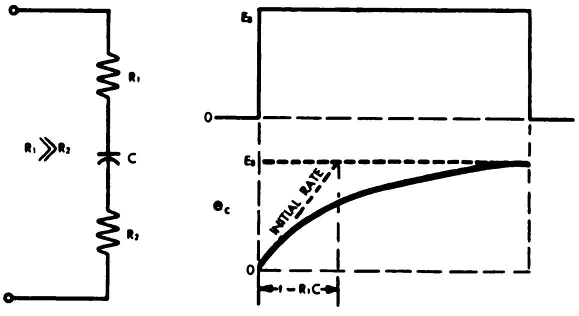


Figure 6–65. Slope of Initial Charge

When R_2 is quite small, you may disregard it for simplicity. Therefore, the slope equation reads,

$$\text{Slope} = \frac{E_B}{R_1C}$$

Using the simplified equation, you can express the relation of the jump to slope as follows:

$$\frac{\text{jump}}{\text{slope}} = \frac{\dfrac{E_B R_2}{R_1}}{\dfrac{E_B}{R_1 C}}$$

$$= \frac{E_B R_2}{R_1} \times \frac{R_1 C}{E_B}$$

$$\frac{\text{jump}}{\text{slope}} = R_2 C$$

You know this is an approximation, but you can see that the jump to slope ratio is nearly equal to the product of R_2 times C. For linear current increase, therefore, the time constant of the deflection coil circuit (L/R) should equal R_2C.

In the practical utilization of the principles just described, it is usually necessary to make the sweep coil a part of the plate circuit of a power amplifier tube in order to obtain the high currents with the desired waveshape. The trapezoidal waveshape is then generated at a low level and amplified by the power amplifier tube.

To see how the details are worked out, study the example of a complete generator and sweep coil circuit in Figure 6–66. The quantitative values worked out are for the coil used for producing the 50-mile sweep voltage described earlier. In studying this circuit, you will find

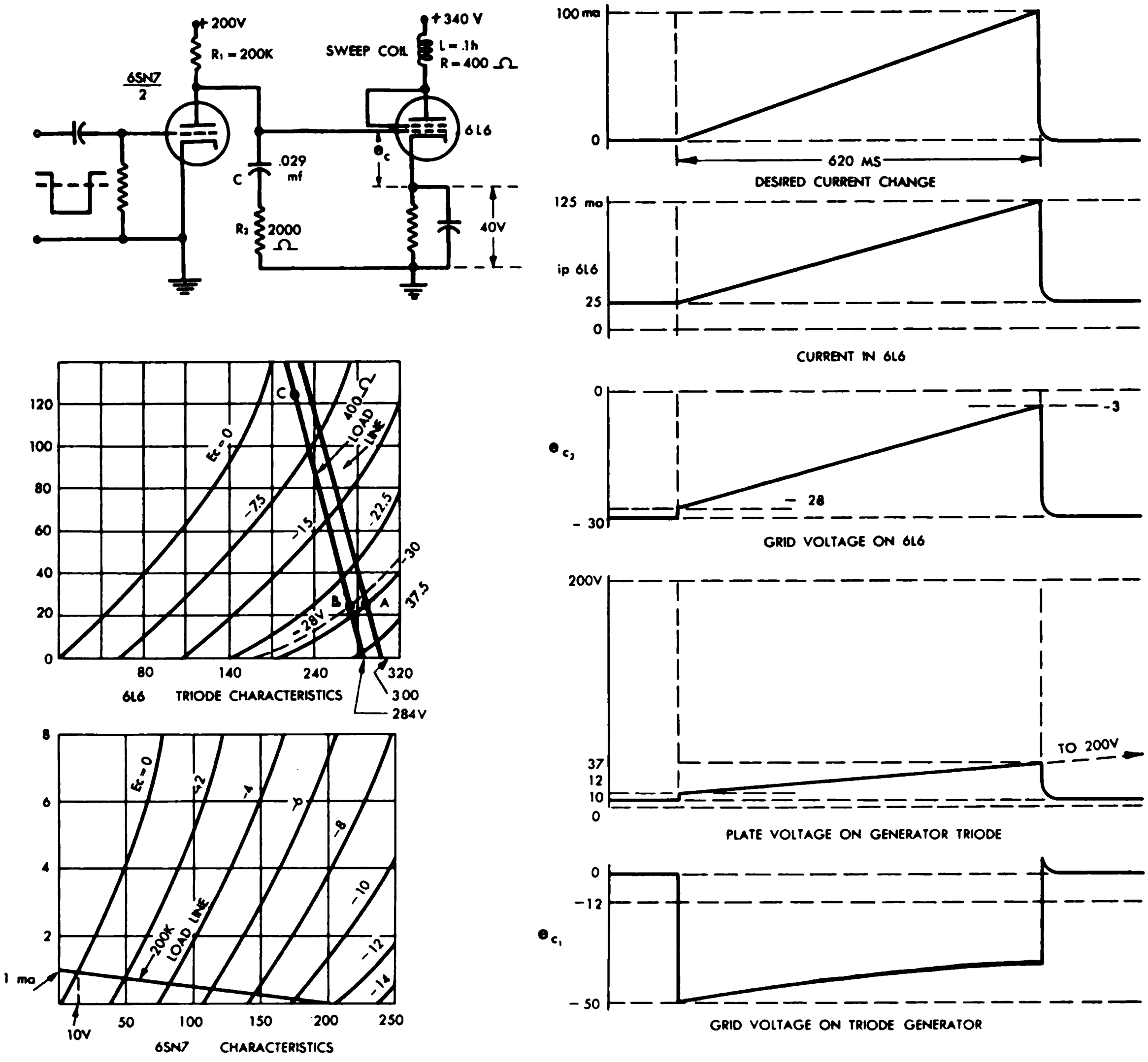


Figure 6–66. Complete Generator and Sweep Coil Circuit

it convenient to start with the sweep coil and work backwards to the sweep generator. Therefore, the waveshapes that are shown and the analysis that follows are in reverse order.

The 50-mile sweep voltage coil has an inductance of 100 millihenrys and a resistance of 400 ohms. A linear time base requires a linear current rise of 100 ma in 620 microseconds, which can be accomplished with a jump voltage of 16 volts (from previous calculations). Since this applied voltage is from the plate circuit and is due to grid voltage change, which is amplified in the tube, the required jump in grid voltage is only a fraction of the plate voltage change.

In the circuit in Figure 6–66, the sweep coil is in the plate circuit of a 6L6 power amplifier. The 100-ma current change actually starts from 25 ma instead of zero to take advantage of the more linear part of the tube characteristics. Notice the 6L6 tube characteristics shown in Figure 6–66. The load line is drawn for the 400-ohm resistance of the coil. The voltage on the grid between sweep voltages (first tube conducting), is plus 10 volts, but 40 volts bias makes the grid-to-cathode voltage minus 30 volts. Therefore, the waveshape starts forming at point A on the load line. Then the sudden grid-voltage change causes a jump of 16 volts in the plate circuit.

The current increase in the coil generates 16 volts of back emf, reducing the effective plate voltage by 16 volts. Thus a new load line is drawn from 300 — 16 or 284 volts. The grid voltage must change to point B on the load line for a 16-volt change in the plate voltage. The current has just started increasing above 25 ma, so point B is at 25 ma plate current and 284 volts plate voltage. The grid voltage change is from —30 to —28 volts as read from these characteristic curves. Therefore, the jump required at the grid of the 6L6 is 2 volts.

After the jump, the slope occurs. The current must rise at a constant rate to 125 ma. At 125 ma the grid voltage, as read from the characteristic curves, is —3 volts. This is point C on the characteristic curves (Figure 6–66). The final slope amplitude is 28 — 3 or 25 volts.

Note the difference in waveshape between the earlier circuit at C in Figure 6–63 and the grid voltage curve for the 6L6 (Figure 6–66). The slope component is many times as large as the jump component at the grid while the slope and jump are almost the same as the circuit in Figure 6–63. The difference is due to the plate resistance of the 6L6 tube in series with the coil. The additional resistance requires a much larger slope component.

The voltage change across resistor R_2 and capacitor C in the circuit must be the same as the required grid voltage change. The sweep generator components must be selected for this waveshape.

First consider the plate voltage required on the 6SN7. The slope must be linear, which means the charge must be limited to the first 10% of the charging curve. The voltage slope component has a final amplitude of 25 volts. The slope starts from 10 volts. If 100 volts is used as the applied voltage, 25 volts is 25% of 100 volts, which would be too nonlinear. Using 200 volts, and subtracting the 10 volts of original charge (plate voltage when the 6SN7 is conducting), 25 volts is 25/190 or 13.3% of full charge. This will be acceptable in this case because a higher voltage will raise the plate voltage, with the 6SN7 conducting, to more than 10 volts, which would require increasing of the 6L6 bias.

With 200 volts plate voltage, a load resistor must be selected that will hold the plate voltage at 10 volts while the 6SN7 is conducting. As shown by the load line on the 6SN7 characteristic curves in Figure 6–66, a 200K resistor will accomplish this.

Next determine the capacitor size. First, determine the capacitor voltage by the formula:

$$e_c = E_B\,(1 - \epsilon^{-t/RC})$$

and then using the following given information:

$$e_c = 25 \text{ volts}$$

$$E_B = 200 - 10 = 190 \text{ volts}$$

$$t = 620 \text{ microseconds}$$

$$R = 200\text{K}$$

Substitute in the equation,

$$1 - \frac{e_c}{E_B} = \epsilon^{-t/RC}$$

$$1 - \frac{25}{190} = \epsilon^{-620 \times 10^{-6}/2 \times 10^{5}\,C}$$

Therefore,

$$C = .029 \text{ mf}$$

The last item, the size of R_2, provides the jump voltage, which is 2 volts. You can calculate R_2 by transposing in the equation,

$$\text{jump} = \frac{E_b R_2}{R_1}$$

$$\text{This gives,} \quad R_2 = \frac{\text{jump} \times R_1}{E_b}$$

$$\text{Substituting, } R_2 = \frac{2\text{v} \times 200\text{K}}{200\text{v}} = 2000 \text{ ohms}$$

The pulse required to cut off the 6SN7 according to the characteristic curves must be more than 12 volts negative. The negative square wave (shown as e_{C1}), having an amplitude of 50 volts, is more than ample for this purpose.

In the cathode-ray tube deflection coil circuit, a separate coil may be provided that will cancel the magnetic field caused by the residual 25-ma current in the 6L6 tube. This causes the magnetic field strength to be zero between sweeps even though 25 ma is flowing in the sweep coil.

In Figure 6–67, which summarizes the effect of varying the sizes of the component parts of the trapezoidal sweep generator, notice that varying R_1, the plate load resistor, affects the over-all amplitude of the waveshape but has little effect on the jump-to-slope ratio. Varying the capacitor size changes the slope only, while varying R_2 changes the jump voltage only.

SQUARE WAVE GENERATORS

The circuits discussed thus far may be broadly classed as waveshaping circuits. Another class of circuits to consider is that of waveshape generating circuits. These circuits either produce a specified waveshape over and over again as long as power is supplied, or produce one wave when pulsed and remain quiescent until another pulse is applied to the circuit when they produce another wave. The output waveshape is the same regardless of the shape or amplitude of the initiating, or "trigger," pulse.

Circuits of this type are called *multivibrators* or *relaxation oscillators*. Those that run continuously with only power applied are called *free-running* multivibrators. Those that must be started for each cycle are called *one-shot* or *start-stop* multivibrators. The frequency of free-running multivibrators, as with other types of oscillators, is influenced by all voltages and component sizes associated with the circuits. Therefore, although such circuits have only fair frequency stability when free running, adequate stability is readily achieved from a reliable AC synchronizing voltage.

These circuits can produce sawtooth waveforms (thyratron sweep generator), triangular, or square waveforms. The square waveshape produced can be differentiated to produce accurately

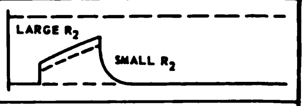
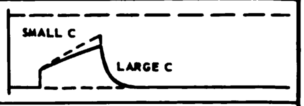
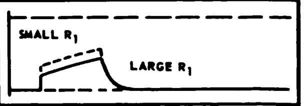
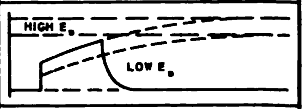
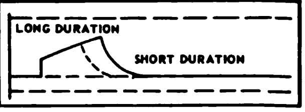
CIRCUIT CHANGE	EFFECT ON JUMP VOLTAGE	EFFECT ON SLOPE AMPLITUDE	EFFECT ON SLOPE LINEARITY	EFFECT ON DURATION	WAVESHAPE
INCREASE R_2	INCREASED BECAUSE RATIO R_2/R_1 IS INCREASED.	NO CHANGE. RC IS STILL APPROXIMATELY THE SAME.	NO CHANGE. RC IS UNCHANGED.	NO CHANGE. DURATION IS DETERMINED BY GRID PULSE.	
INCREASE C	NO CHANGE. RATIO R_2/R_1 IS UNCHANGED.	SLOPE IS LESS. FINAL AMPLITUDE LESS. BECAUSE RC IS LONGER, CHANGE IS SMALLER PERCENTAGE.	IMPROVED BECAUSE SMALLER PART OF CHARGE CURVE IS USED.	NO CHANGE.	
INCREASE R_1	SMALLER. RATIO R_2/R_1 IS LESS.	SMALLER BECAUSE RC IS LONGER, C CHARGES TO LOWER VOLTAGE.	IMPROVED BECAUSE SMALLER PART OF CHARGING CURVE USED.	NO CHANGE.	
INCREASE E_B (PLATE SUPPLY VOLTAGE)	INCREASED DUE TO GREATER APPLIED VOLTAGE.	INCREASED DUE TO HIGHER VOLTAGE CHARGE LIMIT.	NO CHANGE. SAME RC, SAME DURATION, SAME PERCENTAGE.	NO CHANGE.	
DURATION OF DRIVING PULSE INCREASED	NO CHANGE. SAME APPLIED VOLTAGE, SAME R_2/R_1 RATIO.	NO CHANGE. SAME APPLIED VOLTAGE, SAME RC.	DECREASED BECAUSE GREATER PART OF CHARGE CURVE USED.	INCREASED BECAUSE INPUT PULSE IS LONGER.	

Figure 6–67. Effect of Varying Elements in a Trapezoidal Sweep Generator

spaced peaked waveshapes. The duration and amplitude of any of the waveshapes are readily varied. Time delayed pulses can be produced by triggering a multivibrator with a peaked wave, letting it go through a cycle and obtaining somewhat later a new peaked wave from the square wave producing it.

The Plate-Coupled Multivibrator

The plate-coupled multivibrator (Figure 6–68) is the simplest of the multivibrators. It produces almost a square wave when properly designed, and a perfect square wave when additions are made to the basic circuit. Note that basically this multivibrator consists of two triode amplifier stages, where the output of the second stage is introduced back into the input of the first. The phase inversion through the circuit is 360°, and any slight change in the first tube will be amplified through the circuit and cause a greater change in the same direction in the first tube. That is, feedback is "in phase" and the circuit will oscillate. There are no elements in the circuit to limit the feedback to any one frequency, so feedback occurs at all frequencies. Hence, a square wave, containing several hundred frequencies, is produced.

Looking at the action from the point of view of transients — that is, following each voltage change step by step — there are four different conditions that exist during one cycle of operation. They are:

1. An extremely rapid change from V_1 conducting to V_2 conducting.
2. A long period during which V_1 is cut off and the circuit is relaxed.
3. A second violent change as V_1 conducts, driving V_2 beyond cutoff.
4. A long period during which V_2 is cut off and the circuit is relaxed.

The cycle repeats as condition one follows condition four. The rapid changes just described are indicated by the vertical parts of the waveshapes in Figure 6–68. These changes ordinarily occur in a fraction of a microsecond and are so fast that the spot on a CRT becomes invisible when the waveshape is observed on an ordinary oscilloscope. The long relaxation periods are represented by the nearly constant voltages between switchover times.

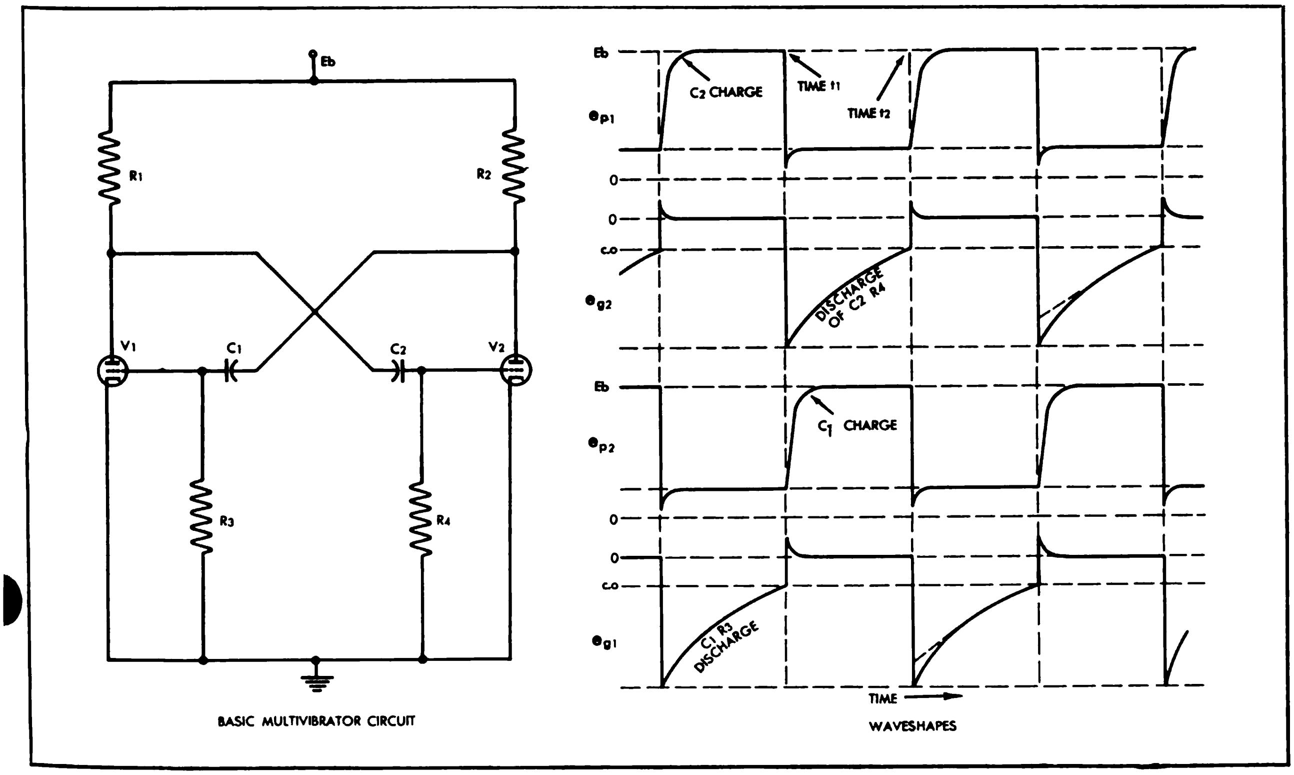


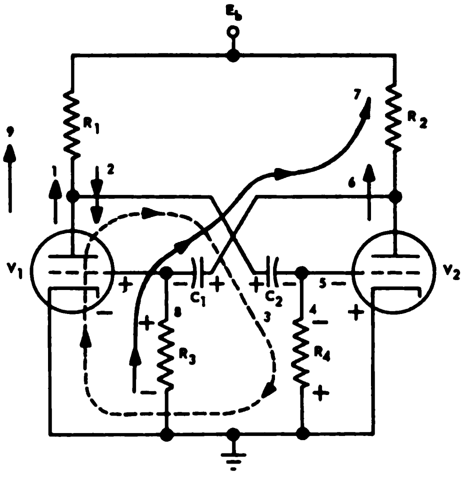
Figure 6–68. Plate-Coupled Multivibrator

Operation. Figure 6–69 shows the operation of a plate-coupled multivibrator. When the filament voltages are on and the plate voltage is applied, current flows in both tubes. If the circuit components are the same in both tube circuits and both tubes are of the same type, the circuit is said to be *symmetrical,* and both currents should be equal. But it is impossible to obtain a perfect balance in the circuit due to the nonuniformity of available parts and the irregular way in which emission occurs from the cathode. Any slight change in current in one tube that is not accompanied simultaneously by a similar change in the other tube will start oscillation.

If a slight current increase occurs in tube V_1, the following series of events then occurs in rapid order: The increase of current in V_1 increases the voltage drop across R_1, which lowers the plate-to-ground voltage of V_1. This lower voltage is now less than the charge on C_2, and capacitor C_2 discharges toward the new plate voltage. The discharge current flows through R_4 making the grid end negative. The negative grid of V_2 decreases the current through V_2. This reduces the drop across R_2, causing the plate-to-ground voltage at V_2 to be higher. Capacitor C_1 is between this plate and ground and therefore must charge to the new higher voltage.

The charging current flows through R_3 toward the plate supply voltage as shown by solid arrows in Figure 6–69. This current causes the top of R_3 to be positive and, therefore, the grid of V_1 to be positive. The positive grid causes the original slight increase in current to become a tremendous current increase. As the changes go around the circuit, a large voltage drop occurs at the plate of V_1; the grid of V_2 is made more negative; current in V_2 is decreased more; plate voltage at V_2 goes up, making the grid of V_1 more positive and further increasing the V_1 plate current.

This is a cumulative process that continues until the current in V_2 is decreased to zero by a grid voltage far below cutoff. With zero current through V_2 the plate voltage becomes equal to the supply voltage since there is no voltage drop across R_2. C_1 quickly charges to this value by grid current flow in V_1. After that, current through R_3 is zero, so the grid-to-cathode voltage is zero. The plate voltage of V_1 remains at a constant low value while the grid voltage is zero, so capacitor C_2 continues to discharge to that value. While discharging, the current (dotted lines) holds V_2 cut off. Everything is at a standstill in the circuit except for the slow discharge of C_2. This corresponds to condition 4 given before.



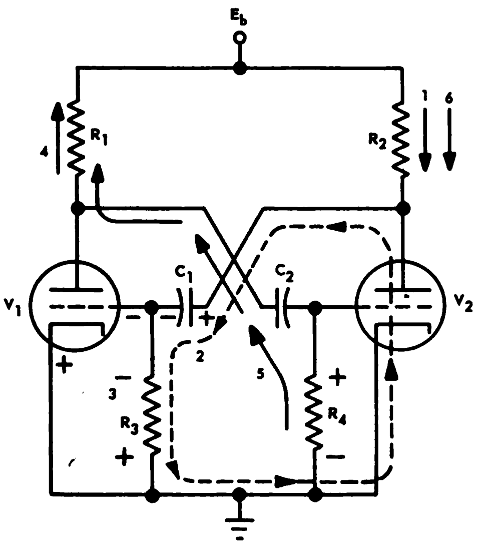
1 CURRENT INCREASES IN V_1.
2 VOLTAGE DROPS AT PLATE OF V_1 DUE TO INCREASED VOLTAGE ACROSS R_1.
3 CAPACITOR C_2, NOW CHARGED HIGHER THAN APPLIED VOLTAGE, DISCHARGES TOWARD NEW LOWER VOLTAGE, DOTTED LINES SHOW CLOSED DISCHARGE CIRCUIT.
4 DISCHARGE CURRENT MAKES TOP OF R_4 NEGATIVE WITH RESPECT TO BOTTOM.
5 SINCE GRID AND CATHODE ARE CONNECTED TO R_4, GRID BECOMES NEGATIVE WITH RESPECT TO CATHODE.
6 MORE NEGATIVE GRID AT V_2 DECREASES CURRENT RAISING VOLTAGE AT PLATE OF V_2.
7 A CURRENT FLOWS THROUGH PATH OF ARROWS TO CHARGE CAPACITOR C_1 TO NEW HIGHER VOLTAGE.
8 THE CHARGING CURRENT MAKES THE TOP OF R_3 POSITIVE WITH RESPECT TO BOTTOM AND CONSEQUENTLY, THE GRID OF V_1 IS POSITIVE.
9 THIS CAUSES A GREAT INCREASE IN CURRENT IN V_1, WHERE THE ORIGINAL SMALL INCREASES OCCURRED.

Figure 6–69. Sequence of Changes in Multivibrator

The slow discharge of C_2 continues and the voltage across C_2 becomes lower and lower. The voltage across R_4 continuously equals the C_2 voltage in accordance with Kirchhoff's voltage law, so the R_4 voltage decreases. Upon reaching a voltage just less than the cutoff voltage for V_2, a slight current starts flowing through V_2. This ends the period of inactivity in the circuit as swift changes follow this slight current.

The current lowers the V_2 plate voltage. C_1 starts to discharge, making the grid of V_1 negative. The resultant decrease in plate current raises the V_1 plate voltage. C_2 now recharges to the new V_1 plate voltage. The C_2 charging current makes the grid of V_2 positive, and the original tiny current is increased tremendously. This regenerative process continues until V_1 is cut off by the discharge of C_1. At this point, V_1 plate voltage rises quickly to the plate supply voltage, C_2 charges swiftly to supply voltage through V_2 grid current, and the plate voltage of V_2 steadies at a low value. This ends the rapid changes during condition one.

The second inactive period occurs as V_1 is held nonconducting by the slow discharge of C_1. The discharge continues until the C_1 voltage is just below cutoff for V_1. This period is as described for condition 2, Figure 6–70.



1 AS C_2 DISCHARGES TO LESS THAN CUTOFF VOLTAGE, CURRENT FLOWS LOWERING VOLTAGE AT V_2.
2 CAPACITOR C_1 DISCHARGES.
3 THE (DOTTED LINE) CURRENT MAKES THE TOP OF RESISTOR AND GRID NEGATIVE.
4 THE NEGATIVE GRID VOLTAGE LOWERS PLATE CURRENT SO PLATE VOLTAGE GOES UP.
5 CAPACITOR C_2 NOW RECHARGES, THE CURRENT (HEAVY ARROWS) MAKES GRID POSITIVE.
6 THIS INCREASES THE CURRENT MORE.

Figure 6–70. Second Switching Action

Then the slight current in V_1 repeats the first rapid change described above. In the waveshapes in the basic circuit diagram in Figure 6–68, the long period between time t_1 and t_2 is the time that V_2 is cut off. The exponential grid-voltage decrease of e_{g2} shows how long the capacitor C_2 can hold V_2 nonconducting. Time t_2 is the rapid change just described, and the half-cycle following that is the time V_1 is cut off.

In summary, the operation of this circuit consists of long periods of time when one tube conducts a high current while the other tube is cut off, followed by an extremely rapid change to the other tube conducting and the first tube cut off.

The slow discharge of the capacitors is due to the long time constant of the discharge circuit. In the waveshape in the basic circuit diagram in Figure 6–68, the grid voltage is shown as going positive for short periods of time. This occurs when the other plate is rising toward the supply voltage. The capacitor is recharging, making the top of the grid resistor positive. The resultant positive voltage on the grid makes the grid act as a diode plate, and part of the emitted electrons flow to the grid and into the capacitor.

This ready source of electrons charges the capacitor quickly. That is, the grid-to-cathode equivalent resistance is very small, forming a short time constant during the capacitor charge. The positive-going grid causes an extra dip in plate voltage, which makes the grid of the other tube somewhat more negative than normal. The effect if the grid did not become positive is shown by dotted lines in the waveshapes in the basic circuit diagram, Figure 6–68.

QUANTITATIVE ANALYSIS. It is fairly easy to calculate the frequency and approximate voltage throughout a given circuit of this type. Consider, for example, the circuit in Figure 6–71A. A plate voltage of 300 volts is applied to a 6SN7 dual triode with the circuit components shown. The plate voltage of V_1 varies between the value due to zero grid voltage and the value due to zero plate current. You can determine these values from a 20K load line on the 6SN7 characteristic curves (Figure 6–71B). The load line crosses the zero grid voltage line at a plate voltage of 97 volts. At zero plate current, the plate voltage is 300 volts. From this information you can construct the plate voltage waveshape.

All this plate voltage change occurs across the grid resistor on V_2 at the first instant. The resistor voltage is the grid voltage for V_2, so the grid will swing negative by the same amount that the plate voltage drops. The plate voltage drops 300 — 97 or 203 volts. The grid is at zero when the plate voltage drop occurs, so the grid immediately goes 203 volts negative. Then the capacitor starts discharging down to 97 volts. The discharge will be exponential. The equivalent circuit during the discharge is shown in Figure 6–71D.

The voltage across r_{p1} (the tube's plate resistance) is 97 volts and the capacitor is discharging from 300 volts to 97 volts. The plate resistance r_{p1} is found by using the plate current and voltage when the grid voltage of V_1 is zero.

$$r_{p1} = e_b/i_b = 97/10.3 \text{ ma} = \text{about } 9.4\text{K}.$$

In parallel with this (through the battery) is the 20K plate load resistor. This parallel combination will have about 6K resistance in series wit 1-megohm. For simplicity the 6K resistance ca be disregarded. This will introduce an error o less than 1%. The discharge time constant with out the 6K resistor will be .001 mf × 1 me = 1000 microseconds.

The discharge continues until the resistor (an grid) voltage at V_2 is just above cutoff. Cuto voltage for this tube with 300 volts applied is —1 volts, as read from the characteristic curves. Th cutoff period is terminated when the voltag drops to 18/203 or 8.86%. The resistor voltag is down to 8.86% in 2.423 time constants. Thi

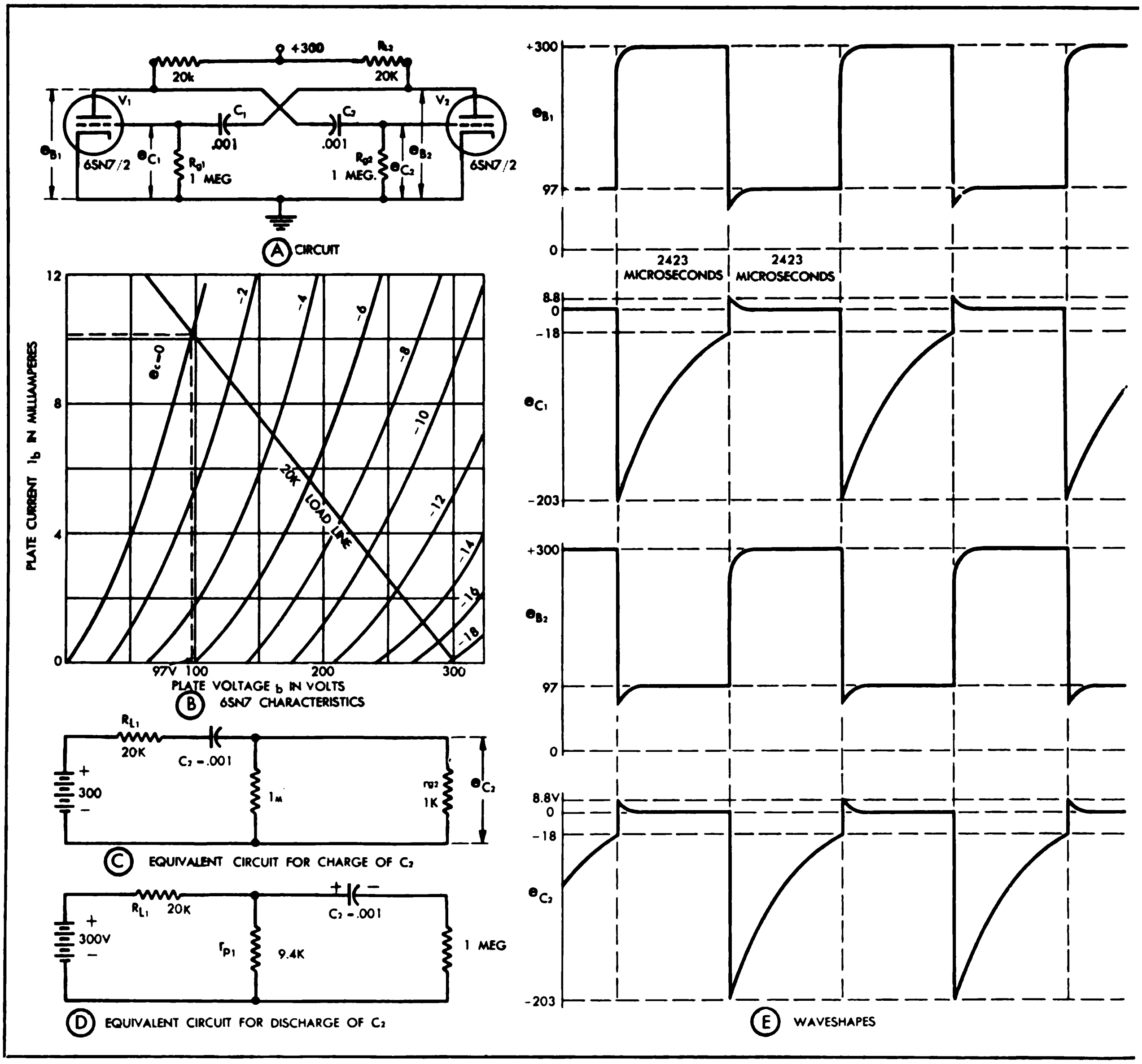


Figure 6–71. Quantitative Analysis of Plate-Coupled Multivibrator

is calculated from the exponential equation for resistor voltage during discharge.

$$e_R = E_B \epsilon^{-t/RC}$$

$$18 = 203 \epsilon^{-t/10^6 \times .001 \times 10^{-6}}$$

$$.0886 = \epsilon^{-t/1000}$$

$$t = 1000 \times 2.423$$

$$t = 2423 \text{ microseconds}$$

The cutoff time for tube V_2 will be 2423 microseconds. Since the resistor and capacitor values are exactly the same, tube V_1 will be cut off for 2423 microseconds during the other half-cycle of operation. The grid and plate voltage values and curves will also be identical. If a difference exists in the circuit of each tube, separate calculations must be carried out for each half-cycle. In this case, 2423 + 2423 = 4846 microseconds. The frequency at which square waves are produced is the reciprocal of the duration.

$$F = 1/4846 \times 10^{-6} = 206 \text{ cycles per second.}$$

The duration and amplitude of the positive peak on the grid waveform can be readily calculated from the equivalent circuit shown in Figure 6–71C. The positive peak occurs as the capacitor is charged to a higher plate voltage. The capacitor charges through a resistance consisting of the plate load resistor for V_1 and the grid-cathode resistance of V_2. This is a total of 21K. The 1-megohm resistor carries only 0.1% of the charging current and is therefore safely disregarded. With the .001 mf capacitor the time constant is $.001 \times 10^{-6} \times 2.1 \times 10^4$, or 21 microseconds. If the charge requires 5 time constants, the positive peak will disappear after 100 microseconds.

The amplitude of this peak can be determined by using Ohm's law. The total voltage rise at the plate of V_1 is 203 volts. This raises the grid voltage from a cutoff of −18 volts to some positive value. Since the first 18 volts of plate-voltage rise change the grid voltage to zero, the remaining rise above zero is 203 − 18 or 185 volts. This causes grid current to flow and the voltage to divide across the series combination consisting of the grid-cathode resistance and the plate load resistance. As the grid-cathode resistance equals 1000 ohms and the plate load resistance equals 20,000 ohms, 1/21 of the 185 volts, or 8.8 volts, appears across the grid-cathode resistance. Thus, the positive peaks at the grid will be 8.8 volts above zero. Because the plate voltage cannot rise faster than the capacitor can charge, the plate-voltage rise is exponential over a period of 100 microseconds. This rounds off the leading edge of the plate waveshape. The positive grid at the other tube causes a dip in plate voltage at that tube.

The output from this circuit is normally taken from either plate although the grid voltage is sometimes useful.

Figure 6–72 is a chart showing the effects of varying the sizes of the component parts of this circuit. The essential data is given, but the detailed and less important effects are omitted because of a lack of space. The effects of changing R_{L2}, R_{g2}, C_2, and V_2 are not listed, since these effects are similar to those that result from changing R_{L1}, R_{g1}, C_1, and V_1 respectively — except that each effect occurs during the other half-cycle of operation.

Synchronization of the Multivibrator

Free-running multivibrators are generally not used as such in radar circuits, because their frequency stability is poor. To avoid this frequency instability, multivibrators are usually synchronized with another frequency that forces the period of the multivibrator oscillaton to be exactly the same as the period of the synchronizing frequency. Such a multivibrator is said to be *driven* by the synchronizing voltage.

SYNCHRONIZATION WITH A SINE WAVE. Although a waveform of almost any shape may be used for synchronization, a sine wave or a pulse is generally used. The circuit of a multivibrator that is synchronized by the injection of a sine wave voltage in the cathode circuit of one tube is shown in Figure 6–73A. The actual grid-to-cathode voltage of V_1, which is the voltage controlling the flow of plate current, is the difference between the grid-to-ground voltage e_{g1} and the cathode-to-ground voltage e_k. The source of sinusoidal voltage should have a low internal impedance, for otherwise the plate current of V_1 flowing through this source causes a voltage drop that alters the sinusoidal shape of the wave. Sometimes a low-voltage winding on a power-supply transformer, such as a filament winding, is used to supply this control voltage.

If the multivibrator in Figure 6–73A were balanced and running freely, the waveform of the voltage at the grid of V_1 would be that shown

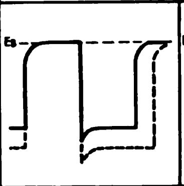
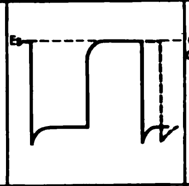
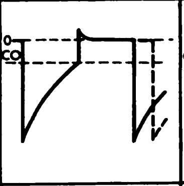
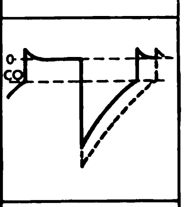
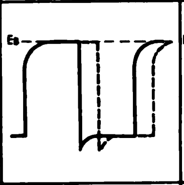
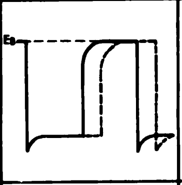
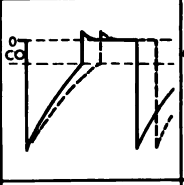
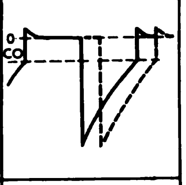
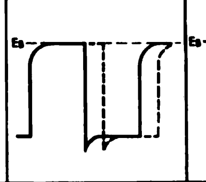
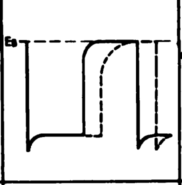
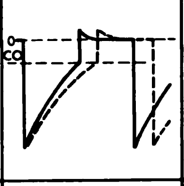
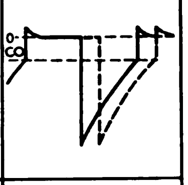
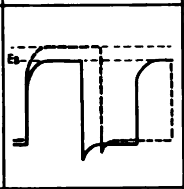
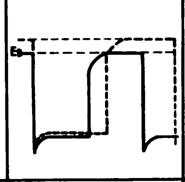
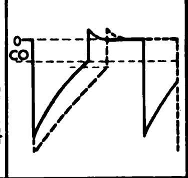
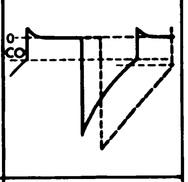
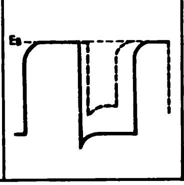
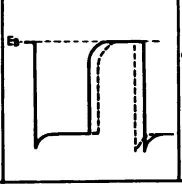
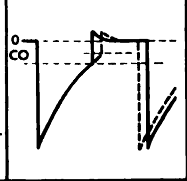
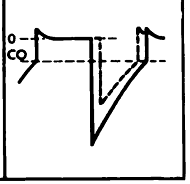
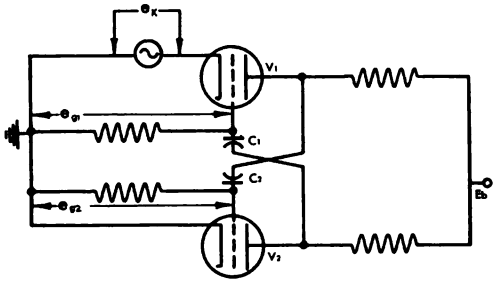
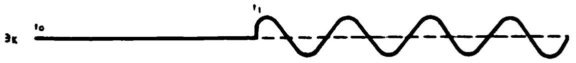
CHANGE	FREQUENCY	AMPLITUDE OF e_{b1}	DURATION e_{b1} AT E_b	AMPLITUDE e_{b2}	DURATION e_{b2} AT E_b	WAVESHAPE CHANGE ——— BEFORE CHANGE ------ AFTER INCREASE			
						e_{b1}	e_{b2}	e_{g1}	e_{g2}
R_{L1} INCREASED	DECREASED BECAUSE V_2 CUT OFF LONGER	INCREASED BECAUSE e_{b1} IS LOWER DURING ZERO GRID VOLTAGE TIMES	UNCHANGED	UNCHANGED	LONGER SINCE GRID IS MORE NEGATIVE AT START OF C.O TIME				
R_{g1} INCREASED	LOWER BECAUSE V_1 CUTOFF LONGER	UNCHANGED	LONGER SINCE TUBE IS CUT OFF LONGER	UNCHANGED	UNCHANGED				
C_1 INCREASED	LOWER BECAUSE V_1 CUT OFF LONGER	UNCHANGED	LONGER	UNCHANGED	UNCHANGED				
E_b INCREASED	LOWER BECAUSE BOTH GRIDS ARE DRIVEN FARTHER BELOW CUT OFF WHILE THE CUT OFF VALUE ONLY LOWERS SLIGHTLY	INCREASED BECAUSE E_b GOES UP A LOT WHILE THE e_{b1} VALUE FOR ZERO GRID VOLTAGE GOES UP ONLY SLIGHTLY	INCREASED BECAUSE V_1 IS CUT OFF LONGER	INCREASED AS E_b GOES UP A LOT WHILE THE e_{b2} VALUE FOR ZERO GRID VOLTAGE GOES UP ONLY SLIGHTLY	INCREASED BECAUSE V_2 CUT OFF LONGER				
V_1 REPLACED WITH TUBE OF HIGHER Mu	HIGHER	DECREASED BECAUSE HIGH r_p OF V_1 CAUSED e_{b1} FOR ZERO GRID VOLTAGE TO BE MUCH HIGHER	SLIGHTLY LONGER SINCE CUTOFF FOR V_1 IS LOWER	UNCHANGED	DECREASED CONSIDERABLY SINCE GRID OF V_2 IS NOT DRIVEN VERY FAR NEGATIVE				

Figure 6–72. Effects of Varying Elements in Multivibrator

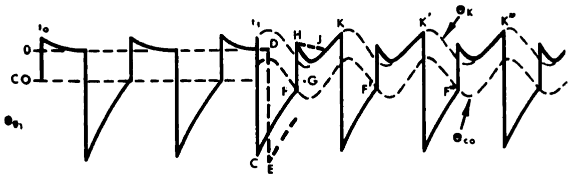
between times t_0 and t_1 in Figure 6–73C. Because there is no synchronizing voltage applied between those two times, the cathode voltage will be constant at ground potential as shown in Figure 6–73B. At time t_1, when the synchronizing voltage is suddenly applied, the cathode potential starts to vary sinusoidally. Although this variation does not affect the voltage on the grid with respect to ground, the grid-to-cathode potential contains this sinusoidal component of voltage. Therefore, the effective cutoff voltage of the tube varies sinusoidally about the normal value in phase with the synchronizing voltage on the cathode. The cathode voltage e_k and the effective cutoff voltage e_{co}, shown at C, explain the synchronizing action by showing that the instant at which V_1 is made to conduct occurs when the e_{g1} curve crosses the e_{co} curve.



A SYNCHRONIZATION AT CATHODE



B CATHODE VOLTAGE



C GRID WAVESHAPE

Figure 6–73. Synchronization with Sine Wave on Cathode

At time t_1 the voltage of the cathode starts to rise, decreasing the conduction of V_1. The positive-going voltage produced at the plate of V_1 initiates the switching action, and the tube is quickly cut off. Thus, e_{g1} drops along t_1 instead of DE as it would in the free-running condition (Figure 6–73C). Capacitor C_1 discharges exponentially along curve CFG. Since this curve intersects the e_{co} curve at F, the switching action, by which V_1 is made conducting and V_2 is cut off, takes place at this time. Thus, the switching takes place at time F instead of at G as would be the case in a free-running multivibrator.

The switching drives the grid of V_1 positive, but the grid current drawn quickly charges C_1 so that the grid returns to cathode potential. If e_{g1} followed curve HJ as it would if the multivibrator were free-running, the grid would draw current because the synchronizing voltage causes the cathode to be negative relative to ground at this time. Therefore, e_{g1} follows the cathode voltage along curve HK. When the cathode voltage begins to rise, the plate current through V_1 starts to decrease. By time K the rise in voltage at the plate of V_1 resulting from this decrease in plate current has become large enough to drive V_2 into conduction, and the tubes are very rapidly switched.

The cycle of the multivibrator is forced to be somewhat shorter than the length of the free-running cycle by the action of the sine wave. Thus, switching in one direction occurs at points F, F′, F″, etc., and switching in the other direction occurs at K, K′, K″, etc. The period of the multivibrator, KK′ or F′F″, is seen to be equal to the period of the sine wave after the short transition interval, therefore, the multivibrator may be said to be synchronized. A sine wave can thus be used to control the frequency of a multivibrator. Synchronizing voltages can make the multivibrator operate either above or below its natural frequency. However, if an attempt is made to pull the multivibrator to higher and higher frequencies, a limit is reached beyond which the multivibrator synchronizes to one-half of the driving frequency. Similarly, the multivibrator may synchronize to one-third or a smaller fraction of the driving frequency and *frequency division* may be obtained.

When the sine wave synchronizing voltage is applied to the grid instead of to the cathode of one of the tubes of the multivibrator (Figure 6–74A), the synchronizing voltage as shown adds directly to the grid voltage. Consequently, the grid-voltage waveform does not resemble that of a free-running multivibrator. Without the synchronizing signal, the multivibrator is free-running and the grid waveform appears as shown between t_0 and t_1 in Figure 6–74B. The synchronizing voltage is applied slightly before time t_1 in the random phase shown, but because of grid

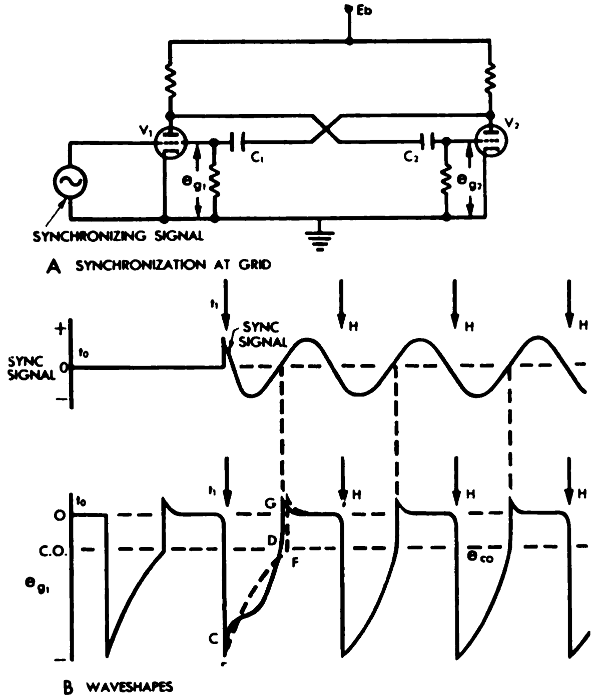


Figure 6–74. Synchronization with Sine Wave on Grid

limiting, it has no effect until V$_1$ cuts off at its natural time t$_1$. Between C and D in Figure 6–74B the synchronizing voltage adds to the normal discharge voltage which would rise along curve EF, and results in the distorted curve shown. Since the synchronizing frequency is higher than the natural frequency of the multivibrator, the distorted curve reaches cutoff sooner than curve EF.

When V$_1$ is conducting during the time between G and H, grid limiting occurs and prevents the synchronizing voltage from adding to the grid voltage. However, when the synchronizing signal drops below zero voltage and starts to decrease the voltage on the grid, the multivibrator regenerative action rapidly cuts off V$_1$. Since this action occurs at the same time, H, on each cycle of the synchronizing signal, the multivibrator is forced to operate at the synchronizing frequency. This latter action occurs in this way only when the synchronizing frequency is higher than the natural frequency of the multivibrator.

The first cycle of the synchronizing signal between t$_1$ and H will not, in general, be the same shape as the steady-state waveshape. The shape of this cycle depends on the time and phase at which the synchronizing signal is applied. The waveshapes at B show only one of many possibl waveshapes for the first cycle. After one or tw cycles, however, the multivibrator adjusts itsel to a steady-state condition in which it operates s that the phase relation between the synchronizin signal and the multivibrator output is correct.

Synchronization with Positive Pulse. Although multivibrators can be synchronized with a sine-wave voltage, a more satisfactory metho of synchronization is obtained with short trigge pulses. These pulses may be either positive o negative. Note the effect of positive pulses on the multivibrator grid waveform in Figure 6–75. A positive pulse such as that at B or C, when applied to a tube that is already conducting, serves to cause only a momentary increase in current flow. *Thus, a positive trigger pulse applied to a conducting tube of a multivibrator has no effect on the action of the multivibrator.*

However, when a positive trigger pulse is applied to a nonconducting tube and is of sufficient amplitude to raise the grid above cutoff, as the pulse at D in Figure 6–75, the tubes are switched as current starts to flow in the tube that was formerly cut off. If the trigger pulse occurs at a time such as A, the grid voltage will not rise to cutoff as shown at A′, and the switching action will not be started. *Thus, a positive trigger pulse applied to a nonconducting tube in a multivibrator can cause switching action to take place only if the pulse is large enough to raise the grid above the cutoff voltage.*

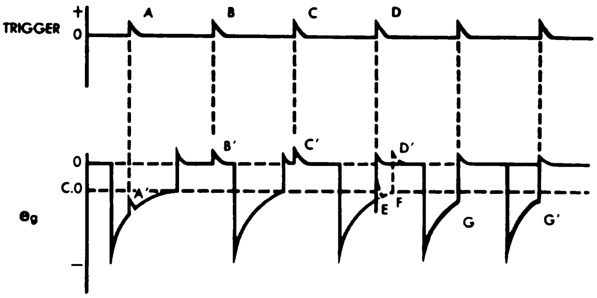


Figure 6–75. Synchronization with Positive Pulses on Grid

With the exception of the trigger pulses, the grid waveform illustrated is that of a free-running multivibrator until time D. The positive pulses which occur at D drive the grid above cutoff, so that the cycle of the multivibrator is shortened by an amount equal to EF. In order for proper synchronization to take place, the period of the multivibrator must be greater than the interval between trigger pulses. Then the trigger pulses cause the multivibrator to switch earlier in the cycle than it would if free-running. As a result, the grid has not reached cutoff by time G when the next trigger pulse is applied. Thus, the frequency of the multivibrator is forced to be the same as the repetition frequency of the trigger pulses, and the multivibrator will be switched consistently at time G after the transition is made from the free-running condition.

SYNCHRONIZATION WITH A SUBMULTIPLE OF TRIGGER FREQUENCY. A multivibrator may also be synchronized to a submultiple of the trigger frequency. As shown in Figure 6–76, trigger A causes switching of the multivibrator but triggers B and C have no effect since they are applied to a conducting tube. Trigger D is applied to a nonconducting tube, but it is not large enough to start conduction so that the next pulse to cause a switching action is trigger pulse E. In the case shown every fourth trigger pulse switches the multivibrator so that the repetition frequency of the multivibrator is one-fourth that of the trigger pulses.

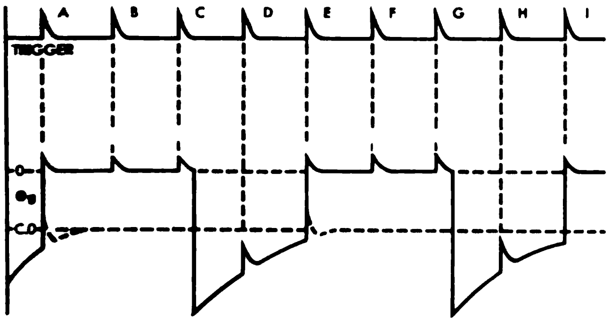


Figure 6–76. Synchronization at a Submultiple of Trigger Frequency

SYNCHRONIZATION WITH NEGATIVE PULSES. Negative pulses can be used to trigger a multivibrator as well as positive pulses. As shown in Figure 6–77, the first few cycles of the waveforms show the voltage variations that take place at the grids of a free-running multivibrator. As shown at B and B′ and at C and C′, *a negative trigger pulse applied to a nonconducting tube of a multivibrator has no effect on the operation of the multivibrator.* However, if the negative trigger pulse is applied to the conducting tube, that tube operates as a single-stage amplifier and applies a larger positive pulse to the grid of the nonconducting tube. The negative trigger pulse A reduces the grid voltage of V_1, as shown at A′, causing an increase in e_{g2}, as shown at A″. Since the change of e_{g2} is not sufficient to cause switching, trigger pulse A has no effect on the circuit.

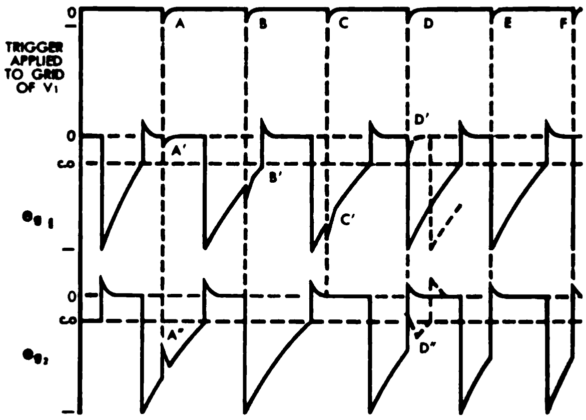


Figure 6–77. Synchronization at Grid with Negative Pulses

When a trigger pulse occurs in a later phase relative to the positive alternation of e_{g1}, as at D′, the amplified pulse applied to V_2, at D″ raises the grid above cutoff, exactly as if a positive pulse were applied directly. *Thus, a negative trigger pulse applied to the conducting tube of a multivibrator can synchronize the multivibrator, provided that amplification by the conducting tube produces a resultant positive pulse large enough to raise the grid voltage of the nonconducting tube to cutoff.* Note that it is not necessary for the negative trigger to reduce the grid voltage of the conducting tube to cutoff.

The synchronizing voltage can be introduced at the plate as well as at the grid or cathode. The voltage has very little effect on the plate current of the tube to which it is introduced but has the usual effect on the *other* grid. The coupling capacitor transfers the synchronizing voltage to the grid of the other tube. Therefore, introducing a pulse of a given polarity at the plate of one tube has the same effect as introducing the pulse on the grid of the other tube.

The Driven Plate-Coupled Multivibrator

You previously saw how the multivibrator circuit can be used either as a free-running or as a synchronized square-wave generator. The circuit can also be used as a start-stop or one-shot multivibrator, if it is held normally inoperative, and then a pulse applied to trigger or drive it into operation for one cycle and return to the inoperative condition.

The one-shot multivibrator circuit shown in Figure 6–78 is the same circuit that you previously studied except that here the bias placed on the grid of V_1 is a negative 50 volts. This bias keeps V_1 normally cut off and permits V_2 to conduct continuously. However, the circuit will operate for one cycle if a voltage is applied to the grid with sufficient amplitude to raise the grid voltage above cutoff. Since cutoff is minus 18 volts, the required amplitude must exceed 50 — 18, or 32 volts positive. Any voltage of more than 32 volts at the grid will cause V_1 to conduct.

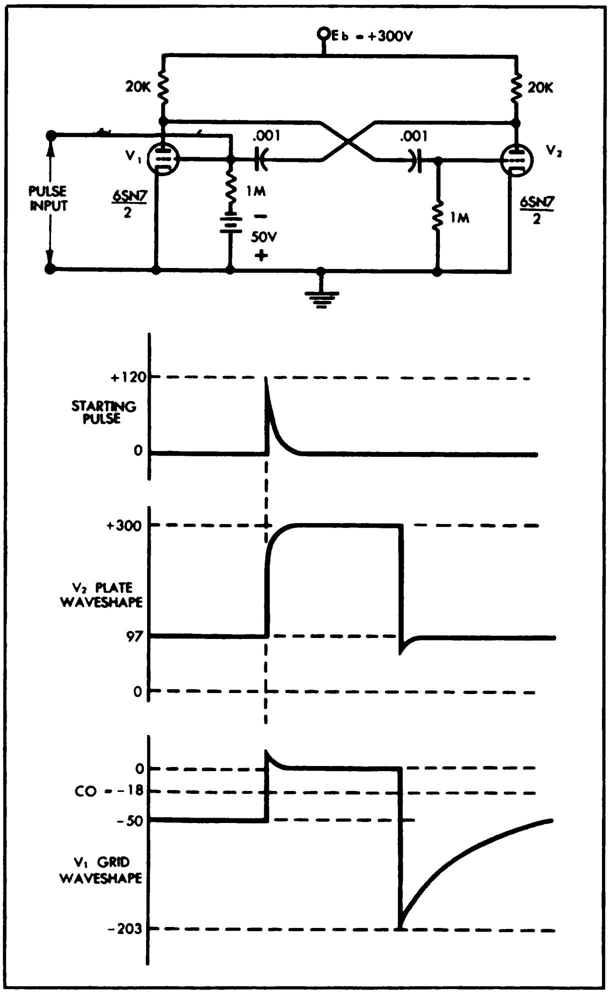


Figure 6–78. One-Shot Plate-Coupled Multivibrator

The resultant drop in plate voltage at V_1 is amplified around the circuit until V_2 is cut off, thus placing the plate of V_2 at supply voltage level. V_2 remains cut off for the usual time and then starts to conduct, driving the grid voltage at V_1 far below cutoff. Because the charge on the V_1 coupling capacitor leaks off to minus 50 volts, which is still below cutoff, the cycle stops as V_1 can not conduct again. Figure 6–78 shows that a positive pulse is applied to the grid to start conduction. One positive square wave is generated at the plate of V_2 for the one pulse. Note in the grid waveshape at V_1 that the V_1 grid voltage starts and ends with minus 50 volts. In order for the circuit to produce another square wave, another pulse must be applied to the grid.

Other methods have been successfully used to keep the driven plate-coupled multivibrator inoperative. The circuit in Figure 6–79 illustrates one method. It is the start-stop plate-coupled multivibrator. This circuit has an advantage over the circuit previously shown in that a separate power supply is not required. It uses fixed bias by a voltage divider from plus 300 volts. With the values shown, the cathode will be 50 volts more positive than the grid. As cutoff is still at −18, 32 volts will again be required to make the tube conduct. When conducting starts due to a pulse on the grid, the circuit will oscillate for one cycle and then quit.

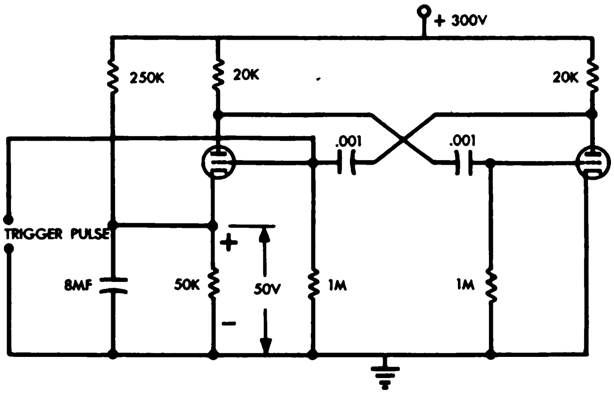


Figure 6–79. Start-Stop Plate-Coupled Multivibrator

A negative pulse can be used to start a cycle in either of the last two circuits illustrated provided it is introduced at the plate of V_1 or the grid of V_2. This causes the negative pulse to be amplified and inverted by V_2 bringing the grid of V_1 above cutoff and thereby starting tube operation.

Improving the Basic Circuit

There are a number of slight changes that can be made in the basic plate-coupled multivibrator circuit to improve its stability and output waveshape. One is to return the grids to a positive voltage, and another is to use series limiting resistors.

RETURNING THE GRIDS TO A POSITIVE VOLTAGE. Returning the grids to a positive voltage improves the stability of a plate-coupled multivibrator. In this arrangement, the grid-leak resistors are often connected to the source of plate voltage, or the grids are returned to B-plus. Under this condition the grid current flow causes the B-plus potential to exist mostly across the resistor while the grid-to-cathode voltage remains within a fraction of a volt of zero.

When the plate voltage of the other tube drops, the capacitor discharges as usual, with a current that is high enough to drive the grid negative by the amount of the plate voltage drop. Due to this condition the exponential discharge of the capacitor appears as though the capacitor were charged to the plate voltage drop plus the plate voltage supply. The net result is that the curve shown in Figure 6–80A, from the negative peak value to cutoff, is much more linear because a smaller percentage of the discharge curve is used.

Returning the grids to a positive voltage produces an advantage in that the discharge curve strikes cutoff at a more vertical angle. This means that if variations of voltage or temperature in the circuit affect the relation of the waveshape to the cutoff value, only a small change of time will occur. In contrast, study the grid voltage curve in Figure 6–80B where the grid resistor is grounded. The discharge curve there is almost horizontal at the cutoff value. In this case a small variation of the cutoff value will cause a large time change. In conclusion, then, returning the grids to B-plus decreases the variation in duration of pulses and the free-running frequency for a given change in circuit constants.

You can increase your understanding of the effect of returning the grid to B-plus by making a quantitative analysis and by studying equivalent circuits. Notice the equivalent circuit in Figure 6–81C during the time that V_1 is conducting. At B, the parts of the circuit involved during the charge of C_1 are shown. Since the grid-cathode resistance is very small compared to the grid-leak resistor, it can be represented as a direct connection, producing the equivalent circuit at C. Here R_2 is small and, as a result, the capacitor will be fully charged before tube V_2 starts to conduct. Thus, at the time V_1 is cut off, capacitor C_1 starts with a charge of 300 volts. The discharge circuit for C_1 is shown at D. V_2 is conducting during this time, causing a voltage of 203 volts across the load resistor R_2.

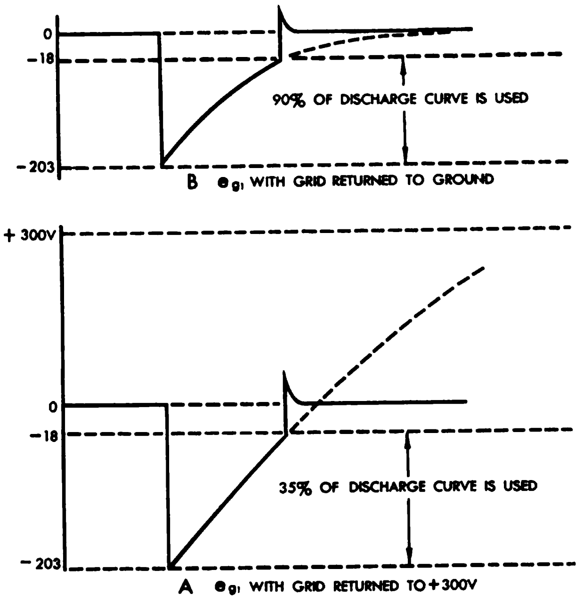


Figure 6–80. Improving Basic Circuit

A closed circuit is formed as shown at E. The voltage across the load resistor may be considered as a source of voltage, or battery, which is of a polarity to aid the capacitor in causing current flow. The total voltage across the resistor R_3 becomes the capacitor charge plus the other voltage ($300 + 203 = 503$ volts). The grid end of resistor R_3 is 503 volts negative with respect to the end connected to 300 volts. As a result, the grid is 203 volts negative with respect to the cathode. Effectively, the capacitor is charged by the plate voltage source. Since the polarity of the source is reversed, the plate voltage source will aid the discharge of the capacitor.

When a source is connected to a charged capacitor with a polarity that discharges it, the capacitor will completely discharge and recharge to the new voltage all in one long exponential

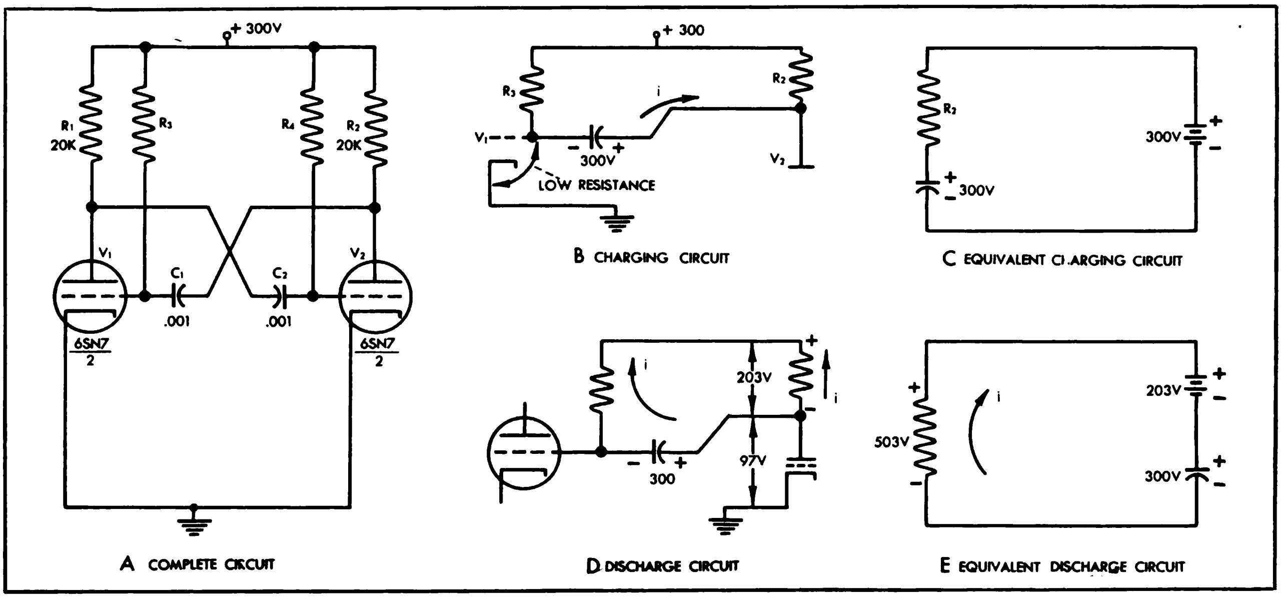


Figure 6–81. Effect of Returning Grid to B-plus

curve. In this case, the curve will have the same amplitude as it would if it charged from zero to 503 volts. The resulting slope is as shown in Figure 6–80B. Of course, the slope due to the more rapid discharge will go through zero sooner if the same RC is used. Therefore, a longer RC must be used when grid resistors are returned to a positive voltage than when the resistors are returned to ground, for the same duration of output pulse.

USE OF SERIES-LIMITING RESISTORS. The use of series-limiting resistors in the grid circuits of this multivibrator increases the squareness of the waveshapes. In the circuit of Figure 6–82, these resistors are R_5 and R_6. The first half-cycle of the plate and grid waveshapes shown is without the grid-limiting resistors in the circuit, and the second half-cycle is with R_5 and R_6 in the circuit. Note that the grid does not go positive when resistors are provided. When the other plate becomes more positive, most of the voltage change occurs across the grid-limiting resistor, while only a small fraction of a volt exists between the grid and cathode, provided the grid-limiting resistor is of the order of 500K or more.

The plate that is going positive cannot normally rise in voltage any quicker than the coupling capacitor can charge, so in the normal

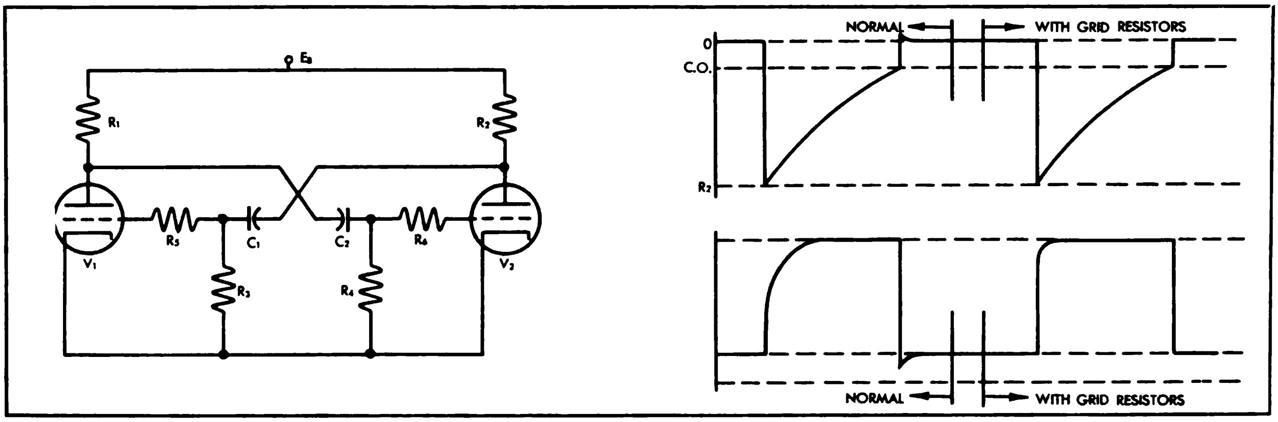


Figure 6–82. Using Series Limiting Resistors

circuit, the plate waveshape resembles the coupling capacitor charging curve. When grid-limiting resistors are inserted, the limiting resistor is in series with the low grid-cathode resistance. These two resistors and the grid-leak resistor form a high-resistance circuit for the capacitor to charge through. The circuit is like the trapezoid sweep generator in that a "jump" voltage will occur across the limiting and grid-leak resistors. Then the exponential rise will start from there. In the waveshape of Figure 6–82 the jump is about 90% of the total amplitude, with only slight rounding of the leading edge of the pulse.

The Cathode-Coupled Multivibrator

In the plate-coupled multivibrator, feedback in the correct phase to sustain oscillation was obtained by coupling the plate of each tube to the grid of the other. Feedback in the correct phase can also be obtained by connecting the two triodes as shown in the cathode-coupled multivibrator in Figure 6–83. The usual plate-to-grid connection in this case is made from V_1 to V_2; the cathode of the second tube is capacitively coupled to the cathode of the first.

If you follow through these changes, you can see that feedback is in phase. Assume that the current in V_1 is decreasing. As the plate becomes more positive, the V_2 grid will become more positive and current will increase in V_2. This makes the cathode of V_2 more positive. Since the cathode of V_1 is coupled to the V_2 cathode, the cathode of V_1 will become more positive. Because a positive-going cathode is like a negative-going grid, the current in V_1 will decrease. Since the original change was a current decrease in V_1, the feedback is in phase to sustain oscillation.

Notice that in this multivibrator the cutoff time of the grid of the second tube is controlled by the discharge of C_2 through R_2, while the cutoff time for the first tube is controlled by the charge of the cathode coupling capacitor through the cathode resistor of the first tube.

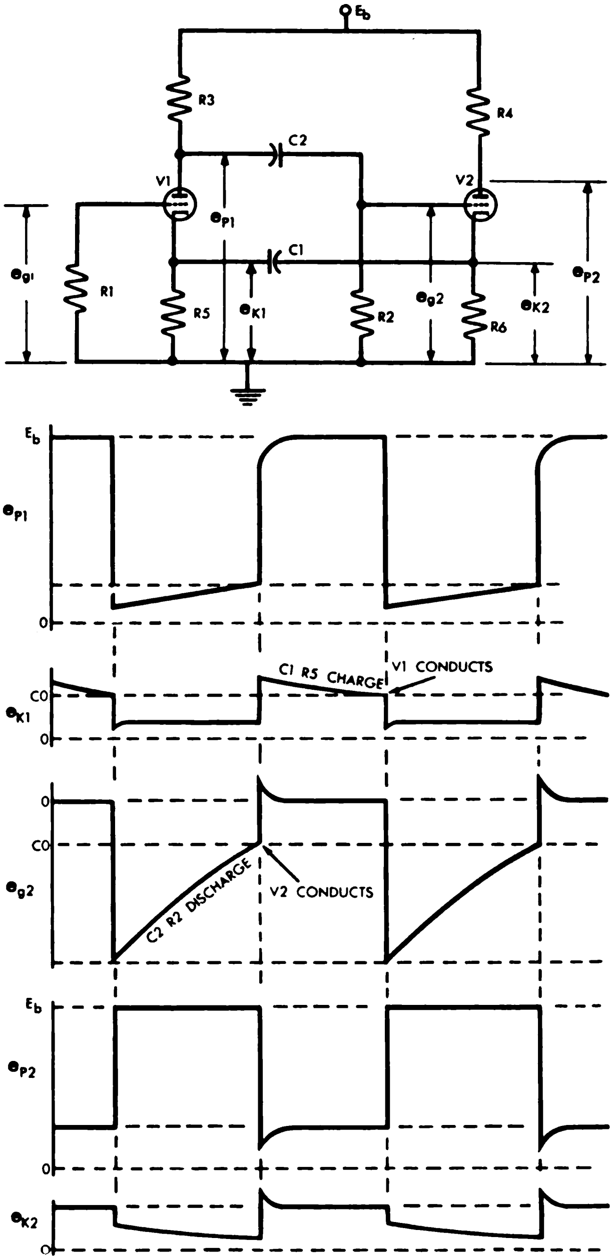


Figure 6–83. Cathode-Coupled Multivibrator

In order to understand the operation of this multivibrator, assume that V_1 is conducting and that V_2 has been cut off by an action similar to that just described. Capacitor C_2 discharges through R_2, the r_p of V_1, and R_5, so that the voltage at the grid of V_2 decreases from a high negative potential toward ground as the discharge current through R_2 decreases. When e_{g2} rises to the cutoff voltage of V_2, the tube starts to draw current through R_6. The rising voltage produced across this resistor is coupled through C_1 to the cathode of V_1 since the voltage across the capacitor cannot change instantaneously.

The positive-going voltage that is coupled to the cathode of V_1 adds to the existing voltage across R_5, reducing the flow of current in V_1. As i_{p1} decreases, the voltage at the plate of V_1 increases, and this positive-going voltage is coupled to the grid of V_2 to increase further the current in V_2. This action around the circuit is regenerative, and it ends with the current in V_1 reduced to zero and the current in V_2 at a maximum.

Capacitor C_1 is charging to the voltage existing across R_6 through the path R_5, the r_p of V_2, and R_4 to the high-voltage supply. The charging current produces across R_5 a voltage that is more than sufficient to hold V_1 beyond cutoff. However, as the voltage across the capacitor rises, the magnitude of the charging current falls. Consequently, the voltage across R_5 decreases and at some time it will fall below the value required to cut off V_1.

At this instant a current starts to flow through V_1 lowering the voltage at the plate of the tube. This negative-going voltage is coupled through C_2 to the grid of V_2, causing i_{p2} to decrease. As a result of the reduction i_{p2} and the decrease in e_{K2}, the voltage across R_5 decreases. Therefore, C_1 discharges through R_6, the r_p of V_1, and R_3 in an effort to charge in the opposite direction to the voltage across R_5. The cumulative effect of the discharge of C_1 through R_6 and the negative-going voltage applied to the grid of V_2 combine to cut off V_2 almost instantaneously.

The waveshapes at the important points are shown in Figure 6–83. While V_1 is cut off, a cathode voltage exists due to the charge of the cathode coupling capacitor. The voltage is visible in the waveshape e_k, since the high initial charging current causes a high cathode voltage. At a certain positive cathode voltage the grid-to-cathode voltage difference will be so great that no plate current will flow. The cathode voltage exceeds this value at first, but as the capacitor charges, the resistor (R_5) voltage decreases exponentially until the cutoff value is reached. At this point, current flows in V_1, which by the drop in e_{p1} cuts off V_2. This discharge of C_2, for holding e_{g2} below cutoff, is the usual action as observed with the previous circuit.

During the time V_2 is cut off, its plate voltage rises instantly to E_b, forming a square waveshape. There is no rounding of the corner because there is no capacitor to charge. When V_2 is cut off, the cathode voltage tends to become zero, but it cannot be zero until the capacitor is discharged. Therefore, the waveshape shows first a sudden drop as plate current ceases, then an exponential drop as the capacitor discharges. The same discharge is indicated by the plate and cathode waveshape of V_1. The discharge is such that it makes the cathode of V_1 more negative, so the plate voltage drops below the zero grid voltage level, and a slight dip is seen in the grid voltage waveshape. As the discharge progresses, capacitor voltage decreases and its effects decrease. The plate voltage gradually rises and a slight rise of cathode voltage is seen.

Generally speaking, increasing the size of any part in the circuit except R_1 will lower the frequency. The duration of each half-cycle is dependent upon every part in the circuit. Changing any one value will affect the duration of both half-cycles. Increasing the power supply voltage will also lower the frequency.

Because of the instability of this circuit when free-running, it should be synchronized if it is to operate at a constant frequency or if its action is to be simultaneous with the action of some other circuit. Like the plate-coupled multivibrator, this circuit can be synchronized at either plate, grid, or cathode if the right polarity of synchronization voltage is used. If, for example, you should desire to start conduction in the second tube at a certain time, a positive voltage should be applied to the second grid, first plate, or first cathode at that time. A negative pulse is required at the first grid, second plate, or second cathode to do the same thing. This circuit can also be kept normally inoperative and be made to operate only once for each trigger pulse.

A practical cathode-coupled multivibrator as used in a bombardment radar set is the one shown in Figure 6–84. The circuit is slightly simplified. Its purpose is to charge a pulse-forming network, which then discharges, forming a positive square wave of exactly ¾ microsecond duration, at an amplitude of 800 volts. This pulse is amplified to 12,000 volts to become the plate voltage for the transmitter.

The circuit can be free-running, but is normally a one-shot multivibrator because of a minus 135 volts that holds the second tube cutoff and a positive grid return that keeps the first tube conducting. When a negative trigger pulse is applied, it reduces the current in the first tube. Now regenerative amplification cuts the tube off completely, while the second tube conducts heavily. The time constants are such that the second tube conducts 4.4 microseconds, then is cut off. A large amount of energy must be stored in the pulse-forming network during this short time so

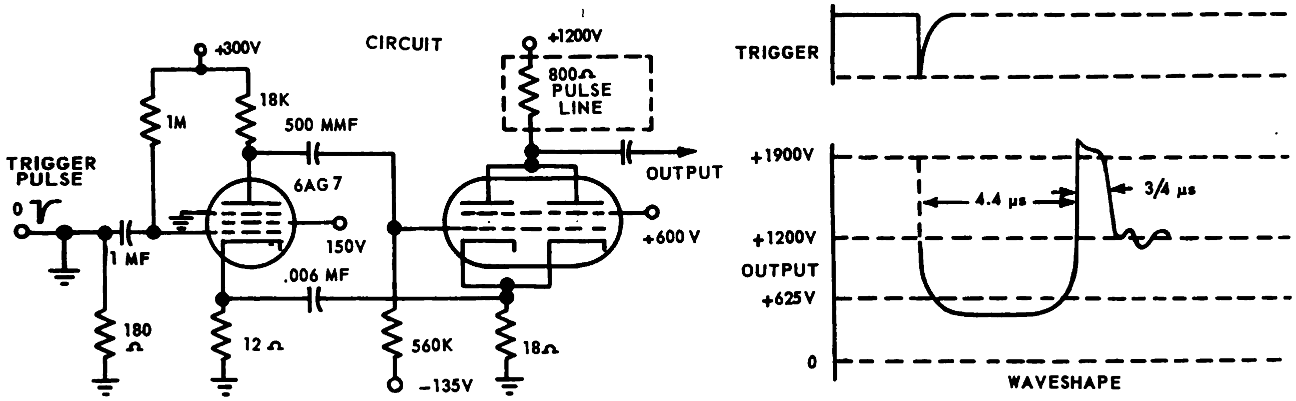


Figure 6–84. Cathode-Coupled Multivibrator in Radar Bombardment Set

a pair of very large tubes in parallel are used to provide a current of 2 amperes for the 4.4 microseconds.

The network presents a resistive impedance to the circuit and is shown as an 800-ohm resistor in the diagram. With this high current only 18 ohms of cathode resistance is required to cut off the small pentode used as the first tube. The network discharges immediately, producing the pulse previously described. In some models a positive pulse follows the negative trigger pulse, which makes the switching action occur at exactly 4.4 microseconds after the start. The positive pulse is introduced at the grid of the first tube.

The Start-Stop Multivibrator

A widely used circuit, which never becomes free-running and must always be triggered for each output pulse, is the start-stop multivibrator. This circuit is often called a *flip-flop*, from British slang, or a *one-shot multivibrator.* As shown in Figure 6–85, the circuit consists essentially of a two-stage resistance-capacitance-coupled amplifier with one tube cut off and the other conducting normally. The balanced condition of the circuit is established by the arrangement for biasing the tubes. The grid of V_2 is connected to its cathode through the resistor R_2, and the resultant voltage drop across R_k biases V_1 to cutoff. When V_2 is not conducting, V_1 cannot be cut off by the self-bias developed across R_k.

The steps in the action of the start-stop multivibrator follow.

1. V_1 is cut off initially by the voltage drop produced across R_k by i_{p2}, the plate current of V_2 (time A).
2. V_2 is conducting heavily because its grid is at cathode potential (time A).
3. A positive trigger pulse (time B) sufficient in amplitude to raise the grid of V_1 above cutoff voltage is impressed on the grid of V_1 through C_1.
4. V_1 begins to conduct and the voltage at its plate decreases. This decrease passes through C_2, as the voltage across a capacitor cannot be changed instantaneously, and appears on the grid of V_2 as a negative-going voltage.
5. The negative-going voltage on the grid of V_2 decreases i_{p2}.
6. The voltage drop across R_k decreases, allowing more current to flow in V_1.
7. The plate voltage of V_1 is still further decreased.
8. The grid of V_2 goes still more negative.
9. This action is repeated until V_2 is cut off and V_1 conducts heavily. The switch from V_2 to V_1 is practically instantaneous. The circuit remains with V_1 conducting and V_2 cut off during the interval from B to C, while C_2 discharges sufficiently toward the lowered value of plate voltage of V_1 to allow the grid of V_2 to rise from its lowest value to cutoff voltage.
10. V_2 begins to conduct (time C).
11. The plate current of V_2 flowing through R_1 raises the cathode voltage of V_1, thus reducing its plate current.
12. The decreased plate current of V_1 allows the plate voltage of V_1 to increase.
13. This increase is coupled to the grid of V_2, increasing still further its plate current.
14. The action described is repeated until V_1 is cut off and V_2 is conducting heavily. This action also is practically instantaneous.

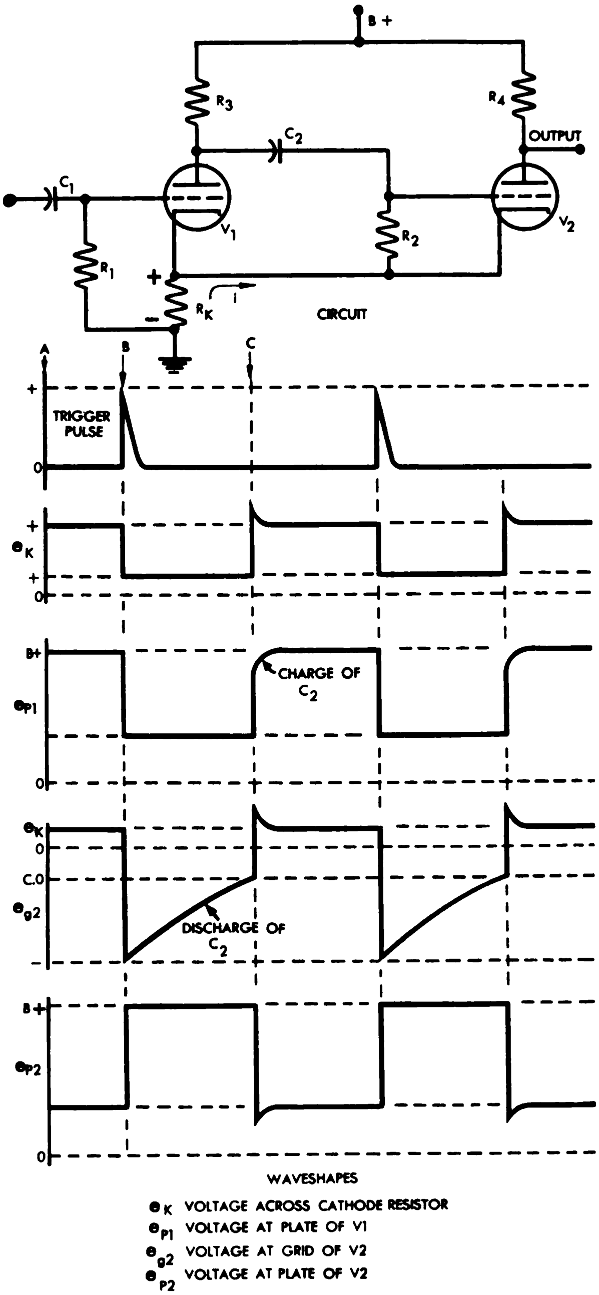


Figure 6–85. Start-Stop Multivibrator

The circuit has now come back to its original balanced state and will remain so until another positive pulse arrives and causes V_1 to conduct. When applied to the input of the one-shot multivibrator, every positive trigger pulse that causes V_1 to conduct results in a large positive-pulse output from the plate circuit of the second tube. The length of the positive output pulse produced at the plate of V_2 is controlled by the time constant of C_2 times R_2. If larger values of C_2 and R_2 are used, the length of the positive output is increased. A positive output voltage pulse is produced for each positive input trigger pulse.

Quantitative Analysis. The quantitative determination of the amplitude and duration of the output pulse is not too difficult to accomplish. Study Figure 6–86 as an example. The circuit shown is the most basic one. The waveshapes are similar to those in Figure 6–85 except that the voltages and time intervals have been determined and the waveshapes drawn to scale.

Start by finding the voltages in the circuit before the trigger pulse arrives. Check to see that V_1 is cut off with the 3K resistor. The circuit will not work if R_k is too small. V_2 is conducting heavily due to zero bias. Now draw a load line on the characteristic curves to find the voltages. In drawing the load line, consider the cathode resistance. Any resistance in series with the tube will affect the current, but if the cathode resistor is quite small, its effect is negligible. You can obtain reasonably accurate results by disregarding the cathode resistance when it is less than 10% of the load resistance. When it is 10% or more, sketch the load line for the total cathode and load resistance.

In the case of V_2 in this problem, the cathode resistance is exactly 10% of the load resistance, so the load line is drawn for 30K + 3K = 33K. Check the plate current and plate voltage at the intersection of the load line and zero grid bias. The plate voltage appears to be 73 volts and the current 6.9 ma. Knowing the current through R_k, find the voltage across it. ($E = IR$, $i_pR_k = .0069 \times 3000 = 20.7$ volts.) Cutoff is 18 volts as read from the curves, so the first tube is actually cut off, and the circuit will function.

Now proceed with the calculations. The waveshape shown is the plate-to-ground voltage, while the characteristic curves show plate-to-cathode voltage. You must add the cathode voltage above ground to the plate voltage to find the actual plate-to-ground voltage. Since the cathode is 20.7 volts above ground, add this to 73 to get 93.7 volts at the plate of V_2. To simplify the picture, the waveshape at the grid of V_2 shows the voltage at the grid with respect to the *cathode* rather than ground. In the waveshape of cathode voltage shown, notice that before the trigger pulse arrives, the cathode voltage is 20.7 volts as previously calculated.

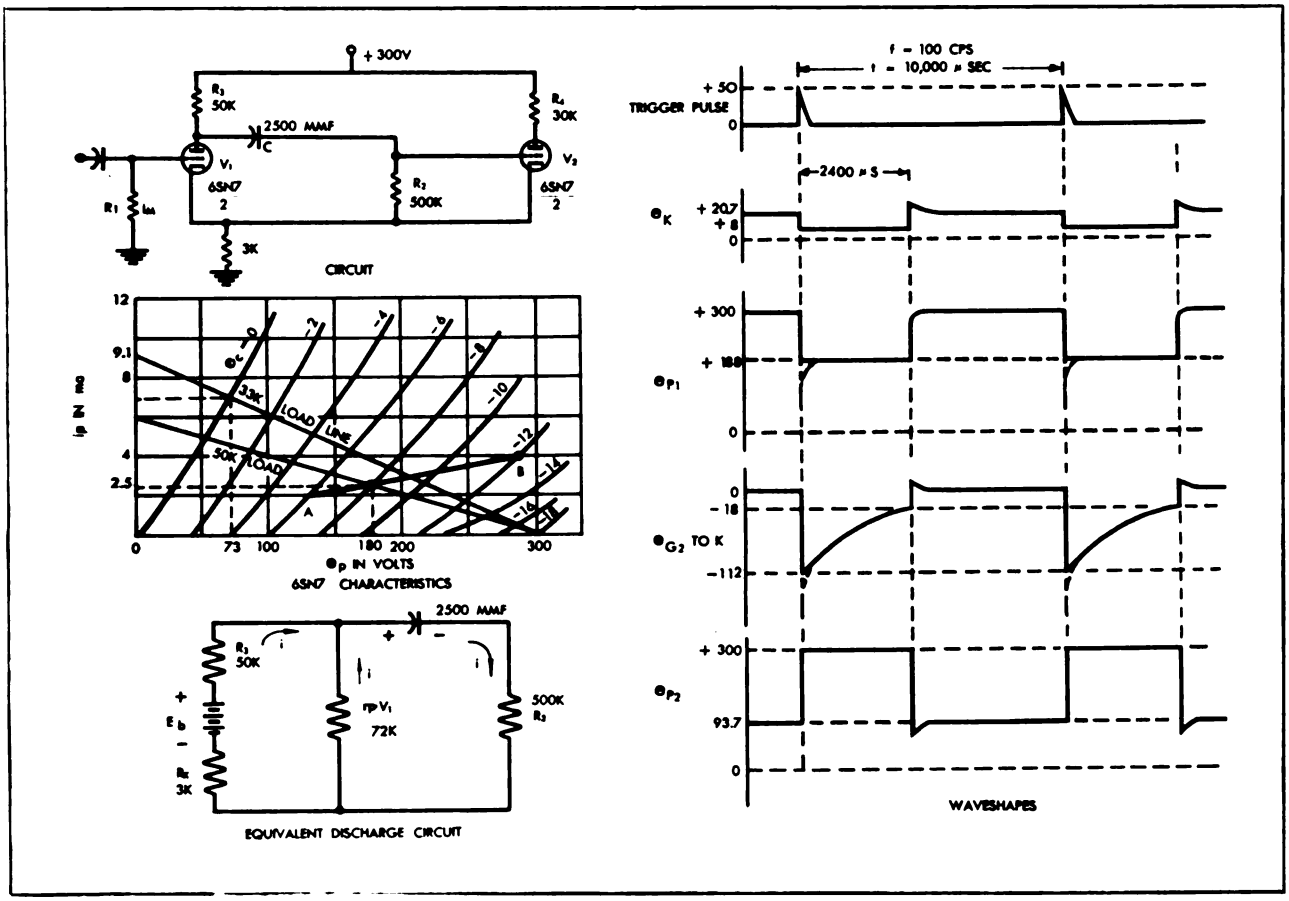


Figure 6–86. Quantitative Analysis of Start-Stop Multivibrator

Now, determine the voltages in the circuit immediately after the trigger pulse and switchover has occurred. During this time the second tube is cut off so the plate voltage is 300 volts. The first tube is conducting as much as self-bias with a 3K bias resistor will allow. To find this value, a load line and bias line are required. Since 3K is less than 10% of 50K, the load line can be drawn for 50K only. For the bias line assume different values of current, find the bias, and plot the points on the characteristic curves.

For example, if the current is 2 ma, the bias would be i_pR_k, or .002 × 3000, or 6 volts. To plot this, find the 2-ma current line and the 6-volt grid voltage curve (point A). Assume a second current of 4 ma. The bias would be 12 volts if 4 ma flowed through the 3K resistor. Mark the intersection of the 4-ma current line and 12-volt grid voltage line (point B). Draw a line through these points and the intersection of the bias line with the load line is the operating point for the tube. This shows that the bias, or cathode, voltage is 8 volts. The plate-to-cathode voltage is 180 volts. The plate voltage has changed from 300 to 180 + 8 or 188 volts. The change is 112 volts in the negative direction. The capacitor must discharge by this amount through the 500K grid resistor R_2.

The new plate voltage at V_1 is shown on the E_{p1} curve. The grid voltage at V_2 goes from zero to negative 112 volts at the switchover time, then decreases exponentially toward zero. This holds V_2 nonconducting until the voltage decreases below cutoff. From the curves, cutoff is −18 volts. In order to calculate the time of this exponential decrease, the RC must be known. C is given as 2500 mmf. The grid resistor of 500K can be considered as the total R for approximate calculations, but the current flows through the 500K resistor, then through two parallel paths in returning to the capacitor. One path is through R_k, the power supply, and R_3 in series. The other path is through the tube.

Assuming that the internal resistance of the power supply is zero, the first path has a resistance of 53K. The other parallel path, through the tube, has a resistance that is somewhat variable due to bias change during the discharge but is quite near the value determined by dividing the plate voltage by the plate current. From the curves the plate voltage is 180 volts, the plate current 2.5 ma, both at the operating point with 8 volts bias. Since $r_p = e_{p1}/i_{p1}$, the tube resistance is 180/.0025, or 72K. This parallel combination (with a net resistance of 30.5K) is in series with the 500K resistor to form an equivalent circuit as shown in the equivalent discharge circuit. From this equivalent circuit you can see that the values for the equation to determine the time constant are as follows:

$$e_c = -18 \text{ volts}$$

$$E_B = -112 \text{ volts}$$

$$R = 530.5\text{K ohms}$$

$$C = 2500 \text{ mmf}$$

Substituting in the exponential equation,

$$e_c = E_B\, \epsilon^{-t/RC}$$

$$-18 = -112\, \epsilon^{-t/530.5 \times 10^3 \times 2500 \times 10^{-12}}$$

$$t = 2400 \text{ microseconds}$$

The positive square wave at the second plate will have a duration of 2400 microseconds, and its amplitude will be 300 − 9.7, or 206.3 volts.

The circuit switches over again at the end of 2400 microseconds. V_2 conducts, raising the cathode voltage to 20.7 volts and cutting off V_1. The plate voltage of V_1 goes to 300 volts as fast as the capacitor can charge. The capacitor charging current raises the grid of V_2 to a positive voltage momentarily and the cathode above 20.7 volts momentarily. This, in turn, causes a momentary dip in the plate voltage at V_2. The amplitude and duration of these momentary charges can be calculated by using the same procedure outlined for the plate-coupled multivibrator.

A large voltage trigger pulse will cause the dip shown by dotted lines in the waveshape of V_1 plate and V_2 grid. This dip is an amplified version of the trigger pulse and lasts no longer than the trigger pulse. It does not affect the discharge time of the capacitor. If the pulse is of the minimum amplitude required to trigger this circuit, this dip will hardly be noticeable. Experience has shown that the trigger pulse should be of the lowest possible amplitude consistent with reliable operation.

The duration of the output pulse can be varied from the time duration of the trigger pulse to the time between pulses. If attempts are made to make the pulse longer than the time between pulses, the second pulse may not start a new cycle, and the circuit would operate on every other trigger pulse.

EFFECT OF VARYING CIRCUIT ELEMENTS. The usual method of varying the output pulse width is to vary the size of the RC in the grid of V_2. The pulse width will increase with the resistance of R_2 and the capacity of C. Another result is that the amplitude of the output pulse will increase with an increase of the plate load resistor. Notice these effects in Figure 6–87, which summarizes the effects of varying the circuit elements in the start-stop multivibrator.

The stability of the pulse width from this circuit is improved by returning the grid of V_2 to B-plus. The capacitor discharges at a higher and more linear rate below the cutoff value, causing the slope to be steeper at cutoff.

Although the trigger pulse is applied to the first grid in this circuit, the circuit can also be triggered at the plate of the first tube or the grid of the second tube if a negative pulse is used. The action will be the same.

APPLICATION. The start-stop multivibrator in Figure 6–88 is used in a radar set to produce a pulse of measured duration that drives a sawtooth sweep generator. The basic circuit is modified in several ways to produce this more efficient circuit. A third tube is incorporated in the circuit to isolate the multivibrator from the source of trigger voltage and to amplify and sharpen the pulse. This third tube (V_3) is biased beyond cutoff so that it acts as an open circuit at all times except when a trigger pulse is on the grid. The grid of the first tube (V_1) has about +14 volts on it from the +300-volt voltage divider, while the grid of the second tube (V_2) is returned to +300 volts through a large resistor. The current through V_2 and the cathode resistor is high enough to cut off V_1 in spite of the positive voltage on the V_1 grid. This positive voltage partly determines the duration of the output pulse.

The circuit operates in this way. A positive trigger pulse, simultaneous with the transmitter pulse, is applied to the grid of V_3. This tube is

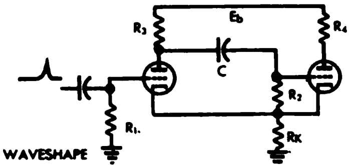
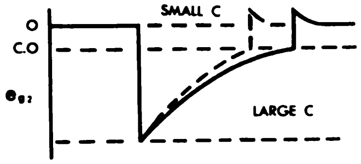
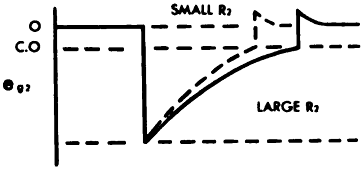
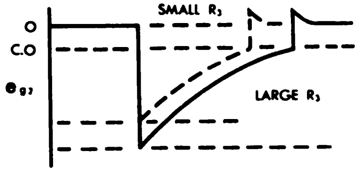
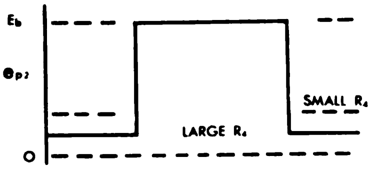
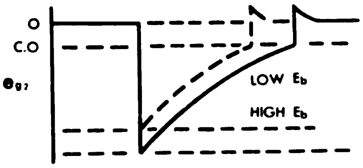
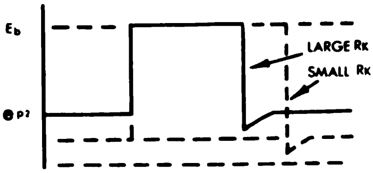
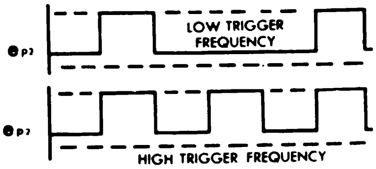
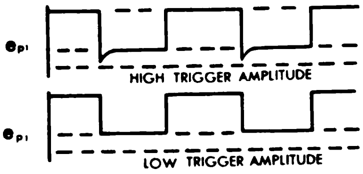
CIRCUIT CHANGE	EFFECT ON DURATION OF OUTPUT PULSE	EFFECT ON AMPLITUDE OF OUTPUT PULSE	EFFECT ON FREQUENCY OF PULSE REPETITION	EFFECT ON STABILITY	
INCREASE C	INCREASED BECAUSE DISCHARGE LASTS LONGER	NO CHANGE.	NO CHANGE. FREQUENCY GOVERNED BY TRIGGER FREQUENCY.	NO CHANGE SAME PERCENTAGE OF RC USED.	
INCREASE R_2	INCREASED DISCHARGE IS SLOWER	NO CHANGE.	NO CHANGE.	NO CHANGE. SAME PERCENTAGE OF RC USED.	
INCREASE R_3.	INCREASE BECAUSE AMPLITUDE OF e_{g2} IS INCREASED.	NO CHANGE	NO CHANGE.	DECREASED BECAUSE DISCHARGE CURVE HAS LESS SLOPE AT CUTOFF.	
INCREASE R_4.	NO CHANGE DOES NOT APPRECIABLY EFFECT PRECEDING CIRCUIT	INCREASED SINCE C_{g2} IS LOWER BETWEEN TRIGGER PULSES.	NO CHANGE.	NO CHANGE.	
INCREASE E_b.	INCREASED BECAUSE V_1 PLATE SWING AND V_2 GRID VOLTAGE CHANGE IS GREATER.	INCREASED DUE TO GREATER V_2 PLATE VOLTAGE CHANGE.	NO CHANGE.	IMPROVED SLIGHTLY, SMALLER PERCENTAGE OF DISCHARGE CURVE USED.	
INCREASE RK	SHORTER SINCE VOLTAGE CHANGE AT PLATE OF V_1 WILL BE DECREASED	NO CHANGE.	NO CHANGE.	DECREASED BECAUSE VOLTAGE CHANGE AT PLATE OF V_2 WILL BE DECREASED	
INCREASE TRIGGER FREQUENCY	NO CHANGE. UP TO POINT WHERE DURATION IS TIME BETWEEN PULSES.	NO CHANGE.	INCREASED	NO CHANGE.	 e_{p2} LOW TRIGGER FREQUENCY e_{p2} HIGH TRIGGER FREQUENCY
INCREASE TRIGGER AMPLITUDE	NO CHANGE.	NO CHANGE.	NO CHANGE.	SMALL INCREASE WILL IMPROVE STABILITY IF ORIGINALLY TOO LOW	 e_{p1} HIGH TRIGGER AMPLITUDE e_{p1} LOW TRIGGER AMPLITUDE

Figure 6–87. Varying Elements in Start-Stop Multivibrator

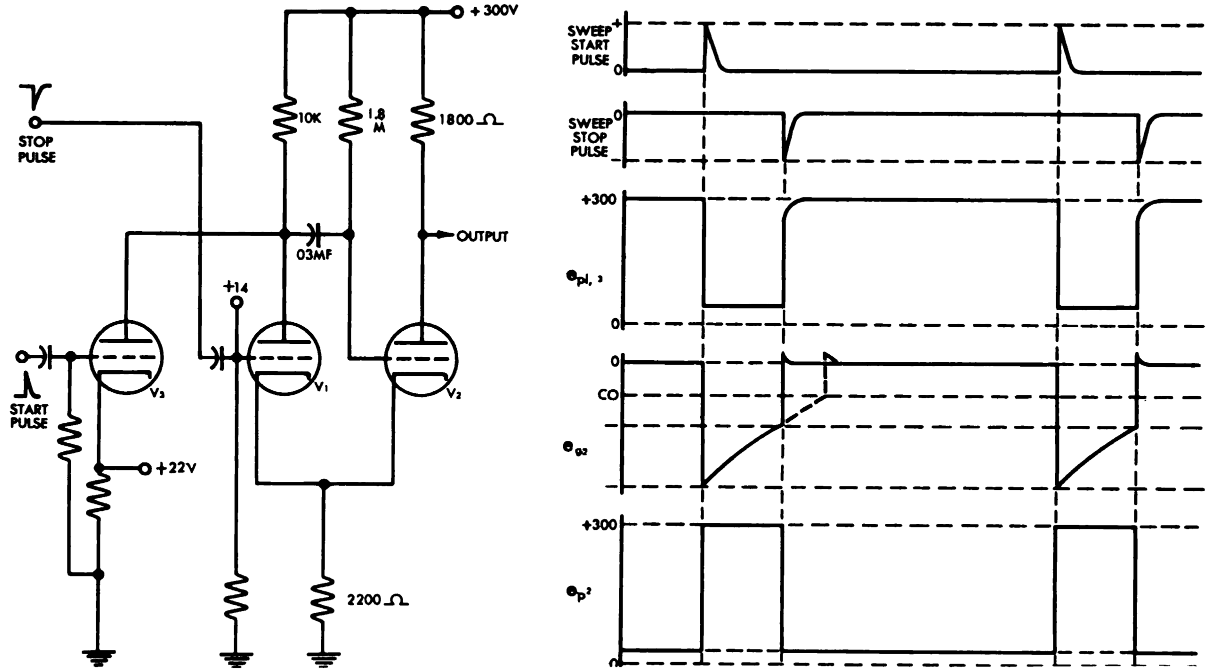


Figure 6–88. Application of Start-Stop Multivibrator

called a clipper, since 22 volts bias keeps it cut off and only the peak of the trigger pulse causes conduction. Due to amplification in V_3, a large negative-going change appears in the plate circuit. This is coupled through the .03 mf capacitor to the grid of V_2. The current in V_2 is reduced, V_1 conducts, the added drop cuts off V_2, and the discharging capacitor keeps V_2 cut off. When the capacitor discharges down to cutoff for V_2, this tube would conduct, ending the output pulse. But this is the sweep circuit and the pulse should end when the spot reaches the range mark for maximum range.

At the time the spot reaches the last range mark, a negative voltage pulse is fed back to the grid of V_1. This reduces the current in V_1, and the small change is amplified around the circuit to cut off V_1 and cause V_2 to conduct again. Dotted lines in the V_2 grid waveshape show the normal changeover time, which is always later than the sweep-stop pulse. This system of stopping the sweep multivibrator with a voltage from the sweep deflection circuits simplifies the adjustments in the sweep circuit. When the range switch is changed to change the duration of the sweep, no part of this multivibrator is changed. But the speed of the spot movement is changed so it reaches the last range mark at a different time and the stop-pulse cuts off the multivibrator at a different time. Thus, the duration of the pulse is determined by the sweep circuit rather than the constants of the multivibrator circuit.

You recall that a hard tube sawtooth generator must be driven with a negative square wave at its grid. The positive square-wave output from this multivibrator is amplified and inverted by a triode amplifier, then is applied to the sweep generator tube in the normal way.

In addition to producing pulses of a certain duration, the start-stop is sometimes used to "delay" a pulse. For example, in one airborne radar set, the process of causing the transmitter to operate consumes about 5 microseconds, and the transmitter pulse occurs 5 microseconds after the original trigger pulse. Although the sweep voltage in the indicator circuit must start at the same time as the transmitter pulse, the original pulse is used for the sweep voltage so that both the sweep and transmitter pulses occur at the same frequency. Since the original synchronization pulse would start the time base 5 microseconds before the transmitter operates, it cannot be used directly. Instead, it must be "delayed" 5 microseconds. The start-stop multivibrator can introduce the delay.

The usual multivibrator circuit is used, with a transformer input (Figure 6–89). The input

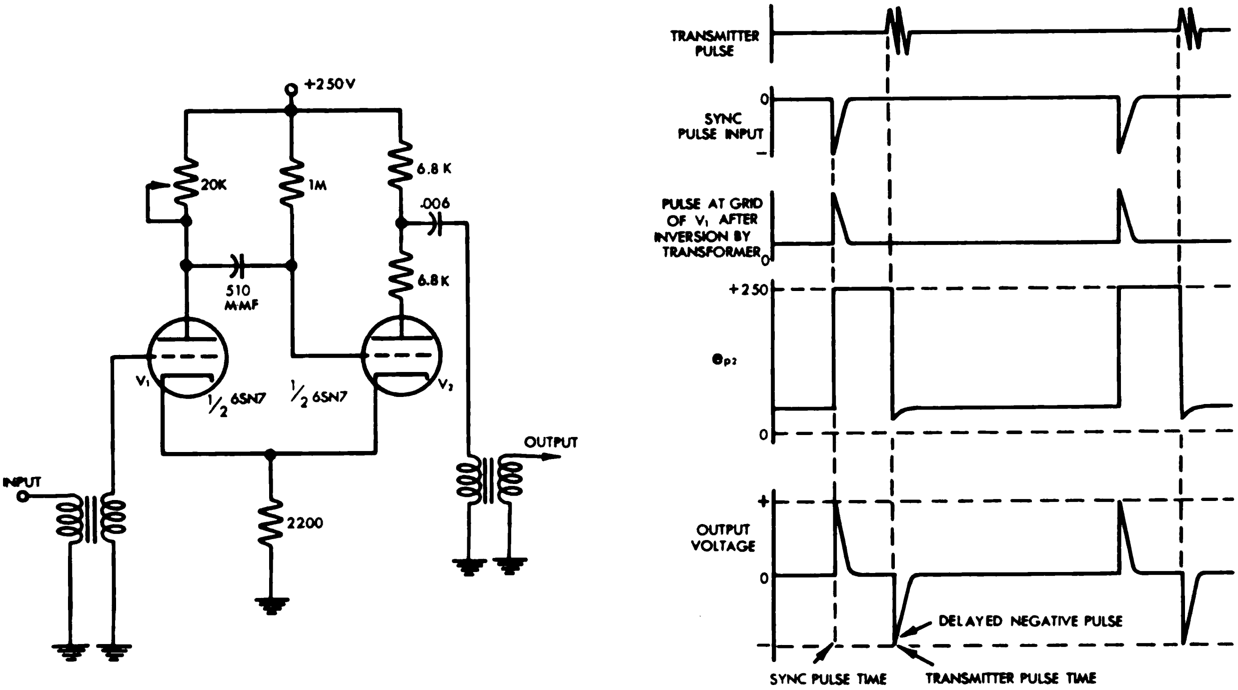


Figure 6–89. Start-Stop Multivibrator as Delay Circuit

pulse is negative and occurs 5 microseconds before the transmitter signal. Due to the transformer connections, the polarity is reversed and the circuit is triggered by the resulting positive pulse. The usual square wave appears at the second plate, with the duration determined by the RC circuit at the grid.

The transformer features a short L/R time constant, so the square wave is differentiated in the secondary. Again by selecting proper secondary connections, the second peak can be made negative. The original negative peak is replaced by a similar negative peaked pulse that is delayed 5 microseconds. Since the delay should be adjustable over a narrow range, the plate load resistor of the first tube is variable. An ingenious built-in calibration indicator permits the operator to adjust this delay time while airborne. The adjustment is checked and readjusted, if necessary, just prior to the bomb run, when the radar set is used as the bomb sight.

The Phantastron Circuit

The phantastron is another circuit that is used to delay a timing pulse. It is classed as a medium precision delay circuit. Its operation and output are very similar to those of the flip-flop circuit. Under power supply voltage changes it is quite stable because its operation depends on the fact that there is specific DC voltage relationship between the tube elements. Any variation of source voltage varies all these voltages in the same proportion, causing a minimum change in the voltage relationship since all voltages are supplied by voltage dividers across the same voltage source. The phantastron has the unique advantage over multivibrator circuits in that the pulse width, or delay, varies directly with one of the DC applied voltages. This control by a DC voltage, makes remote control feasible at any distance through unshielded power cables since there is no signal present in the control circuit.

Study the basic phantastron circuit in Figure 6–90. This circuit is never free-running and is triggered by a negative pulse at the control grid (grid No. 1) or by a positive pulse applied to grid No. 3. Consider first the conditions before a negative trigger pulse arrives. Grids 2 and 4, which are tied together inside the tube, are connected to a voltage divider from +300 volts. The normal output of the divider is 86 volts, but with zero control grid voltage and 86 volts on grid 2, 4 ma of current will flow to grid 2. This additional current in the voltage divider drops grid 2 to 68.7 volts. Grid 3 is connected to a voltage divider which sets its voltage at 27.3

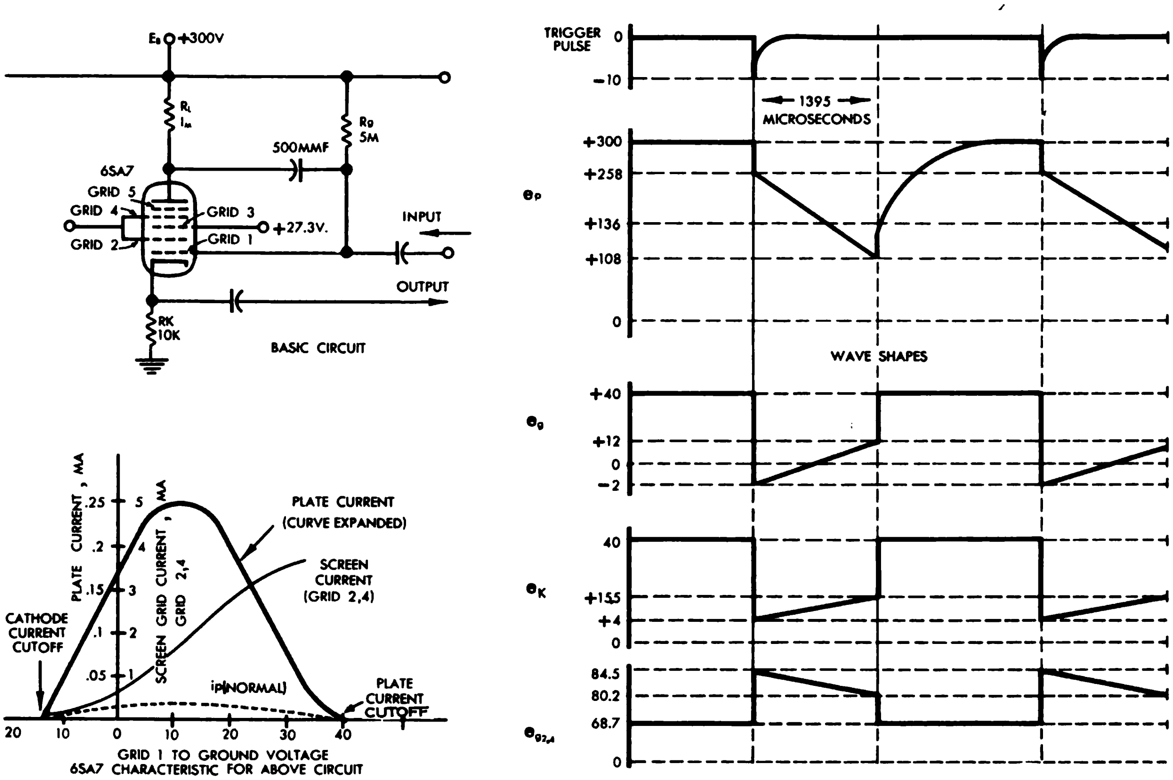


Figure 6–90. Phantastron Circuit and Waveshapes

volts. This grid does not draw current because it is negative with respect to the cathode.

The current through the 10K cathode resistor is 4 ma. Since the cathode is 40 volts above ground, the grid 3-to-cathode voltage is 27.3 — 40 = — 12.7 volts. When grid 3 is more than 12 volts negative with respect to the cathode, it prevents electrons from passing it in their journey to the plate. Therefore, since the —12.7 volts is beyond plate current cutoff for grid 3, no electrons will reach the plate. Thus, the plate voltage is +300 volts. Grid 1 is connected to E_B through a 5-megohm resistor. A very small grid current flows through the cathode resistor, but the grid voltage may be considered the same as the cathode voltage. In other words, the difference in potential between grid 1 and the cathode voltage is zero. Grid 5, the suppressor grid, is connected to the cathode and has no part in the operation of the circuit other than performing the normal function of a suppressor grid.

The phantastron is similar to the start-stop circuit. You can consider the 6SA7 phantastron tube to act as two tubes. The cathode, grid 3, and plate are equivalent to the first triode in the start-stop multivibraor. No plate current flows to the plate and the "V_2" tube is cut off. The cathode, grid 1 and grid 2 form another triode, which is comparable to the second tube of the flip-flop. As mentioned previously, this triode is conducting. Consequently, any voltage change which will cause plate current to flow in the triode that is cut off, will start a regenerative switching action.

You can best understand the action of the phantastron when it is triggered, by dividing it into three successive phases. The first phase is an extremely rapid change which ends at a first balance point. The second phase is a slow linear change which ends at a second balance point. The third phase is the recovery of the circuit to pretrigger condition. First let us consider what happens when the voltage on grid 1 is varied. The 6SA7 characteristic curves in Figure 6–90 show the relationship between the various currents. The dotted curve shows the plate current on the same scale as the screen current curve. The expanded plate current curve uses a scale 20 times as large, in order to show the exact shape of the curve. At 40 volts the voltage between grid 3 and cathode is enough to cut off plate current. Grid 1 and the cathode are at the same potential. Now decrease the voltage on grid 1 from +40 volts to +30 volts. The cathode current decreases, as in any vacuum tube.

This decrease in cathode current produces a decrease in the voltage across the cathode resistor, producing a decrease in the voltage between grid 3 and the cathode. No longer is the voltage on grid 3 sufficient to keep plate current cut off. The plate current rises as the voltage on grid 1 becomes less positive. At the same time, the screen current decreases more rapidly than the plate current is increasing, thus the over-all decrease in total tube current (cathode current). As the voltage on grid 1 becomes less and less positive, the screen current continues to decrease. However, at about 12 volts, the total current becomes so small that the plate current begins to decrease. At about −12 volts, all current in the tube is cut off.

Now, knowing the action of the tube with changes in grid voltage, let us apply a trigger pulse to the circuit. The trigger pulse must have the proper polarity and amplitude to produce these changes. Since grid 3 is 0.7 volt beyond cutoff, a positive, 1-volt pulse applied to grid 3 will do. A negative pulse of 10 volts applied to grid 1 will also work. In Figure 6–90 a negative pulse of 10 volts amplitude is shown. When this is applied to grid 1, the grid 1 voltage will drop to 30 volts. Cathode follower action will cause the cathode voltage to drop to 30 volts right with the grid.

At this point, examine the relation between the cathode and grid 3. Their voltage difference is 27.3 − 30 or −2.7 volts. Grid 3 is not negative enough to prevent electron flow to the plate and plate current will flow as soon as the grid 3-to-cathode voltage drops below 12 volts. Regenerative action starts with this plate current flow. The plate current through R_L causes a drop of plate voltage. Grid 1 is capacitively coupled to the plate so the grid becomes less positive. This drops the cathode voltage still more, which, in turn, further reduces the voltage difference between grid 3 and cathode. The current to the plate increases; the plate voltage drops, and the grid is driven farther in the negative direction.

The regenerative effect just explained is similar to the cutting off of the first multivibrator tube by the second tube. The cutting off process, however, does not go that far in the phantastron, for if the current to the plate were stopped, the plate voltage would have to go up, and this action would be the opposite to the desired feedback. The increase of current to the plate ends at the first balance point. This balance occurs when the negative-going grid 1 voltage decreases the over-all current so much that the plate current cannot continue to increase. Grid 3 is unable to divert enough electrons to the plate to maintain the increase of plate current, so the plate current stops increasing and becomes steady. This occurs at about −2 volts grid 1 voltage, so the plate has dropped 42 volts before the balance point is reached.

Each of these voltages is shown in the curves, under the trigger pulse. The grid voltage has dropped to minus 2 volts. Since this is a 42-volt change, which is caused by the plate, the plate curve shows a 42-volt drop to 258 volts. The cathode voltage drops due to the decrease in plate current until, with −2 volts on the grid, the current is 0.4 ma and the voltage is 4 volts. The voltage at grids 2 and 4 has gone up because the screen current is decreased, reducing the drop across the voltage divider and raising its voltage to 84.5 volts.

Now consider the action resulting from the starting of the second phase. The capacitor from plate to grid at this time starts to discharge to the new lower plate voltage. At this time the grid voltage is −2v and the capacitor discharges from 260 to 96v. In discharging, the grid end of the grid resistor becomes more positive. This increases the plate current, starting another drop in plate voltage. Grid 3 is not involved directly in this stage as it is straight triode action between the plate, grid 1, and the cathode that affect the circuit operation. The plate voltage drop has an effect opposite to that of the discharging capacitor and thereby is degenerative in action.

The capacitor discharges, the grid goes *positive*, plate current increases, plate voltage drops, tending to make the grid go *negative*. But the plate cannot drop enough to exceed the positive change at the grid because it is the positive grid voltage change that causes the plate voltage drop. So the plate voltage change only counteracts *part* of the effect of the capacitor discharge, slowing the discharge considerably. This negative feedback keeps the capacitor discharge in the most linear part of the exponential curve. In fact, mathematically it can be shown that the time constant is lengthened by an amount equal to the amplification factor of this triode. Looking at it still another way, the exponential curvature at the grid is counteracted by opposite curvature at the plate due to the inverting properties of the triode tube.

The discharge continues at a linear rate, as shown by the plate voltage curve illustrated in Figure 6–90. The increase in current raises the cathode voltage to 15.5 volts, and the grid 1 voltage is raised to +12 volts before the second balance point is reached.

Remember that during all this time, the screen grids (2 and 4) have been continually drawing current. As the plate voltage continues to fall, there is a minimum point where 68 volts on the screens will collect more electrons from the space current than a higher voltage on the plate. This occurs at about 108 volts on the plate. This point is also the peak of the current characteristic in the illustration. To the right of this peak, the plate current decreases and the screen current increases. The screen grids take more current than the plate can attract, so the plate current stops increasing and levels off. This leveling off initiates the rapid switchover to recover the circuit to pretrigger condition.

When the plate current levels off, the voltage stops decreasing at the plate. With the counteracting effect of the plate drop removed, the capacitor discharge raises the grid 1 voltage at a very rapid rate. The positive-going grid 1 increases the current from the cathode, which raises the cathode voltage. This has the same effect as making grid 3 more negative; so grid 3 reduces the current to the plate. Actually, it is dividing the increasing current in favor of the screen grids. The screen current increases tremendously while the plate current is actually decreased. The plate voltage starts to go in the positive direction. This is coupled to the grid by the 500-mmf capacitor to make it go in a positive direction. The cathode voltage is raised some more, which brings grid 3 closer to cutoff, reducing the plate current more and raising the plate voltage.

This regenerative action continues until grid 3 is beyond cutoff — which stops plate current completely. During this regenerative action, the grid voltage has increased as fast as the plate voltage, so no change in capacitor voltage occurs. But when the grid reaches +40 volts, the plate current reaches zero and the grid no longer affects the plate voltage. Thus, as the grid voltage jumps from 12 to 40, the plate voltage curve shows a 28-volt jump from 108 to 136 volts. When the grid voltage reaches a constant value, the plate rises only as fast as the capacitor can recharge along a slow exponential curve up to +300 volts. The cathode voltage rises immediately with the grid voltage while the screen grid voltage drops at recovery time because of the increased screen current.

Now consider the circuit from the viewpoint that it is ready for the next trigger pulse. The duration of operation, and consequently, the duration of the negative pulse on the cathode, is fixed in the circuit shown. This pulse width is determined essentially by the RC time constant in the grid 1 circuit, the plate voltage, and the gain of the triodes involved. The pulse width can be calculated, but the calculations are a bit lengthy to be included here.

CHANGING PULSE WIDTH. The duration of the pulse is generally made variable by applying a DC voltage to the plate from a potentiometer. The circuit is as shown in Figure 6–91. A potentiometer can select any voltage from zero to +300 volts. A diode is inserted in series to disconnect the phantastron plate from the low-resistance potentiometer during the operation of the circuit. Before the trigger pulse, the plate voltage and voltage at the arm of the potentiometer arm are the same because the load resistor is so large and the current from the potentiometer so small that there is little voltage drop between the arm and 6SA7 plate. Most of the current therefore passes through the diode. When the circuit operates, the plate voltage drops immediately, the diode plate becomes less positive than its cathode, and diode current stops. So the potentiometer-diode circuit has no further effect on the operation of the phantastron.

To see how the potentiometer affects the pulse width, study the effect of varying the position of the potentiometer contact. With the plate voltage at +300 volts, the situation is as previously described. But when this plate voltage is reduced to +240 volts by the potentiometer, the pulse width is decreased. The plate voltage drops the same 42 volts at the time of the trigger pulse since the grid drops 42 volts to the first balance point. Then the slow linear voltage decrease

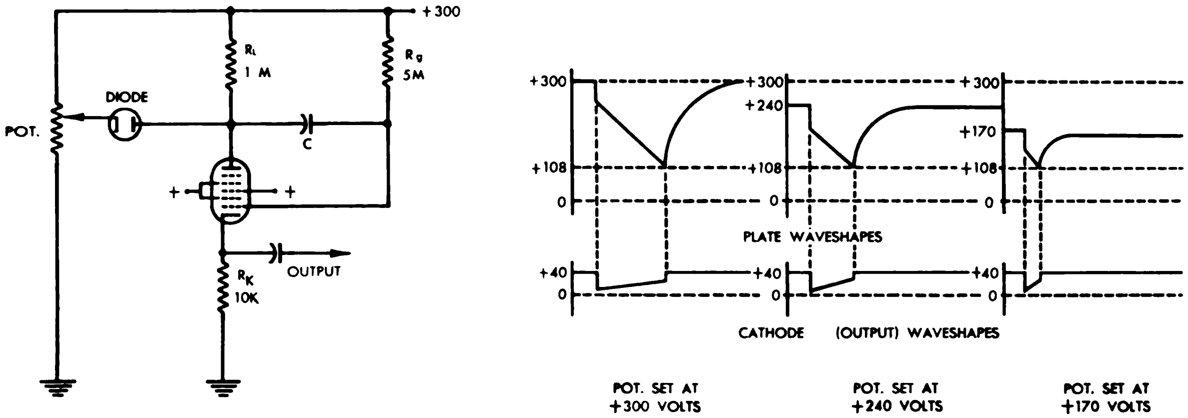


Figure 6–91. Changing Pulse Width in Phantastron Circuit

occurs. The slope of this decrease is the *same* regardless of plate voltage. This terminates at the same 108 volts as the 300-volt waveshape. So the amount of linear decrease is less, which decreases the time required to reach 108 volts. If the potentiometer is set so the plate voltage is 170 volts before the pulse, the plate will again drop 42 volts, and follow the *same* slope to 108 volts. Again the amplitude of the slope is decreased so the slope time is decreased.

Since the initial drop, the slope, and the final value do not change, the decrease in pulse length is linear with the decrease in plate voltage. This relation holds over the range from the full plate voltage (300 volts in this case) to a minimum of 108 + 42 or 150 volts. This is the minimum voltage point because the initial 42-volt drop would bring it down to 108 volts, or zero pulse width. Since the pulse width for the example used is 1395 microseconds, the pulse width is variable from zero to 1395 microseconds.

Typical Use. The phantastron circuit shown in Figure 6–92 is taken from a typical radar set. It is similar to the previous circuit except

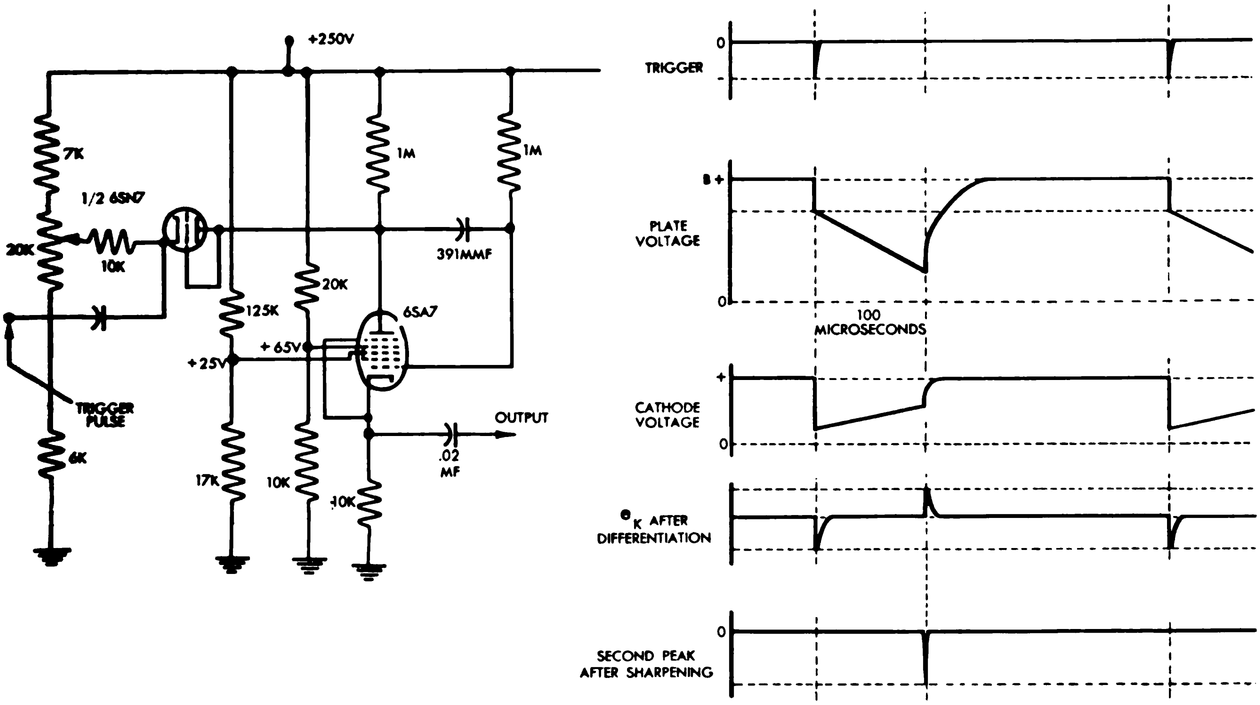


Figure 6–92. Typical Radar Phantastron Circuit

that here the values and voltages are slightly different, the voltage dividers for grids, 2, 3, and 4 are shown, and the disconnector diode is actually a triode with its grid tied to the plate. This last is done in the set to make use of the other half of a 6SN7 that is employed in another circuit. The potentiometer which sets the plate voltage has fixed resistors in each end of it to limit the pulse width to a certain maximum and minimum width which is adequate for the purpose for which it is designed.

The trigger pulse is applied to the cathode of the disconnector diode. The pulse momentarily changes the plate current in the diode, which causes the negative pulse to appear at the plate of the phantastron tube. The plate-to-grid coupling capacitor couples the pulse voltage to the grid where it starts the circuit through a cycle of operation. The output is taken from the cathode of the 6SA7, where a negative, approximately-square waveshape is produced. Actually, the waveshape is not as ideally square as shown in Figure 6–90, because the trailing edge becomes somewhat exponential in following the capacitor charge.

In use, the square wave is usually differentiated and the trailing edge pulse is put through a pulse sharpening circuit to remove the effect of the exponential trailing edge. The resulting sharp pulse is delayed from the original trigger pulse by a few hundred microseconds.

This circuit is satisfactory for short delays, but when the delay becomes nearly as long as the time between pulses, the long exponential recovery of the plate circuit overlaps the next pulse. With the circuit not yet fully recovered, the second pulse may not trigger the circuit. Therefore the circuit is modified for long delay periods as shown in Figure 6–93. The change is the addition of a triode between the plate and grid of the phantastron tube. This triode is called a *cathode follower.*

Before the trigger pulse arrives, the grid of the cathode follower is at the plate potential; it is drawing current, and the cathode is within a fraction of a volt of the grid voltage. Therefore the .005-mf coupling capacitor has the plate voltage on one side and the grid voltage on the other — which is not different from the circuit in Figure 6–90. When the trigger pulse drops the plate voltage, the grid voltage of the cathode follower drops, and consequently its cathode voltage goes down. The .005-mf capacitor must discharge, and in doing so, makes grid 1 go in the negative direction. Thus, by cathode follower action, the grid drops right along with the plate as though capacitive coupling existed between them.

When the end of the pulse arrives, and the plate voltage rises back to 250 volts, it does not have to charge the capacitor mentioned above. The plate can raise the cathode follower grid

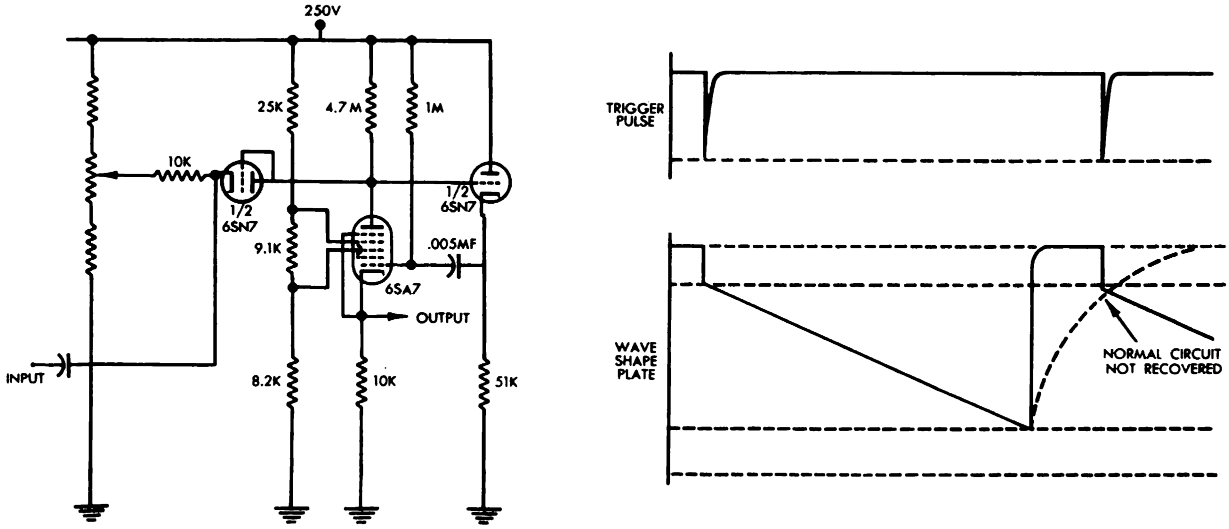


Figure 6–93. Phantastron with Cathode Follower

immediately to +250 volts. Then the capacitor charges through a path from the 6SA7 cathode to grid 1, through the capacitor, from cathode to plate of the cathode follower, to +250 volts. The grid of the cathode follower is not involved, other than through interelectrode capacity, which is very small. Only a slight exponential curvature of the plate waveshape is shown in the figure. This curve is due to stray wiring capacity in the circuit. The dotted curve shows the exponential rise which would interfere with the next pulse if no cathode follower were used. In all other respects, this long delay circuit is the same as the short delay circuits.

The phantastron discussion to this point, in association with other circuits as well as its own theory of operation, has used a negative trigger to activate the circuit. However, a positive trigger can be applied to grid 3 instead of a negative trigger to grid 1. Figure 6–94A is a circuit using a positive trigger input; Figure 6–94B shows the waveshapes for the circuit.

A voltage divider network places a positive voltage on grid 3. Another network places on grids 2 and 4 a positive voltage that is higher than the voltage felt on grid 3. Current flows to grid 2, producing a voltage drop across R_k that is more positive than grid 3. Under these conditions (static) there is no plate current flowing because grid 3 is negative with respect to the cathode. Grid 1 has a small current flow through a 5-megohm resistor, but the potential of grid 1 is considered to be the same as that of the cathode. With no plate current the plate potential will be 300 volts and capacitor C_2 charges exponentially to that value. With these factors in mind, let us now apply a trigger and see the operation of the circuit.

A short duration positive trigger applied to grid 3 makes that grid positive with respect to the cathode. This causes plate current to flow and plate voltage (E_p) to drop sharply. C_2 couples the plate voltage change to grid 1 making it more negative. Grid 1, now more negative, reduces the current through the entire tube. Reduction in tube current causes the voltage drop across R_k to decrease. Where grid 3 had been negative with respect to the cathode, it is now more positive, thus increasing plate current (I_p).

Grid 3 is the controlling factor in the amount of current flowing to the plate or to the screen grids (2 and 4). Therefore, with an increase in plate current, there is a corresponding decrease in screen grid current. However, after the initial increase in plate current and decrease in screen current, both plate and screen currents begin to increase slowly. As plate current increases, plate

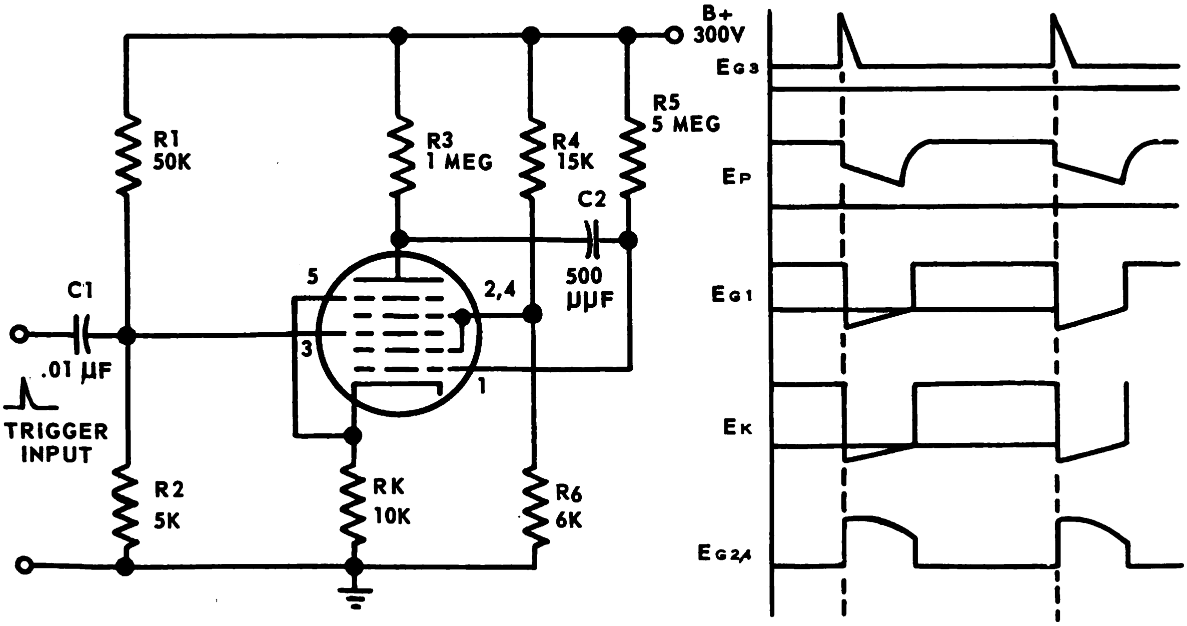


Figure 6–94. Phantastron Delay Circuit

voltage decreases and C_2 discharges slowly through R_5. The discharge of C_2 causes the voltage on grid 1 to rise slowly.

The increase in tube current flowing through R_k causes the voltage across R_k to increase. As soon as the cathode voltage reaches the point where it is positive with respect to grid 3, plate current ceases to flow, with screen grid current quickly going to its maximum value. The plate voltage will rise exponentially to 300 volts as C_2 charges through R_3. Thus, the circuit is again at its static condition.

The Blocking Oscillator

The blocking oscillator is another type of relaxation oscillator. It can do many of the things that a multivibrator does. Thus, it can be free-running, synchronized at each cycle or at a submultiple frequency, or it can be driven in the manner of a one-shot multivibrator.

A blocking oscillator is any oscillator that cuts itself off after one or more cycles due to the accumulation of a negative charge on the grid capacitor. Thus, in an oscillator in which the grid swings positive with respect to the cathode, electrons are attracted to the grid and accumulate on the plate of the grid capacitor nearest the grid. Since these electrons cannot return to the cathode through the tube, they must return through the grid-to-cathode resistor. If this resistor is sufficiently large, electrons may accumulate on the capacitor faster than the resistor permits them to return to the cathode. In this case a negative charge is built up at the grid which may bias the tube beyond cutoff.

After the tube is cut off, it provides no additional electrons to the grid capacitor. The capacitor continues to discharge through the resistor, and a point is reached eventually where the tube again conducts. Thus, the process repeats, and the tube becomes an intermittent oscillator. The rate of the recurrence of operating conditions is determined by the RC time constant of the grid circuit.

There are two general types of blocking oscillators — the *single-swing* type in which the tube is cut off at the completion of one cycle, and the *self-pulsing* type, in which each cycle of oscillation causes the grid to become progressively more negative until the tube is biased out of operation. In radar the single-swing type oscillator usually operates within the audio-frequency range, while the self-pulsing type produces pulses of r-f energy.

The single-swing oscillator circuit shown in Figure 6–95 consists of a transformer-coupled oscillator, with a capacitor in series with the grid of triode V_1. To understand its operation, assume that the grid capacitor C_g has been negatively charged by a preceding cycle. The tube, therefore, is biased well below cutoff. As the charge on the capacitor leaks off, the biasing voltage is reduced to the point where the tube begins to conduct. As plate current starts to flow, a magnetic field is set up around the plate winding P of the transformer. The dots at each winding indicate similar polarities. If, for example, a current flows through a winding so that the dot end is positive, the field set up in the core induces voltages in the other windings that makes the dot end positive in these windings at the same time.

The field builds from zero to a maximum in direct proportion to the plate current, and therefore induces a voltage across winding S_1. This action is impressed upon the grid of the tube through the grid capacitor C_g with a polarity that drives the grid more and more positive as the field in the plate winding is building up. The increased plate current induces a still higher secondary voltage, which causes the tube to conduct more. This regenerative process soon has the tube conducting at its limiting value which is determined by B+ and the resistance in the circuit.

Having reached the point of a steady state of conduction, one would assume that the positive grid voltage would be lost and that plate current consequently would decrease. This is not true, however, as the induced positive secondary voltage continues to rise for a period of time, and plate current continues high for this same time although it decreases slightly.

The reason that the positive secondary voltage continues to rise for a period of time is that the magnetic flux in the transformer core is not precisely in step with the primary current. The magnetic flux in the transformer core *lags* the primary current. Too, as long as secondary current flows there is a counter magnetomotive force. Therefore, as the magnetic flux lags the primary current and as the secondary current decreases, the magnetic flux continues to increase.

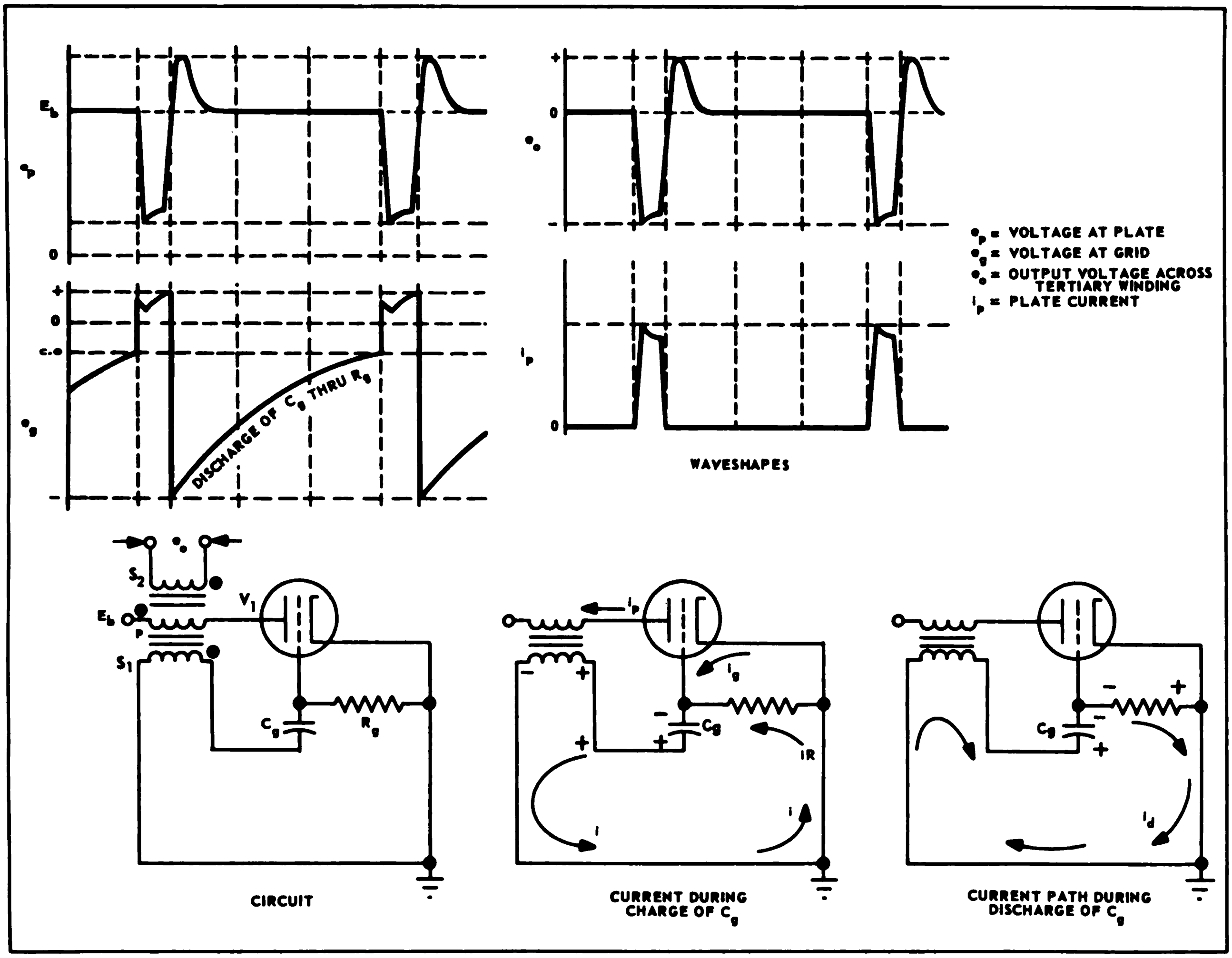


Figure 6–95. Single-Swing Blocking Oscillator

Eventually the magnetic flux stops increasing; it is in phase with the primary current and the high positive voltage on the grid is suddenly withdrawn. The grid is driven negative by the reverse voltage in the grid circuit and also by the coincident collapse of the magnetic flux which induces a reverse voltage in the secondary. As the grid-leak resistor R_g is large, the time for discharge of capacitor C_g is long, keeping the tube cut off so that the oscillation occurs for about a half-cycle.

The time consumed by the rise and decay of plate current depends upon the inductance and resistance of the transformer. The time between pulses depends primarily upon the value of resistance since the grid capacitance is fixed because of pulse width requirements.

Note the current paths during charge and discharge, Figure 6–95. The capacitor charges very quickly through the low resistance of the conducting grid-cathode circuit but discharges very slowly through the large grid-leak resistance. The rounded waveshapes show that the voltage and current changes occur more slowly than in a multivibrator. This is due to the inductance in the transformer. The inductance also is the cause of the plate voltage rising above E_B when the tube is cut off. The current continues to flow in the same direction as it charges the capacity in the transformer windings and causes a voltage drop that adds to E_B. The inductance and distributed capacity form a resonant circuit that tends to oscillate due to shock excitation. If the Q of this LC circuit is high, it may oscillate for several cycles. In the case illustrated in Figure 6–95, damping is at the critical value so that the oscillation occurs for about a half-cycle.

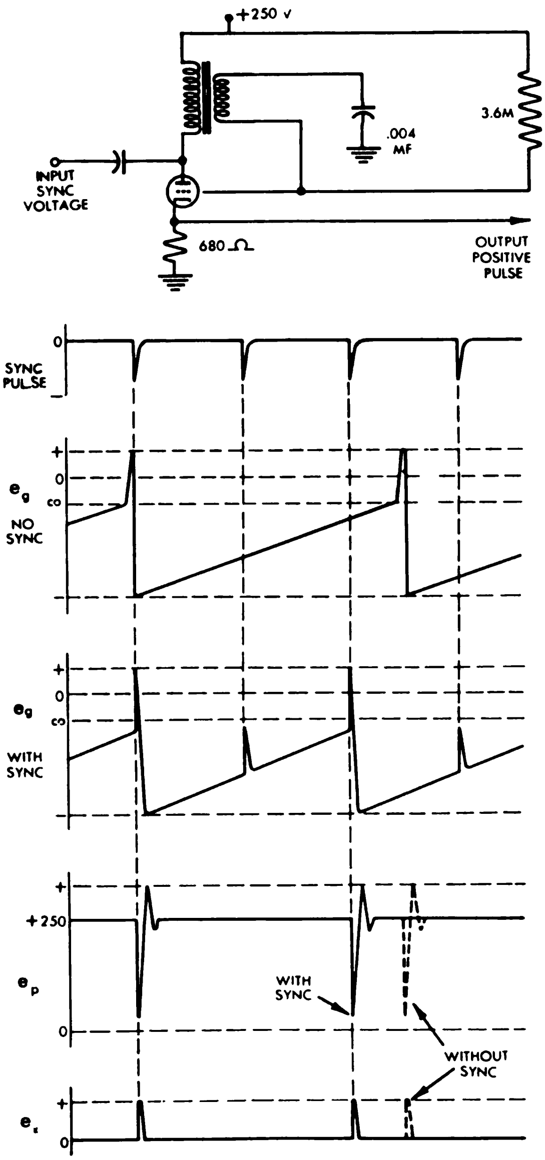


Figure 6–96. Synchronized Blocking Oscillator

The output (e_o) from this circuit is taken from a third winding on the transformer, but the output can also be taken from the plate, where a large negative pulse is produced, or from a small resistor in the cathode circuit, where a small but sharp positive pulse is available. The waveshape of the cathode pulse is the same as the plate current waveshape shown in the figure.

The Synchronized Blocking Oscillator

The synchronized blocking oscillator in Figure 6–96 is a typical circuit. This oscillator can be used as the timing oscillator in a small piece of auxiliary equipment. Normally the set produces its own PRF independently when the oscillator is free-running, as you can see in the waveshapes labeled "e_g, no sync." The natural frequency for its operation is about 300 pps. However, a synchronizing pulse can be introduced at the plate, without making changes in its circuit, that will produce a PRF as high as 1000 pps. The synchronization pulses shown occur at 700 pps. Their amplitude is such that one pulse makes the circuit operate. The next pulse does not bring the grid above cutoff, but the third pulse does make the tube conduct.

Since the oscillator is synchronized on alternate pulses, its PRF becomes 700/2, or 350 pps. If the main radar set has a PRF of 1000 pps and the amplitude is slightly less than that shown, the blocking oscillator can be synchronized on every third cycle, and it will operate at $333\frac{1}{3}$ pps. By synchronizing this auxiliary set with the big set, indications received by the small set can be displayed on the main indicator, relieving the operator of observing two radar indicators at the same time.

The amplitude of the synchronizing pulse has a great effect on the frequency of this circuit. If the amplitude of the synchronization pulse shown is increased 50%, the first pulse that occurs after the circuit begins operation will bring the grid above cutoff and the circuit will be synchronized on every pulse. As stated before, a low amplitude will cause triggering on every third pulse since the first two will not have sufficient amplitude to make the tube conduct.

In each case it was assumed that the natural period of the circuit was longer than the time between the pulses causing the circuit to operate. If the period of the blocking oscillator is shorter than the synchronizing pulse period, long and short cycles will be produced because the circuit will be triggered by some pulses but will conduct at its natural period in between.

Maximum frequency stability is achieved in this circuit by connecting the grid load resistor to the plate voltage supply. This has the same effect as returning the grids to B+ in multivibrators. The useful output from this circuit is taken from a small (680-ohm) resistor in the cathode circuit. This resistor has a negligible effect on the operation of the circuit but develops a voltage pulse from the plate current flowing through it.

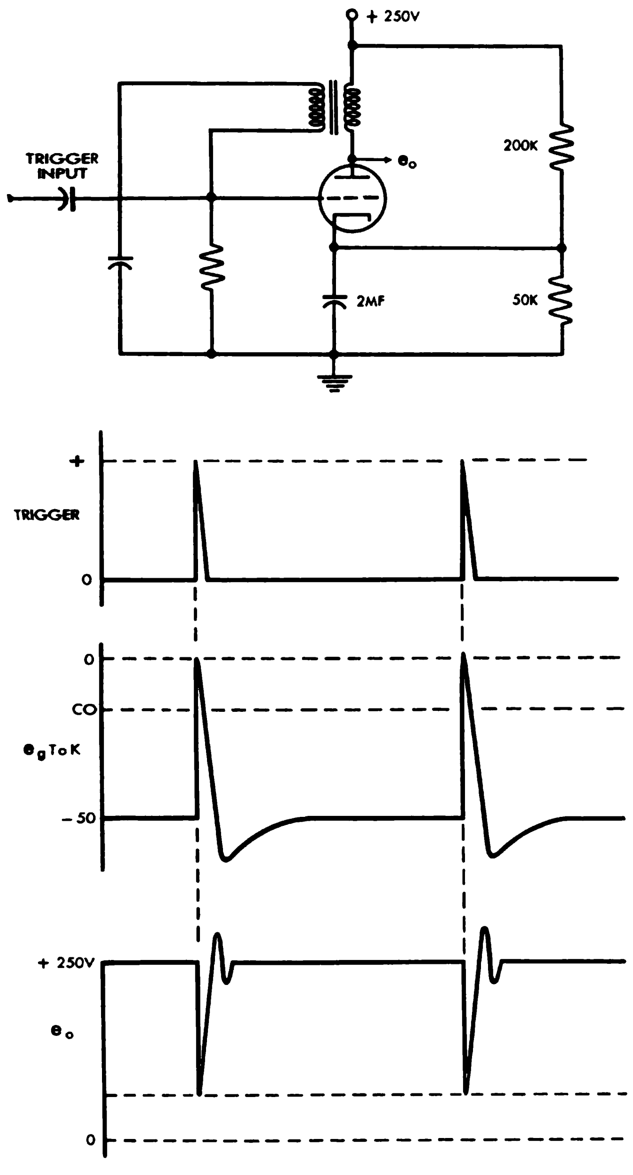


Figure 6–97. Blocked Oscillator

The Driven Blocking Oscillator

Some radar circuits require a blocking oscillator that operates only when triggered (Figure 6–97). Such a circuit is often called a *blocked oscillator* since a permanent DC voltage prevents the circuit from oscillating of its own accord. Only a positive voltage on the grid or a negative voltage on the cathode will cause the tube to conduct. If the voltage is a pulse, the circuit will go through a cycle of operation, cut itself off, and the DC voltage will keep it cut off. Cutoff is maintained by a voltage divider from the plate supply voltage. With the values shown, the cathode voltage will be plus 50 volts with respect to ground. The grid is grounded and is therefore 50 volts negative with respect to the cathode.

The grid waveshape shows voltages with respect to the cathode since these are the important voltages in understanding the circuit operation. Cutoff for most tubes used in this type of circuit occurs at about −18 volts, so the trigger pulse must be more than 50 − 18, or 32 volts in amplitude to start a cycle of operation. In the grid waveshape the normal discharge of the grid capacitor reduces the grid voltage after a cycle of operation, but the discharge is to minus 50 volts, so the circuit does not again conduct without another trigger pulse.

The positive trigger pulse must be applied to the grid only. It is not practical to apply a trigger pulse to the cathode because the filter capacitor is usually found there. But a negative pulse may be applied to the plate, where the transformer will invert the pulse to a positive one on the grid. The output may be taken from the plate, where a large negative pulse is available, or from a third transformer winding where a pulse similar to the plate pulse can be obtained with either polarity.

The driven, or blocked, oscillator is useful as a pulse sharpening circuit. One set uses the circuit in Figure 6–98 to improve the pulse shape from the phantastron. The phantastron produced the square wave labeled input waveshape, which has been amplified and inverted to be the input to the differentiating transformer shown in the circuit. The waveshape at the secondary of the transformer is a peaked wave, as shown in the second waveshape, but because of the inherent imperfections in the trailing edge of the phantastron pulse, the peaked wave due to the trailing edge is not a sharp one. For precise triggering of the circuits following the phantastron, a sharper pulse is required. Therefore the peaked wave is applied to a blocked oscillator.

In place of the voltage divider from +250 volts used in the earlier circuits, a long time constant in the cathode circuit provides the cutoff voltage for the preceding circuit. After several cycles of operation, the grid current due to positive grid voltages charges the cathode capacitor. It will discharge slowly between cycles and be recharged with each cycle. The RC time constant in the grid circuit is rather short and cannot hold

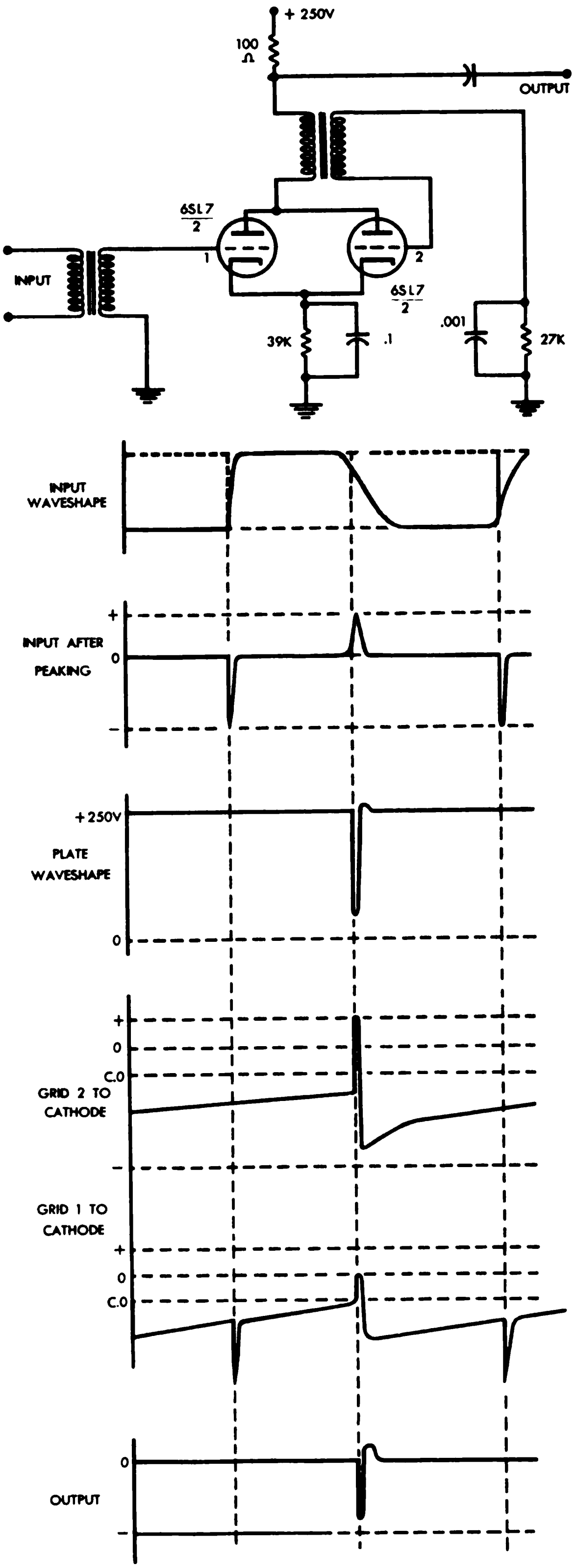


Figure 6–98. Blocked Oscillator as a Pulse-Sharpening Circuit

the grid below cutoff for the several hundred microseconds between pulses.

The negative pulse at the leading edge of the square pulse will not affect the tube already cut off. The positive pulse at the trailing edge will cause the left-hand tube to conduct. The current through the transformer and its associated magnetic field will induce a positive voltage at the grid of the right-hand tube. This starts a regenerative action in the blocking oscillator associated with the right-hand tube and it goes through a normal cycle of operation. Meanwhile the pulse ends at the grid of the left-hand tube and, with no other voltage on the grid, the tube is cut off again.

The left-hand tube is an amplifier and isolation circuit that causes the blocking oscillator to start very soon after the trigger pulse arrives, then disconnects the triggering circuit from the blocking oscillator so it can go through its normal cycle unaffected by the triggering circuit.

The output is taken from a small resistor above the transformer in the plate lead. A sharp negative pulse of low amplitude is produced across the resistor.

COUNTING CIRCUITS

A counting circuit, also known as a frequency divider, is one that receives uniform pulses representing units to be counted and produces a voltage proportional to their frequency. By slight modifications the counting circuit is used in conjunction with a blocking oscillator to produce a trigger pulse that is a submultiple of the frequency of the pulses applied.

The pulses applied to the counting circuit must have the same amplitude and time duration if accurate frequency division is to be made. Counting circuits, therefore, are ordinarily preceded by shaping and limiting circuits to insure this uniformity of amplitude and width. Under these crcumstances the pulse-repetition frequency constitutes the only variable, and frequency variations may be measured.

Positive Counting

Positive pulses, which may vary only in their recurrence frequency, are applied to the input of the positive counter, Figure 6–99. The charge

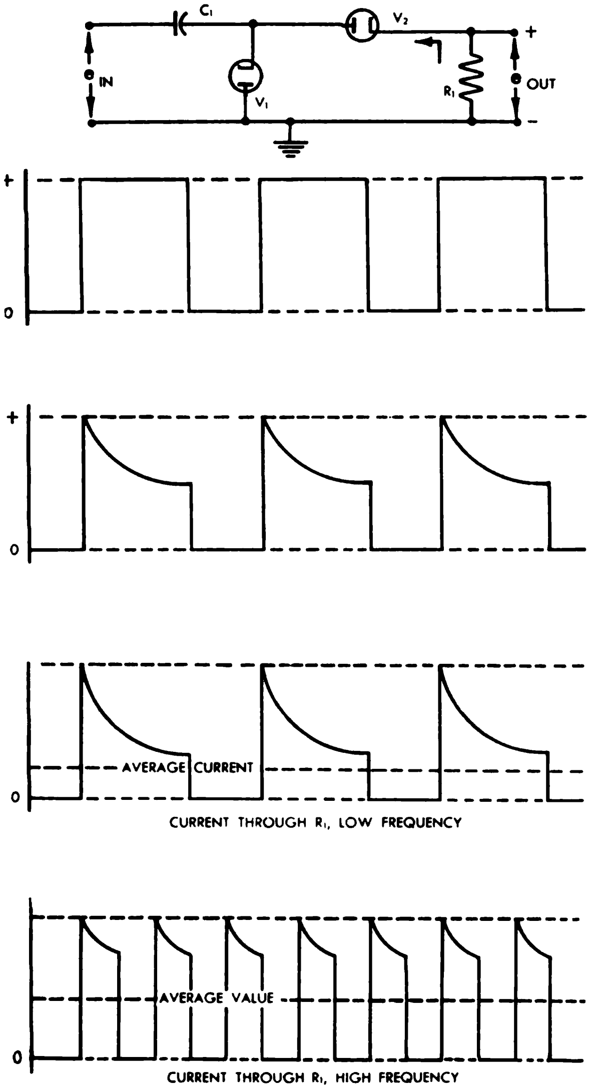

C1
V2
IN
OUT
R1
V1
+
-
+
0
+
0
AVERAGE CURRENT
0
CURRENT THROUGH R1, LOW FREQUENCY
AVERAGE VALUE
0
CURRENT THROUGH R1, HIGH FREQUENCY

Figure 6–99. Positive Counter

on the coupling capacitor C_1 cannot change instantaneously as the positive leading edge is applied, so the plate of V_2 becomes positive and that tube conducts. A charging current flows through R_1 during the pulse time and a small charge is developed on C_1. At the end of the pulse the drop in voltage places the diode side of the capacitor at a negative potential equal to the charge accumulated on C_1.

Tube V_2 cannot conduct, as its plate is negative with respect to its cathode. However, V_1 conducts, discharging the small charge from the capacitor, which would otherwise build up during each succeeding positive pulse, eventually rendering the circuit insensitive to the applied pulses.

Since a certain amount of current flows through R_1 each time a pulse is applied, an average current flows that increases as the pulse repetition frequency increases and decreases as this frequency decreases. The IR drop developed across R_1 can be used to control a succeeding stage as is illustrated in Figure 6–100. The filter in the grid circuit of V_3 aids in obtaining smooth operation by removing too rapid changes in voltage developed across R_1. The voltage at the grid of V_3 varies with changes in the pulse frequency and produces variations in the plate current of V_3. A milliammeter is placed in series with the plate circuit so that changes in the average plate current are indicated as a measure of variations in the recurrence frequency of the input pulses.

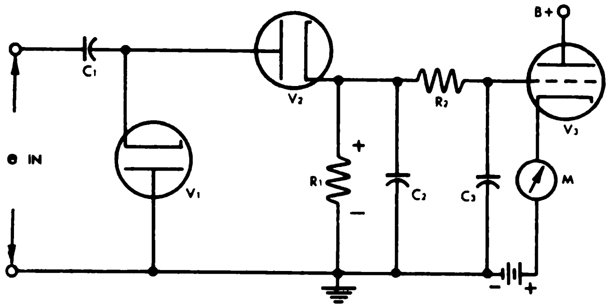


Figure 6–100. Circuit Controlled by Positive Counter

Negative Counting

If the connections to diodes V_1 and V_2 in the positive counting circuit are reversed, the circuit will respond to negative pulses and become a negative counter circuit (Figure 6–101). Diode V_2 conducts during the time the negative pulse is applied and an electron current flows through R_1 in the direction indicated by the arrow. At

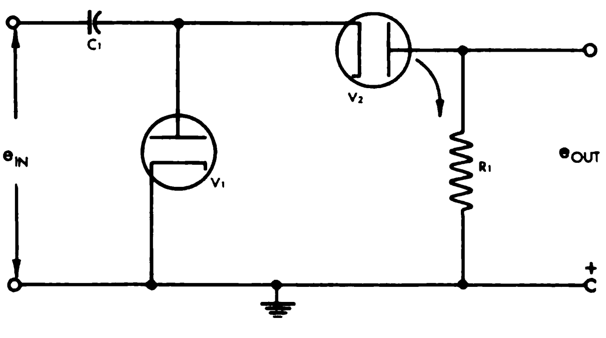


Figure 6–101. Negative Counting Circuit

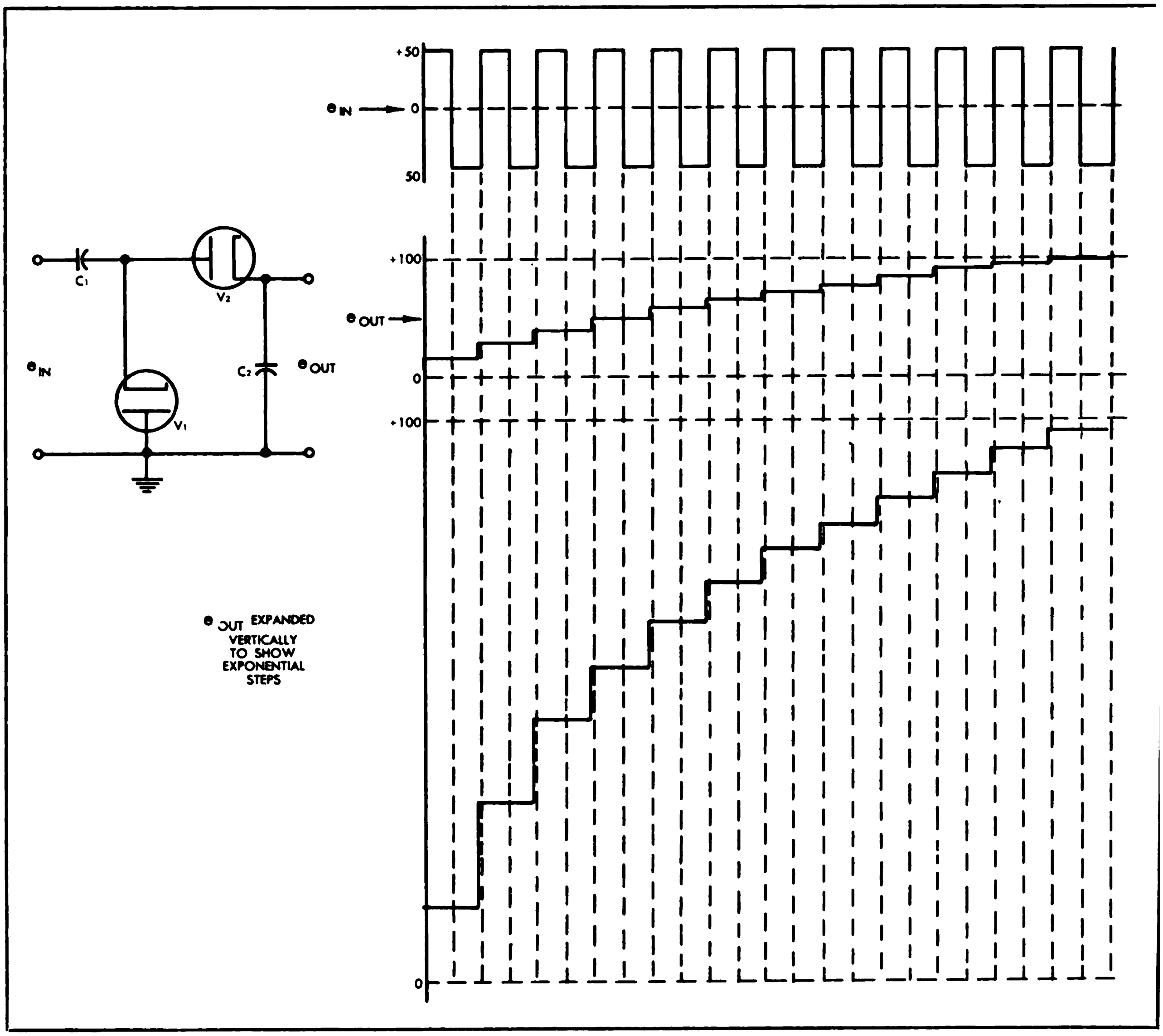


Figure 6–102. Step-by-Step Counting

the end of the negative pulse, V_1 conducts sufficiently to remove the charge that developed on C_1 during the negative pulse while V_2 was conducting.

The current through R_1 increases with an increase in the pulse frequency as before. However, if the voltage developed across R_1 is applied to the same control tube shown previously, the increase in current causes the grid of V_3 to become more negative. This decreases the plate current through V_3 and the meter. Thus, an increase in the frequency of the negative pulse causes a drop in the average plate current measured by the meter. This is opposite to the effect in the positive counter.

Step-by-Step Counting

The step-by-step counting circuit in Figure 6–102 is similar to the positive counting circuit except that a capacitor that is large compared to C_1 replaces the resistor R_1. The charge on this capacitor, C_2, increases slightly during the time of each positive pulse and produces a step voltage across the output. These steps decrease in size exponentially as the voltage across C_2 approaches the final value, the rate being dependent upon the output impedance of the driving circuit. As long as there is no path through which C_2 can discharge, the voltage across it continues to increase with each successive pulse until it is equal to the amplitude of the applied signals. At this point the cathode of V_2 is held at a positive voltage equal to that on the plate during the pulse time, and the tube fails to conduct.

The step-charging circuit is used to trigger a blocked oscillator after a certain number of steps. A simple circuit of this type is shown in Figure 6–103. The step-by-step counter determines the voltage on the blocked oscillator grid. The oscillator cathode has a high fixed bias on it, and a certain number of steps are required to raise the grid near enough the bias level to make the tube conduct. When the tube conducts, the blocked oscillator circuit goes through a normal cycle. The usual grid current during this cycle discharges capacitor C_2. At the beginning of the next input cycle, C_2 begins charging again in steps, starting from zero.

This circuit is not difficult to analyze quantitatively. The following is the calculation of the exact frequency division ratio in this circuit. In the step-charging circuit, the time constant of the charging circuit is short since only the diode plate resistance is in series with the capacitors. Therefore, the capacity is fully charged during each cycle of the input waveshape. But the capacitors are in series, so the full charging voltage will be divided between the two.

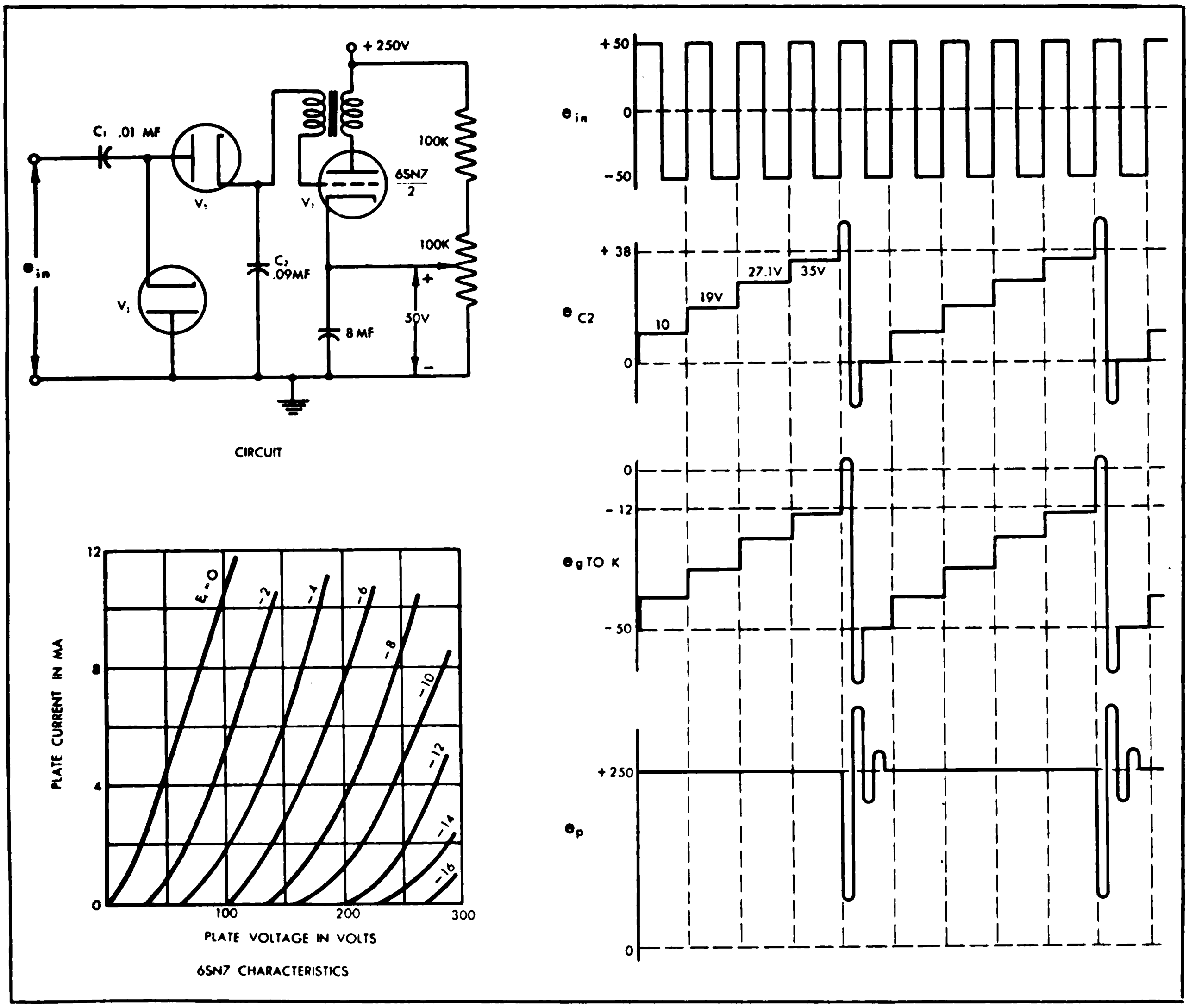


Figure 6–103. Triggering Blocked Oscillator with Step-Charging Circuit

The voltage distribution in capacitors is in inverse ratio to the capacity and is mathematically expressed,

$$\frac{C_1}{C_2} = \frac{E_2}{E_1}$$

In this equation,

C_1 = Capacity of first series capacitor

C_2 = Capacity of second series capacitor

E_2 = Charge on C_2 if applied voltage is $E_1 + E_2$

E_1 = Charge on C_1 if applied voltage is $E_1 + E_2$

The voltage to be found is E_2. Let the applied voltage be E_a. The equation for the E_2 voltage becomes:

$$E_2 = E_a \frac{C_1}{C_1 + C_2}$$

With the values indicated in Figure 6–103, E_2 is,

$$E_2 = 100 \times \frac{.01}{.01 + .09}$$

$$= 100 \times \frac{.01}{0.1}$$

$$E_2 = 10 \text{ volts}$$

The charge on capacitor C_2 with 100 volts applied is 10 volts. With voltage distribution being in inverse ratio to capacitance (the smaller the capacitance, the greater the voltage), C_1 will charge to 90 volts. The charge of 10 volts on C_2 is 10% of the applied voltage. With these capacitors the first step of each cycle will find C_2 charging to 10% of the peak-to-peak applied voltage. The plate of V_2 is held at zero by V_1 during each negative half-cycle of e_{in}. Therefore the plate voltage of V_2 will change from zero to 100v during each input cycle and C_2 will charge to 10% of the total change that is felt on the plate of V_2.

The first input pulse ends; C_1 is discharged, but C_2 holds its 10-volt charge. The second pulse occurs. The 10-volt charge on C_2 will oppose the 100 volts of the second pulse. The total applied voltage for the capacitors to charge to will be 90 volts. E_a then is 90 volts and, from the previous equation, C_2 will again be charged by 10% or 9 volts. This is in addition to the initial 10 volts, so at the end of the second pulse, the C_2 voltage is 10 + 9 or 19 volts. The third pulse will be 100 volts again, but 19 volts will oppose it. Therefore the E_a will be 100 − 19 or 81 volts, and C_2 will charge an additional 8.1 volts. The new E_2 becomes 19 + 8.1 or 27.1 volts. Successive input pulses will raise C_2 by 10% of the remaining voltage toward plus 100 until the blocked oscillator works.

In the oscillator the bias is set at plus 50 volts. The plate supply voltage is 250 volts, so the plate-to-cathode voltage is 200 volts. Using the plate-to-cathode voltage, you can determine the grid voltage for cutoff. In Figure 6–103, this value is near −12 volts, as indicated on the characteristic curves. With the cathode at plus 50 volts, the grid must be within less than 12 volts of this value, that is, above 50 − 12, or 38 volts to allow conduction. The steps continue until 38 volts is exceeded. Since the fourth step is 35 volts and the fifth step 41 volts, the 38-volt level is crossed between the fourth and fifth step.

The blocked oscillator goes through one cycle of operation at the end of which the "bucket" or storage capacitor is discharged. The capacitor stores, step by step, until the desired charge is reached. Since every fifth input pulse results in an output pulse, this circuit is a five to one frequency divider.

To cause the circuit to divide by three, or four, or some other value, the bias is set at different values. For example, a bias of 28 volts would bring the circuit above cutoff at the end of the third pulse, making the circuit a 3 to 1 count divider. The sizes of the steps can be varied by changing the ratio of the two capacitors. For example, if C_2 is changed to .04 mf, it will take 20% of the charge, and the 38-volt level will be crossed at the end of the third pulse, causing division at a 3 to 1 ratio. In practice, this frequency division can be carried to the extent that with six successive frequency dividers, a 100,000 cps sine wave can be reduced to 50 cps. The final circuit will be triggered on each two thousandth cycle of the original voltage.

CHAPTER 7

Amplifiers and Oscillators

Perhaps the two most important circuits in radio and radar equipment are amplifiers and oscillators. They are essentially alike, each relying upon the amplifying property of a transistor or a vacuum tube with a grid. Depending on how its circuit is connected, a grid-type tube can operate either as an amplifier or as an oscillator. Transistors can perform the same functions.

This chapter deals with the types of amplifiers and oscillators commonly installed in radio and radar equipment. It enumerates their uses, describes their operation, analyzes circuits in which they are used and, in general, gives you the basic information required for an understanding of amplifiers and oscillators. Chapter 8 explains the use of transistors in amplifier and oscillator circuits.

AMPLIFIERS

An amplifier is a device whose output is an enlarged reproduction of the essential characteristics of the input wave. It may be a single-stage amplifier; that is, one consisting of a single vacuum tube and its associated circuits, or it may be a number of stages joined together by coupling circuits.

BASIC AMPLIFIER

Basically, an amplifier consists of a vacuum tube with a grid and a number of associated circuits. The principal circuits are the grid-cathode, the grid-bias voltage supply, and the plate circuit.

In the single-stage amplifier circuit shown in Figure 7–1, the signal voltage is applied in series with the grid-cathode circuit, and the grid-bias voltage supply. The changes in the grid-to-cathode voltage due to the changes in the signal voltage cause the plate current to change. The plate current changes flow through R_L, which is in series with the plate-cathode circuit and the plate voltage supply. The resistor R_L is called the plate *load* resistor; the voltage developed across it varies with the plate current. Since the load resistor is fairly large — approximately five times the internal (plate) resistance of the tube — the voltage developed across R_L, because of the varying current flowing through it, is greater in magnitude than the signal voltage applied to the grid. Hence, by the action of the amplifier, a small grid voltage change produces a larger voltage change in the plate circuit, and thus is said to be *amplified.*

The grid of the amplifier is normally biased so that it remains negative with respect to the cathode regardless of the alternating cycle of the input voltage wave. As long as the grid is negative with respect to the cathode, it does not attract any electrons. When no electrons are attraced by the grid, no current flows in the grid circuit. Under this condition, the grid circuit consumes no power. But, if the grid becomes positive with respect to the cathode, current flows and the grid circuit does consume power. In the circuit illustrated the grid is operated at a negative potential with respect to the cathode, and there is no power lost in the grid circuit.

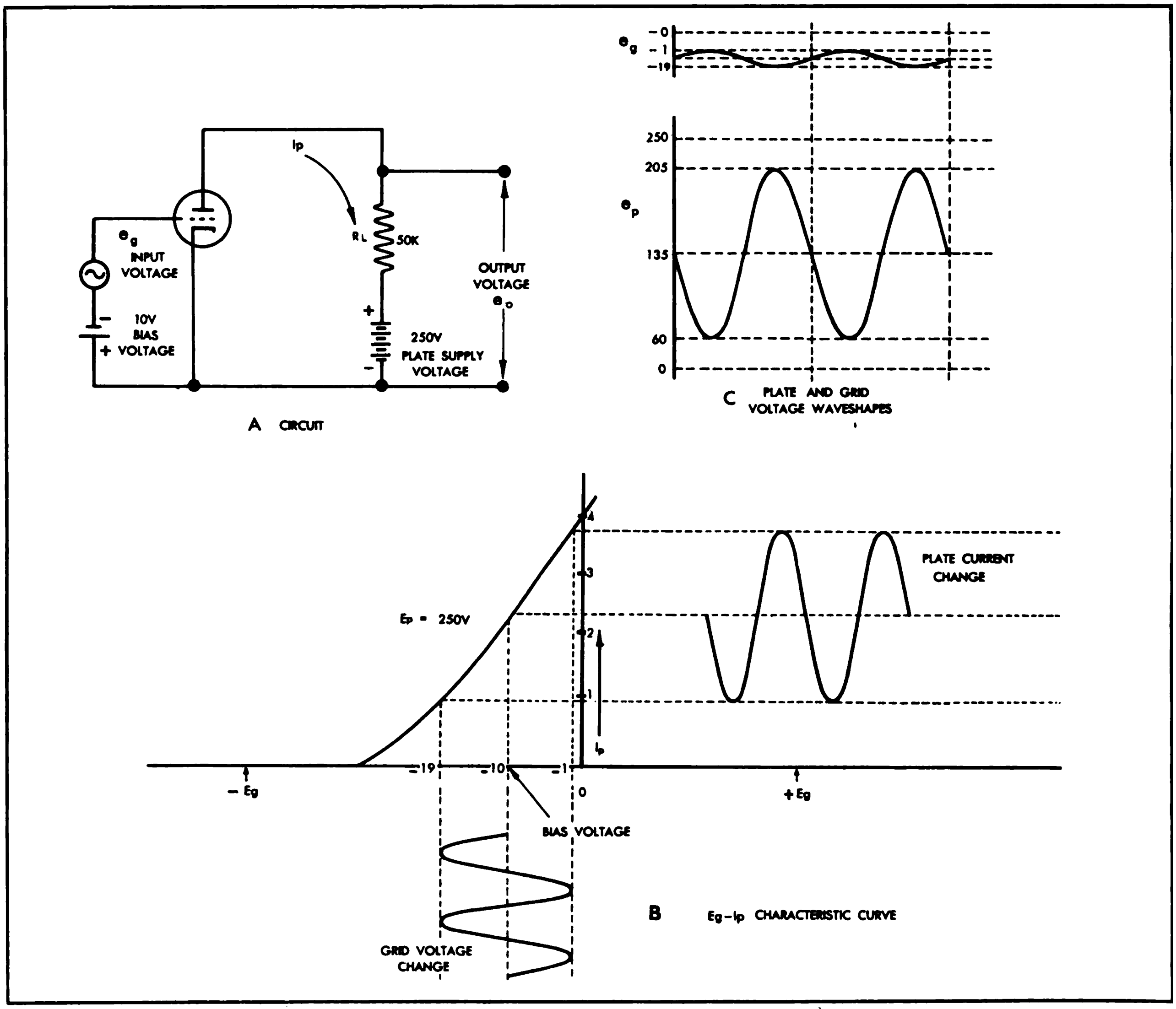


Figure 7–1. Basic Amplifier Circuit and Waveshapes

In dealing with amplifier circuits, knowledge that an amplifier produces a large voltage in its output when a small voltage is applied to its grid is not sufficient. It is also important to know just how much the voltage is increased by amplification. For determining this increase quantitatively, we use characteristic curves like those discussed in Chapter 4.

The characteristic curves which can be used for determining voltage increases in amplifiers are the grid-voltage plate-current curve and the plate-voltage plate-current curve. Although the following discussion is based on the grid-voltage plate-current curve, in actual problems it is more practical to use the plate-voltage plate-current curve, since it does not involve the mathematical calculations which follow.

Notice the grid-voltage plate-current curve at B below the amplifier circuit in Figure 7–1. This curve gives the plate current for any value of grid voltage in the useful range for tube operation. To see how this curve is used assume, for example, that the input voltage to the grid of the amplifier is a sine wave with a peak-to-peak amplitude of 18 volts (about 6.3 volts rms), and that the bias applied to the grid is negative 10 volts. This means that the average grid voltage will be minus 10 volts over a cycle of operation, or that the operating point of the stage is minus 10 volts.

To determine the plate current for any value of grid voltage, follow a vertical projection from a value of grid voltage up to the Eg-Ip curve, and then move across horizontally to the plate current change. The point of intersection is the value of plate current for a specific value of grid voltage. Notice that in the curve the grid voltage varies from —1 to —19 volts, and the resulting plate current varies from 3.8 ma to 0.9 ma. The average plate current is 2.3 ma. At no-signal current condition — that is, when no signal is applied to the grid — the value of plate current is likewise 2.3 ma.

The voltage change across the load resistor (R_L) represents the useful output of the amplifier. From the values of current indicated in the curve, you can calculate the voltage output by multiplying the resistance by the plate current at specific values of grid voltage.

Example

1. Grid at -1 volt
   $E_o = I_pR_L$
   $= 3.8\text{ ma} \times 50\text{K ohms}$
   $= .0038 \times 50000$
   $= 190$ volts
2. Grid at -10 volts
   $E_o = 2.3\text{ ma} \times 50\text{K ohms}$
   $= .0023 \times 50000$
   $= 115$ volts
3. Grid at -19 volts
   $E_o = 0.9\text{ ma} \times 50\text{K ohms}$
   $= .009 \times 50000$
   $= 45$ volts

The preceding calculations have disregarded one factor — the plate supply voltage. Any voltage drop across the plate load resistor (this voltage will be negative at the top of the resistor) will subtract from the 250 volts plate supply voltage and will change the actual output (E_o) of the amplifier.

The amplitude of the IR_L drop is the same as that of the output voltage. Nevertheless, to find the exact value of E_o, add the 250 volts to the voltage drop across the load resistor as in the following:

$-190 + 250 = 60$ volts
$-115 + 250 = 135$ volts
$-45 + 250 = 205$ volts

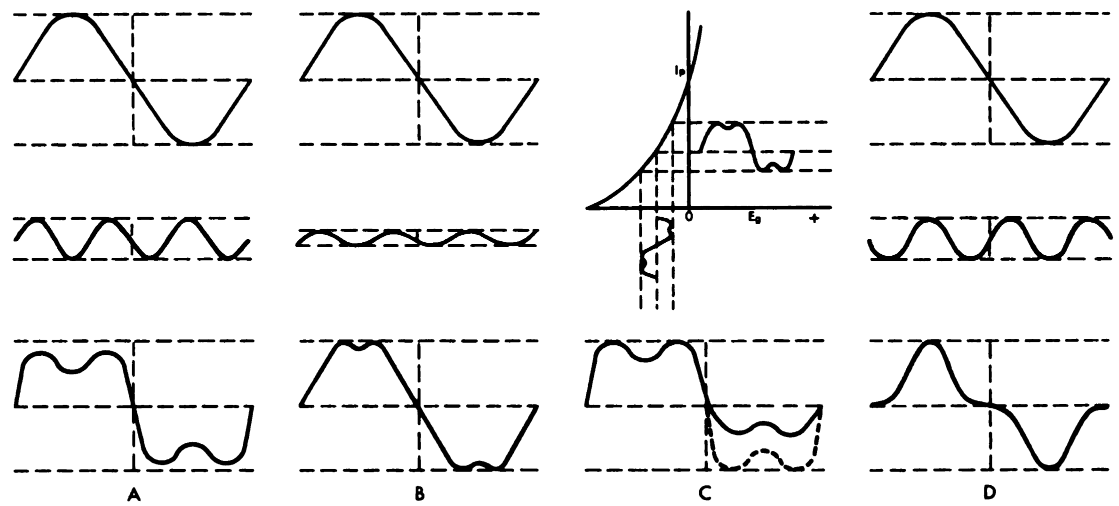
Notice in Figure 7–1C, curves showing the grid and plate voltage curves for the values indicated. These curves show correct amplitude and time relationships. The input voltage (e_g) varies from —1 to —19 — that is, the voltage variation of the input signal is 18 volts. The plate-current grid-voltage curve represents the voltage variation at the top (plate end) of resistor R_L. Its range is from 60 to 205 volts or, in other words, the total output variation is 205 minus 60, or 145 volts.

From this figure it is obvious that the output voltage is considerably greater in amplitude (variation) than the input voltage. The ratio between the two voltages is 8.1 to 1. This ratio is called the *gain* of an amplifier stage. The power output of the stage described is less than 2/10 watt. Thus, you can conclude that the circuit in question produces a fairly large voltage change, but produces very little power. Such a stage can be called a voltage amplifier, as it is concerned chiefly with voltage amplification and not power.

DISTORTION IN AMPLIFIERS

Whenever the plate-current waveshape in an amplifier is not identical to the voltage wave impressed on the grid, the output is said to be *distorted*. There are three types of distortion in amplifiers — frequency distortion, harmonic or nonlinear (also called amplitude) distortion, and phase-shift distortion. Any one of these may be present in amplifiers, either separately or in combination. You can see the effects of each type in Figure 7–2.

Notice in Figure 7–2A the drawing showing a typical pair of original signals that might appear on the grid of the amplifier. The pair includes a sine wave of one frequency and a second sine wave at a frequency three times that of the first. These two waves are combined in the grid circuit and then amplified by the tube. If both frequencies are not amplified the same amount, the waveform appearing in the plate circuit will not resemble the waveform of the signal on the grid — that is, the output wave will be distorted. This type of distortion is called frequency distortion. Notice in Figure 7–2B that the higher frequency signal in the distorted out-



A ORIGINAL SIGNAL COMPOSED OF SINE WAVE PLUS A SECOND WAVE OF HIGHER FREQUENCY.

B ORIGINAL SIGNAL AFTER SUFFERING FREQUENCY DISTORTION. THE HIGHER FREQUENCY WAS REDUCED IN AMPLITUDE, BUT THE LOW FREQUENCY WAS UNAFFECTED.

C ORIGINAL SIGNAL AFTER SUFFERING AMPLITUDE DISTORTION. NEGATIVE PORTION IS NOT AS GREAT IN AMPLITUDE AS IT SHOULD BE.

D ORIGINAL SIGNAL AFTER SUFFERING PHASE DISTORTION. HIGH FREQUENCY SIGNAL IS SHIFTED IN PHASE WITH RESPECT TO THE ORIGINAL SIGNAL.

Figure 7–2. Types of Distortion

put is only about 1/8 the amplitude of the low frequency signal, although it started out with an amplitude about 1/4 as large. The low frequency signal was virtually unaffected by the tube. Obviously, the circuit has discriminated against one frequency. Thus, frequency distortion has occurred.

Harmonic distortion affects the magnitude, or amplitude, of the output wave. This type of distortion is often caused by the plate current changing a large amount for a grid voltage change near zero grid voltages, but only a small amount for the same grid voltage change near cutoff. The effect of nonlinear distortion is that one part of the output wave does not have as great an amplitude as another part. In Figure 7–2C the negative portion of the wave is not as large as it should be. The dotted line shows how the negative part would appear if amplitude distortion were not present.

Phase-shift distortion occurs when reactive components are present in the circuit. Under these conditions, there is a phase shift of one or more of the signals comprising the input signals. As a result, the output signal is not a true replica of the original signal. Figure 7–2D shows the high-frequency component of the signal shifted 180°, producing a distorted output.

CLASSIFICATION OF AMPLIFIERS

Amplifiers are named or classified in several ways. According to the results they achieve, there are two basic types — voltage amplifiers and power amplifiers. According to the conditions under which they operate, amplifiers, especially power amplifiers, are classified as Class A, Class B, and Class C amplifiers. The class at which an amplifier operates depends on the amount of bias voltage applied to the grid of the tube and the portion of AC signal-voltage cycle during which plate current flows. Depending upon the frequency range over which they operate, amplifiers may be further classified as direct current amplifiers, audio frequency amplifiers, intermediate frequency amplifiers, radio frequency amplifiers, and video (wide band) amplifiers.

Voltage Amplifiers

Voltage amplifiers are primarily intended for amplifying voltage. They are designed to develop the greatest amplified voltage possible across the load in the plate circuit of the amplifier. To accomplish this objective, it is necessary that the load resistance be as high as possible so that it will offer a large opposition to the plate-current change. This results in a large voltage being developed in the output circuit.

Power Amplifiers

Power amplifiers are designed to deliver large amounts of power to the load in the plate circuit without regard to voltage. Since power equals voltage times current, a power amplifier must have a large output voltage across the load in addition to a relatively large current flow. In power amplifiers the impedance (resistance) of the load is smaller than that in voltage amplifiers.

Class A Amplifiers

An amplifier is considered as a Class A amplifier when the grid bias and alternating grid voltages have such value that plate current flows continuously throughout the cycle of the applied voltage and never reaches zero. Class A amplifiers are biased to about half of cutoff value. The basic amplifier shown in Figure 7–1 is a Class A amplifier.

Class A amplifiers are characterized by low efficiency. By efficiency is meant the ratio of the power output to the power input. For practical purposes the efficiency of a Class A amplifier ranges between 20% and 25%. Theoretically, it has a maximum efficiency of about 50%. Class A amplifiers are used as audio and radio frequency amplifiers in radio, radar, and sound systems.

Class B Amplifiers

Class B amplifiers are biased to approximately cutoff. Plate current, therefore, flows only during the positive half-cycle of the applied signal voltage. The efficiency is higher and the current consumption is less in a Class B amplifier than in one operating Class A. Power loss in Class B amplifiers is low for two reasons. First, plate current does not flow when there is no signal applied to the grid, and thus very little power is wasted during the non-operating periods. Second, plate current flows only during the positive half of the input cycle. This means that the average current flow will be only 32% of the peak current in the stage.

Figure 7–3 shows the relation between grid voltage and plate current in a tube-operated Class B. Note that plate current flows only during the positive half of the signal voltage. Grid current flows only during the time that the grid is driven positive.

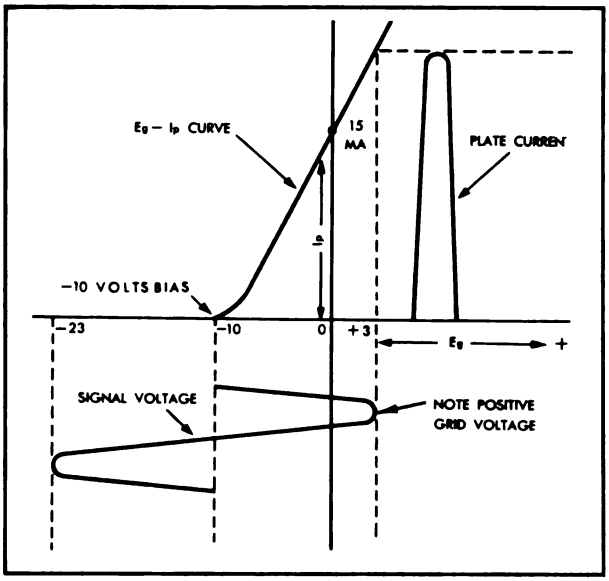


Figure 7–3. Class B Amplification

Class B amplifiers are used mostly as power amplifiers. As power amplifiers their power output is proportional to the square of the grid excitation voltage. The best bias for Class B operation is that which corresponds roughly to the cutoff bias that would be obtained if the main part of the characteristic curves shown in the chart in Figure 7–4 were projected as straight lines. Notice the dotted straight line extended from the straight line part of the characteristic curve labeled $E_b = 300$. The point at which this

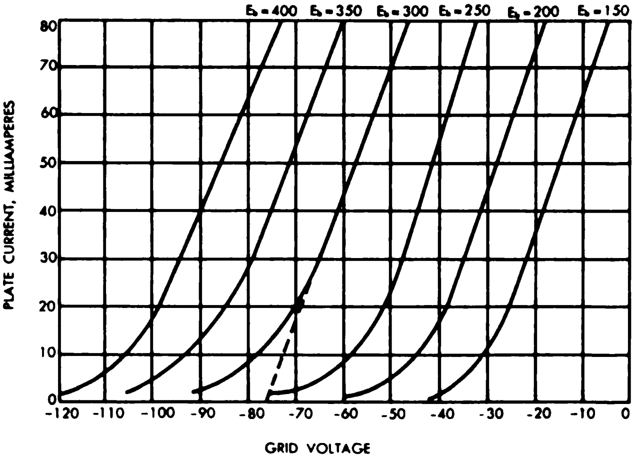


Figure 7–4. Determining Correct Bias Voltage for Class B Amplifier

line strikes the grid-voltage line, approximately −75 volts, gives the cutoff bias for 300 volts plate operation. Curves of this type provide convenient means of determining grid bias for Class B operation for different plate voltage conditions.

Distortion occurs in Class B amplifiers under much the same general conditions as in Class A amplifiers. Frequency distortion and phase-shift distortion are essentially alike in both. Harmonic, or nonlinear, distortion in the output of a Class B amplifier operating with the proper load resistance depends upon the departure of the characteristic curve from straight lines, and upon the operating point. Refer to the Class B amplification curve in Figure 7–3. Obviously, the half-cycle output is far from the distortionless reproduction of the full-cycle input. One-half of the cycle is missing. This missing half-cycle is supplied by one of two methods — by an additional Class B amplifier, or by the flywheel action of a resonant circuit. Distortion of the positive peaks of grid voltage may occur due to flow of grid current if the grid becomes positive.

The use of two tubes to supply both halves of the input cycle in the output constitutes a push-pull Class B amplifier (Figure 7–5). One tube operates during the first half-cycle of the AC signal voltage and the other tube during the other half-cycle. Since plate current flows during one half-cycle in one tube and during the next half-cycle in the other, the plate current waveforms can be combined in the load circuit.

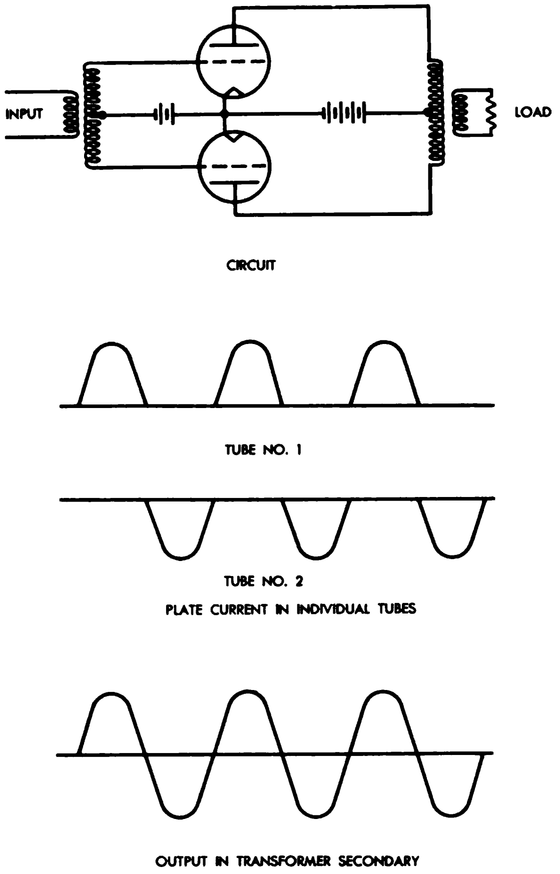

INPUT
LOAD
CIRCUIT
TUBE NO. 1
TUBE NO. 2
PLATE CURRENT IN INDIVIDUAL TUBES
OUTPUT IN TRANSFORMER SECONDARY

Figure 7–5. Using Two Class B Tubes to Produce Full Sine Waves

The load circuit of the push-pull amplifier is the center-tapped primary of the output transformer. During one cycle, one tube generates a voltage across the transformer winding. During the next half-cycle, the other tube generates a voltage of opposite polarity across the winding. Since the plate currents of the two tubes flow in opposite directions through their respective halves of the transformer primary winding, the voltages across the primary windings of each tube combine in the secondary to produce a reasonably undistorted output of the input AC signal voltage.

A single-tube, Class B amplifier (Figure 7–6), can be used successfully in r-f amplifier stages having a parallel-tuned circuit as the plate load. The parallel-tuned circuit is sometimes called a *tank circuit*, because it has the ability to store power. When it is used as the plate load of a single-ended, Class B amplifier stage, the capacitor in the parallel-tuned circuit is charged by the output voltage produced by the flow of plate current through the load on the positive half-cycles. Although no current flows through the tube on the negative half-cycles of the applied signal voltage, the capacitor discharges into the inductor during this period and thus supplies the missing half-cycle in the output voltage. This so-called flywheel effect of the tank circuit occurs only when the resonant frequency of the parallel-tuned circuit is equal to the frequency of the applied signal voltage.

The maximum amount of useful power available from a single power tube is limited by the distortion due to the introduction of undesired harmonic frequencies in the amplifier circuit. A harmonic frequency is a multiple of any given frequency. For example, the second harmonic of 1000 cps is 2000 cps, the third harmonic is 3000 cps, and so on. Second and higher harmonics are generated by a vacuum tube when its

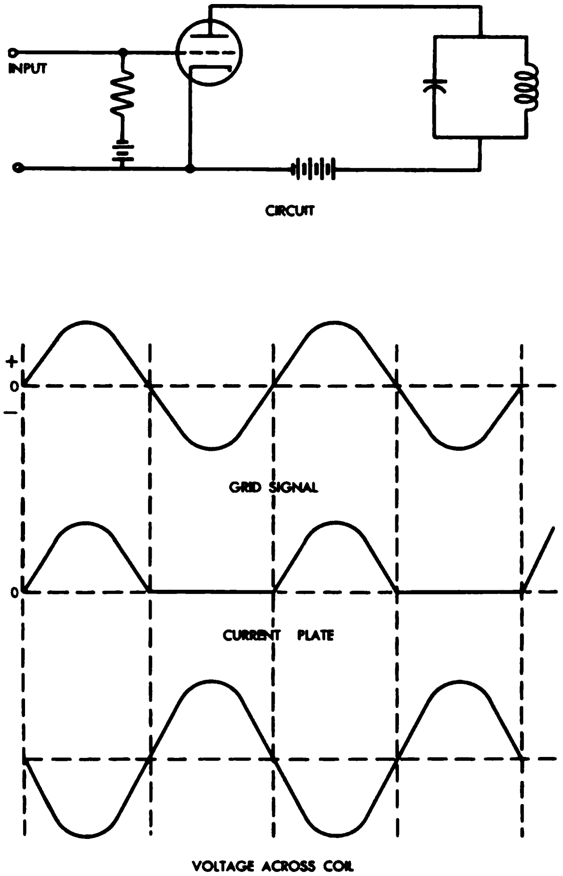
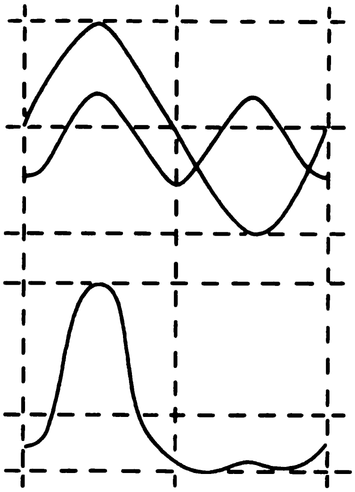
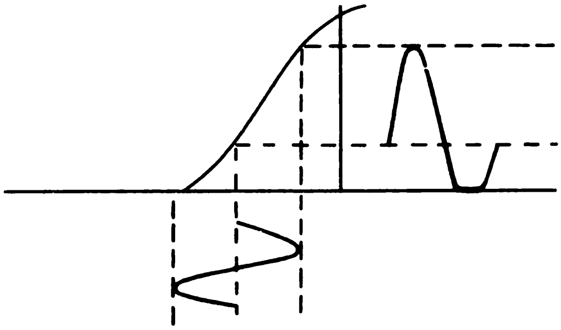


Figure 7–6. Using Resonant Circuit to Produce Full Sine Wave in Class B Amplifier



A ADDITION OF SECOND HARMONIC CAUSES DISTORTION



B $E_g - I_p$ CURVE CAUSES AMPLITUDE DISTORTION

Figure 7–7. Introduction of Harmonics

grid-voltage plate-current changes are non-linear.

Figure 7–7 will help you in understanding the action of harmonics in vacuum tubes or semiconductors. Consider the top diagram in A. It shows two frequencies, a sine wave and its second harmonic. Just below the two frequencies is the waveshape produced by adding the two graphically. Notice that the sum is a distorted waveshape of the sine wave in which there is a large positive half-cycle and a small negative cycle. Diagram B shows similar results in which a generated harmonic is added to the signal frequency (fundamental) which causes the amplitude of the output to be distorted. This action results from operating the tube on the curved portion of the E_g-I_p curve. Here, too, the output wave has a large positive half-cycle and a small negative half-cycle, a condition evidencing distortion. The output waveshape contains both the original fundamental signal waveshape and the harmonic which was generated in the tube and added to the fundamental by tube action.

Elimination of the *even* numbered harmonics, second, fourth, and so on, is possible when a push-pull circuit arrangement is employed. Only the *odd* harmonics (particularly the third) will be left to limit the power output. However, the effect of the *odd* harmonics is minimized by connecting to each tube a load resistance that is more nearly equal to the dynamic plate resistance of the tube. As a result, the amount of undistorted power will approach the maximum amount of power obtainable, that is, the power if there were no distortion. Therefore, two tubes connected in push-pull will give considerably more than twice the undistorted power that a single tube can deliver.

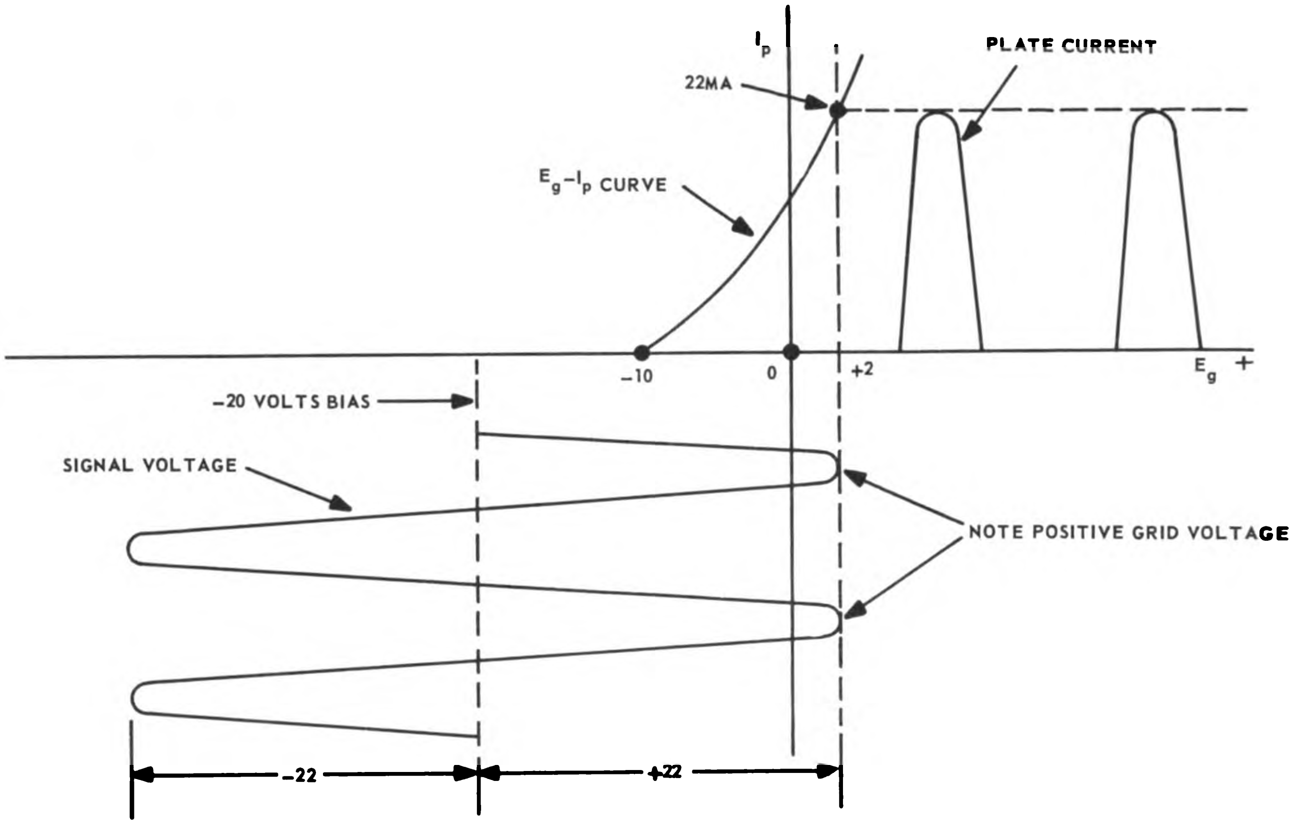


Figure 7–8. Class C Amplification

When two tubes are connected in push-pull, the plate-current pulse of each individual tube flows through only one-half of the transformer primary. This makes the combined output of the two tubes equivalent to an AC current having a crest value $I_{max}/2$ flowing through the entire transformer primary. If R_L is the equivalent load resistance between the primary terminals of the output transformer, the AC voltage drop between the plate and cathode of a single tube is one-half the voltage drop of the current $I_{max}/2$ across the resistance R_L or $R_L I_{max}/4$. The minimum instantaneous plate voltage can be obtained as follows:

$$E_{min} = E_b - R_L I_{max}/4$$

$$4E_{min} = 4E_b - R_L I_{max}$$

$$R_L I_{max} = 4E_b - 4E_{min}$$

The correct load resistance from plate to plate then is equal to:

$$R_L = \frac{4(E_b - E_{min})}{I_{max}}$$

The power output is equal to the square of the load current multiplied by the load resistance. Mathematically, this relationship is expressed as,

$$\text{Power output} = \frac{R_L I_{max}^2}{8}$$

The maximum efficiency possible with Class B power amplifiers is theoretically 78.5%. In most practical applications, however, the efficiency attained is about 60% to 65%. Class B amplifiers are principally used in transmitters as voltage amplifiers for amplitude-modulated waves.

Class C Amplifiers

A Class C amplifier is one in which the grid bias is appreciably greater than cutoff value. When no alternating voltage is applied to the grid, the plate current is zero. When an alternating voltage is applied to the grid, plate current flows for appreciably less than ½-cycle.

Except for grid bias, Class C amplifiers operate in the same way as Class B amplifiers. Since current flows for a small part of a cycle, the distortion in Class C amplifiers is very great (Figure 7–8). A Class C amplifier is also characterized by the fact that it develops its output at a relatively low ratio of power amplification. Still another characteristic is that its grid usually swings sufficiently positive to allow saturation point current to flow through the tube. As a result the plate output waves are not free from harmonics and suitable means must be provided to remove them from the output.

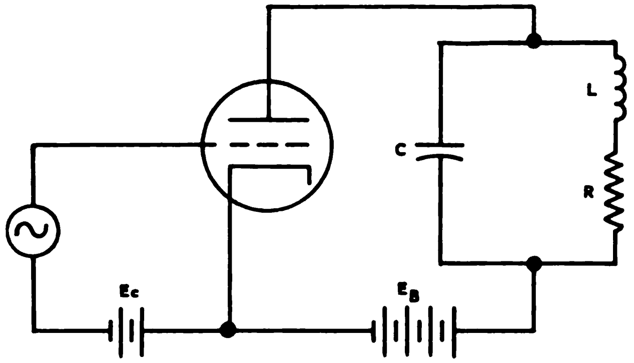


Figure 7–9. Preventing Distortion by Using Resonant Circuit as Plate Load

One means of doing this is demonstrated in Figure 7–9. The tuned tank circuit shown offers an impedance to the operating frequency that is quite high and that has a power factor nearly equal to unity, but to the harmonic frequencies it presents a low impedance, causing them to be attenuated or causing low amplification by the tube. Due to the high impedance offered to the signal frequency, it is amplified to a much greater extent than are the harmonic frequencies.

The Class C amplifier has high efficiency because when plate current is permitted to flow, the instantaneous potential of the plate is low compared to the plate supply voltage. In this way, energy is supplied to the plate circuit only when most of the plate supply voltage is used up as a voltage drop across the tuned circuit. Therefore, most of the energy is delivered to the tuned circuit instead of being wasted at the plate. Principally because of this fact, the efficiency of the Class C amplifier is in the neighborhood of 60% to 80%.

Class C amplifiers are used as r-f amplifiers in transmitters. They are very useful in high-frequency equipment where it is necessary to deliver appreciable power.

Direct Current Amplifiers

Direct current amplifiers are used to amplify very-low-frequency or DC voltages. A simple DC amplifier consists of a single tube with a grid resistor across the input terminals, and with the load in the plate circuit. The load may be some sort of mechanical device such as a relay or a meter, or the output voltage may be used to control the gain of an amplifier.

The DC voltage that is to be amplified must be applied directly to the grid of the amplifier tube. For this reason, only direct coupling can be used in the amplifier input circuit. Coupling is shown in Figure 7–10, where a comparison is made between a capacitor-input circuit and a direct-input circuit. In both cases, direct cur-

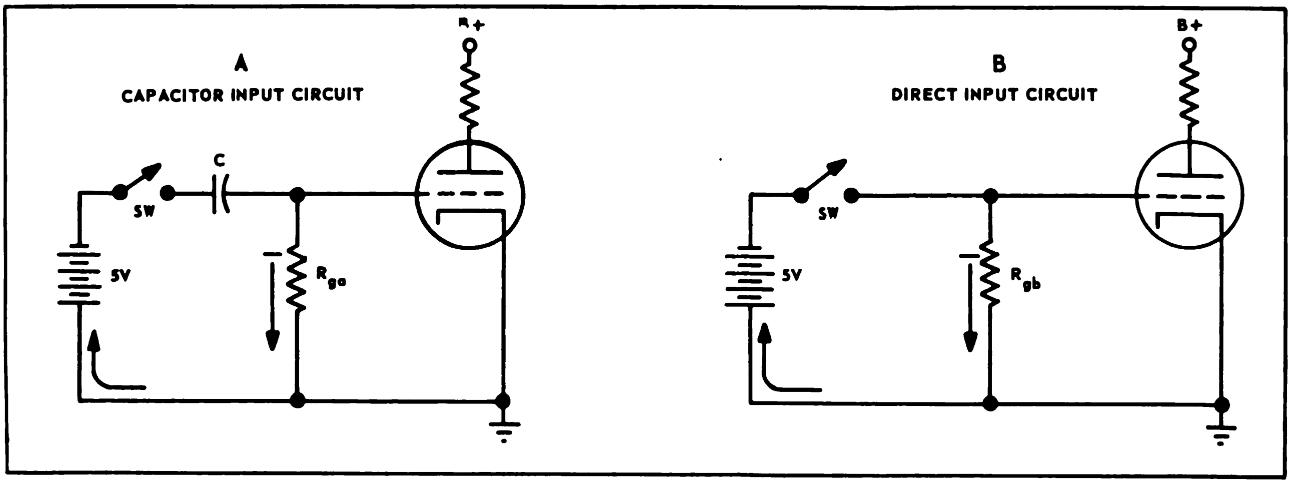


Figure 7–10. Direct and RC Coupling for DC Amplifier

rent voltage is applied to a direct current amplifier.

When the switch in the capacitor-input circuit (Figure 7–10A) is closed, the direct current battery voltage charges capacitor C. The electrons moving in the direction indicated cause a voltage equal to the battery voltage to appear momentarily across resistor R. This voltage, in turn, appears on the grid of the tube. However, as the capacitor continues to charge, and up to the point where its charge is equal to the battery voltage, the voltage across resistor R decreases until it reaches ground potential. During this time, the grid voltage changes from ground to negative 5 volts and then back to ground.

These variations appear in the output of the amplifier as a changing voltage. However, they are only momentary because there are no further changes in the circuit values. If the switch remains closed, the applied DC voltage is constant, and just as soon as the capacitor is fully charged, the output of the amplifier will return to its original level. Therefore, this circuit is incapable of amplifying a direct current voltage.

In the direct input circuit (Figure 7–10B), however, when you close the switch, the battery voltage is applied directly to the grid. Unlike the capacitor-input circuit, therefore, the voltage on the grid remains constant as long as the switch remains closed. Before the switch is closed, a fixed value of current flows in the plate circuit. This results in a fixed voltage drop across the load resistor. When the DC voltage is applied to the input terminals, this voltage is not blocked by a capacitor but is impressed directly upon the grid of the tube. It makes the grid more negative than before and permits less plate current to flow. Accordingly, there is a smaller voltage drop across the load resistor. Since this voltage change is greater than the voltage applied to the grid, the input voltage has been amplified. Therefore, this circuit can serve as a DC amplifier.

One of the most important uses of a DC amplifier is in a DC vacuum tube voltmeter. Notice the circuit of a vacuum tube voltmeter shown in Figure 7–11. The voltage to be measured is applied to the voltage divider made up of R_1, R_2, and R_3. The ratio of voltage division can be varied by a voltage range switch in such a way that several ranges of voltage can be measured. Resistor R_4 is used to prevent damage to the meter if too high a voltage is applied. In the plate circuit, the additional battery and the variable resistor are used to balance the normal plate current of the circuit. The variable resistor can be adjusted so that meter M reads zero when no signal is applied. Whenever a DC voltage is applied to the input, the tube amplifies it and causes a current to flow through the meter. Since the meter reading is proportional to the voltage applied, you can read the amount of voltage on the calibrated scale of the meter.

In addition to their use in vacuum-tube voltmeters, DC amplifiers are also used for amplifying the output of the discriminator in automatic frequency control circuits. One of the problems in microwave receivers is frequency control — that is, keeping the intermediate frequency constant in spite of minor variations in the transmitted frequency or the local oscillator frequency. To accomplish this, special circuits are used to adjust the frequency of the local oscillator in such a manner that the correct i-f will be produced. These circuits, called automatic frequency control circuits, are discussed later, but here the purpose is to show the function of the DC amplifier in the circuit.

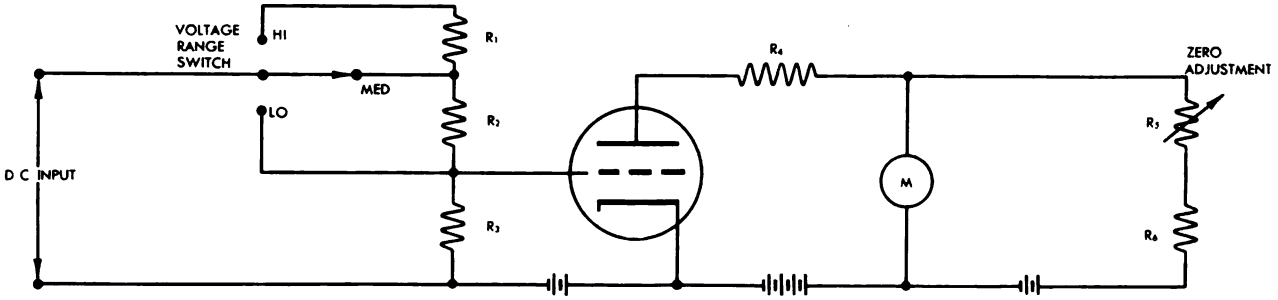


Figure 7–11. Vacuum Tube Voltmeter

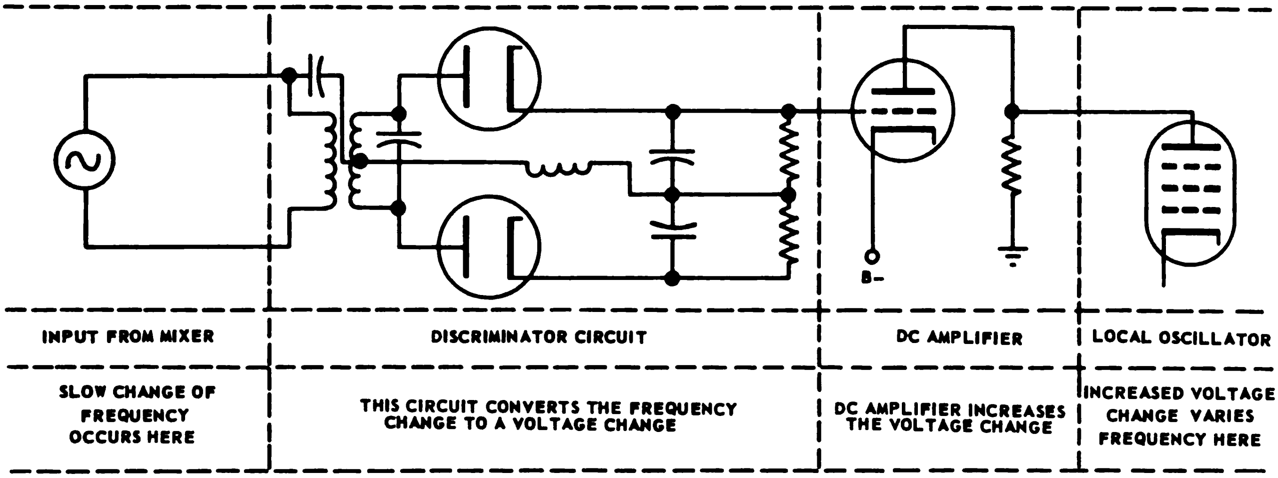

B-
INPUT FROM MIXER
DISCRIMINATOR CIRCUIT
DC AMPLIFIER
LOCAL OSCILLATOR
SLOW CHANGE OF FREQUENCY OCCURS HERE
THIS CIRCUIT CONVERTS THE FREQUENCY CHANGE TO A VOLTAGE CHANGE
DC AMPLIFIER INCREASES THE VOLTAGE CHANGE
INCREASED VOLTAGE CHANGE VARIES FREQUENCY HERE

Figure 7–12. Application of DC Amplifier

To understand the part the DC amplifier plays, first consider the frequency discriminator shown in Figure 7–12. Its purpose is to convert any frequency deviation from the intermediate frequency of the receiver to a DC voltage. This DC voltage has a magnitude proportional to the amount of deviation from the intermediate frequency and a polarity that depends on whether the deviation is above or below the intermediate frequency.

The DC amplifier tube is connected between ground and a negative potential, making the plate positive with respect to the cathode and grid. The two resistors in the cathode circuit of the balanced diodes act as a load for the discriminator output. This output controls the DC amplifier, causing it to conduct more or less, depending on the effect of the intermediate frequency rms voltages on the plates of the balanced diodes. The changes in the output of the DC amplifier, in turn, cause variations in the repeller plate voltage of the local oscillator and change the local oscillator frequency in the direction required to keep the intermediate frequency constant.

Audio Amplifiers

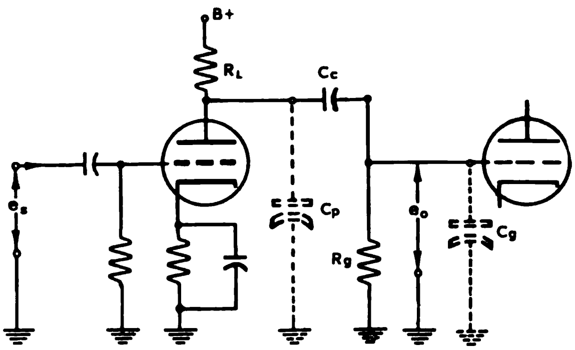
Amplifiers designed for frequencies within the audio spectrum (approximately 15 to 15,000 cycles) are called audio frequency amplifiers.

To obtain sufficient gain it is usually necessary to use one or more stages of audio amplification —that is, to use the output of one tube to control the grid circuit of a second, then to use the amplified output of the second tube to control the grid of the following tube, and so on. When two or more amplifiers are connected in this manner, they are said to be connected in *cascade.*

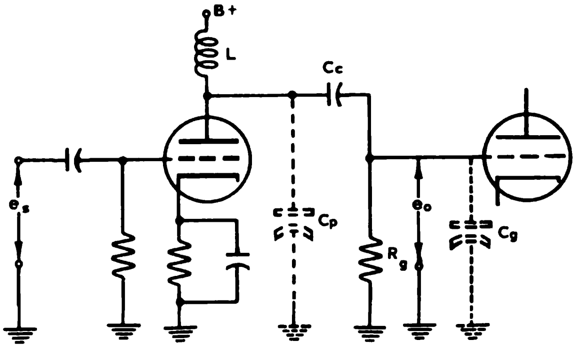
When one stage of audio amplification (a tube and its circuit connections) is coupled to the next, the plate of the amplifier tube is not normally connected directly to the grid of the following stage as with DC amplifiers, but is usually coupled through a special circuit. The coupling circuit transfers the varying voltage between the stages, and at the same time supplies the DC potentials and currents necessary for the operation of the tubes. The main types of audio stage coupling are resistance-capacitance, impedance, and transformer coupling.

Figure 7–13 shows simple circuits of triode audio amplifiers, each having one of these three types of couplings. Since the method most widely used for coupling audio stages is the resistance-capacitance method, it is the only one discussed in this chapter. For information on the other types, refer to Chapter 2.

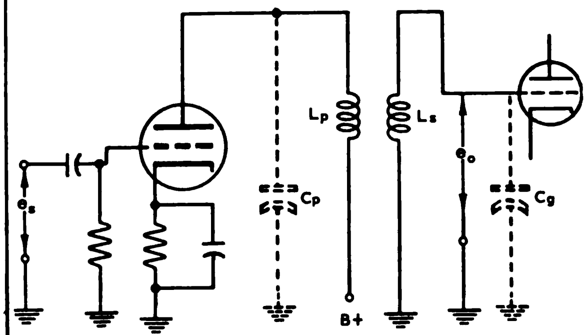
Figure 7–13A is the circuit of a resistance-capacitance, or RC-coupled, amplifier. Notice that cathode bias is used. This is the most commonly used type of bias that you will encounter in RC-coupled circuits. The capacitor C_c, which is called the *coupling capacitor,* is to provide an AC current path to the grid of the next stage. The resistor R_L is called the *coupling resistor.* Its resistance is high so that as much voltage as possible can be transferred to the grid of the following tube.



A RESISTANCE - CAPACITANCE-COUPLED AMPLIFIER



B IMPEDANCE-COUPLED AMPLIFIER



C TRANSFORMER-COUPLED AMPLIFIER

Figure 7–13. Types of Coupling

The coupling of the signal between the two stages in Figure 7–13A takes place in this way. When a signal is applied to the grid of the first stage, voltage variations are produced in the plate circuit of this tube. These variations are impressed upon the grid of the second tube through the coupling capacitor and grid resistor R_g of the second tube.

The coupling capacitor serves two functions. It provides a low impedance path for the AC to the grid of the following tube. In addition, it blocks the DC plate voltage of the first tube from the grid of the second tube.

EQUIVALENT CIRCUITS. The variations produced in the plate current of a vacuum tube when a signal voltage is applied to the control grid are exactly the same as would be produced by a generator developing a voltage of $-\mu e_s$ in a circuit consisting of the tube plate resistance in series with the load impedance. The effect on the plate current of applying a signal voltage e_s to the grid is exactly as if the plate-cathode circuit were a generator developing a voltage and having an internal resistance equal to the plate resistance of the tube. Thus, the RC-coupled amplifier shown in Figure 7–13A can be represented by the equivalent circuit shown in Figure 7–14. The minus sign on the equivalent generator voltage is used only to indicate that the voltage is of opposite polarity to the signal voltage. This indicates the fact that there is a 180° phase shift between the grid voltages and the plate voltages in a vacuum tube.

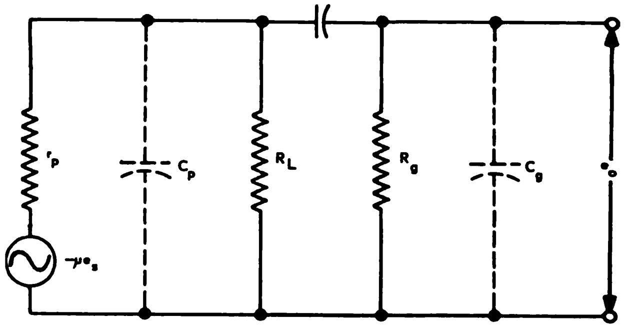


Figure 7–14. Equivalent Circuit for RC-Coupled Amplifier

Note that the cathode bypass capacitor and the biasing resistor are omitted in the equivalent circuit. This omission is permissible; first, because the capacitor is of such size that it places the cathode at ground potential for AC, and, second, because the biasing resistor is considered a part of the plate load. All the circuit constants shown in the equivalent circuit are those which affect any ordinary triode amplifier. In fact, depending upon the frequency, some of the constants shown may be omitted. Thus, for example, in the midfrequency band (200 to

10,000 cycles) the electrode capacities C_p and C_g may be omitted in equivalent circuits because their impedances are so large that they act as open circuits. Similarly, the coupling capacitor may be left out because of the negligible voltage drop across it.

CALCULATING GAIN. Two important facts to remember about audio amplifiers are: that the output of an audio amplifier is always less than the μe_s voltage because of the internal resistance (r_p) of the tube, and second, that the gain is the ratio of the output voltage to the input voltage, which is expressed mathematically as,

$$A = \frac{e_o \text{ (output voltage)}}{e_s \text{ (input voltage)}}$$

The following paragraphs discuss and derive gain formulas for triode audio circuits operated at mid-frequencies, circuits in which the grid resistance R_g is much greater than the load resistance R_L ($R_g \gg R_L$), circuits operated at high frequencies, and circuits operated at low frequencies. In each case the equivalent circuit is represented by a constant-voltage generator circuit.

Gain at Mid-Frequencies (Figure 7–15)

$$\text{Total resistance, } R_t = \frac{R_L R_g}{R_L + R_g} + r_p$$

Plate current,

$$i_p = -\frac{\mu e_s}{R_t} = -\frac{\mu e_s}{\dfrac{R_L R_g}{R_L + R_g} + r_p}$$

$$= -\frac{\mu e_s (R_L + R_g)}{R_L R_g + r_p (R_L + R_g)}$$

The output voltage equals the product of i_p and the effective resistance of R_L and R_g in parallel, therefore,

$$e_o = -i_p \frac{R_L R_g}{R_L + R_g} = -\frac{\mu e_s (R_L + R_g)}{R_L R_g + r_p (R_L + R_g)} \times \frac{R_L R_g}{R_L + R_g}$$

$$= -\frac{\mu e_s R_L R_g}{R_L R_g + r_p (R_L + R_g)}$$

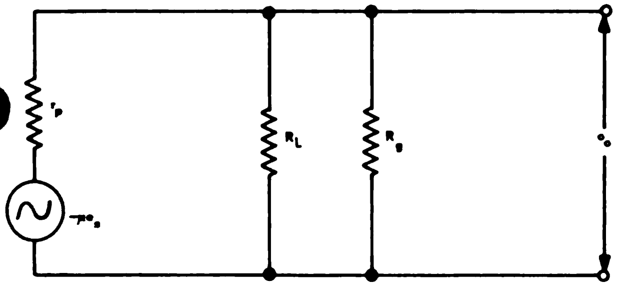


Figure 7–15. Equivalent Circuit at Mid-Frequencies

Gain (A) equals $\frac{e_o}{e_s}$, thus

$$A = \frac{e_o}{e_s} = \frac{\mu e_s R_L R_g}{R_L R_g + r_p (R_L + R_g)} \div e_s$$

$$A = -\frac{\mu R_L R_g}{R_L R_g + r_p (R_L + R_g)}$$

(The minus sign indicates the 180° phase shift between the input and output voltages.)

GAIN WHEN R_g IS MUCH GREATER THAN R_L. The following calculations assume that R_g is at least ten times as large as R_L and that the frequency is in the mid-frequency band. The calculations are considerably simpler than those for the preceding formula since the parallel combination of R_L and R_g is approximated as equal to R_L. Note that the equivalent circuit in Figure 7–16 is a modification of the circuit for mid-frequencies.

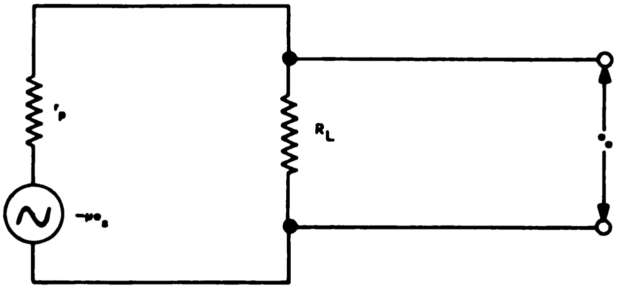


Figure 7–16. Equivalent Circuit when $R_g \gg R_L$

$$\text{Total resistance, } R_t = r_p + R_L$$

$$i_p = -\frac{\mu e_s}{r_p + R_L}$$

$$e_o = i_p R_L = -\frac{\mu e_s R_L}{r_p + R_L}$$

$$\text{Gain } A = \frac{e_o}{e_s} = -\frac{\mu e_s R_L}{r_p + R_L} \div e_s$$

$$\text{Therefore } A = -\frac{\mu R_L}{r_p + R_L}$$

In connection with the gain equation just derived, it is important to remember that since any vacuum tube has internal resistance (plate resistance), only a portion of the equivalent generator voltage is available across the load resistor. It is therefore very often useful to consider that the plate resistance and the load resistor make up a voltage divider across which the voltage generated with the tube is applied. The ratio $\frac{R_L}{r_p + R_L}$ expresses the proportion of the voltage across the load resistor. You can see from this equation that for maximum output, R_L must be many times larger than r_p.

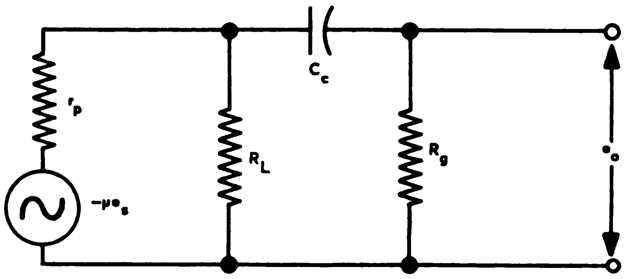


Figure 7–17. Equivalent Circuit at Low Frequencies

GAIN AT LOW FREQUENCIES (15 TO 200 CPS). Notice the coupling capacitor in the equivalent circuit in Figure 7–17. At low frequencies, the reactance X_c of this capacitor becomes appreciable and must be taken into consideration in the gain formula. At low frequencies, therefore, the gain formula reads,

$$A = -\frac{\mu R_L R_g}{R_L R_g + r_p R_L + r_p R_g - jX_{c_c}(r_p + R_L)}$$

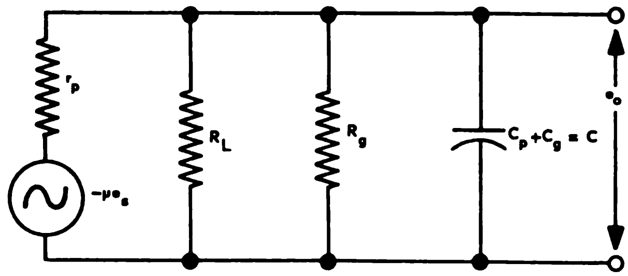


Figure 7–18. Equivalent Circuit at High Frequencies

GAIN AT HIGH FREQUENCIES (10,000 CPS AND ABOVE). At these frequencies, the plate-to-cathode capacity, the input capacity (grid-to-cathode), and the stray wiring capacity effectively shunts the grid resistance, R_g. This means that the reactances of these capacitances are so low that the voltage drop across them is reduced appreciably. See Figure 7–18 for the equivalent circuit. The effect of this reduction is reduced output voltage from the amplifier. The equation for gain at high frequencies is,

$$A = -\frac{\mu R_L R_g}{R_L R_g + r_p(R_L + R_g) + j\omega C r_p R_L R_g}$$

FREQUENCY RESPONSE. The most important property of a resistance-coupled amplifier is the manner in which its amplification varies with frequency. The frequency response curve in Figure 7–19 represents the response of a characteristic amplifier over a wide range of frequencies. The low response at low frequencies is due to the large reactance that the coupling capacitor offers at these frequencies. The low response at high frequencies is due to the tube and stray wire capacities in the amplifier circuit.

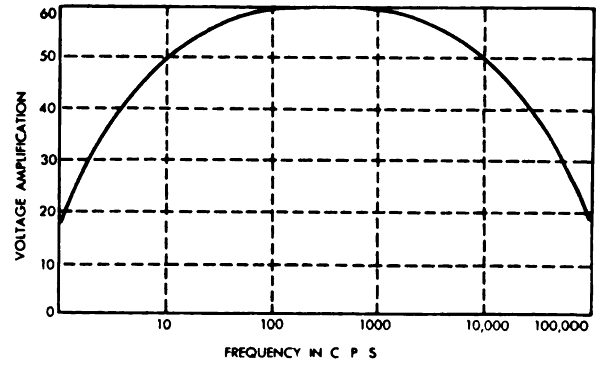
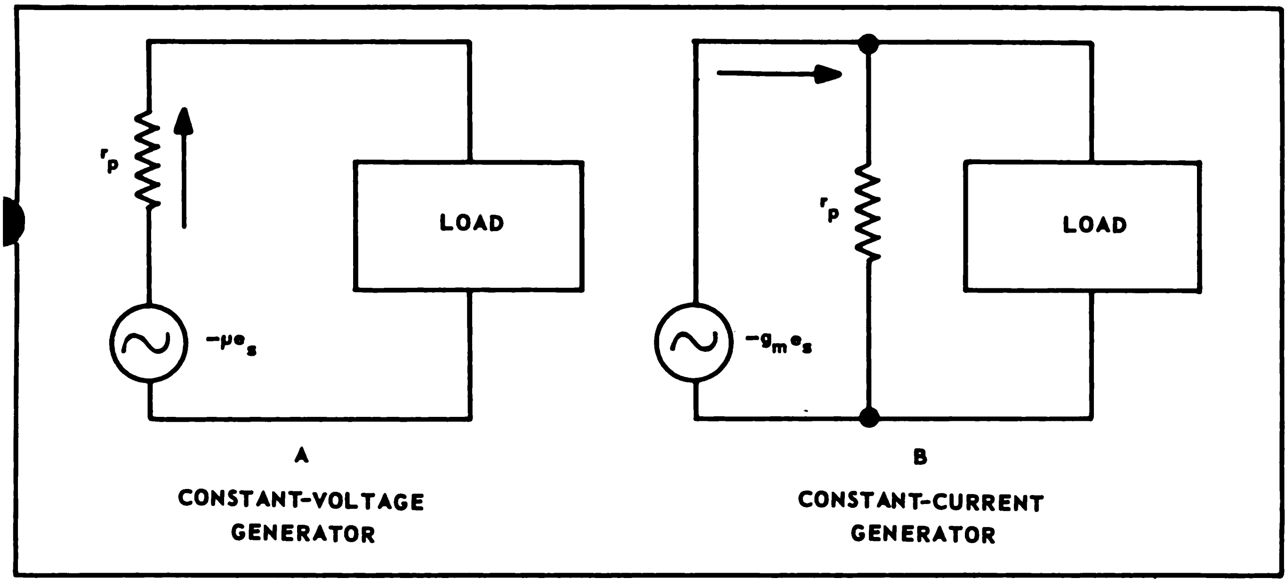


Figure 7–19. Response of Audio Amplifier

PENTODE EQUIVALENT CIRCUITS. The discussion thus far has dealt with triodes in which the equivalent circuit was represented by a constant voltage generator circuit. With some modifications of the equivalent circuit, the principles discussed are applicable also to pentode amplifiers. The major change involves using a constant-current generator rather than the constant-voltage circuit. In pentodes it is assumed that the plate variations are identical to those produced by a generator developing a current $-g_m e_g$ acting in a circuit consisting of the plate resistance in parallel with the load resistance. Notice the illustration in Figure 7–20, comparing the constant-voltage circuit and the constant-current circuit. The calculations previously discussed are applicable, with some modifications, to pentodes when the constant current type equivalent circuit is used.

Video Amplifier

A video amplifier is designed for uniformly amplifying a wide range of frequencies, including the audio range, up to several megacycles per



A
CONSTANT-VOLTAGE GENERATOR

B
CONSTANT-CURRENT GENERATOR

Figure 7–20. Comparison of Equivalent Circuits

second. Typical video amplifiers are RC-coupled circuits. They differ from audio amplifiers only in the values of the circuit components.

The RC-coupled amplifier shown in Figure 7–21A presents several problems that must be solved before the RC-coupled amplifier can be employed as a video amplifier. The high-frequency response is limited by the output capacitance C_o, the distributed capacitance C_d, and the input capacitance of the following stage C_i acting in parallel to shunt the load resistance R_L. The low-frequency response, on the other hand, is limited by the time constant R_gC_c, which must be long as compared to the period of the lowest frequency to be amplified. Tubes with very low interelectrode capacitances must be used, and care must be taken in wiring the circuit to keep leads short and properly spaced in order to reduce distributed capacitance in the wiring. All these problems are reduced by employing pentode tubes and by using small plate load resistors and frequency compensating circuits.

INPUT CAPACITY. Normally, triodes are not adaptable for use in video amplifier circuits, principally because of their high inter-electrode capacity. When an alternating current is applied between the grid and cathode of a triode amplifier tube, an alternating current flows in the small capacitors (C_{gk}, C_{pk}, and C_{gp}, shown in Figure 7–21B), just as AC would flow in any type of capacitor. The current flow in the capacitance between the plate and cathode (C_{pk}) and in the capacitance between the grid and the cathode (C_{gk}) is small, since the size of these capacities is small. Therefore, current flow in them does not seriously affect circuit operation.

On the other hand, the instantaneous voltage between the grid and plate is considerably larger than it would be if there were no amplification taking place in the tube. From the grid end of the tube this increased current is equivalent to an increase in the output capacitance of the tube; the effective capacitance might be several times greater than that which you could expect from the interelectrode capacitances alone. Note in Figure 7–21C that the input capacity consisting chiefly of C_{gk} is increased by the additional capacity (μC_{pg}) which is the capacitance reflected back into the grid circuit by action of the varying and amplified plate current charging the grid-to-plate capacitance. The higher the amplification factor, the greater is the reflected capacitance; and the higher the input capacitance, the greater the attenuation of high frequencies by the amplifier.

The pentode circuit shown in Figure 7–21D shows that the high frequency attenuating effect of high input capacitance is practically eliminated in the pentode tube. The screen grid in the pentode tube forms an electrostatic shield between the plate and the grid. Instead of the varying plate current charging the capacitance between the plate and grid and effectively reflecting a capacitance greater by the μ of the

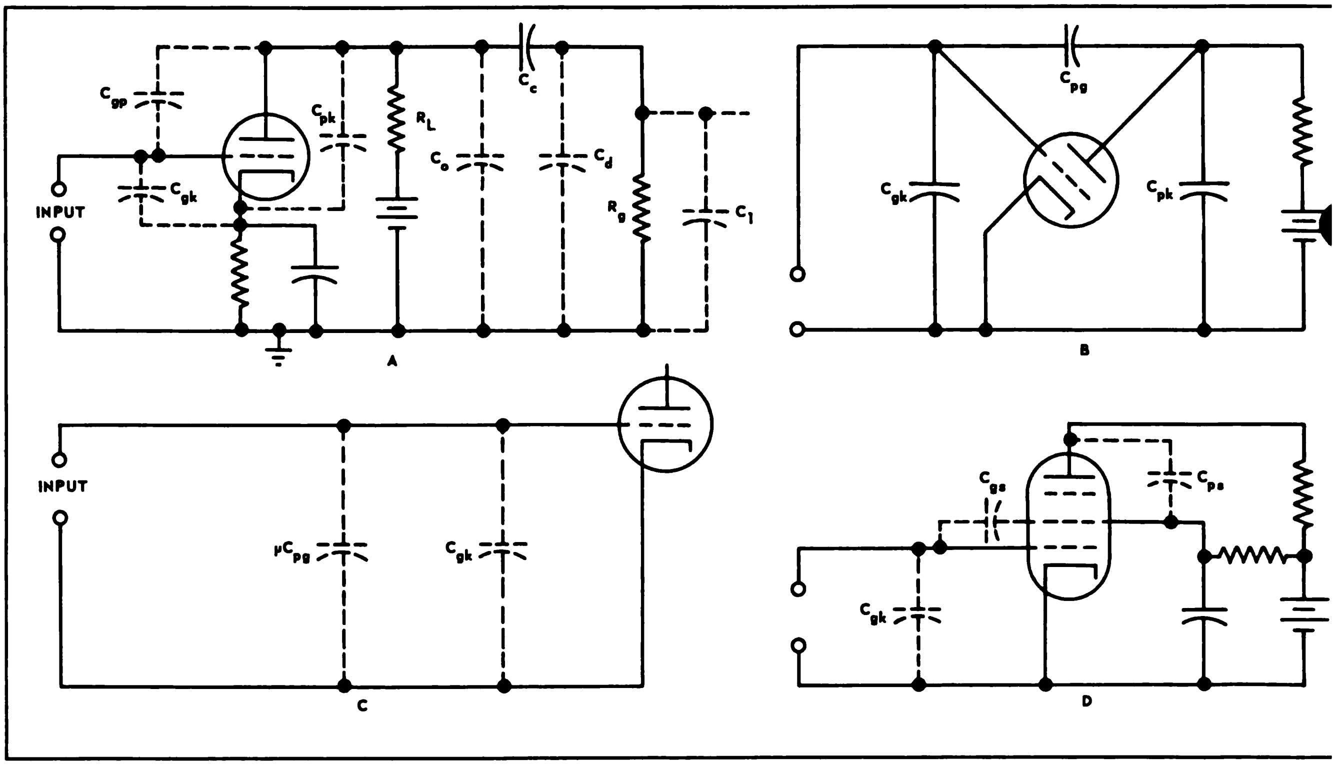


Figure 7–21. Effective Interelectrode Capacitance in Triode Amplifier with Resistive Load

tube into the grid circuit, the plate-to-screen capacitance takes the charge and, by doing so, leaves the signal voltage virtually unaffected. The capacitance between the control grid and the screen grid carries only a very small charging current since the screen grid voltage is maintained at a constant value. Both the C_{gs} capacity and also the C_{gk} (grid-to-cathode capacitance) are small. At high frequencies, therefore, any current flow in these capacitances produces no noticeable attenuation. In addition to low input capacitance, pentodes also offer the advantage of higher amplification than a triode.

Plate Load Resistors. The second circuit feature of video amplifiers is the use of small load resistors. The size of the load resistor in RC-coupled amplifiers is important in that it largely determines the frequency response and the gain of the amplifier. In this connection, Figure 7–22 shows the effect of various sizes of load resistors on frequency response in a pentode amplifier stage. Notice that a large load resistor causes a high output voltage at the middle frequencies and a steep drop in voltage at high frequencies. In small resistors, the proportionate drop in voltage output (gain) at high frequencies is much smaller than the drop at mid-frequencies.

The smaller proportionate drop at high frequencies in the second case mentioned, results chiefly from the use of a small plate-load resistor. The limit to amplification at the high frequency end in an amplifier is controlled by capacitances that shunt the load resistance of the tube. These capacitances, which are called shunt capacitances, consist of the wiring, the socket, and the inherent capacitances of the tube. They act as a low impedance in parallel with the load resistor and effectively reduce the load impedance of the tube. For example, when the reactance of the shunt capacitance is equal to the reactance of the tube, the output voltage is reduced by half (two equal parallel impedances have a total impedance of half the individual impedances). In general, the higher the frequency, the lower the output impedance of the tube, and the lower the voltage output, or gain.

This might lead you to conclude that it is not desirable to use plate load resistors of low ohmic value, since at high frequencies the shunt ca-

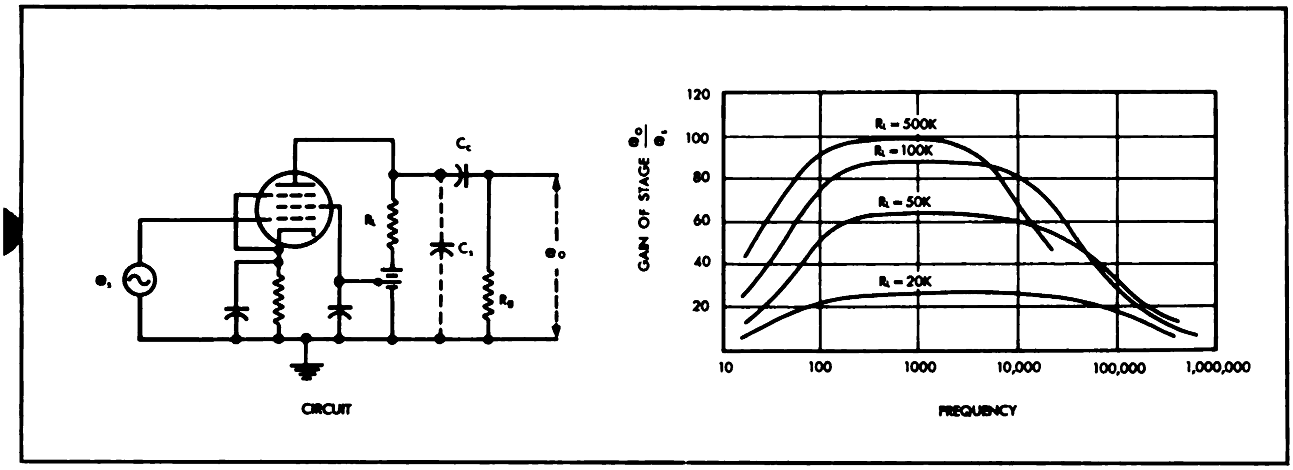


Figure 7–22. Effect of Size of R_L on Frequency Characteristic of Amplification Stage

pacitance reduces the output impedance enough without further decreasing it by small resistors. However, the video amplifier is primarily concerned with uniformly amplifying a wide band of frequencies and only secondarily with gain. To obtain a wide and uniform frequency response, there must be a sacrifice in gain. A small load resistor reduces the gain at high frequencies, but at the same time, it improves frequency response. At the middle frequencies, the reactance of the shunting capacitances is not effective in reducing the output impedance of the tube, and the frequency response is fairly flat there.

Low Frequency Compensation. As previously mentioned, video amplifiers must amplify frequencies at the low end of the band as well as at the high end. According to the frequency-amplification chart, Figure 7–23, amplification of low frequencies shows improvement when the coupling capacitor C_c is large. A large capacitor has less reactance to low frequencies than a small capacitor, and low reactance means a smaller voltage drop. Since the coupling capacitor and the grid resistor form a voltage divider in the input circuit, more voltage appears across the grid resistor when the capacitor has a small reactance. Therefore, greater amplification takes place in the tube. There is a limit, however, to the increase in amplification possible by increasing the size of the coupling capacitor, particularly in radar. A large C_c has capacitance to ground that affects the high frequency response of the circuit.

A second and somewhat more satisfactory means for improving low frequency amplification is by using low frequency compensating circuits, Figure 7–24. A low frequency compensating circuit improves response because of the components C and R in series with the load resistor. Since C is comparatively large, it offers practically zero reactance to middle and high frequencies and, therefore, does not affect these frequencies. At low frequencies, however, its reactance is high, and in parallel with R, it produces a reactance that increases the total load

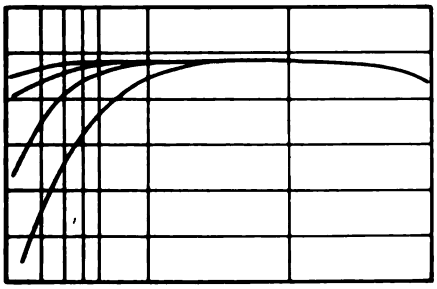


Figure 7–23. Increasing Coupling Capacitor Between Stages Improves Low Frequency Amplification

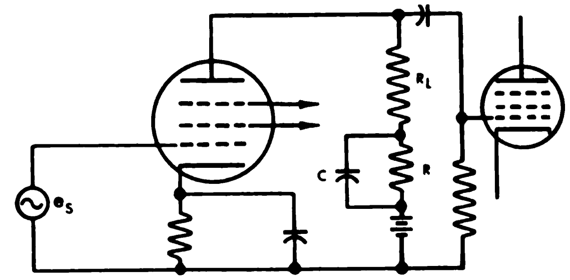


Figure 7–24. Low Frequency Compensation Circuit

impedance. Earlier you saw how the output voltage of an amplifier may be increased by increasing the load resistance. Therefore, since the compensating circuit produces a larger load impedance, it follows that the gain of the stage is higher.

A third method for improving low frequency amplification is a combination of the two just mentioned. It consists of adjusting the coupling capacitor size and adding a capacitive reactance in the plate load in such a manner as to obtain correct proportion between the two. In other words, the voltage drop across C_c is compensated for by the compensating circuit in the plate.

Remember that in frequency amplification, amplification of low frequencies is not as important as amplification of high frequencies. When a radar pulse is very narrow, frequencies below 10,000 cps are of little importance. On the other hand, not all signals are narrow pulses and, since a video signal contains large amplitude, low frequency components, the low frequency response should be fairly good.

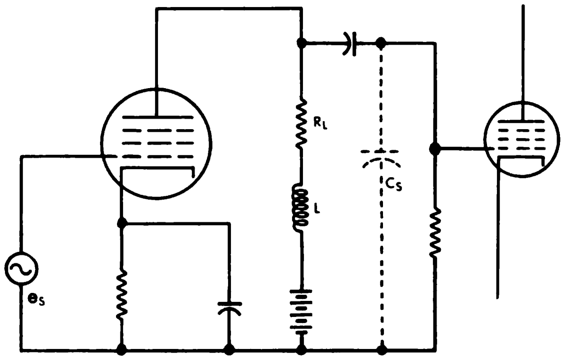


Figure 7–25. Shunt Peaking Coil Provides High Frequency Compensation

High Frequency Compensation. The use of small load resistors in plate circuits of amplifiers does not completely solve the problem of improving high frequency response. Therefore, carefully designed high frequency compensating circuits are often used. One such circuit consists of a low reactance inductor placed in series with the load resistor (Figure 7–25). At low frequencies the inductor L offers a very low reactance, being practically a short circuit. At high frequencies, however, its reactance is high and, since larger voltages are developed, it follows that the gain of the amplifier is increased. Since the inductor sets up a resonant circuit with the stray capacitances in the circuit, it is so designed that the circuit is resonant at a frequency where the gain is low. This design feature gives an added boost to the signal.

Another and similar compensating circuit is the series peaking coil circuit, Figure 7–26. In this circuit coil L is connected in series with coupling capacitor C_c. (Remember that the cause of reduced high frequency voltage is the decrease in reactance of the stray capacitance in the circuit and the grid-to-cathode capacitance in the next tube.) At high frequencies the inductance L resonates with the stray capacitance and the grid-to-cathode capacitance (represented as C_s), causing an increased current through C_s, and hence a greater voltage across C_s which results in a higher gain.

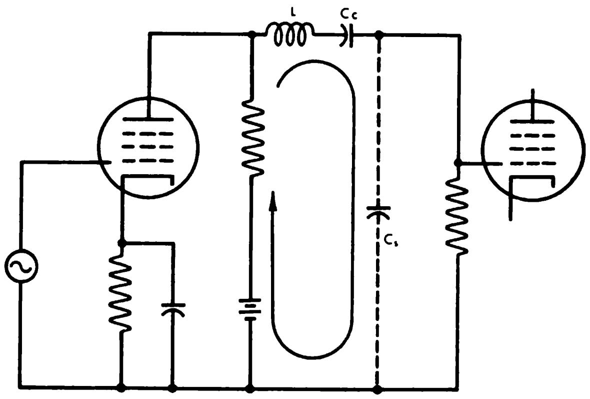


Figure 7–26. Series Peaking Coil Provides High Frequency Compensation

Improving Frequency Response by Negative Feedback. Frequency response can also be improved by negative feedback. This involves feeding a small part of the output voltage back into the input in opposite phase or polarity and proportionately reducing gain. With negative feedback, when the output of an amplifier decreases at high frequencies, less voltage is fed to its input, and when the output increases, more voltage is fed back to the input. When the feedback is large, the gain is proportionately decreased, and when the feedback is small, the gain is proportionately increased. In this way, at high frequencies, when the gain of an amplifier is decreased, less out-of-phase voltage is fed back to the input and the gain is effectively increased.

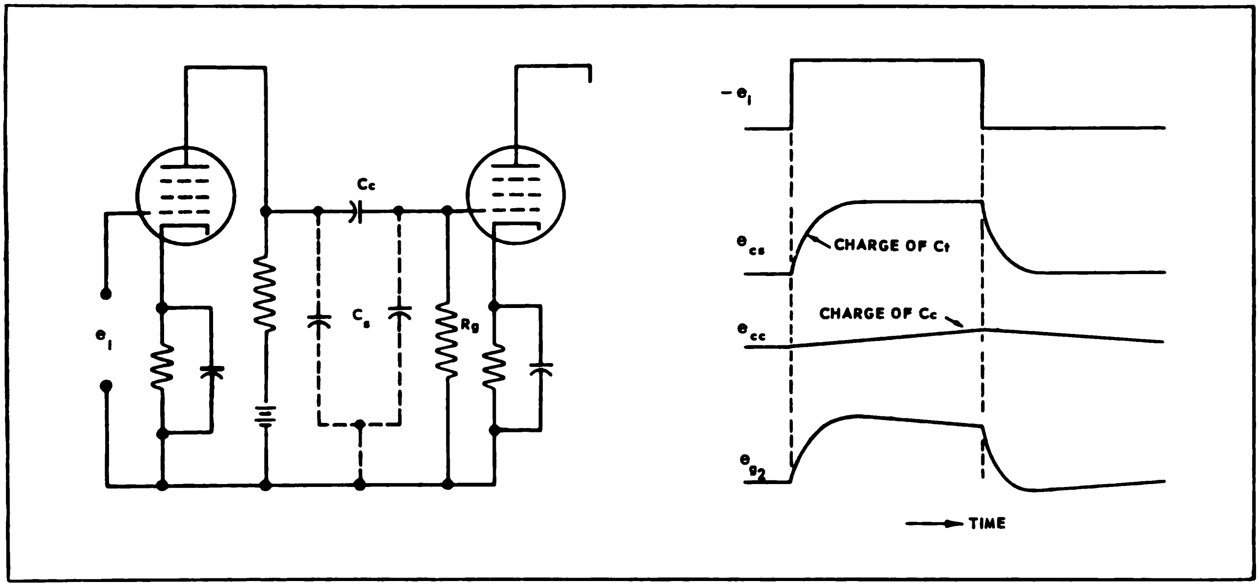


Figure 7–27. Uncompensated Circuit

Analysis of Video Amplifiers

The preceding discussion was an analysis of video amplifiers in the terms of frequencies found in pulse waveshapes. The following is an analysis of video amplifiers in terms of transient voltages.

Figure 7–27 shows a typical uncompensated circuit that serves as the basis for the analysis. The input voltage is labeled e_i. For easier comparison with other waveshapes, it is inverted in the diagram. The inversion is represented by the negative sign in front of e_i. When the steep leading edge of the input square pulse is applied to the first tube, the plate voltage does not rise as sharply as the input, since it cannot rise any faster than the stray capacitance C_s is able to charge. This makes the plate voltage rise along an exponential curve and causes the waveshape on the following grid to be rounded in the same manner as shown at e_{g2}. To get a sharp leading edge, the stray capacitance must be small.

The time constant of this curve is the stray capacitance times R_t, (the total resistance in parallel with the stray capacitance). Mathematically, then, $T\,C = R_tC_s$. R_t is the effective resistance of the load resistor, tube plate resistance, and grid resistance, all in parallel. The relationship between these elements and the frequency (f_2) at which power drops to one-half (and voltage to 70%) is expressed by the equation,

$$T\,C = R_tC_s = \frac{1}{2\pi f_s}$$

During the flat top of the input pulse, it is important that the coupling capacitor C_c is large so that it will charge slowly and keep the second grid voltage from falling rapidly. The time constant of this curve is expressed by the equation,

$$T\,C = R_cC_c$$

where R_c is the total resistance of R_g in series with R_l and r_p in parallel.

The relation between this time constant and the low frequency is expressed by the equation,

$$T\,C = R_cC_c = \frac{1}{2\pi f_1}$$

where f_1 is the low frequency at which the power drops to one-half and the voltage to 70%.

When the end of the pulse arrives, the plate voltage drops only as fast as C_s discharges through the resistors in the circuit. Therefore, this time constant should be short. (The waveshapes in the illustration are exaggerated for purposes of discussion.)

In summary, remember that the square corners of the pulse are caused by the high frequency components in the square wave and that the small shunt capacitances, such as stray wiring capacitance and grid-to-cathode capacitance, are responsible for the squareness of the leading and trailing edges of the pulse. The flat top of the pulse is due to the right amplitude of the low frequencies. The coupling capacitor, and its relation to the resistance in the circuit, directly affect the flat top of a pulse.

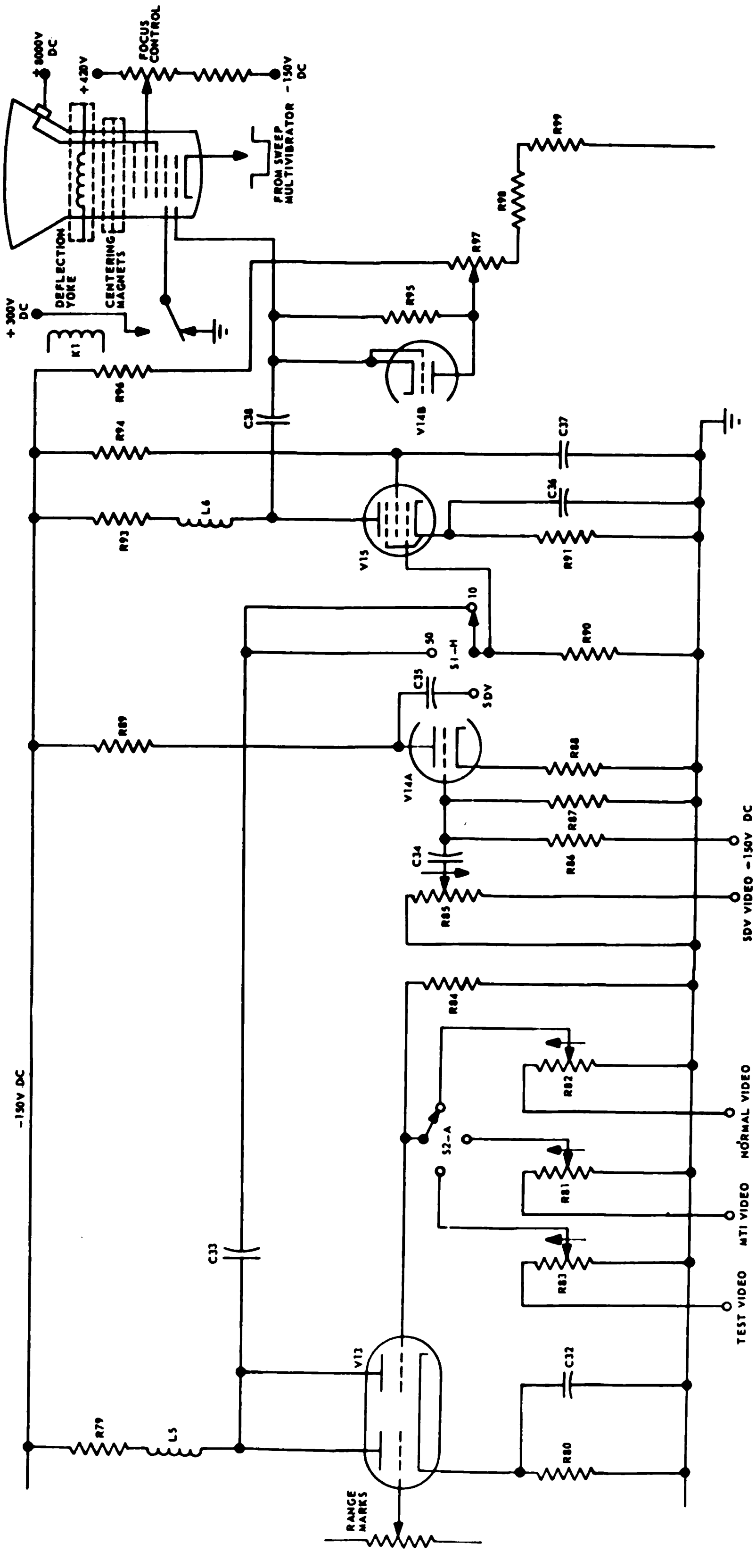


Figure 7–28. Video Amplifier

Video Amplifier Circuit in Typical Radar Set

At first glance the video amplifier circuit shown in Figure 7–28 appears to be the usual video circuit. But a closer inspection will show that new circuitry and terminology have been added.

The video circuits of the indicator mix the range marks signal with the selected video signal and amplify the resultant to the level necessary for application to the cathode-ray tube. Three video inputs are available for mixing with the range marks pulses: the normal video, the MTI video and test video. With switch S2–A you can select the one you wish to view. Three separate gain potentiometers provide low impedance terminations for the various inputs: R82 for the normal video, R81 for the MTI video, and R83 for the test video.

The desired video signal is applied through switch S2–A to the control grid of one half of duo-triode video amplifier mixer tube V13. The range marks signal is applied to the other control grid. The plates of V13 are connected together and to B+ through peaking coil L5 and load resistor R78. Therefore, the voltage developed across the plate load impedance contains both the range marks signals and the video signals. The output of video amplifier mixer tube V13 is coupled through capacitor C33 to the 50-mile and 10-mile contacts of selector switch S1–H.

Also coupled to the contacts of the SDV position of this switch, through capacitor C35, is the output of the SDV amplifier. SDV is *Slowed Down Video*. SDV is one of the newer electronic circuits which is used in the present-day radar systems networks. In a radar network, it is desirable to transmit a video signal from one radar site to another or to a control center. This is accomplished via telephone lines. As the high frequency components of a video signal cannot be carried over a telephone line, electronic circuits convert the video to a low frequency signal. At the receiving station circuits reconvert the video to a high frequency signal appropriate for use in the indicator system.

The SDV is developed across SDV gain control potentiometer R85. Capacitor C34 feeds the SDV signal into the control grid of tube V14A, which is normally biased to cutoff by the —150v DC supply through a voltage divider network which consists of resistors R86 and R87. This is done so that extraneous signals will not be passed on to the CRT. However, the positive-going video signals from the SDV equipment drive the tube into conduction and produce a negative-going amplified signal in the plate circuit of tube V14A. Switch S1–H selects either the output signal of SDV amplifier tube V14A or the output of video amplifier mixer tube V13.

The selected video signal is applied to the control grid of video amplifier tube V15, which is a conventional video amplifier stage. The positive-going output of tube V15 is developed across inductance L6 and resistor R93, and is coupled through capacitor C38 to the cathode of DC restorer tube V14B. Tube V14B is a diode-connected triode which maintains the control grid of the CRT at a DC potential which is determined by the setting of intensity potentiometer R97. The video signal drives the control grid of the electrostatically-focused and magnetically-deflected CRT more positive than this DC potential by an amount equal to its instantaneous amplitude.

The sweep failure protection circuit prevents the phosphor on the CRT screen from being burned if the sweep circuits should fail. A positive 300v DC is necessary on the accelerator grid for the CRT to function properly. If the sweep circuits should fail, the signal that is normally applied to the sweep protection circuit is no longer available, since it is a sample of the sweep circuit voltage. A relay in the sweep circuit, K1, would become de-energized, and there would be no accelerator grid voltage on the CRT. The electron beam could not reach the screen phosphor. If this protective circuit were not provided, the beam would produce a stationary spot whenever the deflection components failed, and burning of the screen would occur.

The azimuth protection circuit is another protective device for the CRT. This circuit protects the CRT from damage if the deflection yoke should cease to rotate. The motor that drives the deflection yoke also drives a tachometer which is the source of voltage for the azimuth protection circuit. If the drive motor should stop, the voltage from the tachometer would no longer exist, and a relay would become de-energized. The de-energizing of the relay places —150v DC on the control grid of the CRT, immediately driving the CRT electron beam to cutoff and preventing a stationary trace from appearing on the CRT screen.

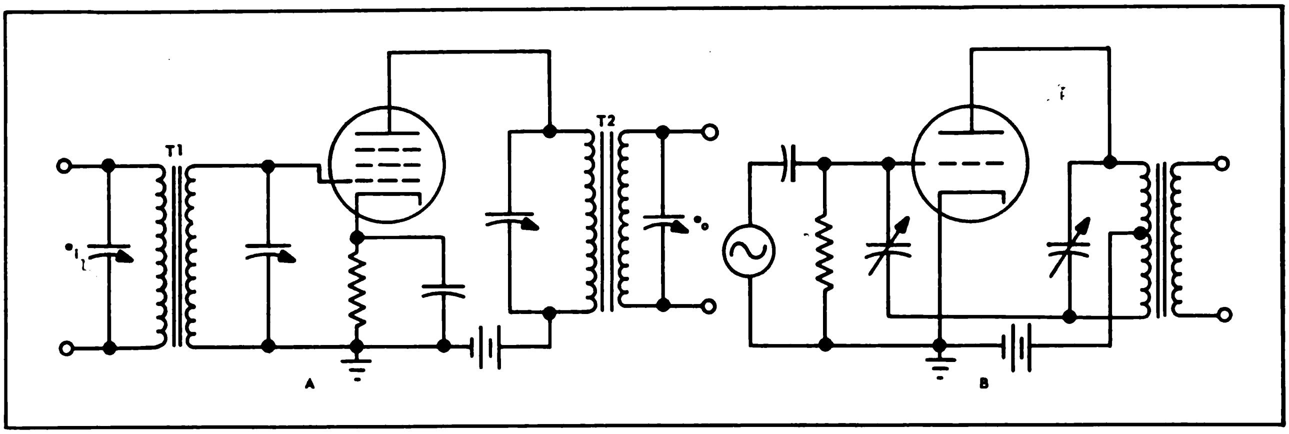


Figure 7–29. Typical RF Amplifier

I–F and R–F Amplifiers

I-f amplifiers and r-f amplifiers are essentially alike. They differ only in their use and frequency. I-f amplifiers are designed for amplifying a single frequency, usually higher than 100 kc, while r-f amplifiers amplify a band of frequencies on each side of the frequency for which they are designed.

Because of the inefficiencies of conventional vacuum tubes at high frequencies, r-f amplifiers are not used for 1000-mc frequencies. For example, the 3-cm and the 10-cm radar sets use r-f amplifiers neither in front of the mixer stage in the receiver, nor for amplifying the output of the oscillator in the transmitter. In contrast, however, some low frequency radar sets do use r-f amplifiers, and all sets, regardless of frequency, use i-f amplifiers.

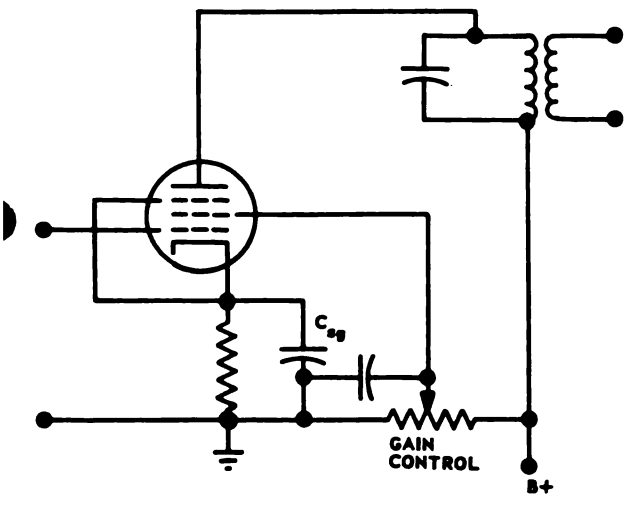
For low-power operation such as required in receivers, pentode r-f amplifiers, because of their low grid-to-plate capacitance, provide the highest gain with the least tendency to break into self-oscillation. The r-f amplifier shown in Figure 7–29A is an i-f amplifier, since it is designed to amplify only one frequency. Single frequency amplification occurs because of the parallel resonant circuits at the input and output. These resonant circuits discriminate against all other frequencies by developing maximum voltage at their resonant frequency, and minimum voltage at other frequencies. This ability of an amplifier to select one frequency and reject others is called *selectivity*. Selectivity in an amplifier is greatest when high Q tuned circuits are used. (A high Q tuned circuit is produced here by using powdered iron cores in the transformers.)

The r-f amplifier in Figure 7–29B is one which is commonly used in transmitters. As contrasted with the circuit at A, which uses a pentode tube, this circuit uses a Class-A-operated triode. Another difference is that the tuned circuit is in the plate circuit only. Note the coupling circuit from the plate tank to the grid. This is a *neutralization* circuit to feed out-of-phase voltage from the output back into the grid circuit. Its function is to prevent the triode from oscillating.

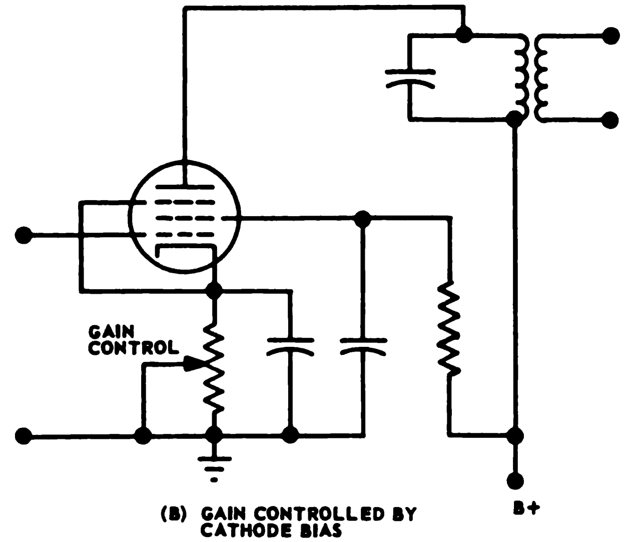
GAIN OF TUNED R-F AMPLIFIER. The gain of a tuned r-f amplifier, using a variable-mu tube, can be controlled by regulating the transconductance (g_m) of the tube. The two most widely used methods of gain control are illustrated in Figure 7–30A and B.

In the circuit of Figure 7–30A, the gain is controlled by the magnitude of screen grid voltage. As the screen grid voltage is increased, the transconductance increases, with a resultant increase of gain. When the screen grid voltage is decreased, the reverse action takes place, with a resultant decrease in gain.

It might be well to mention at this point, that the capacitor (C_{sg}) between screen grid and ground in the circuit of Figure 7–30A is called the screen bypass capacitor. Its purpose is to decrease degeneration due to the AC component of signal voltage causing the screen grid voltage to vary at the same rate. Its action is similar to that of the cathode bypass capacitor previously discussed. The bypass capacitor acts as a filter so that the screen grid voltage, in this case, will be relatively steady, even though there is an AC component in the tube. The effect of removing this capacitor would be evident in the plate circuit by a decrease in gain.



(A) GAIN CONTROLLED BY SCREEN GRID VOLTAGE



(B) GAIN CONTROLLED BY CATHODE BIAS

Figure 7–30. Methods of Amplifier Gain Control

In Figure 7–30B the cathode bias is regulated in order to control amplification. The change of gain with this type of control depends on a change of the operating point. The amplifier is made to operate over a different portion of its characteristic curve so that the mu of the tube, and thus the gain, is changed. This is true within certain limits. In many amplifier circuits using cathode bias to control gain, a variable-mu tube is utilized. This type of tube is constructed so that its characteristic curve does not reach cutoff sharply, but rather at a gradual slope.

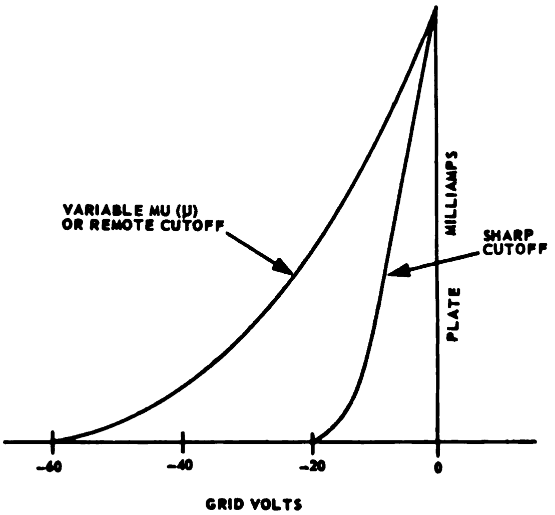


Figure 7–31. Dynamic Characteristic Curves of Both a Sharp Cutoff and Remote Cutoff Vacuum Tube

Figure 7–31 illustrates the characteristic curves of both a sharp cutoff vacuum tube and a variable-mu, or remote cutoff, tube. It can be seen that if the bias is changed in a variable-mu vacuum tube, the operating point will move along the gradually curved portion of this curve, thus providing a change in mu (amplification factor) with little distortion. Remember that the mu of a vacuum tube is equal to

$$\frac{\Delta E_p}{\Delta E_g} \text{ (} I_p \text{ constant).}$$

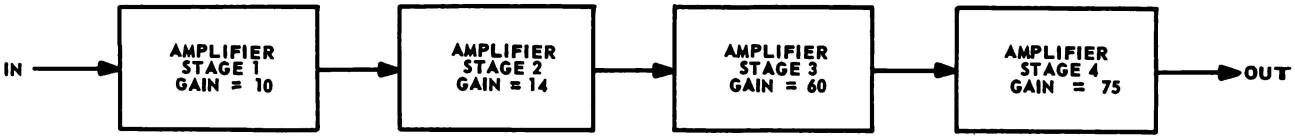
Gain, as previously discussed and as shown on the response curves of the various amplifiers covered thus far, can be expressed or compared by two methods. One method has already been illustrated where,

$$\text{Gain (or amplification)} = A = \frac{\Delta E_{out}}{\Delta E_{in}} \text{ (for voltage)}$$

$$\text{or } \frac{\Delta P_{out}}{\Delta P_{in}} \text{ (for power) or } \frac{\Delta I_{out}}{\Delta I_{in}} \text{ (for current)}$$

If the gains of individual amplifiers in cascade are known, and it is desired to compute their over-all gain by this method, then the gains of all the stages of amplification must be multiplied together. The block diagram of Figure 7–32 illustrates this method of calculating the over-all gain of amplifiers in cascade.

The power or current gain of a number of amplifiers in cascade can be found in the same manner if the amplification of each individual stage is known.


IN
AMPLIFIER STAGE 1 GAIN = 10
AMPLIFIER STAGE 2 GAIN = 14
AMPLIFIER STAGE 3 GAIN = 60
AMPLIFIER STAGE 4 GAIN = 75
OUT

$$\text{TOTAL GAIN } (A_T) = A1 \times A2 \times A3 \times A4$$

$$A_T = 10 \times 14 \times 60 \times 75$$

$$A_T = 630{,}000$$

Figure 7–32. Cascade Amplifiers Showing Over-all Gain

Although current, voltage, and power gain (amplification), as well as their magnitudes relative to reference values, can be expressed as an ordinary ratio, it has been found to be far more convenient to make use of logarithmic ratios. The unit that has been adopted is the decibel, which is indicated by the symbol db, and is defined as

$$\text{Decibels} = 10 \log_{10} \frac{P_{out}}{P_{in}}$$

The sign associated with the number of decibels indicates which power is greater; thus, a negative sign means P_{out} is less than P_{in}.

If decibels are used to express amplification, this simply means that the presence of the amplification increases the power output by the number of decibels attributed to the amplification. Relative power is proportional to the square of voltage or current, thus

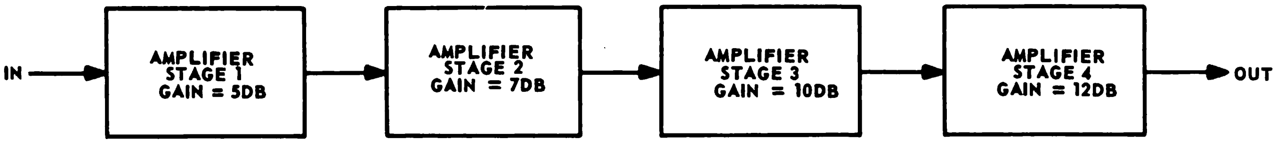
$$\text{Decibels} = 20 \log_{10} \frac{E_{out}}{E_{in}} = 20 \log_{10} \frac{I_{out}}{I_{in}}$$

The practical value of the decibel arises from its logarithmic nature. This permits the enormous ranges of power involved in communications and radar work to be expressed in decibels without running into inconveniently large numbers, while at the same time permitting small ratios to be expressed. Thus, 1 db represents a power ratio of approximately 5:4, while 60 db represents a ratio of 1,000,000:1. The logarithmic character of a decibel also makes it possible to express the input and output powers of a number of amplifiers in cascade as the sum of the db gains of the individual amplifiers. The block diagram in Figure 7–33 illustrates this manner of computing the over-all gain of an amplifier when the db gains of the individual stages are known.

The decibel is also the natural unit for expressing sound intensities, since the effect sound waves have on the ear is roughly proportional to the logarithm of the intensity.

Operation of I-F Amplifiers. I-f amplifiers are used in superheterodyne receivers. In these receivers the incoming signal is converted into a new and lower frequency and amplified by specially-designed, high-gain i-f amplifiers. Great care must be given in the choice of the intermediate frequency. For example, the lower the intermediate frequency, the higher the gain and selectivity. Although communications receivers may use intermediate frequencies as low as 75 kc or 275 kc, the tendency is for them to use much higher frequencies because of poor image rejection at the lower frequencies.

Unlike communications receivers which have a relatively narrow bandpass, radar receivers are


IN
AMPLIFIER STAGE 1 GAIN = 5DB
AMPLIFIER STAGE 2 GAIN = 7DB
AMPLIFIER STAGE 3 GAIN = 10DB
AMPLIFIER STAGE 4 GAIN = 12DB
OUT

$$\text{TOTAL GAIN} = A_T = (DB)\,1 + A\,(DB)2 + A(DN)3 + A\,(DB)4$$

$$A_T = 5 + 7 + 10 + 12$$

$$A_T = 34 \text{ DB}$$

Figure 7–33. Cascade Amplifiers Showing Over-all DB Gain

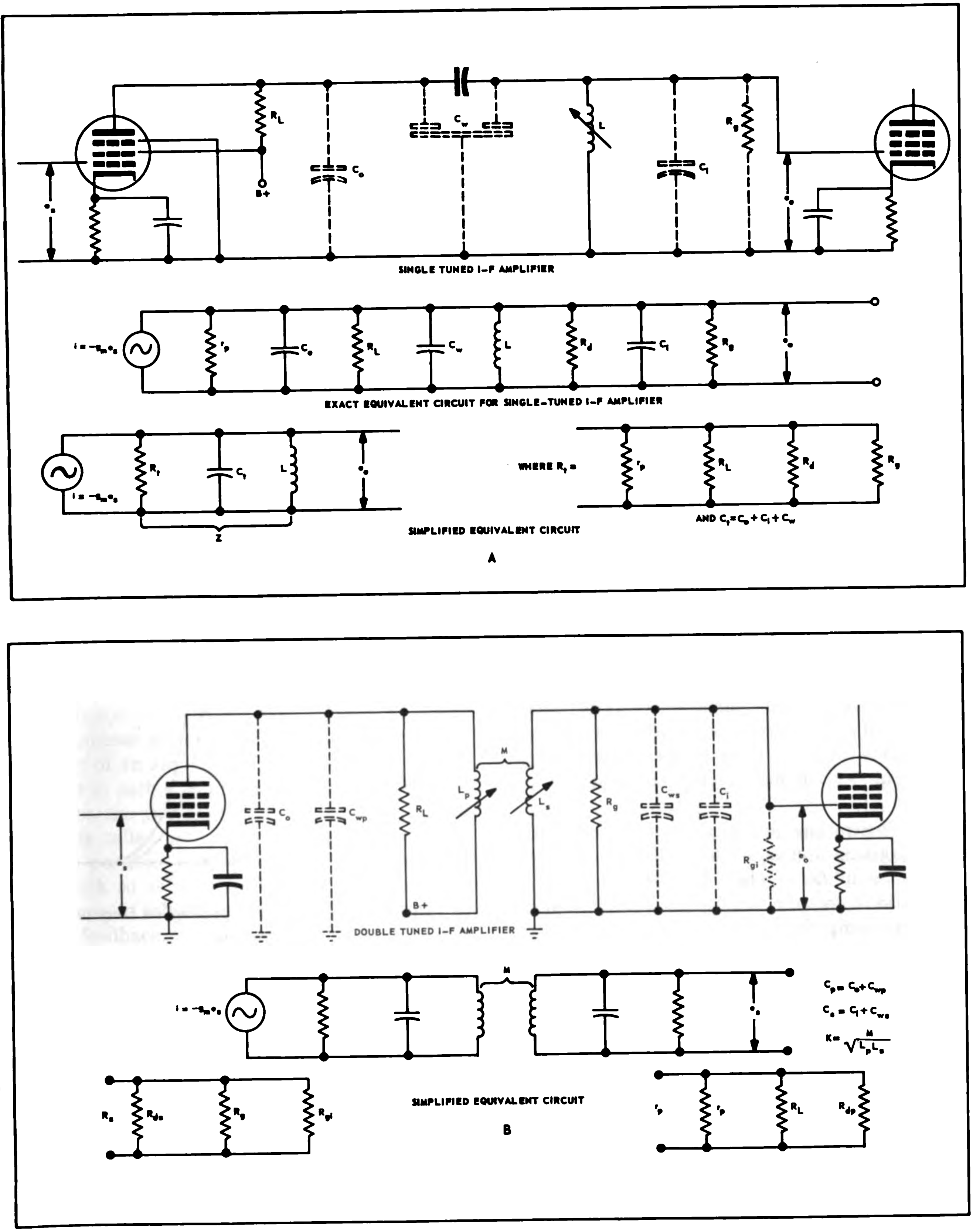


Figure 7–34. Tuned I-F Amplifiers and Equivalent Circuits

designed with bandpasses as wide as 10 to 20 mc. Obviously, with such wide bandpasses, the intermediate frequency must be greater than 10 mc in order to keep the lower side band above zero mc. The usual practice is to use intermediate frequencies of 30 mc and 60 mc in radar receivers. Although high gain circuits are hard to design at these frequencies, the bandpass characteristics nevertheless are good. In fact, the additional bandpass devices which must be built into the set further reduce gain. In a radar receiver, therefore, it is common to find six, eight, or more stages of intermediate frequency amplification.

Previously, you saw that the video amplifier must be specially designed. Similarly, r-f and i-f amplifiers require special design. A pulse-modulated r-f signal contains the r-f frequency (or i-f frequency) plus a sideband frequency for each of the frequencies in the pulse. All these frequencies add up to a very wide band of frequencies. To accommodate them, the amplifier band must be fairly wide. The bandwidth of the amplifier may be increased by lowering the Q, stagger tuning, and overcoupling.

I-f amplifiers are classified as *single-tuned* or *double-tuned* depending upon whether each interstage coupling circuit includes one or two tuned circuits. Double-tuned coupling produces greater gain for a given bandwidth, but is not used often because of the greater simplicity in alignment and in manufacture of single-tuned circuits.

The illustrations in Figure 7–34 show the circuit diagrams and equivalent circuits for both types of amplifiers. The following are the gain calculations for the single-tuned intermediate frequency amplifier:

Gain of One Stage

From the equivalent circuit C,

$$e_o = iz = g_m e_s z$$

$$\text{Then, } A = \frac{e_o}{e_s} = \frac{g_m e_s z}{e_s} = g_m z$$

Gain at which

$$2\pi f L = \frac{1}{2\pi f C}$$

At this frequency, the parallel combination of C_t and L is an infinite impedance in parallel with R_t. Therefore, $A = -g_m R_t$. (Adjusting L in this circuit to the point where maximum gain occurs, denotes the presence of the intermediate frequency.)

Notice in Figure 7–34B that both the primary and secondary circuits are tuned to the intermediate frequency. The formulas for calculating the intermediate frequency are as follows:

$$\text{Primary, } f = \frac{1}{2\pi\sqrt{L_p C_p}}$$

$$\text{Secondary, } f = \frac{1}{2\pi\sqrt{L_s C_s}}$$

Since the primary and secondary tuned circuits must be tuned to the same frequency,

$$\text{I-f} = \frac{1}{2\pi\sqrt{L_p C_p}} = \frac{1}{2\pi\sqrt{L_s C_s}}$$

To get the best results in double-tuned i-f amplifiers, use coupling that is neither too loose nor too close, since it gives a single humped response curve. Such coupling is called critical coupling and is expressed mathematically,

$$k_c = \frac{1}{Q}$$

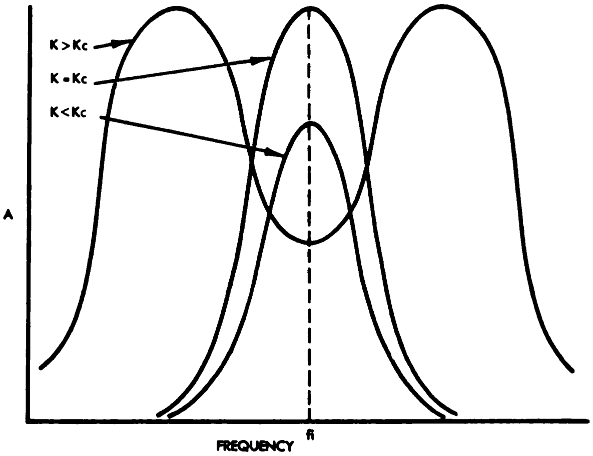


Figure 7–35. Effect of Coupling on Frequency Response

Notice in Figure 7–35 the effect of three types of coupling on frequency response. The following are the effects of each type:

Where the coupling is too loose ($k < k_c$), there is decrease in gain.
When the coupling is too close ($k > k_c$), the response curve is double humped and causes each pulse to be followed by an oscillatory transient.
When the coupling is just right ($k = k_c$), there is maximum gain and bandwidth.
(The absolute value of gain is represented by $|A|$.)

R-f amplifiers are similar to i-f amplifiers except for tubes used at UHF frequencies. An r-f amplifier is used when the signal-to-noise ratio of a radar receiver is not high enough.

Several stages of i-f amplifiers are used in radar receivers. Each succeeding stage narrows the bandwidth of the receiver. Radar receivers usually make use of a bandwidth from 3 to 6 megacycles. Poor fidelity results from use of several stages in cascade, unless the bandwidth is increased. Increasing bandwidth may be accomplished by any of the following methods:

- Close coupling the transformers in double-tuned stages.
- Loading the primary section of the transformer by making the value of R_t smaller (R_t can be decreased only to a certain limit).
- Tuning some i-f stages slightly above the intermediate frequency, and some slightly below.
- Not using a bypass capacitor in the cathode or screen grid circuit.

Each of these reduces the gain and the input capacitance of each stage and results in an increase of the over-all bandwidth. Remember, though, that none of these methods can be used without *reducing* the gain of the intermediate frequency amplifier.

FEEDBACK IN AMPLIFIERS

Whenever a portion of the amplified output energy of an amplifier is fed back into the input circuit in such a manner as to reinforce the input voltage, the gain is greatly increased. Such feedback is called *positive* or *regenerative* feedback. If the portion of the amplified output energy is fed back so as to oppose the input voltage, the feedback is called *degenerative*, *negative*, or *inverse* feedback.

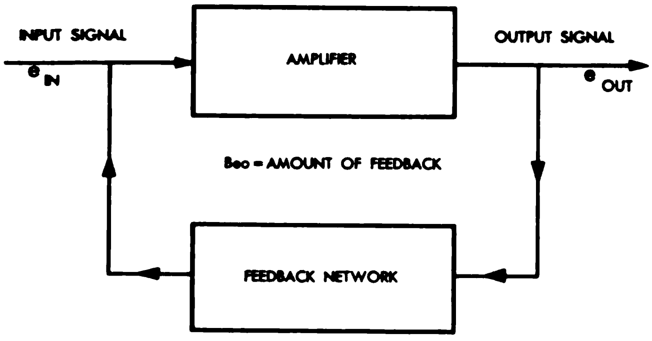


Figure 7–36. Feedback Amplifier

Figure 7–36 shows a simple block diagram of a feedback amplifier with the voltages as indicated. When there is no phase shift in the input voltage e_i, and when amplification factor is A, the net output voltage equals $e_o - \beta e_o$ (βe_o representing the fraction of the output fed back to the input). With βe_o fed back to the input, the total input is $e_i + \beta e_o$. If this input is amplified A times, the equation for the output voltage is,

$$e_o = (e_i + \beta e_o)A$$

Since the gain is $\frac{e_o}{e_i}$, you can derive the gain formula for a feedback amplifier by solving the equation, $e_o = (e_i + \beta e_o)A$ as follows:

Multiplying in the right member,

$$e_o = Ae_i + \beta Ae_o$$

Subtracting βAe_o from both members,

$$e_o - \beta Ae_o = Ae_i$$

Factoring the left member

$$e_o(1 - \beta A) = Ae_i$$

Thus,

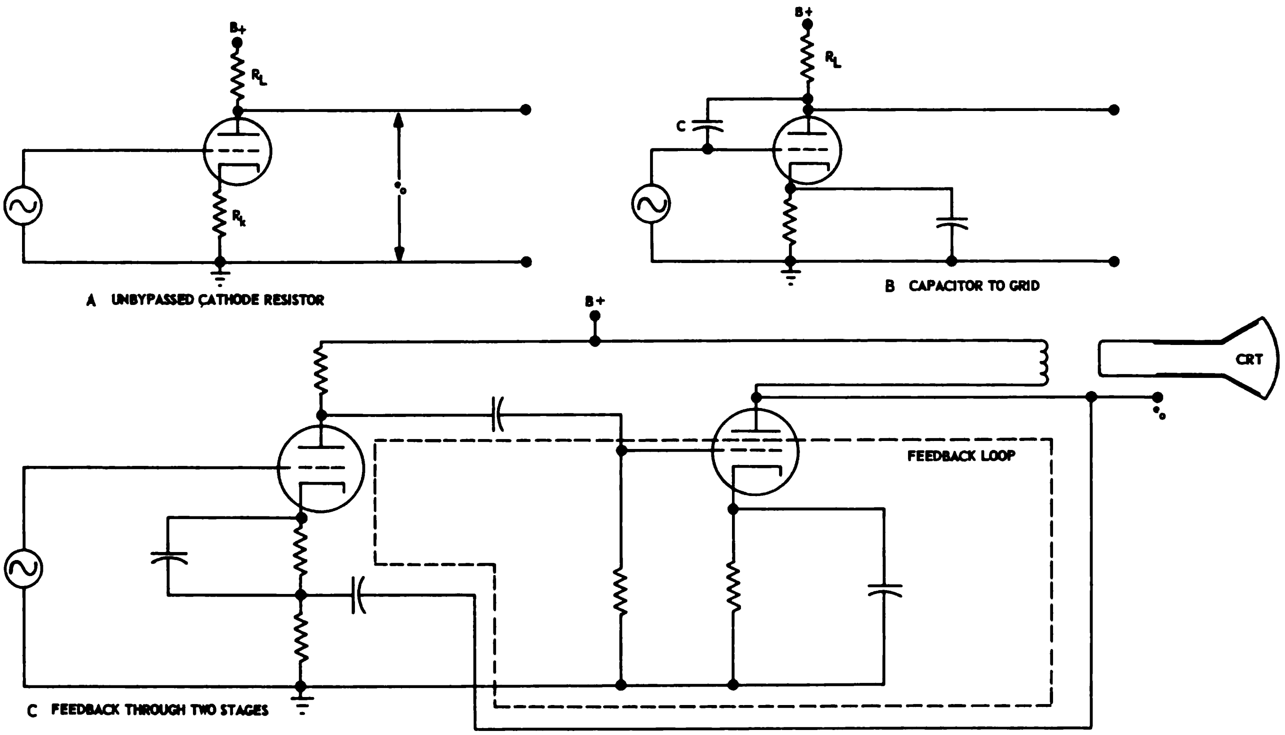
$$\text{Gain} = e_o/e_i = A/(1 - \beta A)$$

The expression $1-\beta A$ is important as it will tell you the nature of the feedback. When the quantity $1-\beta A$ is less than 1, the gain of the amplifier is increased, and the feedback is *regenerative*, or *positive*. When the quantity $1-\beta A$ is zero, that is, $\beta A = 1$, regeneration is large enough to make the stage self-sufficient and the amplifier will act as an oscillator. When the quantity $1-\beta A$ is greater than 1, the gain of the amplifier is decreased and the feedback is *degenerative*, *negative*, or *inverse*.

Degenerative feedback decreases gain, but it reduces distortion introduced by the circuit itself.

Methods of Obtaining Feedback

Feedback can be obtained in amplifiers in several ways. One arrangement used to obtain negative feedback is illustrated in Figure 7–37A. In this circuit feedback voltage is developed across the unbypassed cathode resistor R_k as a result of the plate current flowing through it. Current flow through an unbypassed cathode resistor develops a voltage that varies at the same rate as the plate voltage. Since resistor R_k is located between the cathode and the grid of the tube, any voltage developed across it is in series with



A UNBYPASSED CATHODE RESISTOR

B CAPACITOR TO GRID

C FEEDBACK THROUGH TWO STAGES

Figure 7–37. Methods of Providing Feedback

the input signal of the tube. The phase relation is correct for degeneration to occur.

Thus, when a positive signal appears on the grid of the tube, the plate current increases, and the voltage drop across R_k also increases. A voltage increase across the cathode resistor makes the grid more negative relative to the cathode, the reverse of the action by the signal voltage. This arrangement is very useful in cancelling out distortion in the output signal resulting from the curvature of the E_g-I_p characteristic. Since plate voltage changes in a tube do not follow the grid voltage changes exactly, there is some distortion in the output waveshape. However, degenerative feedback introduces a portion of the distorted output back into the input in reverse, counterbalancing the conditions causing the original distortion.

Similarly, feedback can be obtained by connecting a capacitor from the plate back to the grid of the tube as shown in Figure 7–37B. Here plate voltage changes produced by plate current variations are opposite in polarity to the original grid voltage changes. The capacitor introduces a small part of the plate voltage change back into the grid circuit. Thus, the plate change reflects the distortion in the plate current change in reverse, virtually cancelling the distortion originally introduced into the output.

Use of Feedback

Feedback can be used in amplifiers using more than one stage. Figure 7–37C shows a two-stage feedback amplifier. Not only is it useful for counteracting the effects of distortion introduced by the tubes, but it also can be used to cancel out-of-phase relationships caused by certain circuit components. For example, it compensates for any changes in the inductive reactance of the deflection coil in the circuit. When current flows through this coil, its reactance changes, and this causes a change in the plate voltage of the tube. This change in voltage moves around the feedback loop and counteracts the inductance change. This makes the plate voltage normal and insures an undistorted output voltage.

In some circuits reactance elements introduce positive feedback into the input, and the circuit oscillates. This is desirable in oscillators but not in amplifiers. Therefore, some type of feedback arrangement, similar to that in the two-stage feedback amplifier, must be used to introduce a negative feedback into the input to cancel the effects of the regenerative feedback.

CATHODE FOLLOWER

A cathode follower amplifier is a single-stage degenerative amplifier in which the output is taken from across the cathode resistor as shown in Figure 7–38. This circuit is essentially an impedance matching device for matching a high-impedance circuit to a low-impedance circuit without discriminating against any AC frequency. Its voltage output is always less than the input voltage, but it is capable of power amplification. Two of the advantages of cathode followers are low input capacitance and distortionless output.

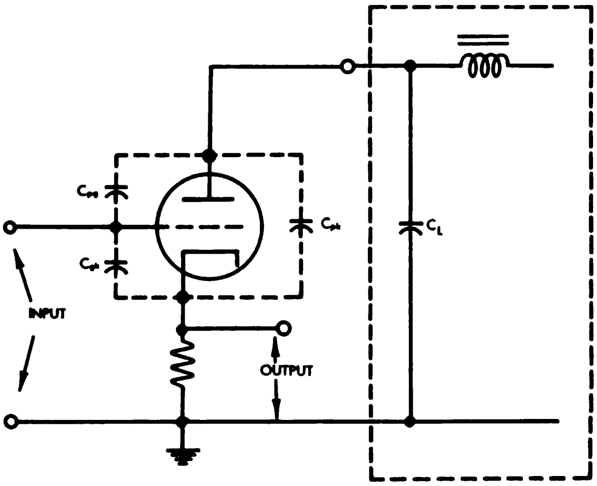


Figure 7–38. Interelectrode Capacitance in Cathode Follower Circuit

Input Capacity

As previously mentioned, serious losses result from the high input capacitance in triode tubes. This is because the resistive load causes a leading current in the grid circuit and the plate-to-grid capacitance adds to the normal grid-to-cathode capacitance to produce a large capacitance at the input of the triode. Unlike the triode, the plate load in the cathode follower is a large capacitance (usually the filter capacitor in the power supply). Nevertheless, as in the triode, the capacitance C_{pg} still must charge when a voltage is applied to it. However, the current in the grid circuit does not lead, because the capacitive load in the cathode follower causes the charging current to shift in phase and puts it exactly in phase with the original grid voltage. Since adding in-phase current is the same as lowering the resistance in the grid circuit, the impedance reflected in the grid circuit is purely resistive, and its effect on all frequencies is the same; that is, there is no frequency attenuation. The input capacitance of a cathode follower is given by $C_{in} = C_{gk} + C_{pg}\ (1–A)$.

Distortionless Output

Another advantage of the cathode follower is that it introduces very little amplitude distortion into the output. It is a degenerative circuit in which negative feedback is always produced by an unbypassed cathode resistor, and its output is taken from across the cathode resistor and not from the plate.

Design Considerations

Three of the most important considerations in the design of cathode followers are gain, input and output impedances, and size of the cathode resistor. Since the gain and the size of the cathode resistor are the only variables in a cathode follower, and impedance is, in turn, related to both, you should be acquainted with the formulas for computing them.

GAIN FORMULA. Notice the use of the following designations in the equivalent circuit in Figure 7–39.

e_s = input voltage
e_g = voltage between grid and cathode
e_o = output voltage

From the cathode follower circuit,

$$e_g + e_o = e_s$$

Transposing and substituting,

$$e_g = e_s - e_o = e_s - i_p R_k$$

Since, $e_o = i_p R_k$

$$\text{and } i_p = \frac{\mu e_g}{r_p + R_k} = \frac{\mu e_s - \mu i_p R_k}{r_p + R_k}$$

$$\text{Therefore, } i_p r_p + i_p R_k = \mu e_s - \mu i_p R_k$$

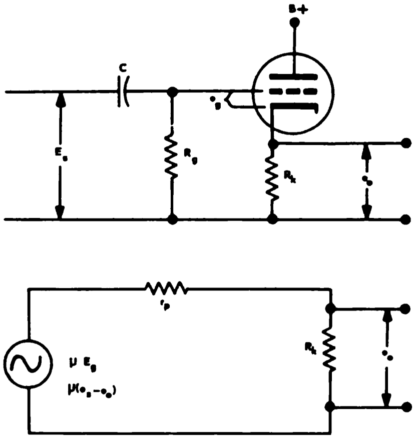
$$\text{and } i_p\ [r_p + (R_k)\ (1 + \mu)] = \mu e_s$$

$$\text{Therefore, } i_p = \frac{\mu e_s}{r_p + R_k(\mu + 1)}$$

$$\text{Then, } e_o = \frac{\mu e_s R_k}{r_p + R_k(\mu + 1)}$$

$$\text{Thus, Gain} = \frac{e_o}{e_s} = \frac{\mu R_k}{r_p + R_k(\mu + 1)}$$

Notice in the equation just derived that the denominator is greater than the numerator. This indicates that the gain of a cathode follower is less than one. Remember, though, that the less-than-one gain applies only to the AC voltages, and not to DC values. Actually, the cathode to ground DC voltage is greater than the input voltage as long as the grid-to-cathode voltage is negative.



CONSTANT-VOLTAGE GENERATOR EQUIVALENT CIRCUIT

Figure 7–39. Cathode Follower

IMPEDANCE. The input impedance in a cathode follower is high. Since cathode followers are operated with the grid negative with respect to the cathode, a high amplitude voltage can be applied between the grid and ground without causing grid current flow. This is due to the degenerative action of the cathode resistor and the high input impedance during the positive input signal. Because of its high input impedance, the cathode follower has negligible loading effect on the circuit which drives it.

The output impedance is low. The expression for the plate current that flows in the constant-voltage generator equivalent circuit of the cathode follower in figure 7–39 is,

$$i_p = \frac{\mu e_s}{r_p + R_k(\mu + 1)}$$

If both the numerator and the denominator of this expression are divided by $\mu + 1$, the expression will have the form of the plate current of a circuit in which the tube has an amplification factor of $\frac{\mu}{\mu + 1}$ and an AC plate resistance of $\frac{r_p}{\mu + 1}$. This expression reads,

$$i_p = \frac{\left(\frac{\mu}{\mu + 1}\right) e_s}{\left(\frac{r_p}{\mu + 1}\right) + R_k}$$

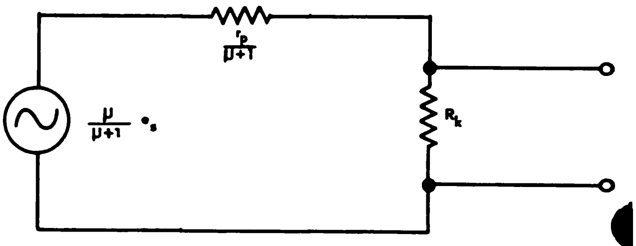


Figure 7–40. Equivalent Circuit

The equivalent circuit is redrawn in Figure 7–40 to show the new circuit constants.

The output impedance of the cathode follower is the parallel combination of the cathode resistance and the effective AC plate resistance. In general, the output impedance is resistive, and is expressed as,

$$Z_o = \frac{\frac{r_p R_k}{\mu + 1}}{\frac{r_p}{\mu + 1} + R_k}$$

$$Z_o = \frac{\frac{r_p R_k}{\mu + 1}}{\frac{R_k\,(\mu + 1) + r_p}{\mu + 1}}$$

$$Z_o = \frac{r_p R_k}{r_p + R_k\,(\mu + 1)}$$

CATHODE RESISTOR. The size of the cathode resistor is an important design consideration in that it largely determines the output impedance, a factor of special concern in impedance matching. To find the value of the cathode resistor, use the following equation:

$$R_k = \frac{Z_o r_p}{r_p - Z_o\,(\mu + 1)}$$

where Z_o equals the output impedance, r_p the plate resistance, and μ equals the amplification of the tube.

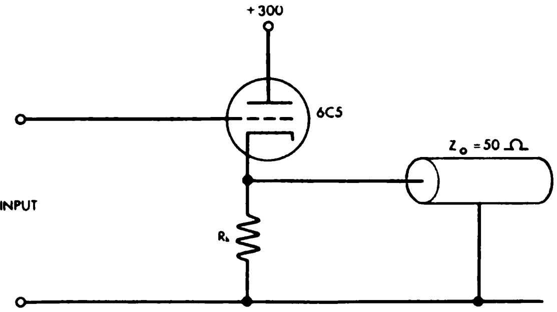


Figure 7–41. Impedance Matching in a Cathode Follower

Example

If the 6C5 triode shown in Figure 7–41 is connected as a cathode follower and has a plate voltage of 300 volts, a plate resistance of 10,000 ohms, and an amplification factor of 20, find the size of the cathode resistor and the gain.

Solution

To find the size of the cathode resistor, substitute in the formula,

$$R_k = \frac{Z_o r_p}{r_p - Z_o\,(\mu + 1)}$$

$$= \frac{50 \times 10^4}{10^4 - 50\,(20 + 1)}$$

$$= \frac{500{,}000}{10{,}000 - 1050}$$

$$= \frac{500{,}000}{8950}$$

$$R_k = 55.8 \text{ ohms}$$

To find the gain, substitute in the formula,

$$\text{Gain} = \frac{\mu R_k}{r_p + R_k\,(\mu + 1)}$$

$$= \frac{20 \times 55.8}{10^4 + 55.8\,(20 + 1)}$$

$$= \frac{1116}{11171.8}$$

$$\text{Gain} = 0.1 \text{ (approximately)}$$

PHASE INVERTERS

A phase inverter is a circuit that produces an output voltage of opposite polarity to the input voltage without distorting the waveshape. Phase inverters are commonly used in radar equipment. For example, sweep voltages are usually applied to the deflection plates in electrostatic cathode-ray tubes in push-pull, since this type of operation reduces the defocusing effects resulting from applying the sweep voltage to only one of the pair of deflection plates. To obtain the push-pull operation, a phase inverter circuit is used.

Literally, the commonly accepted term, *phase inverter*, is something of a misnomer, since phase is ordinarily associated with the time and there is no appreciable time difference or phase shift between the output and input circuits of the ordinary phase inverter. Such a circuit is only an *apparent* phase inverter. In reality it is a *polarity inverter.*

Polarity Inversion by an Untuned Transformer

A simple method of inverting the polarity of a waveshape is by an untuned transformer. To understand how a transformer inverts polarity, assume that the output wave pulse at the plate of tube V_1 in Figure 7–42 is positive and that tube V_2 requires a negative pulse. This requirement can be met by using the polarity-inverting property of an ordinary transformer since, in all transformers, a current through the primary induces a voltage in the secondary of opposite polarity to the primary voltage. (Of course if the output or input connections to the transformer are reversed, the output and input voltage will have the same polarity.) In transformer inversion, it is clearer to think of the output as a voltage whose polarity is inverted relative to the primary voltage, except perhaps when a sine wave signal is used, where the polarity inversion is referred to as 180° phase shift.

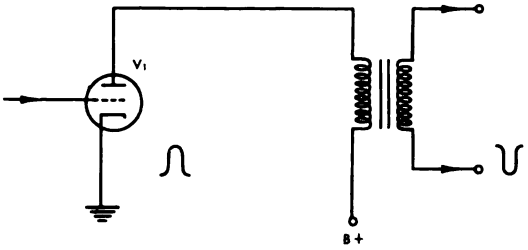


Figure 7–42. Polarity Inversion

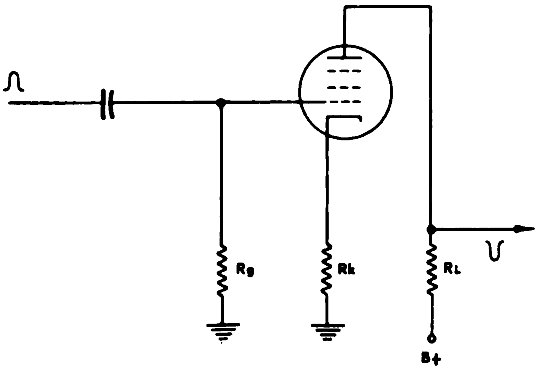


Figure 7–43. Polarity Inversion Without Amplification

Polarity Inversion Without Amplification

Under some conditions, it is necessary to reverse the polarity of a pulse or a waveform without changing its amplitude. Although a transformer may be used, it is much better to use an ordinary RC-coupled amplifier with an unbypassed cathode resistor as shown in Figure 7–43. This circuit inverts the phase since any vacuum tube amplifier with a resistive load has an output of opposite polarity to the input. In other words, a positive-going signal on the grid produces a negative-going signal at the plate. There is no amplification because of the degenerative feedback introduced into the grid by the unbypassed cathode resistor. This degeneration occurs because the cathode voltage rises as the grid voltage rises, preventing the swing of the voltage between the grid and cathode from reaching the amplitude of the applied grid signal.

Paraphase Amplifier

A *paraphase* amplifier is a combination amplifier and phase inverter, which converts a single input into a push-pull output. Paraphase amplifiers are used where waveshapes of equal amplitude and opposite polarity are required for operating circuits in push-pull, as in the electrostatic deflection circuits in cathode-ray tube equipment. There are two types of paraphase amplifiers — the single-tube paraphase amplifier, and the two-tube paraphase amplifier.

Single-Tube Paraphase Amplifier

With a single-tube paraphase amplifier (Figure 7–44) the output is taken from both the cathode and the plate. The cathode resistor R_L and the plate resistor R_L are the load resistors. These resistors are equal and since the same current flows through both, equal voltages appear across them. The voltages across these resistors are opposite in polarity since the output is taken from the positive end of the cathode load resistor and from the negative end of the plate load resistor.

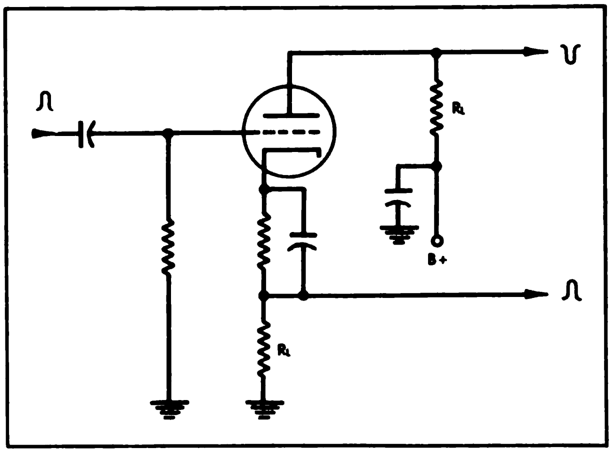


Figure 7–44. Single-tube Paraphase Amplifier

Two-Tube Paraphase Amplifier

The two-tube paraphase amplifier consists of one tube that acts as a conventional amplifier, and a second that inverts the output of the first tube. The two tubes in combination thus produce two equal output voltages opposite in polarity.

Figure 7–45 shows the circuit diagram of a typical two-tube paraphase amplifier. The first tube V_1 amplifies the input waveform shown at its grid and impresses the amplified output across the voltage divider consisting of R_1 and R_2. The resistor R_2 is of such value that the varying voltage across it has the same amplitude as the voltage on the grid of V_1. The voltage across R_2 is impressed on the grid of V_2, the phase inverter tube, where it is amplified. Since the plate load resistors, R_5 for V_1 and R_4 for V_2, are equal, the outputs of the two tubes are equal. The phase inverter inverts the phase of the voltage applied to its grid, making it opposite in phase to the voltage output of V_1. Note in this connection that the waveshape in the output of V_2 is in phase with the grid voltage to V_1. Phase inversion has occurred in V_1 and again in V_2, thus shifting the phase of this voltage back to its original polarity.

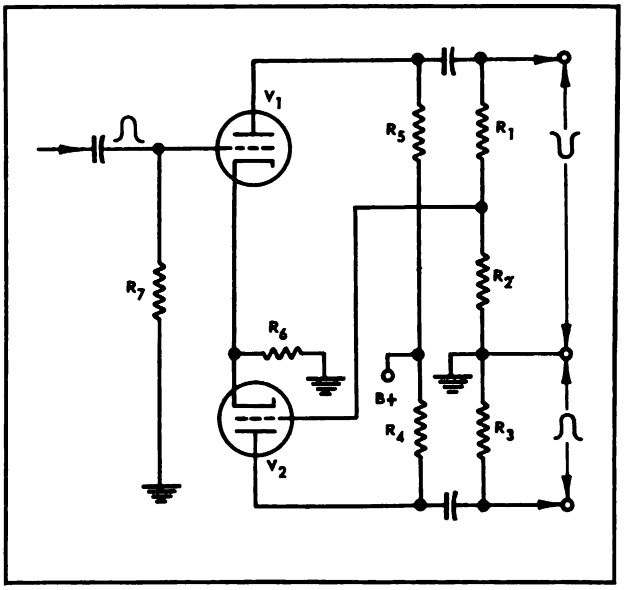


Figure 7–45. Two-tube Paraphase Amplifier

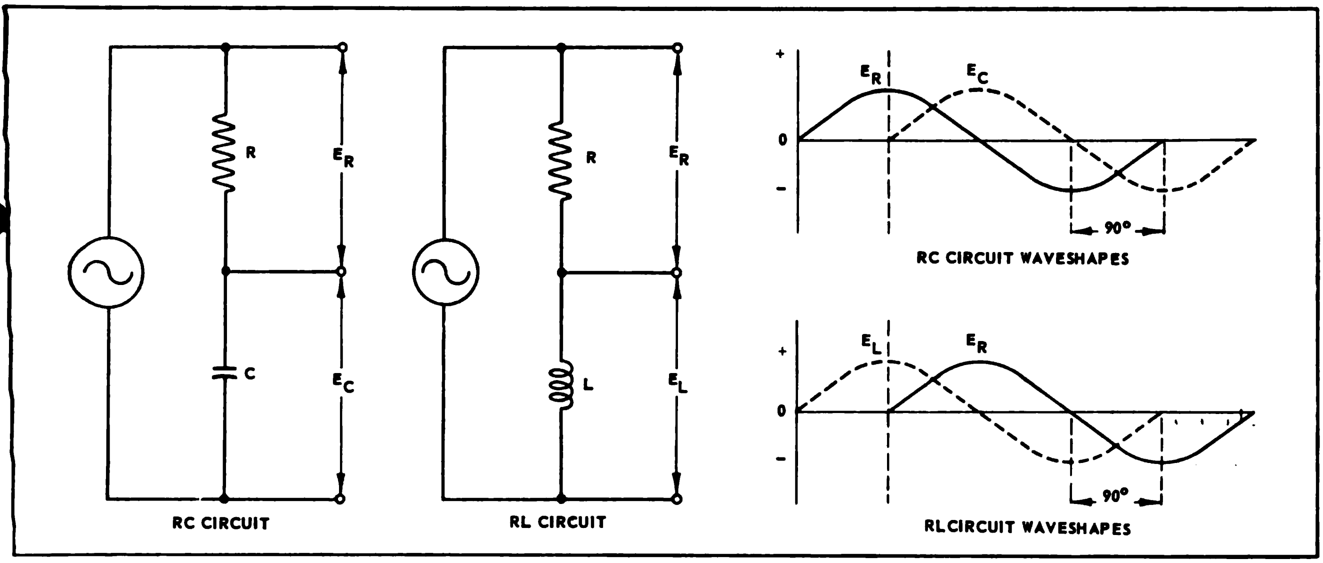


Figure 7–46. RC and RL Phase Splitting

PHASE-SPLITTING CIRCUITS

A phase-splitting circuit is one which produces, from the same input, two output waveforms which differ in phase from one another. An example of the use of a phase-splitting circuit is in the generation of the circular sweep in the type J scan indicator. The sweep voltage for this scope is generated by two out-of-phase voltages applied to the deflector plates. The two out-of-phase voltages are produced by a phase-splitting circuit.

Two types of circuits used for producing two out-of-phase output waveforms from the same input are the RC phase-splitting circuit, and the RL phase-splitting circuit.

Referring to the RC circuit in Figure 7–46, notice that R and C, which are connected in series, represent the load impedance. The same current flows through both R and C. The voltage across R is always in phase with the current in it since, in a resistive circuit, current and voltage are in phase. In a capacitive circuit, current and voltage are 90° out of phase with current leading. Therefore, the voltage across C is 90° out of phase with the current. Thus, the two voltages — that across R and that across C — are 90° out of phase with each other. An identical phase shift occurs in an RL circuit. The only difference is that the current lags the voltage in L by 90°. The voltages, however, like those across R and C in the RC circuit, are 90° apart.

OSCILLATORS

An oscillator (Figure 7–47) is a device capable of converting direct current into an alternating current at a frequency determined by the values of the constants in the device. Generally, this device is a vacuum tube since a vacuum tube has the ability to amplify — one requirement of an oscillator. When a portion of the amplified output energy of a vacuum tube is fed back into the grid circuit in the correct phase, oscillations occur. Because of this property, an oscillator may be regarded as a self-excited amplifier.

Oscillators in radar sets perform a large number of functions. They are used as UHF generators in transmitters, as local frequency oscillators in receivers, and as master oscillators in timing circuits, pulse-forming circuits, gating circuits, and sweep-producing circuits.

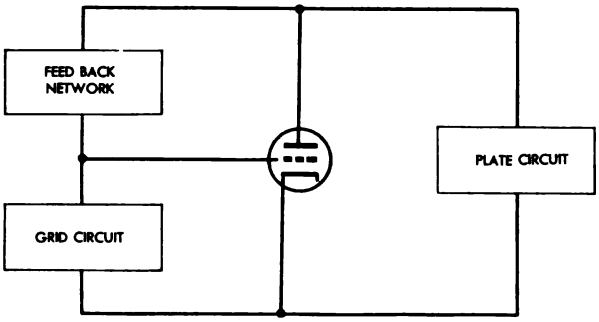


Figure 7–47. Block Diagram of an Oscillator

As previously mentioned, a vacuum tube is able to oscillate because of its amplifying ability. A vacuum tube oscillates because any small voltage change in the plate or grid circuit can be transferred from one circuit to the other by the process of amplification. Whenever a portion of the amplified output of an oscillator is fed back into the grid circuit in the proper phase, oscillations will start. They will continue as long as in-phase voltage is applied to the grid circuit by the output circuit. Feedback of this kind is called regenerative, or positive, since it is in phase with the grid voltage and sustains oscillations.

In a vacuum tube there is a normal phase shift between the grid and plate. Hence, without an additional shift in phase, the voltage returned to the grid would not have the correct polarity, or phase, to sustain oscillations in an oscillator. Therefore, a feedback network must be used to shift the feedback voltage another 180° so that it is in phase with the initial grid voltage. The following devices are employed as feedback networks for this purpose:

- Transformers
- RC networks or LC networks
- Interelectrode capacitances
- Additional vacuum tubes

A very important characteristic of regenerative feedback is that in supplying the input power to an oscillator, it effectively introduces a negative resistance into the input circuit of the oscillator. Although negative resistance cannot be represented by a physical resistor, it provides a concept that is convenient in discussing oscillators, since it represents the source from which energy is obtained to replace the energy losses in the oscillating circuit.

This means that an oscillator, to keep on oscillating, must have energy supplied from some source to make up for losses in the feedback circuit. Otherwise, it will stop oscillating after a period because of the accumulated losses. Negative resistance at the input explains how these circuit losses are replaced. A circuit with negative resistance is one which transfers energy from one place to another with an increase in effective energy rather than a decrease. A negative resistance circuit is a generator of energy and not a user of energy.

There are a number of oscillator types that you should know something about. Two basic types are the Hartley oscillator and the Colpitts oscillator. Other oscillators are the tuned-plate tuned-grid oscillator, the crystal oscillator, the ultraudion oscillator, the multivibrator (which was discussed in the preceding chapter), the resistance-capacitance coupled oscillator, and the shock-excited oscillator.

Hartley Oscillator

There are two forms of Hartley oscillators — the series-fed and the shunt-fed. In the series-fed Hartley, direct current flows through part of the tank circuit. In Figure 7–48A, note that the tube, a portion of the tank circuit, and the power-supply form a series circuit. In the shunt-fed Hartley, Figure 7–48B, direct current does not flow through any part of the tank circuit. The plate supply voltage is in parallel with the tube and tank circuit. R-f is kept out of the plate supply by an r-f choke, and DC out of the tank circuit by a blocking capacitor.

In Hartley oscillators, the frequency of oscillations is determined by the LC constant of the resonant tank circuit. Bias for the tube is provided by the grid-leak combination R_1 and C_1 (Figures 7–48A and B). Grid-leak bias or a combination of grid-leak bias and fixed bias is used, since oscillators operate Class C to secure high output and efficiency. If only a high fixed bias were used, plate current would not flow when plate voltage was applied, and thus it would be impossible for oscillations to occur. However, when grid-leak bias is used, the bias is initially zero and plate current flows immediately. After oscillations build up, the grid-leak bias becomes proportional to the r-f voltage across the grid tank circuit and maintains a value providing a stable oscillatory condition.

Voltage for sustaining oscillations in the Hartley oscillator is fed from the tank circuit to the grid circuit by mutual inductance between the plate and grid coils. Adjusting the position of the taps connecting the grid, cathode, and plate leads determines the magnitude of the feedback.

Colpitts Oscillator

The Colpitts oscillator is essentially the same circuit as the Hartley oscillator except that a pair of capacitances in series is connected across the tank coil. This combination, C_1 and C_2 in the circuit at Figure 7–48C, forms a voltage divider circuit that divides the voltage across the resonant circuit into two parts. The voltages at

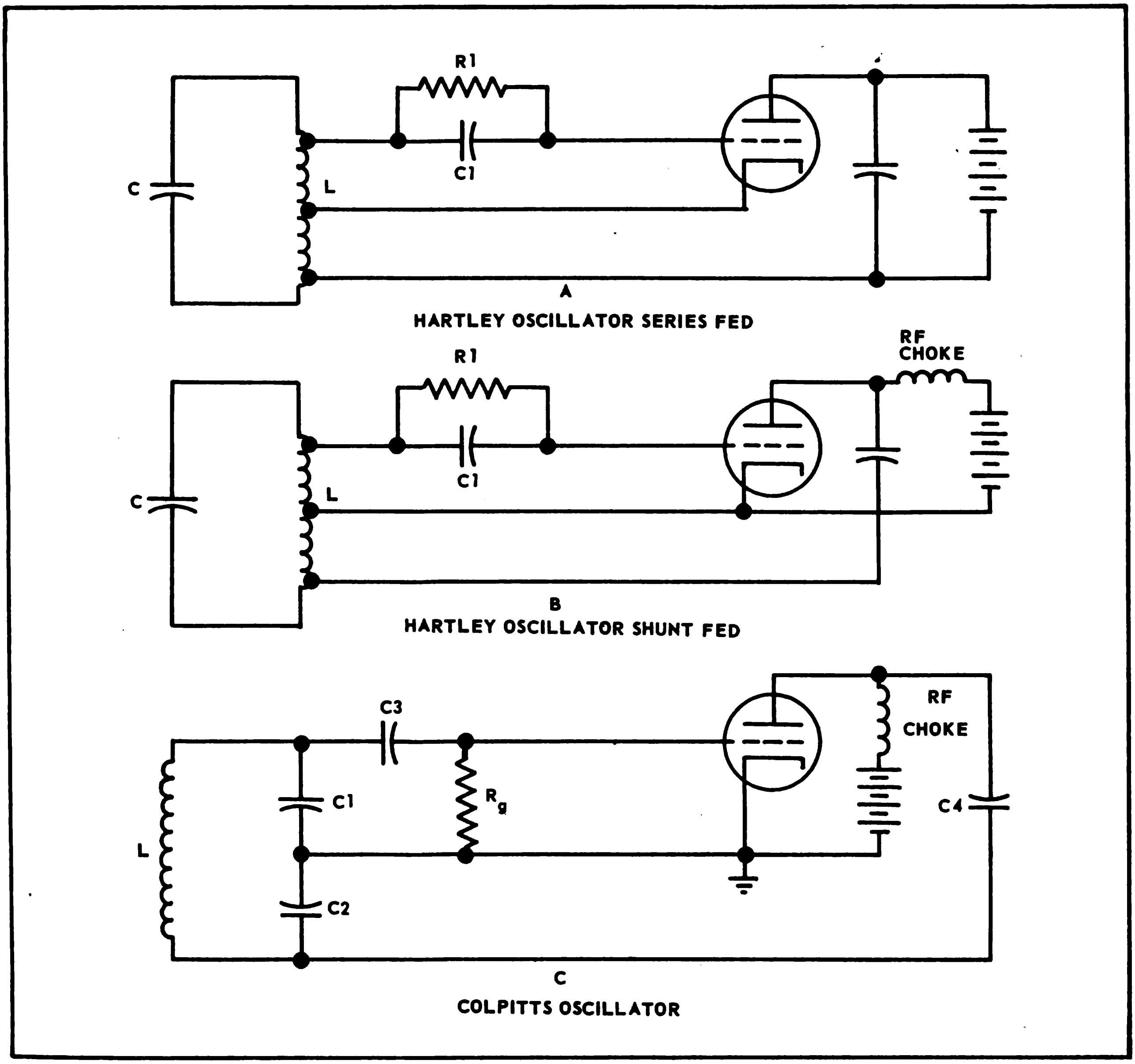


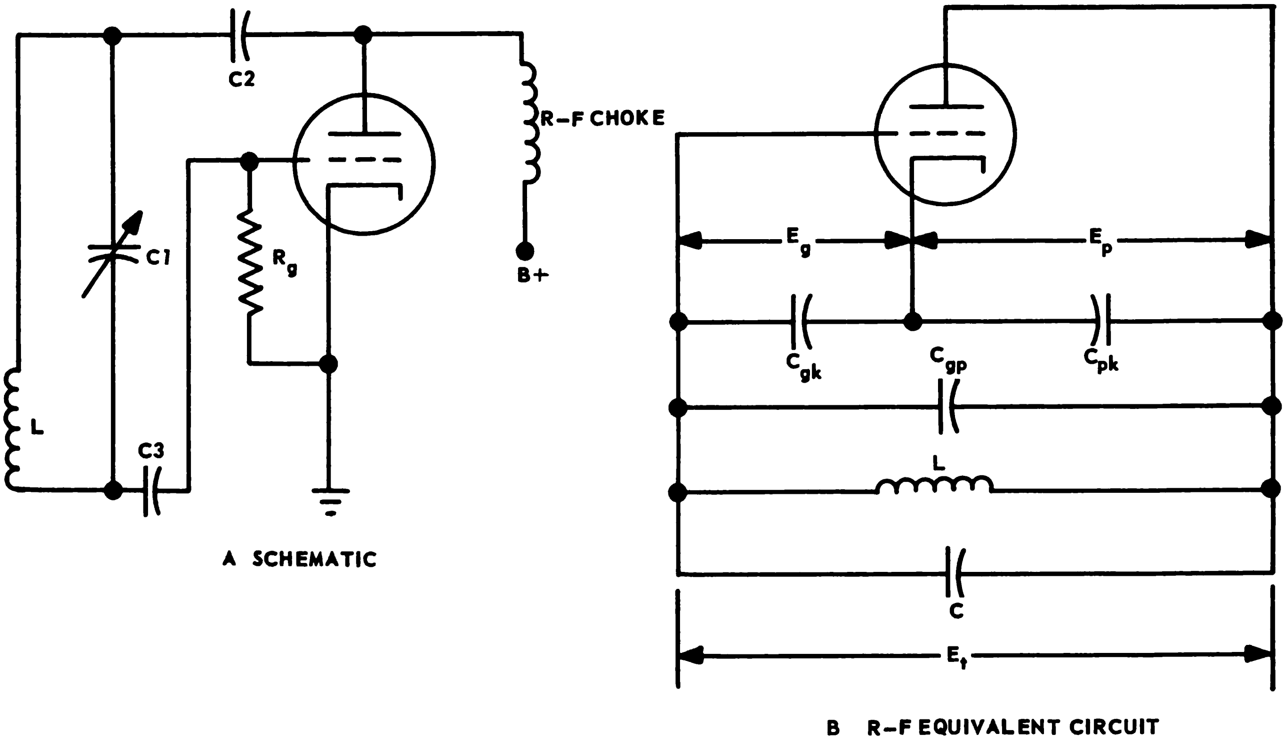
Figure 7–48. Basic Oscillator Circuits

the ends of the resonant circuit are opposite in polarity with respect to the cathode and in the right phase to sustain oscillation. The total tank capacitance consists of C_1 and C_2. C_3 and R_g constitute the grid-leak bias combination.

Ultraudion Oscillator

A type of oscillator frequently used at ultra high frequencies is the ultraudion oscillator illustrated in Figure 7–49. The required polarity inversion of the instantaneous voltage on the grid with respect to the plate results from the grid and plate being connected to opposite ends of the tank circuit. The plate is parallel fed, the r-f choke preventing the alternating component of the plate voltage from entering the power supply. The blocking capacitor C_2 is large enough to be effectively a short circuit to the r-f current but is an open circuit to the direct current.

This oscillator, as well as most other feedback oscillators, uses self-bias instead of fixed bias so that the oscillations can start more easily. If the high negative bias required for proper operation were applied at the time the oscillator



A SCHEMATIC

B R-F EQUIVALENT CIRCUIT

Figure 7–49. Ultraudion Oscillator

was turned on, the tube would be completely cut off and oscillations could not start. As oscillations are built up, a small charge is put on capacitor C_3 each time the lower end of the tank circuit swings positive. Part of the charge leaks off during the time the grid is not positive relative to the cathode. The voltage to which C_3 is charged ultimately makes the grid sufficiently negative so that only a small amount of charge is added to the capacitor at the peak of each cycle. This small increase of charge leaks off through R_g during the remaining time of the cycle. Thus, the grid is automatically maintained at the proper bias for good operation.

The ultraudion circuit is equivalent to the Colpitts oscillator. The combination of the grid-cathode capacitance and the plate-cathode capacitance forms a voltage dividing network which, in effect, grounds one point on the tank circuit. The voltage drop across C_{gk}, Figure 7–49B, provides the grid excitation. The total tank capacitance consists of the tuning capacitor, C_1, in parallel with the series combination of C_2, C_{pk}, C_{gk}, and C_3. However, C_2 and C_3 are made large in order to offer negligible reactance to the r-f current. Thus, they do not materially affect the frequency at which the tank circuit oscillates.

Tuned-Plate Tuned-Grid Oscillator

The tuned-plate tuned-grid oscillator, sometimes abbreviated as TPTG, has tuned circuits in both the grid and plate circuits. This oscillator may be used at ultra-high frequencies as well as at low frequencies.

In Figure 7–50, which depicts the TPTG oscillator, notice that the inductance in the plate tank circuit is not inductively coupled to the inductance in the grid circuit. The feedback

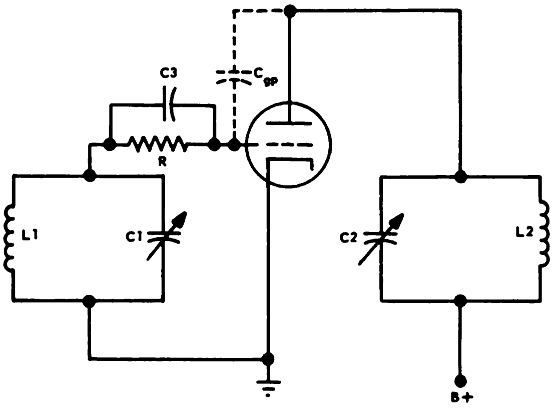


Figure 7–50. Tuned-Plate Tuned-Grid Oscillator

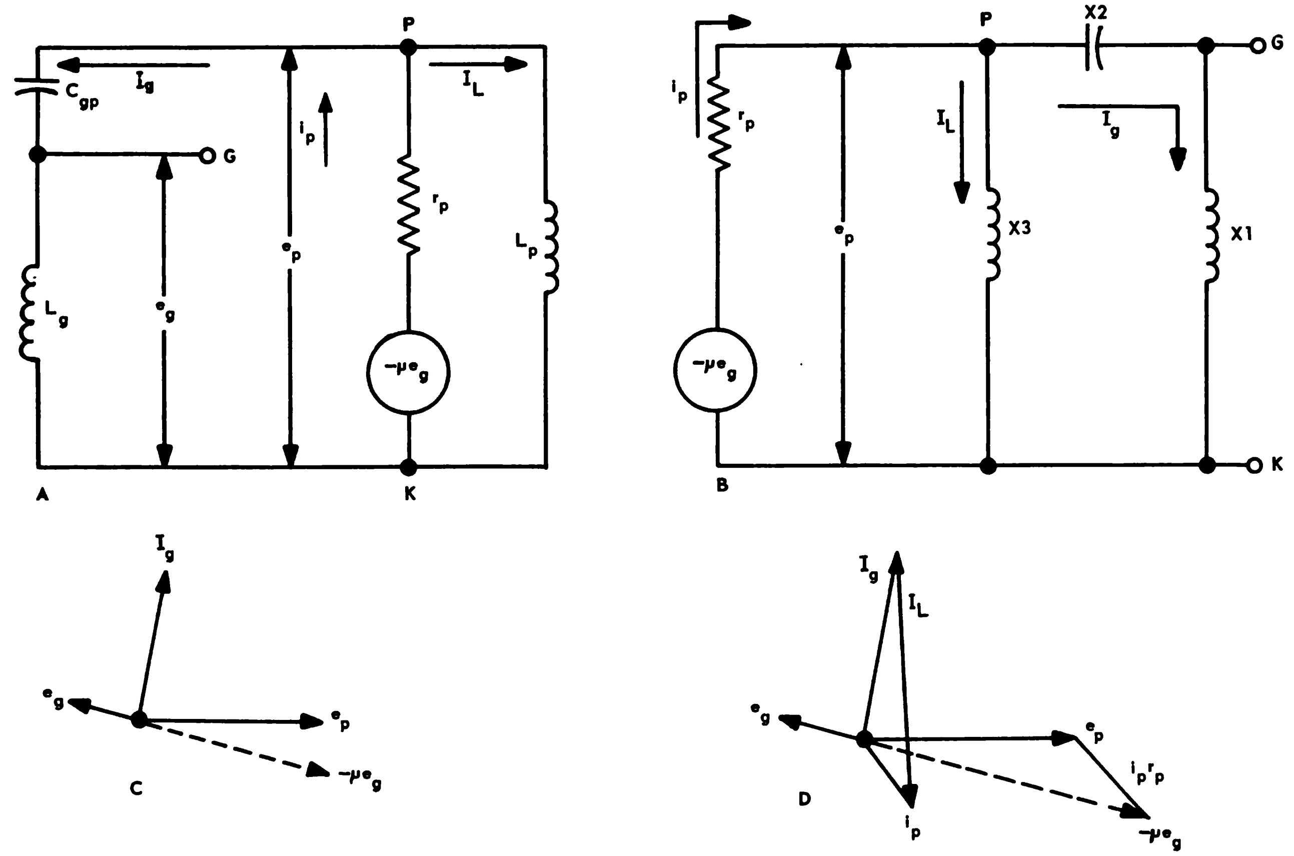


Figure 7–51. Equivalent Circuit of TPTG Oscillator

necessary for sustaining oscillations occurs through the interelectrode capacitance between the plate and grid of the tube (C_{gp}).

This circuit automatically oscillates at a frequency lower than the natural frequency of both the plate tank (L_2 and C_2) and the grid tank (L_1 and C_1). The tank that is tuned to the lower frequency, whether it is the plate or the grid tank, controls the oscillator frequency. The other circuits mainly affect the magnitude of the feedback voltage.

Figure 7–51 contains the equivalent circuit of the TPTG oscillator and vector diagrams which explain the operation of this oscillator. In B the circuit has been reduced to a π network, considering only the resultant grid-plate, grid-cathode, and plate-cathode impedances. The combination of these impedances acting as a π-section must produce approximately a 180° phase shift through the network. The reactances shown are the total equivalent grid-plate, grid-cathode, and plate-cathode reactances, including strays, interelectrode capacitances, and connected reactances. For the circuit to oscillate, the following considerations must hold. $X1 + X2 + X3 = 0$ and $X1 = X3/\mu$. From this it can be seen that X1 and X3 are of the same sign and X2 is of the opposite sign and equal to their sum.

In constructing vector diagram C, the plate-to-cathode voltage e_p was taken as a reference vector. In diagram B the current I_g flowing through the grid-plate and the grid-cathode impedances is capacitive, since the reactance due to C_{gp} (X2) is greater than the inductive reactance of the tank circuit from grid to cathode (X1). I_g leads the plate voltage e_p by less than 90° because of the power absorbed in this circuit branch. The grid-to-cathode voltage e_g, due to the current flowing through L_g, leads the current I_g. The amount of lead is less than 90° because of the grid driving power required. It can be seen from vector diagram C that e_g leads e_p by slightly less than 180°.

The equivalent voltage acting in the plate circuit, $-\mu e_g$, is drawn in the opposite direction from e_g and is shown as a dotted line. From vector diagram D we can see that the vector $-\mu e_g$ is equal to the sum of the vector e_p plus the vector $i_p r_p$. From diagrams A and B we can see that plate current i_p is equal to $I_L + I_g$. Thus we draw on diagram D a vector lagging e_p by 90°, originating at the end of the current vector I_g. This vector is extended until it meets a line drawn from the origin and parallel to the vector $i_p r_p$. The vector drawn is the current I_L, and the sum of I_g and I_L is the plate current i_p. The plate current is definitely not in phase with e_p and cannot be so long as there is a load in the grid circuit. However, the higher the Q of the circuits, the more nearly in phase these quantities are.

Resistance-Capacitance Coupled Oscillators

Many oscillators use resistance-capacitance networks to provide regenerative coupling between their output and input circuits and to determine the oscillation frequency. Such oscillators are called resistance-capacitance, or simply RC oscillators. Examples of RC oscillators are the phase-shift oscillator, the Wien Bridge oscillator, and the free-running multivibrator, which was discussed in an earlier chapter.

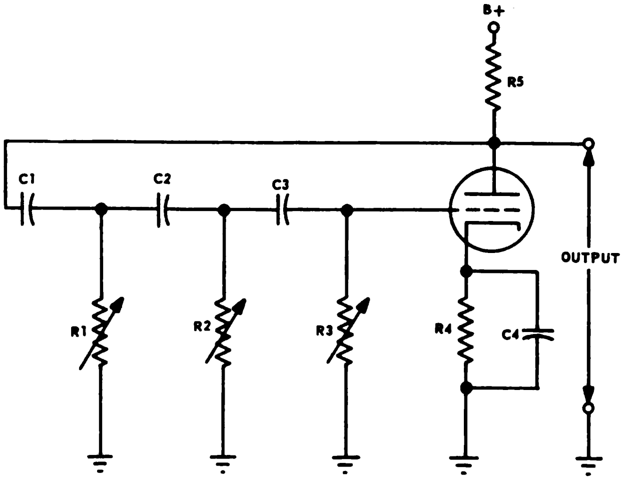


Figure 7–52. Phase-Shift Oscillator

Phase-Shift Oscillator. The phase-shift oscillator circuit, Figure 7–52, consists of a single amplifier tube and a phase-shifting feedback circuit. An oscillator requires that the signal fed back from the plate to the grid be shifted 180° in order to sustain oscillations. In phase-shift oscillators this function is performed by resistance-capacitance sections.

One of these sections is shown in Figure 7–53A. When an alternating voltage is applied to this circuit, current flows in the circuit. Since the circuit impedance is capacitive, the current leads the impressed voltage E. The voltage drop E_R across resistor R is in phase with the current that flows through it. Therefore, the voltage E_R also leads the impressed voltage by the same phase angle (60° is shown in B and C below).

When the resistance of R is varied, the phase angle of the current flowing in the circuit is also varied. If R were reduced to zero, the current would theoretically lead the applied voltage by 90°. However, adjusting R to zero would be impractical. First, because reducing R to zero would leave no impedance for developing a useful voltage, and second, because with R equal to

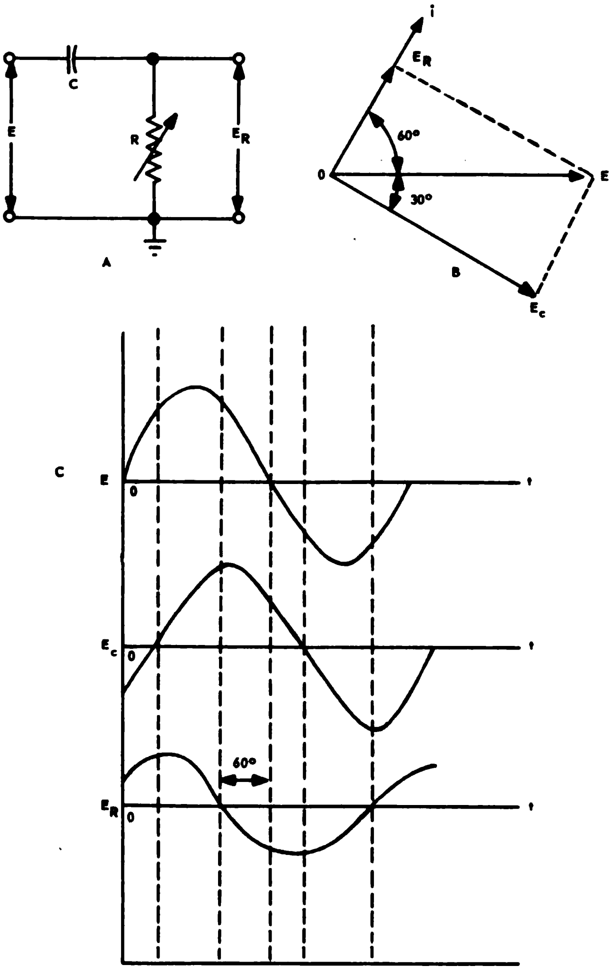


Figure 7–53. RC Phase Shifter

zero, the current would lead the voltage by slightly less than 90° due to the inherent circuit resistance. This means that there must be a minimum of three RC sections in the phase-shifting network to produce a 180° phase shift.

When the output of the first section is applied to a second phase shifter, another shift of the output voltage occurs. R_2 can be adjusted to make the output voltage of the second section the desired phase angle ahead of the voltage applied to the first phase shifter. When the output of the second phase shifter is fed to the third, another phase shift occurs. R_3 is adjusted to make the voltage output 180° ahead of the voltage at the input to the first phase shifter.

The oscillations are started in this oscillator by any slight circuit changes such as variations in the plate supply or random noises. When such disturbances occur, the slight change is amplified, inverted 180° at the plate, and one frequency is inverted another 180° by the RC network, from which it is returned in phase to the grid of the tube for reamplification. This process continues until plate-current saturation is reached where the tube cannot amplify further.

The frequency for which the network furnishes a 180° phase shift may be adjusted by varying one or more of the capacitors or resistors, the frequency being inversely proportional to the product RC. RC oscillators are widely used below 30 kc, where the elimination of inductances saves in weight, size, and cost.

WIEN BRIDGE OSCILLATOR. The Wien Bridge oscillator shown in Figure 7–54A is a two-tube RC oscillator, tuned by a resistance-capacitance bridge. One tube, V_1, serves as an oscillator and amplifier, and the other, V_2, as an inverter. This circuit would oscillate even without the resistance-capacitance bridge because of the 180° phase shift produced by tubes V_1 and V_2. However, this system would oscillate over a very wide range of frequency. The bridge circuit, therefore, is used to insure the elimination of all feedback voltages except that having the desired frequency.

Refer to the circuit in Figure 7–54B, which is another way of drawing the same circuit. A degenerative feedback voltage is provided by resistor R_3 and lamp L_{p1}. The amplitude of this feedback voltage remains nearly constant for all frequencies since the resistances are practically constant for all frequencies and since there is no phase shift across the voltage divider.

The inverter shifts the output of V_1 180°. Thus, the voltage appearing across R_2 of the bridge is in proper phase to sustain oscillations. This action occurs when $R_1 C_1 = R_2 C_2$. The frequency at which the circuit oscillates is,

$$\frac{1}{2\pi R_1 C_1}$$

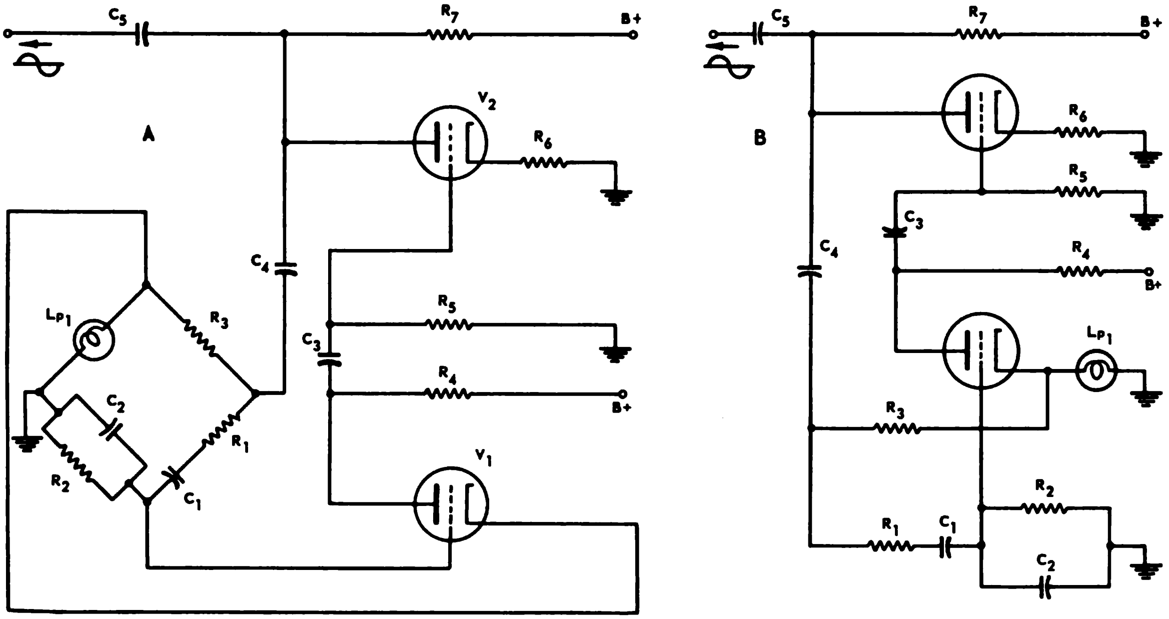


Figure 7–54. Wien Bridge Oscillator

Lamp L_{p1} is a thermistor, a device in which resistance varies directly with current. Its function is to compensate automatically for changes in the circuit in order that sufficient feedback will always be applied to V_2. When the current in the circuit increases, the filament of the lamp becomes hotter, making its resistance larger. A greater negative feedback voltage is developed across the increased resistance. Thus, more degeneration is provided which reduces the gain of V_1, and thereby holds the output voltage at a nearly constant amplitude.

Crystal Oscillators

When the frequency of a transmitter must be held within close limits, a crystal-controlled oscillator is used. This is an oscillator in which a specially cut crystal controls the frequency. Crystal-controlled oscillators are the standard means used for maintaining the frequency of radio transmitting stations within assigned limits. For the accuracy in timing needed in radar, a crystal-controlled oscillator is usually used as a basic circuit to generate an output at a highly stable, accurately measured frequency. Crystals are used also as the active element in high-fidelity microphones, loud-speakers, and electric pickup arms.

PIEZOELECTRIC EFFECT. Crystals exhibit a characteristic which makes possible their various uses. This characteristic, known as *piezoelectricity* or the *piezoelectric effect*, is the property of a crystal by which mechanical stresses produce electric charges, and conversely, electric charges produce mechanical stresses. If a voltage is applied to a crystal, it will vibrate at the frequency of the applied voltage. The amplitude of vibration of a given crystal, however, is much larger at one frequency than at all others. This frequency is the *resonant* frequency of the crystal.

This piezoelectric effect is exhibited to different degrees by crystals of various substances such as quartz, tourmaline, and Rochelle salt. Of the three, Rochelle salt exhibits this effect to the greatest degree. Quartz crystals, however, are used for frequency control in most oscillators, because quartz has greater mechanical strength.

A natural quartz crystal has a hexagonal cross-section and pointed ends, as shown in Figure 7–55. Sections cut from such a crystal exhibit the property of piezoelectricity, which may be expressed in terms of three sets of axes. The axis joining the points at the ends of the crystal is known as the *optical* or *Z-axis*. Mechanical stresses along this axis produce no piezoelectric effect. A hexagonal cross-section results when a quartz crystal is cut perpendicular to its optical axis. The three axes passing through the corners of this hexagon are called *electrical* or *X-axes*, while the three axes perpendicular to the faces of the crystal are known as *mechanical* or *Y-axes*. One X-axis and one Y-axis are shown in the illustration in Figure 7–55.

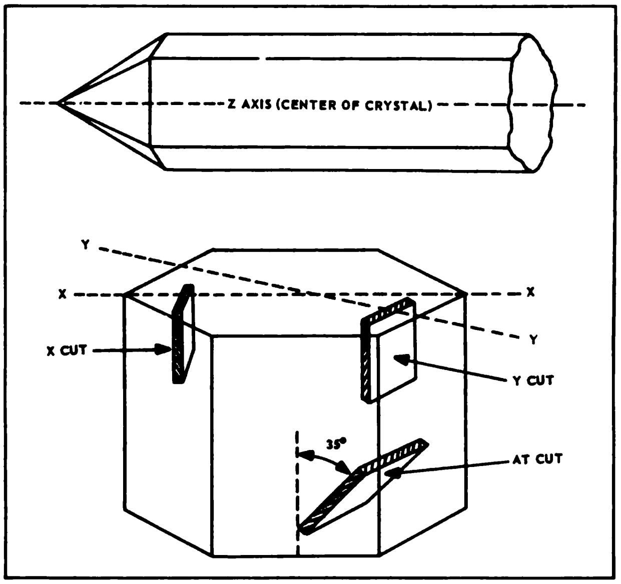


Figure 7–55. Quartz Crystal Cuts

CRYSTAL CUTS. The crystals used in electrical circuits are thin sheets cut from the natural crystal and ground to the proper thickness for the desired resonant frequency. For any given crystal cut, the thinner the crystal, the higher the resonant frequency. The cut of the crystal means the definite way in which the usable crystal is cut from the natural crystal. Crystals are cut at various angles with respect to the X, Y, or Z-axis. The various types of cuts are X, Y, AT, BT, CT, DT, ET, FT, and GT. Only the X, Y, and AT cuts are shown in Figure 7–55. In the X and Y cuts, the face of the crystal is cut parallel to the Z-axis. AT, BT, CT, DT, ET, FT, and GT are called special cuts. They are cut with the face of the crystal at an angle to the Z-axis.

Table 7-1
Characteristics of Various Crystal Cuts

Type Cut	Frequency Range	Temperature Coefficient	Remarks and/or Characteristics
X	100 kc to 1100 kc	Negative values −10 to −25 parts per million per degrees C.	Complicated spectrum. Low activity. Obsolete cut.
Y	Up to 10 mc	−20 to +100 parts per million per degrees C.	Discontinuities in frequency with temperature changes. Obsolete cut.
AT	500 kc to 10 mc	Zero, 40° to 50° C.	High activity, extensively used.
BT	500 kc to 10 mc	Zero, 20° to 30° C.	High activity, extensively used.
CT	50 kc to 500 kc	Zero, 20° to 30° C.	Extensively used cuts. Avoids use of large quartz plates for low frequencies. These two cuts are directly related to AT & BT cuts; however, they are physically smaller for these low frequencies.
DT	50 kc to 500 kc	Zero, 20° to 40° C.	
ET	100 kc to 1 mc	Zero, 70° to 80° C.	Harmonic types. Crystal produces output voltages at 3rd, 4th, 5th, 6th, and 7th harmonics of fundamental vibration.
FT	100 kc to 1 mc	Zero, 70° to 80° C.	
GT	100 kc	Zero, 10° to 100° C.	Excellent secondary standard of frequency. Excellent temperature-versus-frequency characteristics. Operates best at 100 kc.

The purpose of the special cuts is to increase the stability of the crystal by reducing its coefficient of temperature drift. Temperature drift is the change in the resonant frequency of the crystal with changes in temperature. The *temperature coefficient* is the relationship between the frequency change and the temperature change. If the frequency increases with a temperature rise, the crystal has a negative temperature coefficient. If variations in temperature have little or no effect on resonant frequency, the crystal has a zero temperature coefficient. The temperature coefficient is usually given as the number of cycles per megacycle change, per degree centigrade temperature change.

The temperature coefficient is determined by the cut. The X-cut has a negative temperature coefficient. The Y-cut temperature coefficient is both negative and positive, depending upon the temperature. The AT-cut has a very low or practically zero temperature coefficient. The GT-cut has a zero temperature coefficient over a wide temperature range. The characteristics of various crystal cuts may be seen in Table 7–1.

Transmitters which require a very high degree of frequency stability, such as broadcast transmitters, use temperature-controlled ovens to maintain the crystal temperature in the proper range. These ovens are thermostatically controlled containers in which the crystals are placed. Of course, crystals with special cuts are used with a temperature coefficient as close to zero as possible.

The type of cut also determines the activity of the crystal. Some crystals vibrate at more than one frequency and thus will operate at harmonic frequencies. These crystals are commonly called crystals with multiple peaks. Crystals which are not of uniform thickness may have two or more resonant frequencies. Usually one resonant frequency is more pronounced than the others, and the others are called spurious frequencies. Sometimes such a crystal oscillates at two frequencies at the same time. In other cases, the crystal jumps frequency as the plate tank capacitor is varied past a certain point.

The amount of current that can safely pass through a crystal ranges from 50 to 200 milliamperes. When the voltage amplitude is too high, the amplitude of mechanical vibration becomes too great, and the crystal cracks. The crystal temperature increases when the current is too high, and the crystal scorches or burns.

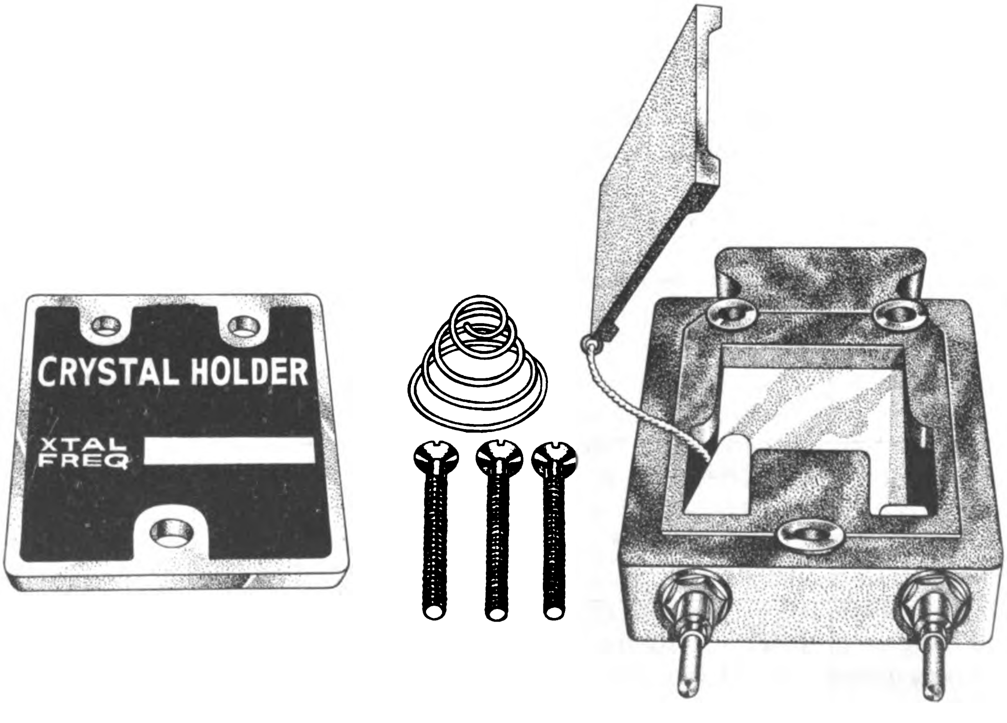


Figure 7–56. Typical Crystal Holder

CRYSTALS IN CIRCUITS. Crystals are mounted in holders which support them physically, and provide the electrodes by which voltage is applied to them. In addition, the holder must allow the crystal maximum freedom of vibration at all times. Many types of crystal holders are used for this purpose. The one shown in Figure 7–56 is one type of holder used in mounting crystals. Note that in this holder the crystal is placed between two parallel plates with a small air gap between the crystal and the top electrode.

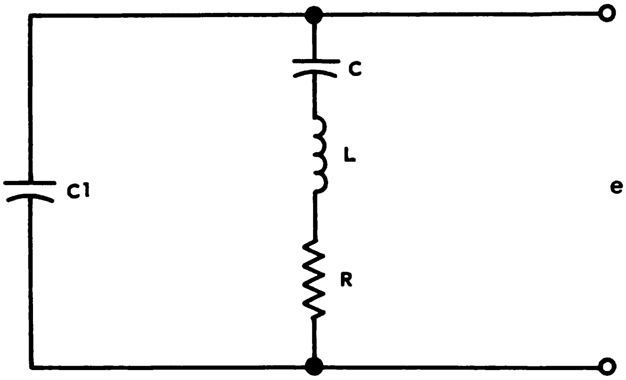


Figure 7–57. Equivalent Circuit of Quartz Crystal and Holder

This arrangement may be used with AT and BT crystal cuts as well as in cases where the crystal is not subjected to mechanical shock. Crystal holders have a significant effect on the frequency. In fact, some variable gap holders shift the frequency of a 14-mc crystal as much as 20 to 50 kc when the air gap is varied.

The crystal and its mounting are usually enclosed in a sealed container for protection from moisture and other climatic hazards. In some cases, the holder is evacuated and provided with a thermostat and heater (electric oven) to maintain the temperature of the crystal constant.

The frequency stability of crystal-controlled oscillators depends on the Q of the crystal and its temperature coefficient. The Q of a crystal is very high. It may be more than 100 times greater than that obtained with an equivalent electrical circuit. The Q of the crystal is determined by the cut, the type of holder, and the accuracy of grinding. Commercially produced crystals range in Q from 5000 to 30,000 while some laboratory experimental crystal range in Q up to 400,000.

The following data show the relative size and characteristics of a typical crystal.

Dimensions:

Thickness	0.25 inch
Width	1.3 inches
Length	1.08 inches
Resonant frequency . . .	430 kilocycles

Equivalent electrical characteristics:

Inductance	3.3 henrys
Capacitance	.042 mmf
Capacitance of holder . .	5.8 mmf
Q	23,000

When placed in an electrical circuit, a crystal can act like a very high Q series resonant circuit. The electrical circuits associated with a vibrating crystal can be represented by an equivalent circuit composed of resistance, inductance, and capacitance, as shown in Figure 7–57. In this circuit, C_1 represents the capacitance between the metal plates of the crystal holder. When the crystal is not vibrating, the circuit acts only as this capacitance. The series combination of L, C and R represents the electrical equivalent of the vibrating crystal characteristics.

The frequency stamped on a crystal holder is usually the parallel resonant frequency of the crystal and its holder as applied in an oscillator circuit. This does not mean that an ordinary coil and capacitor could be substituted for the crystal to give the same electrical characteristics. The Q of the crystal circuit is many times higher than that of an equivalent LC-type circuit. This high Q factor gives circuit stability to the crystal which could never be obtained in an ordinary tank circuit.

Crystal-Controlled Oscillator

Since a vibrating quartz crystal is equivalent to a resonant circuit, it can be used in place of the usual tuned circuit as a frequency controlling element in an electron tube oscillator. The circuit shown in Figure 7–58 represents a common type of crystal oscillator. It is similar to the tuned-plate, tuned-grid oscillator, with the crystal replacing the tuned-grid circuit. The feedback takes place through the plate-to-grid interelectrode capacitance. The oscillations occur at the resonant frequency of the crystal, and the plate circuit is tuned slightly higher, or inductively.

The operation of this circuit is similar to that of the tuned-plate, tuned-grid oscillator. When the filament has heated the cathode to the emission point, the plate voltage is applied, and the tube starts conducting. The surge of plate current flowing through L releases energy to the resonant plate circuit and starts it oscillating. The r-f voltage developed by the alternating current in the plate tank is applied to the crystal through the plate-to-grid capacitance and bypass capacitor C_1.

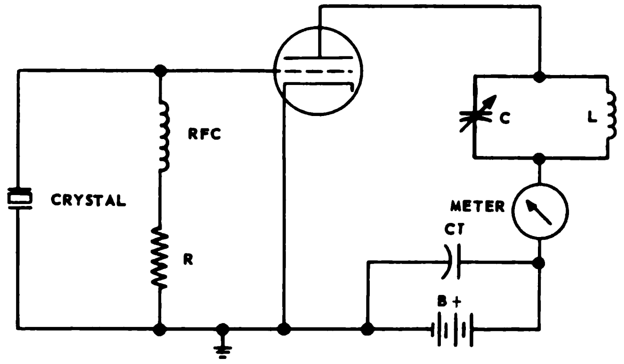


Figure 7–58. Crystal Controlled Oscillator

Placing this voltage across the two plates of the crystal holder causes the crystal to bend physically. When the first surge of plate current is over, the crystal springs back to its normal shape. In doing this, it generates a voltage of its own, causing the grid to become positive. The grid then draws current, becomes negative, and the cycle is repeated. If enough energy is applied back to the circuit in the proper phase to make up for resistive losses, the process continues at the mechanically resonant frequency of the crystal.

A resistor is connected from grid to ground through a choke to allow electrons which have been attracted to the grid on the positive swing to return to the cathode ground. The current flowing through this resistor develops the bias voltage. The capacitor usually associated with the grid-leak resistor is not necessary, because the crystal and holder serve in its place.

A meter is generally used in tuning a crystal oscillator. In Figure 7–58, the meter is placed in series with the plate circuit. DC plate current flows from the cathode to the plate, then through the meter. Any change in plate current is thus shown on the meter (a DC milliammeter). When the oscillator is generating its greatest amount of r-f current, the grid draws its greatest current, and the bias is maximum. This holds the plate current to a minimum, and the meter reading is at a minimum. In tuning, the plate is first tuned for minimum current by adjusting the tuning capacitor in the plate tank; then it is detuned (the capacitance is reduced) to cause about 3 to 5 ma more plate current to flow than at minimum.

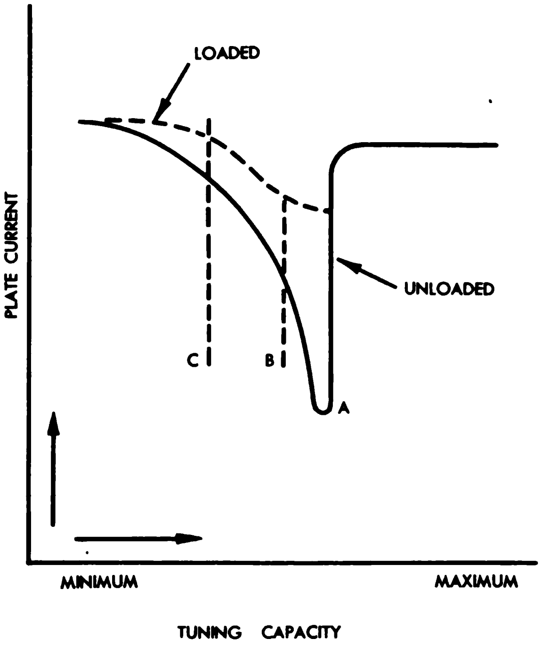


Figure 7–59. Variation of Plate Current During Crystal Oscillator Tuning

According to the frequency response curve in Figure 7–59, there is a pronounced drop in plate current when a crystal oscillator goes into oscillation. Although maximum output occurs then, a crystal oscillator should be operated between points B and C on the response curve. The reason for this is that the point of maximum output, A on the curve, produces unstable operation. This means that if the crystal is operated at this point, the oscillator becomes very erratic; it may operate intermittently, or stop after a brief oscillation. A further point is that when cathode bias is used, the plate current, under load, may rise during tuning and exceed the nonoscillating value of current. When this happens, operate the oscillator between the equivalent points B and C on the corresponding rising plate current curve.

The power output that can be obtained from a crystal oscillator is limited. If too high an

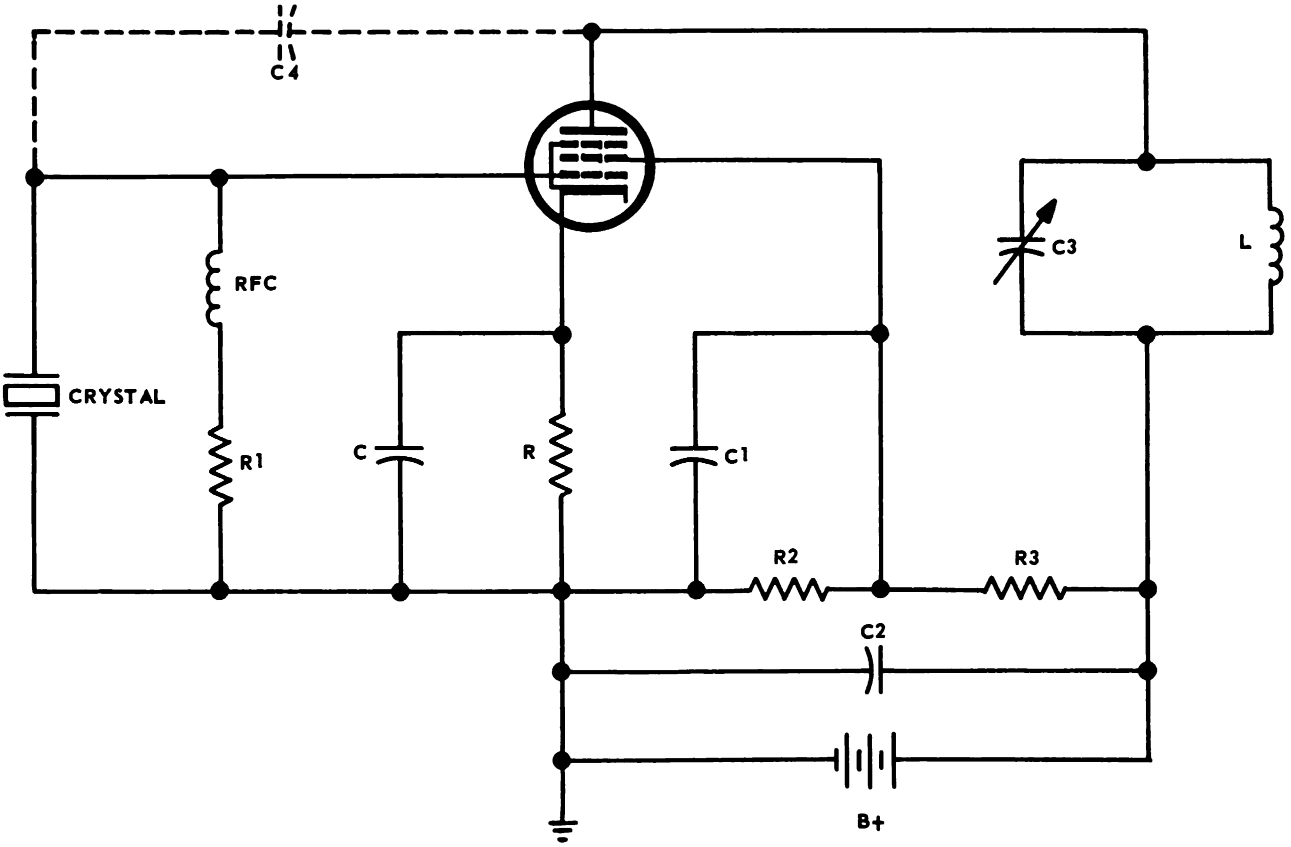


Figure 7–60. Pentode Crystal Oscillator

excitation voltage is applied to an active crystal, it may vibrate strenuously enough to shatter itself. For this reason the triode tube crystal oscillator shown is seldom used. Usually, a pentode or tetrode is used instead, for pentodes and tetrodes are tubes of high power sensitivity. With them, a comparatively high power output may be obtained from a small excitation voltage.

This means that only a small amount of excitation voltage is needed to drive the tube to full power output, and a relatively small amount of feedback is enough to meet the requirements for oscillation. The small grid-to-plate capacitance in these tubes offers high impedance to r-f and limits the grid excitation to the small required value. In fact, this feedback by the interelectrode capacitance may be too small, and it may be necessary to add an additional capacitance between grid and plate.

The operation of the typical pentode crystal oscillator circuit, shown in Figure 7–60, is identical to that of the triode just explained. Capacitor C_4 represents a capacitor that may have to be added to supply sufficient feedback to keep the circuit oscillating.

Note that grid-leak bias is supplemented by cathode bias in this circuit. Cathode bias is developed by resistor R. The DC flowing from cathode to plate develops a voltage across the resistor that makes the end connected to the cathode positive with respect to ground. Since the lower end of the cathode resistor is effectively connected to the grid by R_1 and the choke, any voltage developed across R appears between the grid and the cathode. This makes the grid negative with respect to the cathode by an amount equal to the voltage drop across R.

Capacitor C is a bypass capacitor to bypass the r-f component of plate current around the cathode resistor. The cathode resistor R is sometimes referred to as a minimum bias resistor, for bias voltage is developed across it any time plate current flows. Thus, it is independent of oscillation and prevents excessive tube current if oscillation stops. Figure 7–61 illustrates another configuration of a crystal oscillator.

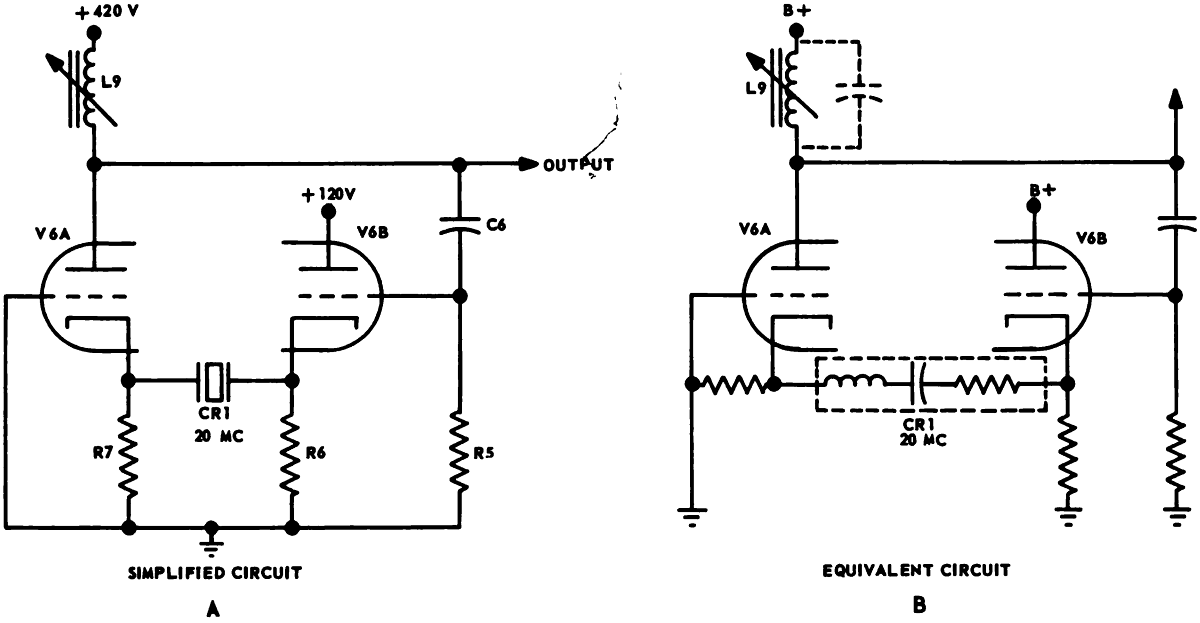


Figure 7–61. 20-Megacycle Crystal Oscillator

Shock-Excited Oscillators

A shock-excited oscillator is a type of oscillator that uses a vacuum tube as a switch to interrupt the steady flow of plate current through a resonant LC tank circuit as a means of exciting oscillations in the tank. This oscillator is used in operations where periodic oscillations of a certain frequency occurring over short intervals of time are required. Sometimes it is called a ringing oscillator.

One use of the shock-excited oscillator in radar is in range measurement operation. At the time the transmitter generates a pulse, transient oscillations are generated by a shock-excited oscillator. These oscillations continue for the duration of the range sweep. Before the transmitter sends out another pulse, the transient oscillation set up by the shock-excited oscillator is damped out. In this way the range sweep is divided into a series of known intervals, each determined by the oscillatory period of the shock-excited oscillator.

Another use of the shock-excited oscillator is in the peaker circuit. In this use the sine wave output of the oscillator is used to overdrive an amplifier stage having a low Q resonant tank. The amplifier produces very sharp, narrow peaks at a rate controlled by the voltage applied to its grid by the oscillator. The peaked output may

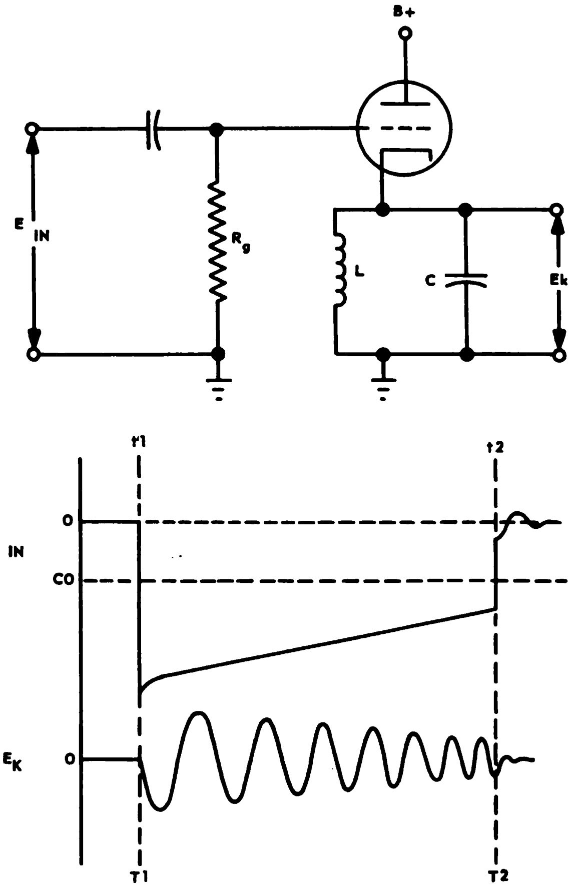


Figure 7–62. Shock-Excited Oscillator

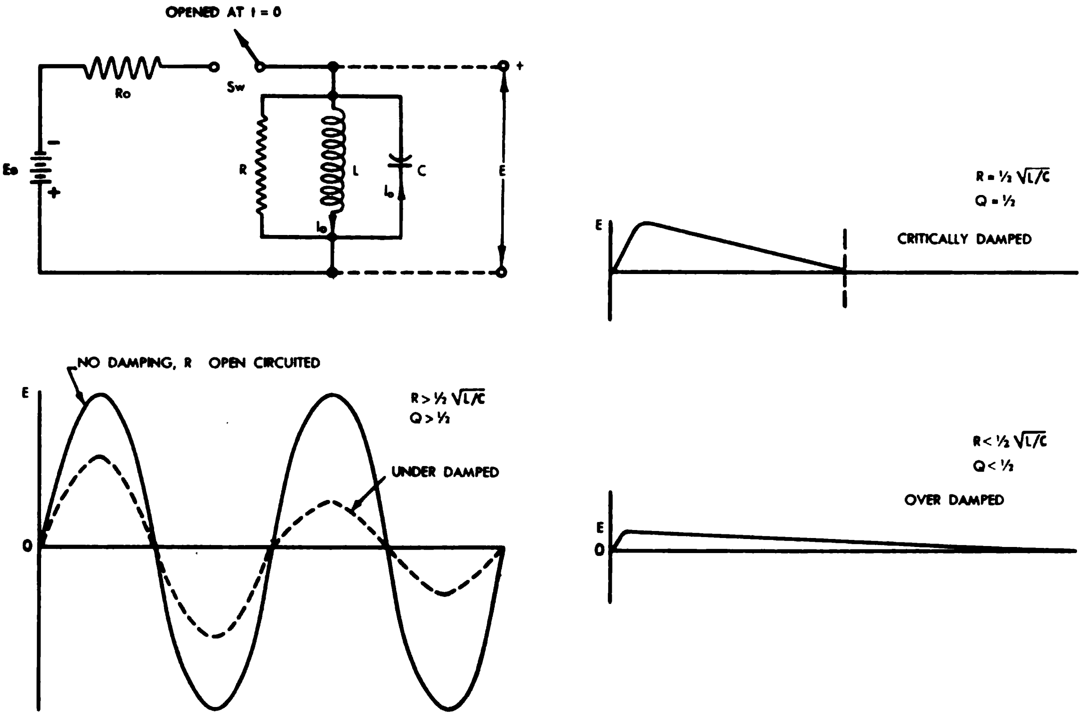


Figure 7–63. RLC Transients

be used as trigger or modulator pulses. The peaker circuit is discussed later.

Notice the circuit of a shock-excited oscillator (Figure 7–62) and the input and output waveforms of the voltages applied and produced. Before each transmitter pulse, the tube is conducting and a steady current flows through it and inductance L. At the instant (t_1) the transmitter sends out a pulse, a negative gate pulse is applied to the grid of the oscillator tube and the tube is cut off. However, the current through the inductor cannot stop instantly and flows into the capacitor, charging it in a direction making it negative, and starting the oscillations. Losses in the LC circuit are made as small as possible, with the result that the amplitude of oscillation is nearly constant throughout the duration of the input gate pulse. At the end of the gate pulse, time t_2, the tube again conducts and a second oscillation is started. However, the conducting tube is equivalent to a resistance shunted across the LC circuit and causes the oscillations to be damped quickly.

Before discussing the peaker circuit it is well to first investigate transients in the RLC circuit. Transients in RLC circuits are not all of the same shape. Their form depends upon the Q of the circuit. In Figure 7–63 the initial steady current I_o is assumed to be built up in the inductance L. The plotted variations of E are those following the opening of the switch. The curves are all drawn for the same values of L and C but for different values of Q corresponding to different values of R.

If R is open circuited, so that Q is infinite, the transient is a sinusoidal wave. Because the inductor current cannot change instantly when the switch is opened, the current I_o flows from L into C and causes the voltage to increase because of the charge placed on C. The voltage across the capacitor causes the current to decrease. The voltage reaches its maximum value when the current is zero, because a further decrease of current means a reversal of direction and a discharge of the capacitor. At $t = 0$ energy is stored in the inductance because of the magnetic field set up by the current I_o. When the inductor current decreases to zero and the capacitor voltage is at its maximum, this energy is all transferred to the electric field of the capacitor.

As the capacitor voltage decreases after reaching its peak, the energy is returned to the inductance, the return of energy being complete when $E = 0$. The reversed current in L then

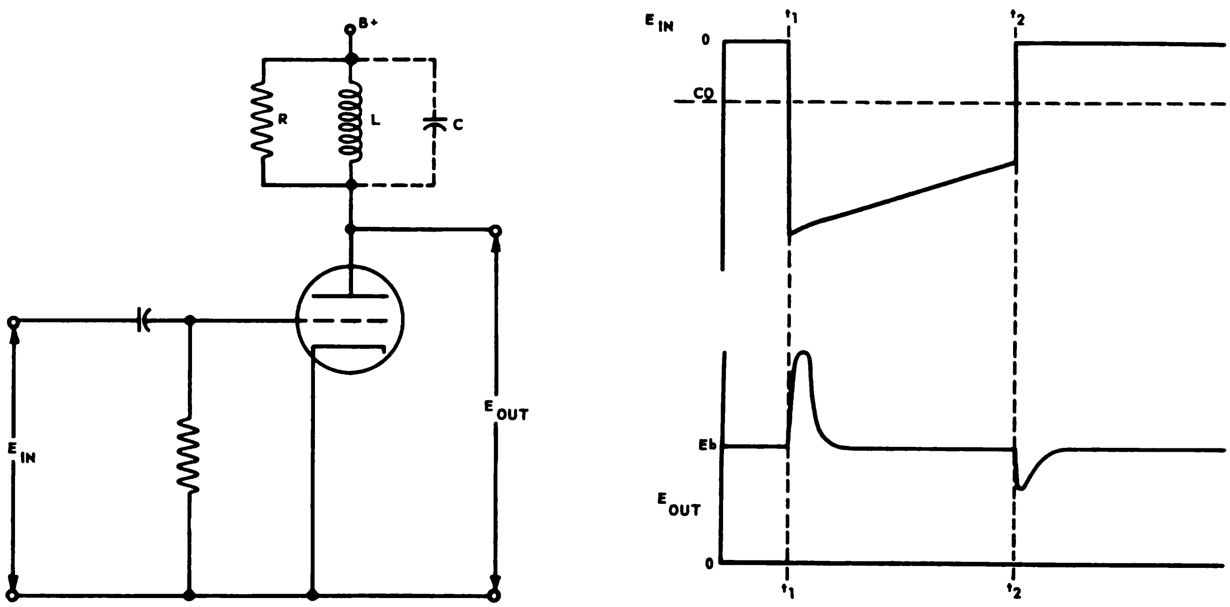


Figure 7–64. RLC Peaker

charges C in the direction to make *E* negative, and so the process continues, energy passing from one element to the other and back, and *E* oscillating above and below the zero axis. Since the circuit is assumed to be lossless, the amount of energy transferred does not change, and the oscillation continues at a constant amplitude.

If a high resistance R is used in the circuit, oscillating variations of *E* are still obtained but the amplitude of oscillation decreases from one cycle to the next, as shown by the dotted curve. Here Q is greater than one-half.

If the value of R is reduced, the rate of delay of the oscillations is increased. When R is reduced to $\frac{1}{2}\sqrt{LC}$ so that $Q = \frac{1}{2}$, the damping is so rapid that no negative swing of *E* is obtainable, and the circuit is said to be critically damped.

If the value of R is further reduced so that Q is less than one-half, the over damped transient is obtained.

The RLC peaker circuit in Figure 7–64 is actually another form of shock-excited oscillator. The capacitance is usually a lumped inductor and circuit capacitance. A resistance is connected across the LC circuit to produce nearly critical damping when the tube is cut off. Instead of an oscillation, therefore, a single sharp, positive peak is produced at time t_1, and a negative peak at time t_2. The negative peak is smaller than the positive one because of the additional damping provided by the conducting tube after time t_2. The circuit is connected to the plate rather than the cathode in order to obtain an output pulse of greater amplitude and steeper leading edge than the input pulse.

CHAPTER 8

Transistors

HISTORY OF TRANSISTORS AND SEMICONDUCTORS

Basically, the transistor is a device that controls the flow of electrical charges. The materials of which the transistor is made are classified as semiconductors. A transistor transfers the signal current from a low resistance circuit to a high resistance circuit. The transistor has a number of significant advantages over the electron tube, which enhance its use in modern electronic equipment. Transistors are replacing electron tubes in most of the usual applications.

Crystal Rectifier

The crystal was first used as a rectifier (detector) in the early days of radio. The crystal was fastened in a small cup with a flexible wire called a cat-whisker lightly touching the crystal. The adjusting arm was moved until the cat-whisker was in the spot on the crystal that produced a sound in the headset. This was the point of maximum receiver sensitivity. Since this adjustment was very critical, it had to be made over and over. In addition, a variable capacitor in parallel with the antenna coil was used to tune to the desired signal.

Point Contact Diode

During World War II electron tube diodes were replaced by point contact diodes — germanium rectifiers — in many radar and other high-frequency applications. Advantages over the electron tube were that the point contact diode did not require heater power and had a very low shunt capacitance. Figure 8–1 shows the point contact diode to be identical with the crystal rectifier. You will see that the point contact diode consists of a semiconductor, a metal base, and a metallic point contact. The two connections consist of an external lead welded to the metallic point contact, and an external lead welded to the metal base.

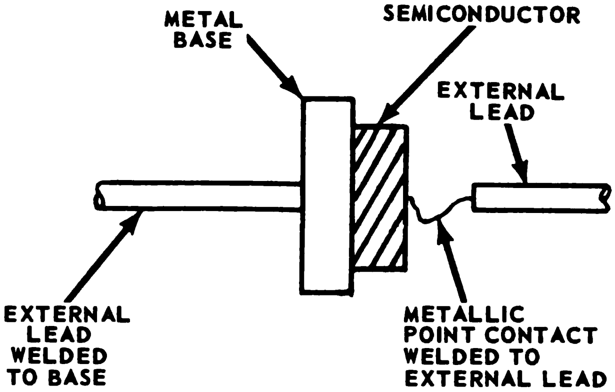

METAL BASE
SEMICONDUCTOR
EXTERNAL LEAD
EXTERNAL LEAD WELDED TO BASE
METALLIC POINT CONTACT WELDED TO EXTERNAL LEAD

Figure 8–1. Point Contact Diode

Point Contact Transistor

In 1948 the development of the point contact transistor was announced. Figure 8–2 shows that the physical construction of the point contact transistor is very similar to the point contact diode. However, a third lead with a metallic point contact is located near the other metallic point contact on the crystal semiconductor.

The two point contact leads are called the *emitter* lead and the *collector* lead. The proper

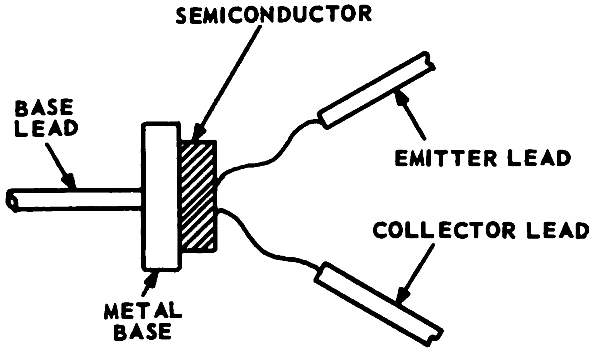


Figure 8–2. Physical Construction of Point Contact Transistor

bias applied to the two metallic points with respect to the metal *base* enables the point contact transistor to produce a power gain.

Junction Diode

In 1949 the development of the junction diode, Figure 8–3, was announced. Two dissimilar sections of semiconductor material joined together form a junction diode. Because of their characteristics, one section is called a P-type semiconductor; the other, an N-type semiconductor. The junction diode has two connections; a lead to the P-type semiconductor and a lead to the N-type semiconductor. While the junction diode has a higher power-handling capability than the point contact diode, it also has a larger shunt capacitance.

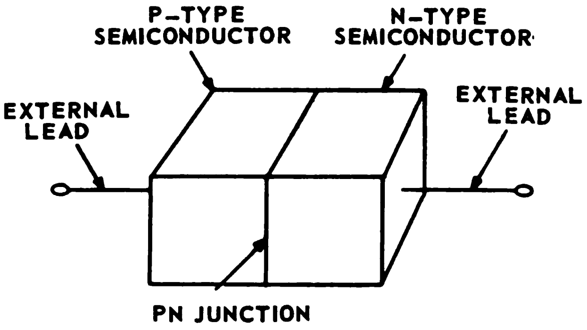


Figure 8–3. Junction Diode

Junction Transistor

The development of the junction transistor and the junction diode was announced at the same time. In Figure 8–4 you can see that the junction transistor consists of *two* PN junctions.

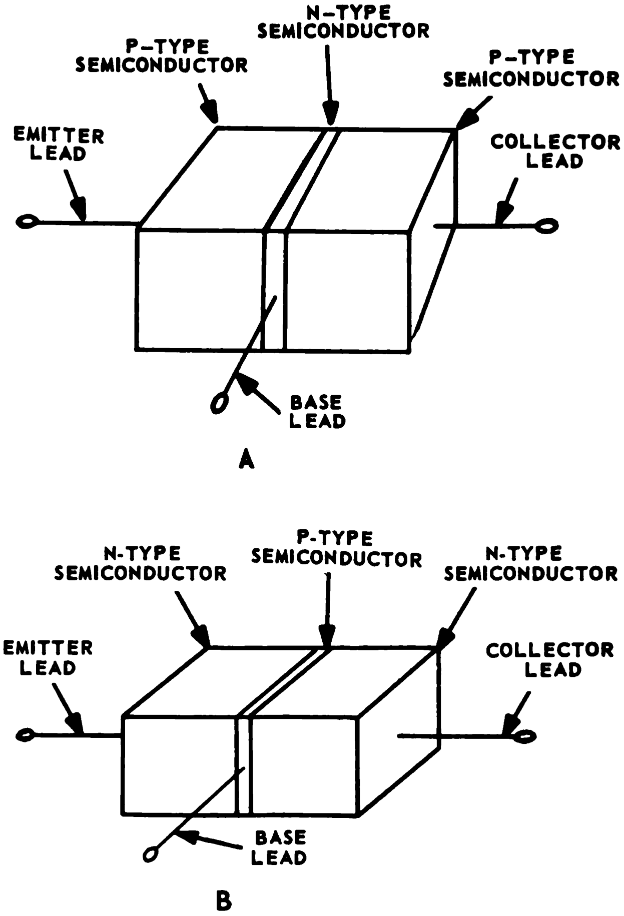


Figure 8–4. Transistor Functions

While there is great similarity in the operation of the junction transistor and that of the point contact transistor, it is easier to predict accurately a circuit's performance using a junction transistor. Also, the junction transistor has a lower signal-to-noise ratio, and can handle more power than the point contact transistor.

The transistor, like the vacuum tube, may be used as a current, a voltage, or a power amplifier. A small signal input controls a larger output. As an example, a signal input of one milliampere may control 20-milliamperes in the output. As in any electronic circuit, varying the arrangement will vary the amount of amplification.

The transistor may also be used as an oscillator, where energy from a DC source is converted to AC.

A transistor, with proper circuitry, may be used to provide amplitude modulation or frequency modulation, that is, to provide variations in the amplitude or the frequency of an r-f signal. While long range transmitters have been developed, the transistor is generally used as a substitute for electron tubes in low power applications. The transistor can also be used to demodulate (detect) amplitude-modulated or frequency-modulated signals.

The various types of radar, teletypewriter, computer, and television circuits use the transistor to modify the shape of signal waveforms. Transistor circuitry can be used to square sine waves and to clip the negative or the positive alternations of a series of positive and negative pulses.

USE IN MILITARY AND COMMERCIAL EQUIPMENT

Portable Equipment

An entirely new field in the development of portable equipment was opened when transistors came into use, for example, the transistorized "walkie-talkie." Compactness and ruggedness are characteristics of transistorized equipment that permit its military application in varied types of portable equipment. Other military uses of the transistor are in test equipment, tape recorders, meteorological equipment, and photographic equipment. In commercial portable equipment we find the transistor used in tape recorders, radios, television sets, hearing aids, automobile radios, and marine radios.

Fixed Equipment

Transistorized fixed equipment takes up less space and uses less power than older types. Military and commercial fixed equipment that use transistors are wire systems (telephone and teletypewriter), facsimile, radar, computers, public address systems, intercommunications systems, both television and radio receivers and transmitters.

COMPARISON WITH ELECTRON TUBES

Efficiency and Voltage Requirements

Transistor power efficiency is greater than that of the vacuum tube because it operates with small DC voltage and currents and, in addition, does not require heater power. We find other advantages of the transistor to be its useful life, low noise level, small size, and rugged construction.

Useful Life

A very important consideration in the application of any electronic device is its useful life expectancy. As the transistor is hermetically sealed in glass or metal, it will withstand a variety of abuses that an electron tube cannot withstand. For example, even though the transistor is completely immersed in water, it will continue to operate for long periods of time with little noticeable change in characteristics. Centrifugal force, gravity, and impact tests that would shatter an electron tube do not harm the transistor. Complete data on the life expectancy of transistors is not available at this time as they are a comparatively new development; however, it has been estimated they can operate continuously for approximately eight years, a much longer time than the average vacuum tube.

Noise Level

With an input frequency of 1000 cycles per second, the noise level of a transistor is approximately 20 db. The noise level of the vacuum tube for the same frequency input is a lower value. Although the noise level of a transistor is higher than that of an electron tube at this particular frequency, the noise level becomes considerably lower at a higher frequency. The noise level of the transistor is inversely proportional to the audio-frequency input.

Size and Construction

The power amplifier electron tube and the power amplifier transistor, which is much smaller, are designed for the efficient dissipation of heat. Heat dissipation is provided by the flange type construction of the power transistor cover, but a specially constructed metallic heat dissipator must be used in some cases.

Since the power transistor, among the largest of transistors, is smaller than the miniature electron tube, transistors are especially adaptable to the miniaturization of electronic equipment. Capacitors, resistors, batteries, transformers, and associated circuit components of transistors may be made smaller than the corresponding components used in electron-tube circuits.

Transistor Material

Conductors, as defined in Chapter 2, are those materials that provide, with little resistance, a good path for electron flow. Examples are copper, silver, gold, and iron. *Insulators* are those materials, such as rubber, porcelain, and glass, that offer high opposition to electron flow. *Semiconductors* are those materials which provide a path for electron flow but offer moderate resistance. Examples are carbon in diamond form, germanium, and silicon. Transistors are composed of materials such as germanium or silicon.

FUNDAMENTAL THEORY OF TRANSISTORS

Structure of Matter

To understand the theory of the internal conduction mechanism that occurs in the transistor, you will need a knowledge of the theory of the structure of matter. While Chapter 2 briefly explained the structure of matter, let us review this specific subject here. Matter is defined as any substance that has weight (mass) and occupies space. Examples are air, water, clothing, rocks, and wood. These examples illustrate the fact that matter is found in any one of three stages: gaseous, liquid or solid. Matter is found in nature as elements, compounds or mixtures. These are made up of molecules, atoms, and subatomic particles.

Element

Matter is composed of one or more basic materials which are called *elements.* There is definite proof that over 100 elements exist and the belief is that there are several additional elements that are yet to be discovered. The definition of an element is that substance which cannot be decomposed by ordinary chemical changes, nor made by chemical union of a number of substances. Examples of metallic elements are copper, iron, and gold; of nonmetallic elements, oxygen and hydrogen.

Compound

A *compound* is a substance containing more than one constituent element and having properties different from those of its elemental constituents. The simplest example of a compound is water, composed of two parts of hydrogen and one part of oxygen.

Molecule

The smallest particle of matter which can exist by itself and still retain all the proporties of the original substance is called a *molecule.* Take a drop of water and divide it until the smallest possible particle that is still water is obtained; that particle is known as a molecule.

Atom

The atom is the smallest part of an element that can participate in ordinary chemical changes. Atoms of different elements differ in their average mass, but atoms of the same element are of the same average mass. The atom may be considered to be the smallest particle that retains its identity as part of the element from which it is divided. As there are over 100 known elements, there must be over 100 known different atoms.

Subatomic Particles

It was once believed that the atom was the smallest particle of matter; it has since been determined that the atom itself can be subdivided into still smaller, or *subatomic particles.* A single atom is similar to the solar system in that there is a central body, called a nucleus, about which a number of smaller particles (electrons) move in approximately elliptical orbits. Because of the way electrons move in their elliptical orbits, it is hard to determine the number of electrons in an atom.

Careful study and research has disclosed that each electron in an atom has a charge of electricity that is identical with the charge on any of the other electrons. The charge that is associated with an electron is the smallest electrical charge that has been discovered. The name given to that charge is *elemental charge.* R. A. Millikan, an American physicist, was the first to measure this charge, and it was arbitrarily called a *negative* charge.

If we could magnify the nucleus enough, we would see that it is composed of several different kinds of particles, two of which are the proton and the neutron. The proton has an elemental positive charge, whereas the neutron is an uncharged particle. The positive charge of a proton is equal in magnitude to the negative charge of an electron, but opposite in nature. The atom is said to be electrically neutral when there is an equal number of positive and negative charges, that is, when the number of protons in the nucleus is equal to the number of electrons in orbit.

Electrical Balance and Unbalance

Protons and electrons are particles of energy because they carry positive and negative charges of electricity, respectively. These charges form the electric field of force within the atom. The atom is said to be electrically balanced when the number of positive charges is equal to the number of negative charges. This is the normal condition. Electrons, however, can be transferred from one atom to another. When an electron is transferred, the normally equal distribution of charges in the atom no longer exists. The atom now contains more of one kind of electricity than the other and is said to be electrically charged or in a state of unbalance.

A typical example that is often given in the classroom is the experiment in which a glass rod is rubbed with silk. When the glass rod is rubbed with silk, the electrons which are loosely held to the atoms in the glass are transferred to atoms of the silk. After separation of the rod and the silk, the rod will be positively charged because it will have more positive charges (protons) than negative charges (electrons). The silk, having acquired the electrons from the glass rod, will have an excess of electrons; therefore, it will be negatively charged.

In conductor type material, all electrons except those that revolve in the outer shell are tightly bound to the nucleus. Figure 8–5 illustrates the germanium atom and shows the electrons in the outer orbits that are free to move from one atom to another in a haphazard manner. The atomic structure of an atom of material determines the number of free electrons. When an electromotive force is applied to conductor-type material, some of the free electrons in the outer shell of each atom leave their orbits and tend to drift or flow in the same general direction through the conductor. This electron drift is generally referred to as current flow.

CONDUCTORS, SEMICONDUCTORS, AND INSULATORS

You will find that all materials may be divided into these three categories. The division into these groupings is based on the varying degrees that these materials allow electric current to flow. The ease with which electric current flows depends upon the atomic structure of the material.

Conductors

The material that has a large number of free electrons is a good conductor. These free electrons serve as charge carriers in an electric current. It has been found that all metals are conductors to some extent, but that some metals conduct better than others. Among the most commonly known metals used as conductors are

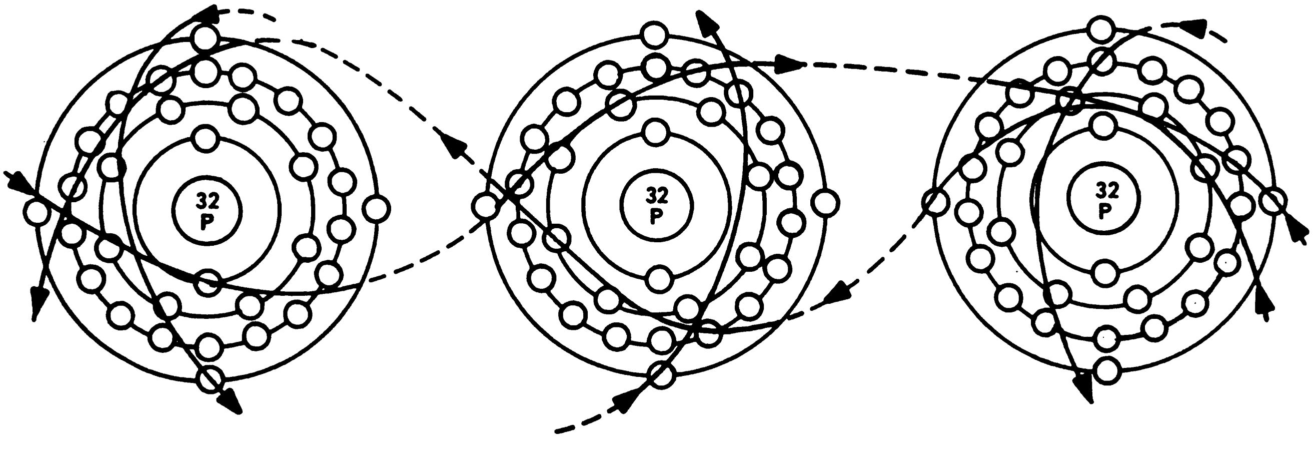


Figure 8–5. Movement of Free Electrons

silver, copper, and aluminum. The best conductor is silver, then copper, but copper is used more extensively because it is less expensive. Where weight is the major consideration, aluminum is used. A material's ability to conduct electricity also depends on its dimensions.

Insulators

An insulator is a single material, or combination of materials, whose atomic structure prevents most movement of electrons from atom to atom. Another explanation of an insulator is that it has few loosely held electrons. Examples of insulators are glass, dry wood, rubber, mica, and certain plastics.

Semiconductors

A semiconductor is a material that is neither a good conductor nor a good insulator. Germanium and silicon are substances that fall in this category. These materials, due to their peculiar crystalline structure, under certain conditions may act as a conductor, under others as an insulator. At absolute zero, a pure semiconductor has all of its electrons tightly bound and acts as an insulator. As the temperature is raised, however, a limited number of electrons become available for conduction.

As the number of available charge carriers in semiconductors is limited and dependent on temperature, conduction properties in these materials may be found to be quite different from those of good conductors. Over some temperature range, semiconductors may have a negative temperature coefficient of resistance. Pure germanium is a fairly poor conductor, and pure silicon is somewhat worse. For use in transistors, carefully controlled amounts of impurities are added to germanium and silicon. While there are many other semiconductors, only germanium and silicon will be discussed in this manual.

CRYSTALS, DONORS, ACCEPTORS, AND HOLES

Crystals

A microscopic study will show that most solids have a crystal structure. The exception is material exhibiting a biological structure of cells such as leaves, branches, and bone. Each material has a characteristic form such as a cube, a rhomboid, a needle, a geometric pattern, or variations of hexagonal or rectangular structure. It has also been found that inconsistencies in conduction from crystal to crystal have been introduced by the crystal boundaries. The atoms of crystals are arranged in a specific pattern or *lattice*. This relationship is not only from one atom to another, but also to a number of adjacent, equidistant atoms. The size and number of atoms present and the electrical forces between them determine the specific arrangement or pattern. Silicon and germanium are elements which crystallize in the cubic-lattice form; carbon crystallizes in the diamond-lattice.

Silicon, which has the atomic number of 14, has its electrons held in groups of energy levels, or shells, containing two, eight, and four electrons respectively. The inner shells of two and eight electrons are stable and complete; they have no excess orbits or energy levels. The outer four electrons are known as *valence* electrons and as such contribute to the chemical properties of the atom.

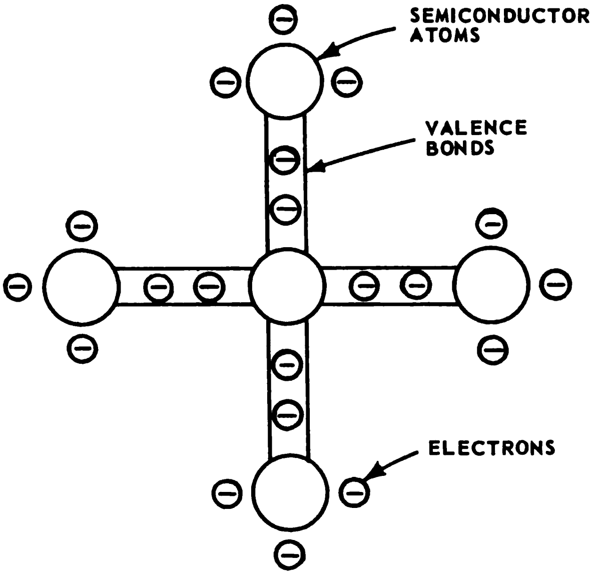


Figure 8–6. Pure Germanium Crystal, Lattice Structure

Germanium, which has the atomic number of 32, has its electrons held in shells, or energy levels, containing two, eight, eighteen, and four electrons. It has chemical and electrical properties similar to silicon because of the four valence electrons in the outer shell. As shown in Figure 8–6, each atom of germanium is joined in its lattice to its nearest neighbor atom by four valence bonds, each having two valence electrons, one from each atom. The coordinated rotation of two valence electrons (one from each of two adjacent atoms) forms an *electron-pair bond* between each pair of atoms. Another name given to electron-pair bonds is valence bonds.

The electron-pair bonds cause the cores (the sphere which contains the nucleus and the orbiting electrons) to be attracted toward each other. The cores tend to repel each other because the cores have a positive charge. When the balance of the forces of attraction and repulsion is equal, the crystal is said to be in a state of equilibrium. When germanium and silicon are in this state of equilibrium, they have no free electrons and therefore behave as insulators. When the temperature is raised, kinetic energy is transferred to the electrons and a few of the valence bonds are broken, freeing these electrons. With these electrons freed, the substance has more properties of a conductor than of an insulator.

Donor and Acceptor Impurities

Atoms of substances other than germanium may join the crystal lattice structure of germanium. Whether these substances are found in germanium in its natural state or are added intentionally, they are referred to as *impurities.* They are added intentionally during the processing of germanium for use in transistors. One group of substances that exhibits the important characteristic of joining the lattice structure of germanium is called donors; the second group is called acceptors.

N-Type Germanium, Donors

A germanium crystal where one of the germanium atoms has been replaced by a donor impurity is shown in Figure 8–7. The dark sphere in the center of the figure represents the nucleus of the donor atom and all of the electrons that orbit about the nucleus. Electron-pair bonds are formed using four of the valence electrons of the donor with four electrons of the neighboring atoms of germanium. As there are no adjacent electrons available, the fifth valence electron of the donor cannot form an electron-pair bond. Thus, this electron is an excess electron.

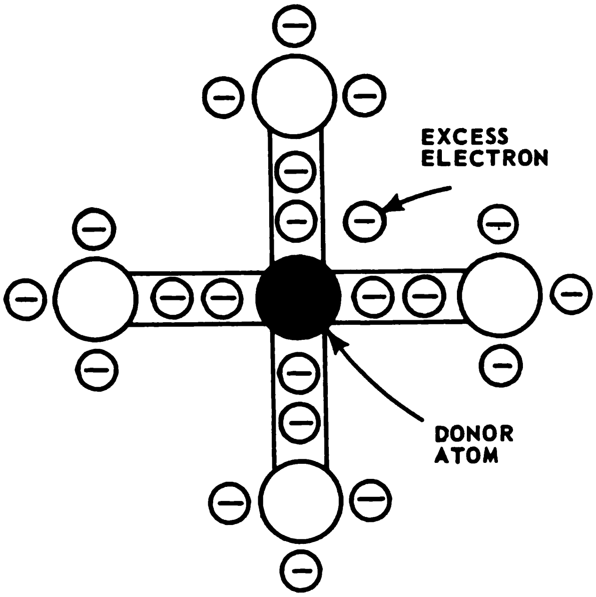


Figure 8–7. N-Type Germanium Crystal

The donor nucleus has very little influence on the excess electron and at normal room temperature there is sufficient thermal energy to cause the excess electron to break away and move through the space between the crystal lattices. When the excess electron leaves the donor atom, the donor atom has a positive charge equal to the negative charge of one electron. The sphere that represents the donor is then called a donor ion because any atom that gains or loses an electron is called an ion. When germanium contains donor impurities (arsenic, phosphorus and antimony), it is called N-type germanium; the letter N represents the negative charge of the excess electron.

P-Type Germanium, Acceptors

An inspection of Figure 8–8 shows a germanium crystal where an acceptor impurity has replaced one of the germanium atoms. The illustration again represents the nucleus of the acceptor atom and all the electrons that orbit about the nucleus. Note that there are three valence elecrons in the acceptor impurity. The three valence electrons of the acceptor impurity form electron-pair bonds with an electron from three of the neighboring atoms of germanium; one valence electron of the fourth germanium atom stands ready to accept another electron to form an electron-pair bond. This constitutes an electron-hole arrangement.

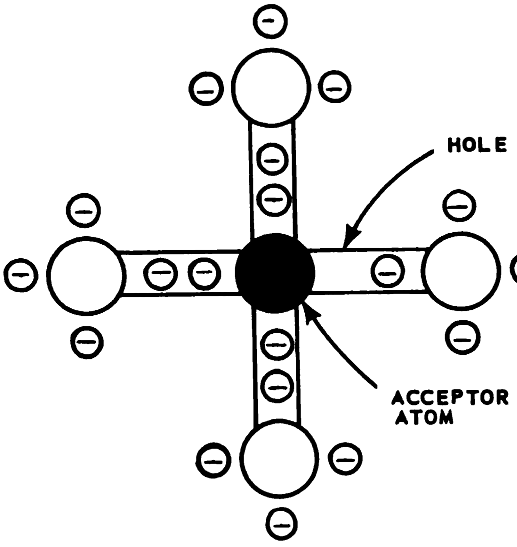


Figure 8–8. P-Type Germanium Crystal

The spot where an electron would normally be in pure germanium is designated a *hole*. If an electron in a nearby electron-pair bond absorbs enough energy to break the bond and move to fill in the hole in an electron-hole arrangement, the hole moves to a new position. The acceptor atom becomes ionized. The acceptor now possesses a negative charge. The germanium atom now has only three valence electrons. Since it requires four, the germanium atom is now positively charged with the positive charge concentrated in the hole in the electron-hole arrangement. The understanding of the concept of holes is very important in the study of the operation of transistors. Experiments have shown that the positive hole moves about in the crystal lattice in much the same way that an electron moves within the crystal itself.

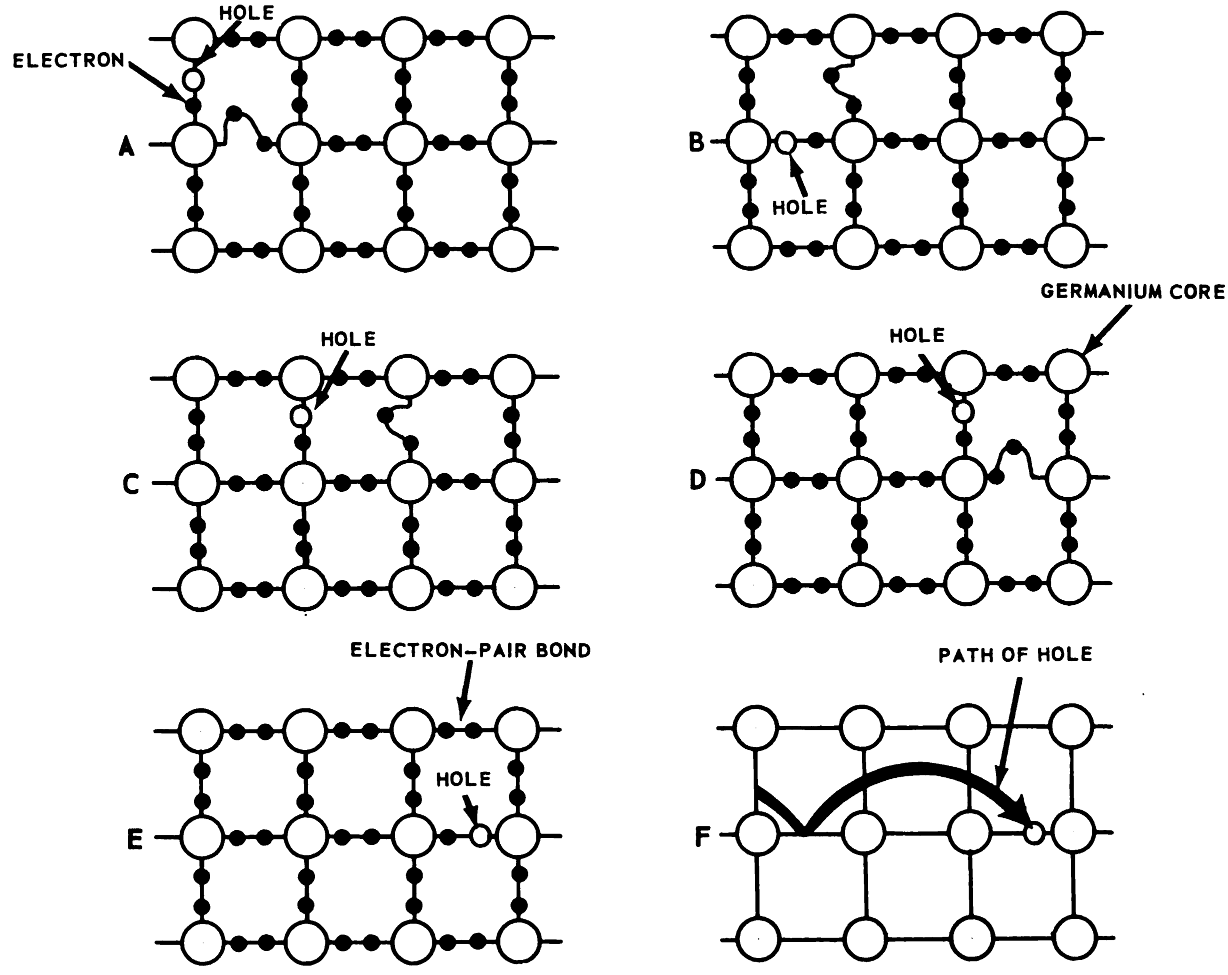


Figure 8–9. Movement of Hole Through Crystal

Germanium material containing acceptor impurities is called P-type germanium. In using the letter P, the reference is to the positive charge of the hole. Substances used as acceptors are aluminum, gallium, boron, and indium.

Holes

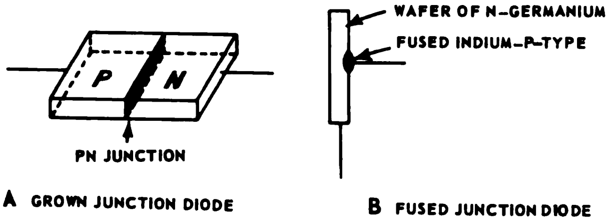
To understand transistors and transistor theory, it is convenient to think of holes as specific particles. Just as electrons in motion constitute an electrical current, holes in motion constitute an electrical current. We must remember, however, that there are several differences between the two.

First, holes can exist only in a semiconductor material such as germanium or silicon. The hole's existence depends upon a specific arrangement of electrons and atoms such as are found only in crystal substances. Holes are not found in conductors such as copper and aluminum. Second, while electric and magnetic fields deflect the hole, the direction of deflection is opposite to that of the electron because the hole has a charge equal and opposite to that of the electron. The electron will move toward the positive pole in an electric field; the hole will move toward the negative pole. Third, the electron, at least in the field of electronics, cannot be destroyed. But when a hole is filled by an electron moving away from an electron-pair bond, that hole is no longer there. A new hole appears in the position the moving electron just left.

Figure 8–9 shows how a hole appears to move from one position to another. In A the hole exists in the upper left atom. An electron starts to move to that hole. In B that hole has been filled and a hole now appears in the first atom of the 2nd row. Another electron moves to fill this hole, leaving a hole in another atom. Figure 8–9F shows the apparent path of the hole.

PN JUNCTIONS

A very important and unusual phenomenon occurs at the surface where contact is made between the N-type and P-type germanium, when they are joined in the same crystal. The surface where contact is made is called a *PN junction.* The junction is made by inserting into the growing crystal first one kind of impurity and then another. This produces regions of N and P conductivity, as illustrated in Figure 8–10A.



A GROWN JUNCTION DIODE **B** FUSED JUNCTION DIODE

Figure 8–10. PN Junctions

The fusing of the impurity material onto thin germanium wafers, by vapor diffusion (Figure 8–10B), and by the electroplating of the impurity material onto the germanium or silicon are two other methods of forming junctions. By careful manufacturing procedures the transition between the two regions can be made quite sudden.

If you could look through a microscope and actually see inside the bulk germanium material, you would see the motion of the germanium cores and the impurity ions in their lattice positions due to thermal energy. But this motion does not constitute a current. Because the cores and ions do not leave their lattice positions, they may be thought of as being stationary. The holes and the excess electrons have a haphazard movement which is also due to thermal energy. This movement of charges is called *diffusion.*

There is an even distribution of holes throughout the P-type germanium even though the holes are in motion; the same is true of the excess electrons; they are evenly distributed through

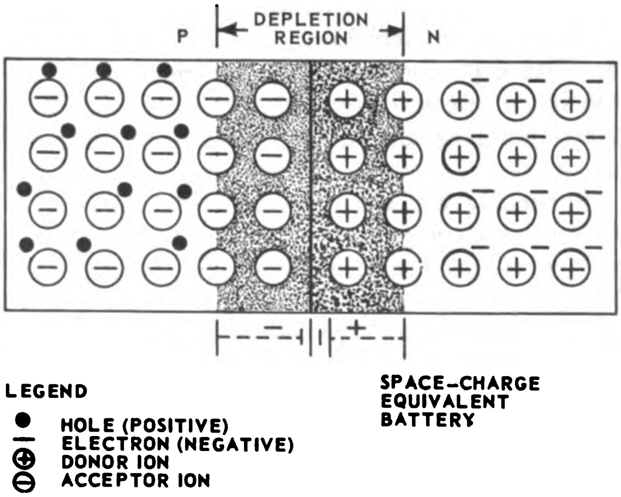


Figure 8–11. Joined Sections of P-Type and N-Type Germanium

the N-type germanium. You might expect that when the N- and P-type germaniums are joined to form a junction, Figure 8–11, that the holes in the P-type germanium and the electrons in the N-type germanium would move toward each other, combine, and thus eliminate all holes and excess electrons. This is not true, however, for after the two types of germanium are joined, only a few holes and electrons combine. Then a restraining force called a *barrier* is set up automatically to prevent total combination.

Junction Barrier

As stated, some of the holes in the P-type and some of the excess electrons in the N-type germanium diffuse toward each other and combine when these two types of germanium are joined. One hole and one excess electron are eliminated in each combination. (The excess electron becomes a part of an electron-pair bond.) This action, which takes place in the immediate vicinity of the junction, lasts only a short time. The negative acceptor ions in the P-region and the positive donor ions in the N-region that are near the junction remain uncompensated.

The electrons that would diffuse into the P-region are repelled by the uncompensated negative charge on the acceptor ions. The holes that would diffuse into the N-region are repelled by the uncompensated positive charge of the donor ions. Because of the restraining force that is set up, all of the holes and electrons cannot combine. The name given to the region containing the uncompensated acceptor and donor ions is the *depletion region,* because there is a depletion of holes and a depletion of excess electrons in it. The depletion region is also referred to as the *space charge region,* since the acceptor and the donor ions are charged electrically and do not move.

Previously we said that the restraining force between the two types of germanium is called a barrier, specifically we can now say that the *electric field* between the acceptor and the donor ions is the barrier. The *width* of the barrier is the physical distance from one side of the barrier to the other. The density of holes and excess electrons in the germanium crystal determines the width of the barrier. The *height* of the barrier is the difference of potential from one side of the barrier to the other side, or the intensity of the electric field. It is measured in volts. The barrier height, with no external batteries connected, is only a fraction of a volt. Figure 8–12 shows graphically the width, height, and difference of potential of joined sections of P-type and N-type germanium.

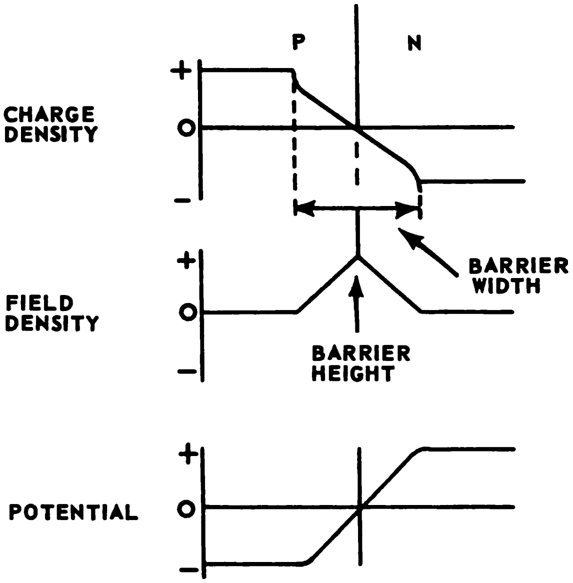


Figure 8–12. Static Curves of PN Junction

PN Junction, Reverse Bias

Let us connect a battery to a PN junction as shown in Figure 8–13. The negative terminal of the battery is connected to the P-type germanium and the positive terminal of the battery is connected to the N-type germanium. The holes are attracted toward the negative terminal and away from the junction; the electrons are attracted toward the positive terminal and away

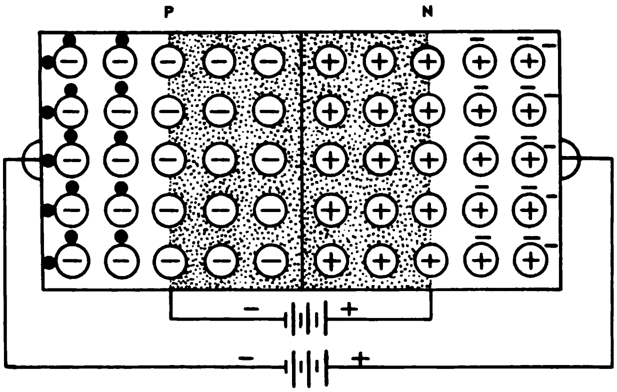


Figure 8–13. Reverse Bias

from the junction. This type of bias widens the depletion region and increases the height (potential) of the barrier.

The depletion region widens until the barrier height equals the potential of the external battery; at this time no current flow of holes or electrons will occur because the voltages oppose each other. In Figure 8–13 note that the depletion region has been given a space-charge equivalent battery. The PN junction is said to be biased in the *reverse* direction, or a reverse bias is placed across the PN junction when the battery is connected this way. If a reverse bias greater than the largest possible barrier height is applied across the junction, the crystal structure will break down. For this reason such large voltages are not normally used.

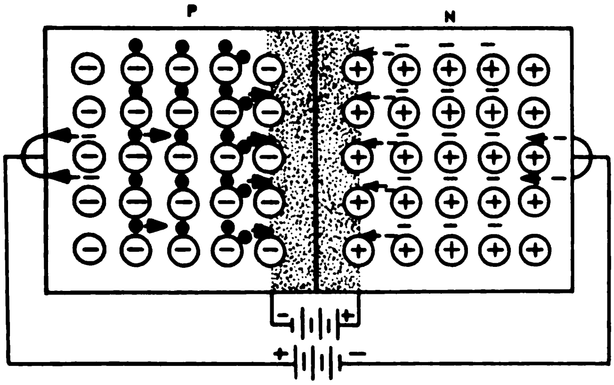


Figure 8–14. Forward Bias

PN Junction, Forward Bias

In Figure 8–14 the positive terminal of the battery is connected to the P-type germanium, and the negative terminal of the battery is connected to the N-type germanium in a PN junction. With these polarities the holes are repelled from the positive terminal and drift toward the junction; the electrons are repelled from the negative terminal and drift toward the junction. Since the holes and excess electrons have acquired some energy, they penetrate the depletion region and combine. Every time an excess electron combines with a hole, an electron from the negative terminal of the external battery enters the N-type germanium and drifts toward the junction. At the same time, an electron from an electron-pair bond in the crystal that is near the positive terminal of the external battery breaks its bond and enters the positive terminal of the battery.

This action creates a hole that drifts toward the junction. This recombination of excess electrons and holes in and about the space charge (depletion) region continues as long as the external battery is connected. An electron current flows in the external circuit as long as this action takes place. Note the different currents occurring at the same time. A continuous electron current flows in the external circuit; a current consisting of holes flows in the P-type germanium, and an electron current flows in the N-type germanium. With these battery polarities the PN junction is biased in the *forward direction.* Increasing the forward bias increases the current.

Since we stated that the barrier potential, with no external battery connected, was a fraction of a volt, you might think that an external battery of very low voltage, on the order of 1 volt, would eliminate the barrier completely. But increasing the voltage of the external battery increases the current flow through the crystal. The crystal has a relatively high resistance; therefore, increasing the current increases the voltage drop on both sides of the barrier. This does not leave enough voltage of the external battery to overcome the barrier. If a mistake is made and excessive forward bias is used, then excessive current will cause thermal agitation resulting in the breakdown of the crystal structure.

Diode Action

We have seen what happens when voltage is applied in either direction to a PN junction. From these two actions we can see how a PN germanium diode rectifies. Figure 8–15 shows the amount of current for different voltages applied to a practical PN junction. Current flow in the forward bias direction is quite high, but current flow in the reverse bias direction, although very low, is not zero. The two scales used in the graph in Figure 8–15 are different so that you can see the actual shape of the curve. Current in the forward bias direction (above the base line) is measured in milliamperes, while current in the reverse bias direction (below the base line in the chart) is measured in microamperes.

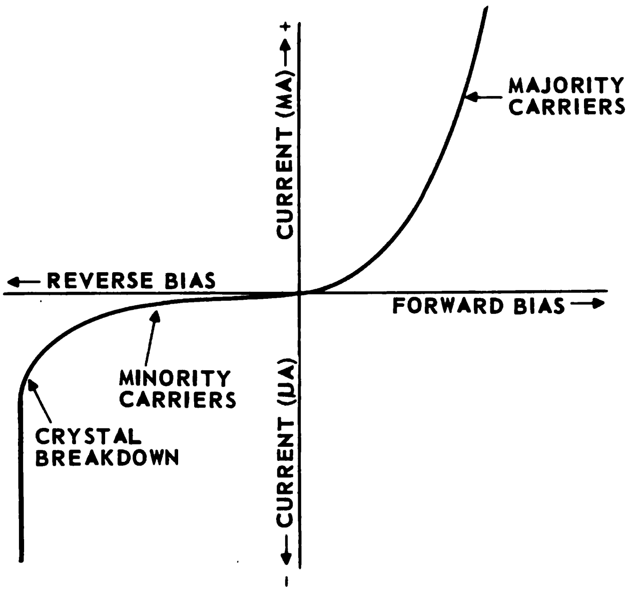


Figure 8–15. PN Junction Current and Voltage Graph

The reverse bias current is easily explained. Some acceptor ions and their associated holes occur in the N-type germanium and some donor ions and their associated excess electrons occur in the P-type germanium. These holes and excess electrons are called *minority carriers.* They are very few in number compared to the *majority carriers,* which are the holes in the P-type germanium and the excess electrons in the N-type germanium. When reverse bias is applied to a PN junction, this voltage acts as forward bias for the minority carriers. The minority carrier holes and excess electrons combine and produce a current, just as described previously for the majority carriers. Notice in Figure 8–15 that a high reverse current flows for high reverse bias voltages. The breakdown of the single-crystal structure causes this high current, not the minority carriers.

Figure 8–15 shows that for the same amplitude of applied voltage, the current for the forward bias is much greater than for the reverse bias. Therefore, a PN junction biased in the forward direction is the equivalent of a low resistance, and the PN junction biased in the reverse direction is the equivalent of a high resistance. From the power formula ($P = I^2R$), you can see that the power developed in a high-resistance element, for a given current, is greater than that developed in a low-resistance element.

Assume that we have two PN junctions in one crystal, one being biased in the forward direction and the other in the reverse direction. Then apply a signal to the low-resistance section (PN junction biased in the forward direction) and take it from the high resistance element (PN junction biased in the reverse direction). When the output is developed in the external circuit, we will find that we have increased the power. We have produced a power gain. This combination of PN junctions has *transferred* the signal from a low-resistance circuit to a high-resistance circuit. The word TRANSISTOR is a contraction of the terms TRANSfer and resISTOR.

PNP Transistor

It takes three sections of germanium to form two PN junctions. The combination of the three sections forms a transistor. The combination of two sections of P-type germanium with one section of N-type germanium is shown in Figure 8–16. This formation is referred to as a PNP transistor. Two depletion regions (barriers) occur at the junctions when three sections of germanium are combined even though no external

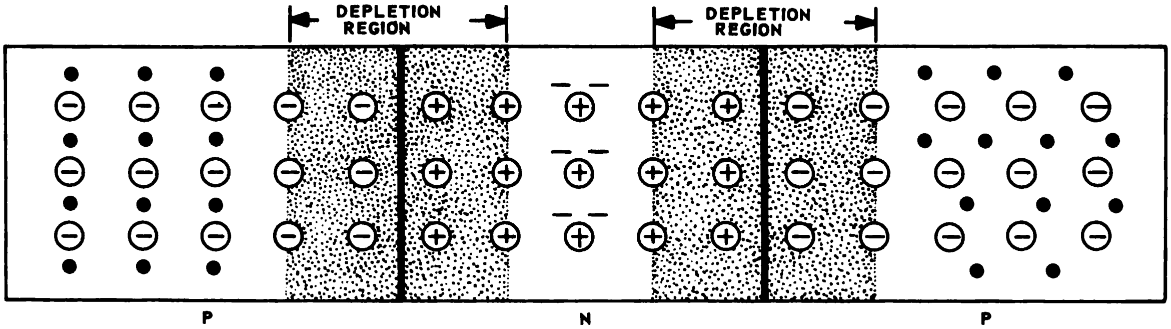


Figure 8–16. PNP Theoretical Transistor

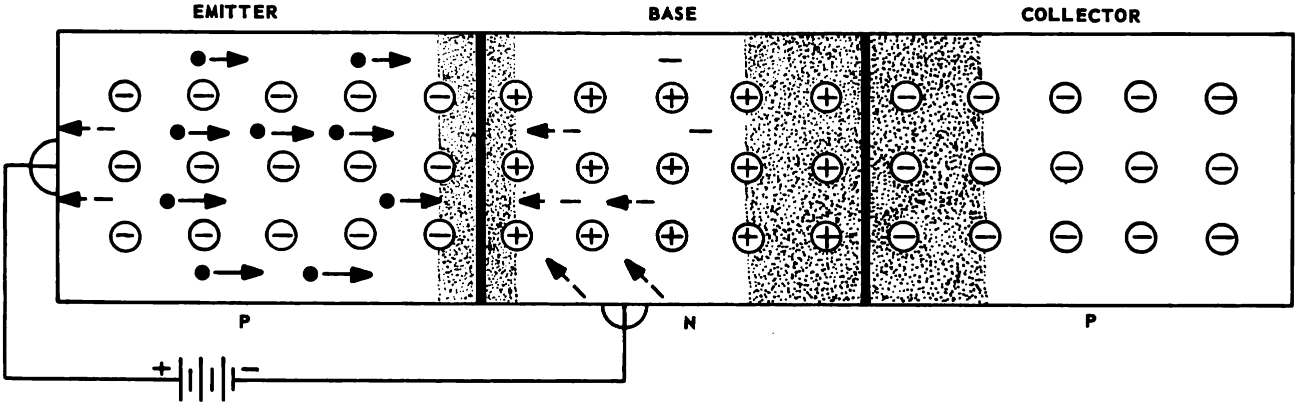


Figure 8–17. Forward Bias between Emitter and Base

voltage has been applied. This phenomenon is the same as that which occurs when two sections of germanium are combined.

In an electron tube the main current flow is from cathode to plate; in a junction transistor the main current flow is from *emitter* to *collector*. (See Figure 8–17.) The emitter and collector of a transistor are comparable, then, to the cathode and plate, respectively, of the electron tube. The current in the electron tube passes through the grid; in the transistor the current passes through the *base*. The input signal is applied to the grid of an electron tube, and to the base of a transistor, which may be compared to the grid of the electron tube. One difference between the electron tube and the transistor is that the emitter of the transistor requires no heater, whereas the cathode of a vacuum tube does.

In developing our transistor we applied forward bias to one junction and reverse bias to the other. The emitter-base junction is normally biased in the forward (low resistance) direction; the base-collector junction is normally biased in the reverse (high resistance) direction.

Figure 8–17 shows the biasing of the first junction (emitter-base) in the forward direction with the second junction (base-collector) unbiased. A comparison with Figure 8–16 will show you that the depletion region at the first junction has been greatly reduced; the depletion region at the unbiased junction remains unchanged. The transistor in Figure 8–18 has the second junction (base-collector) biased in the reverse direction, with the first junction unbiased. A comparison with the transistor in Figure 8–16 shows that the biased junction has a much larger depletion region, while the unbiased junction remains as before. Except for a few minority carriers, which are not shown here, there will be no current flow across the junction that is biased in the reverse direction.

In Figure 8–19 both junctions are biased at the same time. With these biases many of the holes from the emitter that enter the base do

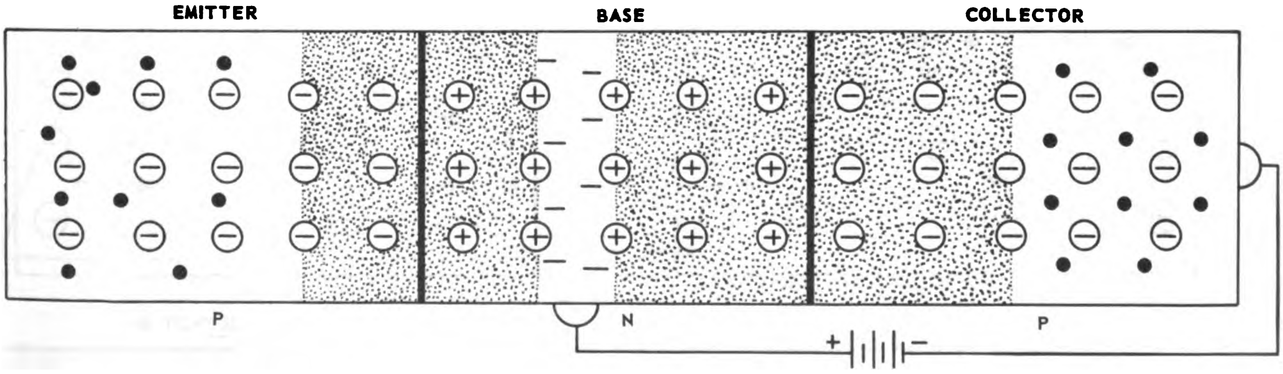


Figure 8–18. Reverse Bias between Base and Collector

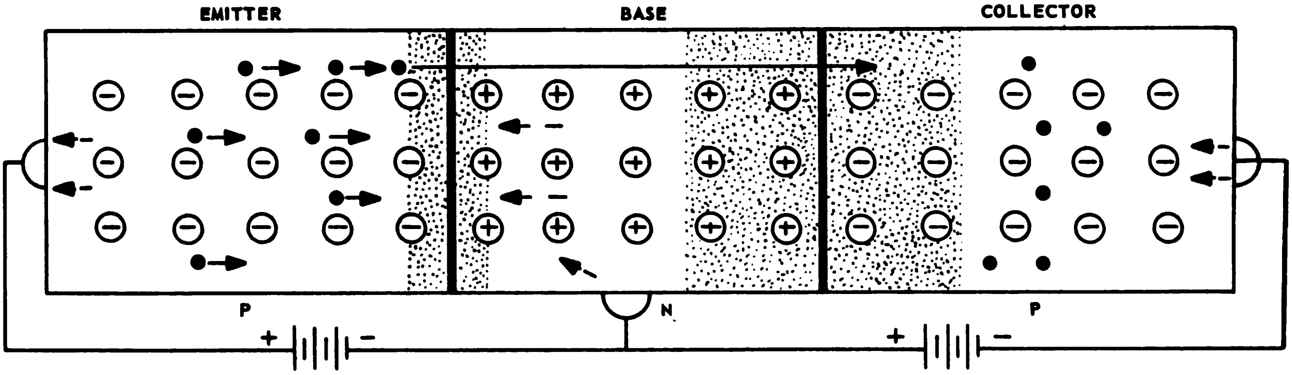


Figure 8–19. Simultaneous Application of Forward and Reverse Bias

not combine with the electrons that enter the base from the battery beween the emitter and the base. Instead, they diffuse through the base and enter the base-collector depletion region. When these holes enter the collector, they combine with electrons entering the collector from the battery between the base and the collector. Other holes that enter the base from the emitter-base junction and do not combine with electrons entering the base from the battery are attracted to the collector by both the negative potential of the base-collector battery and the negative acceptor ions in the collector.

Unless most of the holes diffuse through the base into the collector, we cannot get maximum power gain in a transistor. This is accomplished in a practical transistor by making the base region very narrow in comparison with the other two regions, as shown in Figure 8–20. In the actual operation of transistors, approximately 92% to 99% of the current from the emitter reaches the collector.

NPN Transistor

The NPN transistor is very similar in its operation to that of the PNP transistor. However, if you study Figures 8–20 and 8–21 very closely, you will find two very important differences. In the PNP transistor the emitter-to-collector current carrier is the hole, whereas in the NPN transistor the emitter-to-collector current carrier is the electron. The bias voltage polarities are reversed, because of the different arrangement of P- and N-type material in the two transistors.

COMPARISON OF TRANSISTORS AND ELECTRON TUBES

There are differences and similarities between electron tubes and transistors in addition to those that have already been discussed. In an electron tube the plate current is determined mainly by the grid-cathode voltage; in a transistor the collector current is determined mainly by the

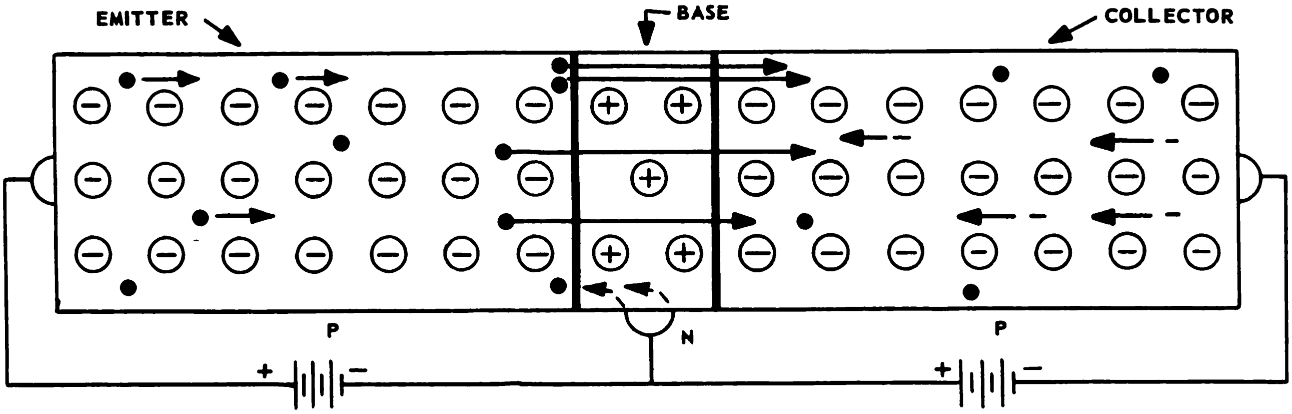


Figure 8–20. Practical PNP Transistor

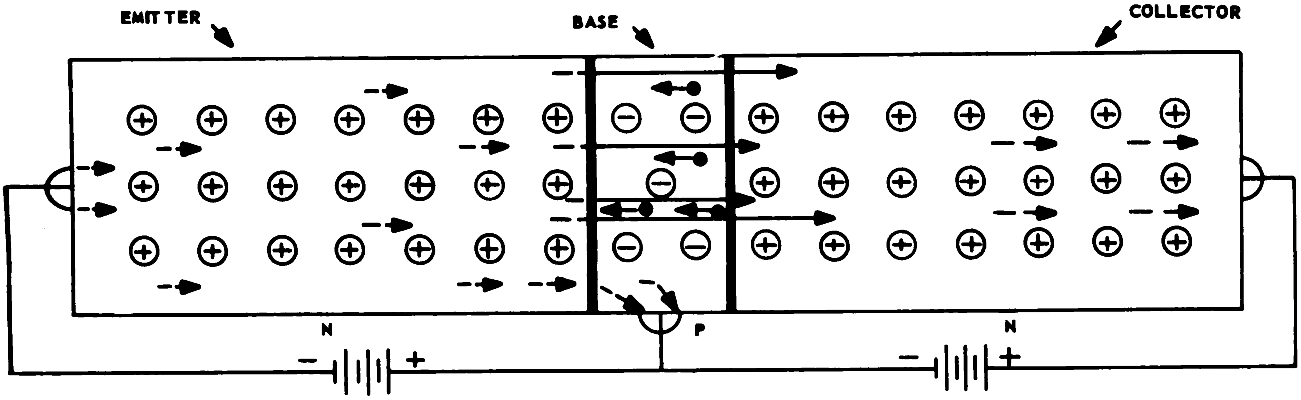
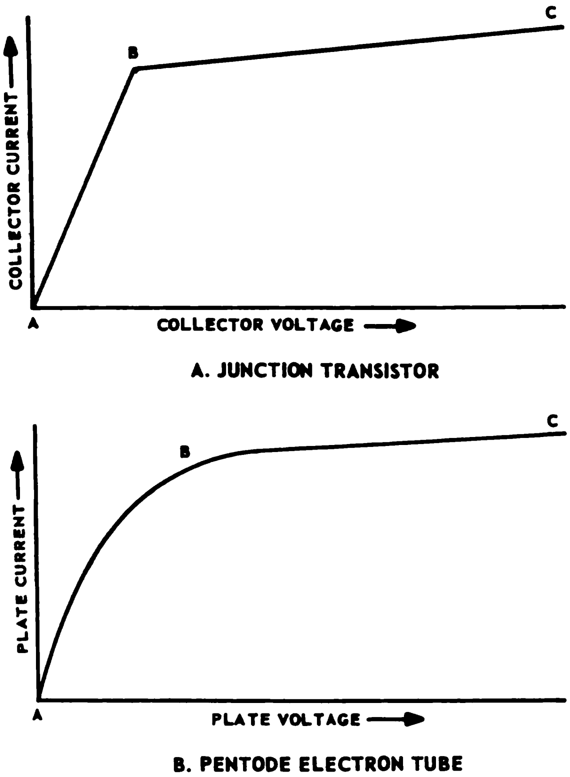


Figure 8–21. Practical NPN Transistor

emitter-base voltage. The collector current-collector voltage characteristic of a transistor with fixed emitter-base voltage is similar to the plate current-plate voltage characteristic of the pentode tube with fixed grid-cathode voltage. The two characteristic curves are illustrated in Figure 8–22.

In the transistor curve (Figure 8–22A) with a steady rise of collector voltage there is a con-



A. JUNCTION TRANSISTOR

B. PENTODE ELECTRON TUBE

Figure 8–22. Collector and Plate Characteristics for Transistor and Electron Tube

current steady rise of current from A to B. This indicates that there are enough carriers moving from the emitter-base barrier through the base-collector barrier to satisfy the applied collector voltage. The section from A to B shows low collector resistance. Although the collector voltage is rising from B to C, collector current remains relatively constant because there is no increase in the supply of carriers through the emitter-base. The section of curve from B to C represents high collector resistance. The section of the pentode curve from B to C in Figure 8–22B shows the leveling off of plate current due to the action of the screen and the suppressor grids.

The plate must be positive with respect to the cathode in an electron tube for current to flow. It is quite different in a transistor; the collector may be positive or negative with respect to the emitter for current to flow. The determining factor is whether electrons or holes are the emitter-to-collector carriers.

Unless the grid is positive with respect to the cathode in electron tube applications, grid-cathode current does not flow. In most transistors, however, there is current flow between emitter and base. Thus, we can conclude that in most cases the input resistance of a transistor is much lower than its output resistance and the input resistance of an electron tube is much higher than its output resistance.

TRANSISTOR AMPLIFIER FUNDAMENTALS

Graphical Symbols and References

Figure 8–23 shows the reference designations and graphical symbols that are used for tran-

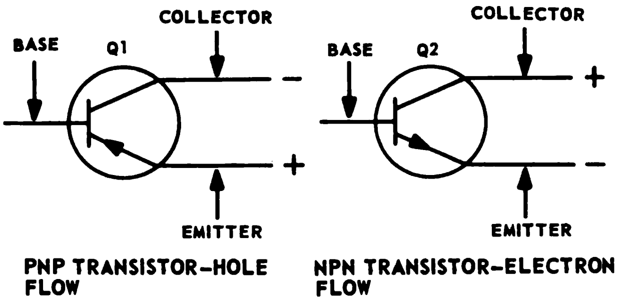


Figure 8–23. Graphical Symbol of a Transistor

sistors. V is used when we refer to an electron tube. Where there are more than one tube, a whole number is added, such as V1, V4702, etc. The letter Q is the reference designation for the transistor and the number portion may be any whole number.

In the PNP transistor (Figure 8–23A) the hole is the emitter-to-collector current carrier in the crystal. The collector must be negative with respect to the emitter for holes to flow internally from emitter to collector (direction of emitter arrow). In the external circuit, electrons flow from emitter to collector which is in a direction opposite to that of the emitter arrow.

In the NPN transistor (Figure 8–23B) the electron is the emitter-to-collector current carrier in the crystal. The collector must be positive with respect to the emitter for electrons to flow internally from emitter to collector (opposite to emitter arrow direction). Externally, electrons flow from the collector to the emitter which again is opposite to the direction of the emitter arrow.

General Transistor Amplifier Theory

In analyzing the qualitative behavior of transistor circuitry, the following generalizations should be very helpful. It should be noted, however, that these generalizations apply only to a transistor circuit that is operated as a Class A amplifier.

The first letter of the transistor type indicates the polarity of the emitter voltage, with respect to the base. Thus a PNP transistor has a positive DC voltage applied between emitter and base, and an NPN transistor has a negative DC voltage applied between the emitter and the base. The second letter of the transistor type indicates the polarity of the collector with respect to the base. Thus a PNP transistor has a negative DC voltage applied to the collector, and an NPN transistor has positive DC voltage applied to the collector.

From these two facts you can see that the first and second letters of the transistor types indicate the relative polarities between the emitter and the collector. In a PNP transistor the emitter is positive with respect to the collector or the collector is negative with respect to the emitter. In an NPN transistor the emitter is negative with respect to the collector or the collector is positive with respect to the emitter.

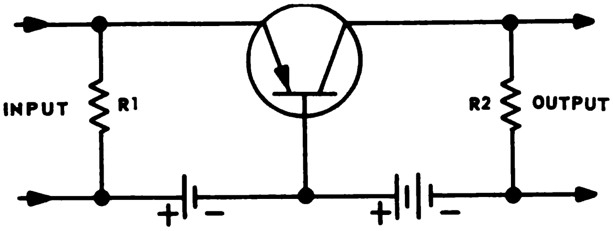
You will note that the DC electron current direction is always *against* the direction of the arrow on the emitter. If the electrons flow into the emitter, they flow out of the collector; if they flow out of the emitter, they flow into the collector.

The base-emitter junction is always biased in the forward direction; the collector-base junction is always biased in the reverse direction. The emitter and collector currents will increase with an input voltage that increases the forward bias. Likewise the emitter and collector currents will decrease with an input voltage that opposes or decreases the forward bias.

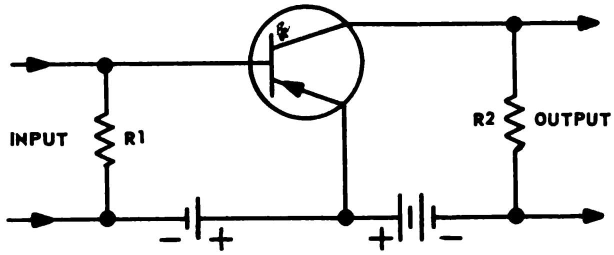
Transistor Arrangements

There are several amplifier circuit arrangements possible for the transistor. Figure 8–24A shows the common-base amplifier where the signal is introduced into the emitter-base circuit, and taken from the collector-base circuit. Since the base element of the PNP transistor is common to both the input circuit and the output circuit, this arrangement is referred to as a *common-base* (CB) amplifier. The common-base circuit is also referred to as the grounded-base amplifier.

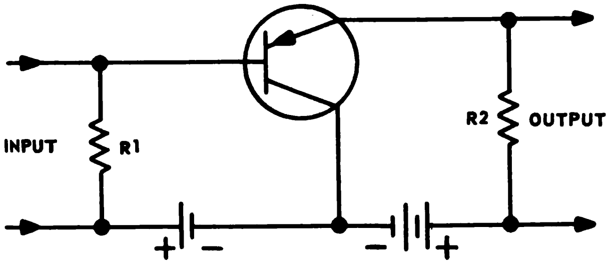
In the common-emitter amplifier of Figure 8–24B the signal is introduced into the base-emitter circuit and taken from the collector-emitter circuit. Here the emitter element of the PNP transistor is common to both the input and the output circuit giving rise to the name *common-emitter* (CE) or grounded-emitter amplifier. A common-emitter amplifier using an NPN transistor is similar to the circuit of Figure 8–24B; the polarities of the biasing batteries are reversed in order to maintain forward bias in the base-emitter circuit and reverse bias in the collector-base circuit.



A. COMMON BASE AMPLIFIER



B. COMMON EMITTER AMPLIFIER



C. COMMON COLLECTOR AMPLIFIER

Figure 8–24. PNP Amplifier Circuits

Figure 8–24C shows the common-collector amplifier. Because the collector element of the transistor is common to the input and output circuits, the circuit is called a *common-collector* (CC) amplifier or grounded-collector amplifier. The signal is fed into the base-collector circuit and removed from the emitter-collector circuit.

Additional Bias Arrangement

Regardless of the type of circuit, the proper bias for the Class A transistor amplifier consists of forward bias on the emitter-base circuit and reverse bias on the collector-base circuit. You have just seen in Figure 8–24 one method of biasing each circuit configuration. Other biasing methods can be used.

Figure 8–25 shows a common-base amplifier biased by voltages from a voltage divider network across a single battery. The entire battery supplies the collector-base bias. As the transistor in the circuit shown is a PNP type, the required reverse bias is achieved by placing the battery so that the collector is negative with respect to the base. The forward bias, required to make the emitter positive with respect to the base, is tapped off the voltage divider network consisting of resistors R3 and R4. If the PNP transistor is replaced by an NPN transistor, only the battery would have to be reversed.

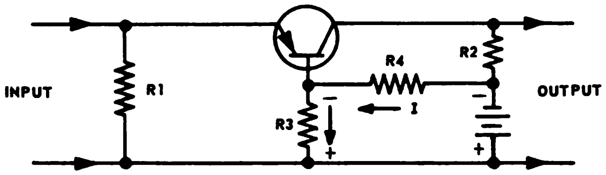


Figure 8–25. Common-Base Amplifier with Single Battery Bias

There are two methods of biasing the common-emitter amplifier with separate batteries. Figure 8–24B illustrates the method used to place the emitter at both AC and DC ground potential.

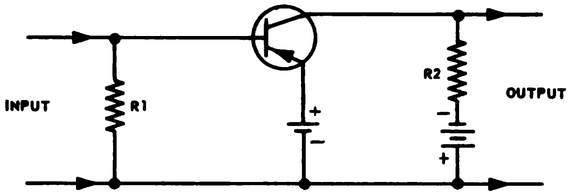


Figure 8–26. Common-Emitter Amplifier Using Two Bias Batteries

Figure 8–26 illustrates the method used in order that the base-emitter bias battery series aid the collector-emitter battery. Both batteries would have to be reversed if an NPN transistor were used.

Figure 8–27 shows the common-emitter amplifier biased with a single battery. The required reverse bias in the collector-base circuit is produced directly by the single battery. The in-

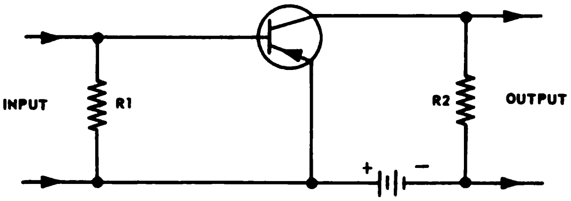
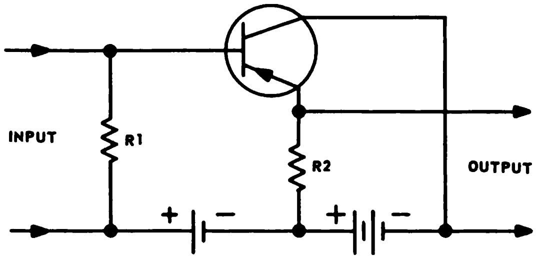
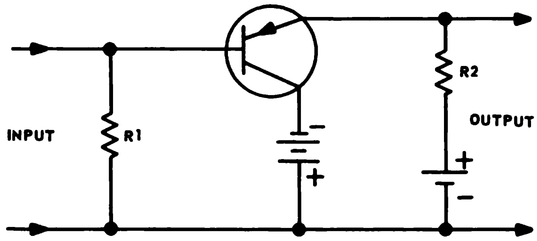


Figure 8–27. Common-Emitter Amplifier Using Single Bias Battery

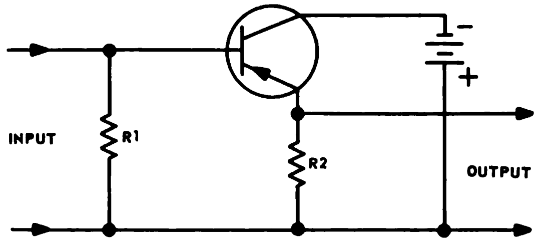
ternal structure of the transistor produces the forward bias between the emitter and the base. Remember that the forward bias for the PNP transistor means that the base is negative with respect to the emitter. The collector is at the highest negative potential in a PNP transistor, and the emitter is at the highest positive potential.



A. COMMON-COLLECTOR AMPLIFIER USING TWO BIAS BATTERIES



B. COMMON-COLLECTOR AMPLIFIER USING TWO BIAS BATTERIES



C. COMMON-COLLECTOR AMPLIFIER USING A SINGLE BIAS BATTERY

Figure 8–28. Methods of Bias for Common-Collector Amplifier

Since the base is physically located between the emitter and the collector, the base must assume a voltage somewhere between the two. Therefore the base is less positive than the emitter, or it is negative with respect to the emitter. This meets the requirement of forward bias. The amplitude of the voltage between the emitter and the base must be very small compared to that between the collector and the base. The two PN junctions act as a voltage divider internally. The high resistance represented by the junction between the collector and the base develops the larger voltage drop. The low resistance represented by the junction between emitter and base develops a low voltage drop.

There are three arrangements of the two batteries used to provide bias in the common-collector amplifier. One method has been shown in Figure 8–24C. The second and third methods are shown in Figures 8–28A and 8–28B. The proper forward bias (base-emitter) and reverse bias (base-collector) are established in each case by the batteries. To use an NPN transistor, reverse the polarities of both batteries in each circuit. Figure 8–28C illustrates the biasing of the common-collector amplifier with a single battery. The battery directly establishes the reverse bias (base-collector). The internal structure of the transistor determines the forward bias (base-emitter). The internal mechanism for establishing the forward bias is the same as that covered for the common-emitter amplifier.

Current Flow and Voltage Phase Relations, CB Amplifier

Figure 8–29 illustrates a common-base amplifier using an NPN transistor biased with two batteries. The arrows indicate the direction of electron current flow. The letter I represents the total emitter current in Figure 8–29. The flow

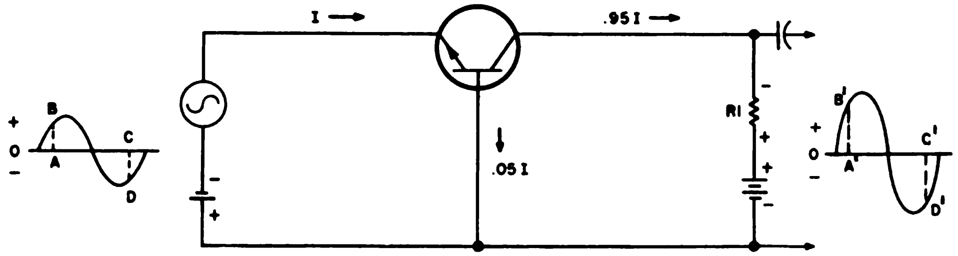


Figure 8–29. Common-Base (CB) Amplifier, Current Flow and Voltage Waveforms

of the minority carriers across the reverse-biased base-collector has been ignored in order to simplify the explanation of the circuit. The majority of the current from the emitter flows toward the collector. Transistors in practical use have from 92% to 98% of the emitter current reaching the collector with the remainder flowing through the base. Let us assume in this discussion that 95% of the current (0.95I) reaches the collector and 5% (.05I) flows to the base.

The waveforms in Figure 8–29 represent voltage waveforms. The input signal is the one on the left of the figure, and the output signal is on the right. With no signal applied a certain steady current flows through R1 producing a steady voltage drop. Let us apply a signal as shown in Figure 8–29. During the first, or positive, alternation, the signal opposes the battery voltage between the base and the emitter. This decreases the forward bias. The decrease in forward bias cuts down the total current (I) flowing through the emitter. The collector and base currents are also reduced a corresponding amount. The smaller current flow through R1 produces a smaller voltage drop and, as in any electron tube amplifier, the output voltage rises. $(E_o = E_{app} - IR1)$. The input voltage AB produces the output voltage A′B′. During the time the input signal is positive and opposes the forward bias, the output signal is also positive.

During the second, or negative, alternation, the signal aids the battery bias. The forward bias increases and allows more current to flow through the emitter. This increase is divided between the collector and the base current. Since the current flow through R1 has increased, the voltage drop across R1 increases. The output voltage decreases a corresponding amount. The input voltage CD produces the output voltage C′D′. During the negative alternation, which aids the forward bias, the output is negative going. There is no phase reversal between the input and the output signal voltages of a CB amplifier.

If a PNP transistor had been used in the circuit the batteries would have to be reversed. The general theory of operation would be the same, however.

Current Flow and Voltage Phase Relations, CE Amplifier

Figure 8–30 shows a common emitter amplifier using an NPN transistor. The arrows indicate the direction of electron current flow. Again, as in the previous discussion, the flow of the minority carriers across the reverse-biased base-collector junction has been ignored. The portions of emitter current that flow through the collector and the base are again 95% and 5%, respectively, as in the CB amplifier.

The positive alternation of the input voltage AB (Figure 8–30) from the signal generator aids the forward bias that is produced by the base-emitter battery. This gives the effect of increasing the voltage, which increases the total current (I) through the emitter. The collector and the base currents are increased correspondingly. The increasing current flow through resistor R1 increases the voltage drop, resulting in a decreased output voltage. The output waveform is a negative-going wave.

During the second negative alternation, the input signal opposes the forward bias of the emitter-base battery, decreasing the forward voltage. This action also decreases the emitter current and, as a result, the collector and base

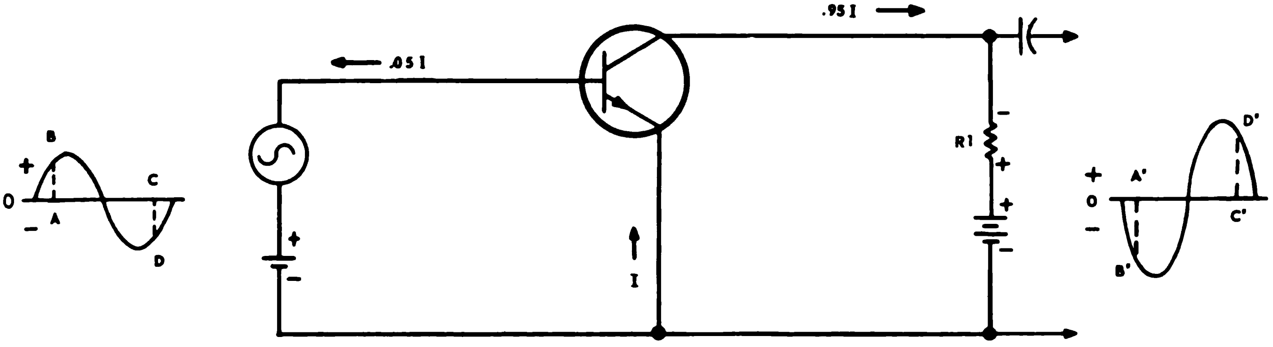


Figure 8–30. Common-Emitter (CE) Amplifier, Current Flow and Voltage Waveforms

currents are decreased. Decreasing the current through resistor R1 decreases the voltage drop and, as previously explained, the output voltage rises. A check of the input signal voltage and the output waveshape in Figure 8–30 shows a 180° shift in polarity across the amplifier.

Current Flow and Voltage Phase Relations, CC Amplifier

Figure 8–31 illustrates a common-collector amplifier using an NPN transistor; the flow of minority carriers across the reverse-biased collector-base junction is not considered. The portions of emitter current that flow through the collector and the base are the same as for CB and CE amplifiers. (Using a PNP transistor would require a reversal of the polarities of the batteries, but the voltage phase relationship would remain the same.)

As in the previous explanations, when the input (signal) voltage aids the forward bias voltage, the increased forward bias increases the total emitter current (I). The increase of current through the load resistor R1 increases the voltage drop across R1. The output voltage illustrates this action. During the positive alternation of the input signal, the output signal is positive. A negative input signal reduces the forward bias. This, in turn, reduces the emitter current. The reduction in emitter current reduces the current through resistor R1 and reduces the voltage drop across it. The output waveform at C′D′ shows this effect. During the time that the input signal is negative, the output signal is also negative.

From the preceding discussion, we can conclude that there is no reversal in the polarity of the signal from input to output in the CC amplifier.

The circuit configuration of the common-emitter amplifier is similar to that of the conventional (grounded-cathode) amplifier. The circuit configuration of the common-base amplifier is similar to that of the grounded (common) grid amplifier. The circuit configuration of the common-collector amplifier is similar to that of the cathode follower.

Current in the Base Lead

Figures 8–29, 8–30, and 8–31 show electron current in the base lead for an NPN transistor flowing away from the base region. This may not always be true in some specific circuit applications. We have neglected the reverse current flow between the collector and the base in these three circuits.

If we ignore the reverse bias collector-base current in a PNP transistor circuit, the electron current flow in the base lead will be toward the base region at all times. The direction of electron current in the base region for either a PNP or an NPN transistor actually depends on the percentage of emitter current that enters the base lead and the magnitude of the reverse-bias collector-base current that enters the base lead. In turn, these values depend on the characteristics of the specific transistor and on the operating point of the transistor.

Letter Symbols

The ratio of the collector current to the emitter current determines the current gain of the CB amplifier. In transistor electronics this ratio is

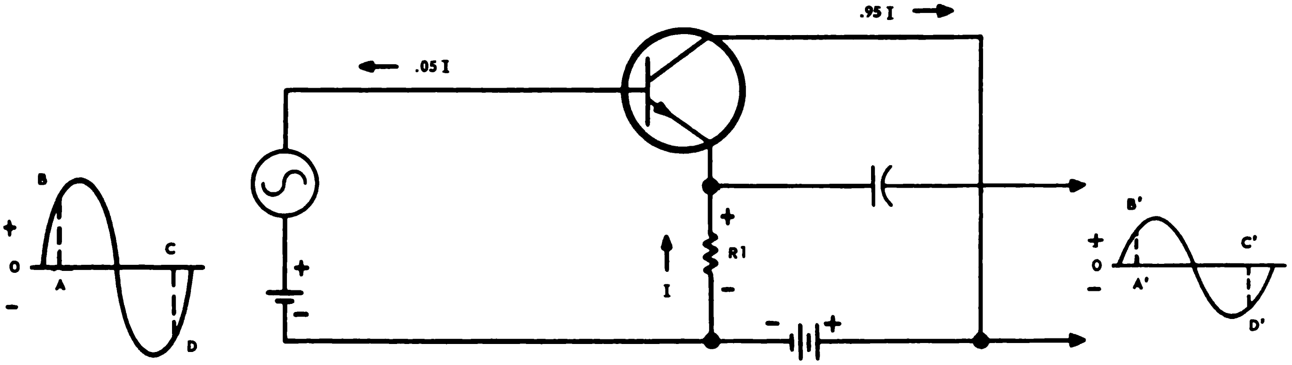


Figure 8–31. Common-Collector (CC) Amplifier, Current Flow and Voltage Waveforms

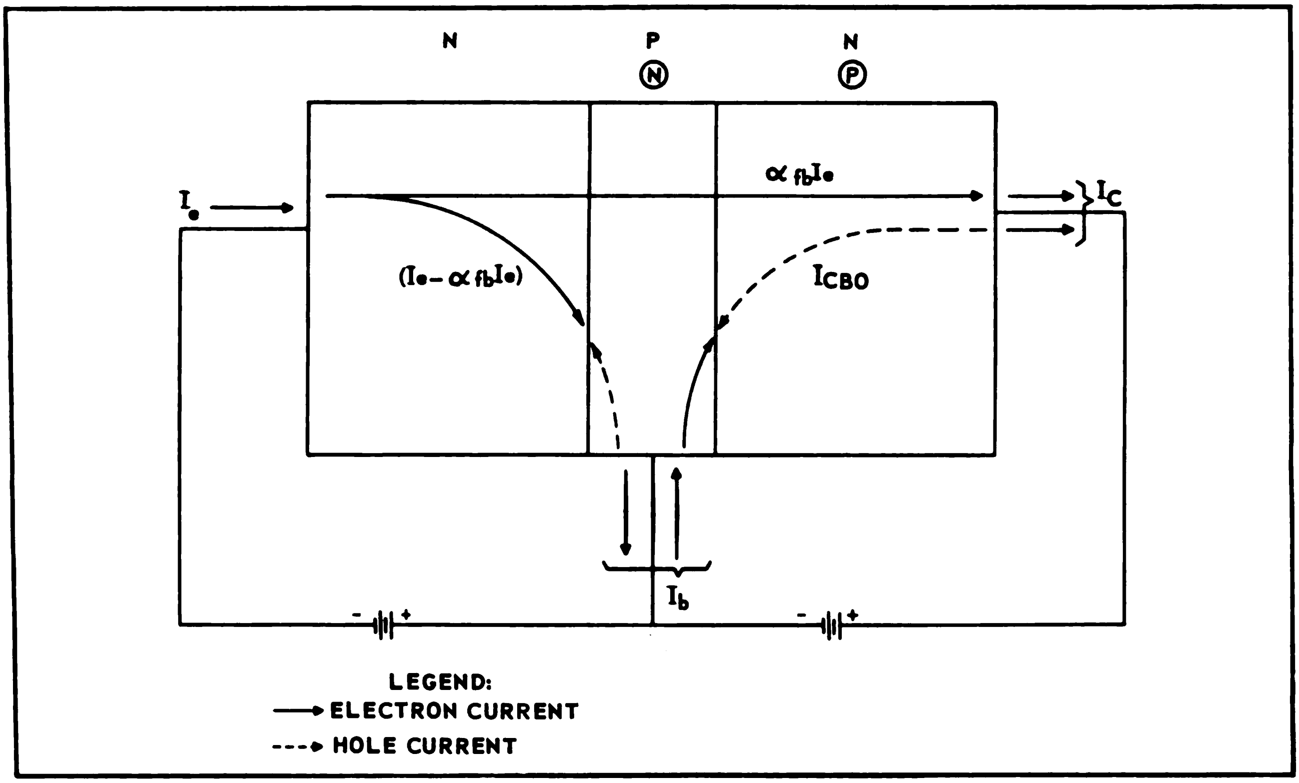


Figure 8–32. Current Flow in NPN Transistor

important and is referred to as alpha$_{fb}$ (α_{fb}), with *f* indicating the current from the emitter to the collector (forward direction), and *b* indicating that a common-base amplifier is used. (α_{fb} is the forward short circuit current amplification factor.) In a general sense, it represents the percentage of emitter current that reaches the collector.

The designation for total emitter current is I_e. I_{CBO} is the designation for reverse-bias current flow between base and collector. This current is also referred to as *saturation* or *leakage* current. The letters in the subscript indicate a collector-*b*ase current, measured with the emitter *o*pen.

The NPN transistor in Figure 8–32 has proper bias applied. The base region acts as N-type germanium to the minority carriers (electrons) in the base region. The circled *N* in the illustration indicates this condition. The collector region acts as P-type germanium to the minority carriers (holes) in the collector region, as indicated by the circled *P*.

The emitter current (I_e) in the NPN transistor consists of electrons. Between 92% and 99% of the emitter electron current passes through the base and collector regions and enters the positive terminal of the collector-base battery. This current can be expressed as $\alpha_{fb}I_e$. This leaves from 1% to 8% of the emitter electrons or $I_e—\alpha_{fb}I_e$, which is the total emitter current minus the current that flows to the collector. These electrons combine in the base-emitter junction with holes from the base region. The holes just mentioned are generated in the base region and near the base lead. As a result of the generation of the holes, electrons flow *away from* the base into the base lead.

Reverse-bias current, I_{CBO}, which is the flow of electrons in the base region that combine with holes from the collector region, also flows between the base and the collector. The combining of holes and electrons takes place at the base-collector junction. The result of the reverse-bias current is a flow of electrons in the base lead *toward* the base. The electron flow in the base lead is designated as I_b. As you can see from Figure 8–32, the direction of electron flow in the base lead depends on which of the two currents is larger. If I_{CBO} is greater than $I_e—\alpha_{fb}I_e$, electrons flow toward the base.

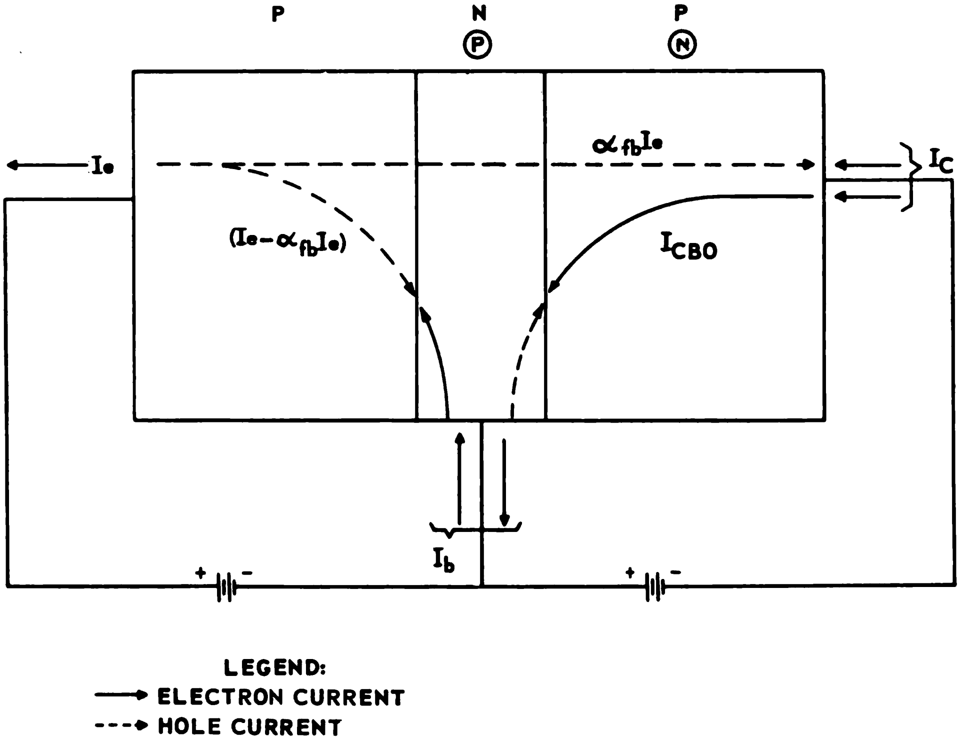


Figure 8–33. Current Flow in PNP Transistor

The preceding discussion also applies to the PNP transistor, Figure 8–33, but the electron and hole currents are interchanged internally. The polarity of the biases and the directions of external electron current flow are also reversed.

BIAS STABILIZATION

Operating Point

The bias (operating point) for a transistor is determined by specifying the quiescent (DC, no-signal) values of collector voltage and emitter current. To have reliable operation of a transistor over a wide range of temperatures requires stable bias voltages and current. In an NPN transistor, reverse-bias collector current or saturation current (I_{CBO}) is the flow of holes (minority carriers) from the collector region toward the base region (Figure 8–32). The holes from the collector can accumulate in the base region if the resistance of the base region is high, or if high value external resistors are connected to the base.

As the holes increase in number, *emitter* electron flow into and through the base increases. This, in turn, increases the collector current. Increasing the collector current raises the temperature of the collector-base junction, increasing the saturation current. Figure 8–34 is a graph of the relationship between temperature and satu-

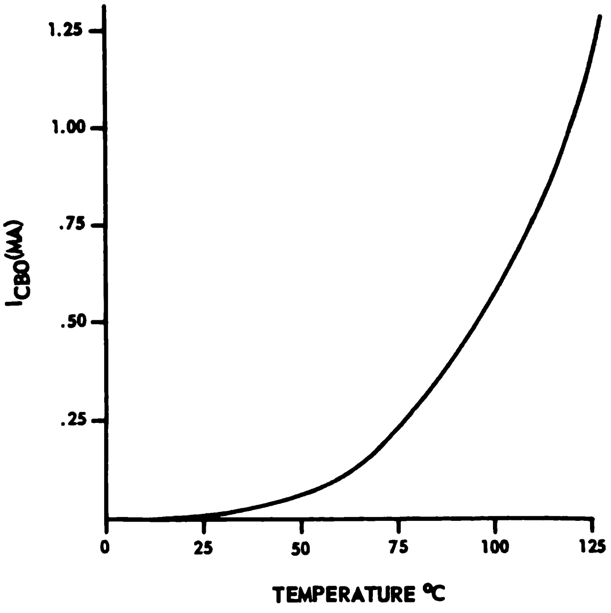


Figure 8–34. Variation of Junction Temperature with Saturation Current

ration current. It is possible for the temperature and current to increase until severe distortion occurs, the transistor becomes inoperative, or destroys itself. To prevent this happening, high-valued resistors should not be used in the base lead. The condition just described is the same for a PNP transistor, except that the parts the electrons and holes play are interchanged.

Emitter-Base Junction Resistance

The family of curves in Figure 8–35 indicates the variation of collector current with temperature. Each curve is plotted with a fixed collector-base voltage (V_{CB}) and a fixed emitter-base voltage (V_{EB}). If only the saturation current caused the collector current variations with temperature, then the collector current should be constant at temperatures below 10° C ($V_{EB} = 200$ mv and $V_{EB} = 300$ mv). But collector current does vary with temperature even when the saturation current is close to zero. The decrease in emitter-base junction resistance when the temperature is *increased* causes this variation. This means that the emitter-base junction resistance has a *negative temperature-coefficient of resistance.*

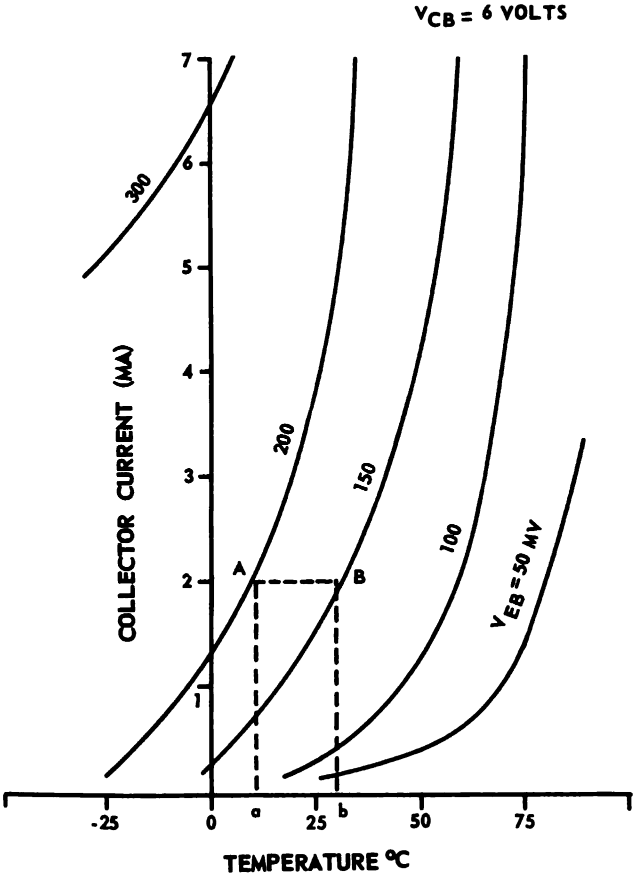


Figure 8–35. Variation of Collector Current with Transistor Temperature

To reduce the effect of the negative temperature coefficient of resistance, you can put a *swamping* resistor (resistor of large value) in the emitter lead. In effect, the variation of the emitter-base junction resistance then becomes a small percentage of the *total* resistance in the emitter circuit. The external resistor is said to *swamp* or overcome the variation in the junction resistance.

Another method you can use to reduce the effect of the negative temperature coefficient of resistance is reducing the emitter-base bias as the temperature increases. To illustrate the second method refer to Figure 8–35. In order to maintain the collector current at 2 ma while the transistor temperature varies from 10° (a) to 30° (b), the forward bias must be reduced from 200 mv to 150 mv. The temperature difference is approximately 20°; the voltage difference is 50 mv. You can calculate the variation in forward bias per degree centigrade by

$$\frac{\text{Difference in forward bias}}{\text{Difference in temperature}} = \frac{50\text{ mv}}{20°\text{ C}}$$

$$= 2.5\text{mv}/°\text{C}.$$

TRANSISTOR ANALYSIS

In the graphical analysis of the transistor, voltage, current and power gain are calculated from the output static characteristic curves. Dynamic transfer characteristic curves, constructed from the forward transfer static characteristic curves and the output characteristic curves, are used to determine distortion under overload conditions. The constant power dissipation line, from which the operation of the transistor within its power dissipation ratio is determined, is constructed on the output static characteristic curve from collector voltage and current readings at a number of points. In addition, a graphical explanation can be made of the effect of collector current and voltage on the capacitance existing between the collector and the base.

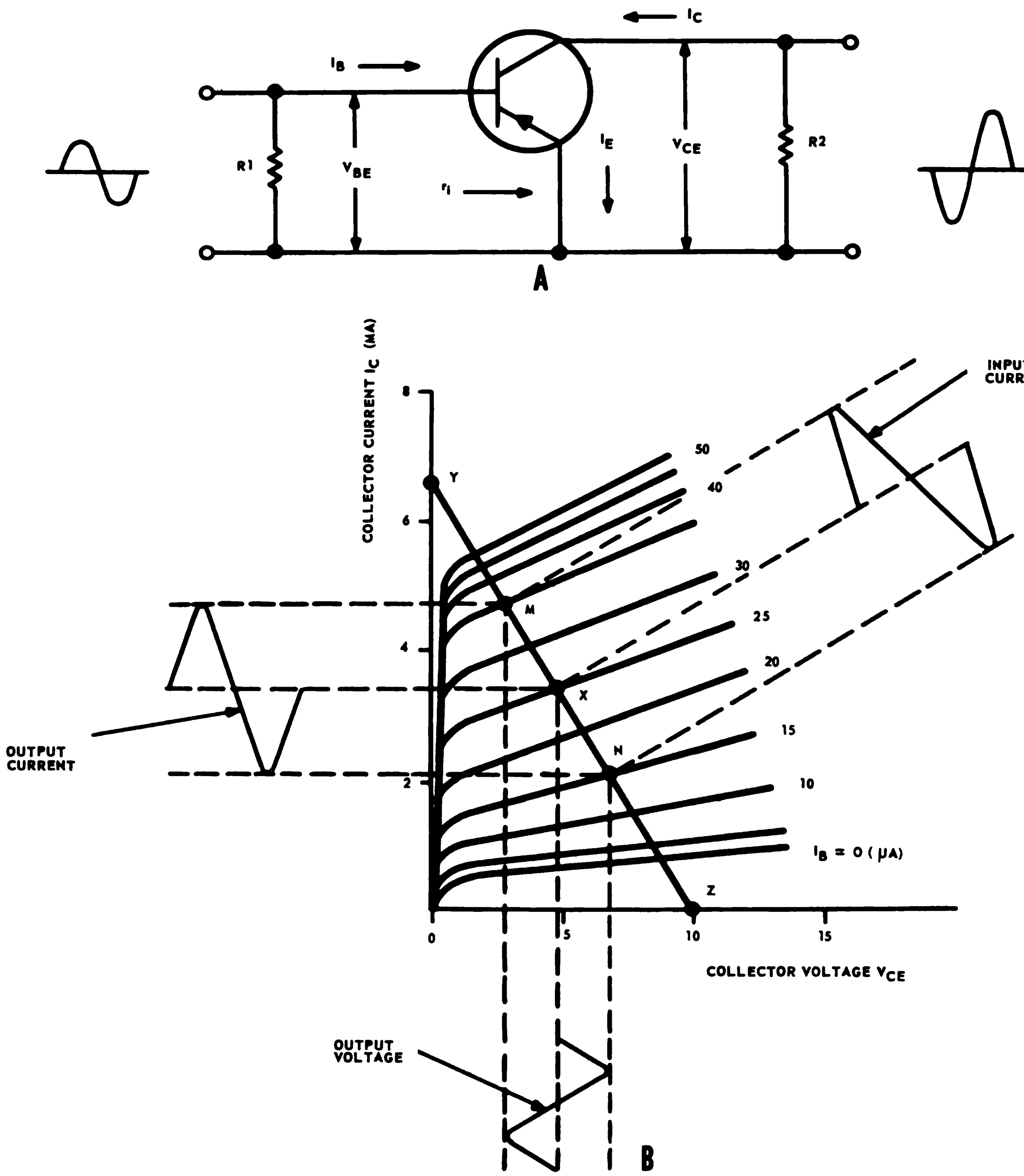


Figure 8–36. CE Amplifier and Its Output Characteristic Curves with Loadline

Output Characteristic Curves and the Calculation of Gain

Output characteristic curves are obtained by plotting the collector current against the collector voltage with the base current held constant. The common-emitter static characteristic curves are used to calculate the current, voltage and power gain of a common-emitter transistor amplifier. The accompanying example shows how this is accomplished.

Example

Find the voltage, current and power gain for the common emitter transistor amplifier of Figure 8–36A, given the following:

Collector supply voltage = 10 volts
Load resistance R_2 = 1500 ohms
Emitter-base input resistance (r_i) = 500 ohms
Peak-to-peak input current = 20 microamperes

Step 1:

Establish the loadline for R_2 on the output characteristic curves in Figure 8–36A. (This is done in the same way as for an electron-tube amplifier.)

At zero collector current, collector voltage (V_{CE}) = 10v, the collector supply voltage. Place point Z at the 10-volt mark on the horizontal axis. At maximum current the voltage on the collector is zero, dropping the total collector supply voltage across R_L.

$$I_C = \frac{10}{1500} = .0067 \text{ ampere}$$

$$I_C = 6.7 \text{ ma}$$

Place point Y at the 6.7–ma mark on the vertical axis. Connect point Y with point Z.

Step 2:

Locate the operating point.

A base current of 25 μa will provide Class A operation at normal operating temperatures. Mark point X where the loadline crosses the 25-μa curve. This is the operating point. A line dropped from point X to the horizontal axis shows a collector voltage of 4.8 volts provides this base current; a line drawn from X perpendicular to the vertical axis shows that a collector current of 3.4 ma flows at this point.

Step 3:

Plot the input waveshape.

Since the peak-to-peak current is 20 μa, the deviation is 10 μa above (35 μa) and 10 μa below (15 μa) the operating point. Place point M at the intersection of the loadline and the 35-μa curve. Place point N at the intersection of the loadline and the 15-μa curve. The input current waveform is drawn between these limits to the right of the characteristic curves in Figure 8–36B.

Step 4:

Plot the output current and voltage waveshapes.

Draw horizontal lines from points M, X and N to the vertical axis. The points where the lines through M and N cross the axis are the upper and lower limits of the output current variations. The output current waveshape is drawn between these limits to the left of the curves in Figure 8–36B. Drop lines from M, X and N to the horizontal axis. The points where the lines from M and N cross the voltage axis mark the limits of the output voltage variation. The output voltage waveshape is drawn between these limits below the characteristic curves in Figure 8–36B.

Step 5:

Determine the current gain.

Current gain is the ratio of the change in collector current to the change in base current.

$$A_i = \frac{\Delta I_C}{\Delta I_B}$$

$$= \frac{I_{C(max)} - I_{C(min)}}{I_{B(max)} - I_{B(min)}}$$

Substitute the values in the formula

$$A_i = \frac{4.7 \text{ ma} - 2.1 \text{ ma}}{35\,\mu a - 15\,\mu a}$$

$$= \frac{2.6 \text{ ma}}{20\,\mu a}$$

$$= \frac{2.6 \text{ ma}}{0.02 \text{ ma}}$$

$$= 130$$

Therefore, the current is amplified 130 times.

Step 6:

Determine the voltage gain.

Voltage gain is the ratio of the change in collector voltage to the change in base voltage.

$$A_v = \frac{\Delta V_{CE}}{\Delta V_{BE}}$$

$$\Delta V_{BE} = \Delta I_B r_i$$

$$A_v = \frac{V_{CE(max)} - V_{CE(min)}}{\Delta I_B r_i}$$

Substitute values in the formula.

$$A_v = \frac{6.7v - 2.7v}{20\,\mu a \times 500 \text{ ohms}}$$

$$= \frac{4v}{.00002 \times 500}$$

$$= \frac{4}{.01}$$

$$= 400$$

Therefore, the voltage is amplified 400 times.

Step 7:

Determine the power gain.

Power gain is the voltage gain times the current gain.

$$G = A_v A_i$$

Substitute values obtained in Steps 5 and 6.

$$G = 130 \times 400$$

$$G = 52{,}000$$

Thus, we see that the power input has increased 52,000 times in going through the transistor.

The power gain converted to db is:

$$G = 10 \log 52{,}000$$

$$= 10 \times 4.7$$

$$= 47 \text{ db}$$

Dynamic Transfer Characteristic Curve

You can learn a great deal from the loadline on the output static characteristic curve, but it is more difficult than getting the information from the dynamic characteristic curve. The curve shown in Figure 8–37 is formed when the effects of the loadline are added to the forward transfer static characteristic curves. When you study how the collector current acts with a signal current applied to the base, you will find it helpful to plot the input signal and the resulting collector current along the dynamic transfer characteristic.

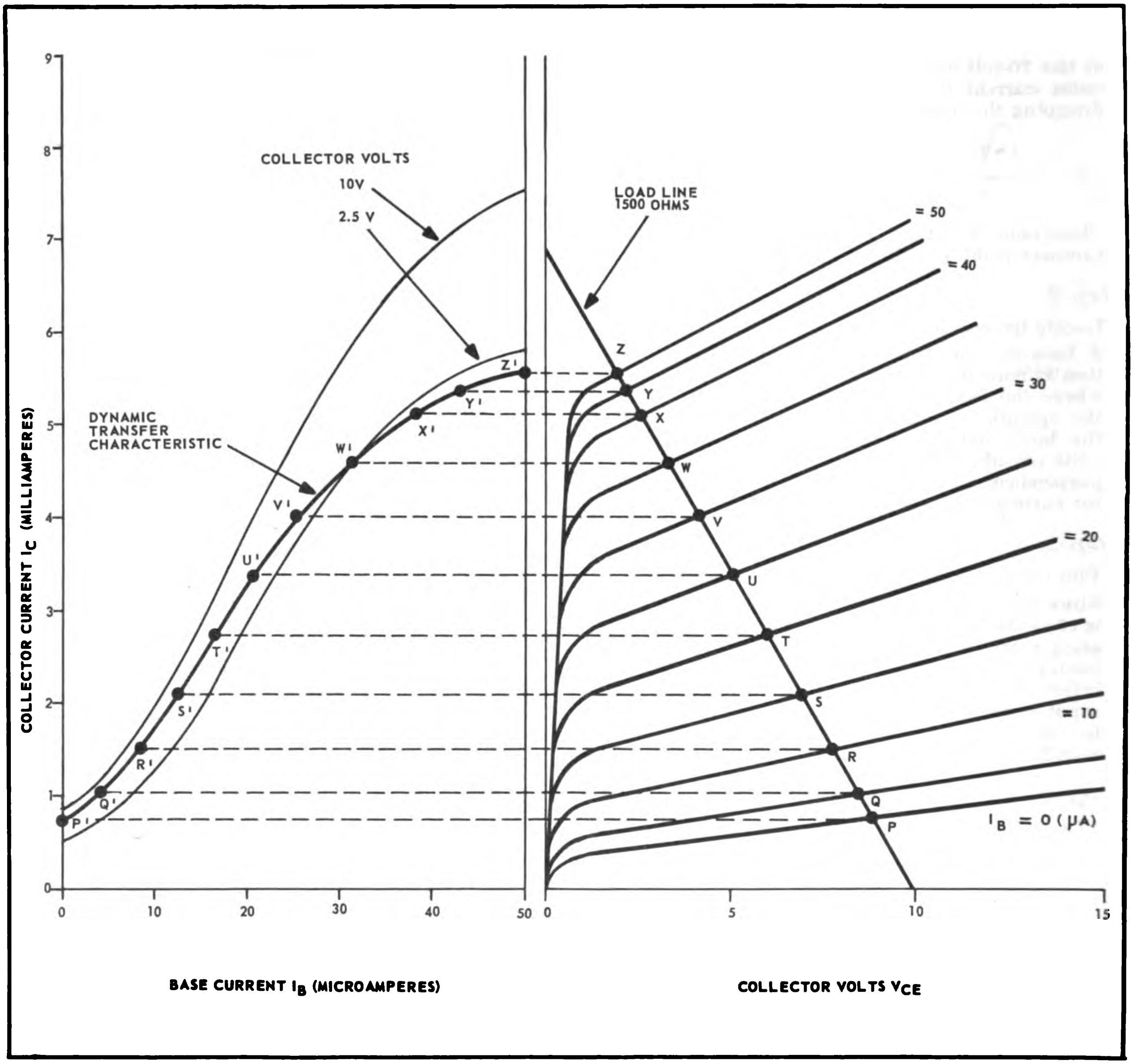


Figure 8–37. Construction of Dynamic Transfer Characteristic Curve

In Figure 8–37 the output characteristic with the 1500-ohm loadline is shown to the right. The transfer static cnaracteristic is drawn to the left. The 1500-ohm loadline is projected on the transfer static characteristic so that we may show the collector current-base current curve which represents the collector current corresponding to certain base currents and the effect of the load in creating the effective collector voltages. The two families of curves have three related characteristics — a common collector-current axis and, although shown differently, common values of base current, and common values of collector-supply voltage.

In projecting the effect of the loadline on the transfer static characteristics, we will first plot the collector current values (points P through Z) for each value of base current shown on the output characteristic curve. This is done in the following manner:

(1) Extend a line horizontally from point P of the output characteristic curve to point P′ of the transfer static characteristic curves. ($I_B = 0$ and $I_C = 0.85$ ma.)

(2) Extend a line horizontally from each of the remaining points (Q through Z) to establish Q′ through Z′ of the transfer static characteristic curves.

(3) Connect points P′ through Z′ in series to establish the *dynamic* transfer characteristic curve.

(4) The two curves marked 2.5v and 10v on the transfer characteristics are obtained with collector voltage held constant and the output AC short-circuited, the static condition. Since the collector voltage now varies because of the presence of the load resistor, the collector voltage of the dynamic transfer characteristic curve is not shown as a constant value.

The transistor will operate linearly when the proper operating point is established, and if the change of base current is within the linear portion of the dynamic transfer characteristic curve. Under these conditions the amplified output signal will be an exact reproduction of the input signal, which is Class A operation. Figure 8–38 shows input and output waveshapes during linear operation, drawn on the dynamic characteristic curve. The output shows that no distortion has been introduced.

If the change of base current exceeds the linear portion of the dynamic transfer characteristic curve when the proper operating point is used, the input signal will overdrive the transistor and the amplified output signal will be distorted. In Figure 8–39 the distorted output waveshape is shown drawn on the dynamic characteristic curve for a transistor. Notice how both the top and the bottom of the output waveshape have been flattened.

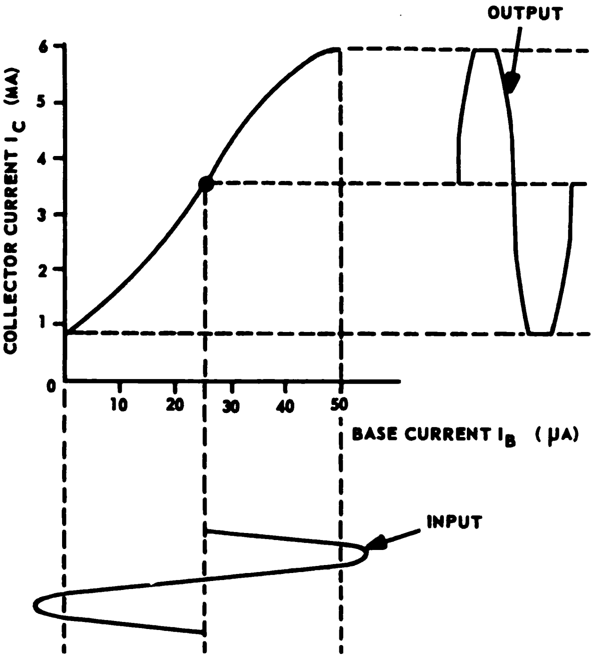


Figure 8–39. Overdriven Operation

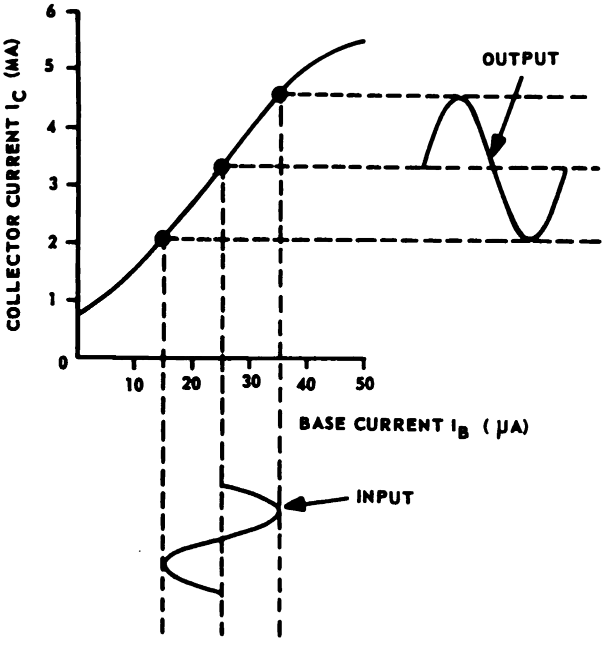


Figure 8–38. Linear (Class A) Operation

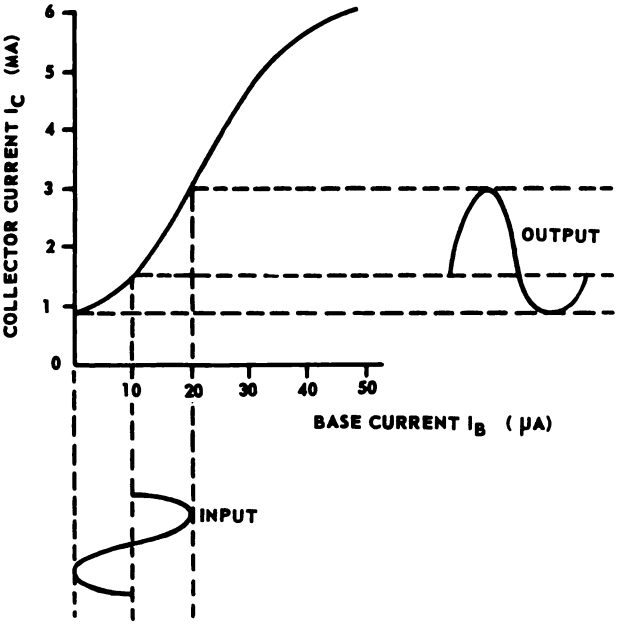


Figure 8–40. Nonlinear Operation

If the operating point is either too low or too high, the change of base current will go beyond the linear portion of the dynamic transfer characteristic curve. When the operating point is too low, as in Figure 8–40, the output signal is distorted during the negative change in collector current. When the operating point is too high, the positive-going portion of the input is not on the linear portion of the characteristic curve.

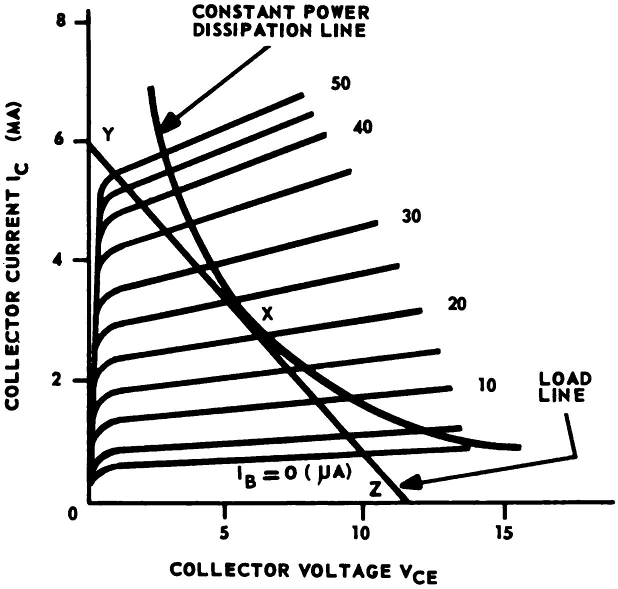


Figure 8–41. Output Static Characteristic Curves with Constant Power Dissipation Line

Constant Power Dissipation Line

The maximum collector power of a transistor is the power it can safely dissipate without damage. To be certain that you do not exceed the maximum collector power dissipation rating, you first draw a constant power dissipation line on the output static characteristic curves (Figure 8–41). Then you choose a collector load resistor that will have a loadline falling in the area bounded by the vertical and horizontal axes and the constant power dissipation line.

Let us assume that the rating for a given transistor is 18 mw. The output static characteristic curves are as shown in Figure 8–41. The constant power dissipation line for this transistor is drawn through the points of collector voltage and collector current whose product ($P = EI$) is 18 mw. To find these points the following procedure may be used.

Solve the power formula $P = EI$ for I.

$$I = \frac{P}{E}$$

Substitute 18 mw for power in the formula, and for a series of values of voltage find corresponding values of current. For instance, if 15 volts is one value,

$$I = \frac{18\text{ mw}}{15\text{ v}} = 1.2\text{ ma}$$

Use enough voltage values to insure construction of the line.

E (volts)	I (ma)	E (volts)	I (ma)
15	1.2	6	3.0
12	1.5	5	3.6
10	1.8	4	4.5
8	2.25	3	6.0

Table 8–1

Locate these voltage-current points and mark them on the output static characteristic curves. The curve produced by connecting these points is the constant power dissipation line.

Any loadline that is tangent to the constant power dissipation line will allow maximum permissible power gain of the transistor while operating without exceeding the maximum collector power dissipation rating. In the use of power amplifiers this is very important.

Interelement Capacitances

The interelement capacitances associated with the transistor are shown in Figure 8–42. The indicated capacitances are shown externally, although the actual capacitance effects are produced by the PN junctions within the transistor. The capacitance values vary because the widths of the PN junctions vary according to the voltages across them and the current flow through them. Since this is true, the recommended collector supply voltage specified by the transistor manufacturer must be maintained within very close tolerances.

The collector-base capacitance depends on the collector voltage and the emitter current.

As the reverse bias voltage (V_{CB}) increases, the width of the PN junction between base and col-

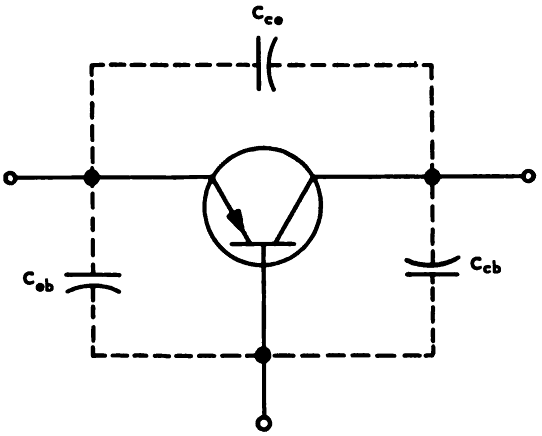


Figure 8–42. Effective Interelement Capacitances of the Transistor

lector increases. This increase in width lowers the interelement capacitances. This phenomenon is equivalent to the spreading apart of the plates of a capacitor to lower the capacitance. An increase in emitter current, most of which flows to the collector through the base-collector junction, effectively reduces the width of the PN junction. This results in an increase in collector-base capacitance (C_{cb}). This phenomenon is equivalent to reducing the distance between the plates of a capacitor to increase the capacitance.

The collector-base capacitance causes positive feedback in high-frequency amplifier applications that may result in oscillation. To keep a high-frequency amplifier from oscillating, external circuits must be used. The average value of collector-base capacitance (C_{cb}) varies from 2$\mu\mu$f for high-frequency transistors to 50 $\mu\mu$f for low-frequency (audio) transistors.

The collector-emitter capacitance (C_{ce}) of a transistor is also caused by the PN junction and varies with emitter and collector voltages. Common-base audio amplifiers have a better high-frequency response than common-emitter amplifiers because the collector-emitter capacitance is normally 5 to 10 times greater than the collector-base capacitance.

The variation of interelement capacitance with current and voltage is used in frequency-modulating oscillators.

Although the emitter-base capacitance (C_{eb}) is very high because of normal forward bias and the resultant small width of the PN junction, it does not offer many problems in amplifier design because it is normally shunted by a low input resistance.

Typical Circuit Values

Three different transistor circuit arrangements have been discussed thus far. Table 8–2 gives typical values for input and output resistances, current, voltage and power gain for medium-power transistors in the three transistor circuit arrangements.

Item	CB Amplifier	CE Amplifier	CC Amplifier
Input resistance	30-150 ohms	500-1500 ohms	20K-500K ohms
Output resistance	300K-500K ohms	30K-50K ohms	50-1000 ohms
Voltage gain	500-1500	300-1000	Less than 1
Current gain	Less than 1	25-50	25-50
Power gain	20-30 db	25-40 db	10-20 db

Table 8–2

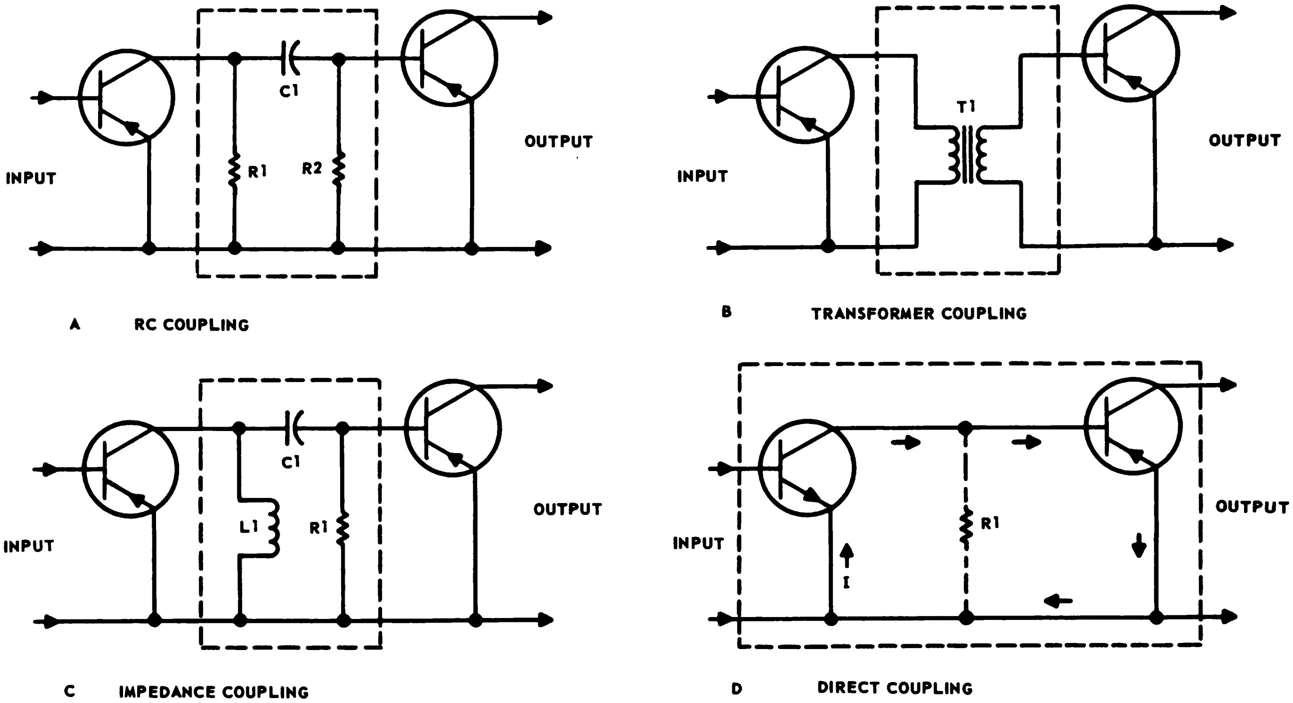
COUPLING NETWORKS

The same types of coupling are used between transistor stages as between electron tube stages. RC networks, transformers, impedance-coupling networks, and direct coupling circuits are illustrated in Figure 8–43 as interstage coupling between transistor stages. Variable resistors in the coupling networks are used to change the current, voltage and power gains of the transistor stages. The variable resistor is usually referred to as the gain control. At audio frequencies the gain control is referred to as a volume control.

RC-Coupled Amplifiers

RC coupling is used extensively with junction transistors. In Figure 8–43A, *R1*, *C1*, and *R2* form the RC network between the two transistor stages. *R1* is the collector load resistor for the first stage; *C1* blocks DC and couples the AC signal; *R2*, the DC return resistor for the input element of the second stage, develops the signal applied to the input of the second stage.

C1 prevents the DC voltage of the collector of the first stage from appearing at the input



A RC COUPLING

B TRANSFORMER COUPLING

C IMPEDANCE COUPLING

D DIRECT COUPLING

Figure 8–43. Interstage Coupling Networks

terminal of the second stage. The reactance of the capacitor, which is in series with the input resistance of the following stage, must be small compared to the input resistance. Otherwise much of the signal voltage will be dropped across the capacitor. A high value of capacitance is used because the input resistance of the following stage is low (usually lower than 1,000 ohms). But the physical size of the capacitor can be kept small because of the low voltages used. The ohmic value of the DC return resistor is usually 7 to 15 times the input resistance of the second stage. This ratio of resistance is necessary to prevent shunting the signal current around the input circuit of the second stage.

Since the reactance of the coupling capacitor increases as the frequency decreases, the very low frequencies are attenuated. The shunting effect of the collector-emitter capacitance of the first stage and the base-emitter capacitance of the second stage limits the high-frequency response of the amplifier. The efficiency of the RC-coupled amplifier (ratio of AC power output to DC power delivered to the stage) is low due to the dissipation of DC power in the collector load resistor.

RC coupling is used to a large extent in audio amplifiers, from low-level, low-noise preamplifiers to high-level power amplifiers. Other advantages of RC coupling with transistors are high gain, economy of circuit parts, and small size. In battery-operated equipment, however, RC coupling is usually limited to low-power operation to limit battery drain.

Transformer-Coupled Amplifiers

Transformer coupling between stages is shown in Figure 8–43B. The collector load impedance of the first stage is the primary winding of transformer T1 (including the AC reflected load from the secondary winding). The secondary winding of transformer T1 performs two functions; it introduces the AC signal to the base and acts as the base DC return path. The power efficiency of the transformer-coupled amplifier approaches the theoretical maximum of 50% as there is no collector load resistor to dissipate power. This explains the extensive use of the transformer-coupled amplifier in portable, battery-operated equipment.

As explained earlier, variations in temperature produce variations in the operating point of a transistor. The very low resistance in the base path helps temperature stabilize the DC operating point. A swamping resistor in the emitter lead maintains the current stability. Transformers can be used to match the load to the output impedance of the transistor and to match the source to the input impedance of the transistor. Impedance matching is necessary for obtaining maximum available power gain in a given stage.

At low frequencies the shunt reactance of the primary winding of the transformer reduces the low-frequency response. At high frequencies the collector capacitance and the leakage reactance between primary and secondary windings reduce the high-frequency response. Compared to the resistors and capacitors used for coupling, transformers are more expensive, heavier, and larger in size. Therefore, the use of transformers is normally restricted to those applications where high-power efficiency and high output power are required.

Impedance-Coupled Amplifier

An impedance coupling network is very similar to the RC network in an RC-coupled amplifier. Either one or both resistors are replaced by inductors. Figure 8–43C illustrates one of the more frequently used arrangements with the collector load resistor replaced by an inductor. This provides high power efficiency since there is practically no DC power loss. The shunt reactance of the inductor reduces the low-frequency response, while the collector capacitance reduces the high-frequency response. There is no leakage reactance to produce a loss of the high frequencies in the impedance-coupled amplifier, as in the transformer-coupled amplifier. The frequency response of the impedance-coupled amplifier lies between that of the transformer-coupled amplifier and that of the RC-coupled amplifier.

Direct-Coupled Amplifier

The direct-coupled amplifier is used for amplification of DC signals and of very low frequencies. In the circuit of Figure 8–43D, NPN and PNP transistors are directly connected; the arrows indicate the direction of current flow. A collector load resistor (R1) must be connected as indicated by the dotted lines if the collector current of the first stage is larger than the base current of the second stage.

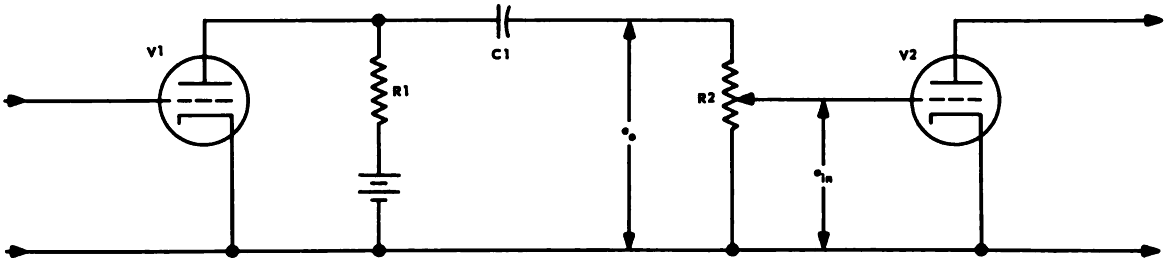
The direct-coupled amplifier is the most economical of the four types because so few circuit parts are required. But there is a limit to the number of stages that can be directly coupled. Severe temperature instability is produced because the temperature variation of the bias current in one stage is amplified by all the stages.

Volume Controls

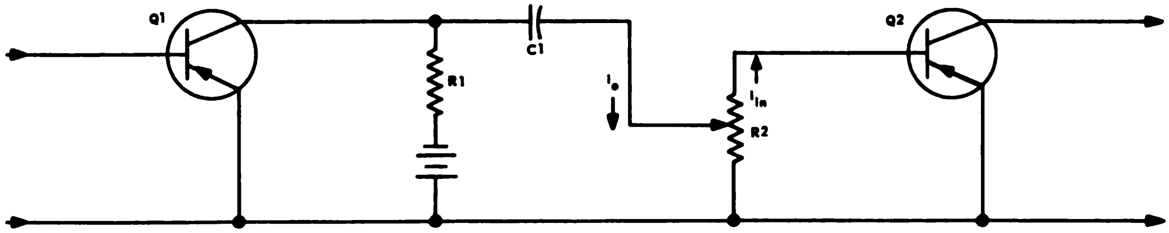
Whether a volume control is a voltage divider or a current divider depends on the magnitude of the source impedance compared to the magnitude of the load impedance. When the source impedance is low, we can consider the source to be a constant-voltage source and the volume control is a voltage divider. In the case of the electron tube circuit in Figure 8–44A, the source impedance (plate impedance of V1) is low compared to the load (grid-cathode input impedance of V2). The volume control (R2) in this case is a voltage divider. Normally the grid-cathode circuit does not draw current. The voltage (e_{in}) that is developed between the movable arm and the lower end of the variable resistor R2 controls the signal current flow in the cathode-plate circuit of V2. By moving the arm from the upper to the lower end of variable resistor R2 the signal input to V2 can be varied from maximum (V1 output voltage $e_o = e_{in}$) to zero ($e_{in} = 0$) respectively.

If the source impedance is high compared to the load impedance, the volume control is considered a current divider because the source is considered a constant-current source. In the case of the transistor circuit shown in Figure 8–44B the source impedance (Q1 collector) is high compared to the load (Q2 base-emitter impedance). In this circuit the volume control (R_2) is a current divider. While the grid-cathode circuit of the electron tube normally does not draw current, the input circuit of the transistor normally draws considerable current.

Collector signal output current i_o from Q1 divides at the movable arm and in this manner supplies a varying amount of input current (i_{in}) to the Q2 base-emitter circuit. Output current i_o is equal to input current i_{in} when the variable arm is in the upper position, since the base-emitter junction resistance can be considered negligible compared to the normal resistance value of variable resistor R_2. When the variable arm is in this position, maximum gain is obtained. When



A VOLTAGE DIVIDER CIRCUIT



B CURRENT DIVIDER CIRCUIT

Figure 8–44. Volume Control Circuits

the variable arm is in the lower position, no signal input current ($i_o = 0$) enters Q2 and the gain is zero.

Certain requirements are necessary in a volume control circuit. First, the circuit should be so arranged that the volume control introduces either no noise or a minimum amount of noise. This requirement can be achieved by avoiding a heavy flow of direct current through the volume control. Second, the volume control and its associated circuit should permit the gain to be varied from zero to maximum. In addition, the volume control should be so designed for a given circuit that all frequencies are attenuated equally for all positions of the variable arm of the control.

Tone Controls

A tone control is used to manually adjust an amplifier's frequency response. Like volume control circuits, tone control circuits may be used as current dividers. To change any fixed low-frequency, or high-frequency, compensating network to a tone control circuit, substitute a manually variable circuit element for one of the fixed circuit elements.

Capacitors C1 and C2 and resistor R3 in Figure 8–45 form a high-frequency compensating network. If R3 is made variable, you have a treble (high-frequency) boost tone control circuit.

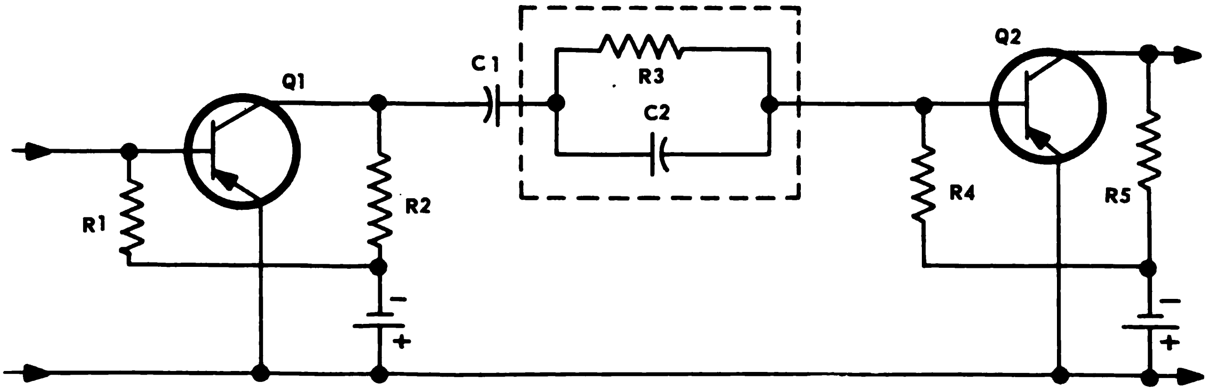


Figure 8–45. Two-Stage Preamplifier, Providing High-Frequency Compensation for Transducer

TRANSISTOR LIMITERS

One circuit which lends itself particularly well to transistorization is the limiter circuit. In our study of transistors, we have determined that by applying the correct bias voltage to the emitter-base junction and collector-base junction, the transistor can be made to amplify a small input signal. This may be accomplished by using either a PNP or NPN transistor. The type of transistor used will determine the polarity of the biasing voltages. The transistor amplifier, like the vacuum tube amplifier, can be operated at cutoff or at saturation. This being true, a transistor limiter may be constructed to operate as a cutoff limiter or as a saturation limiter.

Cutoff Limiting

The circuit of Figure 8–46A is a typical transistor limiter circuit. The electron current flow is indicated by the direction of the arrows. To simplify the explanation of the circuit, the flow of minority carriers across the reverse-biased

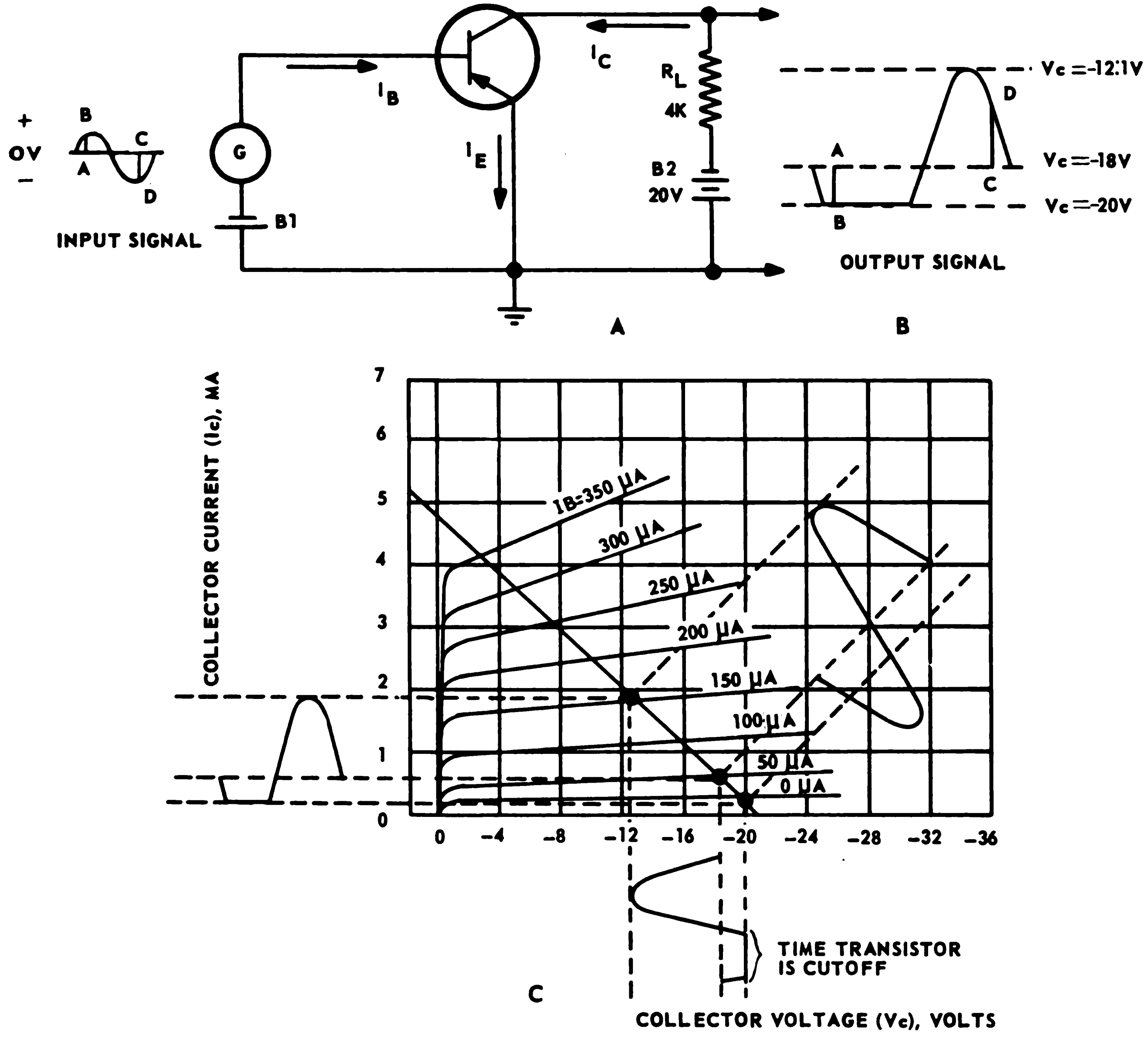


Figure 8–46. Cutoff Limiting Using a PNP Transistor

base-collector junction has been ignored. The portions of emitter current that flow through the collector and the base have been discussed earlier in this chapter. (If an NPN transistor were used in place of the PNP transistor, the polarities of the batteries would have to be reversed, but the phase relationship would remain the same.)

Let us take the first alternation of the input signal voltage, which opposes the forward bias produced by the base-emitter battery (B1). This effectively decreases bias voltage, thereby decreasing the emitter current. The collector and base currents are decreased by corresponding amounts. The decreased collector current flowing through the load resistor decreases the voltage drop across this resistor.

As the current and voltage drop across the load resistor continue decreasing, the collector voltage (V_q) approaches the collector-emitter bias voltage, in this case —20v. Since the operating point of the circuit has been placed on the loadline at a point that will cause the collector current to reach 0 ma with the input signal shown, the transistor will be cut off during a portion of the alternation. This condition is called cutoff limiting. When the amplitude of the input signal is AB as shown in Figure 8–46A, the signal drives the circuit beyond cutoff and no current flows. The output is at —20v as shown.

Now let us take the second alternation; the input voltage aids the forward bias produced by the emitter-base battery (B1). As a result the forward voltage is increased, thereby increasing the emitter current. The collector and base currents are increased by corresponding amounts. The increased collector current flowing through the load resistor increases the voltage drop across the resistor. Thus, the collector voltage (V_c) rises to —12.1v. This effect is shown on the output waveform, Figure 8–46B. As the input signal begins decreasing (CD) the output waveshape begins to go more negative. During the entire negative alternation the input signal aids the forward bias; the output signal is positive going, although, as shown, the entire output is negative due to the applied —20 volts.

The amount of cutoff limiting in the output is determined chiefly by the amount of forward bias. However, the type of transistor as well as the input circuit resistance and the amplitude of the input signal are also determining factors that affect the amount of cutoff limiting. A decrease in the forward bias shifts the operating point to a lower value of quiescent base current, thereby increasing the amount of cutoff limiting. Increasing the forward bias shifts the operating point to a higher value of quiescent base current, thereby decreasing the amount of cutoff limiting.

Saturation Limiting

The circuit in Figure 8–47A uses a PNP transistor amplifier biased to provide saturation limiting. The electron current flow through this common-emitter amplifier is indicated by the direction of the arrows. If an NPN transistor were to be used, the polarities of the batteries would have to be reversed but the input and output phase relationships would remain the same as for the PNP transistor.

During the first (positive) alternation the input signal voltage (Figure 8–47A) opposes the forward bias produced by the base-emitter battery (B1). The resultant forward voltage is decreased, thereby decreasing the emitter current. The collector and base currents are decreased by corresponding amounts. The decreased collector current flowing through load resistor R_L produces a smaller voltage drop across R_L. The collector voltage V_c changes from a no signal value of —1 volt to —6v as shown in Figure 8–47B. This may be determined from the characteristic curve of the limiter circuit illustrated in Figure 8–47C. For the entire half-cycle that the input goes positive, the output is negative going. No limiting action occurs.

During the second alternation the input signal voltage aids the forward bias produced by the emitter-base battery. The resultant forward voltage is increased, thereby increasing the emitter current. The collector and base currents are increased by corresponding amounts. The operating point of the circuit in Figure 8–47C has been established on the loadline above the point for Class A operation. The increase in amplitude of the input signal between X and Y does not increase the collector current, which is at saturation. The increased collector current flowing through load resistor R_L increases the voltage drop across R_L. At saturation a steady voltage of about 15.9 volts opposes the battery voltage B2.

The output voltage levels off at —0.1v. This may be determined from the characteristic curve of the limiter circuit illustrated in Figure 8–47C. This effect, shown on the output waveform, is

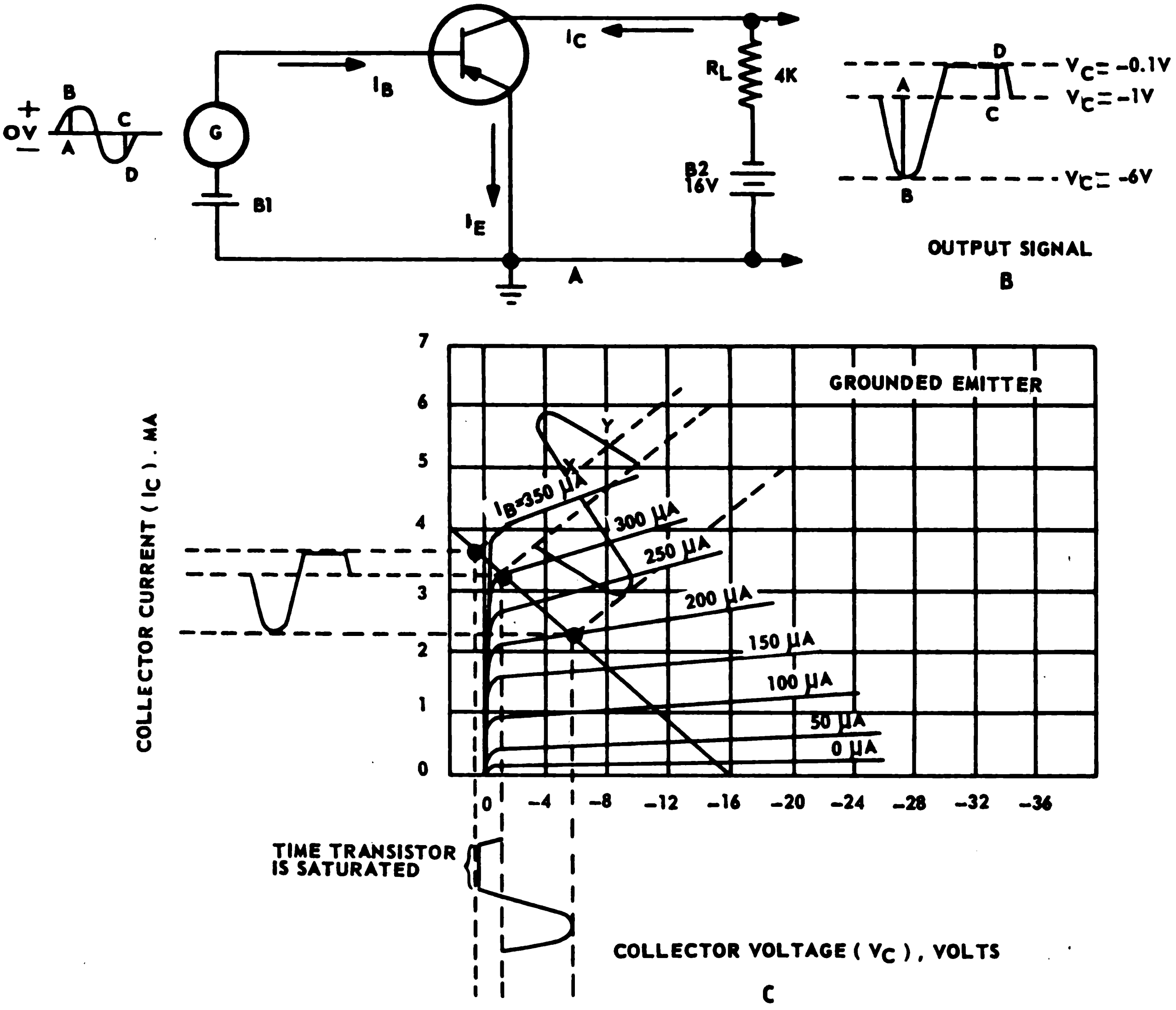


Figure 8–47. Saturation Limiting Using a PNP Transistor

called saturation limiting. Increasing the forward bias shifts the operating point to a higher value of quiescent base current, thereby increasing the amount of saturation limiting. Decreasing the forward bias shifts the operating point to a lower value of base current, thereby decreasing the amount of saturation limiting. Other factors determining the amount of saturation limiting are the type of transistor used, the input circuit resistance, and the amplitude of the input signal.

Although saturation limiting may be achieved easily by using transistor circuitry, it is seldom used because a transistor operating at, or near, saturation generates considerable heat. This is undesirable because it increases the collector cutoff current (I_{CO}) of the transistor. With a high I_{CO}, the transistor is very likely to develop a short circuit across the junction. Once this has occurred, the transistor is useless and must be replaced.

Cutoff and Saturation Limiting

Figure 8–48 illustrates a conventional common-emitter amplifier circuit that may be adjusted for either cutoff or saturation limiting. Capacitor C1 is a coupling capacitor for the AC signal component input as well as a blocking capacitor for the DC component of the preceding stage. Resistor R7 is the collector load resistance

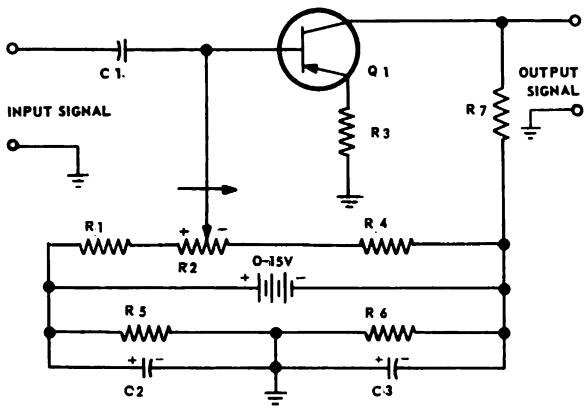


Figure 8–48. Transistor Triode Limiter

and serves to develop the output signal from collector to ground.

Resistors R1, R2 and R4 make up a voltage divider network. Potentiometer R2 varies the amount of forward bias applied to the base of the transistor. Resistors R5 and R6 serve as an additional voltage divider that is bypassed to ground through capacitors C2 and C3. Capacitor C3 is used to maintain a steady supply voltage for the collector and base sections of the transistor; C2 is used to keep emitter-to-base voltage constant. The operation of this network (R5, C2 and R6, C3) is identical to that of a cathode resistor and bypass capacitor in a vacuum tube cathode circuit. Resistor R3 is a new component in the study of transistor circuit analysis and warrants further discussion.

As was mentioned briefly under Bias Stabilization, the collector cutoff current of a transistor is extremely sensitive to temperature variations. That is, any increase in temperature will immediately increase the I_{CO}. Since the change in I_{CO} is effectively coupled to the base circuit, the change will be amplified by transistor action, thus increasing collector current. If the increased value of collector current exceeds the rated value of collector current, the transistor will be destroyed. To compensate for this effect, a resistor is inserted in the emitter leg. This resistor is called a stabilizing or swamping resistor.

The emitter current flowing through the stabilizing resistor (R3) develops a voltage at the emitter that is negative with respect to ground. Since the base is also negative with respect to ground, the emitter-base voltage has been reduced by an amount equal to the voltage drop across the stabilizing resistor. In Figure 8–48, assume that a temperature change causes a resultant rise in I_{CO}, which also increases the collector current.

Since the collector current makes up the major portion of emitter current flowing through R3, the emitter voltage will become more negative with respect to ground. The increase in voltage developed across R3 serves to oppose the forward bias across the emitter-base junction. With the forward bias decreased, the base current will also decrease, thereby decreasing the collector and emitter currents by corresponding amounts. We have stabilized the limiter circuit by reducing the collector current.

Circuit operation of the limiter circuit illustrated in Figure 8–48 is relatively easy to understand. The horizontal arrow bisecting the arm of potentiometer R2 indicates the direction the arm moves when rotated in a clockwise direction. When the arm is rotated clockwise, the base is made more negative with respect to ground. This increases the forward bias across the emitter-base junction. This increases the base current which, in turn, increases collector current. In this position of potentiometer R2 the circuit produces saturation limiting.

When the arm of potentiometer R2 is rotated in a counterclockwise direction, the base is made less negative with respect to ground. This decreases the forward bias across the emitter-base junction. The lower forward bias voltage decreases the base current which, in turn, decreases collector current. Now the circuit produces cutoff limiting.

By adjusting potentiometer R2 to about midrange you can obtain an undistorted output waveform. The limiter circuit is now operating as a normal amplifier with fair fidelity.

TRANSISTOR AMPLIFIER CIRCUITS

Push-Pull Amplifiers

Figure 8–49 illustrates a simplified circuit of a Class B amplifier with the emitter-base junctions zero biased. Neither transistor conducts with no signal input. With an input signal, the transistors conduct on alternate half-cycles. Collector current flows during the half-cycle of the input signal voltage that aids the forward bias of the transistor. The output signal is combined in the secondary of the output transformer.

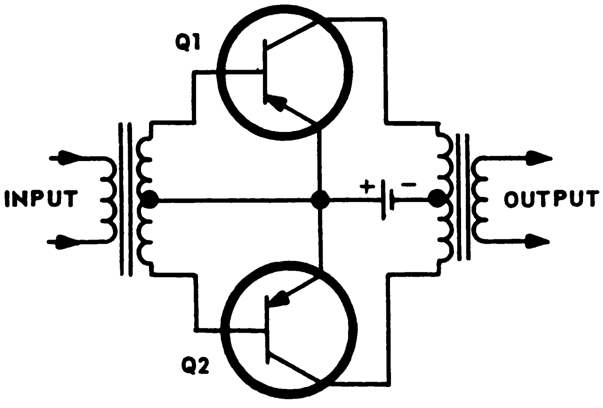


Figure 8–49. Class B Transistor Push-Pull Amplifier with No Forward Bias

Maximum efficiency is obtained because the transistors conduct only when there is a signal input.

To obtain an indication of the output current waveform for a given signal current input, use the dynamic transfer characteristic curve for the amplifier. This curve is derived by plotting the variations in collector (output) current for one transistor against base (input) current under load conditions (Figure 8–50A). Assuming that the two transistors are identical and have identical characteristics, you can place two of the curves of Figure 8–50A back to back and see the over-all dynamic transfer characteristic for the push-pull amplifier. Figure 8–50B shows the two curves combined back-to-back with the zero line of each curve lined up vertically to reflect the zero bias current. In Figure 8–50C a sine wave (input base current) is projected vertically onto the characteristic curve.

The projected points are determined and then projected horizontally to form the output collector current waveform. The distortion which occurs at the crossover points — the points where the signal passes through zero value — is known as *crossover distortion.* The lower the signal input currents, the greater the distortion. This type of distortion can be eliminated, however, by using a small amount of forward bias on both transistors of the push-pull amplifier.

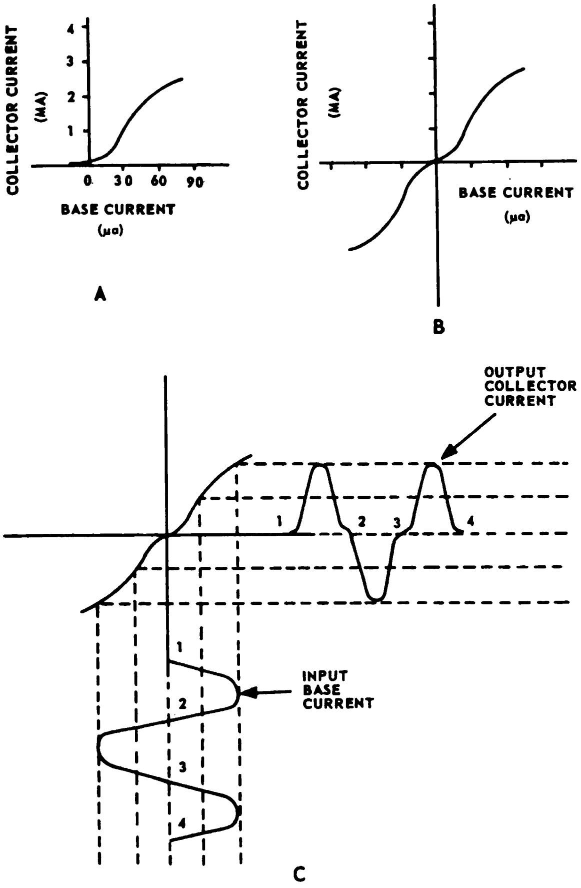


Figure 8–50. Transistor Push-Pull Amplifier Characteristics with Zero Bias

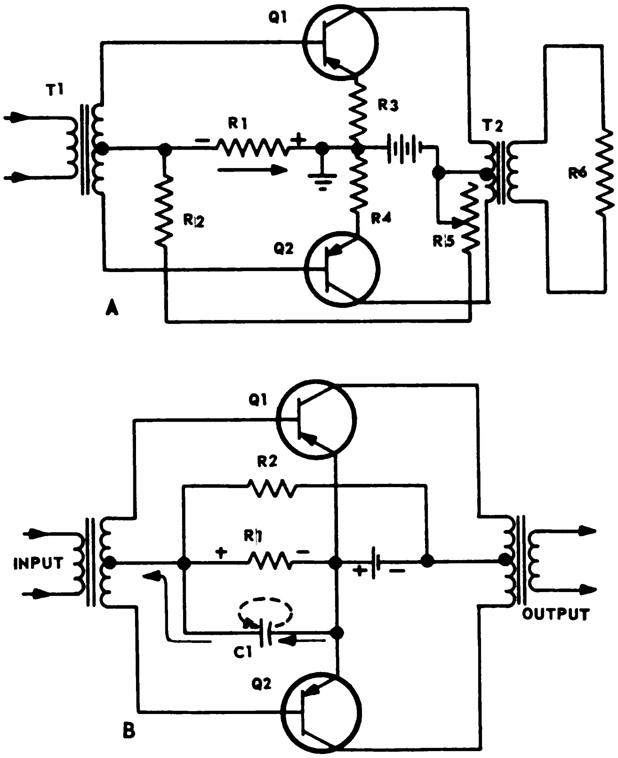


Figure 8–51. Transistor Push-Pull Amplifier with Forward Bias

Figure 8–51 depicts a Class B push-pull amplifier that has a small forward bias applied to the base-emitter junctions. Resistors R1 and R2 form a voltage divider. Crossover distortion is eliminated by the small forward (base-emitter) bias developed across R1 for both transistors.

The curves in Figure 8–52 show how crossover distortion is eliminated in the amplifier circuit of Figure 8–51. Figure 8–52A shows, placed back to back, the dynamic characteristic curve of each transistor under zero base current bias conditions. The two curves are not combined. The base current obtained when forward bias is applied is shown by the dashed lines. This is done to obtain the over-all dynamic characteristic curve of the amplifier. Then the separate curve of each transistor, with forward bias applied, is placed back-to-back and aligned at the base current line (dashed line), Figure 8–52B. Now the zero base current lines (solid lines) are offset. The two characteristic curves form a nearly straight line.

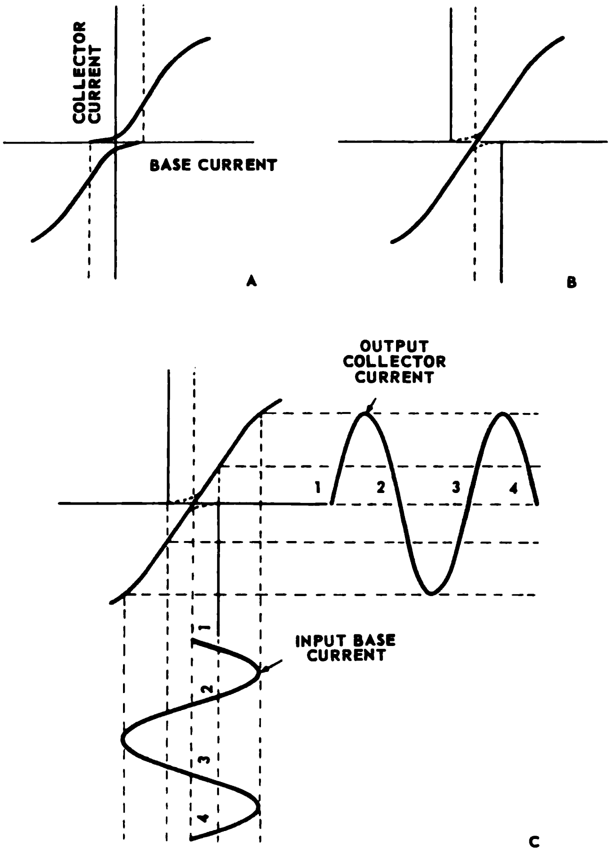


Figure 8–52. Transistor Push-Pull Amplifier Characteristics with Forward Bias

In Figure 8–52C, points on the sine wave of input base current are projected vertically onto the characteristic curve. From these points, projection is made horizontally to form the output collector current waveform. After comparing this output current waveform with that in Figure 8–50C, you will note that crossover distortion has been eliminated by applying a small forward bias.

In the circuit shown in Figure 8–51A the arrow indicates the direction of electron current flow from the battery in the voltage divider. The direction of current establishes the polarity of voltage across resistor R1. This voltage is the required small forward bias. You will note that no bypass capacitor has been placed across R1. Figure 8–51B shows what happens with a bypass capacitor across R1. When there is an input signal, the capacitor charges (solid line arrow) through the base-emitter junction of the conducting transistor and discharges (dashed line arrow) through resistor R1. During discharge, a DC voltage with the indicated polarity is developed across R1. This voltage acts as reverse bias and could drive the amplifier into Class C operation, which would produce greater distortion than crossover distortion. As you can see, a bypass capacitor must not be used in a transistor push-pull amplifier for distortionless operation.

Wide-Band or Video Amplifiers

Transistor wide-band, or video, amplifiers have the same general characteristics as vacuum tube wide-band, or video, amplifiers. That is, the transistor video amplifier must also use compensating networks to compensate for the interelement capacitances of a transistor.

Figure 8–53 illustrates a typical transistor video amplifier using both high- and low-frequency compensation. L1 provides shunt peaking and L2 series peaking. R_f and C_f provide low-frequency compensation. These circuits do not interfere with each other as they operate independently. The reactance of inductor L1 in series with the load is very small at low frequencies and has little effect on the collector load impedance. Likewise, the reactance of inductor L2 in series with the coupling capacitor is very small at low frequencies and has little effect on the input circuit to the second stage.

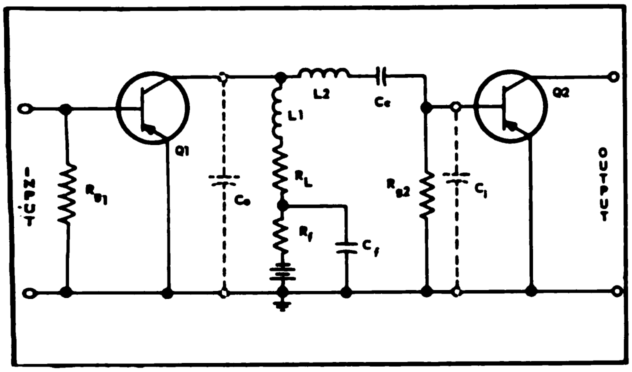


Figure 8–53. Transistor Wide-Band Amplifier with Interstage Compensation

At high frequencies the reactance of C_f is very small, and C_f can be considered a short circuit across resistor R_f.

The losses at low frequency are caused by the increasing reactance of C_c. As the frequency decreases, the ratio of the reactance of C_c to the resistance of R_{g2} provides a smaller and smaller voltage drop across R_{g2} as the input to the second stage. As in the electron tube amplifier, R_f in series with the load provides additional gain at low frequencies. Since the resistance would have the same effect for all frequencies, C_f is shunted across R_f to reduce the gain at the high frequency end of the bandpass. At the higher frequencies, R_f is effectively shorted out.

The losses at the high frequency end of the bandpass are due to the stray capacitance and the capacitance of the output impedance, represented by C_o in Figure 8–53, and the capacitive effect of the input impedance, represented by C_i. These two capacitances are in parallel, which increases their effect. The shunt peaking circuit, consisting of L1 in series with R_L, forms a low-Q parallel resonant circuit with C_o and C_i. Instead of dropping off at the high-frequency end of the bandpass as before, the response remains flat until a considerably higher frequency is reached. We can consider that the inductance of L1 has canceled the capacitance of C_o and C_i. Placing L2 in series with the coupling capacitor provides series compensation, or series peaking. The two capacitances causing the losses are no longer in parallel, which reduces their effect. Series peaking circuits are more difficult to adjust than the shunt peaking type.

The effectiveness of the compensating circuits can be checked by applying a square-wave input to the circuit in Figure 8–53. The waveshapes on the left-hand side of Figure 8–54 show the composition of the square-wave input. The right-hand waveshapes show the output with various types of distortion. If the value of resistor R_L is too small, the amplification of high frequencies may not fall off as soon, but the output waveshape may look like that in Figure 8–54A. This waveshape shows the effect of too little amplification of the low frequencies or too much amplification of the high frequencies.

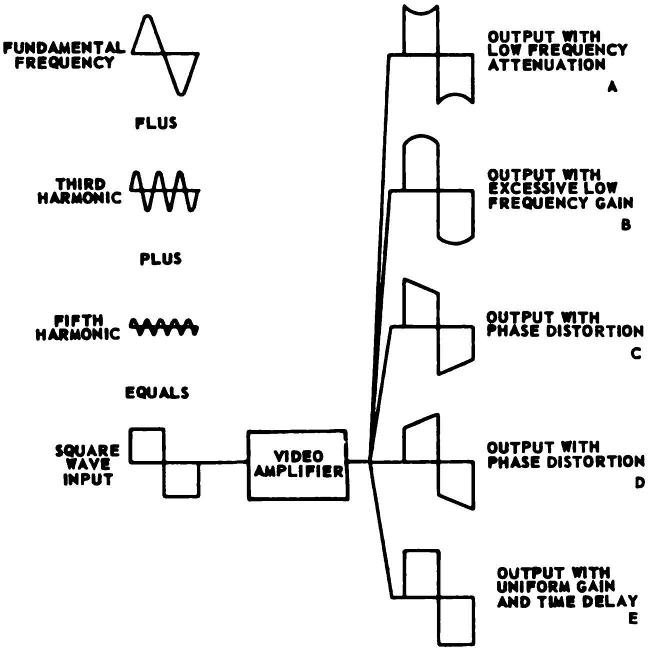

FUNDAMENTAL FREQUENCY
PLUS
THIRD HARMONIC
PLUS
FIFTH HARMONIC
EQUALS
SQUARE WAVE INPUT
VIDEO AMPLIFIER
OUTPUT WITH LOW FREQUENCY ATTENUATION
A
OUTPUT WITH EXCESSIVE LOW FREQUENCY GAIN
B
OUTPUT WITH PHASE DISTORTION
C
OUTPUT WITH PHASE DISTORTION
D
OUTPUT WITH UNIFORM GAIN AND TIME DELAY
E

Figure 8–54. Square-Wave Input and Output Characteristics

If either the compensating resistor R_f or the capacitor C_f is too small, the attenuation of the low frequencies will not be taken care of. If R_L is too large, or if the compensating circuit comprised of R_f and C_f is too large, the low frequencies will have more gain than the high frequencies, resulting in the waveshape at B. This is the same waveshape that is produced by too much attenuation of the high frequencies. These two effects can also be produced by the wrong amount of high frequency compensation. If either L1 or L2 provide too little compensation, the output will appear similar to that in Figure 8–54B. If they provide too much compensation, the output will be similar to that in Figure 8–54A.

If the effect of the phase shift across C_c and R_{g2} is not eliminated by the time constant of

R_fC_f, phase distortion occurs. Figure 8–54C shows the output when the phase leads at low frequencies, and Figure 8–54D shows the output when the phase lags at low frequencies.

Transistor Common-Collector Amplifier

The transistor common-collector amplifier, Figure 8–55, is also a type of single-stage amplifier. The input signal is applied to the base and the output signal is taken from the emitter. Since the output emitter signal follows the input base signal in both phase and time, the transistor common-collector amplifier is also referred to as a transistor cathode follower.

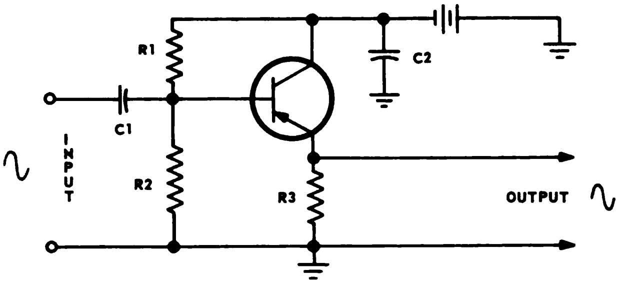


Figure 8–55. Single-Stage Common-Collector Amplifier

The common-collector amplifier, like the common-base and common-emitter amplifiers, can be biased with a single bias battery. However, the method used in obtaining forward bias for the base-emitter junction in Figure 8–55 is a voltage divider network comprising resistors R1 and R2. This arrangement provides the necessary forward bias across the base-emitter junction by making the base negative with respect to ground. If an NPN transistor were to be used instead of the PNP transistor, the polarity of the bias battery would have to be reversed, but the voltage phase relationship would be the same.

The input signal of Figure 8–55 is applied between base and ground. However, since bypass capacitor C2 effectively places the collector at AC ground, the signal is actually being applied between base and collector.

The output signal is taken across the emitter load resistor, R3, which is between the emitter and ground. The input impedance of the common-collector amplifier is very high, being 50,000 ohms or more. The output impedance is very low, usually less than 100 ohms. The transistor common-collector amplifier, like the electron-tube cathode follower, has a voltage gain of less than 1. Power gain, however, is fairly good.

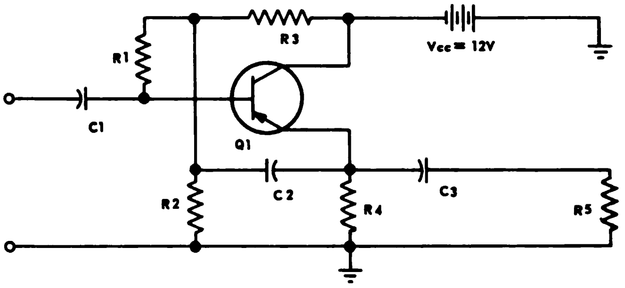


Figure 8–56. Common-Collector Amplifier with Feedback

Another variation of the common-collector amplifier is shown in Figure 8–56. The output signal is taken across the emitter load resistor R4. The signal is then coupled to load resistor R5, through capacitor C3. The addition of capacitor C2 furnishes feedback between the emitter and the base to reduce the shunting effect of the base voltage divider, resistors R2 and R3.

Figure 8–57 shows the frequency response of the common-collector amplifier in Figure 8–56. Notice that by increasing the bias voltage, V_{cc} from —3 volts to —12 volts, the flat response of the amplifier is extended toward the high end of the frequency spectrum. Although the voltage gain increased slightly with the increase in bias voltage, the gain remains less than one.

Tuned Amplifiers

An electrical device that transmits energy in one direction only is called a unilateral electrical device. The transistor is not such a device. The

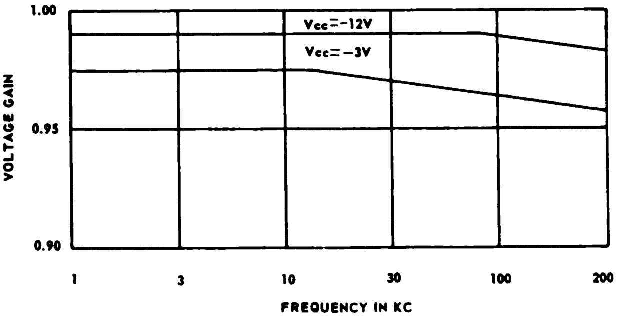


Figure 8–57. Frequency Response of a Common-Collector Amplifier

voltage variations in the output circuit cause voltage variations in the input circuit. This internal voltage feedback to the input circuit from the output circuit gives the same effect as placing an additional generator in the input circuit. The feedback voltage, which aids the input voltage, is positive, or regenerative, feedback. If enough regenerative voltage is fed back, the amplifier will oscillate. Since the feedback voltage at low frequencies is usually small, special circuit precautions are not ordinarily required to prevent oscillation. On the other hand, high-frequency amplifiers with tuned coupling networks are susceptible to oscillations and special circuit precautions must be taken.

Either positive or negative feedback alters the impedance of the input circuit of an electrical device. It affects both resistive and reactive components of the input impedance. The change caused by the internal feedback can be eliminated by using an external feedback circuit. When both the resistive and the reactive changes in the input circuit are canceled by the external feedback circuit, the amplifier is considered *unilateralized.* If only the reactive changes in the input circuit are canceled by the feedback circuit, the amplifier is considered to be *neutralized.* This is a special case of unilateralization. The external feedback circuit prevents oscillation in the amplifier in either case.

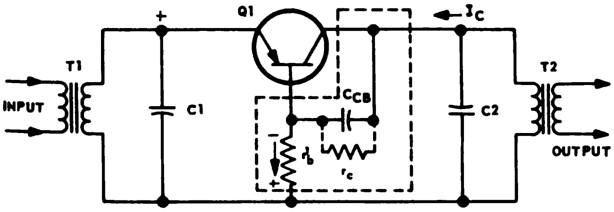


Figure 8–58. Common-Base Amplifier Showing Internal Feedback Elements

Figure 8–58 illustrates a tuned common-base amplifier. Feedback through the internal elements of the transistor shown in dashed lines may cause oscillations. For simplicity the DC biasing voltages have been omitted.

Resistor r'_b in Figure 8–58 is known as the base-spreading resistance. It represents the resistance of the bulk material of the base. The capacitance of the base-collector junction is represented by capacitor C_{cb}. The resistance of the base-collector junction is represented by resistor r_c. The reverse bias on the base-collector junction makes this resistance very large. Capacitor C_{cb} effectively shunts resistor r_c at very high frequencies.

If the forward bias is aided by the incoming signal, the emitter becomes more positive with respect to the base. This causes collector current (I_C) to increase in the direction indicated by the arrow in Figure 8–58. Part of the collector current flows through capacitor C_{cb} and resistor r'_b aids the incoming signal and therefore is considered a positive feedback that may cause the amplifier to oscillate.

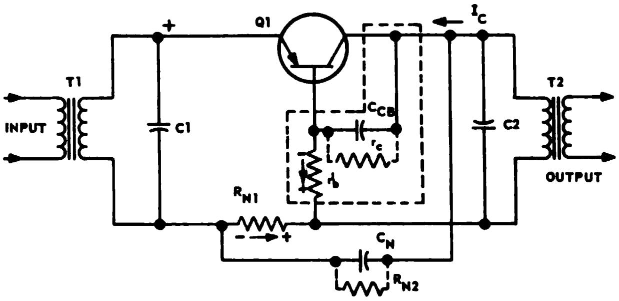


Figure 8–59. Common-Base Amplifier Showing Internal and External Feedback Elements

Figure 8–59 shows the addition of an external circuit, R_{N1}, and R_{N2} and C_N, to the high-frequency amplifier circuit. Resistors R_{N1} and R_{N2} and capacitor C_N correspond to resistors r'_b and r_c and capacitor C_{CB}. The need for resistor R_{N2} is dependent upon the frequency to be amplified; as the frequency increases, the need for resistor R_{N2} decreases.

The collector current increases when the incoming signal aids the forward bias. Part of the collector current flows through capacitor C_{CB} and resistor r'_b. The resultant voltage drop across r'_b has the indicated polarity. A portion of the collector current also flows through capacitor C_N and resistor R_{N1}. The polarity of the voltage across R_{N1} opposes the voltage across r'_b. When the voltages are equal there is no positive or negative feedback from the output circuit to the input circuit, and the amplifier is unilateralized.

Emitter degeneration is used to unilateralize the common-emitter amplifier shown in Figure 8–60. Transformer T1 couples the input signal

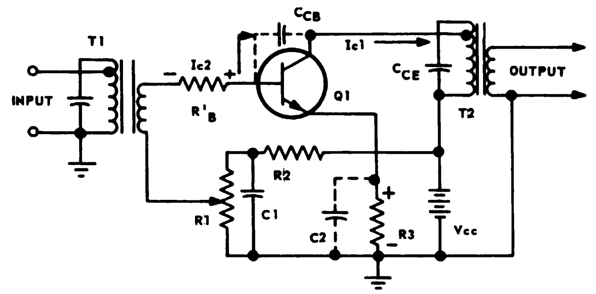


Figure 8–60. Common-Emitter Amplifier with Emitter Degeneration

to the base of transistor Q1, while transformer T2 couples the output signal to the following stage. Notice that the primaries of both the input and output transformers are tapped. This is done to achieve the proper impedance match between the output of one stage and the input of the following stage. The tapped coil of T2 presents a lesser DC loading effect on the transistor; thus, it is possible to obtain the desired Q which, in turn, lends better selectivity to the amplifier. R1 and R2 form a voltage divider network across the collector supply voltage (V_{cc}).

Potentiometer R1 is used as an r-f gain control in this circuit. Since transistor Q1 is an NPN transistor, a positive voltage applied to the base will increase gain. When the potentiometer arm is moved to the top of R1, the larger positive voltage applied to the base increases the gain of the stage. Capacitor C1 is used to maintain a constant potential across resistor R1.

When the input signal aids the forward bias, the collector current (I_{c1}) increases in the direction shown in Figure 8–60. Internally, a portion of the collector current (I_{c2}) is coupled to the base-spreading resistance through the collector-base junction capacitance (C_{CB}). The voltage developed across the base-spreading resistance aids the incoming signal and constitutes a positive feedback. To offset the positive feedback, the collector current I_{c1} flowing through stabilizing resistor R3 develops a degenerative voltage equal and opposite to that developed across the base-spreading resistance. The net voltage feedback to the input circuit is zero, and the amplifier is thus unilateralized. A large regenerative feedback from collector to emitter through interelement capacitance C_{CE} is present at all times and at all frequencies. This feedback is effectively shunted to ground when C2 is added to the circuit.

TRANSISTOR OSCILLATORS

Audio Oscillator

The transistor audio oscillator, shown in Figure 8–61, may have the output signal taken across the output terminals of transformer T2 or directly at the collector through capacitor C2. The loading effect on the circuit is decreased by C2. The common-emitter circuit for the oscillator has moderate input and output impedances. This reduces the requirement for a feedback network to provide matching. Current, power and voltage gains are all greater than one. In fact, the common-emitter transistor circuit provides the highest power gains, which makes this circuit a good one to use in a transistor oscillator.

In designing the transistor audio oscillator, the primary winding of transformer T1 (Figure 8–61) must match the high collector impedance of the transistor, while the secondary winding must match the low impedance of the base-emitter circuit. The frequency of oscillation will mainly depend upon the inductance of the windings in transformer T1 and their distributed capacitance. The frequency of the oscillator may be lowered by placing a capacitor (C1) in parallel with the secondary of T1. This will also decrease the amplitude of the oscillations.

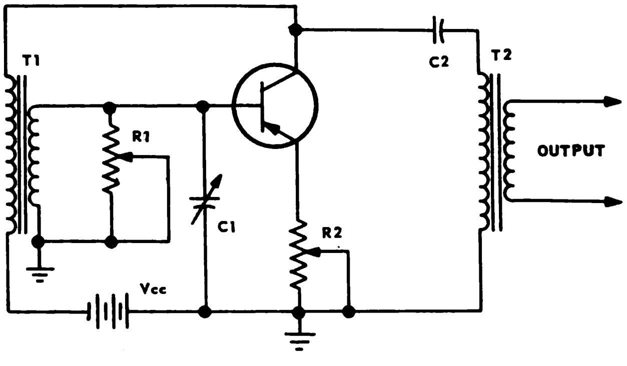


Figure 8–61. Transistor Audio Oscillator

Feedback for the oscillator in Figure 8–61 is obtained across transformer T1. Since a PNP transistor is used in the circuit, transformer T1 must be connected so that a negative voltage (forward bias) is applied to the base. Resistor R1 is a feedback control. When R1 is adjusted to a high value, a relatively high amplitude of feedback is applied to the base causing the oscillator to block. The feedback control should be adjusted for a sinusoidal output.

Frequency instability is caused by transistor characteristics rather than by the external circuit elements. The DC operating point is usually chosen so that the operation of the transistor circuit is over the linear portion of the transistor characteristic curve. When the operation of the circuit falls into nonlinear portions of the characteristic curve because of variations in bias voltages, the characteristics of the transistor vary. Since these characteristics are basic to the transistor and therefore affect frequency of oscillation, frequency variations will occur with changes in bias voltages. Consequently, the prime requirement for frequency stability is a constant supply voltage.

The operating point varies with changes in temperature in a transistor oscillator. Since the transistor oscillator is an amplifier with additional circuitry, the temperature can be stabilized in the same way for both oscillators and amplifiers. Resistor R2 of Figure 8–61 is the temperature stabilization resistor. Decreasing the value of resistor R2 effectively increases the forward bias across the emitter-base junction. This has the same effect as increasing the regenerative feedback to the base, thereby increasing the amplitude of the oscillation and causing a blocking condition. If resistor R2 is too large, degeneration will occur in the emitter circuit, thereby decreasing the amplitude of oscillation.

Several methods may be used to determine whether or not a transistor audio oscillator is oscillating. Headsets connected in the circuit should produce an audible sound during oscillation; a DC ammeter in the collector circuit will show a slight rise in I_c when the oscillator is oscillating. The most effective and accurate method of determining whether or not oscillations are present is the use of an oscilloscope connected to the output.

Series-Fed Hartley Oscillator

A transistor oscillator that lends itself well to use in a receiver is the Hartley. Figure 8–62 shows a transistor series-fed Hartley oscillator whose output is connected to a transistor buffer amplifier.

The manner in which oscillation is developed in the transistor series-fed Hartley oscillator is similar to that of the vacuum tube version. Transistor Q1 and the associated circuitry make up

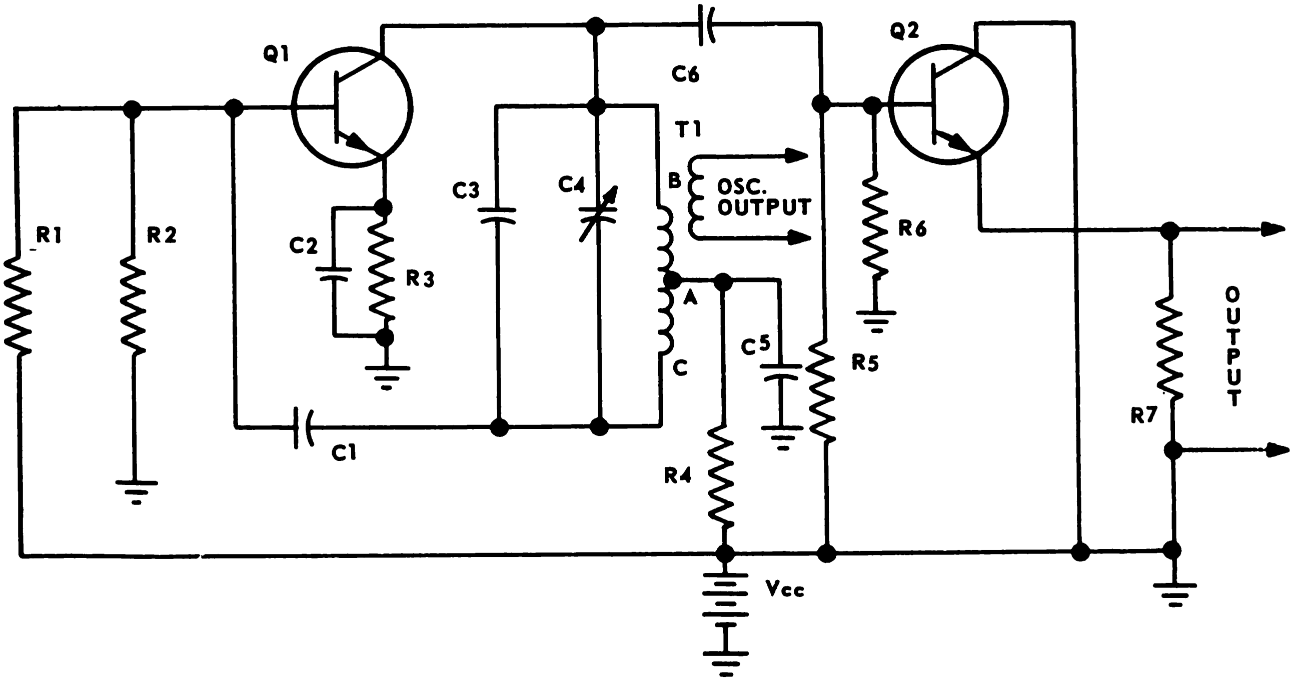


Figure 8–62. Transistor Series-Fed Hartley Oscillator and Buffer Amplifier

the oscillator portion of Figure 8–62. Capacitor C6 couples the oscillations to the base of transistor Q2. Transistor Q2 and the associated circuitry make up the transistor buffer amplifier.

In the oscillator section resistors R1 and R2 provide the necessary forward bias across the base-emitter junction. Since an NPN transistor is used in the oscillator section, a positive voltage will be applied to the base to provide the forward bias. Collector bias for transistor Q1 is obtained through the tapped transformer (T1) winding A to B. Capacitor C2 is the emitter bypass capacitor which provides the AC bypass around stabilizing resistor R3.

The regenerative feedback is obtained from the induced voltage in the autotransformer (T1) winding A to C, coupled through capacitor C1 to the base of the transistor. Capacitor C5 places the tapped terminal (A) of transformer T1 at AC ground potential. Resistor R4 is a DC dropping resistor to limit the value of collector current flowing through transformer T1.

Frequency of the oscillator is predominantly determined by the value of inductance in transformer T1 and the capacitance of C3 and C4. Capacitor C4 is variable to provide a means of varying the frequency output. In most applications the output is taken from the buffer amplifier, but the oscillator is sometimes provided with an auxiliary output. The auxiliary output from the oscillator would be across the secondary of transformer T1.

The buffer amplifier performs the same function that a vacuum tube buffer amplifier does. It acts as a one-way coupling device. Close observation of Figure 8–62 reveals that transistor Q2 and its associated circuitry is connected as a common-collector amplifier with the output taken across resistor R7. Resistors R5 and R6 provide the necessary forward bias across the base-emitter junction.

Shunt-Fed Hartley Oscillator

Figure 8–63 shows a version of the transistor shunt-fed Hartley oscillator. Resistors R1, R2, and R3 provide the necessary bias for the oscillator. Capacitor C4 in parallel with the primary of T1 forms the frequency determining network. The oscillator can be tuned to cover a wide range of frequencies since C4 is variable. The function of C2 is to block DC; capacitor C3 provides an AC bypass around stabilizing resistor R4.

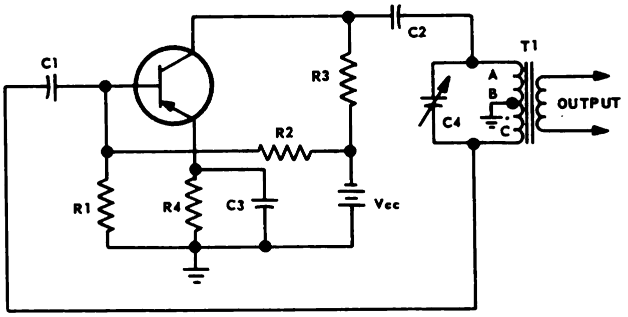


Figure 8–63. Transistor Shunt-Fed Hartley Oscillator

The primary of T1 acts as an autotransformer to supply the regenerative feedback signal. The feedback obtained from the induced voltage in transformer winding B to C is coupled through capacitor C1 to the base of the transistor. Direct-current flow through transformer T1 is avoided by shunt feeding the collector current through R3.

Like any other oscillator, a common-collector buffer amplifier may be added to the transistor shunt-fed Hartley oscillator to prevent undue loading of the oscillator.

Colpitts Oscillator

A transistor Colpitts oscillator circuit is illustrated in Figure 8–64. Capacitors C1 and C2 in parallel with the primary of transformer T1 constitute the tuned circuit. These capacitors form a voltage divider network. Both C1 and C2 are variable; either may be adjusted to control the frequency and the amount of feedback voltage. Regenerative feedback is obtained from C2 in the tank circuit and applied to the emitter of the transistor. For minimum feedback loss, the ratio of X_{c1} and X_{c2} should be approximately

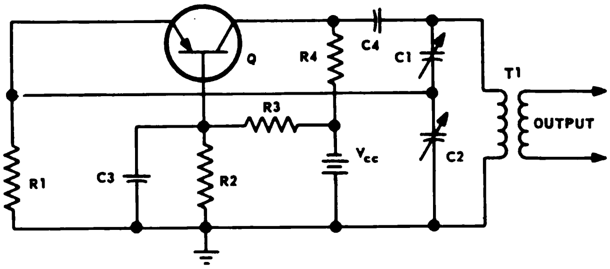


Figure 8–64. Transistor Colpitts Oscillator

equal to the ratio of the output impedance of the transistor to its input impedance. Base bias is provided by the voltage divider network of R2 and R3; resistor R4 acts as the collector load resistor. Resistor R1 develops the emitter input signal and also acts as the emitter stabilizing resistor.

Crystal Oscillator and Frequency Multiplier

The transistor crystal oscillator has the same advantages as an electron-tube crystal oscillator. A quartz crystal is normally used in an oscillator circuit because of its extremely high Q and good frequency stability over a given temperature range. Figure 8–65A is the circuit of a common-base Pierce oscillator. The bias voltages are supplied by the voltage divider consisting of R2 and R3 across the battery (V_{cc}) and R4 between the collector and the battery. If you redraw the circuit of Figure 8–65A replacing the crystal with its equivalent circuit (Figure 8–65B) you will see that the circuit closely compares to that of the Colpitts oscillator.

Its operation is similar. The voltage developed across capacitor C1 provides the regenerative voltage necessary from collector to emitter. R1 is the emitter swamping resistor. While the oscillatory frequency is determined mainly by crystal CR1, the parallel capacitance of C1 and C2 also affects the frequency. These capacitances are usually quite large to swamp the input and output capacitances of the transistor.

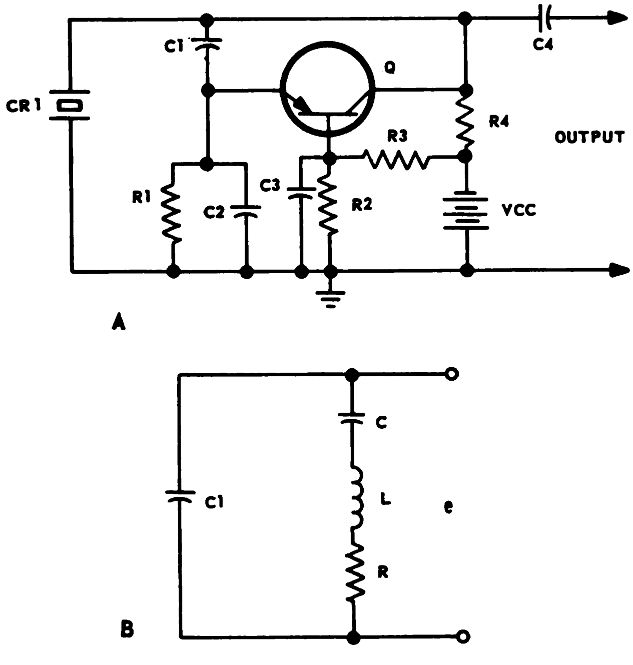


Figure 8–65. Pierce Oscillator and Equivalent Circuit

Figure 8–66 shows a transistor crystal oscillator whose output is transformer coupled to a frequency multiplier. Basically the crystal oscillator, transistor Q1 and its associated circuitry, is a modified Pierce oscillator. Resistors R2, R1, and R4 provide the necessary forward bias across the emitter-base junction. Capacitor C1 provides an AC bypass to ground and resistor R3 is the stabilizing resistor. The oscillating frequency of the oscillator is determined not only by crystal CR1, but also by the interelement capacitance existing between the base-emitter section and collector-emitter section of the transistor. The voltage developed across the base-emitter capacitance provides the necessary feedback for the oscillator. When transformer T1 is tuned to the oscillator frequency, the crystal, or fundamental frequency, will appear at the base of transistor Q2.

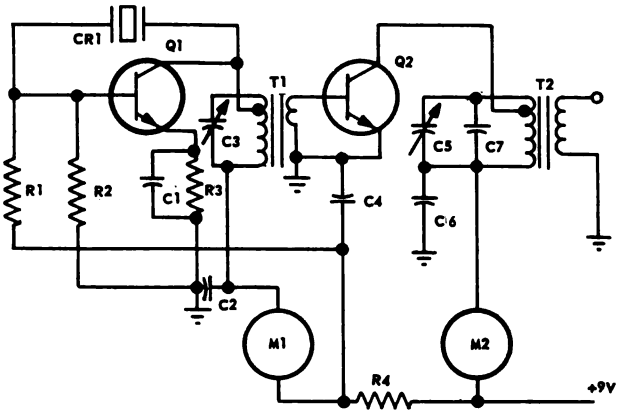


Figure 8–66. Transistor Crystal Oscillator and Doubler

Transistor Q2 and its associated circuitry make up the frequency multiplier stage. In this particular circuit the frequency multiplier is acting as a frequency doubler; the fundamental frequency applied to the circuit is doubled in the output circuit. Other oscillators may have frequency tripler or quadrupler stages.

Transistor Q2 is essentially a Class C r-f amplifier with no fixed bias. To operate the stage as a frequency doubler, the collector tank (transformer T2 and capacitors C5 and C7) is tuned to the second harmonic instead of the fundamental frequency applied to the base. Thus the frequency of the output voltage developed across the collector circuit and the circulating tank current, is twice the frequency of the AC current applied to the base of the transistor. The current flowing in the collector circuit is a pulsating DC, like the collector pulses in the Class C amplifier. Figure 8–67 shows the collector current pulses and the voltage relationships for the doubler amplifier in Figure 8–66.

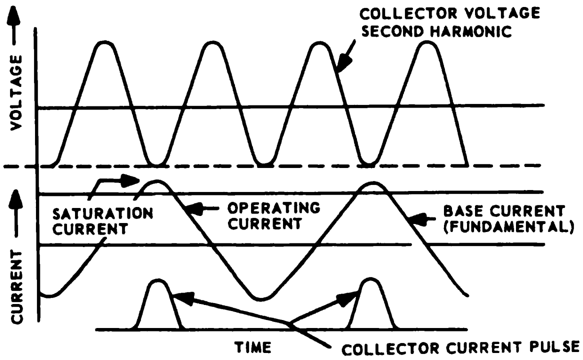


Figure 8–67. Voltage and Current Waveforms in a Doubler

In the doubler, the pulses of direct current flow for only half the time they would flow in a normal Class C amplifier. If the current flow were not reduced, the collector losses would be high because collector current would flow while the collector voltage was high. If the time of collector current flow is reduced, the average collector current is likewise reduced. Hence, the power input and power output are also reduced. In practice, the power output of a doubler is less than the output power of a conventional Class C amplifier. This is true for both transistor and vacuum tube amplifiers.

Unlike the Class C amplifier, the frequency multiplier need not be neutralized. Since its input and output circuits are tuned to different frequencies, the energy fed back through the interelement capacitance is not of the proper phase to sustain oscillations.

Frequency doublers are generally used in transmitters with a crystal oscillator to obtain higher frequencies than those at which a crystal can normally operate. Doublers are also found in transmitters using variable frequency oscillators. Here they help provide a wide band of frequencies.

Though it is possible to tune the output of an amplifier to the fourth multiple of the driving frequency, it is usually better to use two doublers in place of one quadrupler. The quadrupler requires a greater input signal and greater driving power, which increases the power losses in the input circuit.

For tuning the output circuit the crystal oscillator in Figure 8–66 uses a meter placed in series with the collector circuit. DC collector current flows from the emitter to the collector, then through the meter. If a PNP transistor were used, the direction of the current flow would be just the reverse, but phase relationships would remain the same. In tuning, the collector tank of the oscillator is first tuned for minimum current on M1 by adjusting tuning capacitor C3 in the collector tank; then C3 is detuned (the capacitance is increased) until about 0.1 ma more collector current flows. The reason for not tuning the collector circuit to the exact crystal frequency is the same as for the electron tube crystal oscillator; exact tuning might result in unstable operation.

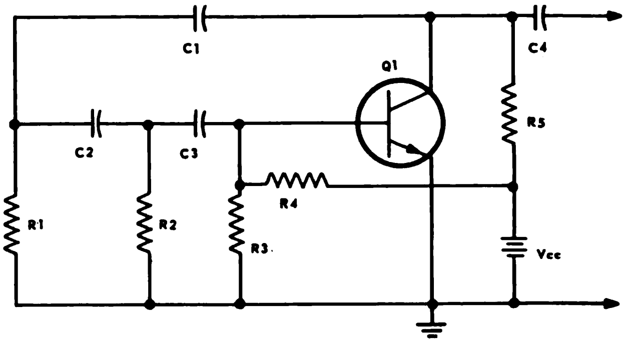


Figure 8–68. Transistor Lag-Line RC Oscillator

Lag-Line RC Oscillator

Figure 8–68 shows a transistor lag-line, or ladder phase-shift, oscillator; resistors R3, R4 and R5 provide the necessary base and collector bias for the oscillator. Capacitors C1, C2 and C3 and resistors R1, R2 and R3 constitute the feedback and phase-shifting networks.

Like the vacuum tube lag-line oscillator, oscillations are started by any random noise or transistor noise when power is applied. A change in the flow of base current results in an amplified change in collector current, phase shifted

180°. The signal returned to the base is inverted 180° by the action of the phase shift circuit, making it regenerative. An important factor governing feedback voltage in a transistorized RC oscillator is the type of transistor used in the circuit. Since losses incurred in the RC network are high, a high gain transistor must be used.

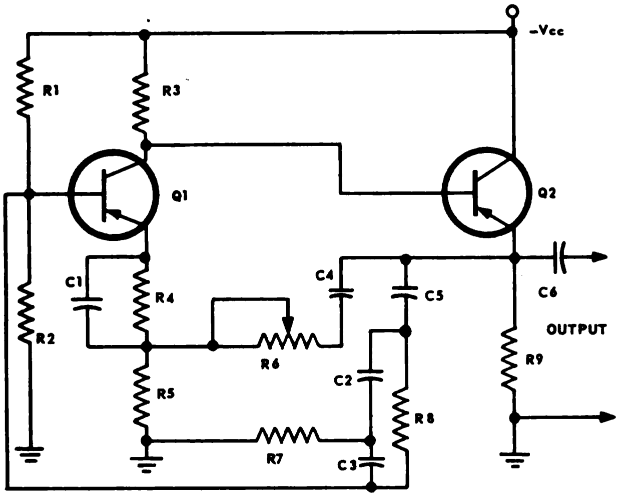


Figure 8–69. Transistor Bridged-T Oscillator

Bridged-T Oscillator

Another transistor circuit that uses a resistance-capacitance network for development of a sinusoidal output is the bridged-T oscillator. The circuit illustrated in Figure 8–69 uses two transistors. Transistor Q1 is connected as a common-emitter circuit that is directly coupled to the base of transistor Q2. Transistor Q2 is connected as a common-collector circuit with the output taken across the emitter resistor, R9. Resistors R1, R2 and R3 provide the necessary base and collector bias voltage for transistor Q1.

The bridged-T oscillator is similar in operation to that of a Wien bridge oscillator. Since two transistors are used to obtain the full 360° phase shift for proper regenerative feedback, an RC phase shift network is not necessary. Degenerative feedback is provided to obtain frequency stability and distortionless output. Regenerative voltage is fed from the emitter of transistor Q2 to the emitter of transistor Q1 through capacitor C4 and resistor R6.

The amplitude of this positive feedback may be controlled by adjusting resistor R6, the feedback control. In some applications an incandescent lamp having a tungsten filament may be substituted for R6 to stabilize the amplitude of the generated oscillations. The resistance of a lamp filament increases with a rise in current, thus maintaining a constant positive-feedback voltage.

A degenerative path for the undesired frequencies is provided from the emitter of transistor Q2, through capacitor C5 and the bridged-T network (resistors R7 and R8, capacitors C2 and C3) to the base of transistor Q1. The bridged-T network produces minimum voltage and zero phase shift at the desired frequency. This voltage is applied to the base of transistor Q1. At all other frequencies, the bridged-T network produces a voltage with a 180° phase shift. This degenerative feedback prevents oscillations at the undesired frequencies. The frequency at which the bridged-T produces zero phase shift and minimum voltage is determined by the values of R and C in the bridged-T network.

Blocking Oscillator

A transistor blocking oscillator, like a vacuum tube blocking oscillator, conducts for a short period of time and is cut off (blocked) for a much longer period of time. A basic circuit for the transistor blocking oscillator using a PNP transistor is shown in Figure 8–70. (If an NPN transistor were used in the circuit, the polarities of the collector supply battery V_{CC} would have to be reversed to maintain reverse bias across the collector-base junction.) Note the similarity between this circuit and the transistor audio oscillator previously discussed.

To understand the circuit operation, begin with the instant the circuit is first energized. Current rises rapidly in the base circuit due to the forward bias established across the base-emitter junction by the DC power supply V_{CC}. Collector current (I_c, Figure 8–70B) also increases. The increasing collector current induces a negative voltage in the secondary winding of transformer T1. This negative voltage (V_{B1}, Figure 8–70B) appearing at the base of the transistor serves to increase the forward bias across the base-emitter junction. Capacitor C1 charges through the small forward resistance of the base-emitter junction; when the transistor is saturated, the rapid regenerative action stops.

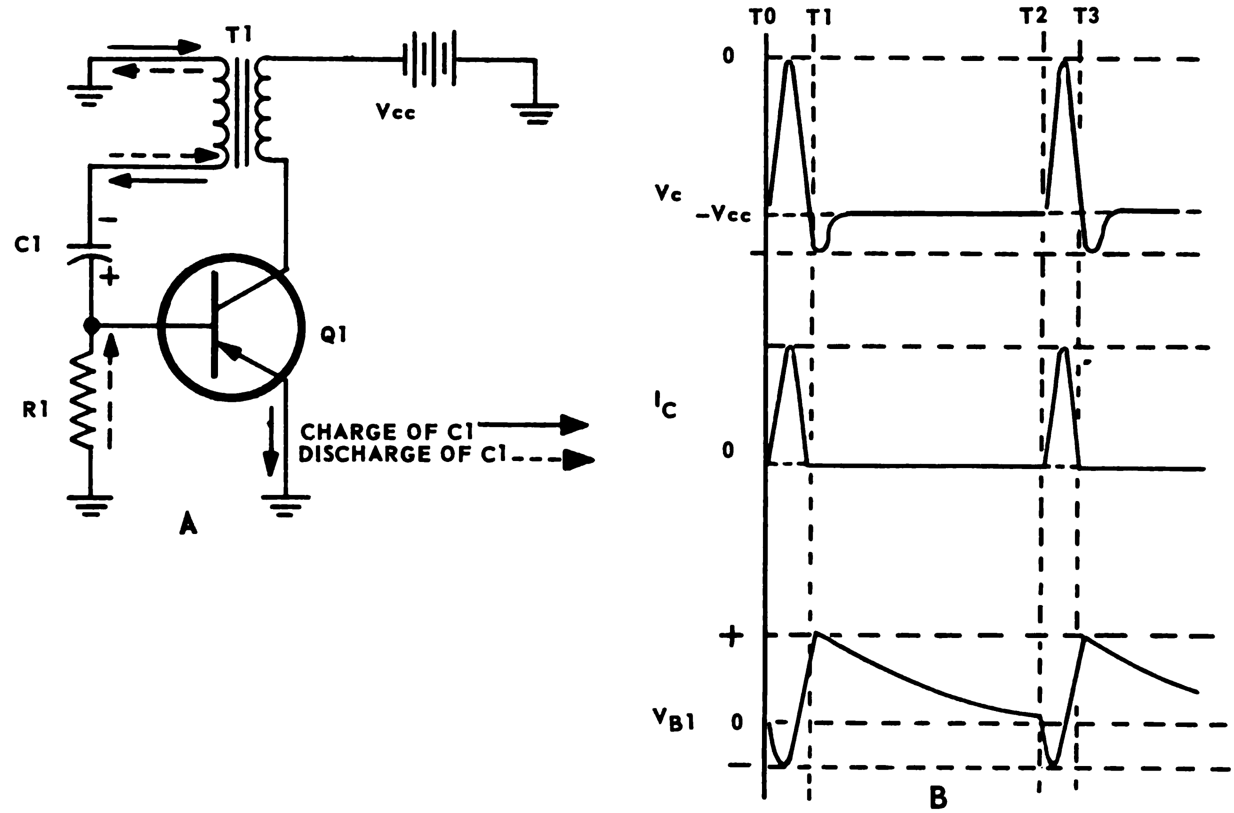


Figure 8–70. Transistor Blocking Oscillator with Waveform

At saturation the collector current (I_c) becomes constant and collector voltage (V_c) is nearly zero volts. As soon as there is no longer any induced voltage in the secondary winding of transformer T1, a charging potential is no longer applied across capacitor C1. Capacitor C1 begins discharging through resistor R1. Just before time T1 (Figure 8–70B), the field around the secondary winding of transformer T1 collapses and induces a voltage in the reverse direction driving the base positive (reverse bias). Base and collector current (I_c) decrease to zero. At the same time, collector voltage (V_c) increases in a negative direction and exceeds the collector supply voltage battery (V_{CC}) during the time the transistor is cut off (from time T1 to T2, Figure 8–70B). The transistor is held at cutoff until capacitor C1, discharging through R1, reaches the point at which the transistor is forward-biased and conduction begins again (time T2, Figure 8–70B).

A transistor blocking oscillator circuit with synchronization is shown in Figure 8–71. Although the circuit shown is a free-running (astable) blocking oscillator, it may also be synchronized with an incoming signal. In order to properly synchronize the circuit, the input signal frequency must be of a higher repetition rate than the free-running frequency of the oscillator. Resistor R5, in series with resistor R4, is a damping control used to determine the amplitude of the output positive pulse. Diode CR2 is placed across the primary of transformer T1 to protect the transistor. At some instant, the backswing voltage produced by the collapsing field of transformer T1 may exceed the breakdown voltage of the transistor. When this action occurs, crystal CR2 will conduct and eliminate the negative-going portion of the alternation. Resistor R3 is the stabilization resistor, which may also be used to develop a negative-going output trigger pulse.

In many applications an emitter bypass capacitor (C4) is also used. This serves to further stabilize the voltage across resistor R3 and, at the same time, decreases the natural free-running frequency.

The natural free-running frequency of the blocking oscillator is determined chiefly by transformer T1, capacitors C2 and C3, and resistors

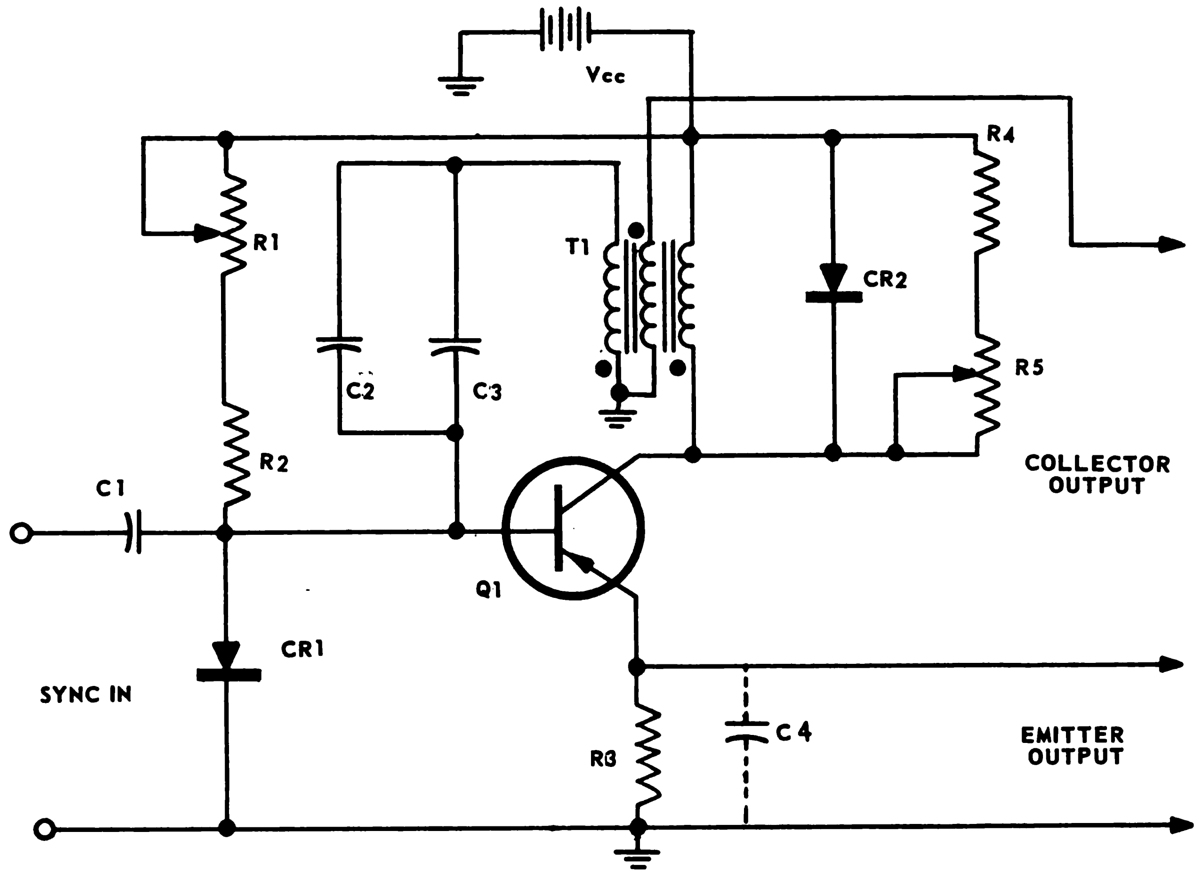


Figure 8–71. Synchronized Transistor Blocking Oscillator

R1 and R2. When synchronized, the input synchronizing signal is coupled through capacitor C1 to the base of the transistor. Since a negative pulse is necessary to synchronize the oscillator, CR1 is placed across the input circuit to eliminate any positive pulse which may appear at the input. The output signal is obtained across the tertiary winding of transformer T1. This is done to prevent undue loading within the collector circuit.

MULTIVIBRATORS

Free-Running (Collector-Coupled) Multivibrator

The transistor free-running (astable) multivibrator, like its counterpart, the electron-tube multivibrator, is a two-stage oscillator with a nonsinusoidal output. While one stage conducts, the other stage is cut off. At a certain point in their operation, the stages reverse their conditions — the conducting stage is now cut off while the other stage conducts. A square wave output is normally produced by this process. The emitter-coupled transistor multivibrator is similar to the cathode-coupled electron tube multivibrator circuit, and the collector-coupled transistor multivibrator (Figure 8–72) is similar to the plate-coupled electron tube multivibrator.

Only the collector-coupled transistor multivibrator is discussed in this manual, as most multivibrator circuits function similarly. The waveshapes for this circuit are given in Figure 8–73.

The basic collector-coupled transistor multivibrator of Figure 8–72 is a two-stage, RC-coupled, common-emitter amplifier using NPN transistors. If PNP transistors were used, the only circuit change necessary would be the reversal of the collector supply battery (V_{CC}). The output of the first stage is coupled to the input of the second stage; the output of the second stage is coupled to the input of the first stage. You recall that there is a 180° phase shift between the output and the input of a common-

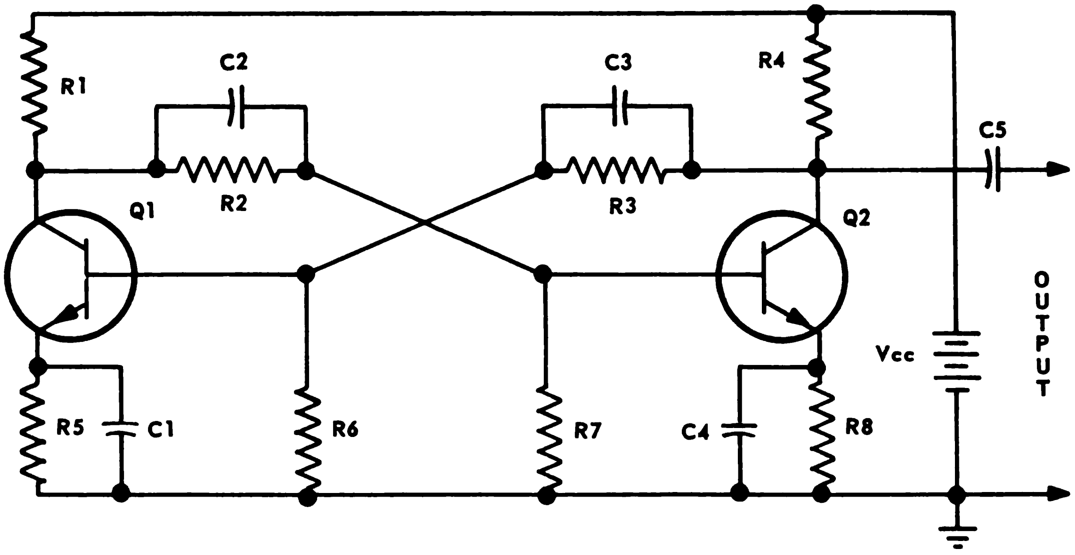


Figure 8–72. Transistor Multivibrator

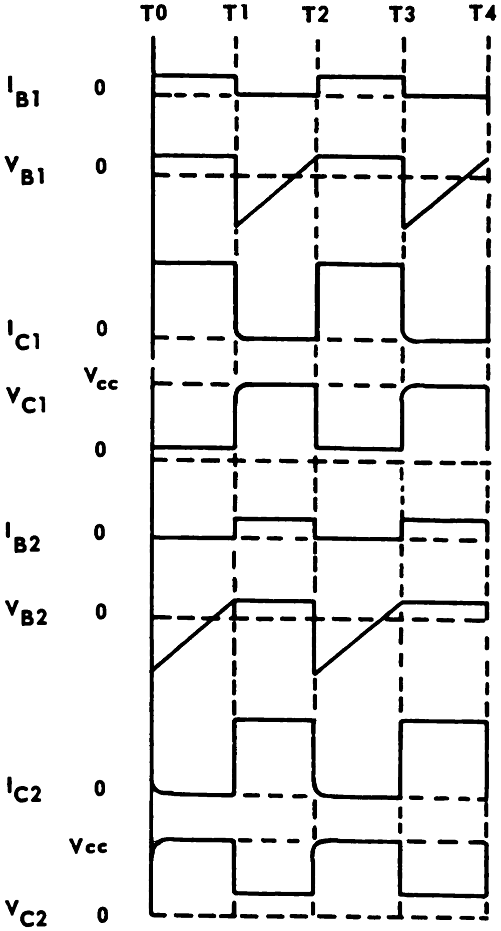


Figure 8–73. Transistor Common-Collector Multivibrator Waveforms

emitter amplifier. This means that the output of each stage is in phase with the input to the other stage. The part of the output fed back from one stage to the other is the regenerative feedback required for oscillation. Bias, as well as temperature stabilization, is established identically for both transistors.

Circuit operation of the transistor collector-coupled multivibrator is very similar to that of the electron-tube plate-coupled multivibrator. As in electron tube circuits, the variation in tolerances of the components causes one transistor to conduct more heavily than the other transistor. Let us assume that transistor Q1 is conducting more heavily than transistor Q2 at the beginning.

During the period of time from T0 to T1 (Figure 8–73), collector current (I_{c1}) and collector voltage (V_{c1}) remain constant; capacitor C2 discharges through resistor R7. As C2 discharges through R7, the voltage drop across R7, which is the base voltage of transistor Q2 (V_{b2}), decreases, which effectively decreases the reverse bias on transistor Q2. This action continues until time T1 is reached, and forward bias is reestablished across the emitter-base junction of transistor Q2. Now transistor Q2 begins to conduct.

As collector current (I_{c2}) in transistor Q2 increases, the collector voltage (V_{c2}) becomes less positive during the period of time from T1 to T2. This voltage, coupled through capacitor C3 to the base of transistor Q1, drives it more negative and causes a decrease in collector current (I_{c1}) through Q1. The resulting increased positive voltage at the collector of Q1 is coupled through C2 and appears across R7. The collector current (I_{c2}) of transistor Q2 therefore increases. This process continues rapidly until Q1 is cut off. Transistor Q1 remains cut off and transistor Q2 conducts until C3 discharges enough through resistor R6 to decrease the reverse bias (V_{b1}) on the base of transistor Q1 (time T2). At time T2 the complete cycle is repeated.

Resistors R1 and R4 of Figure 8–72 are the collector load resistors for transistors Q1 and Q2 respectively. The necessary base bias of transistor Q1 is provided by a voltage divider network consisting of resistors R3 and R6. Likewise, the necessary base bias of transistor Q2 is provided by a voltage divider network consisting of resistors R2 and R7. Resistors R5 and R8 are temperature stabilization resistors for transistors Q1 and Q2 respectively. Capacitors C1 and C4 are the emitter bypass capacitors. The output frequency of the transistor multivibrator is usually determined by the values of resistance and capacitance in the circuit.

The output signal is coupled through capacitor C5 to the load. Although the output is obtained across transistor Q2 in Figure 8–72, it could also be obtained from the collector of transistor Q1. Like the electron tube plate-coupled multivibrator, the output of the transistor multivibrator is essentially a square wave (Figure 8–73).

One-Shot Multivibrator

A basic transistor one-shot multivibrator is shown in Figure 8–74 with the accompanying voltage waveshapes in Figure 8–75. This circuit is similar to the electron tube version of the one-shot, single-shot, or single-swing multivibrator. Another term used for this circuit is *monostable* multivibrator since it has only one stable condition. When triggered it goes through a complete cycle and returns to its original quiescent state. (A bistable multivibrator changes from one stable state to the other when triggered.) When the circuit in Figure 8–74 is triggered by an external pulse, its operating point is moved from the initial stable region to the other stable region.

The time constant of the circuit elements holds the operating point in the new stable region for a period of time. The operating point then moves back to the original stable region. When this circuit is in its initial stable region (quiescent condition), transistor Q2 is at saturation and Q1 is cut off. When a positive trigger pulse of short duration is applied to the base of transistor Q1, the multivibrator circuit functions until one full cycle is completed. Then the circuit remains quiescent until another trigger pulse is fed in.

Battery V_{CC} provides the necessary collector bias voltages for both transistors and the forward bias for the emitter-base junction of transistor Q2. Transistor Q2 is at saturation during the quiescent period; hence, the collector voltage

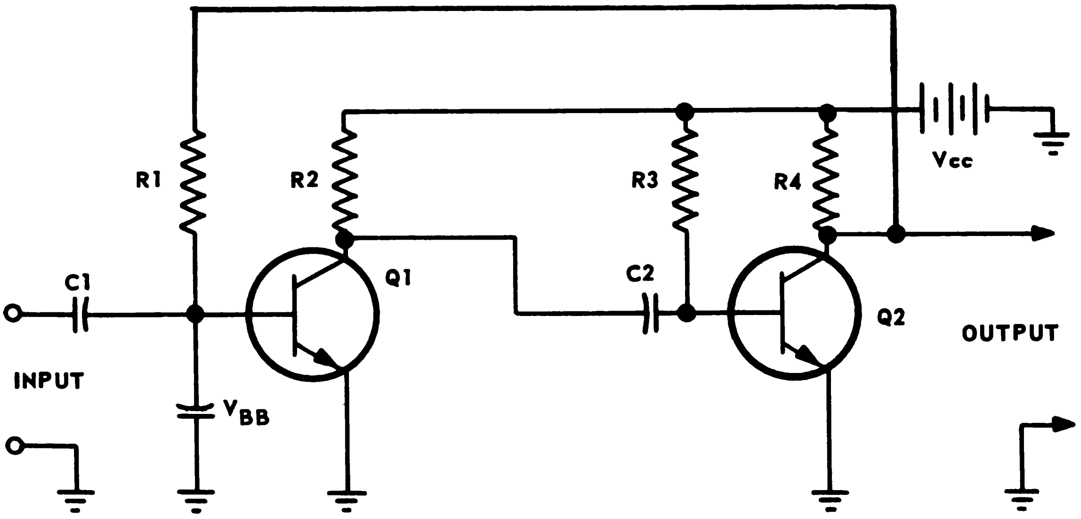


Figure 8–74. Transistor One-Shot Multivibrator

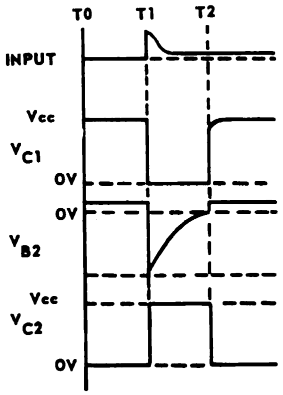


Figure 8–75. Transistor Monostable Multivibrator Waveforms

is effectively zero volts. Figure 8–75 shows the collector waveform (V_{c2}) during this time (T0 to T1). The reverse bias across the emitter-base junction provided by battery V_{BB} maintains transistor Q1 at cutoff. Collector voltage (V_{c1}) is positive and equal to the value of battery voltage V_{CC}. The collector waveform (C_{c1}) during this same time is also shown in Figure 8–75. Capacitor C2 is charged to battery voltage V_{CC} through the essentially shorted emitter-base junction of forward-biased transistor Q2 and collector load resistor R2.

Apply a positive input trigger pulse at time T1 to the base of transistor Q1 through coupling capacitor C1. This causes transistor Q1 to conduct. The collector voltage (V_{c1}) at time T1 decreases (becomes less positive). Capacitor C2 immediately begins to discharge through resistor R3. The regenerative signal from the collector of transistor Q1 is quickly applied to the base of transistor Q2 through C2. This places a maximum negative voltage (V_{b2}) at the base of transistor Q2 at time T1. This negative voltage at the base of transistor Q2 effectively places reverse bias across the emitter-base junction, thereby driving transistor Q2 toward cutoff.

As collector current through Q2 decreases, the collector voltage rises. A portion of this collector voltage is coupled through resistor R1 to the base of transistor Q1, increasing its positive voltage. This regeneration results in a rapid change of both transistors; it drives transistor Q1 further toward saturation and transistor Q2 further toward cutoff. When collector current ceases to flow (time T1), the collector voltage V_{c2} immediately rises to equal the battery voltage of V_{CC}.

From time T1 to T2, capacitor C2 is discharging through R3 and the low saturation resistance of transistor Q1. When capacitor C2 reaches zero volts at time T2, transistor Q2 no longer has the necessary reverse bias across its emitter-base junction to keep the collector current cut off. Thus, at time T2, collector current rises, thereby driving transistor Q2 into saturation and the collector voltage (V_{c2}) is again at zero volts. At this time, the portion of regenerative voltage from the collector of Q2 coupled to the base of transistor Q1, through R1, is now zero volts, and battery V_{BB} once again provides the necessary reverse bias across the emitter-base junction to drive transistor Q1 back to cutoff.

The collector voltage (V_{c1}) at time T2 again equals the collector supply voltage (V_{CC}), and capacitor C2 recharges through the essentially shorted emitter-base junction of transistor Q2 and resistor R2. Capacitor C2 continues to charge until it equals the collector supply voltage, when all further action ceases, or until another input trigger pulse appears at the base of transistor Q1. Of course, when another trigger pulse appears at the base of transistor Q1, the circuit then completes another full cycle of operation.

The output is taken from the collector of transistor Q2. As shown in Figure 8–75, the collector waveform (V_{c2}) at this point is essentially square. The duration of the output pulse (time T1 to T2) is primarily determined by the time constant of resistor R3 and capacitor C2 during discharge. Output frequency is directly proportional to the frequency of the input trigger pulses.

Eccles-Jordan Multivibrator

A basic Eccles-Jordan (bistable) multivibrator is shown in Figure 8–76. This circuit differs from the collector-coupled one-shot multivibrator primarily in that it requires two input triggers to complete one cycle. The Eccles-Jordan circuit is in a stable state when either transistor is conducting while the other is cut off. The states of the transistor are switched with the application of a properly applied trigger pulse.

When DC power is first applied, one transistor conducts, while the other transistor is cut off. The conducting transistor quickly saturates. Each transistor remains in this stable state, held there by the condition of the other, until a trigger pulse is applied. A negative trigger pulse applied

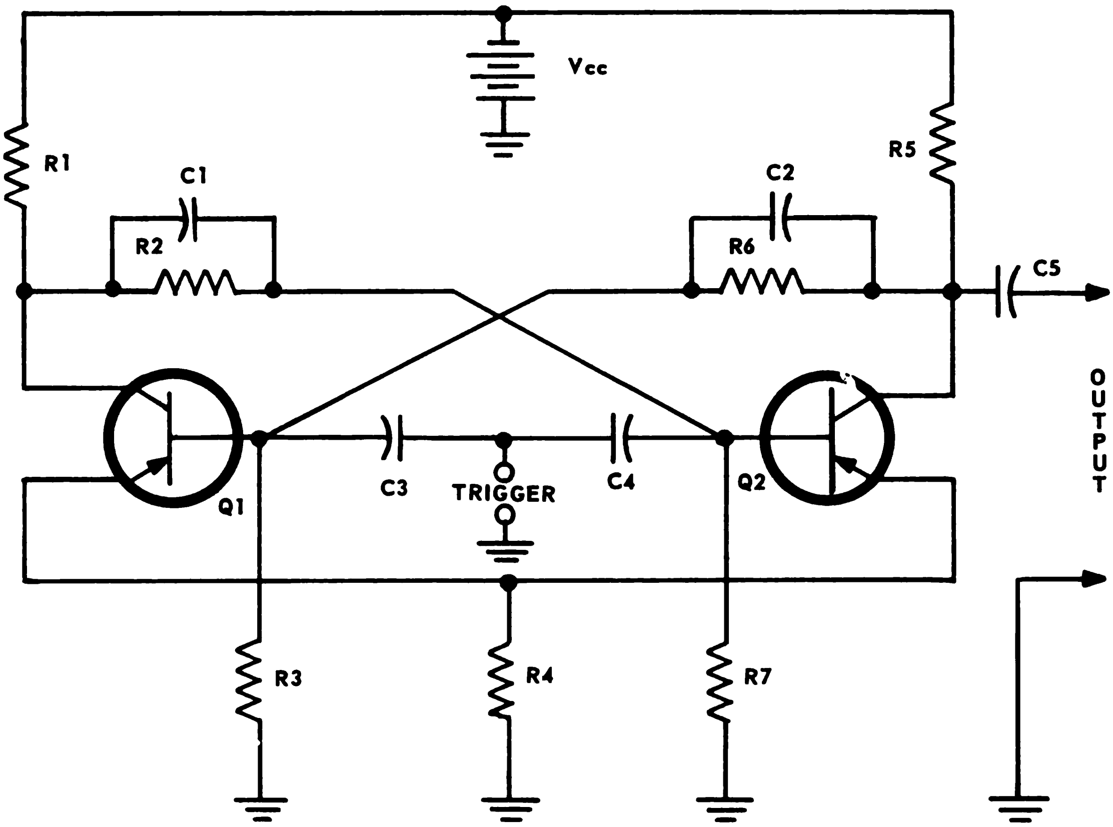


Figure 8–76. Transistor Eccles-Jordan Multivibrator

to the base of the conducting transistor will not change the state of operation. On the other hand, the same negative trigger pulse applied to the base of the cut off transistor will change the conducting state of the circuit. That is, the transistor which was originally conducting will now be cut off, and the transistor that was cut off will now be conducting. A positive pulse applied to the base of the conducting transistor will also trigger the circuit.

Fundamentally, the method used to trigger the circuit is determined by the polarity and magnitude of the trigger pulse available and the desired repetition rate of the output. Although the transistorized Eccles-Jordan multivibrator shown in Figure 8–76 uses a negative trigger pulse applied to the base circuit of both transistors, a trigger pulse of sufficient magnitude and proper polarity may be directed to the collector of either transistor.

For circuit operation of the Eccles-Jordan multivibrator, assume transistor Q1 is conducting and transistor Q2 is cut off. A negative trigger pulse applied to the base of each transistor at

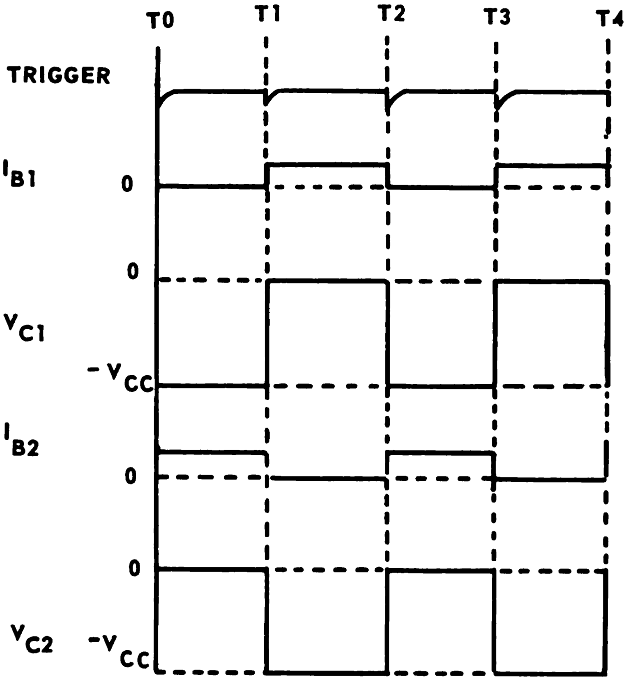


Figure 8–77. Eccles-Jordan Multivibrator Waveforms

time T0 (Figure 8–77) causes transistor Q2 to conduct. The increasing collector current causes the collector voltage (V_{c2}) of transistor Q2 to decrease (become less negative). This change in voltage is coupled to the base of transistor Q1 and reduces its forward bias. Conduction in transistor Q1 begins to decrease. As the collector current decreases, collector voltage (V_{c1}) changes from zero to a negative value (approaching the value of battery voltage V_{CC}). This change in voltage is coupled to the base of transistor Q2, making the base more negative and increasing the conduction of the transistor. The regenerative feedback continues until transistor Q2 is saturated and transistor Q1 is cut off.

Transistor Q2 continues conducting and transistor Q1 remains cut off. They remain in this state until the next negative trigger pulse is applied to the base (time T1, Figure 8–77). At time T1 transistor Q1 conducts and Q2 is cut off. This completes one full cycle of the output square wave. The time constant of capacitor C1 and resistor R2 and that of capacitor C2 and resistor R6 essentially determine the time from conduction to cutoff of transistors Q1 and Q2 respectively. In addition, the capacitors rapidly couple the changing voltages to the bases to insure rapid switching action of the transistors.

Like the transistor one-shot multivibrator, Figure 8–72, the output signal of the bistable multivibrator is coupled through a capacitor (C5) to the load. Although the output is taken across transistor Q2, it could have also been taken across transistor Q1. Frequency division of this transistor multivibrator is the same as that in the electron tube version, that is, a ratio of 2 to 1.

Resistors R1 and R5 provide the DC collector load for their respective transistors. The necessary forward bias for transistors Q1 and Q2 is provided by a voltage divider network consisting of resistors R3 and R6 for Q1 and R7 and R2 for Q2. Resistor R4 has a twofold purpose; the first is to provide emitter temperature stabilization, and the second to insure a bistable condition within the circuit. Capacitors C3 and C4 serve to couple the input trigger pulse to the base of each transistor.

METHODS OF AMPLITUDE-MODULATING TRANSISTOR AMPLIFIERS

Up to this point, amplitude and frequency modulation have not been discussed in this manual. You will find that Chapter 9 covers the theory of modulation and demodulation.

As shown in Figure 8–78, a transistor amplifier may be modulated by injecting a modulating signal into the base, the emitter, or the collector. In this circuit, transformer T1A provides base injection, transformer T1B provides emitter injection, and transformer T1C provides collector

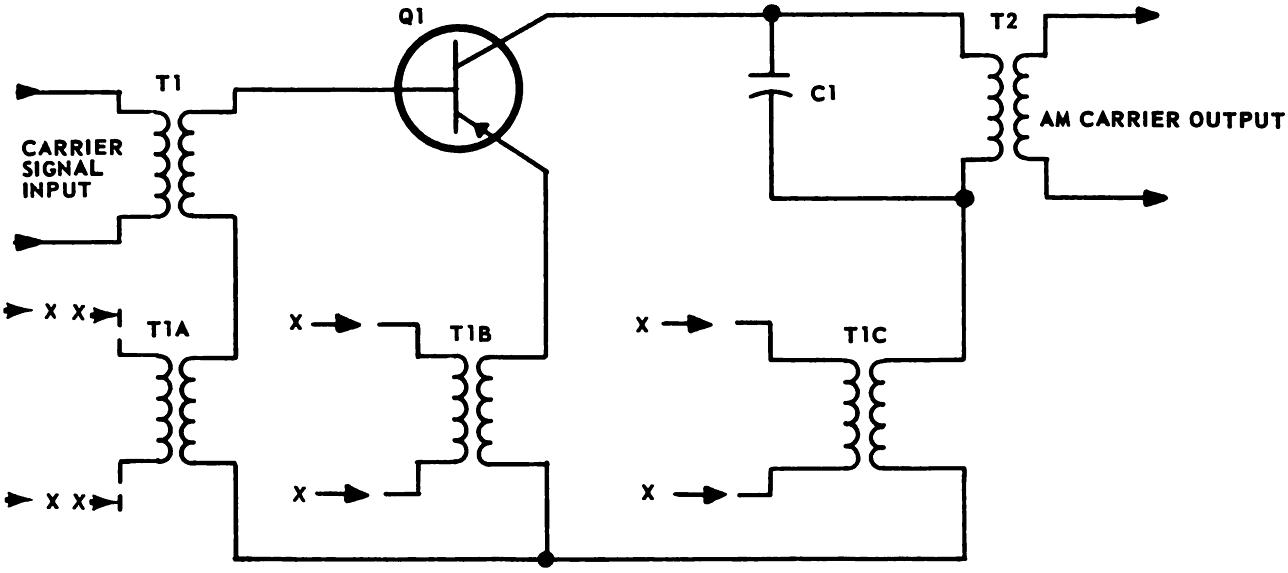


Figure 8–78. Methods of Injecting AM Signal into a Transistor Amplifier

injection. Since a change in bias will change the gain of the transistor, the amplitude of the output signal will vary.

The carrier signal is applied to the base circuit of Q1 through transformer T1. The parallel circuit of capacitor C1 and the primary of transformer T2 are resonant to the carrier signal, which has been amplified by the transistor circuit. Any of the modulating methods just described may vary the gain of the transistor in accordance with the modulating signal. When the bias is increased, the amplitude of the signal on the collector is increased; when the bias is decreased, the amplitude of the signal is decreased. The output, as a result, is an amplitude-modulated carrier that is coupled through transformer T2 to the following stage.

The modulated output would be distorted if the strength of the r-f carrier signal were greater than the emitter-to-base bias with modulation by either base or emitter injection. The distortion is a result of the carrier signals determining the emitter-base bias. Therefore, the carrier signal strength must always be smaller than the emitter-base bias when an amplifier is modulated with base or emitter injection. Low-level modulation only will prove satisfactory when base or emitter injection is used. Collector injection will prove satisfactory with either high-level or low-level modulation.

The requirements for transmitting an amplitude-modulated carrier using either electron tubes or transistors are essentially the same. Figure 8–79 shows a block diagram of a typical transmitter using amplitude-modulated amplifiers. Waveforms are shown for each stage. Transmitters that require a large power output normally use this type of modulation. The fundamental carrier frequency generated by the oscillator is increased in the multiplier stage. To provide sufficient driving power for final amplification, the amplitude of the carrier frequency is increased in the driver stage. The power amplifier stage is the amplitude-modulated amplifier.

The microphone picks up the sound and converts it into an electrical signal which is amplified in the audio amplifier stage. The strength

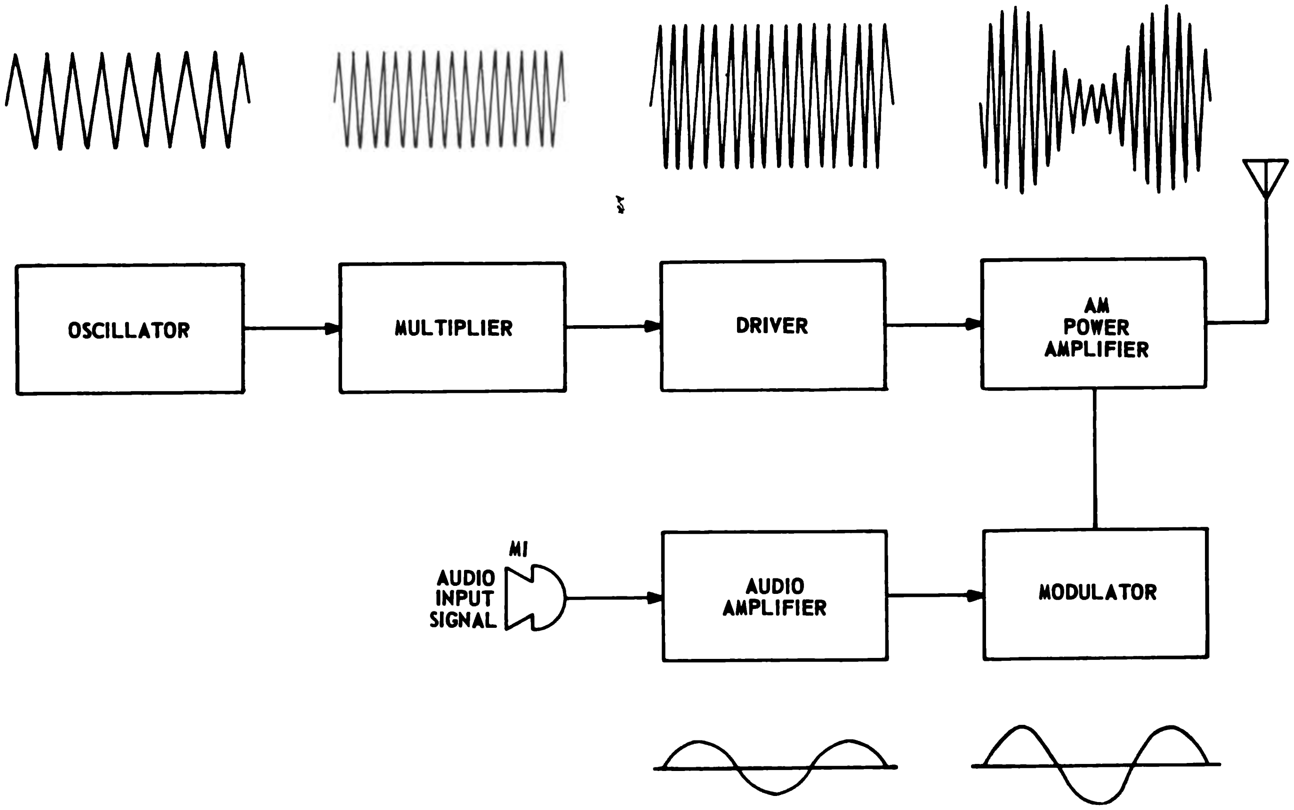


Figure 8–79. Block Diagram of an Amplitude-Modulated Transmitter

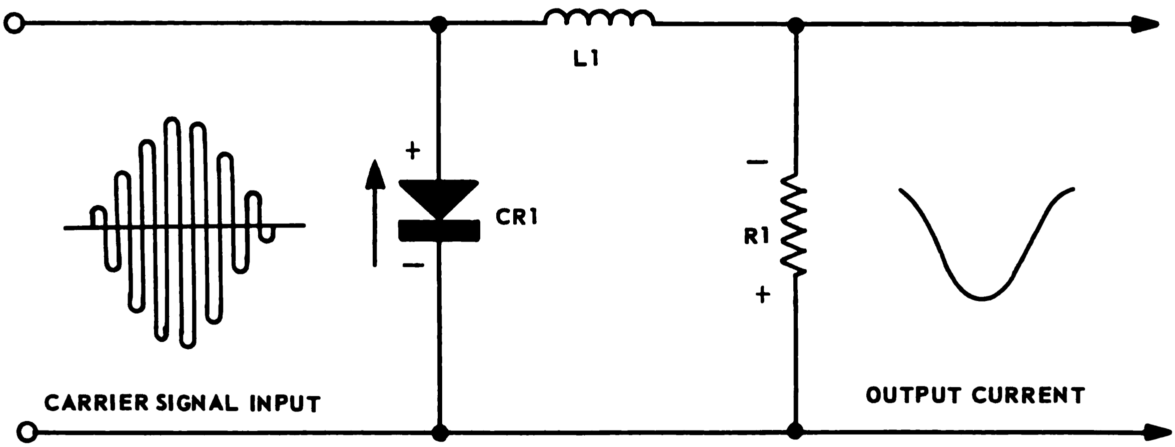


Figure 8–80. Current Output-Type Diode Detector

of the audio signal is then sufficient to drive the modulator stage. The amplified audio signal from the modulator stage modulates the power amplifier stage. This amplitude-modulated carrier is then applied to the antenna. For high-level modulation the modulating signal is normally injected into the plate circuit of an electron tube power amplifier or the collector circuit of a transistor power amplifier.

TRANSISTOR DETECTORS

The detection of amplitude-modulated signals may be accomplished by transistor diodes or triodes. The transistor detector rectifies and filters the amplitude variations of the carrier signal in much the same way as electron tube detectors.

Current Output Diode Detector

Figure 8–80 illustrates a current output-type diode detector. CR1 is the detector diode; resistor R1 is the load, and choke L1 is the filter. Since transistors are current amplifying devices, this detector may be used in transistor circuits.

Diode CR1 will not conduct on the negative alternation; all the current flows through choke L1 and resistor R1. On the positive alternation of the carrier signal, the diode conducts and ef-

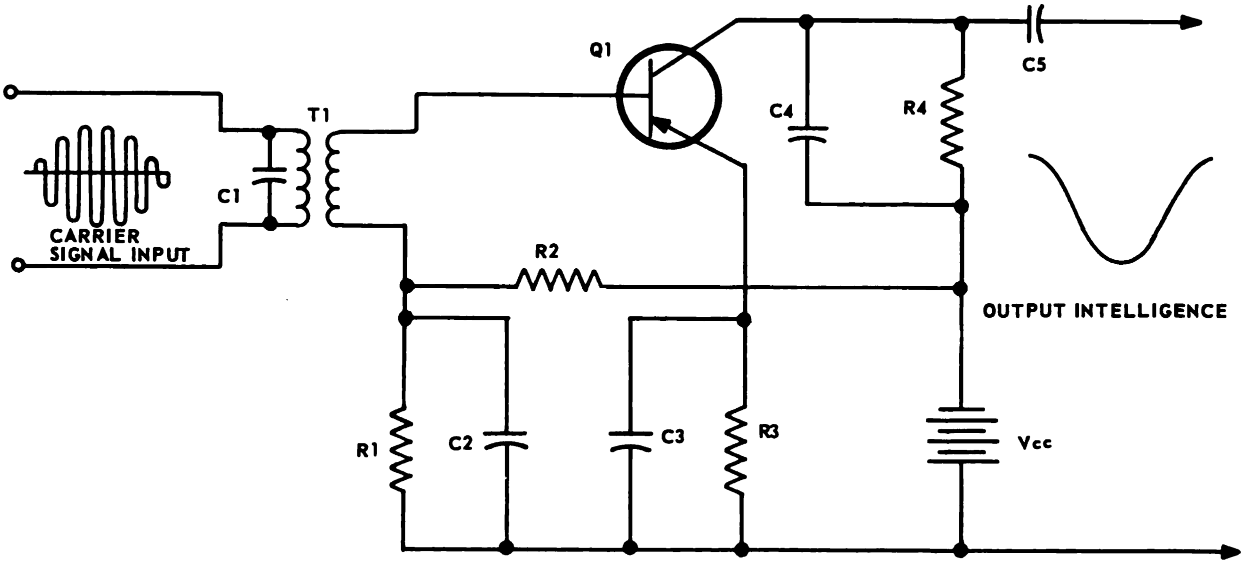


Figure 8–81. Common-Emitter Transistor Detector

fectively shorts R1 and L1. No current flows through these circuit elements. When the current tends to stop flowing through L1, the collapsing field about the choke tends to keep current flowing through the resistor in the same direction. The choke filters the carrier signal current flow through resistor R1. The filtered output current increases and decreases at a rate that is equivalent to the amplitude changes in the carrier signal. The amplitude changes in the carrier signal have produced current changes in the output of the detector.

Common-Emitter Transistor Detector

The operation of a common-emitter transistor detector is similar to that of an electron-tube plate detector. Rectification of the carrier input signal is accomplished by biasing the emitter-base portion of the transistor for Class B operation. Essentially, this causes the emitter-base section to act as a diode. In turn, amplification takes place in the emitter-collector portion of the transistor.

Figure 8–81 illustrates a common-emitter transistor detector circuit. The carrier signal is transformer coupled to the base-emitter circuit (biased for nonlinear operation). The signal is rectified by the diode portion (base-emitter) of transistor Q1.

The primary of T1 and capacitor C1 form a parallel resonant circuit for the input carrier signal. R1 and R2 form a voltage divider to furnish the correct emitter-base bias. Resistor R1 is the diode load resistor; capacitor C2 filters the carrier signal. Capacitor C4 is the bypass capacitor for R4. Capacitor C3 is the emitter bypass capacitor for audio variations. Resistor R3, sometimes referred to as a swamping resistor, serves to stabilize the circuit by compensating for differences between transistors and by reducing the effects caused by temperature drift. Capacitor C5 is the coupling capacitor and also blocks the DC voltage from the following stage.

The advantage of the common-emitter transistor detector is that it provides amplification. Also, a very small load is placed on the high frequency source supplying the detector. Of course, some limiting action may occur on the input signal, thus causing distortion on the output signal.

With very small input signals, the crystal and transistor common-emitter detectors have square-law operation, due to the nonlinearity of the lower portion of their characteristic curves. This means that the output of these two detectors is distorted with small input signals. Either a positive-going or a negative-going output may be obtained from the crystal detector. Since the crystal detector places a load on the input circuit, its Q, and thus its selectivity, is lowered. This is due to the rectifier currents flowing through the input circuit.

The transistor common-emitter detector does not load the input circuit unless the transistor is overdriven. Since there is no loading, the selectivity is good. The crystal detector is less rugged than electron-tube detectors, thus requiring greater care in handling. The crystal and transistor detectors are smaller than electron tubes and require fewer connecting leads and associated circuit elements.

FREQUENCY MODULATION WITH TRANSISTORS

Frequency-Modulated Oscillator

Figure 8–82 illustrates a typical frequency-modulated oscillator using a reactance modulator (Q2). The circuit of Q1 is a tickler coil oscillator. The frequency of the oscillator is determined by the parallel resonant circuit comprising capacitor C1 and winding 1–3 of transformer T1. Feedback is provided by winding 4–5 of the transformer. Winding 6–7 of the

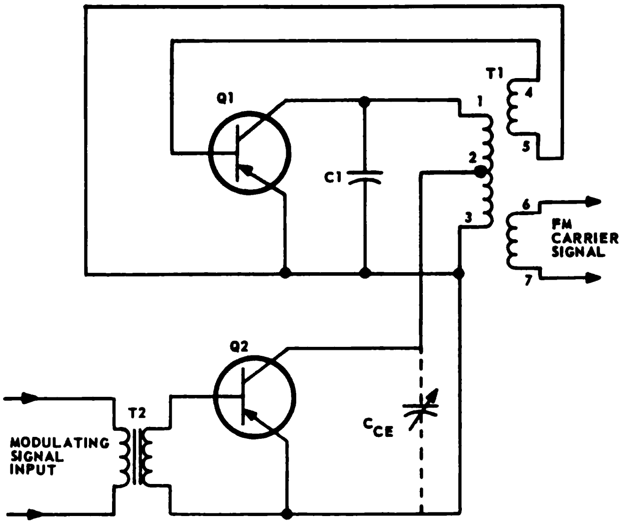


Figure 8–82. Transistor Frequency-Modulated Oscillator

transformer couples the signal to the following stage.

The emitter-base bias of the reactance modulator Q2 is varied by the modulating signal, which is coupled to the circuit through transformer T2. The collector voltage increases and decreases as the bias is increased and decreased, both at the modulating rate. The output capacitance (C_{CE}) of the modulator decreases as the collector voltage increases and increases as the collector voltage decreases.

Since this capacitance is in parallel with part of the transformer T1 winding used for the oscillator, changes in C_{CE} affect the oscillator frequency. When the capacitance of C_{CE} decreases, the oscillator frequency increases, and when the capacitance of C_{CE} increases, the oscillator frequency decreases. You can see, then, that the resonant frequency of the oscillator tank circuit is increasing and decreasing at the modulation rate; thus the frequency of the generated signal is a frequency-modulated carrier signal.

FM Detectors Using Transistors

Figure 8–83 shows a transistorized version of an amplifier and a discriminator using germanium diodes. Detailed vector analysis of this discriminator is the same as that used for the electron-tube version (Chapter 9). Amplifier Q1 amplifies the carrier signal input to the discriminator. C1 is an r-f bypass capacitor and R1 is the emitter stabilizing (swamping) resistor. A parallel circuit composed of capacitor C2 and the primary of transformer T1, which is resonant to the carrier signal, is the load for the amplifier.

The amplified output is coupled through the transformer to the discriminator. Capacitor C3 couples the carrier signal to the secondary of transformer T1 for phase shift comparison with the carrier signal being developed across coil L1. Capacitor C4 and the secondary of transformer T1 form a resonant circuit for the carrier signal. The top half of the transformer T1 secondary, diode CR1, coil L1, load resistor R2, and filter capacitor C5 form half of the comparison network. The other half of the comparison network is composed of the bottom half of the transformer T1 secondary, diode CR2, coil L1, load resistor R3, and filter capacitor C6. The output of the discriminator circuit is taken from the top of capacitor C5 and the bottom of capacitor C6. Capacitor C7 couples the signal to the primary of transformer T2, the output transformer.

Figure 8–84 shows a single transistor used as an FM detector. The design of a transistor used for an FM detector differs somewhat from other transistors. A transistor used for an FM detector is a symmetrical transistor; that is, the emitter and collector sections are identical. This allows either section of the transistor to operate as the emitter or the collector.

When the carrier signal appears across L1, it is also applied to the base of the transistor. By transformer action the same signal from coil L1

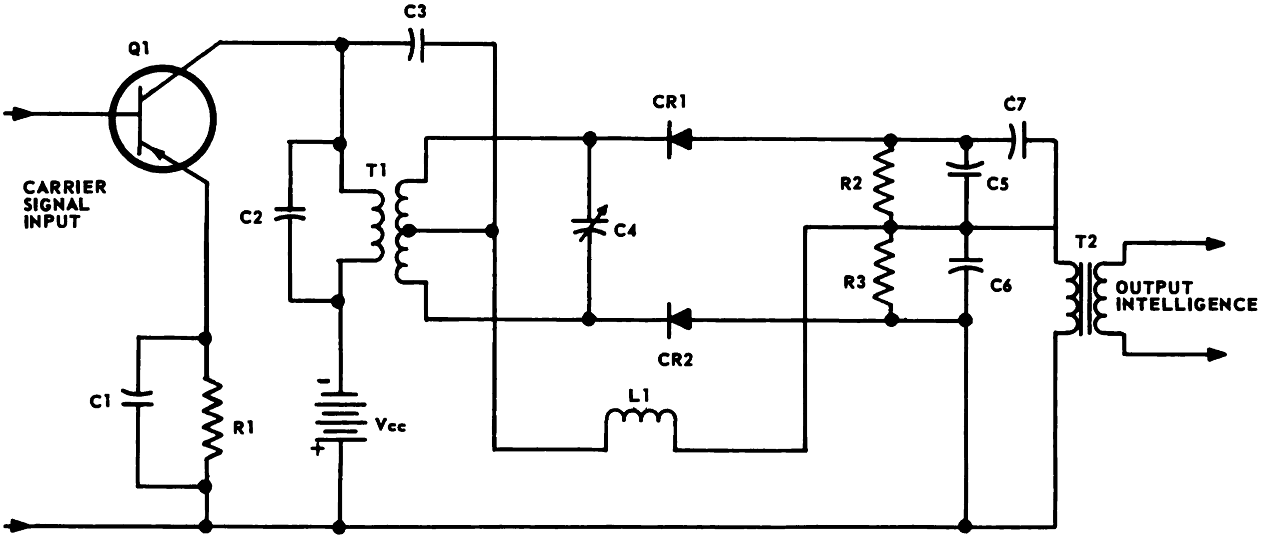


Figure 8–83. Transistorized Discriminator Using Germanium Diodes

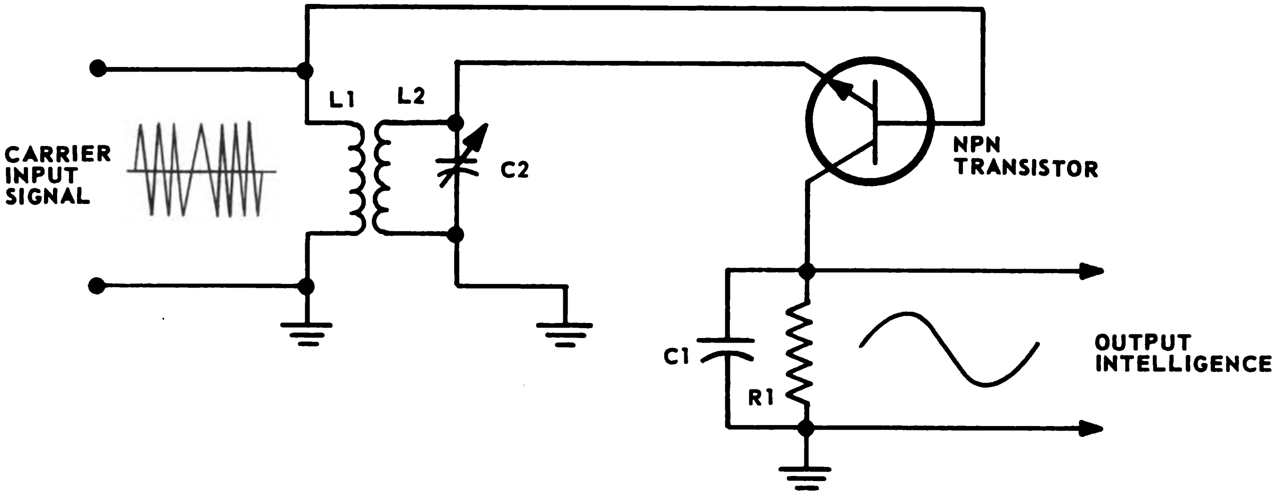


Figure 8–84. A Transistor FM Detector

is coupled to coil L2. This signal, in turn, is applied to the emitter. During the negative alternation of the input signal, the emitter-base junction is biased in the reverse direction; thus no current will flow through the transistor or through R1. During the positive alternation, the emitter-base junction is biased in the forward direction, and current will flow through the transistor and R1. Of course, if a PNP transistor were used, the period of conduction would be reversed. So long as the carrier signal frequency does not vary, no audio or video variations will be developed across resistor R1.

When a carrier signal frequency above the resonant frequency of L2 and C2 appears across L1, the secondary voltage lags the primary voltage by an angle less than 90°. If the carrier signal frequency goes below the resonant frequency of L2 and C2, the secondary voltage lags the primary voltage by an angle greater than 90°. The changing carrier signal frequency applied to coil L1 and the transistor actually sets up a varying phase relationship. As a result, the varying phase relationship produces within the transistor and R1 a fluctuating current which is in step with the input carrier signal frequency. The polarity of the output signal at time zero will depend upon the input carrier signal frequency. Resistor R1 develops the audio or video output intelligence and capacitor C1 is an r-f bypass.

One of the difficulties found in the operation of a receiver is the tendency of the local oscillator to drift in frequency; that is, it varies above or below the predetermined frequency. This causes a corresponding loss of gain and fidelity. An arrangement that will automatically keep a receiver, or a transmitter, accurately tuned to a predetermined frequency is termed Automatic Frequency Control, abbreviated AFC. The circuit just discussed can be used for AFC.

MIXERS

The combining of a radio frequency and an oscillator frequency to produce a new frequency, the *intermediate frequency*, is defined as *mixing* or *frequency conversion.* Two methods explained in Chapter 9 (employing electron tubes) are used with transistors, too. The first method we shall explain uses a transistor to mix the radio frequency signal and the oscillator frequency, both fed in from external sources. The second method to be explained uses one transistor to produce the oscillator frequency and to mix it with a radio frequency from an external source.

Mixing, in either case, produces four major frequencies in the output — the original radio frequency, the original oscillator frequency, a frequency equal to the sum of these two frequencies, and a frequency equal to the difference between them. The difference frequency, which is the one normally desired, is called the intermediate frequency.

The Mixer

A schematic diagram of a transistor mixer stage is shown in Figure 8–85. The r-f signal is injected into the base circuit, and the local os-

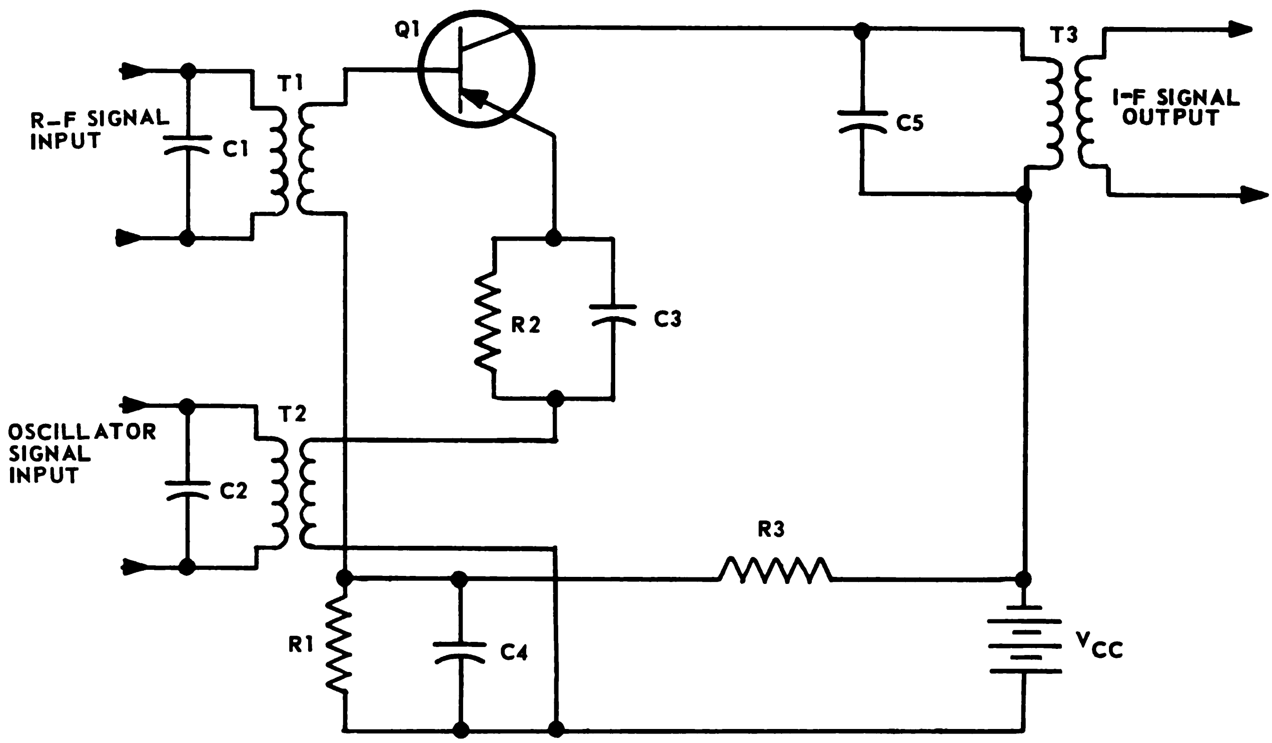


Figure 8–85. Transistor Mixer

cillator frequency is injected into the emitter circuit. The r-f signal and local oscillator signal are then heterodyned in the mixer Q1. The intermediate frequency is selected by the collector tank circuit.

Capacitor C1 and the primary of transformer T1 form a parallel resonant circuit for the r-f signal. The r-f signal is then coupled through transformer T1 to the base circuit of mixer Q1. Capacitor C2 and the primary of transformer T2 form a parallel resonant circuit for the oscillator signal input, which is coupled to the emitter circuit of mixer Q1. The emitter-base bias is provided by resistor R1 which, with resistor R3, forms a voltage divider. Capacitors C3 and C4 are bypass capacitors and resistor R2 is the emitter swamping resistor. C5 and the primary of transformer T3 form the collector parallel resonant circuit for the i-f. The signal is coupled through transformer T3 to the following stage.

The Converter

A schematic drawing of a converter stage is illustrated in Figure 8–86. The r-f, which is injected into the base circuit of Q1, and the oscillator frequency generated by the converter Q1 are heterodyned in the converter.

The r-f input signal is developed across the parallel resonant circuit composed of capacitor C1 and the primary of transformer T1. The signal is then transformer-coupled to the base circuit of Q1. Emitter-base bias is developed across resistor R1. (Resistors R1 and R3 form a voltage divider.) R2 is the emitter swamping resistor, and C4 is a bypass capacitor. Capacitor C2 and the primary of T2 are the parallel resonant circuit for the oscillator section of Q1.

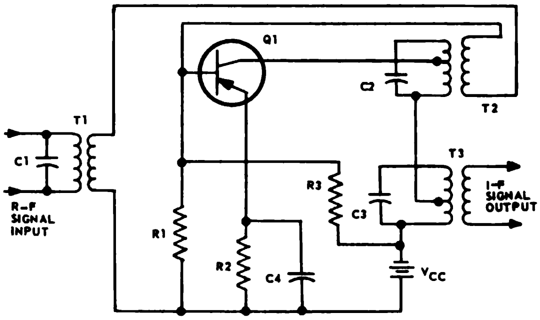


Figure 8–86. Transistor Converter

The secondary of transformer T2 provides the required feedback for the oscillator. Capacitor C3 and the primary of transformer T3, a parallel resonant circuit, select the desired intermediate frequency. The desired selectivity is determined by the tapped primaries of transformers T2 and T3. Selectivity is the characteristic that determines the ability of a resonant circuit to reject undesired signals.

TRANSISTOR SERVO AMPLIFIER

Most low power servomechanisms use two-phase AC induction motors as servo motors. Since there has been a tendency in recent years to reduce the over-all power requirements, the transistorized version of the torque-amplifying device lends itself well to servo amplifier applications. Chapter 14 contains a complete discussion of servomechanisms, which discloses that the same general theory of operation applies to the transistorized versions. Figure 8–87 illustrates a transistorized version of a servo amplifier.

The necessary forward bias for transistor Q1 is provided by the bridge rectifier consisting of diodes CR3, 4, 5, and 6, and the voltage divider network consisting of resistors R1, 2, 3, and 4. The direction of current flow through the voltage divider network is indicated by I4 in Figure 8–87. This arrangement places a small positive

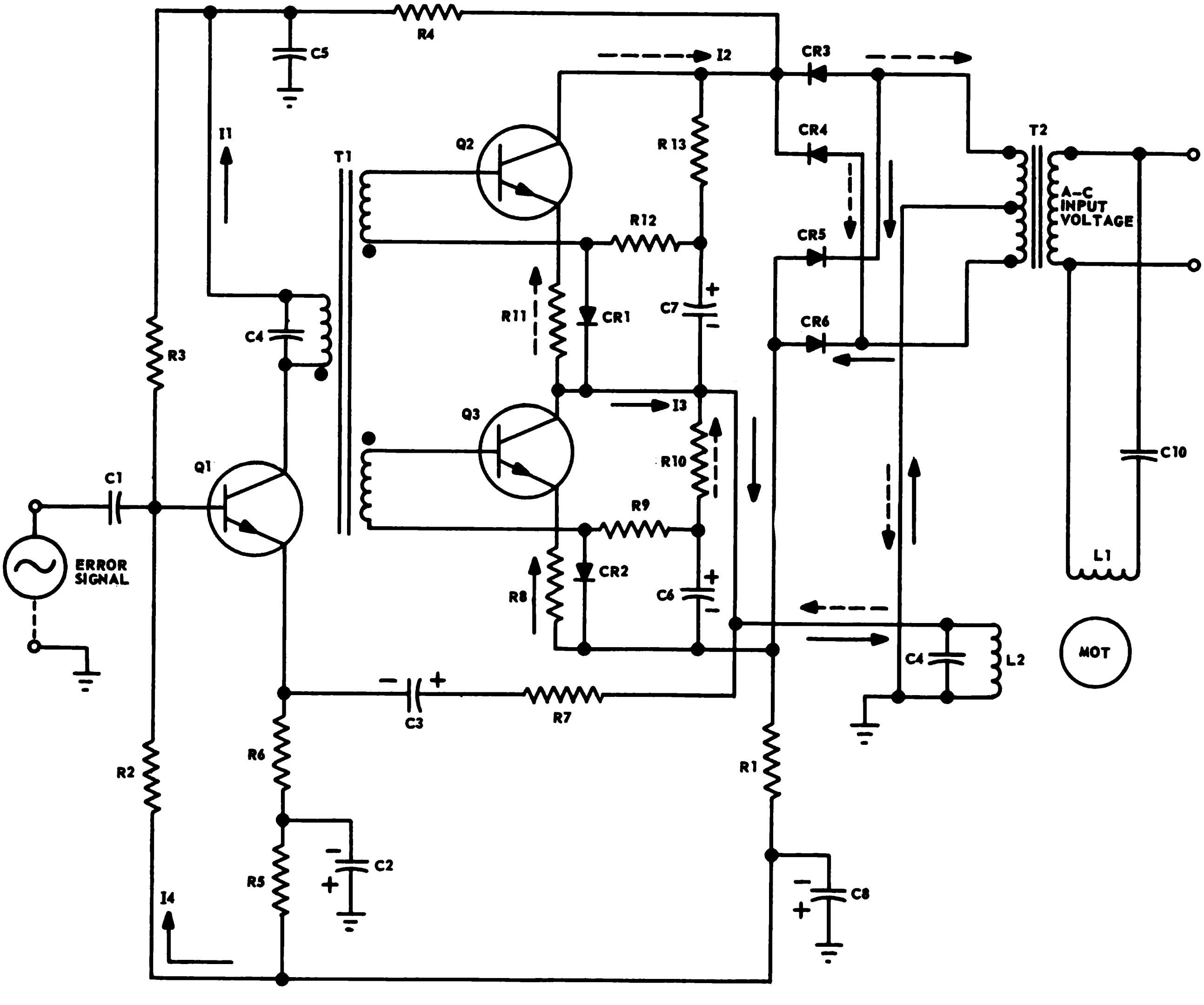


Figure 8–87. Transistor Servo Amplifier

voltage on the base of transistor Q1, which is a necessary condition for NPN transistors. If PNP transistors were used, the polarity of the forward bias would have to be reversed.

In this particular system, the two-phase induction motor M is the servo motor. The motor speed and direction of rotation vary as the phase and magnitude of the error voltage changes. This error voltage is amplified by transistors Q1 and either Q2 or Q3 before it is applied to the control phase coil (L2) of the servo motor. If there is no error voltage, the motor remains stationary. If there is an error voltage, it is an AC voltage which is either in phase, or 180° out of phase, with the voltage applied to the synchro generator. This condition sets up a rotating magnetic field that causes the motor to turn in the proper direction and null the error voltage. By addition of the transistor torque-amplifier device, a small torque may be used to turn a heavy load from a remote position.

ADDITIONAL SEMICONDUCTOR DEVICES

Continuous research in the field of semiconductors, since the first development of the point-contact and the junction transistor, has produced many additional semiconductor devices. Military and commercial equipments are now using some of these devices. Many of the devices now undergoing research and development will be used in future equipments. The theory and application of the junction transistor have been discussed to this point. The point-contact transistor, the first transistor developed, is covered briefly in the following paragraphs. Also to be discussed briefly are the theory and application of some of the additional semiconductor devices such as photosensitive semiconductors, tetrode transistor, spacistor, unijunction transistor, four-layer diode, tunnel diode, zener diode, and molecular electronics.

Point-Contact Transistor

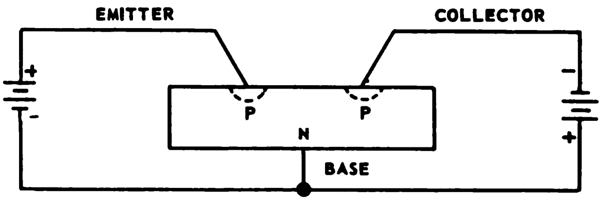
In Figure 8–2 the point-contact transistor consisted of a single piece of P-type or N-type material. At the base contact there is a large area connection (low resistance); however, each of the emitter and collector contacts form small area (high resistance) connections. The point-contact transistor may use either an N-type or a P-type material, but under the contact points P-type layers are present in the N-type material and N-type layers are present under the contact points in the P-type material. These layers are formed during the manufacturing process.

Figure 8–88A shows the N-type point-contact transistor with P-type layers under the contact points. Because the N-type point-contact transistor is similar to a PNP junction transistor, the biasing polarities are the same. Figure 8–88B shows the P-type point-contact transistor with N-type layers under the contact points; it is similar to the NPN junction transistor. The point-contact transistor and the junction transistor use the same bias; the emitter-base junction is biased in the forward direction, and the base-collector junction is biased in the reverse direction.

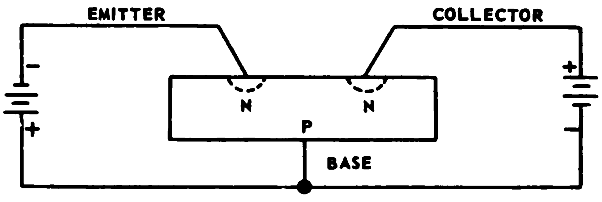
Because of the high resistance that the small area connections present, the point-contact transistor has higher input and output impedances than the junction transistor. The base current in a junction transistor is very small, while in a point-contact transistor it is fairly large. The emitter current in a junction transistor is larger than the collector current; in a point-contact transistor the collector current is greater than the emitter current. The characteristic curves of the two types of transistors are similar.

Photosensitive Semiconductors

In a semiconductor material, the movement of the carriers (holes or electrons) is controlled by electrical, heat, or light energy. In photosensitive



A N-TYPE POINT-CONTACT TRANSISTOR



B P-TYPE POINT-CONTACT TRANSISTOR

Figure 8–88. Bias Arrangement for N-Type and P-Type Point-Contact Transistors

semiconductor material, light energy controls the movement of the carriers, but the light energy must be concentrated on a sensitive area. A concentrated beam of light striking the sensitive area increases the current flow through the semiconductor. Conversely, a decrease of light energy produces a decrease of current flow through the photosensitive semiconductor. Figure 8–89A shows the point-contact photosensitive semiconductor, and Figure 8–89B shows the junction photosensitive semiconductor. These two types of semiconductors are used in various circuits for detection, translation, switching, and so forth.

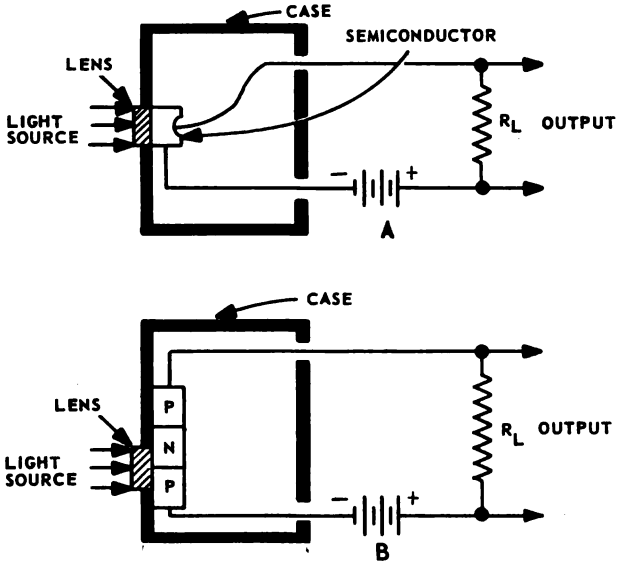


Figure 8–89. Photosensitive Semiconductors

Point-Contact. Notice in Figure 8–89A that the lens concentrates the energy from the light source into a concentrated beam that strikes the sensitive area of the semiconductor. By varying the intensity of the light source, the light energy is increased or decreased which, in turn, increases or decreases the current flow through the point-contact transistor. The current flow through the load resistor (R_L) represents the light energy, which has been converted into electrical energy. The output signal is taken across the load resistor. So that the light energy may be concentrated on the sensitive area, the material (N-type or P-type) is hollow ground on one side.

Junction. A single junction or a double junction transistor may be used as a photosensitive semiconductor. Figure 8–89B illustrates a double junction photosensitive semiconductor circuit. The operation of this circuit is identical to that of the point-contact circuit. The only difference in the circuits is in the physical construction of the semiconductor. The operation of the double junction circuit is comparable to the operation of the junction transistor amplifier.

The light source, striking the first PN junction, is equivalent to an input signal that is applied to the emitter-base junction. The current is amplified in the second PN junction, which is equivalent to the collector-base junction of a transistor amplifier. Therefore, there is a greater variation of collector current in the double junction photosensitive semiconductor than in the point-contact photosensitive semiconductor

Tetrode Transistor

The highest frequency that can be amplified by a conventional three-terminal junction transistor is limited by the input and output capacitance of the transistor. One way to reduce these capacitances is to reduce the size of the material. But this is not practical because the semiconductor material must be large enough to attach the leads to the three semiconductor regions. Another way is to limit the operation of the transistor to a small portion of the semiconductor material. In this manner the effect of a small size transistor that has low input and output capacitance is obtained. To obtain this effect a tetrode, or four terminal, transistor is used.

Figure 8–90 illustrates the tetrode transistor. Notice that the tetrode transistor construction is primarily the same as the three-terminal junction transistor with the exception of a second terminal attached to the base. Terminals 1, 2,

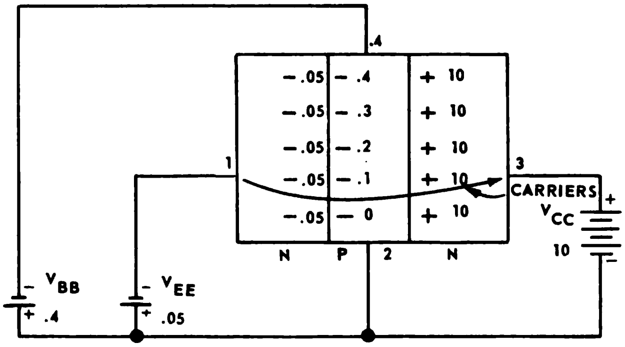


Figure 8–90. Tetrode Transistor

and 3 are emitter, base and collector terminals, respectively, and are biased in the conventional manner. Terminal 4 is the second terminal attached to the base. Battery V_{BB} places a negative bias on terminal 4 and causes the indicated voltage gradient within the base region.

A study of the voltage gradient will show that the emitter is more negative than the base (forward bias) only over a small portion of the emitter-base junction. Current flow between base and collector will now flow only over a small portion of the collector-base junction. Since only a small portion of each junction is involved, the effective input and output capacitances have been reduced. Only the active portions of each junction affect the interelement capacitance. From this we see that the high-frequency response of the tetrode transistor is much greater than that of the three-terminal transistor. One particular application of the tetrode transistor is in wide-band amplifier circuits.

Spacistor

In addition to the interelement capacitances limiting the high-frequency amplification range, one other factor that affects the high-frequency response is the transit time of current carriers from emitter to collector. The gain falls off rapidly as the time of one cycle of the frequency to be amplified approaches the transit time. Transit time also limits the high-frequency response in electron tubes. Two methods to reduce the transit time in the electron tube are: (1) increase the plate voltage, and (2) decrease the spacing between cathode and plate. The transit time in the conventional transistor is long, because there is very little voltage gradient existing in the base region; therefore, current carriers diffuse in the base region (migrate) from the emitter junction to the collector junction.

While the physical size of the transistor is small, a partial solution to the long transit time

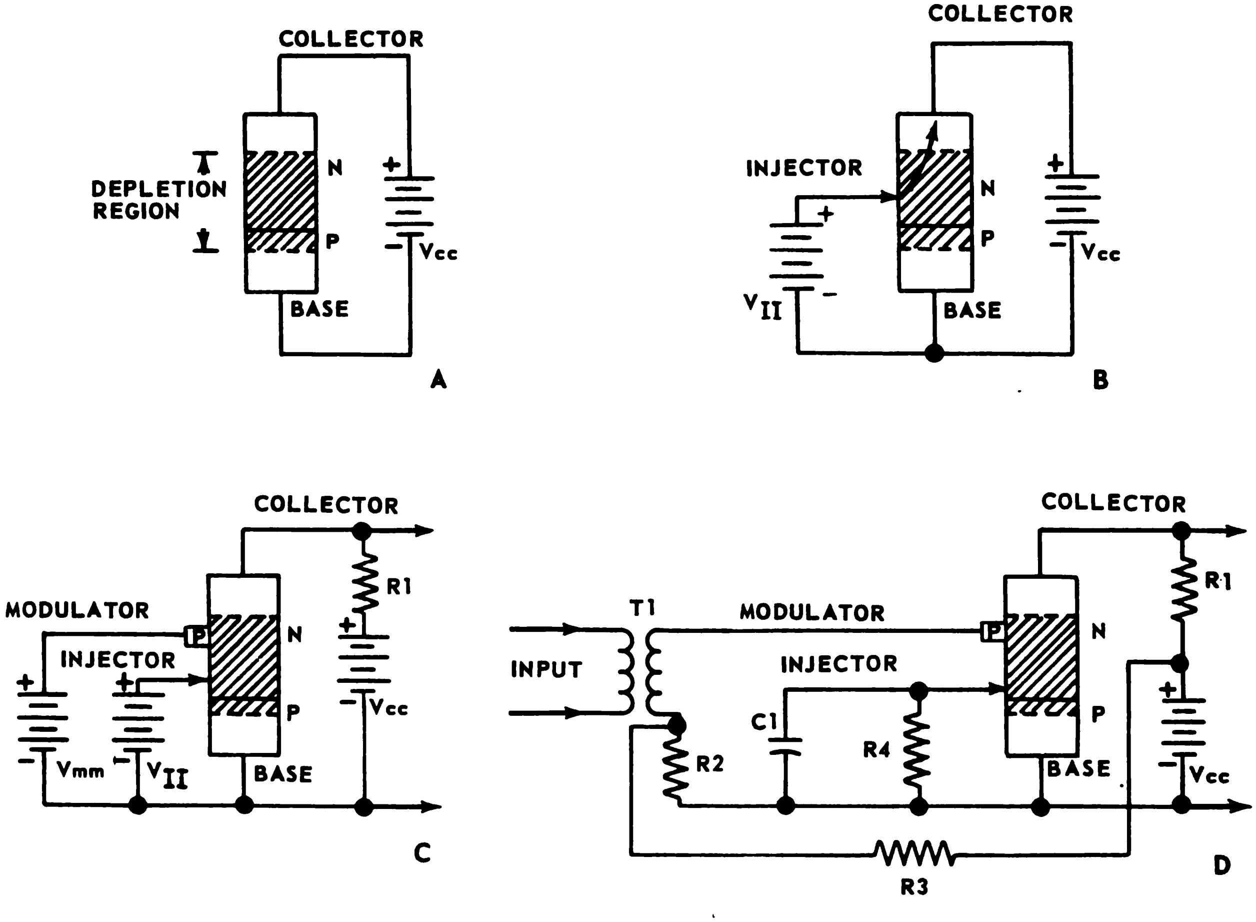


Figure 8–91. Spacistor and Spacistor Amplifier

is the reduction of the size of the base region. Decreasing the size of the base region has increased the response of transistors to approximately 75 megacycles. Methods that employ nonuniform distribution of impurities in the base region have increased the response of transistors to approximately 350 megacycles. The spacistors illustrated in Figure 8–91 eliminate the diffusion of carriers through the base region. Because of the resultant low transit time, the spacistor may be operated up to several kilomegacycles.

Figure 8–91A illustrates just a portion of the spacistor using a reverse-biased PN junction. By using a lower percentage of donor impurities in the N-type material than the percentage of acceptor impurities used in the P-type material, the depletion region in the N-type material is the larger of the two regions. Since two connections are required in one ot the regions, one of the regions must necessarily be larger than the other. With reverse bias on the junction, only a very little reverse-biased current flows through the junction. Widening the depletion region increases the barrier height of the junction; thus a strong electric field exists on either side of the junction. Since the physical distance of the depletion region is very short, a reverse bias of 100 volts will produce an electric field of 60,000 to 120,000 volts per centimeter. This very intense electric field is used to reduce the transit time.

One of the connections made to the larger upper region (Figure 8–91B) is called the injector. Battery V_{II} biases this terminal with less voltage than that of Battery V_{cc}, so that an electron current flows from the injector to the collector. The magnitude of the current depends upon the difference in voltage between battery V_{II} and battery V_{cc}; the intense electric field through which the electrons flow has made their transit time very short.

The second connection made to the larger upper region is called the modulator (Figure 8–91C). A small section of P-type material forming a PN junction with the collector region forms the modulator. Battery V_{MM} supplies bias for the modulator; this bias voltage is less than that of battery V_{cc}. The polarity of the bias voltage is such that the PN junction formed by the modulator and collector regions is reverse biased. A high input impedance at the modulator terminal is produced by this reverse-biased PN junction. Varying V_{MM} varies the current flow between the injector and the collector. If a varying voltage (sine wave) is introduced between the modulator and the injector, then a similarly varying current to the collector is the result.

Under these conditions the spacistor can be used as a very high-frequency or a low-frequency amplifier as shown in Figure 8–91C. The modulator (acting as a control grid) draws no current; therefore the input impedance between the modulator and the injector is extremely high. The injector acts as a cathode. Because of the small area each covers, the input capacitance between modulator and injector is very low. Because of the large depletion region, the output capacitance (collector to base) is very low. The large depletion region acts as a capacitor whose plates are far apart. Under these conditions, current, voltage, and power gain can be achieved.

Unijunction Transistor

The unijunction (single junction) transistor, also referred to as a double-base diode, is actually a diode with two connections made to one part of the semiconductor, as shown in Figure 8–92A. If terminals 2 and 3 were connected together, the resulting device would have the characteristics of a conventional junction diode.

The bias of battery V_{BB} causes electron current flow from terminal 3 to terminal 2 as indicated. The PN junction is reverse biased with terminal 1 connected to terminal 3. The only current that flows through terminal 1 is a reverse-bias current. The solid arrow indicates the current consisting of electron flow from terminal 1 to the PN junction, and the dashed-line arrow indicates the current consisting of holes from terminal 2 to the PN junction.

The voltage gradient from terminal 2 to terminal 3 (Figure 8–92B) is established in the N-type semiconductor by battery V_{BB}. Battery V_{EE}, with the indicated polarity, is inserted between terminals 1 and 3. If the battery voltage of V_{EE} is less than the voltage gradient opposite the P-type semiconductor, the condition as illustrated in Figure 8–92A prevails. If the battery voltage of V_{EE} is greater than the voltage gradient opposite the P-type semiconductor, the PN junction is forward biased. Then heavy electron current (solid-line arrows) flows in the N-type material, and heavy hole current (dashed-line arrows) flows in the P-type material. Electrons will flow out of terminal 1.

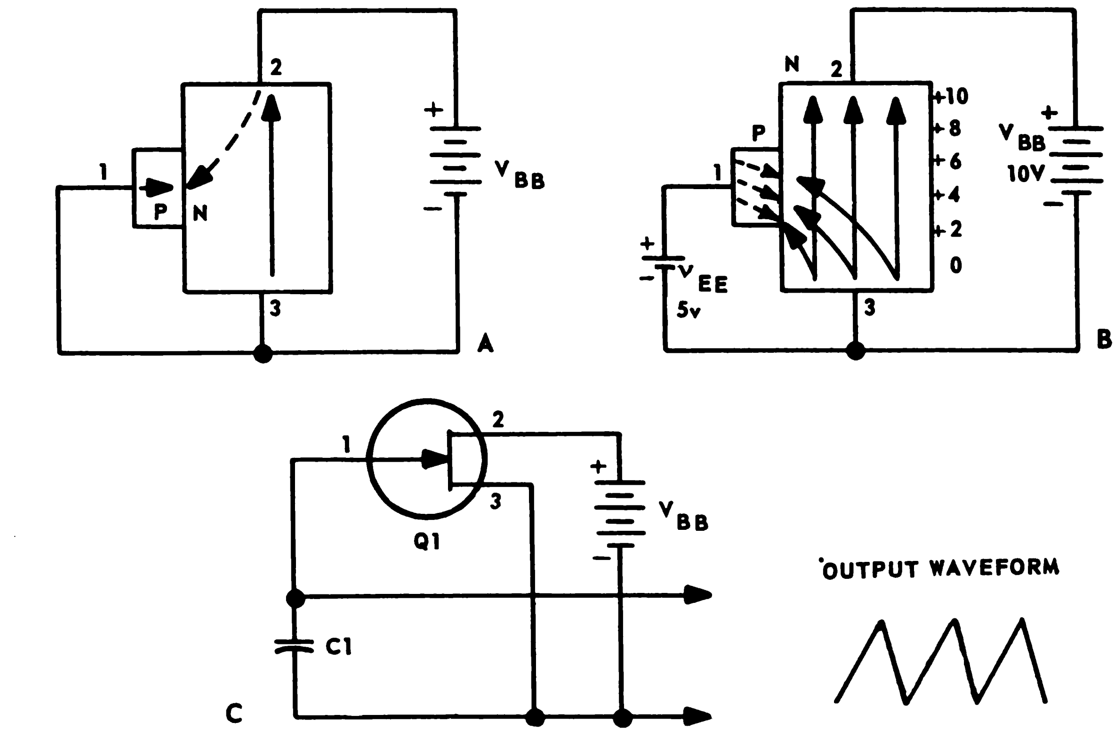


Figure 8–92. Unijunction Transistor and Sawtooth Generator

The characteristics of the unijunction transistor just discussed make it very adaptable for use in multivibrators and sawtooth generators.

Figure 8–92C shows a sawtooth generator using a unijunction transistor. When power is applied to the circuit, the conditions are the same as those indicated in Figure 8–92A. Capacitor C1 is charged very slowly by the small current through the reverse-biased PN junction. The charging time of capacitor C1 is equal to the product of the capacitance of the capacitor and the resistance of the reverse-biased PN junction. During the charge of the capacitor, the positive DC voltage across it rises.

When this positive voltage is greater than the voltage gradient opposite the P-type material, the PN junction becomes forward biased as indicated in Figure 8–92B, and the capacitor discharges very rapidly. The discharge time of capacitor C1 now equals the product of the capacitance of the capacitor and the resistance of the forward-biased PN junction. After the discharge of the capacitor, the conditions are as indicated in Figure 8–92A again, and C1 charges and establishes a forward bias for the PN junction. This sequence continues as long as power is applied, and the slow charging and rapid discharging of capacitor C1 produce an output sawtooth waveform.

Four-Layer Diode

Figure 8–93 illustrates the four-layer diode, which consists of four layers of semiconductor material. Three PN junctions are formed by the four layers of N-type and P-type material as indicated in Figure 8–93A. When the proper bias is used, the two outer PN junctions are forward-biased, and the center PN junction is reverse-biased. A point to be remembered is that the emitter-base junction of a three-terminal transistor is always forward-biased. and the collector-base junction is always reverse-biased. Therefore, the four-layer diode can be analyzed as two separate junction transistors.

The dotted line in the center of the PN material indicates that the four-layer diode is divided into a PNP transistor and an NPN transistor. Figure 8–93B shows the PN junction of the PNP transistor connected to the positive side of battery V_{EE}; since it is forward-biased, it is the emitter-base junction. The remaining PN junction of the PNP transistor is the collector-base junction because it is reverse-biased.

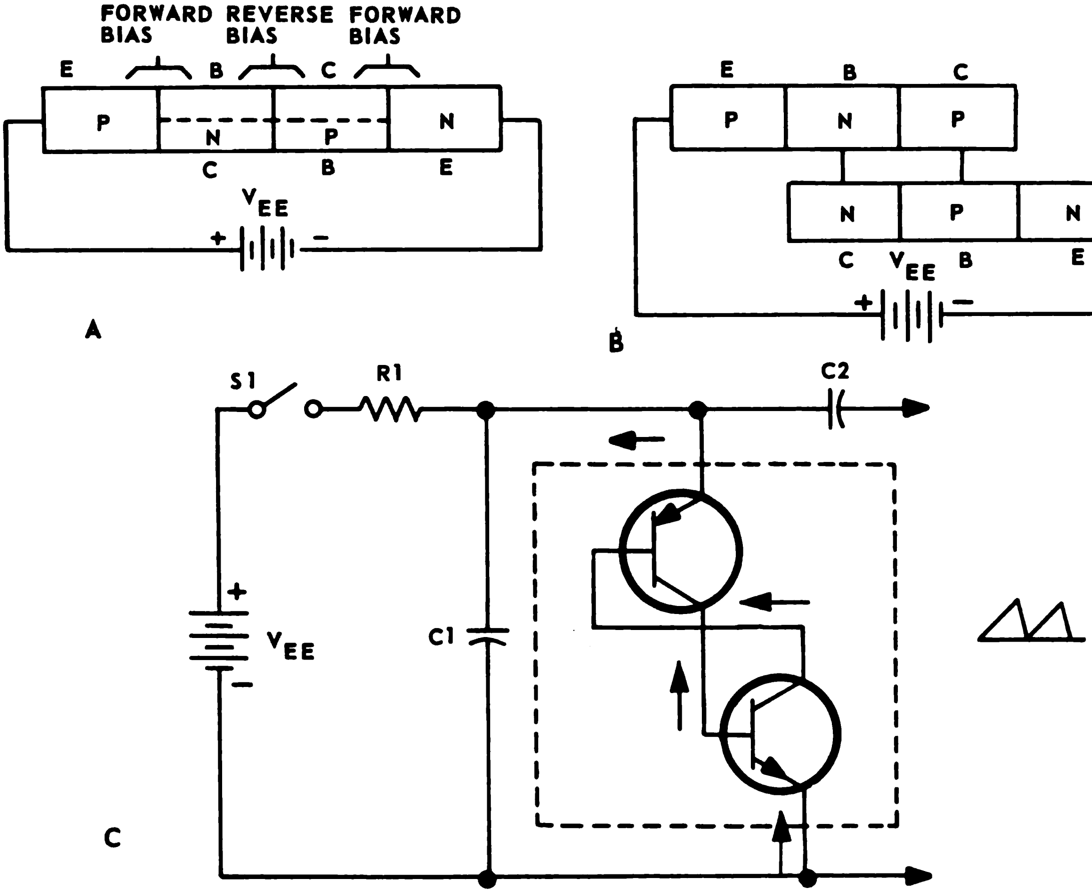


Figure 8–93. Four-Layer Diode, and Sawtooth Generator

Figure 8–93B also shows the PN junction of the NPN transistor connected to the negative side of battery V_{EE}. Thus it is forward-biased and is the emitter-base junction. The remaining PN junction of the NPN transistor is the collector-base junction because it is reverse-biased. Notice in Figure 8–93B how the base of the PNP transistor is connected directly to the collector of the NPN transistor, and that the collector of the PNP transistor is connected directly to the base of the NPN transistor.

Figure 8–93C depicts the four-layer diode connected as a sawtooth generator. The power for the oscillator circuit is furnished by battery V_{EE} when switch S1 is closed, completing the circuit. The time constant of resistor R1 and capacitor C1 determines the frequency of the sawtooth output. The four-layer diode is the equivalent of a thyratron (switch) tube which provides a quick discharge path for C1.

When switch S1 is closed, applying power to the circuit, current flows through resistor R1, and capacitor C1 slowly begins to charge. When the voltage across capacitor C1 is enough to sufficiently bias the four-layer diode, current will flow. The current through the four-layer diode rapidly discharges capacitor C1. The discharge of C1 decreases the bias applied to the four-layer diode, which stops conducting when the bias reaches the minimum value for conduction. At this time, with switch S1 closed, capacitor C1 again starts to charge, repeating the cycle. As long as switch S1 remains closed, the sequence will repeat itself. The output is a sawtooth waveform which is coupled through capacitor C2 to the following stage.

Tunnel Diode

One of the more recent semiconductor developments, the tunnel diode, is drawing a great deal of attention. This new device is exhibiting many advantages over the transistor. It is faster, smaller, less expensive, more reliable, and has more resistance to radiation, a higher cutoff frequency, and a much greater temperature range.

The tunnel diode gets its name from the quantum-mechanical tunneling that characterizes its unusual operation. In high quality semiconductor crystals, the PN junction is considered to be a broad barrier that prevents the direct transfer of energy. In highly impure PN junctions, the majority charge carriers have the ability to tunnel through the junction barrier with the speed of light and appear at the other side. When this phenomenon, or tunneling, occurs, it is possible to decrease the flow of current through the junction with an increase in voltage, or to increase the current with a decrease in voltage. (This effect will be explained later.) However, the depletion region, or forbidden region, between the two types of material is extremely thin.

This thinness is produced by the use of less perfect materials in the manufacturing process. As the semiconductor crystal becomes less perfect, the barrier becomes thinner, less than one-millionth of an inch. It is this thin barrier that gives the diode its ability to respond to frequencies in the kilomegacycle range. Other qualities of the tunnel diode, such as the slope of its current-voltage characteristic, are attained by extreme care in processing the diode. The tunnel diode exhibits a *voltage controlled negative conductance*, which makes it suitable for many applications.

The tunnel effect describes the ability of electrons to travel across the barrier with the speed of light even though the electrons do not seemingly have the energy to overcome the barrier. It appears as though "phonons" (ultrasonic vibrations of the diode crystal lattice) may contribute the energy required to penetrate the barrier.

The tunnel diode uses negative resistance, which is well known, but as yet has not been put to widespread practical application. When the diode is back biased, it conducts highly in the reverse direction. With a small forward biasing, the current increases linearly in the forward direction until it reaches a peak (Figure 8–94). With an increase in forward bias, the forward current then drops to a minimum. This indicates the negative conductance peculiar to the tunnel diode. This change of forward current in the negative direction is called the *Esaki* current. The negative resistance portion of the curve is used to amplify small input voltages. In Figure 8–94 the voltage and current of an ordinary germanium diode are compared with those of a germanium tunnel diode.

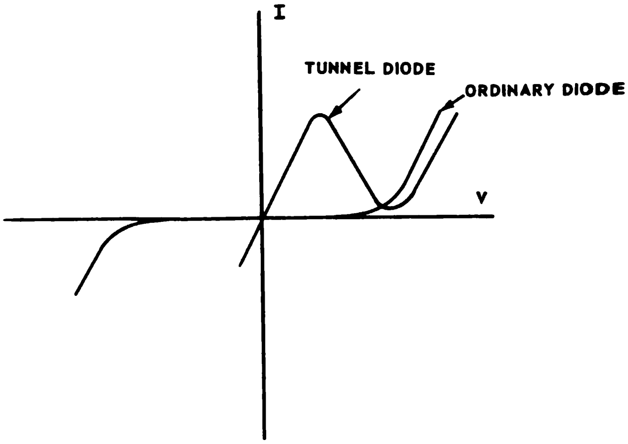


Figure 8–94. Voltage-Current Curves for Tunnel Diode and Ordinary Diode

The tunnel diode amplifier is just two specks of crystal, one slightly different from the other, fused together. The key to its operation is the modern picture of the way electricity acts. Electrons can have many different energy levels, but not *all* different energy levels. They're like paper money, which comes in values of one, two, five or ten dollars, but not 2½ or seven dollars.

Electrons are restricted to certain "permitted" levels of energy. Intermediate energies, between the groups, are "forbidden." The exact values of the permitted and forbidden energy levels are different for different materials. The tunnel diode exploits the differences in energy levels to make a weak signal voltage control a strong current. The voltage does this by shifting electron energies on one side of the diode relative to the other so that sometimes current flows easily between the two, and sometimes it doesn't.

We will use Figure 8–95 to give a more detailed operation of the tunnel diode. In A, zero voltage is applied. Electron energies are equal on both sides of the diode. Electrons "tunnel" back and forth across the joint, but just as many go one way as the other, so the net current is zero. Notice that the P-type semiconductor on the left side has room for more electrons at low levels, below the forbidden region. The N-type semiconductor on the right side has its low levels filled with electrons and surplus electrons in its high levels, above its forbidden region.

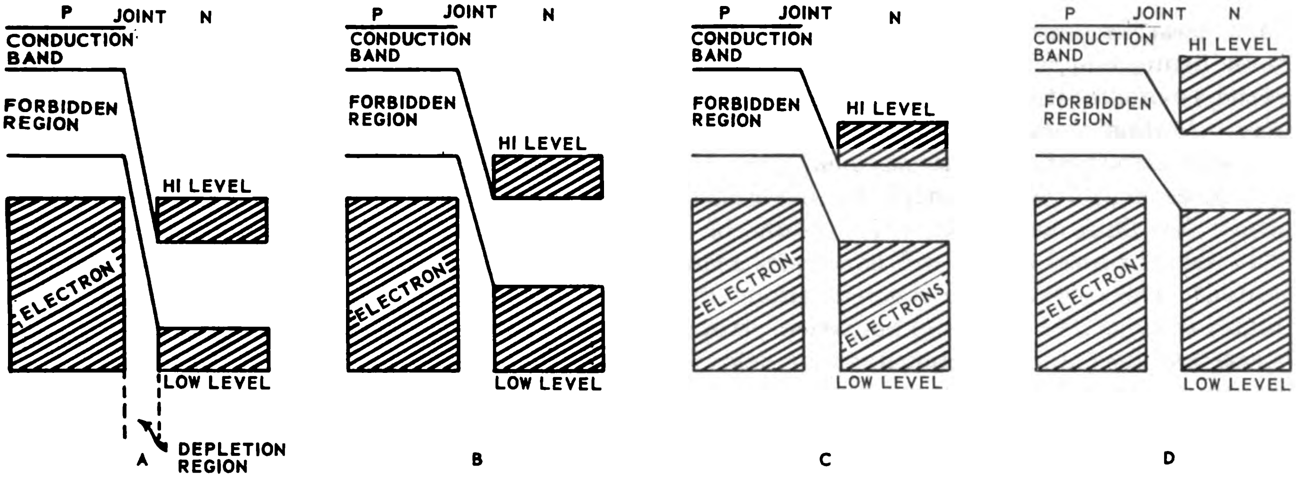


Figure 8–95. Tunnel Diode Theory

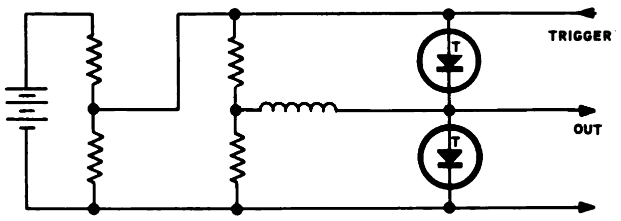
In B, a small voltage has been applied. The energies of the electrons on the N-type side are raised by this increase in voltage, bringing them opposite empty spaces in the P-type low levels. More electrons tunnel from right to left (N-type to P-type) than go the other way. A strong net current flows from right to left. At this point, current is increasing with increasing voltage — just as it does ordinarily in a wire — and the tunnel diode doesn't amplify any more than a copper wire would.

The applied voltage has been increased in Figure 8–95C. The energies of electrons on the N-type side are raised still more. Some now face the forbidden region of the P-type side. Those electrons cannot enter the P-type transistor because they can't exist in a forbidden region. The current from right to left decreases. A steady bias voltage will keep the electrons in this energy range, where the addition of small signal voltages causes sharp drops in current. Here the diode works as an amplifier to magnify the signals.

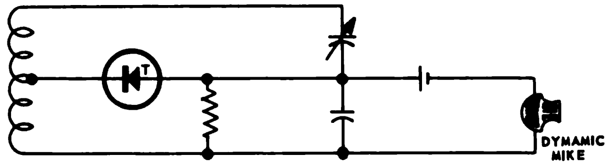
Energies of electrons on the N-type side are raised above the P-type's forbidden region by the high voltage applied in Figure 8–95D. They now face the P-type's higher levels and can flow into the P-type again. Current flow from right to left goes up once more. Here increasing the voltage increases the current, and the diode is no longer useful as an amplifier. Because the tunnel diode works only over a narrow range of voltage, special radio and TV circuits must be designed for it.

Two general types of operation are possible with tunnel diodes; small signal operation about a bias point in the negative resistance range, and large signal switching between two stable points that are separated by a region of instability. The first type of operation gives the diode a possible use as an amplifier or an oscillator. The second type of operation is used in high-speed digital computer systems. One of the outstanding characteristics of the tunnel diode is that its negative resistance remains constant up to extremely high frequencies.

The following characteristics give the tunnel diode remarkable promise for use in communication and computing equipment; high-frequency response (over 100 kmc is conceivable), low



A TUNNEL DIODE INDUCTIVELY COUPLED



B HARTLEY OSCILLATOR

Figure 8–96. Typical Tunnel Diode Circuits

input-power requirement (about 1 mw), low noise figure, simplicity, and the ability to withstand environmental extremes (withstands nuclear radiation, works fine at a few degrees above absolute zero, performs the same at 32°F and 600°F as at 70°F). Although present tunnel diodes handle 5 amperes at a few tenths of a volt, future tunnel diodes will handle far more current. The most adverse characteristic of the tunnel diode is its inability to be used in several stages and have a unidirectional circuit. At the present, this has not been accomplished. The problem of isolating output from input is inherent in two-terminal devices.

Figure 8–96 shows how the tunnel diode may be used in different circuits. Figure 8–96A shows inductive coupling used in a multivibrator (flip-flop) circuit. The B part of the figure illustrates the tunnel diode used in a Hartley oscillator circuit.

Zener Diodes

Development. Zener diodes are transistor PN junctions that change resistance greatly with a small change in applied voltage. A scientist named Zener performed many experiments with PN junctions and discovered what is known as the *Zener effect*. This may be described as the condition that develops when reverse bias is applied to a PN junction in sufficient amounts to cause crystal breakdown and a charge emission.

A Zener diode differs from normal PN junctions in that this emission does not destroy the diode. When a proper load resistor is placed in series with a Zener diode, large changes in applied voltage may occur without causing a change in voltage across the Zener diode. Therefore, the Zener diode has the basic characteristic of the conventional gas-filled voltage regulator tube that will start conducting at a particular voltage and continue to conduct varying amounts of current at a fixed voltage across the tube.

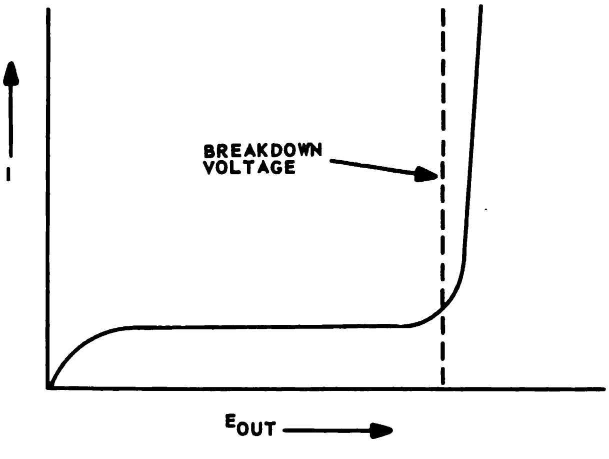


Figure 8–97. Zener Diode Current-Voltage Characteristic Curve

Application. At present Zener diodes are used extensively for voltage regulation in transistor circuits. They are also used as limiters and clampers. The future utilization of these devices will depend upon the use of transistorized circuits and their adaptability for use in solid state electronic devices.

Due to their small size, the size of associated components, and their excellent regulation of the low voltages used in transistor circuitry, Zener diodes are better in transistor circuits than VR tubes. Presently, however, these diodes are limited to relatively low power level applications, in the order of a few watts. Research is now being carried on to achieve the Zener effect in silicon crystal diodes. If this is accomplished, these diodes would have extensive application in power supplies as rectifiers with high power capabilities as well as excellent voltage regulation.

Figure 8–97 shows the Zener diode current-voltage characteristic curve. Note that at a certain value of reverse-bias voltage, (E_{out}), the current increases rapidly, while the voltage across the diode remains almost constant. The voltage at which this action takes place is called the breakdown voltage. This characteristic curve is similar to the curve of some gas-filled tubes that start conducting at a certain voltage and continue to conduct varying amounts while the voltage remains almost fixed.

MOLECULAR ELECTRONICS

The increased technology in several areas has created a great demand on electronics. The electronic equipment required for air and space communications and navigation has greatly increased in complexity and in its need of special skills. The space available for electronic gear has been drastically reduced; in addition, less weight has become highly desirable. Molecular electronics is one answer to these problems.

By definition, *molecular electronics* is the complete merger of function and material. In a single crystal, or monolithic block, various technological developments provide those modifications or changes in the properties of the crystal to make it perform an electronic function in the control or transformation of energy. There is no need for the modified structure to bear any relationship to the original circuit.

In miniaturization there is a one-to-one correspondence of elements between the large and small circuitry. Since there is a limit to how small an individual component can be fabricated, a point is reached which cannot be exceeded in miniaturizing a circuit. But, to obtain a desired system performance, molecular electronics goes into the modification of various material characteristics dealing with the fundamental properties of matter. The result is infinitely smaller than the miniaturization of conventional circuits.

During the early years of electronics, the crystal radio was very close to being a molecular system. However, the performance of the crystal radio did not meet a required standard. Components were added to meet this standard, thus increasing the size and complexity of the system. In the future it is foreseen that we can take a single piece of material, with modifications, and obtain a quality result that is usually expected of a component system.

Recently it was discovered that multizoned semiconductor crystals, in ribbon form, could be grown by the dendrite process. Crystals in ribbon form were drawn in a continuous process from a molten mass of semiconductor material. The size of the ribbon is approximately one-eighth of an inch wide and a few thousandths of an inch thick. An outstanding feature is that the ribbons are ready for use immediately after being grown; there is no need for lapping or finishing in the construction of a function-block.

There are a number of methods used in the molecular approach to constructing a function-block. Interfaces and junctions for desired effects and performance can be grown into the ribbon (dendrite) as it is drawn from the molten mass. After an analysis of the system requirements to determine the function to be performed by the block, a topologist, a mathematician who works with shapes, then designates on paper the arrangements of the interfaces and domains for the most efficient control of energy flow.

The construction of a function-block begins with a basic semiconductor wafer. All of the domains and interfaces are produced by using the present day techniques employed in producing conventional semiconductor devices. The finished block is then enclosed in a capsule that protects the block against shock, temperature changes, vibrations, and ionizing radiation. The end product is a single function-block that is entirely composed of semiconductor material and which performs the function of a circuit or subsystem.

Molecular electronics is still in the developmental stage. However, several different classes of function-blocks have already been successfully developed. Among these are: audio amplifier, video amplifier, tuned amplifier, multivibrator circuits, relaxation oscillators optically tuned (light telemetry system), variable potentiometer, analog-to-digital converter, and DC power supply. The classes just mentioned are the ones most familiar in our study here. There are other classes that have been developed and, as in any scientific field, the possibilities are infinite. In the airspace program the molecular electronic concept will be widely used.

DC POWER SUPPLY USING SILICON RECTIFIERS

Vacuum tube rectifiers are rapidly being replaced by silicon rectifiers and diodes in many electronic and electrical engineering applications. Some of these applications include power supply units, magnetic amplifiers, servo systems, modulators and demodulators, DC restoration and DC stabilization. Reasons for the change to silicon devices are their small size and weight, long-term reliability under adverse conditions of vibration and shock, and independence of external excitation such as filament heating. Silicon rectifiers have high-voltage ratings, extremely small reverse current, low forward voltage drop, and can be used at high ambient temperatures.

This discussion will give you a general review of the more common DC power supply circuits and the precautions necessary to realize fully the advantages offered by the unique properties of silicon rectifiers.

General

A rectifier is considered to be a nonlinear resistive device which possesses low electrical resistance in the forward direction, and high resistance in the reverse direction. These conditions are more closely approached by silicon rectifiers than by any other rectifying device.

In the basic half-wave circuit, which consists of a rectifier in series with a single phase AC source and a resistive load, the rectifier functions as a synchronized switch. Current is allowed to flow when the load and source terminals have the same polarity and is interrupted when the source polarity reverses. Load current flows in half-cycle pulses. Means of smoothing the pulsating current are: (1) passing the current through a filter, (2) increasing the number of phases, and (3) increasing the number of cycles of operation.

A filter is used to smooth the rectified current in most low and medium power applications. This is contingent upon the source being a single-phase AC supply. The amount of ripple desired governs the complexity of the filter. There are two types of filters that may be used. The capacitor-input filter presents a shunt capacitance to the rectifier output, while the choke-input filter presents a series inductance to the rectifier output. The voltage waveforms and the current waveforms differ considerably for the two types of filters.

In certain applications of lightweight power supply units for airborne equipment, three-phase circuits are commonly used. Because the output requires very little filtering, the weight of the power supply can be reduced. High efficiency and good power factors are characteristics of the three-phase circuit.

Large currents are involved in high power rectification, and the filter components tend to be bulky and expensive. Here it is more convenient to increase the number of phases and dispense with the filter entirely.

Single-Phase Rectifier Circuits

The half-wave circuit is simple. Its use is restricted to low output powers, as its efficiency is low and the output voltage has a large low-frequency ripple. The smoothing of the ripple is more difficult under these conditions. Figure 8–98 illustrates the basic half-wave circuit. Its operation is the same as that of an electron tube or metallic rectifier circuit.

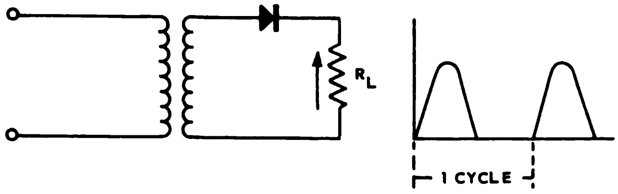


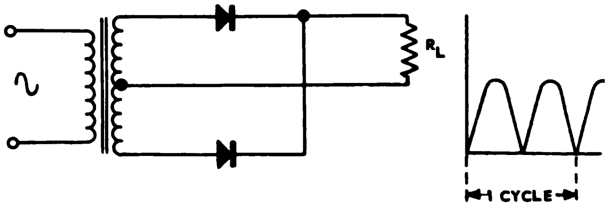
Figure 8–98. Half-Wave Rectifier Circuit

There are two main types of *full-wave* circuits; one uses a center-tapped transformer, while the other is a bridge circuit. These are illustrated in Figure 8–99.

A transformer with a center tapped secondary winding can be used with the bridge circuit when a three-wire DC supply is required. These circuits are used for the larger output powers since they are more efficient than the half-wave circuit, give a smaller ripple voltage at twice the supply frequency, and produce only negligible DC magnetization in the core of the transformer.

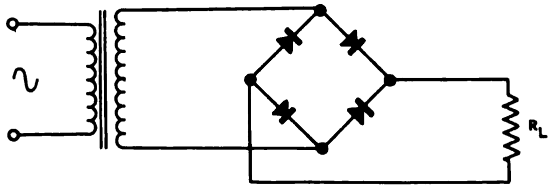
The *center-tapped transformer circuit* (Figure 8–99A) consists of two half-wave circuits operating in push-pull on alternate half-cycles. The single-phase supply is effectively split into two phases. This circuit is used extensively with vacuum tubes. With semiconductor rectifiers its use is normally restricted to fairly low output voltages and to cases where a circuit has been modified to use semiconductor rectifiers in place of tubes.

Where higher DC output voltages are required, the bridge circuit (Figure 8–99B) has several distinct advantages over the circuit in Figure 8–99A. For a rectifier of a given inverse voltage rating, the DC output voltage obtainable using the



A CENTER-TAPPED CIRCUIT

B



C SINGLE-PHASE BRIDGE CIRCUIT

Figure 8–99. Full-Wave Rectifier Circuits

bridge circuit is twice that using the conventional full-wave circuit. The bridge circuit allows the use of transformers without the complication of a center tapped secondary winding. At high voltages this is particularly advantageous, as the number of turns in the secondary of the bridge circuit transformer is half that of the center tapped secondary for a given output. Thus, a simpler and smaller transformer can be designed for the bridge circuit.

The bridge circuit is seldom used with vacuum tubes, principally because of the difficulty of insulating the tube filament supply windings. In addition, two tubes conduct in series and the forward voltage drop, significant for a single tube, is doubled. With silicon rectifiers, the forward voltage drop is too small to have any effect on the performance of the circuit.

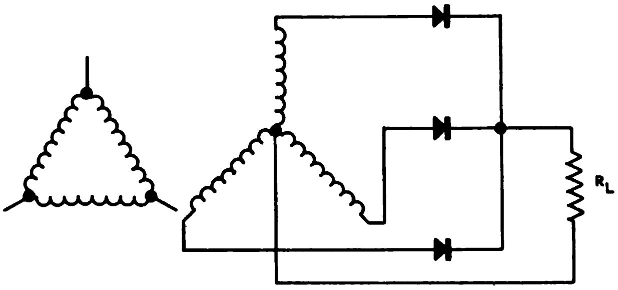


Figure 8–100. Three-Phase Half-Wave Circuit

Three-Phase Rectifier Circuit

Many arrangements are possible for three-phase rectifier circuits. Figure 8–100 illustrates the simplest of the three-phase circuits using a minimum number of rectifiers. Essentially it consists of three single-phase half-wave circuits. Each rectifier conducts for one-third of the cycle. The advantages of using a three-phase AC supply are higher efficiency and lower ripple content; the ripple frequency is three times the supply frequency. Also, if the transformer windings for all three phases are suitably arranged on the same core, no DC magnetization of the core will be produced. This circuit is used for comparatively low power outputs at voltages less than half the rated peak inverse voltage of the rectifiers.

Capacitor-Input Filter

The full-wave circuit of Figure 8–101 has a capacitor input filter, with capacitor C across the load R_L. The total source resistance through which the capacitor charging current flows is represented by R_T. This includes the resistance of one-half of the secondary winding, the reflected primary resistance, the forward resistance of the rectifier unit, and any series resistance added.

Figure 8–101B shows the output voltage across R_L due to the rectified current. Filter capacitor C charges and acts as a reservoir. As the secondary voltage approaches peak value and exceeds the voltage across C, one of the rectifiers conducts, charging the capacitor almost to the AC peak voltage. This charging period is only a small part of the cycle; during the rest of the cycle, C discharges through R_L. The voltage across R_L falls almost linearly. As a result, the voltage across R_L fluctuates around a value E_o and never drops back to zero.

Choke-Input Filter

Choke input circuits are illustrated in Figure 8–102A and 8–102B. Ideally, the choke would have infinite reactance and zero resistance. Then

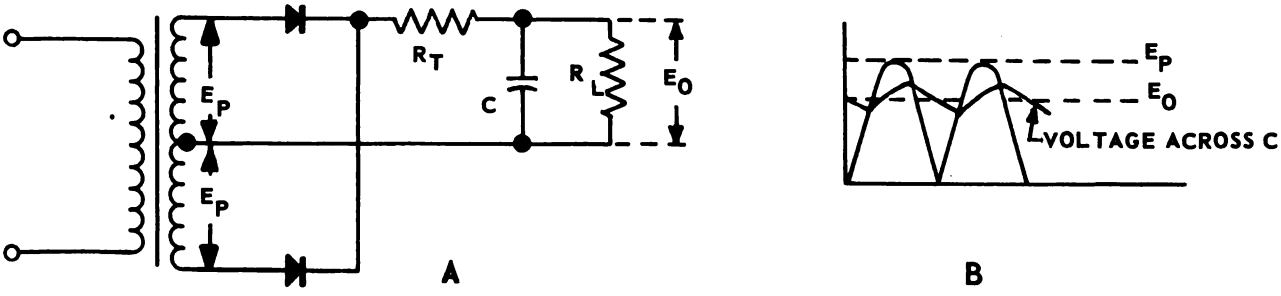


Figure 8–101. Center-Tap Circuit with Capacitor-Input Filter

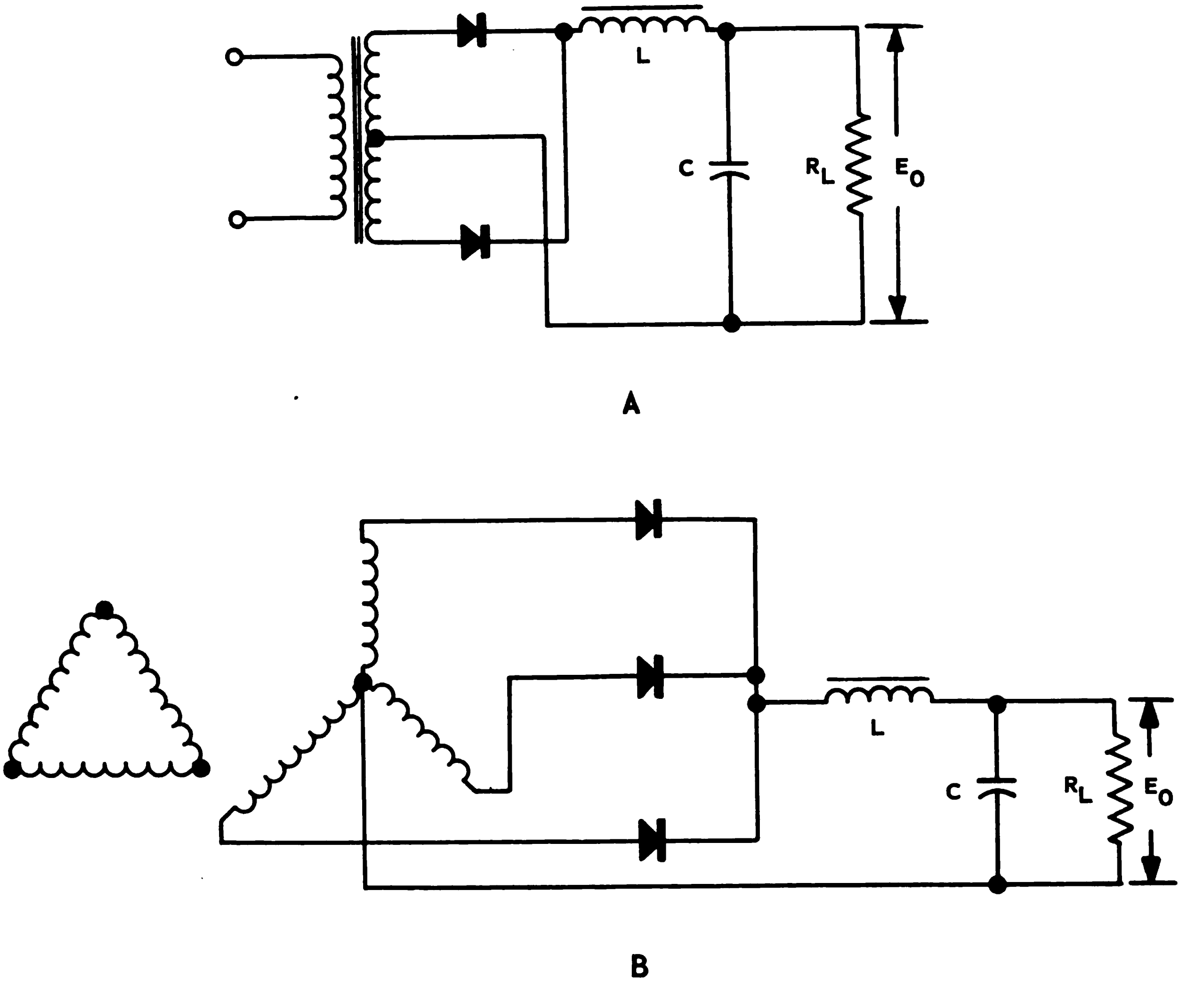


Figure 8–102. Choke-Input Circuits

the load current would be constant and all the ripple voltage would be developed across the choke.

The current is transferred instantly from one rectifier to another each time the supply voltage passes through zero.

The choke inductance must be above that value which just causes the pulsating current from the rectifiers to become continuous. This prevents the current in the filter from falling to zero in any part of the cycle. This value is known as *critical inductance.* When the inductance is less than the critical value, the circuit behaves as a capacitor-input circuit. In practice an inductance double the critical value, known as the *optimum inductance,* is used.

Since the effectiveness of the choke depends on the time constant L/R_L, the value of critical inductance varies inversely as the load current. Hence, as the minimum load current decreases, the value of choke inductance required increases. In view of this, a *swinging choke* can be used to advantage. With this type of choke, the inductance decreases with increase of DC current, due to saturation of the core. Its value at low currents can be much larger than that of a constant inductance choke (such as is required in additional filter sections) of the same dimensions. The decrease of choke inductance as load current increases is not convenient since the critical inductance also decreases at high load currents.

Usually the choke is designed to have optimum inductance at minimum load current so that filtering is effective over the entire current range. If the load current is decreased below the normal minimum, it eventually reaches a value at which the critical inductance becomes equal to that of the choke used.

Precautions

Silicon rectifiers are extremely reliable provided they are used correctly, and adequate precautions are taken against voltage and current overload conditions. The very high current density at which they operate has made it possible to produce an extremely compact device. But their small size means that they have a low thermal capacity, which limits the overloads they can carry even when used with a heat sink or cooling fins. When a semiconductor rectifier starts to break down, its incremental resistance becomes extremely small, possibly negative, and failure is sudden and catastrophic.

In a circuit with a capacitor-input filter, the worst forward current condition occurs at switch-on when a large initial charging current flows through the rectifier to the input capacitor. At the instant of switching on the AC supply, the capacitor charging current is equal to the instantaneous secondary voltage divided by the total source resistance, R_s. It therefore has a maximum possible value E_p/R_s. The total source resistance is the sum of the resistance of the transformer secondary winding, the reflected primary resistance, the forward resistance of the rectifier, and current limiting resistance.

In the majority of circuits, the time constant, R_sC, is small compared with the half-cycle period of the AC supply, so that the charging current falls very rapidly to a value much lower than that at switch-on. If the time constant is large compared with the half-cycle period, the charging current will be approximately a half-sine wave pulse of peak value less than E_p/R_s, and will be repeated in successive alternate half-cycles with exponentially decreasing amplitude.

The presence of inductance in a rectifier circuit can create high voltage transients when power is switched on and off. If a good transformer is used, transients in the AC supply can also appear across the rectifiers. Although these voltage transients do not usually have sufficient energy to destroy the device, their presence across the rectifiers is definitely undesirable. The size of such transients can be reduced sufficiently by shunting the secondary winding with a small capacitor. Transients caused by a choke in the circuit can be reduced by connecting a small capacitor across the input to the filter. The capacitance value required is best determined experimentally, but a typical value for a circuit delivering 500 ma at 300 volts is .005 microfarad.

The choke-input filter is to be preferred with circuits using silicon rectifiers, since the peak current conditions are less severe, and full advantage can be taken of the low forward resistance of the rectifiers to give good regulation at high output currents. At the same time, care should be taken to see that no undesirable resonances occur, since the low resistance in the rest of the circuit will permit large currents to circulate if there is no current limiting resistance. The choke should retain some inductance at currents much greater than normal full load current so that it offers appreciable reactance to the surge current at switch-on.

The reverse voltage across a rectifier unit should not be allowed to exceed the rated peak inverse voltage under any conditions. In a single-phase bridge circuit, the reverse voltage across the rectifier is normally 1.414 times the rms voltage across the transformer secondary. However, this value may be subject to circuitry errors. If all these errors are to be taken into consideration, the nominal peak inverse voltage must be multiplied by a safety factor of 1.6. Thus, the rating of the rectifiers in a bridge circuit should be at least 2.3 times the rms voltage across the secondary.

The forward current in a semiconductor rectifier consists mainly of minority carriers injected into the P-type region. When the voltage across the rectifier is suddenly reversed, those which have not recombined return to the injecting electrode causing an excessive reverse current to flow for a few microseconds. The current overshoots in the reverse direction for a few microseconds and is then suddenly reduced to zero. This rapid decrease of current produces sharp peaks. These can be reduced by shunting the secondary of the transformer with a small capacitor into which the reverse current suddenly released by the rectifiers can flow.

When two rectifiers are connected in series, differences in the reverse current characteristics will cause unequal division of the reverse voltage. This means that series operation can present problems, especially when voltage transients occur. For complete reliability, a shunt resistor or capacitor is placed across each rectifier. Shunt capacitors give the added advantage of protection against voltage transients, and they do not cause additional power loss and local heating.

The reactance of the capacitors at the AC supply frequency should be considerably less than the reverse resistance of the rectifiers at the maximum operating junction temperature.

Connection of rectifiers in parallel can result in considerable differences in the forward currents due to slight variation in the forward characteristics. Equal distribution of forward currents can be achieved by selected rectifiers for parallel operation or placing small resistances in series with each rectifier. A typical resistor of about 3 ohms reduces circuit efficiency very slightly.

Silicon rectifiers have made possible considerable advances in power rectification, but as yet their advantages have not been fully exploited. In the past, circuit engineers have generally turned to silicon rectifiers only in applications where electron tubes or selenium rectifiers would be too bulky, or where germanium could not be used because of high operating temperatures and large reverse voltages. This discussion has endeavored to remove some of the misconceptions which have existed concerning silicon rectifiers, their capabilities and characteristics. In conclusion, it is useful to summarize the advantages which derive from their use.

Among the outstanding features of silicon rectifiers are: high operating temperature, low forward resistance, high reverse voltage rating, small size, and absence of heater supplies. These features lead to a much smaller power supply unit and to appreciable economies. The low forward resistance makes circuit efficiency high by comparison with tube and selenium rectifier circuits. This, in combination with the absence of heater supplies, makes the use of a bridge circuit an attractive proposition. In addition, the low forward resistance and reduced need for series connection, due to the high reverse voltage ratings, lead to very good regulation. In fact, when a choke-input filter is used, regulation can be sufficiently good to make additional voltage stabilization components unnecessary.

The high circuit efficiency and the use of a bridge circuit also mean that a smaller transformer is needed for a given output power compared with selenium or electron-tube circuits. Finally, the combination of high permissible operating temperature and low internal power dissipation reduces the extent and cost of cooling arrangements by comparison with equivalent germanium and selenium circuits.

Thus, in addition to the saving of space that the small size and high reverse voltage rating of the rectifiers permit, the transformer can be smaller; stabilization components may be unnecessary, and cooling equipment can also be smaller. These economies are significant and at present largely offset the high cost of silicon rectifiers in suitable applications. The economies and advantages of power supply units incorporating silicon rectifiers will become continually more important as device manufacturing costs are reduced by improved techniques and increased demand. It can be seen that additional effort is necessary for the design of circuits to avoid misuse of silicon rectifiers, but this is fully justified by increased reliability and reduced running costs.

NEW TECHNIQUES

Maser

Maser means *M*icrowave *A*mplification by *S*timulated *E*mission of *R*adiation. This technique is the most promising development of recent years along lines of eliminating electron flow and still realizing gain.

In the maser technique, either gases or solids (semiconductor types) are supercooled in order to create the proper conditions in molecular or atomic structures for accomplishing either oscillatory or amplifying effects. Supercooled ammonia at near absolute zero is used in the maser technique to provide energy for sustaining oscillations or amplifying signals. Electrostatic focusing is used to separate the high and low energy molecules which are obtained at this temperature.

The high energy molecules are fed into a resonant cavity which is tuned to the resonant frequency of the molecules. A signal fed into the cavity is sustained in very precise oscillations or amplified by the energy released by the high energy molecules returning to the more stable low energy state. Oscillations produced by this technique achieve stabilities on the order of one to ten billion or a thousand times the stability of the crystal controlling the time standard used by the National Bureau of Standards Station WWV.

The solid state masers use a semiconductor solid which is supercooled and placed in a strong magnetic field. This operation places most of the unpaired electrons in a low energy state, making them more responsive to a *pump signal* which is able to lift them to a high energy state. A lower frequency signal, at the resonant frequency of these disturbed electrons, will be reinforced by the energy radiated as the electrons shift back to their lower energy state, thus amplifying the input signal.

One of the present applications of the maser is that of a very stable oscillator in an *atomic clock*, high gain, low noise amplifier, which is used in long-range tracking radars and in radio telescopes. It has a definite need in the future to launch space platforms.

At the present it represents the greatest improvement in signal-to-noise ratio in high sensitivity receivers. Its stability, when used as an oscillator, compares to a watch which loses only one second in 300 years. A disadvantage is that it is an extremely bulky device, as complex cooling arrangements have to be made to keep the host material near absolute zero.

The principal theory of operation is that the maser utilizes radiated energy, from disturbed supercooled atoms or molecules, to reinforce and amplify microwave signals or to sustain microwave signals in very regular oscillations.

Mavar

A development that has even more promise along the lines of noiseless amplifiers is the *mavar* (*M*icrowave *A*mplification by *VA*riable *R*eactance) or parametric amplifier. This technique makes use of purely reactive components in resonant circuits to eliminate the noise generated by resistive devices. The reactive component is made to act like a time-varying inductance or capacitance in a resonant circuit that has the reactance varied at the ebb of energy in the system. Thus, the energy is replaced or built up at the proper frequency in order to sustain oscillations or to amplify a signal.

In a typical arrangement, the reverse bias on a PN junction diode is varied by a pumping signal. This increases and decreases the potential barrier across the junction, making it act like a capacitor whose plates are pulled apart and pushed closer together to vary its capacitance. This pumping action releases some energy into the circuit at the input signal frequency, and this frequency is reinforced by the process.

Raser

Raser, radio frequency amplification by the maser technique, is also being explored and developed.

PRINTED CIRCUIT BOARDS

Transistors, as well as electron tubes, are used extensively with printed circuits; thus, the technician should become familiar with the various types and construction of printed circuit boards.

Types of Printed Circuits

Many devices have been developed in the past 50 years, but the development of electronic devices is perhaps the greatest of them all. In the electronic field the one thing that has provided automation is the printed circuit. The mechanization of electronic products was deterred by the maze of wires and components commonly characteristic of the point-to-point wiring method used in electron-tube circuits. In a printed circuit, electrical connections are made without wires. The printed board also provides a supporting medium for components in a form that is easily adapted to machine assembly. This manual will discuss the fabrication of only one type.

As a technician you must realize that improvement in electronic equipment is a constant endeavor. For instance, at the present time plated circuits, etched circuits, ceramic based printed circuits, stamped circuits, pressed powder circuits, embossed wiring, die blanked molded circuits, and transfer plating are in use. These are only a few of many different methods of applying metals to insulators to make printed circuits.

Etched Circuits

Two types of insulation board that are generally used for the construction of printed boards are laminated plastics and fiberglass. Laminated plastics are constructed by bonding several layers together. This discussion will take into consideration the use of laminated plastic.

Copper foil that approximates .002-inch in thickness or one ounce per square foot, is laminated to the plastic sheet at the time that the plastic is originally made. The copper foil has an adhesive coating which makes this possible.

The first step is to produce a master drawing. This, in turn, is photographed to obtain a negative in the final size of the printed circuit. The negative is then used to produce a stencil on a silk screen frame. Ink is then forced through the stencil onto the copper foil which faithfully reproduces the desired circuit. The heavy coating of ink resists the action of the acid when the board is placed in the etching reagent (ferric chloride). When the printed board is placed in the etching reagent, chemical reaction will remove all copper foil that is not covered with the ink. After the etching process, the board is rinsed with water to remove all etching chemicals, and then a solvent is used to remove the ink. A printed board is the finished product.

The final process involves placing the necessary holes, notches, or slots in the circuit and the cutting of the plate from the manufactured panel. Remember that this is only one of many methods that might be used to construct a printed circuit.

BY ORDER OF THE SECRETARY OF THE AIR FORCE

OFFICIAL

CURTIS E. LEMAY
Chief of Staff

R. J. PUGH
Colonel, USAF
Director of Administrative Services

ATTACHMENT 1

Symbols and Formulas

ELECTROSTATICS

$F = \frac{Q_1Q_2}{K\,d^2}$	Force between charges
$E = \frac{Q}{Kd^2}$	Electric field intensity
$E = \frac{V}{d}$	Field between plates
$F = QE$	Force on Q in field E
$W = QV = eV$	Work
$C = \frac{Q}{V}$	Capacitance
$V = \frac{Q}{d}$	Potential

DIRECT CURRENT

$E = IR$	Ohm's Law
$P = EI = I^2R = \frac{E^2}{R}$	Power

Series Circuits

$R_t = R_1 + R_2 + R_3 + \ldots$	Total resistance in a series circuit
$E_t = E_1 + E_2 + E_3 + \ldots$	Total voltage in a series circuit
$I_t = I_1 = I_2 = I_3 = \ldots$	Total current in a series circuit
$P_t = P_1 + P_2 + P_3 + \ldots$	Total power in a series circuit

Parallel Circuits

$R_t = \frac{R_1\,R_2}{R_1 + R_2}$	Total resistance — Two resistors in parallel
$R_2 = \frac{R_tR_1}{R_1 - R_t}$	Unknown resistance — Two resistors in parallel with total resistance known
$G_t = \frac{1}{R_t}$	Total conductance
$G_t = G_1 + G_2$	Total conductance
$R_t = \cfrac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots}$	Total resistance with any number of resistors in parallel
$E = E_1 = E_2 = E_3 = \ldots$	Total voltage in a parallel circuit
$I_t = I_1 + I_2 + I_3 + \ldots$	Total current in a parallel circuit
$P_t = P_1 + P_2 + P_3 + \ldots$	Total power in a parallel circuit

Resistance

$R = P\frac{l}{d^2}$	Value of resistance as a function of the material and its physical dimensions

where R = resistance in ohms
P = resistance in ohms per *circular-mil-foot* of the material (specific resistance)
l = length of conductor *in feet*
d = diameter of wire *in mils*

Meters

$R_{shunt} = \frac{I_mR_m}{I_t - I_m}$	Shunt required to extend range of ammeter

where I_m = current for full scale deflection
R_m = meter resistance
I_t = extended range of current

$$\text{Ohms per volt} = \frac{\text{Resistance of meter}}{\text{Full scale reading in volts}}$$ Sensitivity of a meter

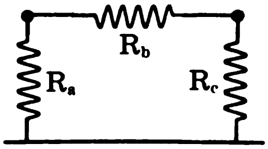
$$\text{or} = \frac{1}{\text{full scale current}}$$

$$R_m = (E \times (E\, x\Omega/V) - R_V$$ Multiplier required to extend range of voltmeter

where E = extended range of voltage
Ω/V = sensitivity of meter
R_V = meter resistance

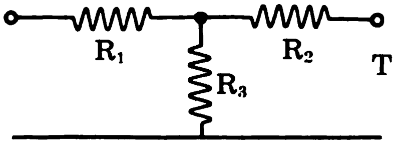
Networks

$$R_1 = \frac{R_aR_b}{R_a + R_b + R_c}$$



π to T transformation

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$



T to π transformation

MAGNETISM AND ELECTROMAGNETS

$$F = \frac{m_1\, m_2}{\mu d^2}$$ Force between two magnetic poles

where μ = permeability of the medium

$$H = \frac{F}{m} \text{ oersteds}$$ Magnetic field intensity

$$H = \frac{2I}{10r} \text{ oersteds}$$ Magnetic intensity about a conductor

where I = current in amperes
r = radius in centimeters

$$H = \frac{4\pi NI}{10\, l} \text{ oersteds}$$ Magnetic intensity for long coils

$$H = \frac{2\pi NI}{10r} \text{ oersteds}$$ Magnetic intensity for short coils

$$\text{mmf} = \frac{4\pi NI}{10} \text{ gilberts}$$ Magnetomotive force

$$\phi = \frac{\text{mmf}}{R} \text{ maxwells}$$ Flux

$$R = \frac{L}{\mu A} \text{ rels}$$ Reluctance

$$B = \frac{\phi}{A} \text{ gauss}$$ Flux Density

$$F = \frac{B\, I\, l}{10} \sin\theta \text{ dynes}$$ Force exerted on conductor

$$\mu = \frac{\beta}{H}$$ Permeability

Induction

$$e = -N\frac{\Delta\phi}{\Delta t} \times 10^{-8}$$ Average Induced voltage

$$e = -M\frac{\Delta i}{\Delta t}$$ Induced voltage

where M is mutual inductance

$$M = K\sqrt{L_1L_2}$$ Mutual Inductance

where k is the coefficient of coupling

$$L = \frac{N\phi}{I} \times 10^{-8}$$ Inductance

where N is number of turns
I is current in amperes

$$e_L = -L\frac{\Delta i}{\Delta t}$$ Voltage across inductance

$$e_L = -N\frac{\Delta\phi}{\Delta t} \times 10^{-8}$$ Voltage across inductance

ALTERNATING CURRENT

General

$$c = E_{max} \sin\omega\, t$$ Instantaneous voltage

where $\omega = 2\pi f$

$i = I_{max} \sin\omega t$ Instantaneous current

where $\omega = 2\pi f$

$P = \frac{E^2}{R} = I^2R = EI \cos\theta$ Power in AC circuits

where θ is the phase angle

$PF = \frac{R}{Z} = \cos\theta$ Power factor

$\theta = \text{arc tan } \frac{X}{R} =$ angle of lead or lag (degrees) Phase angle

$C = K\frac{A}{d}$ Capacitance in micromicrofarads

where A is area of plates
d is distance between plates
K is dielectric constant

$X_L = 2\pi fL$ Inductive reactance

$X_c = \frac{1}{2\pi fC}$ Capacitive reactance

$Z = \sqrt{R^2 + (X_L - X_c)^2}$ Impedance — triangular form

$Z = R \pm jX$ Impedance — rectangular form

$F_r = \frac{1}{2\pi\sqrt{LC}}$ Frequency at resonance

$Q = \frac{X_L}{R}$ Merit of a coil or tank

where R is total resistance in series with coil

$Q = \frac{R}{X_L}$ Merit of a coil or tank

where R is total resistance in parallel with coil

$L_T = L_1 + L_2 \pm 2M$ Total inductance of two coils

where M is the mutual inductance

$L_t = L_1 + L_2 + L_3 \cdots$ Inductance in series

$1/L_t = 1/L_1 + 1/L_2 + 1/L_3 \cdots$ Inductance in parallel

$C_t = C_1 + C_2 + C_3 \cdots$ Capacitances in parallel

$1/C_t = 1/C_1 + 1/C_2 + 1/C_3 \cdots$ Capacitances in series

$Q_t = Q_1 = Q_2 = Q_3 \cdots$ Total charge on condensers in series

$Q_t = Q_1 + Q_2 + Q_3 \cdots$ Total charge on condensers in parallel

$+$ is used when the fields add
$-$ is used when the fields oppose

Transformers

$\frac{E_p}{E_s} = \frac{N_p}{N_s} = \frac{I_s}{I_p}$ Relationships among voltages, turns, and currents in transformers

$\frac{Z_p}{Z_s} = \left(\frac{N_p}{N_s}\right)^2$ Impedances and turns

$Z = Z_1 + Z_p - \frac{Z_m^2}{Z_2 + Z_s}$ Impedance when looking into primary of a transformer

where Z_1 is in series with primary
Z_p is impedance of primary windings
Z_m is mutual impedance
Z_s is impedance of secondary windings
Z_2 is impedance in series with secondary

$Z_r = \frac{-Z_m^2}{Z_2 + Z_s}$ Reflected impedance

Transients

$t = R \times C$ Time constant for RC circuits

where t is in seconds
R is in ohms
C is in farads

$t = \frac{L}{R}$ Time constant for L/R circuits

where t is in seconds
R is in ohms
L is in henries

$$e_c = E_a\left(1 - \epsilon^{-\frac{t}{RC}}\right)$$ Voltage change across a capacitor in an RC circuit

where E_a is the applied voltage

$$e_R = E_a\epsilon^{-\frac{t}{RC}}$$ Voltage across a resistor in an RC circuit

$$i = \frac{E_a}{R}\epsilon^{-\frac{t}{RC}}$$ Current in an RC circuit

$$e_L - E_a\epsilon^{-\frac{Rt}{L}}$$ Voltage across coil in an L/R circuit

$$e_R = E_a\left(1 - \epsilon^{-\frac{Rt}{L}}\right)$$ Voltage change across a resistor in an L/R circuit

$$i = \frac{E_a}{R}\left(1 - \epsilon^{-\frac{Rt}{L}}\right)$$ Current in an L/R circuit

$$E_R = \left(E_a - E_{co}\right)\epsilon^{-\frac{t}{RC}}$$ Voltages in a direct current restorer

where E_{co} is the voltage on the capacitor at the start

VACUUM TUBES AND AMPLIFIERS

$$\mu = \frac{\Delta e_p}{\Delta e_g} \text{ (} i_p \text{ constant)}$$ Amplification factor

$$r_p = \frac{\Delta e_p}{\Delta i_p} \text{ (} e_g \text{ constant)}$$ Plate resistance

where r_p is in ohms
e_p is in volts
i_p is in amperes

$$g_m = \frac{\Delta i_p}{\Delta e_g} \text{ (} e_p \text{ constant)}$$ Transconductance

where g_m is in mhos
i_p is in amperes
e_g is in volts

$$\mu = g_m r_p$$ Relationship among g_m, r_p and μ

where g_m is in mhos
r_p is in ohms

$$A = \frac{E_o}{E_s} = \frac{E_o}{E_g} = \frac{\mu Z_L}{r_p + Z_L}$$ Voltage gain of triode operated Class A

where E_o is the output voltage
E_s is the input voltage
E_g is the grid-to-cathode voltage
Z_L is the equivalent load impedance
r_p is the plate resistance

$$db = 20 \log \frac{E_o}{E_{in}} = 20 \log \frac{I_o}{I_{in}}$$ Voltage or current gain in decibels, if R_{in} equals R_{out}

$$db = 10 \log \frac{P_o}{P_{in}}$$ Power gain of stage operated Class A

where $P_o =$ output power
$P_{in} =$ input power

$$A_t = A_1 \times A_2 \times A_3 \ldots\ldots A_n$$ Total voltage gain of stages in cascade

$$db_t = db_1 + db_2 + db_3 \ldots db_n$$ Total power gain of stages in cascade

$$F_h = \frac{1}{2\pi R_L C_s}$$ Highest frequency in band passed by a video amplifier

where R_L is the total load resistance
C_s is total shunt capacitance

$$F_l = \frac{1}{2\pi R_g C_c}$$ Lowest frequency in band passed by a video amplifier

where R_g is the grid resistor
C_c is the coupling capacitor

$$A' = \frac{E_o}{E_g} = \frac{A}{1 - BA}$$ Gain in a feedback amplifier

where A is the gain without feedback
B is the fraction of the output voltage that is fed back

Cathode Followers

$$Z_o = \frac{R_k r_p}{r_p + R_k(\mu + 1)}$$ Output impedance of a cathode follower

$$A = \frac{E_o}{E_s} = \frac{\mu R_k}{r_p + R_k(\mu + 1)}$$ Gain of a cathode follower

OSCILLATORS

$$F_o = \frac{1}{2\pi\sqrt{LC}}$$

Output frequency of LC oscillator

$$F_o = \frac{1}{2\pi\sqrt{R_1 C_1 R_2 C_2}}$$

Output frequency of a Wien bridge oscillator

$$F_o = \frac{1}{2\pi R_1 C_1}$$

Output frequency of a Wien bridge oscillator when $R_1C_1 = R_2C_2$

$$F_o = \frac{1}{2\pi RC\sqrt{6}}$$

Output frequency of a phase shift oscillator

Shock Excited Oscillators

$$Q = \frac{R_s}{X_L} = \frac{R_s}{L/C}$$

Merit of oscillator tank

where R_s is the shunt resistance

$$X_L = X_C = \sqrt{L/C} \text{ at resonance}$$

$$F_o = \frac{1}{2\pi\sqrt{LC}}$$

Frequency of oscillator when Q is 5 or greater

$$F_o = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \left(\frac{1}{2Q}\right)^2}$$

Frequency of oscillator when Q is less than 5

$$E_{max} = I_p \sqrt{L/C}$$

Maximum output voltage

TRANSMISSION LINES

$$Z_o = \sqrt{L/C} = \frac{E_i}{I_i}$$

Characteristic impedance

where E_i is the incident voltage
I_i is the incident current

$$SWR = \frac{E_{max}}{E_{min}} = \frac{I_{max}}{I_{min}} = \sqrt{\frac{Z_{max}}{Z_{min}}} = \frac{1 + |r|}{1 - |r|}$$

Standing wave ratio

$$\text{where } |r| = \frac{p-1}{p+1}$$

$$SWR = \frac{Z_L}{Z_o}$$

Standing wave ratio (Resistive load only)

where Z_L is larger than Z_o

$$SWR = \frac{Z_o}{Z_L}$$

Standing wave ratio (Resistive load only)

where Z_o is larger than Z_L

$$Z_o = \sqrt{Z_{in} Z_L}$$

Impedance of matching transformer

where Z_{in} is input impedance
Z_o = characteristic impedance
Z_L is output impedance

$$t = n\sqrt{LC}$$

Time for a wave to travel one length of a delay line

where n is the number of sections in the line

$$PW = 2n\sqrt{LC}$$

Pulse width of a pulse forming line

$$f_{co} = \frac{1}{\pi\sqrt{LC}}$$

Cutoff frequency of an artificial line

WAVE GUIDES AND CAVITY RESONATORS

$$\lambda_a = k\frac{300}{f_{mc}}$$

Wavelength in free space (air)

where k is the velocity correction factor
f_{mc} is frequency in megacycles

$$V_p = f\lambda_g$$

Phase velocity

where λ_g is wavelength in guide

$$V_a = f\lambda_a = \sqrt{V_p V_g}$$

Velocity in free space

where λ_a is wavelength in free space (air)

$$\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

Wavelength at cutoff

where m is the first subscript in TE or TM mode designations

n is the second subscript in TE or TM mode designations

a is the short dimension of the wave guide

b is the long dimension of the wave guide

MICROWAVE OSCILLATORS

$F_e = 1.6 \times 10^{-12}\ E$ Force in dynes acting on an electron in an electric field

where E is the electric field strength in volts/cm

$F_m = 1.6 \times 10^{-20}\ V_pB$ Force in dynes acting on an electron in a magnetic field

where V_p is the component of electron velocity perpendicular to the direction of the magnetic field

B is magnetic flux density in gausses

$K.E. = 1/2\ mv^2 = eV$ Kinetic energy in ergs

where m is mass of electron in grams

v is velocity of electron in cm/sec

e is charge on electron in coulomb

V is potential difference through which the electron travels

ATTACHMENT 2

Tables

Powers of Ten

10^{10} = 10,000,000,000
10^{9} = 1,000,000,000
10^{8} = 100,000,000
10^{7} = 10,000,000
10^{6} = 1,000,000
10^{5} = 100,000
10^{4} = 10,000
10^{3} = 1,000
10^{2} = 100
10^{1} = 10
10^{0} = 1
10^{-1} = .1
10^{-2} = .01
10^{-3} = .001
10^{-4} = .000 1
10^{-5} = .000 01
10^{-6} = .000 001
10^{-7} = .000 000 1
10^{-8} = .000 000 01
10^{-9} = .000 000 001
10^{-10} = .000 000 000 1

COMMON LOGARITHMS

N	0	1	2	3	4	5	6	7	8	9
0	....	0000	3010	4771	6021	6990	7782	8451	9031	9542
1	0000	0414	0792	1139	1461	1761	2041	2304	2553	2788
2	3010	3222	3424	3617	3802	3979	4150	4314	4472	4624
3	4771	4914	5051	5185	5315	5441	5563	5682	5798	5911
4	6021	6128	6232	6335	6435	6532	6628	6721	6812	6902
5	6990	7076	7160	7243	7324	7404	7482	7559	7634	7709
6	7782	7853	7924	7993	8062	8129	8195	8261	8325	8388
7	8451	8513	8573	8633	8692	8751	8808	8865	8921	8976
8	9031	9085	9138	9191	9243	9294	9345	9395	9445	9494
9	9542	9590	9638	9685	9731	9777	9823	9868	9912	9956
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4115	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
N	0	1	2	3	4	5	6	7	8	9

COMMON LOGARITHMS

N	0	1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996
100	0000	0004	0009	0013	0017	0022	0026	0030	0035	0039
N	0	1	2	3	4	5	6	7	8	9

NATURAL LOGARITHMS

N	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
	....	0.0000	0.6931	1.0986	1.3863	1.6094	1.7918	1.9459	2.0794	2.1972
10	2.3026	2.3979	2.4849	2.5649	2.6391	2.7081	2.7726	2.8332	2.8904	2.9444
20	9957	3.0445	3.0910	3.1355	3.1781	3.2189	3.2581	3.2958	3.3322	3.3673
30	3.4012	4340	4657	4965	5264	5553	5835	6109	6376	6636
40	6889	7136	7377	7612	7842	8067	8286	8501	8712	8918
50	9120	9318	9512	9703	9890	4.0073	4.0254	4.0431	4.0604	4.0775
60	4.0943	4.1109	4.1271	4.1431	4.1589	1744	1897	2047	2195	2341
70	2485	2627	2767	2905	3041	3175	3307	3408	3567	3694
80	3820	3944	4067	4188	4308	4427	4543	4659	4773	4886
90	4998	5109	5218	5326	5433	5539	5643	5747	5850	5951
100	6052	6151	6250	6347	6444	6540	6634	6728	6821	6913
110	7005	7095	7185	7274	7362	7449	7536	7622	7707	7791
120	7875	7958	8040	8122	8203	8283	8363	8442	8520	8598
130	8675	8752	8828	8903	8978	9053	9127	9200	9273	9345
140	9416	9488	9558	9628	9698	9767	9836	9904	9972	5.0039
150	5.0106	5.0173	5.0239	5.0304	5.0370	5.0434	5.0499	5.0562	5.0626	0689
160	0752	0814	0876	0938	0999	1059	1120	1180	1240	1299
170	1358	1417	1475	1533	1591	1648	1705	1761	1818	1874
180	1930	1985	2040	2095	2149	2204	2257	2311	2364	2417
190	2470	2523	2575	2627	2679	2730	2781	2832	2883	2933
200	2983	3033	3083	3132	3181	3230	3279	3327	3375	3423
210	3471	3519	3566	3613	3660	3706	3753	3799	3845	3891
220	3936	3982	4027	4072	4116	4161	4205	4250	4293	4337
230	4381	4424	4467	4510	4553	4596	4638	4681	4723	4765
240	4806	4848	4889	4931	4972	5013	5053	5094	5134	5175
250	5215	5255	5294	5334	5373	5413	5452	5491	5530	5568
260	5607	5645	5683	5722	5759	5797	5835	5872	5910	5947
270	5984	6021	6058	6095	6131	6168	6204	6240	6276	6312
280	6348	6384	6419	6454	6490	6525	6560	6595	6630	6664
290	6699	6733	6768	6802	6836	6870	6904	6937	6971	7004
300	7038	7071	7104	7137	7170	7203	7236	7268	7301	7333
310	7366	7398	7430	7462	7494	7526	7557	7589	7621	7652
320	7683	7714	7746	7777	7807	7838	7869	7900	7930	7961
330	7991	8021	8051	8081	8111	8141	8171	8201	8230	8260
340	8289	8319	8348	8377	8406	8435	8464	8493	8522	8551
350	8579	8608	8636	8665	8693	8721	8749	8777	8805	8833
360	8861	8889	8916	8944	8972	8999	9026	9054	9081	9108
370	9135	9162	9189	9216	9243	9269	9296	9322	9349	9375
380	9402	9428	9454	9480	9506	9532	9558	9584	9610	9636
390	9661	9687	9713	9738	9764	9789	9814	9839	9865	9890
400	9915	9940	9965	9989	6.0014	6.0039	6.0064	6.0088	6.0113	6.0137
410	6.0162	6.0186	6.0210	6.0234	0259	0283	0307	0331	0355	0379
420	0403	0426	0450	0474	0497	0521	0544	0568	0591	0615
430	0638	0661	0684	0707	0730	0753	0776	0799	0822	0845
440	0868	0890	0913	0936	0958	0981	1003	1026	1048	1070
450	1092	1115	1137	1159	1181	1203	1225	1247	1269	1291
460	1312	1334	1356	1377	1399	1420	1442	1463	1485	1506
470	1527	1549	1570	1591	1612	1633	1654	1675	1696	1717
480	1738	1759	1779	1800	1821	1841	1862	1883	1903	1924
490	1944	1964	1985	2005	2025	2046	2066	2086	2106	2126
500	2146	2166	2186	2206	2226	2246	2265	2285	2305	2324
N	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0

N	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
500	6.2146	6.2166	6.2186	6.2206	6.2226	6.2246	6.2265	6.2285	6.2305	6.2324
510	2344	2364	2383	2403	2422	2442	2461	2480	2500	2519
520	2538	2558	2577	2596	2615	2634	2653	2672	2691	2710
530	2729	2748	2766	2785	2804	2823	2841	2860	2879	2897
540	2916	2934	2953	2971	2989	3008	3026	3044	3063	3081
550	3099	3117	3135	3154	3172	3190	3208	3226	3244	3261
560	3279	3297	3315	3333	3351	3368	3386	3404	3421	3439
570	3456	3474	3491	3509	3526	3544	3561	3578	3596	3613
580	3630	3648	3665	3682	3699	3716	3733	3750	3767	3784
590	3801	3818	3835	3852	3869	3886	3902	3919	3936	3953
600	3969	3986	4003	4019	4036	4052	4069	4085	4102	4118
610	4135	4151	4167	4184	4200	4216	4232	4249	4265	4281
620	4297	4313	4329	4345	4362	4378	4394	4409	4425	4441
630	4457	4473	4489	4505	4520	4536	4552	4568	4583	4599
640	4615	4630	4646	4661	4677	4693	4708	4723	4739	4754
650	4770	4785	4800	4816	4831	4846	4862	4877	4892	4907
660	4922	4938	4953	4968	4983	4998	5013	5028	5043	5058
670	5073	5088	5103	5117	5132	5147	5162	5177	5191	5206
680	5221	5236	5250	5265	5280	5294	5309	5323	5338	5352
690	5367	5381	5396	5410	5425	5439	5453	5468	5482	5497
700	5511	5525	5539	5554	5568	5582	5596	5610	5624	5639
710	5653	5667	5681	5695	5709	5723	5737	5751	5765	5779
720	5793	5806	5820	5834	5848	5862	5876	5889	5903	5917
730	5930	5944	5958	5971	5985	5999	6012	6026	6039	6053
740	6067	6080	6093	6107	6120	6134	6147	6161	6174	6187
750	6201	6214	6227	6241	6254	6267	6280	6294	6307	6320
760	6333	6346	6359	6373	6386	6399	6412	6425	6438	6451
770	6464	6477	6490	6503	6516	6529	6542	6554	6567	6580
780	6593	6606	6619	6631	6644	6657	6670	6682	6695	6708
790	6720	6733	6746	6758	6771	6783	6796	6809	6821	6834
800	6846	6859	6871	6884	6896	6908	6921	6933	6946	6958
810	6970	6983	6995	7007	7020	7032	7044	7056	7069	7081
820	7093	7105	7117	7130	7142	7154	7166	7178	7190	7202
830	7214	7226	7238	7250	7262	7274	7286	7298	7310	7322
840	7334	7346	7358	7370	7382	7393	7405	7417	7429	7441
850	7452	7464	7476	7488	7499	7511	7523	7534	7546	7558
860	7569	7581	7593	7604	7616	7627	7639	7650	7662	7673
870	7685	7696	7708	7719	7731	7742	7754	7765	7776	7788
880	7799	7811	7822	7833	7845	7856	7867	7878	7890	7901
890	7912	7923	7935	7946	7957	7968	7979	7991	8002	8013
900	8024	8035	8046	8057	8068	8079	8090	8101	8112	8123
910	8134	8145	8156	8167	8178	8189	8200	8211	8222	8233
920	8244	8255	8265	8276	8287	8298	8309	8320	8330	8341
930	8352	8363	8373	8384	8395	8405	8416	8427	8437	8448
940	8459	8469	8480	8491	8501	8512	8522	8533	8544	8554
950	8565	8575	8586	8596	8607	8617	8628	8638	8648	8659
960	8669	8680	8690	8701	8711	8721	8732	8742	8752	8763
970	8773	8783	8794	8804	8814	8824	8835	8845	8855	8865
980	8876	8886	8896	8906	8916	8926	8937	8947	8957	8967
990	8977	8987	8997	9007	9017	9027	9037	9048	9058	9068
1000	9078	9088	9098	9108	9117	9127	9137	9147	9157	9167
N	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0

Values of ϵ^{x} and ϵ^{-x}

x	Function	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	ϵ^{x} ϵ^{-x}	1.0000 1.0000	1.0101 0.9900	1.0202 0.9802	1.0305 0.9704	1.0408 0.9608	1.0513 0.9512	1.0618 0.9418	1.0725 0.9324	1.0833 0.9231	1.0942 0.9139
0.1	ϵ^{x} ϵ^{-x}	1.1052 0.9048	1.1163 0.8958	1.1275 0.8869	1.1388 0.8781	1.1503 0.8694	1.1618 0.8607	1.1735 0.8521	1.1853 0.8437	1.1972 0.8353	1.2093 0.8270
0.2	ϵ^{x} ϵ^{-x}	1.2214 0.8187	1.2337 0.8106	1.2461 0.8025	1.2586 0.7945	1.2712 0.7866	1.2840 0.7788	1.2969 0.7711	1.3100 0.7634	1.3231 0.7558	1.3364 0.7483
0.3	ϵ^{x} ϵ^{-x}	1.3499 0.7408	1.3634 0.7334	1.3771 0.7261	1.3910 0.7189	1.4049 0.7118	1.4191 0.7047	1.4333 0.6977	1.4477 0.6907	1.4623 0.6839	1.4770 0.6771
0.4	ϵ^{x} ϵ^{-x}	1.4918 0.6703	1.5068 0.6637	1.5220 0.6570	1.5373 0.6505	1.5527 0.6440	1.5683 0.6376	1.5841 0.6313	1.6000 0.6250	1.6161 0.6188	1.6323 0.6126
0.5	ϵ^{x} ϵ^{-x}	1.6487 0.6065	1.6653 0.6005	1.6820 0.5945	1.6989 0.5886	1.7160 0.5827	1.7333 0.5769	1.7507 0.5712	1.7683 0.5655	1.7860 0.5599	1.8040 0.5543
0.6	ϵ^{x} ϵ^{-x}	1.8221 0.5488	1.8404 0.5434	1.8589 0.5379	1.8776 0.5326	1.8965 0.5273	1.9155 0.5220	1.9348 0.5169	1.9542 0.5117	1.9739 0.5066	1.9939 0.5017
0.7	ϵ^{x} ϵ^{-x}	2.0138 0.4966	2.0340 0.4916	2.0544 0.4868	2.0751 0.4819	2.0959 0.4771	2.1170 0.4724	2.1383 0.4677	2.1598 0.4630	2.1815 0.4584	2.2034 0.4538
0.8	ϵ^{x} ϵ^{-x}	2.2255 0.4493	2.2479 0.4449	2.2705 0.4404	2.2933 0.4360	2.3164 0.4317	2.3396 0.4274	2.3632 0.4232	2.3869 0.4190	2.4109 0.4148	2.4351 0.4107
0.9	ϵ^{x} ϵ^{-x}	2.4596 0.4066	2.4843 0.4025	2.5093 0.3985	2.5345 0.3946	2.5600 0.3906	2.5857 0.3867	2.6117 0.3829	2.6379 0.3791	2.6645 0.3753	2.6912 0.3716
1.0	ϵ^{x} ϵ^{-x}	2.7183 0.3679	2.7456 0.3642	2.7732 0.3606	2.8011 0.3570	2.8292 0.3535	2.8577 0.3499	2.8864 0.3465	2.9154 0.3430	2.9447 0.3396	2.9743 0.3362
1.1	ϵ^{x} ϵ^{-x}	3.0042 0.3329	3.0344 0.3296	3.0649 0.3263	3.0957 0.3230	3.1268 0.3198	3.1582 0.3166	3.1899 0.3135	3.2220 0.3104	3.2544 0.3073	3.2871 0.3042
1.2	ϵ^{x} ϵ^{-x}	3.3201 0.3012	3.3535 0.2982	3.3872 0.2952	3.4212 0.2923	3.4556 0.2894	3.4903 0.2865	3.5254 0.2837	3.5609 0.2808	3.5966 0.2780	3.6328 0.2753
1.3	ϵ^{x} ϵ^{-x}	3.6693 0.2725	3.7062 0.2698	3.7434 0.2671	3.7810 0.2645	3.8190 0.2618	3.8574 0.2592	3.8962 0.2567	3.9354 0.2541	3.9749 0.2516	4.0149 0.2491
1.4	ϵ^{x} ϵ^{-x}	4.0552 0.2466	4.0960 0.2441	4.1371 0.2417	4.1787 0.2393	4.2207 0.2369	4.2631 0.2346	4.3060 0.2322	4.3492 0.2299	4.3929 0.2276	4.4371 0.2254
1.5	ϵ^{x} ϵ^{-x}	4.4817 0.2231	4.5267 0.2209	4.5722 0.2187	4.6182 0.2165	4.6646 0.2144	4.7115 0.2122	4.7588 0.2101	4.8066 0.2080	4.8550 0.2060	4.9037 0.2039
1.6	ϵ^{x} ϵ^{-x}	4.9530 0.2019	5.0028 0.1999	5.0531 0.1979	5.1039 0.1959	5.1552 0.1940	5.2070 0.1920	5.2593 0.1901	5.3122 0.1882	5.3656 0.1864	5.4195 0.1845
1.7	ϵ^{x} ϵ^{-x}	5.4739 0.1827	5.5290 0.1809	5.5845 0.1791	5.6407 0.1773	5.6973 0.1755	5.7546 0.1738	5.8124 0.1720	5.8709 0.1703	5.9299 0.1686	5.9895 0.1670
1.8	ϵ^{x} ϵ^{-x}	6.0496 0.1653	6.1104 0.1637	6.1719 0.1620	6.2339 0.1604	6.2965 0.1588	6.3598 0.1572	6.4237 0.1557	6.4883 0.1541	6.5535 0.1526	6.6194 0.1511
1.9	ϵ^{x} ϵ^{-x}	6.6859 0.1496	6.7531 0.1481	6.8210 0.1466	6.8895 0.1451	6.9588 0.1437	7.0287 0.1423	7.0993 0.1409	7.1707 0.1395	7.2427 0.1381	7.3155 0.1367

Values of ϵ^x and ϵ^{-x}

x	Function	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.0	ϵ^x ϵ^{-x}	7.3891 0.1353	7.4633 0.1340	7.5383 0.1327	7.6141 0.1313	7.6906 0.1300	7.7679 0.1287	7.8460 0.1275	7.9248 0.1262	8.0045 0.1249	8.0849 0.1237
2.1	ϵ^x ϵ^{-x}	8.1662 0.1225	8.2482 0.1212	8.3311 0.1200	8.4149 0.1188	8.4994 0.1177	8.5849 0.1165	8.6711 0.1153	8.7583 0.1142	8.8463 0.1130	8.9352 0.1119
2.2	ϵ^x ϵ^{-x}	9.0250 0.1108	9.1157 0.1097	9.2073 0.1086	9.2999 0.1075	9.3933 0.1065	9.4877 0.1054	9.5831 0.1044	9.6794 0.1033	9.7767 0.1023	9.8749 0.1013
2.3	ϵ^x ϵ^{-x}	9.9742 0.1003	10.074 0.0993	10.176 0.0983	10.278 0.0973	10.381 0.0963	10.486 0.0954	10.591 0.0944	10.697 0.0935	10.805 0.0926	10.913 0.0916
2.4	ϵ^x ϵ^{-x}	11.023 0.0907	11.134 0.0898	11.246 0.0889	11.359 0.0880	11.473 0.0872	11.588 0.0863	11.705 0.0854	11.822 0.0846	11.941 0.0837	12.061 0.0829
2.5	ϵ^x ϵ^{-x}	12.182 0.0821	12.305 0.0813	12.429 0.0805	12.554 0.0797	12.680 0.0789	12.807 0.0781	12.936 0.0773	13.066 0.0765	13.197 0.0758	13.330 0.0750
2.6	ϵ^x ϵ^{-x}	13.464 0.0743	13.599 0.0735	13.736 0.0728	13.874 0.0721	14.013 0.0714	14.154 0.0707	14.296 0.0699	14.440 0.0693	14.585 0.0686	14.732 0.0679
2.7	ϵ^x ϵ^{-x}	14.880 0.0672	15.029 0.0665	15.180 0.0659	15.333 0.0652	15.487 0.0646	15.643 0.0639	15.800 0.0633	15.959 0.0627	16.119 0.0620	16.281 0.0614
2.8	ϵ^x ϵ^{-x}	16.445 0.0608	16.610 0.0602	16.777 0.0596	16.945 0.0590	17.116 0.0584	17.288 0.0578	17.462 0.0573	17.637 0.0567	17.814 0.0561	17.993 0.0556
2.9	ϵ^x ϵ^{-x}	18.174 0.0550	18.357 0.0545	18.541 0.0539	18.728 0.0534	18.916 0.0529	19.106 0.0523	19.298 0.0518	19.492 0.0513	19.688 0.0508	19.886 0.0503
3.0	ϵ^x ϵ^{-x}	20.086 0.0498	20.287 0.0493	20.491 0.0488	20.697 0.0483	20.905 0.0478	21.115 0.0474	21.328 0.0469	21.542 0.0464	21.758 0.0460	21.977 0.0455
3.1	ϵ^x ϵ^{-x}	22.198 0.0450	22.421 0.0446	22.646 0.0442	22.874 0.0437	23.104 0.0433	23.336 0.0429	23.571 0.0424	23.807 0.0420	24.047 0.0416	24.288 0.0412
3.2	ϵ^x ϵ^{-x}	24.533 0.0408	24.779 0.0404	25.028 0.0400	25.280 0.0396	25.534 0.0392	25.790 0.0388	26.050 0.0384	26.311 0.0380	26.576 0.0376	26.843 0.0373
3.3	ϵ^x ϵ^{-x}	27.113 0.0369	27.385 0.0365	27.660 0.0362	27.938 0.0358	28.219 0.0354	28.503 0.0351	28.789 0.0347	29.079 0.0344	29.371 0.0340	29.666 0.0337
3.4	ϵ^x ϵ^{-x}	29.964 0.0334	30.265 0.0330	30.569 0.0327	30.877 0.0324	31.187 0.0321	31.500 0.0317	31.817 0.0314	32.137 0.0311	32.460 0.0308	32.786 0.0305
3.5	ϵ^x ϵ^{-x}	33.115 0.0302	33.448 0.0299	33.784 0.0296	34.124 0.0293	34.467 0.0290	34.813 0.0287	35.163 0.0284	35.517 0.0282	35.874 0.0279	36.234 0.0276
3.6	ϵ^x ϵ^{-x}	36.598 0.0273	36.966 0.0271	37.338 0.0268	37.713 0.0265	38.092 0.0263	38.475 0.0260	38.861 0.0257	39.252 0.0255	39.646 0.0252	40.045 0.0250
3.7	ϵ^x ϵ^{-x}	40.447 0.0247	40.854 0.0245	41.264 0.0242	41.679 0.0240	42.098 0.0238	42.521 0.0235	42.948 0.0233	43.380 0.0231	43.816 0.0228	44.256 0.0226
3.8	ϵ^x ϵ^{-x}	44.701 0.0224	45.150 0.0221	45.604 0.0219	46.063 0.0217	46.525 0.0215	46.993 0.0213	47.465 0.0211	47.942 0.0209	48.424 0.0207	48.911 0.0204
3.9	ϵ^x ϵ^{-x}	49.402 0.0202	49.899 0.0200	50.400 0.0198	50.907 0.0196	51.419 0.0195	51.935 0.0193	52.457 0.0191	52.985 0.0189	53.517 0.0187	54.055 0.0185

Values of ϵ^x and ϵ^{-x}

4.00–5.99

x	Function	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
4.0	ϵ^x	54.598	55.147	55.701	56.261	56.826	57.397	57.974	58.557	59.145	59.740
	ϵ^{-x}	0.0183	0.0181	0.0180	0.0178	0.0176	0.0174	0.0172	0.0171	0.0169	0.0167
4.1	ϵ^x	60.340	60.947	61.559	62.178	62.803	63.434	64.072	64.715	65.366	66.023
	ϵ^{-x}	0.0166	0.0164	0.0162	0.0161	0.0159	0.0158	0.0156	0.0155	0.0153	0.0151
4.2	ϵ^x	66.686	67.357	68.033	68.717	69.408	70.105	70.810	71.522	72.240	72.966
	ϵ^{-x}	0.0150	0.0148	0.0147	0.0146	0.0144	0.0143	0.0141	0.0140	0.0138	0.0137
4.3	ϵ^x	73.700	74.440	75.189	75.944	76.708	77.478	78.257	79.044	79.838	80.640
	ϵ^{-x}	0.0136	0.0134	0.0133	0.0132	0.0130	0.0129	0.0128	0.0127	0.0125	0.0124
4.4	ϵ^x	81.451	82.269	83.096	83.931	84.775	85.627	86.488	87.357	88.235	89.121
	ϵ^{-x}	0.0123	0.0122	0.0120	0.0119	0.0118	0.0117	0.0116	0.0114	0.0113	0.0112
4.5	ϵ^x	90.017	90.922	91.836	92.759	93.691	94.632	95.583	96.544	97.514	98.494
	ϵ^{-x}	0.0111	0.0110	0.0109	0.0108	0.0107	0.0106	0.0105	0.0104	0.0103	0.0102
4.6	ϵ^x	99.484	100.48	101.49	102.51	103.54	104.58	105.64	106.70	107.77	108.85
	ϵ^{-x}	0.0101	0.0100	0.0099	0.0098	0.0097	0.0096	0.0095	0.0094	0.0093	0.0092
4.7	ϵ^x	109.95	111.05	112.17	113.30	114.43	115.58	116.75	117.92	119.10	120.30
	ϵ^{-x}	0.0091	0.0090	0.0089	0.0088	0.0087	0.0087	0.0086	0.0085	0.0084	0.0083
4.8	ϵ^x	121.51	122.73	123.97	125.21	126.47	127.74	129.02	130.32	131.63	132.95
	ϵ^{-x}	0.0082	0.0081	0.0081	0.0080	0.0079	0.0078	0.0078	0.0077	0.0076	0.0075
4.9	ϵ^x	134.29	135.64	137.00	138.38	139.77	141.17	142.59	144.03	145.47	146.94
	ϵ^{-x}	0.0074	0.0074	0.0073	0.0072	0.0072	0.0071	0.0070	0.0069	0.0069	0.0068
5.0	ϵ^x	148.41	149.90	151.41	152.93	154.47	156.02	157.59	159.17	160.77	162.39
	ϵ^{-x}	0.0067	0.0067	0.0066	0.0065	0.0065	0.0064	0.0063	0.0063	0.0062	0.0062
5.1	ϵ	164.02	165.67	167.34	169.02	170.72	172.43	174.16	175.91	177.68	179.47
	ϵ^{-x}	0.0061	0.0060	0.0060	0.0059	0.0059	0.0058	0.0057	0.0057	0.0056	0.0056
5.2	ϵ^x	181.27	183.09	184.93	186.79	188.67	190.57	192.48	194.42	196.37	198.34
	ϵ^{-x}	0.0055	0.0055	0.0054	0.0054	0.0053	0.0052	0.0052	0.0051	0.0051	0.0050
5.3	ϵ^x	200.34	202.35	204.38	206.44	208.51	210.61	212.72	214.86	217.02	219.20
	ϵ^{-x}	0.0050	0.0049	0.0049	0.0048	0.0048	0.0047	0.0047	0.0047	0.0046	0.0046
5.4	ϵ^x	221.41	223.63	225.88	228.15	230.44	232.76	235.10	237.46	239.85	242.26
	ϵ^{-x}	0.0045	0.0045	0.0044	0.0044	0.0043	0.0043	0.0043	0.0042	0.0042	0.0041
5.5	ϵ^x	244.69	247.15	249.64	252.14	254.68	257.24	259.82	262.43	265.07	267.74
	ϵ^{-x}	0.0041	0.0040	0.0040	0.0040	0.0039	0.0039	0.0038	0.0038	0.0038	0.0037
5.6	ϵ^x	270.43	273.14	275.89	278.66	281.46	284.29	287.15	290.03	292.95	295.89
	ϵ^{-x}	0.0037	0.0037	0.0036	0.0036	0.0036	0.0035	0.0035	0.0034	0.0034	0.0034
5.7	ϵ^x	298.87	301.87	304.90	307.97	311.06	314.19	317.35	320.54	323.76	327.01
	ϵ^{-x}	0.0033	0.0033	0.0033	0.0032	0.0032	0.0032	0.0032	0.0031	0.0031	0.0031
5.8	ϵ^x	330.30	333.62	336.97	340.36	343.78	347.23	350.72	354.25	357.81	361.41
	ϵ^{-x}	0.0030	0.0030	0.0030	0.0029	0.0029	0.0029	0.0029	0.0028	0.0028	0.0028
5.9	ϵ^x	365.04	368.71	372.41	376.15	379.93	383.75	387.61	391.51	395.44	399.41
	ϵ^{-x}	0.0027	0.0027	0.0027	0.0027	0.0026	0.0026	0.0026	0.0026	0.0025	0.0025

Natural Sines, Cosines and Tangents

0°-14.9°

Degs.	Function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
0	sin	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157
	cos	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999
	tan	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157
1	sin	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332
	cos	0.9998	0.9998	0.9998	0.9997	0.9997	0.9997	0.9996	0.9996	0.9995	0.9995
	tan	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332
2	sin	0.0349	0.0366	0.0384	0.0401	0.0419	0.0436	0.0454	0.0471	0.0488	0.0506
	cos	0.9994	0.9993	0.9993	0.9992	0.9991	0.9990	0.9990	0.9989	0.9988	0.9987
	tan	0.0349	0.0367	0.0384	0.0402	0.0419	0.0437	0.0454	0.0472	0.0489	0.0507
3	sin	0.0523	0.0541	0.0558	0.0576	0.0593	0.0610	0.0628	0.0645	0.0663	0.0680
	cos	0.9986	0.9985	0.9984	0.9983	0.9982	0.9981	0.9980	0.9979	0.9978	0.9977
	tan	0.0524	0.0542	0.0559	0.0577	0.0594	0.0612	0.0629	0.0647	0.0664	0.0682
4	sin	0.0698	0.0715	0.0732	0.0750	0.0767	0.0785	0.0802	0.0819	0.0837	0.0854
	cos	0.9976	0.9974	0.9973	0.9972	0.9971	0.9969	0.9968	0.9966	0.9965	0.9963
	tan	0.0699	0.0717	0.0734	0.0752	0.0769	0.0787	0.0805	0.0822	0.0840	0.0857
5	sin	0.0872	0.0889	0.0906	0.0924	0.0941	0.0958	0.0976	0.0993	0.1011	0.1028
	cos	0.9962	0.9960	0.9959	0.9957	0.9956	0.9954	0.9952	0.9951	0.9949	0.9947
	tan	0.0875	0.0892	0.0910	0.0928	0.0945	0.0963	0.0981	0.0998	0.1016	0.1033
6	sin	0.1045	0.1063	0.1080	0.1097	0.1115	0.1132	0.1149	0.1167	0.1184	0.1201
	cos	0.9945	0.9943	0.9942	0.9940	0.9938	0.9936	0.9934	0.9932	0.9930	0.9928
	tan	0.1051	0.1069	0.1086	0.1104	0.1122	0.1139	0.1157	0.1175	0.1192	0.1210
7	sin	0.1219	0.1236	0.1253	0.1271	0.1288	0.1305	0.1323	0.1340	0.1357	0.1374
	cos	0.9925	0.9923	0.9921	0.9919	0.9917	0.9914	0.9912	0.9910	0.9907	0.9905
	tan	0.1228	0.1246	0.1263	0.1281	0.1299	0.1317	0.1334	0.1352	0.1370	0.1388
8	sin	0.1392	0.1409	0.1426	0.1444	0.1461	0.1478	0.1495	0.1513	0.1530	0.1547
	cos	0.9903	0.9900	0.9898	0.9895	0.9893	0.9890	0.9888	0.9885	0.9882	0.9880
	tan	0.1405	0.1423	0.1441	0.1459	0.1477	0.1495	0.1512	0.1530	0.1548	0.1566
9	sin	0.1564	0.1582	0.1599	0.1616	0.1633	0.1650	0.1668	0.1685	0.1702	0.1719
	cos	0.9877	0.9874	0.9871	0.9869	0.9866	0.9863	0.9860	0.9857	0.9854	0.9851
	tan	0.1584	0.1602	0.1620	0.1638	0.1655	0.1673	0.1691	0.1709	0.1727	0.1745
10	sin	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822	0.1840	0.1857	0.1874	0.1891
	cos	0.9848	0.9845	0.9842	0.9839	0.9836	0.9833	0.9829	0.9826	0.9823	0.9820
	tan	0.1763	0.1781	0.1799	0.1817	0.1835	0.1853	0.1871	0.1890	0.1908	0.1926
11	sin	0.1908	0.1925	0.1942	0.1959	0.1977	0.1994	0.2011	0.2028	0.2045	0.2062
	cos	0.9816	0.9813	0.9810	0.9806	0.9803	0.9799	0.9796	0.9792	0.9789	0.9785
	tan	0.1944	0.1962	0.1980	0.1998	0.2016	0.2035	0.2053	0.2071	0.2089	0.2107
12	sin	0.2079	0.2096	0.2113	0.2130	0.2147	0.2164	0.2181	0.2198	0.2215	0.2232
	cos	0.9781	0.9778	0.9774	0.9770	0.9767	0.9763	0.9759	0.9755	0.9751	0.9748
	tan	0.2126	0.2144	0.2162	0.2180	0.2199	0.2217	0.2235	0.2254	0.2272	0.2290
13	sin	0.2250	0.2267	0.2284	0.2300	0.2318	0.2334	0.2351	0.2368	0.2385	0.2402
	cos	0.9744	0.9740	0.9736	0.9732	0.9728	0.9724	0.9720	0.9715	0.9711	0.9707
	tan	0.2309	0.2327	0.2345	0.2364	0.2382	0.2401	0.2419	0.2438	0.2456	0.2475
14	sin	0.2419	0.2436	0.2453	0.2470	0.2487	0.2504	0.2521	0.2538	0.2554	0.2571
	cos	0.9703	0.9699	0.9694	0.9690	0.9686	0.9681	0.9677	0.9673	0.9668	0.9664
	tan	0.2493	0.2512	0.2530	0.2549	0.2568	0.2586	0.2605	0.2623	0.2642	0.2661
Degs.	Function	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'

Natural Sines, Cosines and Tangents

15°–29.9°

Degs.	Function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
15	sin	0.2588	0.2605	0.2622	0.2639	0.2656	0.2672	0.2689	0.2706	0.2723	0.2740
	cos	0 9659	0.9655	0.9650	0.9646	0.9641	0.9636	0.9632	0.9627	0.9622	0.9617
	tan	0.2679	0.2698	0.2717	0.2736	0.2754	0.2773	0.2792	0.2811	0.2830	0.2849
16	sin	0.2756	0.2773	0.2790	0.2807	0.2823	0.2840	0.2857	0.2874	0.2890	0.2907
	cos	0.9613	0.9608	0.9603	0.9598	0.9593	0.9588	0.9583	0.9578	0.9573	0.9568
	tan	0.2867	0.2886	0.2905	0.2924	0.2943	0.2962	0.2981	0.3000	0.3019	0.3038
17	sin	0.2924	0 2940	0.2957	0.2974	0.2990	0.3007	0.3024	0.3040	0.3057	0.3074
	cos	0.9563	0.9558	0.9553	0.9548	0.9542	0.9537	0.9532	0.9527	0.9521	0.9516
	tan	0.3057	0.3076	0.3096	0.3115	0.3134	0.3153	0.3172	0.3191	0.3211	0.3230
18	sin	0.3090	0.3107	0.3123	0.3140	0.3156	0.3173	0.3190	0.3206	0.3223	0.3239
	cos	0.9511	0.9505	0.9500	0.9494	0.9489	0.9483	0.9478	0.9472	0.9466	0.9461
	tan	0.3249	0.3269	0.3288	0 3307	0.3327	0.3346	0.3365	0.3385	0.3404	0.3424
19	sin	0.3256	0.3272	0.3289	0.3305	0.3322	0.3338	0.3355	0.3371	0.3387	0.3404
	cos	0.9455	0.9449	0.9444	0.9438	0.9432	0.9426	0.9421	0.9415	0.9409	0.9403
	tan	0.3443	0.3463	0.3482	0.3502	0.3522	0.3541	0.3561	0.3581	0.3600	0.3620
20	sin	0.3420	0.3437	0.3453	0.3469	0.3486	0.3502	0.3518	0.3535	0.3551	0.3567
	cos	0.9397	0.9391	0.9385	0.9379	0.9373	0.9367	0.9361	0.9354	0.9348	0.9342
	tan	0.3640	0.3659	0.3679	0.3699	0.3719	0.3739	0.3759	0.3779	0.3799	0.3819
21	sin	0.3584	0.3600	0.3616	0.3633	0.3649	0.3665	0.3681	0.3697	0.3714	0.3730
	cos	0.9336	0.9330	0.9323	0.9317	0.9311	0.9304	0.9298	0.9291	0.9285	0.9278
	tan	0.3839	0.3859	0.3879	0.3899	0.3919	0.3939	0.3959	0.3979	0.4000	0.4020
22	sin	0.3746	0.3762	0.3778	0.3795	0.3811	0.3827	0.3843	0.3859	0.3875	0.3891
	cos	0.9272	0.9265	0.9259	0.9252	0.9245	0.9239	0.9232	0.9225	0.9219	0.9212
	tan	0.4040	0.4061	0.4081	0.4101	0.4122	0.4142	0.4163	0.4183	0.4204	0.4224
23	sin	0.3907	0.3923	0.3939	0.3955	0.3971	0.3987	0.4003	0.4019	0.4035	0.4051
	cos	0.9205	0.9198	0.9191	0.9184	0.9178	0.9171	0.9164	0.9157	0.9150	0.9143
	tan	0.4245	0.4265	0.4286	0.4307	0.4327	0.4348	0.4369	0.4390	0.4411	0.4431
24	sin	0.4067	0.4083	0.4099	0.4115	0.4131	0.4147	0.4163	0.4179	0.4195	0.4210
	cos	0.9135	0.9128	0.9121	0.9114	0.9107	0.9100	0.9092	0.9085	0.9078	0 9070
	tan	0.4452	0.4473	0.4494	0.4515	0.4536	0.4557	0.4578	0.4599	0.4621	0.4642
25	sin	0.4226	0.4242	0.4258	0.4274	0.4289	0.4305	0.4321	0.4337	0.4352	0.4368
	cos	0.9063	0.9056	0.9048	0.9041	0.9033	0.9026	0.9018	0.9011	0.9003	0.8996
	tan	0.4663	0.4684	0.4706	0.4727	0.4748	0.4770	0.4791	0.4813	0.4834	0.4856
26	sin	0.4384	0.4399	0.4415	0.4431	0.4446	0.4462	0.4478	0.4493	0.4509	0.4524
	cos	0.8988	0.8980	0.8973	0.8965	0.8957	0.8949	0.8942	0.8934	0.8926	0.8918
	tan	0.4877	0.4899	0.4921	0.4942	0.4964	0.4986	0.5008	0.5029	0.5051	0.5073
27	sin	0.4540	0.4555	0.4571	0.4586	0.4602	0.4617	0.4633	0.4648	0.4664	0.4679
	cos	0.8910	0.8902	0.8894	0.8886	0.8878	0.8870	0.8862	0.8854	0.8846	0.8838
	tan	0.5095	0.5117	0.5139	0.5161	0.5184	0.5206	0.5228	0.5250	0.5272	0.5295
28	sin	0.4695	0.4710	0.4726	0.4741	0.4756	0.4772	0.4787	0.4802	0.4818	0.4833
	cos	0.8829	0.8821	0.8813	0.8805	0.8796	0.8788	0.8780	0.8771	0.8763	0.8755
	tan	0.5317	0.5340	0.5362	0.5384	0.5407	0.5430	0.5452	0.5475	0.5498	0.5520
29	sin	0.4848	0.4863	0.4879	0.4894	0.4909	0.4924	0.4939	0.4955	0.4970	0.4985
	cos	0.8746	0.8738	0.8729	0.8721	0.8712	0.8704	0.8695	0.8686	0 8678	0.8669
	tan	0.5543	0.5566	0.5589	0.5612	0.5635	0.5658	0.5681	0.5704	0.5727	0.5750
Degs.	**Function**	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′

Natural Sines, Cosines and Tangents

Degs.	Function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
30	sin	0.5000	0.5015	0.5030	0.5045	0.5060	0.5075	0.5090	0.5105	0.5120	0.5135
	cos	0.8660	0.8652	0.8643	0.8634	0.8625	0.8616	0.8607	0.8599	0.8590	0.8581
	tan	0.5774	0.5797	0.5820	0.5844	0.5867	0.5890	0.5914	0.5938	0.5961	0.5985
31	sin	0.5150	0.5165	0.5180	0.5195	0.5210	0.5225	0.5240	0.5255	0.5270	0.5284
	cos	0.8572	0.8563	0.8554	0.8545	0.8536	0.8526	0.8517	0.8508	0.8499	0.8490
	tan	0.6009	0.6032	0.6056	0.6080	0.6104	0.6128	0.6152	0.6176	0.6200	0.6224
32	sin	0.5299	0.5314	0.5329	0.5344	0.5358	0.5373	0.5388	0.5402	0.5417	0.5432
	cos	0.8480	0.8471	0.8462	0.8453	0.8443	0.8434	0.8425	0.8415	0.8406	0.8396
	tan	0.6249	0.6273	0.6297	0.6322	0.6346	0.6371	0.6395	0.6420	0.6445	0.6469
33	sin	0.5446	0.5461	0.5476	0.5490	0.5505	0.5519	0.5534	0.5548	0.5563	0.5577
	cos	0.8387	0.8377	0.8368	0.8358	0.8348	0.8339	0.8329	0.8320	0.8310	0.8300
	tan	0.6494	0.6519	0.6544	0.6569	0.6594	0.6619	0.6644	0.6669	0.6694	0.6720
34	sin	0.5592	0.5606	0.5621	0.5635	0.5650	0.5664	0.5678	0.5693	0.5707	0.5721
	cos	0.8290	0.8281	0.8271	0.8261	0.8251	0.8241	0.8231	0.8221	0.8211	0.8202
	tan	0.6745	0.6771	0.6796	0.6822	0.6847	0.6873	0.6899	0.6924	0.6950	0.6976
35	sin	0.5736	0.5750	0.5764	0.5779	0.5793	0.5807	0.5821	0.5835	0.5850	0.5864
	cos	0.8192	0.8181	0.8171	0.8161	0.8151	0.8141	0.8131	0.8121	0.8111	0.8100
	tan	0.7002	0.7028	0.7054	0.7080	0.7107	0.7133	0.7159	0.7186	0.7212	0.7239
36	sin	0.5878	0.5892	0.5906	0.5920	0.5934	0.5948	0.5962	0.5976	0.5990	0.6004
	cos	0.8090	0.8080	0.8070	0.8059	0.8049	0.8039	0.8028	0.8018	0.8007	0.7997
	tan	0.7265	0.7292	0.7319	0.7346	0.7373	0.7400	0.7427	0.7454	0.7481	0.7508
37	sin	0.6018	0.6032	0.6046	0.6060	0.6074	0.6088	0.6101	0.6115	0.6129	0.6143
	cos	0.7986	0.7976	0.7965	0.7955	0.7944	0.7934	0.7923	0.7912	0.7902	0.7891
	tan	0.7536	0.7563	0.7590	0.7618	0.7646	0.7673	0.7701	0.7729	0.7757	0.7785
38	sin	0.6157	0.6170	0.6184	0.6198	0.6211	0.6225	0.6239	0.6252	0.6266	0.6280
	cos	0.7880	0.7869	0.7859	0.7848	0.7837	0.7826	0.7815	0.7804	0.7793	0.7782
	tan	0.7813	0.7841	0.7869	0.7898	0.7926	0.7954	0.7983	0.8012	0.8040	0.8069
39	sin	0.6293	0.6307	0.6320	0.6334	0.6347	0.6361	0.6374	0.6388	0.6401	0.6414
	cos	0.7771	0.7760	0.7749	0.7738	0.7727	0.7716	0.7705	0.7694	0.7683	0.7672
	tan	0.8098	0.8127	0.8156	0.8185	0.8214	0.8243	0.8273	0.8302	0.8332	0.8361
40	sin	0.6428	0.6441	0.6455	0.6468	0.6481	0.6494	0.6508	0.6521	0.6534	0.6547
	cos	0.7660	0.7649	0.7638	0.7627	0.7615	0.7604	0.7593	0.7581	0.7570	0.7559
	tan	0.8391	0.8421	0.8451	0.8481	0.8511	0.8541	0.8571	0.8601	0.8632	0.8662
41	sin	0.6561	0.6574	0.6587	0.6600	0.6613	0.6626	0.6639	0.6652	0.6665	0.6678
	cos	0.7547	0.7536	0.7524	0.7513	0.7501	0.7490	0.7478	0.7466	0.7455	0.7443
	tan	0.8693	0.8724	0.8754	0.8785	0.8816	0.8847	0.8878	0.8910	0.8941	0.8972
42	sin	0.6691	0.6704	0.6717	0.6730	0.6743	0.6756	0.6769	0.6782	0.6794	0.6807
	cos	0.7431	0.7420	0.7408	0.7396	0.7385	0.7373	0.7361	0.7349	0.7337	0.7325
	tan	0.9004	0.9036	0.9067	0.9099	0.9131	0.9163	0.9195	0.9228	0.9260	0.9293
43	sin	0.6820	0.6833	0.6845	0.6858	0.6871	0.6884	0.6896	0.6909	0.6921	0.6934
	cos	0.7314	0.7302	0.7290	0.7278	0.7266	0.7254	0.7242	0.7230	0.7218	0.7206
	tan	0.9325	0.9358	0.9391	0.9424	0.9457	0.9490	0.9523	0.9556	0.9590	0.9623
44	sin	0.6947	0.6959	0.6972	0.6984	0.6997	0.7009	0.7022	0.7034	0.7046	0.7059
	cos	0.7193	0.7181	0.7169	0.7157	0.7145	0.7133	0.7120	0.7108	0.7096	0.7083
	tan	0.9657	0.9691	0.9725	0.9759	0.9793	0.9827	0.9861	0.9896	0.9930	0.9965
Degs.	Function	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′

Natural Sines, Cosines and Tangents

45°-59.9°

Degs.	Function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
45	sin	0.7071	0.7083	0.7096	0.7108	0.7120	0.7133	0.7145	0.7157	0.7169	0.7181
	cos	0.7071	0.7059	0.7046	0.7034	0.7022	0.7009	0.6997	0.6984	0.6972	0.6959
	tan	1.0000	1.0035	1.0070	1.0105	1.0141	1.0176	1.0212	1.0247	1.0283	1.0319
46	sin	0.7193	0.7206	0.7218	0.7230	0.7242	0.7254	0.7266	0.7278	0.7290	0.7302
	cos	0.6947	0.6934	0.6921	0.6909	0.6896	0.6884	0.6871	0.6858	0.6845	0.6833
	tan	1.0355	1.0392	1.0428	1.0464	1.0501	1.0538	1.0575	1.0612	1.0649	1.0686
47	sin	0.7314	0.7325	0.7337	0.7349	0.7361	0.7373	0.7385	0.7396	0.7408	0.7420
	cos	0.6820	0.6807	0.6794	0.6782	0.6769	0.6756	0.6743	0.6730	0.6717	0.6704
	tan	1.0724	1.0761	1.0799	1.0837	1.0875	1.0913	1.0951	1.0990	1.1028	1.1067
48	sin	0.7431	0.7443	0.7455	0.7466	0.7478	0.7490	0.7501	0.7513	0.7524	0.7536
	cos	0.6691	0.6678	0.6665	0.6652	0.6639	0.6626	0.6613	0.6600	0.6587	0.6574
	tan	1.1106	1.1145	1.1184	1.1224	1.1263	1.1303	1.1343	1.1383	1.1423	1.1463
49	sin	0.7547	0.7559	0.7570	0.7581	0.7593	0.7604	0.7615	0.7627	0.7638	0.7649
	cos	0.6561	0.6547	0.6534	0.6521	0.6508	0.6494	0.6481	0.6468	0.6455	0.6441
	tan	1.1504	1.1544	1.1585	1.1626	1.1667	1.1708	1.1750	1.1792	1.1833	1.1875
50	sin	0.7660	0.7672	0.7683	0.7694	0.7705	0.7716	0.7727	0.7738	0.7749	0.7760
	cos	0.6428	0.6414	0.6401	0.6388	0.6374	0.6361	0.6347	0.6334	0.6320	0.6307
	tan	1.1918	1.1960	1.2002	1.2045	1.2088	1.2131	1.2174	1.2218	1.2261	1.2305
51	sin	0.7771	0.7782	0.7793	0.7804	0.7815	0.7826	0.7837	0.7848	0.7859	0.7869
	cos	0.6293	0.6280	0.6266	0.6252	0.6239	0.6225	0.6211	0.6198	0.6184	0.6170
	tan	1.2349	1.2393	1.2437	1.2482	1.2527	1.2572	1.2617	1.2662	1.2708	1.2753
52	sin	0.7880	0.7891	0.7902	0.7912	0.7923	0.7934	0.7944	0.7955	0.7965	0.7976
	cos	0.6157	0.6143	0.6129	0.6115	0.6101	0.6088	0.6074	0.6060	0.6046	0.6032
	tan	1.2799	1.2846	1.2892	1.2938	1.2985	1.3032	1.3079	1.3127	1.3175	1.3222
53	sin	0.7986	0.7997	0.8007	0.8018	0.8028	0.8039	0.8049	0.8059	0.8070	0.8080
	cos	0.6018	0.6004	0.5990	0.5976	0.5962	0.5948	0.5934	0.5920	0.5906	0.5892
	tan	1.3270	1.3319	1.3367	1.3416	1.3465	1.3514	1.3564	1.3613	1.3663	1.3713
54	sin	0.8090	0.8100	0.8111	0.8121	0.8131	0.8141	0.8151	0.8161	0.8171	0.8181
	cos	0.5878	0.5864	0.5850	0.5835	0.5821	0.5807	0.5793	0.5779	0.5764	0.5750
	tan	1.3764	1.3814	1.3865	1.3916	1.3968	1.4019	1.4071	1.4124	1.4176	1.4229
55	sin	0.8192	0.8202	0.8211	0.8221	0.8231	0.8241	0.8251	0.8261	0.8271	0.8281
	cos	0.5736	0.5721	0.5707	0.5693	0.5678	0.5664	0.5650	0.5635	0.5621	0.5606
	tan	1.4281	1.4335	1.4388	1.4442	1.4496	1.4550	1.4605	1.4659	1.4715	1.4770
56	sin	0.8290	0.8300	0.8310	0.8320	0.8329	0.8339	0.8348	0.8358	0.8368	0.8377
	cos	0.5592	0.5577	0.5563	0.5548	0.5534	0.5519	0.5505	0.5490	0.5476	0.5461
	tan	1.4826	1.4882	1.4938	1.4994	1.5051	1.5108	1.5166	1.5224	1.5282	1.5340
57	sin	0.8387	0.8396	0.8406	0.8415	0.8425	0.8434	0.8443	0.8453	0.8462	0.8471
	cos	0.5446	0.5432	0.5417	0.5402	0.5388	0.5373	0.5358	0.5344	0.5329	0.5314
	tan	1.5399	1.5458	1.5517	1.5577	1.5637	1.5697	1.5757	1.5818	1.5880	1.5941
58	sin	0.8480	0.8490	0.8499	0.8508	0.8517	0.8526	0.8536	0.8545	0.8554	0.8563
	cos	0.5299	0.5284	0.5270	0.5255	0.5240	0.5225	0.5210	0.5195	0.5180	0.5165
	tan	1.6003	1.6066	1.6128	1.6191	1.6255	1.6319	1.6383	1.6447	1.6512	1.6577
59	sin	0.8572	0.8581	0.8590	0.8599	0.8607	0.8616	0.8625	0.8634	0.8643	0.8652
	cos	0.5150	0.5135	0.5120	0.5105	0.5090	0.5075	0.5060	0.5045	0.5030	0.5015
	tan	1.6643	1.6709	1.6775	1.6842	1.6909	1.6977	1.7045	1.7113	1.7182	1.7251
Degs.	Function	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′

Natural Sines, Cosines and Tangents

60°-74.9°

Degs.	Function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
60	sin	0.8660	0.8669	0.8678	0.8686	0.8695	0.8704	0.8712	0.8721	0.8729	0.8738
	cos	0.5000	0.4985	0.4970	0.4955	0.4939	0.4924	0.4909	0.4894	0.4879	0.4863
	tan	1.7321	1.7391	1.7461	1.7532	1.7603	1.7675	1.7747	1.7820	1.7893	1.7966
61	sin	0.8746	0.8755	0.8763	0.8771	0.8780	0.8788	0.8796	0.8805	0.8813	0.8821
	cos	0.4848	0.4833	0.4818	0.4802	0.4787	0.4772	0.4756	0.4741	0.4726	0.4710
	tan	1.8040	1.8115	1.8190	1.8265	1.8341	1.8418	1.8495	1.8572	1.8650	1.8728
62	sin	0.8829	0.8838	0.8846	0.8854	0.8862	0.8870	0.8878	0.8886	0.8894	0.8902
	cos	0.4695	0.4679	0.4664	0.4648	0.4633	0.4617	0.4602	0.4586	0.4571	0.4555
	tan	1.8807	1.8887	1.8967	1.9047	1.9128	1.9210	1.9292	1.9375	1.9458	1.9542
63	sin	0.8910	0.8918	0.8926	0.8934	0.8942	0.8949	0.8957	0.8965	0.8973	0.8980
	cos	0.4540	0.4524	0.4509	0.4493	0.4478	0.4462	0.4446	0.4431	0.4415	0.4399
	tan	1.9626	1.9711	1.9797	1.9883	1.9970	2.0057	2.0145	2.0233	2.0323	2.0413
64	sin	0.8988	0.8996	0.9003	0.9011	0.9018	0.9026	0.9033	0.9041	0.9048	0.9056
	cos	0.4384	0.4368	0.4352	0.4337	0.4321	0.4305	0.4289	0.4274	0.4258	0.4242
	tan	2.0503	2.0594	2.0686	2.0778	2.0872	2.0965	2.1060	2.1155	2.1251	2.1348
65	sin	0.9063	0.9070	0.9078	0.9085	0.9092	0.9100	0.9107	0.9114	0.9121	0.9128
	cos	0.4226	0.4210	0.4195	0.4179	0.4163	0.4147	0.4131	0.4115	0.4099	0.4083
	tan	2.1445	2.1543	2.1642	2.1742	2.1842	2.1943	2.2045	2.2148	2.2251	2.2355
66	sin	0.9135	0.9143	0.9150	0.9157	0.9164	0.9171	0.9178	0.9184	0.9191	0.9198
	cos	0.4067	0.4051	0.4035	0.4019	0.4003	0.3987	0.3971	0.3955	0.3939	0.3923
	tan	2.2460	2.2566	2.2673	2.2781	2.2889	2.2998	2.3109	2.3220	2.3332	2.3445
67	sin	0.9205	0.9212	0.9219	0.9225	0.9232	0.9239	0.9245	0.9252	0.9259	0.9265
	cos	0.3907	0.3891	0.3875	0.3859	0.3843	0.3827	0.3811	0.3795	0.3778	0.3762
	tan	2.3559	2.3673	2.3789	2.3906	2.4023	2.4142	2.4262	2.4383	2.4504	2.4627
68	sin	0.9272	0.9278	0.9285	0.9291	0.9298	0.9304	0.9311	0.9317	0.9323	0.9330
	cos	0.3746	0.3730	0.3714	0.3697	0.3681	0.3665	0.3649	0.3633	0.3616	0.3600
	tan	2.4751	2.4876	2.5002	2.5129	2.5257	2.5386	2.5517	2.5649	2.5782	2.5916
69	sin	0.9336	0.9342	0.9348	0.9354	0.9361	0.9367	0.9373	0.9379	0.9385	0.9391
	cos	0.3584	0.3567	0.3551	0.3535	0.3518	0.3502	0.3486	0.3469	0.3453	0.3437
	tan	2.6051	2.6187	2.6325	2.6464	2.6605	2.6746	2.6889	2.7034	2.7179	2.7326
70	sin	0.9397	0.9403	0.9409	0.9415	0.9421	0.9426	0.9432	0.9438	0.9444	0.9449
	cos	0.3420	0.3404	0.3387	0.3371	0.3355	0.3338	0.3322	0.3305	0.3289	0.3272
	tan	2.7475	2.7625	2.7776	2.7929	2.8083	2.8239	2.8397	2.8556	2.8716	2.8878
71	sin	0.9455	0.9461	0.9466	0.9472	0.9478	0.9483	0.9489	0.9494	0.9500	0.9505
	cos	0.3256	0.3239	0.3223	0.3206	0.3190	0.3173	0.3156	0.3140	0.3123	0.3107
	tan	2.9042	2.9208	2.9375	2.9544	2.9714	2.9887	3.0061	3.0237	3.0415	3.0595
72	sin	0.9511	0.9516	0.9521	0.9527	0.9532	0.9537	0.9542	0.9548	0.9553	0.9558
	cos	0.3090	0.3074	0.3057	0.3040	0.3024	0.3007	0.2990	0.2974	0.2957	0.2940
	tan	3.0777	3.0961	3.1146	3.1334	3.1524	3.1716	3.1910	3.2106	3.2305	3.2506
73	sin	0.9563	0.9568	0.9573	0.9578	0.9583	0.9588	0.9593	0.9598	0.9603	0.9608
	cos	0.2924	0.2907	0.2890	0.2874	0.2857	0.2840	0.2823	0.2807	0.2790	0.2773
	tan	3.2709	3.2914	3.3122	3.3332	3.3544	3.3759	3.3977	3.4197	3.4420	3.4646
74	sin	0.9613	0.9617	0.9622	0.9627	0.9632	0.9636	0.9641	0.9646	0.9650	0.9655
	cos	0.2756	0.2740	0.2723	0.2706	0.2689	0.2672	0.2656	0.2639	0.2622	0.2605
	tan	3.4874	3.5105	3.5339	3.5576	3.5816	3.6059	3.6305	3.6554	3.6806	3.7062
Degs.	Function	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′

Natural Sines, Cosines and Tangents

75°–89.9°

Degs.	Function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
75	sin	0.9659	0.9664	0.9668	0.9673	0.9677	0.9681	0.9686	0.9690	0.9694	0.9699
	cos	0.2588	0.2571	0.2554	0.2538	0.2521	0.2504	0.2487	0.2470	0.2453	0.2436
	tan	3.7321	3.7583	3.7848	3.8118	3.8391	3.8667	3.8947	3.9232	3.9520	3.9812
76	sin	0.9703	0.9707	0.9711	0.9715	0.9720	0.9724	0.9728	0.9732	0.9736	0.9740
	cos	0.2419	0.2402	0.2385	0.2368	0.2351	0.2334	0.2317	0.2300	0.2284	0.2267
	tan	4.0108	4.0408	4.0713	4.1022	4.1335	4.1653	4.1976	4.2303	4.2635	4.2972
77	sin	0.9744	0.9748	0.9751	0.9755	0.9759	0.9763	0.9767	0.9770	0.9774	0.9778
	cos	0.2250	0.2232	0.2215	0.2198	0.2181	0.2164	0.2147	0.2130	0.2113	0.2096
	tan	4.3315	4.3662	4.4015	4.4374	4.4737	4.5107	4.5483	4.5864	4.6252	4.6646
78	sin	0.9781	0.9785	0.9789	0.9792	0.9796	0.9799	0.9803	0.9806	0.9810	0.9813
	cos	0.2079	0.2062	0.2045	0.2028	0.2011	0.1994	0.1977	0.1959	0.1942	0.1925
	tan	4.7046	4.7453	4.7867	4.8288	4.8716	4.9152	4.9594	5.0045	5.0504	5.0970
79	sin	0.9816	0.9820	0.9823	0.9826	0.9829	0.9833	0.9836	0.9839	0.9842	0.9845
	cos	0.1908	0.1891	0.1874	0.1857	0.1840	0.1822	0.1805	0.1788	0.1771	0.1754
	tan	5.1446	5.1929	5.2422	5.2924	5.3435	5.3955	5.4486	5.5026	5.5578	5.6140
80	sin	0.9848	0.9851	0.9854	0.9857	0.9860	0.9863	0.9866	0.9869	0.9871	0.9874
	cos	0.1736	0.1719	0.1702	0.1685	0.1668	0.1650	0.1633	0.1616	0.1599	0.1582
	tan	5.6713	5.7297	5.7894	5.8502	5.9124	5.9758	6.0405	6.1066	6.1742	6.2432
81	sin	0.9877	0.9880	0.9882	0.9885	0.9888	0.9890	0.9893	0.9895	0.9898	0.9900
	cos	0.1564	0.1547	0.1530	0.1513	0.1495	0.1478	0.1461	0.1444	0.1426	0.1409
	tan	6.3138	6.3859	6.4596	6.5350	6.6122	6.6912	6.7720	6.8548	6.9395	7.0264
82	sin	0.9903	0.9905	0.9907	0.9910	0.9912	0.9914	0.9917	0.9919	0.9921	0.9923
	cos	0.1392	0.1374	0.1357	0.1340	0.1323	0.1305	0.1288	0.1271	0.1253	0.1236
	tan	7.1154	7.2066	7.3002	7.3962	7.4947	7.5958	7.6996	7.8062	7.9158	8.0285
83	sin	0.9925	0.9928	0.9930	0.9932	0.9934	0.9936	0.9938	0.9940	0.9942	0.9943
	cos	0.1219	0.1201	0.1184	0.1167	0.1149	0.1132	0.1115	0.1097	0.1080	0.1063
	tan	8.1443	8.2636	8.3863	8.5126	8.6427	8.7769	8.9152	9.0579	9.2052	9.3572
84	sin	0.9945	0.9947	0.9949	0.9951	0.9952	0.9954	0.9956	0.9957	0.9959	0.9960
	cos	0.1045	0.1028	0.1011	0.0993	0.0976	0.0958	0.0941	0.0924	0.0906	0.0889
	tan	9.5144	9.6768	9.8448	10.02	10.20	10.39	10.58	10.78	10.99	11.20
85	sin	0.9962	0.9963	0.9965	0.9966	0.9968	0.9969	0.9971	0.9972	0.9973	0.9974
	cos	0.0872	0.0854	0.0837	0.0819	0.0802	0.0785	0.0767	0.0750	0.0732	0.0715
	tan	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95
86	sin	0.9976	0.9977	0.9978	0.9979	0.9980	0.9981	0.9982	0.9983	0.9984	0.9985
	cos	0.0698	0.0680	0.0663	0.0645	0.0628	0.0610	0.0593	0.0576	0.0558	0.0541
	tan	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46
87	sin	0.9986	0.9987	0.9988	0.9989	0.9990	0.9990	0.9991	0.9992	0.9993	0.9993
	cos	0.0523	0.0506	0.0488	0.0471	0.0454	0.0436	0.0419	0.0401	0.0384	0.0366
	tan	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27
88	sin	0.9994	0.9995	0.9995	0.9996	0.9996	0.9997	0.9997	0.9997	0.9998	0.9998
	cos	0.0349	0.0332	0.0314	0.0297	0.0279	0.0262	0.0244	0.0227	0.0209	0.0192
	tan	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08
89	sin	0.9998	0.9999	0.9999	0.9999	0.9999	1.000	1.000	1.000	1.000	1.000
	cos	0.0175	0.0157	0.0140	0.0122	0.0105	0.0087	0.0070	0.0052	0.0035	0.0017
	tan	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0
Degs.	**Function**	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′

Squares, Cubes and Roots

Roots of numbers other than those given directly may be found by the following relations:

$\sqrt{100n} = 10\sqrt{n}$; $\sqrt{1000n} = 10\sqrt{10n}$; $\sqrt{\frac{1}{10}n} = \frac{1}{10}\sqrt{10n}$; $\sqrt{\frac{1}{100}n} = \frac{1}{10}\sqrt{n}$, $\sqrt{\frac{1}{1000}n} = \frac{1}{100}\sqrt{10n}$; $\sqrt[3]{1000n} = 10\sqrt[3]{n}$; $\sqrt[3]{10{,}000n} = 10\sqrt[3]{10n}$; $\sqrt[3]{100{,}000n} = 10\sqrt[3]{100n}$; $\sqrt[3]{\frac{1}{10}n} = \frac{1}{10}\sqrt[3]{100n}$; $\sqrt[3]{\frac{1}{100}n} = \frac{1}{10}\sqrt[3]{10n}$; $\sqrt[3]{\frac{1}{1000}n} = \frac{1}{10}\sqrt[3]{n}$.

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
1	1	1.000 000	3.162 278	1	1.000 000	2.154 435	4.641 589
2	4	1.414 214	4.472 136	8	1.259 921	2.714 418	5.848 035
3	9	1.732 051	5.477 226	27	1.442 250	3.107 233	6.694 330
4	16	2.000 000	6.324 555	64	1.587 401	3.419 952	7.368 063
5	25	2.236 068	7.071 068	125	1.709 976	3.684 031	7.937 005
6	36	2.449 490	7.745 967	216	1.817 121	3.914 868	8.434 327
7	49	2.645 751	8.366 600	343	1.912 931	4.121 285	8.879 040
8	64	2.828 427	8.944 272	512	2.000 000	4.308 869	9.283 178
9	81	3.000 000	9.486 833	729	2.080 084	4.481 405	9.654 894
10	100	3.162 278	10.00000	1 000	2.154 435	4.641 589	10.00000
11	121	3.316 625	10.48809	1 331	2.223 980	4.791 420	10.32280
12	144	3.464 102	10.95445	1 728	2.289 428	4.932 424	10.62659
13	169	3.605 551	11.40175	2 197	2.351 335	5.065 797	10.91393
14	196	3.741 657	11.83216	2 744	2.410 142	5.192 494	11.18689
15	225	3.872 983	12.24745	3 375	2.466 212	5.313 293	11.44714
16	256	4.000 000	12.64911	4 096	2.519 842	5.428 835	11.69607
17	289	4.123 106	13.03840	4 913	2.571 282	5.539 658	11.93483
18	324	4.242 641	13.41641	5 832	2.620 741	5.646 216	12.16440
19	361	4.358 899	13.78405	6 859	2.668 402	5.748 897	12.38562
20	400	4.472 136	14.14214	8 000	2.714 418	5.848 035	12.59921
21	441	4.582 576	14.49138	9 261	2.758 924	5.943 922	12.80579
22	484	4.690 416	14.83240	10 648	2.802 039	6.036 811	13.00591
23	529	4.795 832	15.16575	12 167	2.843 867	6.126 926	13.20006
24	576	4.898 979	15.49193	13 824	2.884 499	6.214 465	13.38866
25	625	5.000 000	15.81139	15 625	2.924 018	6.299 605	13.57209
26	676	5.099 020	16.12452	17 576	2.962 496	6.382 504	13.75069
27	729	5.196 152	16.43168	19 683	3.000 000	6.463 304	13.92477
28	784	5.291 503	16.73320	21 952	3.036 589	6.542 133	14.09460
29	841	5.385 165	17.02939	24 389	3.072 317	6.619 106	14.26043
30	900	5.477 226	17.32051	27 000	3.107 233	6.694 330	14.42250
31	961	5.567 764	17.60682	29 791	3.141 381	6.767 899	14.58100
32	1 024	5.656 854	17.88854	32 768	3.174 802	6.839 904	14.73613
33	1 089	5.744 563	18.16590	35 937	3.207 534	6.910 423	14.88806
34	1 156	5.830 952	18.43909	39 304	3.239 612	6.979 532	15.03695
35	1 225	5.916 080	18.70829	42 875	3.271 066	7.047 299	15.18294
36	1 296	6.000 000	18.97367	46 656	3.301 927	7.113 787	15.32619
37	1 369	6.082 763	19.23538	50 653	3.332 222	7.179 054	15.46680
38	1 444	6.164 414	19.49359	54 872	3.361 975	7.243 156	15.60491
39	1 521	6.244 998	19.74842	59 319	3.391 211	7.306 144	15.74061
40	1 600	6.324 555	20.00000	64 000	3.419 952	7.368 063	15.87401
41	1 681	6.403 124	20.24846	68 921	3.448 217	7.428 959	16.00521
42	1 764	6.480 741	20.49390	74 088	3.476 027	7.488 872	16.13429
43	1 849	6.557 439	20.73644	79 507	3.503 398	7.547 842	16.26133
44	1 936	6.633 250	20.97618	85 184	3.530 348	7.605 905	16.38643
45	2 025	6.708 204	21.21320	91 125	3.556 893	7.663 094	16.50964
46	2 116	6.782 330	21.44761	97 336	3.583 048	7.719 443	16.63103
47	2 209	6.855 655	21.67948	103 823	3.608 826	7.774 980	16.75069
48	2 304	6.928 203	21.90890	110 592	3.634 241	7.829 735	16.86865
49	2 401	7.000 000	22.13594	117 649	3.659 306	7.883 735	16.98499
50	2 500	7.071 068	22.36068	125 000	3.684 031	7.937 005	17.09976

SQUARES, CUBES AND ROOTS

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
50	2 500	7.071 068	22.36068	125 000	3.684 031	7.937 005	17.09976
51	2 601	7.141 428	22.58318	132 651	3.708 430	7.989 570	17.21301
52	2 704	7.211 103	22.80351	140 608	3.732 511	8.041 452	17.32478
53	2 809	7.280 110	23.02173	148 877	3.756 286	8.092 672	17.43513
54	2 916	7.348 469	23.23790	157 464	3.779 763	8.143 253	17.54411
55	3 025	7.416 198	23.45208	166 375	3.802 952	8.193 213	17.65174
56	3 136	7.483 315	23.66432	175 616	3.825 862	8.242 571	17.75808
57	3 249	7.549 834	23.87467	185 193	3.848 501	8.291 344	17.86316
58	3 364	7.615 773	24.08319	195 112	3.870 877	8.339 551	17.96702
59	3 481	7.681 146	24.28992	205 379	3.892 996	8.387 207	18.06969
60	3 600	7.745 967	24.49490	216 000	3.914 868	8.434 327	18.17121
61	3 721	7.810 250	24.69818	226 981	3.936 497	8.480 926	18.27160
62	3 844	7.874 008	24.89980	238 328	3.957 892	8.527 019	18.37091
63	3 969	7.937 254	25.09980	250 047	3.979 057	8.572 619	18.46915
64	4 096	8.000 000	25.29822	262 144	4.000 000	8.617 739	18.56636
65	4 225	8.062 258	25.49510	274 625	4.020 726	8.662 391	18.66256
66	4 356	8.124 038	25.69047	287 496	4.041 240	8.706 588	18.75777
67	4 489	8.185 353	25.88436	300 763	4.061 548	8.750 340	18.85204
68	4 624	8.246 211	26.07681	314 432	4.081 655	8.793 659	18.94536
69	4 761	8.306 624	26.26785	328 509	4.101 566	8.836 556	19.03778
70	4 900	8.366 600	26.45751	343 000	4.121 285	8.879 040	19.12931
71	5 041	8.426 150	26.64583	357 911	4.140 818	8.921 121	19.21997
72	5 184	8.485 281	26.83282	373 248	4.160 168	8.962 809	19.30979
73	5 329	8.544 004	27.01851	389 017	4.179 339	9.004 113	19.39877
74	5 476	8.602 325	27.20294	405 224	4.198 336	9.045 042	19.48695
75	5 625	8.660 254	27.38613	421 875	4.217 163	9.085 603	19.57434
76	5 776	8.717 798	27.56810	438 976	4.235 824	9.125 805	19.66095
77	5 929	8.774 964	27.74887	456 533	4.254 321	9.165 656	19.74681
78	6 084	8.831 761	27.92848	474 552	4.272 659	9.205 164	19.83192
79	6 241	8.888 194	28.10694	493 039	4.290 840	9.244 335	19.91632
80	6 400	8.944 272	28.28427	512 000	4.308 869	9.283 178	20.00000
81	6 561	9 000 000	28.46050	531 441	4.326 749	9.321 698	20.08299
82	6 724	9.055 385	28.63564	551 368	4.344 481	9.359 902	20.16530
83	6 889	9.110 434	28.80972	571 787	4.362 071	9.397 796	20.24694
84	7 056	9.165 151	28.98275	592 704	4.379 519	9.435 388	20.32793
85	7 225	9.219 544	29.15476	614 125	4.396 830	9.472 682	20.40828
86	7 396	9.273 618	29.32576	636 056	4.414 005	9.509 685	20.48800
87	7 569	9.327 379	29.49576	658 503	4.431 048	9.546 403	20.56710
88	7 744	9.380 832	29.66479	681 472	4.447 960	9.582 840	20.64560
89	7 921	9.433 981	29.83287	704 969	4.464 745	9.619 002	20.72351
90	8 100	9.486 833	30.00000	729 000	4.481 405	9.654 894	20.80084
91	8 281	9.539 392	30.16621	753 571	4.497 941	9.690 521	20.87759
92	8 464	9.591 663	30.33150	778 688	4.514 357	9.725 888	20.95379
93	8 649	9.643 651	30.49590	804 357	4.530 655	9.761 000	21.02944
94	8 836	9.695 360	30.65942	830 584	4.546 836	9.795 861	21.10454
95	9 025	9.746 794	30.82207	857 375	4.562 903	9.830 476	21.17912
96	9 216	9.797 959	30.98387	884 736	4.578 857	9.864 848	21.25317
97	9 409	9.848 858	31.14482	912 673	4.594 701	9.898 983	21.32671
98	9 604	9.899 495	31.30495	941 192	4.610 436	9.932 884	21.39975
99	9 801	9.949 874	31.46427	970 299	4.626 065	9.966 555	21.47229
100	10 000	10.00000	31.62278	1 000 000	4.641 589	10.00000	21.54435

Index

A

Absolute value 1–1
AC generator 2–32, 33, 34
AC measuring instruments .... 3–9 to 3–15
AC network theorems .... 2–85 to 2–91
Acceptor, P-type germanium 8–7
Addition, algebraic 1–2
Air-core transformers 2–98, 99
Alternating current, defined 2–32
Alternators 2–41, 42
Algebra, principles of .... 1–1 to 1–12
Algebraic expression 1–1
Ammeter, alternating current 3–9, 10, 11
  direct current .... 3–1 to 3–5
Ampere, defined 2–6
Amplification 4–6; 7–3
Amplification factor 4–7
Amplifier, audio .... 7–11 to 7–14
  basic .... 7–1 to 7–3
  cascade 7–11
  class A 7–5
  class B .... 7–5 to 7–8
  class C 7–8, 9
  direct current 7–9, 10, 11
  feedback 7–27, 28
  impedance-coupled 7–11
  intermediate frequency .... 7–22 to 7–27
  paraphase 7-32
  power 7–5
  radio frequency .... 7–22 to 7–27
  resistance-capacitance 7–11
  transformer-coupled 7–11
  video .... 7–14 to 7–21
  voltage 7–3, 4
Amplitude distortion 7–3, 4
Amplitude Modulation 8–54
Analysis of transient voltages .... 6–1 to 6–4
Analysis of video amplifiers 7–19, 20
Antilogarithms 1–13, 14
Apparent power 2-65
Artificial magnets 2–19, 20
Atom 2–1
Audio transformers 2–100, 101
Average value of AC 2–53
Axioms 1–4

B

Bandwidth 2–84
Base 8–2
Barrier, junction 8–10
  width 8–10
  height 8–10
Battery 2–8
Beam power tube 4–15
Bias, types of 4–8, 9, 10
Biased clampers 6–42
Bias stabilization 8–22
Binomial, defined 1–2
  products 1–5
  square 1–5
Blocking oscillator .... 6–86 to 6–88
Braces 1–2
Brackets 1-2
Bridge rectifier 4–25

C

Capacitance, distributed 2–58
  measuring 3–16
  properties .... 2–55 to 2–57
Capacitance input filter 4–24
Capacitive reactance 2–57
Capacitor, charge 6–6
  discharge 6–11, 12
  electrolytic 2–59, 60
  properties 2–55, 56, 57
  variable 2-58, 59
Carriers, majority 8–12
  minority 8–12
Cathode 4–2
Cathode bias 4–8, 9

Cathode-coupled multivibrator 6–71, 72, 73
Cathode follower, application.....6–84, 85; 7–29
  defined 7–29
  gain 7–29
  impedance 7–30
  input capacity 7–29
Cathode-ray tube, construction....4–17 to 4–22
  electromagnetic deflection 4–20, 21
  electrostatic deflection 4–19, 20
  focusing action 4–18, 19
Cell faults 2–8
Cells ....2–7 to 2–11
Characteristic 1–12, 13, 14
Characteristic curves of triode.........4–6, 7, 8
Characteristics of radar pulse........5–9, 10, 11
Charge, unit of.......................... 2–6
Charged body, defined.................. 2–3
Circuit loading 3–6, 7
Circuits, alternating current........2–61 to 2–85
  complex 2–79, 80
  counting ....6–90 to 6–94
  coupled ....2–91 to 2–104
  LR ....6–16 to 6–25
  RC ....6–5 to 6–16
Circuit Q 7–22
Clampers ....6–38 to 6–43
Class A amplifier....................... 7–5
Class B amplifier....................7–5 to 7–8
Class C amplifier...........................7–8, 9
Classification of amplifiers..........7–4 to 7–27
Coefficient 1–1, 3
Coils 2–60, 61
Collector 8–1
Colpitts oscillator 7–34, 35
Commutator 2–34, 35, 36
Complex fractions 1–8
Complex numbers 1–11
Compound generators 2–39, 40
Compound motor, DC................. 2–47
Conductance 2–16
Conductor, defined 2–2
Conductors 8–5
Conjugate 1–10
Connecting cells 2–11
Constant power dissipation line......... 8–28
Copper oxide rectifier.................. 4–25
Coulomb's law 2–3
Counting circuits ....6–90 to 6–94
Counter electromotive force............2–44, 45
Coupling capacitor 7–12
Coupling resistor 7–12
Coupling, effect of....................7–11, 12
  LC 2–93
  RC 2–92
  types 2–92
Crystals 8–6
Crystal controlled oscillator.........7–43, 44, 45
Crystal cuts 7–40, 41
Crystal diode limiter..................6–31, 32
Crystal oscillator 7–40, 41, 42, 43
Current and voltage in LR circuit.......6–24, 25
Current and voltage in RC circuit....6–5 to 6–12
Current, defined 2–6
Current in parallel circuits..........2–14, 15, 16
Cutoff 4–6
Cycle 2–52

D

D'Arsonval meter movement............. 3–2
DC measuring instruments...........3–1 to 3–9
DC motor, speed and direction
of rotation control..................... 2–47
Degenerative feedback 7–27
De-ionizing voltage 4–16
Depletion region 8–10
Detector 8–56, 58
Dielectric, defined 2–3
  fields 2–3, 4, 5
Differentiation 6–16
Diffusion 8–9
Diode characteristics 4–4
Diode, point contact.................... 8–1
  junction 8–2
Diode tubes 4–4, 5
Direct current amplifier.............7–9, 10, 11
Direct current, defined................. 2–32
Direct current generator..........2–34 to 2–38
Direct current restorers...........6–38 to 6–43
Direction, determining 5–5
Discriminant 1–12
Distortion in amplifiers.................7–3, 4
Distributed capacitance 2–58
Distribution of charges................. 2–5
Donor, N-type germanium.............. 8–7
Doppler effect 5–3
Double diode limiting.................. 6–34
Driven blocking oscillator.............6–89, 90
Driven plate-coupled multivibrator.....6–67, 68
Dry cell 2–7
Duty cycle, defined.....................5–10
Dynamic curves 4–7

E

Earth's magnetic field 2–22
Eddy current losses 2–60, 61
Effect of varying R and L in RL circuits 6–21 to 6–24
Electrical quantities, charge 2–6
  current 2–6
  resistance 2–6
Electrodes 2–7
Electrodynamometer ammeter 3–11, 12
Electrolytic capacitors 2–59, 60
Electromagnetism 2–24 to 2–28
Electromotive force 2–6, 7
Electron emission 4–1, 2
Electronic voltage regulator 4–30
Electron-pair bond 8–7
Electron theory 2–1, 2
Electroscope 2–5
Electrostatics 2–2 to 2–5
Element 2–1
Elements of pulse radar system 5–5, 6, 7, 8
Emission saturation 4–5
Emitted electron 4–1
Emitter 8–1
Equations, defined 1–4
  exponential 6–9, 10, 18, 19
  fractional 1–8
  quadratic 1–12
  simultaneous 1–8, 9
  solution 1–4, 5
Etched circuits 8–77
Exponential curves 6–6, 7
Exponents, defined 1–1
  negative 1–3
  zero 1–3

F

Factor 1–1
Factoring, defined 1–5
  polynomials 1–5
  trinomials 1–5, 6
Farad 2–55
Faraday's law 2–28
Fields of force 2–3, 4
Filament, connecting 4–3, 4
  voltages 4–3
Filter, capacitor-input 8–73
  choke-input 8–73
Filter circuits 4–23, 24, 25
Firing voltage 4–16
Fixed bias 4–8
Fly back time 3–18
Force between charged bodies 2–3
Force between magnetic poles 2–20
Forward bias 8–11
Four layer diode 8–66
Fractional equations 1–8
Fractional exponent 1–9
Fractions, addition 1–7
  complex 1–8
  defined 1–7
  improper 1–7
  subtraction 1–7
Free electrons 2–1, 2
Frequency, defined 2–52
  distortion 7–3, 4
  modulation 5–2, 3; 8–57
Frequency response 7–14, 22, 26
Frequency-shift radar system 5–3
Full-wave rectifier 4–23

G

Gain, calculating 7–13
  defined 7–3
  high frequencies 7–14
  I–F amplifiers 7–22 to 7–26
  low frequencies 7–14
  mid-frequencies 7–13
  $R_g \gg R_L$ 7–13
Gas-filled tubes 4–16, 17
Generator, AC 2–32, 33, 34
  types 2–38
  DC 2–34 to 2–38
  self-excited 2–38
  time-base 6–43 to 6–58
Grid clamping 6–43
Grid current limiting 6–32, 33
Grid-leak bias 4–9, 10
Grouping, signs of 1–2

H

Half-wave rectifier 4–22, 23
Hard tube sweep generator 6–49 to 6–51
Harmonics 6–3; 7–7
Hartley oscillator 7–34
Henry, defined 2–30, 31
High frequency compensation 7–18
Holes 8–9

Hot-wire instruments 3–10, 11
Hyperbolic functions 6–10
Hysteresis 2–27, 28

I

I-F amplifiers ....7–22 to 7–27
Imaginary numbers 1–11
Impedance 2–61, 62
Impedance-coupled amplifier 7–11
Impurities 8–7
Impedance triangle 2–65
Improper fraction 1–7
Indicators 5–6, 7, 8
  A scan 5–6
  B scan 5–6
  C scan 5–6
  G scan 5–8
  J scan 5–8
  L scan 5–8
  P scan 5–6
Inductance, critical 8–74
  optimum 8–74
Inductance input filter................4–24, 25
Inductance, measuring 3–11
  properties 2–54, 55
Induction ....2–28 to 2–31
Inductive reactance 2–55
Input capacity 7–15, 16
Insulator, defined 2–2; 8–6
Integration 6–16
Interelectrode capacitance 4–12; 7–29
Internal resistance in battery............2–7, 8
Ion 2–1
Inverse function 1–16

J

J-operator 1–11
Jump voltage, defined 6–53

K

Kirchhoff's laws 2–17, 18, 72

L

LC coupling 2–93
Least common denominator............. 1–7
Lenz's law 2–30
Like charges 2–1
Limiting circuits ....6–28 to 6–37
Lines of force 2–4, 21
Literal coefficient 1–1
Literal number 1–1
Lissajou's figures ....3–23 to 3–26
Load lines 4–10, 11, 12
Logarithms, characteristic in...........1–12, 13
  common 1–12
  defined 1–12
  division 1–14
  extracting root 1–14
  mantissa 1–13, 14
  multiplication 1–14
  natural 1–12, 15
  raising to power 1–14
Log-log decitrig slide rule........1–16, 17, 18, 19
Long LR time constant................ 6–23
Long RC time constant...............6–15, 16
LR charging circuits.................6–17, 18
LR discharging circuits...............6–19, 20
LR time constant....................6–17, 18

M

Magnetic circuits 2–25
Magnetic field, about loop.............. 2–24
  force on conductor.................2–25, 26
  in solenoid 2–24, 25
  properties 2–20, 21
Magnetic flux 2–21
Magnetic lines of force................ 2–21
Magnetic poles 2–20
Magnetism, molecular theory..........2–22, 23
Magnetization curve 2–23
Magnets 2–19, 20
Mantissa 1-12, 13
Maser 8–76
Matter, structure 8–4
  element 8–4
  compound 8–4
  molecule 8–4
  atom 8–4
  subatomic 8–4
  electrical balance 8–5
Mavar 8–77
Maximum power transfer theorem......2–90, 91
Medium LR time constant.............. 6–23
Medium RC time constant.............. 6–15
Mercury vapor tubes.................. 4–17
Metallic oxide rectifiers................. 4–25

Meters, AC ....3–9 to 3–15
  DC ....3–1 to 3–9
Meter sensitivity 3–4, 6, 7
Mho 2–16
Mile, radar 5–3
Mixed expression 1–7
Molecule 2–1
Molecular electronics 8–70
Motor, alternating current.........2–48 to 2–52
  series 2–48
  induction 2–48, 49, 50, 51
  squirrel cage 2–51
  split-phase 2–51
  synchronous 2–51, 52
Motor, direct current..............2–43 to 2–47
Moving iron vane meter...............3–9, 10
Moving target indicator................ 5–3
Multiplication, symbols 1–2
Multipurpose tubes 4–16
Multivibrator, cathode-coupled 6–71, 72, 73
  driven plate-coupled 6–67, 68
  plate-coupled ....6–59 to 6–63
  synchronization ....6–63 to 6–67
Mutual induction 2–30, 31

N

N-type semiconductor 8–2
Natural magnets 2–19, 20
Negative clamping circuit..........6–38 to 6–41
Negative counting 6–91, 92
Negative temperature coefficient......... 8–23
Networks ....2–17, 18; 2–85 to 2–91
Neutralization circuit 7–22
Numbers, complex 1–11
  imaginary 1–11
  real 1–11
Numerical coefficient 1–1, 2

O

Ohm, defined 2–6
Ohmmeter 3–8, 9
Ohm's law 2–7
Oscillator, block diagram............... 7–33
  Colpitts 7–34, 35
  crystal 7–40, 41, 42, 43
  Hartley 7–34
  multivibrator ....6–58 to 6–90
  phase-shift 7–38, 39
  resistance-capacitance coupled ...7–38, 39, 40
  tuned-plate tuned-grid 7–36, 37, 38
  ultraudion 7–35, 36
Oscillographs ....3–19 to 3–27
Oscilloscope 3–17, 18
Oscilloscope uses 3–22
  voltage and current measurements 3–22, 23, 24
  frequency measurement 3–24, 25, 26
  determining phase relations
    of voltages 3–26, 27

P

P-type semiconductors 8–2
Parallel AC circuits, with resistance...... 2–72
  resistance and capacitance........2–73, 74, 75
  inductance and resistance........2–75, 76, 77
  inductance and capacitance..........2–77, 78
Parallel DC circuits...............2–14 to 2–16
Parallel resonance ....2–81 to 2–84
Paraphase amplifier 7–32
Parenthesis 1–2
Peaked wave 6–4, 5
Pentode equivalent circuits.............. 7–14
Pentodes 4–14, 15
Permeability 2–21, 22, 25
Phantastron circuit ....6–79 to 6–86
Phase angle 2–74, 75
Phase, defined 2–53, 54
Phase inverters 7–31, 32
Phase-shift distortion 7–3, 4
Phase shift oscillator.................7–38, 39
Phase-splitting circuits 7–33
Phosphor 4–19
Photosensitive semiconductors 8–62
  point contact 8–63
  junction 8–63
Piezoelectric effect 7–40
Plate coupled multivibrator........6–59 to 6–63
Plate load resistor...................7–16, 17
Plate resistance 4–7
PN junction 8–9
Polarization in cell..................... 2–8
Polynomials 1–2, 3
Polyphase rectifiers 4–27, 28
Positive clamper 6–41
Positive counting 6–90, 91
Positive feedback 7–27
Potential difference 2–7
Power amplifier 7–5
Power, defined 2–7
  formulas 2–12, 13, 14, 74, 75, 76

in AC circuits..................2–62 to 2–66
in DC circuits..................2–12 to 2–14
Power rating of resistors.............. 2–11
Primary cell 2–7
Printed circuits 8–77
Proton 2–1
Pulse, radar 5–3, 4, 5
Pulse recurrence time.................. 5–9
Pulse repetition frequency.............5–9, 10
Pulse shape 5–8
Pulse system, power requirements........ 5–10
Pulse width 5–8

Q

Q of circuit..........................2–69, 70
Quadratic equation, general form........ 1–12
Quantitative law of induced EMF......2–28, 29

R

Radar, definition 5–1
average power 5–10
frequency modulation system..........5–2, 3
peak power 5–10
principles of operation..............5–1, 2
pulse system 5–3, 4, 5
types and uses....................5–11, 12
Radicals 1–9, 10, 11
Radicand 1–10
Radio frequency amplifier..........7–22 to 7–27
Range, determining 5–3
Raser 8–77
Rating of storage cells.............2–10, 11
Rationalizing denominator 1–10, 11
RC coupling 2–92
RC time constant....................... 6–6
Real numbers 1–11
Rectangular notation 2–77, 78
Rectifier type meter................... 3–14
Rectifier, vacuum tube.............4–22, 23
crystal 8–1
metallic oxide 4–25
Reduction to lowest terms.............. 1–6
Remote cutoff tube.................4–15, 16
Resistance, defined 2–6
in AC circuits....................... 2–54
parallel circuits 2–14, 15
Resistance-capacitance coupled amplifier 7–11
Resistors, defined 2–11
variable 2–11, 12
Resonance, frequency 2–69
parallel ....2–81 to 2–84
series ....2–69 to 2–72
Restorer circuits ....6–38 to 6–43
Reverse Bias 8–10
R-F Amplifiers ....7–22 to 7–27
Rheostat 2–12
Ripple frequency 2–37
Ripple voltage 4–23
RLC peaker 7–49
RLC transients 7–47, 48
Roots 1–4, 5

S

Saturation limiting 6–33, 34
Sawtooth wave 6–4
Scans, types of...................5–6, 7, 8
Searching, defined 5–5
Secondary cell 2–8, 9
Secondary emission 4–13
Selectivity 2–84; 7–22
Selenium rectifier 4–26
Self inductance 2–31
Semiconductor 8–4, 6
Series circuits, alternating current..2–61 to 2–72
direct current ....2–12 to 2–14
Series diode limiter...............6–28, 29
Series generator 2–40, 41
Series motor, DC...................2–46, 47
Series-parallel, AC circuits............ 2–80
DC circuits 2–16, 17, 18, 19
Series resonance ....2–69 to 2–72
Series resonant filter.................. 2–72
Shock-excited oscillator 7–46, 47
Shunt diode limiters...........6–29, 30, 31
Shunt generator 2–38, 39
Shunt motor, DC...................2–45, 46
Short LR time constant................ 6–23
Short RC time constant............6–14, 15
Sign, algebraic 1–2, 3
Signs of grouping...................... 1–2
Silicon rectifiers 8–71
single-phase 8–72
three-phase 8–73
precautions 8–75
Simultaneous linear equations........1–8, 9
Slide rule, log-log decitrig.......1–16, 17, 18, 19
Slope formulas 6–55, 56
Space charge region................... 8–10
Special purpose tubes...........4–15 to 4–22

Solving networks by superposition......2–18, 19
Solid state rectifiers.................... 4–25
Spacistor 8–64
Start-stop multivibrator ....6–73 to 6–79
Static curves 4–7
Static electricity 2–2
Step-by-step counting 6–92, 93, 94
Storage cell 2–8, 9, 10
Squarer and peaker...............6–34 to 6–38
Square waves 6–2, 3, 4
Square wave generators............6–58 to 6–90
Subtraction, algebraic 1–2
Suppressor grid 4–14
Swamping resistor 8–23
Synchronized blocking oscillator......... 6–88
Synchronization 6–46, 47
Synchronization of multivibrator...6–63 to 6–67
Synchroscope 3–27, 28

T

T- and Pi-networks................2–86, 87, 88
Tank circuit 7–6
Target 5–2
Temperature coefficient, crystals......... 7–42
Terms, algebraic 1–2, 6
Test Unit I–176......................3–16, 17
Tetrodes 4–12, 13, 14
Tetrode transistor 8–63
Thermal emission 4–1, 2
Thermocouple ammeter 3–13
Thevenin's theorem 2–88, 89, 90
Three-phase generator 2–42
Thyratron 4–17
Thyratron sweep generator........6–43 to 6–49
Time-base, formation 5–4
Time-base generators ....6–43 to 6–58
Time constant of RC circuit............. 6–6
Timer 5–5
Timing circuits, function................ 6–1
Tone controls 8–32
Transconductance 4–7
Transformer, coupling ....2–93 to 2–96
  formulas 2–94, 95, 96
  power losses 2–99
  types ....2–100 to 2–104
Transformer-coupled amplifier 7–11
Transient voltage, defined............... 6–1
Transient voltages in LR circuits....6–16 to 6–25
Transient voltages in RC circuits....6–5 to 6–16
Transistor, amplifiers 8–18, 36
  common-base 8–18
  common-collector 8–20, 40
  common-emitter 8–19
  push-pull 8–36
  tuned 8–40
  video 8–36
Transistor arrangements 8–16
  common-base 8–16
  common-collector 8–17
  common-emitter 8–16
Transistor characteristic curves.......... 8–24
  dynamic transfer 8–25
  gain calculation 8–25
Transistor converter 8–60
Transistor coupling 8–29
Transistor efficiency 8–3
Transistor junction 8–2
Transistor life 8–3
Transistor limiters 8–33
  cutoff limiters 8–33
  saturation limiting 8–34
Transistor mixer 8–59
Transistor multivibrators 8–49
  Eccles-Jordan 8–52
  free-running 8–49
  one-shot 8–51
Transistor noise 8–3
Transistor oscillators 8–42
  audio 8–42
  blocking 8–47
  bridged-T 8–47
  Colpitts 8–44
  crystal 8–45
  RC 8–46
  series Hartley 8–43
  shunt Hartley 8–44
Transistor, NPN 8–12
  PNP 8–14
  comparison 8–14
  graphical symbols 8–15
  letter symbols 8–20
Transistor point-contact 8–1, 62
Transistor size 8–3
Transistor servo amplifier............... 8–61
Transistor theory 8–16
Transistor uses 8–3
Trapezoidal sweep generator........6–54 to 6–58
Trapezoidal waveshape generator...6–51 to 6–54
Triangular waves 6–4, 26, 27
Trigonometric functions 1–15, 16
Trinomial, defined 1–2

Triodes ....4–5 to 4–12
True power 2–64, 65
Tuned-plate tuned-grid oscillator........ 7–36
Tuned transformers 2–101, 102, 103
Tunnel diode 8–67

U

UHF tubes 4–16
Ultraudion oscillator 7–35, 36
Uncharged body 2–3
Uncompensated circuit 7–19
Unijunction transistor 8–65
Universal time constant curves.........6–7, 8, 9
Unlike charges 2–1
Unlike terms 1–2
Uses of oscillograph...............3–17 to 3–27

V

Vacuum tube, cathode-ray.........4–17 to 4–22
  diodes 4–4, 5
  directly heated 4–2
  pentodes 4–14, 15
  power supplies ....4–22 to 4–30
  special purpose ....4–15 to 4–22
  tetrodes 4–12, 13, 14
  triodes ....4–5 to 4–12
Vacuum tube voltmeter.............3–16; 7–10
Values of AC......................... 2–53
Variable capacitors 2–58, 59
Variable resistors 2–11, 12
Varying R and C in RC circuits....6–13 to 6–16
Vector quantities and vector
  algebra ....1–19 to 1–22
  addition and subtraction.............. 1–21
  division 1–22
  multiplication 1–22
  polar and rectangular forms........... 1–21
Vectors ....1–11 to 1–22
Video amplifier, analysis...............7–19, 20
  high frequency compensation.......... 7–18
  improving frequency response......... 7–18
  input capacity 7–15, 16
  low frequency compensation.........7–17, 18
Vinculum 1–2
Voltage amplifier 7–3, 4
Voltage doubler 4–28, 29
Voltage divider 2–13
Voltage drop 2–6
Voltage regulator circuits..............4–29, 30
Voltage regulator tube.................4–16, 17
Voltmeter, AC 3–9
  DC 3–5, 6, 7, 8
Volume controls 8–31

W

Weston-type instrument 3–2, 3
Wet cell 2–7
Wien bridge oscillator.................. 7–39

Z

Zener diode 8–70
Zero exponent 1–3

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AF MANUAL 52-8
VOLUME II

ELECTRONIC

CIRCUIT ANALYSIS

DEPARTMENT OF THE AIR FORCE

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AIR FORCE MANUAL
NUMBER 52-8

DEPARTMENT OF THE AIR FORCE
WASHINGTON 25, D.C.
15 January 1963

Technical Training

ELECTRONIC CIRCUIT ANALYSIS

This manual, published in two volumes, of which this is Volume II, provides the graduate electronics repairman or technician with reference information on the fundamentals of electronics. It also makes available a basic coverage of the subject to the student repairman. A prerequisite to understanding these basic concepts is a knowledge of mathematics and electricity. Students who are not too familiar with these subjects may have to consult other sources of information for greater detail. Both Volume I and Volume II include the same attachments of Symbols and Formulas, and Tables, preceding the Index. The Index pertains only to that volume in which it appears.

CONTENTS

—VOLUME I—

Chapter		*Page*
1	Mathematics for Electronics	1–1
2	DC and AC Circuits	2–1
3	Measuring Instruments	3–1
4	Electron Tubes and Power Supplies	4–1
5	Principles of Radar	5–1
6	Timing Circuits	6–1
7	Amplifiers and Oscillators	7–1
8	Transistors	8–1
	Attachments	
	1 Symbols and Formulas	*1*
	2 Tables	*7*
	3 Index (Volume I)	*23*

—VOLUME II—

Chapter		*Page*
9	Modulation, Detection, and Single Sideband Transmission	9–1
10	Transmission Lines	10–1
11	Waveguides, Cavity Resonators, and Duplexers	11–1
12	UHF Oscillators	12–1

This manual supersedes AFM 52-8, Radar Circuit Analysis, 30 June 1951

OPI: AFPTR
DISTRIBUTION: F

This training manual is published by Air Training Command in accordance with AFR 5-5.

For sale by the Superintendent of Documents, U. S. Government Printing Office
Washington 25, D. C.

CONTENTS (Continued)

Chapter *Page*
13 Antennas 13–1
14 Synchros and Servomechanisms 14–1
15 Special Circuits 15–1
Attachments
1 Symbols and Formulas *1*
2 Tables *7*
3 Index (Volume II) *23*

Comments or recommendations for the improvement of this manual should be sent to Headquarters Air Training Command, Randolph Air Force Base, Texas.

CHAPTER 9

Modulation, Detection, and Single Sideband Transmission

In radio transmission, the steady r-f voltage wave generated by a transmitter alone cannot produce an intelligible signal at a receiver. It merely carries the intelligence to the receiver. In other words, it is the *carrier*. At the transmitter, the intelligence is impressed on the carrier by a process called *modulation*. When the modulated carrier is picked up by the receiver, the reverse of modulation occurs. At the receiver, the intelligence is removed from the carrier. This process is called *detection*. This chapter gives you the basic principles of modulation and detection, and explains the operation of a large number of detector circuits commonly employed in radar and communications equipment.

THEORY OF MODULATION

Various methods are used for impressing the signal voltage (intelligence) on the r-f voltage (the carrier) generated by the transmitter. Sometimes, this process (modulation) is accomplished by varying the *amplitude* of the carrier in accordance with the signal voltage. Sometimes, the *frequency* of the carrier is varied in accordance with the signal voltage, and sometimes it is the *phase* of the carrier that is varied. Thus, there are three common types of modulation — amplitude, frequency, and phase.

Amplitude Modulation

The definition of the popular concept of amplitude modulation is *the process by which the amplitude of an r-f carrier wave is varied at the audio rate of the modulating signal.* This definition has been accepted for many years because it easily describes the composite AM waveform as viewed on an oscilloscope. But what appears to be an r-f carrier varying at an audio rate is actually the instantaneous frequency-voltage relationship of the output frequency components of a nonlinear device.

Amplitude modulation is more accurately defined as *a type of modulation that results when a modulating voltage is applied to a carrier voltage. It produces amplitude and frequency change of the r-f sidebands.* The carrier itself *does not* change in amplitude or frequency during the process of amplitude modulation. The radio-frequency carrier is defined as *a single radio frequency radiated by a transmitter operating without modulation.* Since the purpose of this manual is to provide the technician with a basic coverage of the subject, a thorough understanding of the technical terms used is required.

The typical amplitude-modulated carrier is illustrated in Figure 9–1B. You can see that the amplitude of an amplitude-modulated wave is high during the positive peaks of the signal voltage, but is near zero during the negative peaks. The shape of this wave is the result of applying the carrier and signal voltages to a nonlinear device. A nonlinear device is one in which the current does *not* vary in direct proportion to the voltage.

Whenever two signals are impressed on a linear device (one in which the current *does* vary in direct proportion to the voltage), the output waveshape contains the same frequencies as the input and looks like the waveshape in Figure

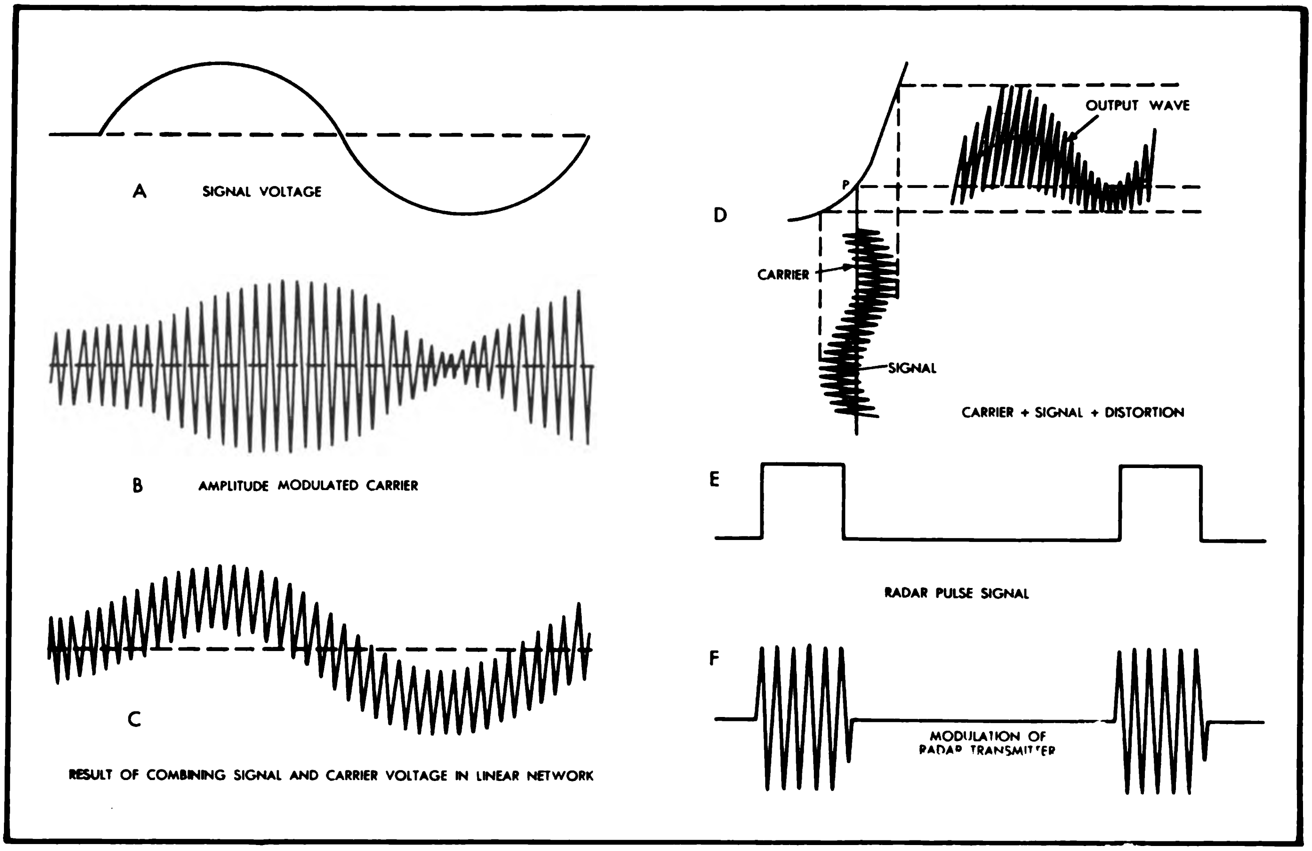

A SIGNAL VOLTAGE
B AMPLITUDE MODULATED CARRIER
C
RESULT OF COMBINING SIGNAL AND CARRIER VOLTAGE IN LINEAR NETWORK
D
OUTPUT WAVE
P
CARRIER
SIGNAL
CARRIER + SIGNAL + DISTORTION
E
RADAR PULSE SIGNAL
F
MODULATION OF RADAR TRANSMITTER

Figure 9–1. Amplitude Modulation

9–1C. In this case the r-f amplifiers in the transmitter amplify the carrier, but discriminate sharply against the modulating frequency. This causes only the carrier to be transmitted. Such an arrangement would defeat the purpose of modulation, as the signal arriving at the receiver would not contain a modulation component at all.

Impressing the carrier and the modulating signal frequencies on a nonlinear network, however, produces the output waveshape shown in Figure 9–1D. The network used in producing this waveshape is a vacuum tube biased to operate on the nonlinear portion of its characteristic curve (point P). The input signal at D is the result of simultaneously impressing the modulating signal and the carrier signal onto the grid circuit of this nonlinear network. The instantaneous grid voltage varies around the bias value as indicated by the input signal. The distorted plate voltage current produces the distorted output waveshape. This is the waveshape you would see on an oscilloscope.

The output of a wave impressed on a nonlinear network contains both the carrier and the signal frequencies and direct current. As you can see in the output waveshape in Figure 9–1D, the negative half-cycles are altered differently than the positive half-cycles. A sine wave altered in this manner is said to be a combination of the fundamental frequencies and additional frequencies caused by distortion. The distortion is introduced by the nonlinear resistance.

When a carrier frequency and a signal frequency are applied to a nonlinear device, the output wave contains the following frequencies:

1. The carrier frequency.
2. The *upper side frequency*, which is equal to the carrier *plus* the signal frequency.
3. The *lower side frequency*, which is equal to the carrier *minus* the signal frequency.
4. Direct current.

When an output wave containing these frequencies is present in a transmitter, the tuned circuits respond to the carrier, the upper side frequency, and the lower side frequency, but not to the modulating signal. The result of this frequency response is a composite waveshape, like the amplitude-modulated carrier shown in Figure 9–1B. The direct current signal indicates the presence of modulation by varying the plate current meter in the modulator stage of the transmitter.

When an 870-kilocycle carrier frequency of constant amplitude is modulated by a 5000-cps signal, the frequencies in the output are the carrier-frequency of 870 kc, an upper sideband frequency of 875 kc, and a lower sideband frequency of 865 kc. Of these, only the 865 kc, 870 kc, and 875 kc components will be accepted by the output tuned circuits of the transmitter. Therefore, they are the ones that are transmitted. The bandwidth of a signal is measured from the lowest to the highest frequency in the sidebands. In this case it is 10 kc (865 to 875 kc).

An important point to remember in amplitude modulation is that the transmitted signal, which the receiver picks up, does *not* contain the modulating frequency at all. It contains three separated, but related, frequency components. Because of this, receivers require a nonlinear device such as a detector or demodulator to *generate*, or reproduce, the signal frequency intelligence in its original form when the carrier and side frequencies are impressed upon it.

For radar systems, the same principles hold true. A typical signal frequency, such as shown in Figure 9–1E, is a 1-microsecond squarewave pulse which recurs at about 1000-microsecond intervals. This wave is composed of many harmonic (sine wave) frequencies. When a carrier is modulated by these pulses, waveshapes like those at F result which contain the carrier and a wide band of side frequencies. The side frequencies are located on each side of the carrier and are called *sidebands.*

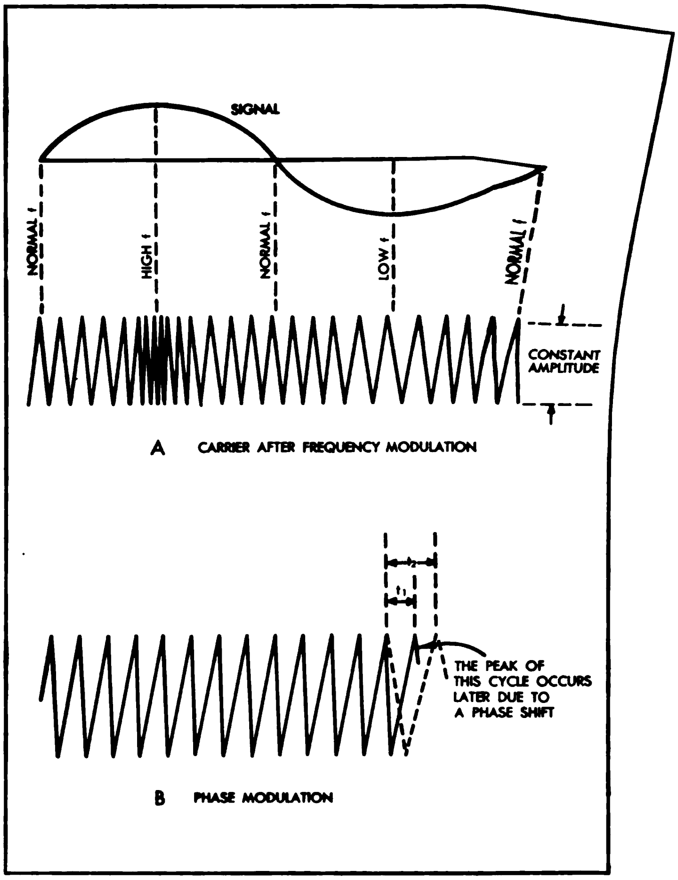

SIGNAL
NORMAL f
HIGH f
NORMAL f
LOW f
NORMAL f
CONSTANT AMPLITUDE
A CARRIER AFTER FREQUENCY MODULATION
t2
t1
THE PEAK OF THIS CYCLE OCCURS LATER DUE TO A PHASE SHIFT
B PHASE MODULATION

Figure 9–2. Frequency and Phase Modulation

Frequency Modulation

In frequency modulation the signal is impressed on the carrier by varying the frequency of the carrier in accordance with the frequency of the signal (Figure 9–2). The frequency of the carrier when it is not being modulated is called the *center frequency.* When the carrier is modulated by a positive signal voltage, its frequency becomes higher in proportion to the amount of positive signal voltage, and when it is modulated by a negative signal voltage, its frequency becomes lower in proportion to the amplitude of the negative signal voltage. Therefore, when the sine wave shown in Figure 9–2A is applied to a carrier, the carrier frequency changes from the normal center frequency to a higher frequency, back to the normal, then to a lower frequency, and back to the normal frequency again in accordance with the voltage of the applied sine wave. The maximum frequency change from the center frequency, which depends upon the amplitude of the signal, is called the *deviation.* (Note that the amplitude of the modulated carrier is constant.)

One advantage of frequency modulation over amplitude modulation is in its noise reducing capabilities. If the receiver is responsive only to changes in frequency, frequency modulation makes possible a considerable increase in the signal-to-noise ratio. Most noise signals produce amplitude modulation of the carrier, or the carrier-plus-modulation signal, which is applied to the demodulating circuit in the receiver.

Another advantage of frequency modulation over amplitude modulation is that there is no power variation with the degree of modulation, as there is with an amplitude-modulated wave. To obtain 100% modulation with an amplitude-modulated transmitter, the average power output must be increased by 50%. A frequency-modulated transmitter requires an insignificant amount of additional power to produce modulation; thus it needs no provision for increased power output on modulation peaks.

The principle of frequency modulation was developed as early as 1920, but was not put to practical use until the 1940's. Radar equipment uses a special circuit, called a *discriminator*, for detecting frequency-modulated radio signals. This circuit gets its name because it discriminates between different frequencies. The discriminator is taken up later in this chapter.

Phase Modulation

Phase modulation, which is a slight modification of frequency modulation, is characterized by changes in the phase of the AC carrier voltage instead of in its frequency. This means that the phase of a cycle shifts with respect to each preceding cycle and that the last cycle is ahead of or behind its normal phase. This causes the peak to occur earlier or later than it would normally occur, as you can see in Figure 9–2B, which shows the normal period as t_1 and the longer period as t_2. The shorter period corresponds to the higher frequency. A frequency modulation detector responds to phase changes in the same manner as it responds to frequency changes.

REACTANCE TUBE MODULATOR

Reactance tubes serve the function of introducing reactance into associated networks. If the associated network is an ordinary oscillator whose frequency is not stabilized, then the reflected reactance will change the frequency of the oscillations that are generated. Therefore, when the reactance swings or varies, frequency modulation is produced. Considerable frequency deviation can be produced by this method.

Figure 9–3A shows a reactance tube, sometimes called a quadrature tube, which, with its load R_1C_1, causes reactance fluctuations in the oscillator's frequency-determining LC branch.

From the equivalent network of Figure 9–3B, note that a reactance $X + \Delta X \sin\omega T$ is generally introduced across terminals 1 and 2 of the frequency determining network of the oscillator. The portion X represents the steady or fixed reactance in the absence of a modulating signal. The portion $\Delta X \sin\omega T$ represents the variable reactance component that is introduced by the reactance tube when the modulating signal is applied to it. The variable component changes according to the modulating signal and brings about frequency modulation. The fixed component X helps stabilize the center frequency of the oscillator.

In Figure 9–3A, capacitors C_2, C_3, and C_4 are coupling elements, while C_5, and C_6 are bypass capacitors. They can have, therefore, no effect on the frequency when their sizes are properly chosen. Capacitor C_4 acts as a feedback coupling capacitor to the oscillator grid. Capacitor C_2 acts as a coupling capacitor to the grid of V_1.

Effect With No Modulation Signal

The first effect to consider is that of the fixed value of reactance, X, on the frequency of the oscillator. This occurs when the modulator tube is connected across the tank circuit with no modulating signal applied. There is a voltage, E, across the tank circuit (L and C) which is connected between points 1 and 2. R_1 and C_1 in series are also connected between points 1 and 2; therefore, the same voltage appears across them; they are in parallel with the tank circuit. This voltage causes a current (I_1) to flow through R_1 and C_1. If R_1 is at least five times as large as the value of the reactance of C_1, this branch of the circuit is essentially resistive. The current I_1, then, can be considered as being in phase with the driving voltage E. Therefore, the vectors representing I_1 and E in Figure 9–3C are shown in the same direction (vertical). I_1 flowing through C_1 produces voltage drop E_1 across C_1.

Across the capacitor E_1 lags I_1 by 90°, as illustrated by their vectors in Figure 9–3C. The voltage E_1 is applied to the control grid of the modulator tube V_1, producing a corresponding plate current variation, I_p, through V_1. The plate current variations of V_1 essentially flow only through the LC tank circuit back toward the cathode of V_1, since the value of R_1 is high in comparison to the multiple impedance across points 1 and 2. Theoretically, the impedance

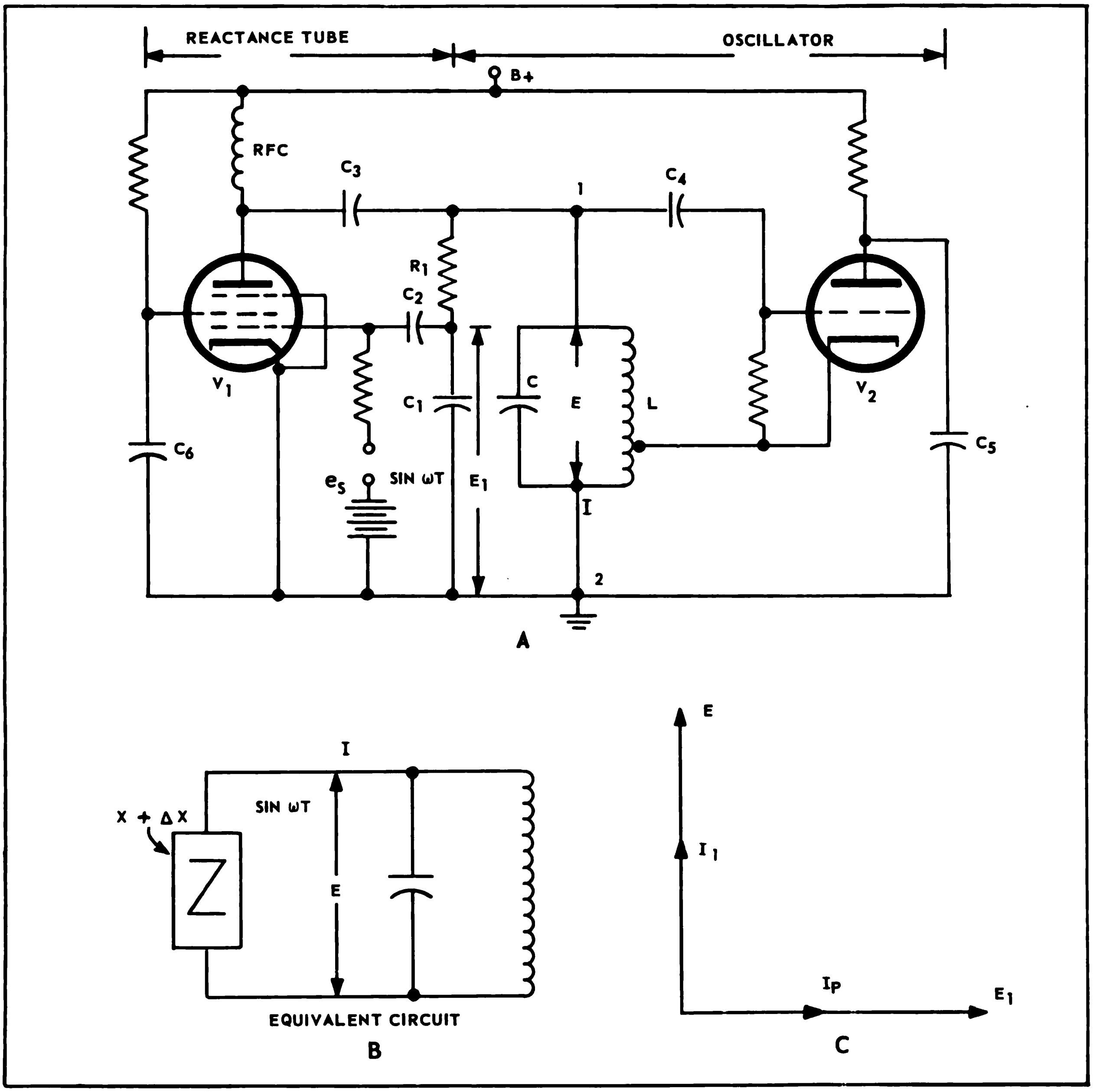


Figure 9–3. Reactance Tube Modulator

across the parallel LC circuit is infinite at resonance if there is no resistance in the circuit. However, in practice this parallel combination of L and C has a much smaller impedance than that offered by R_1C_1 branch. Since current is inversely proportional to impedance, most of I_p flows through the tank circuit.

As explained previously, E is the voltage across the tank circuit. The original tank current, labeled I, is in phase with E since the tank circuit is resonant. E_1 lags E by 90°; the plate current I_p, is in phase with the grid voltage E_1. In the LC circuit, then, I_p, is a superimposed current that lags the original tank current (I) by 90°. (This 90° out-of-phase action is why V_1 is sometimes called a quadrature tube.)

I_2, the resultant current due to I_p and I (Figure 9–4) lags I and E by some angle depending upon the relative amplitudes of I and I_p. Since I_2 is a lagging current, the impedance across the

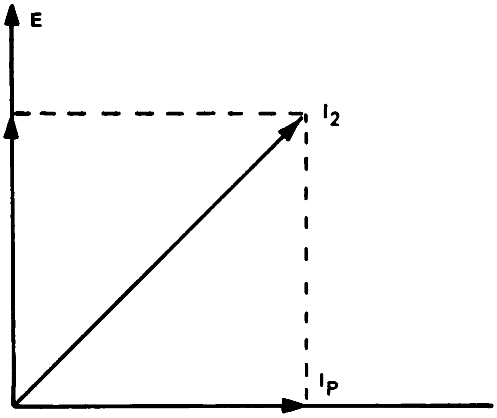


Figure 9–4. Effect of I_p

tank circuit cannot be at its maximum unless something happens. What does happen is that the frequency of oscillation of the tank changes until the voltages across the tank and the current flowing into it are again in phase. At this time the impedance across the tank is again maximum. This is the same result as if a reactance had been introduced in parallel with the LC tank circuit. Since the current I_p lags the voltage E by 90°, the introduced reactance is inductive. Earlier you learned that inductance decreases as inductors are added in parallel. Thus, in this case the frequency of the oscillator increases to a new fixed value due to the introduced reactance X.

If the components R_1 and C_1 (Figure 9–3A) were reversed in position, the grid voltage E_1 would be that across R_1. In this case, the reactance of C_1 should be at least five times the value of R_1, so the voltage E_1 will lead E by approximately 90°, as shown in Figure 9–5. The plate current I_p will then lead E by 90°. The introduced reactance X has a capacitive effect on the oscillating circuit, just as if a capacitor had been connected in parallel with it. A decrease in frequency occurs.

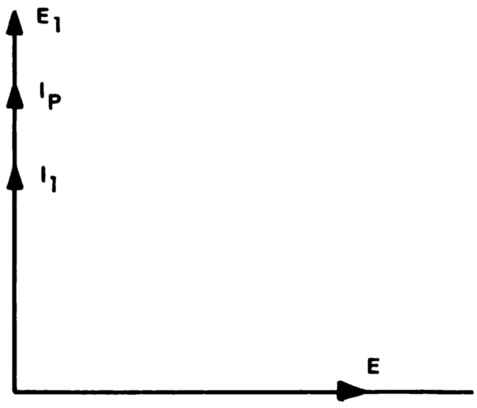


Figure 9–5. Effect of Reversing R_1 and C_1

Summary

The modulator tube is connected across an oscillator tank circuit. The modulator tube is made to appear as either a capacitance or an inductance by having its grid excited with a voltage which either leads or lags the oscillator voltage by 90°.

A leading grid voltage causes a corresponding leading plate current which makes the reactance tube appear as a capacitor in parallel with the oscillator tank circuit. This results in a decrease in the center frequency of the oscillator. A lagging grid voltage causes a corresponding lagging plate current which makes the reactance tube appear as an inductance in parallel with the oscillator tank circuit. This results in an increase in the center frequency of the oscillator. A reactance tube which acts as an inductance can be made to act as a capacitance only by making the proper circuit changes so the grid voltage leads the oscillator tank voltage.

Effect With Modulating Signal

The effect of connecting the modulator tube across the tank circuit when no modulating signal is applied has been shown. Now see what happens when a modulating signal is applied. The magnitude of the introduced reactance is determined by the magnitude of I_p for a given amplitude of grid voltage, E_1. The magnitude of I_p for a given E_1 is determined by the transconductance of V_1 (Figure 9–3). That is, $g_m = I_p/E_1$.

Therefore, it may be said that the value of reactance introduced into the tuned circuit varies directly with the g_m of the reactance tube. When the modulating signal is applied, both E_1 and I_p change, thus causing g_m to vary with the modulating signal. At this time a variable reactance, $\Delta X \sin\omega T$, is introduced into the tuned circuit. ω in this variable reactance either adds to, or subtracts from, the fixed value of reactance that is introduced in the absence of the modulating signal. It is this variable reactance that changes the frequency of the oscillator in accordance with the modulating signal. When properly designed and operated, the reactance tube modulator gives linear frequency modulation, and produces great frequency changes in accordance with the modulating signal.

Only one reactance tube modulator circuit has been presented. There are various types of reactance tube circuits that will produce frequency modulation. The main difference in the various arrangements is the type of phase-shifting circuit used to give a grid voltage that is 90° out of phase with the r-f voltage of the oscillator. All reactance tube modulators produce the same thing — linear deviation over quite a wide frequency band.

DETECTION

Previously it was stated that an amplitude-modulated waveshape contains the radio frequencies, but not the signal frequencies, and that it is the function of the detector in the receiver to reproduce the signal frequencies from the variation of the carrier and its sidebands.

Detection of amplitude-modulated signals requires a nonlinear electrical network. An ideal nonlinear curve for this is one that affects the positive half-cycles of the modulated wave differently from the negative half-cycles, and that so distorts a wave with an applied voltage of zero average value that the average resultant current varies as the signal frequency. The curve shown in Figure 9–6A is called an ideal curve because it is linear on each side of the operating point P and does not introduce harmonic frequencies.

When the input to an ideal nonlinear curve is a carrier and its sidebands, the output contains the following frequencies:

1. The carrier frequency.
2. The upper sideband.
3. The lower sideband.
4. A frequency equal to the carrier minus the lower sideband (or, a frequency equal to the upper sideband minus the carrier), which is the original *signal frequency.*

The detector reproduces the signal frequency by producing a distortion of a desirable kind in its output circuit. When the output voltage of the detector is impressed upon a low-pass filter,

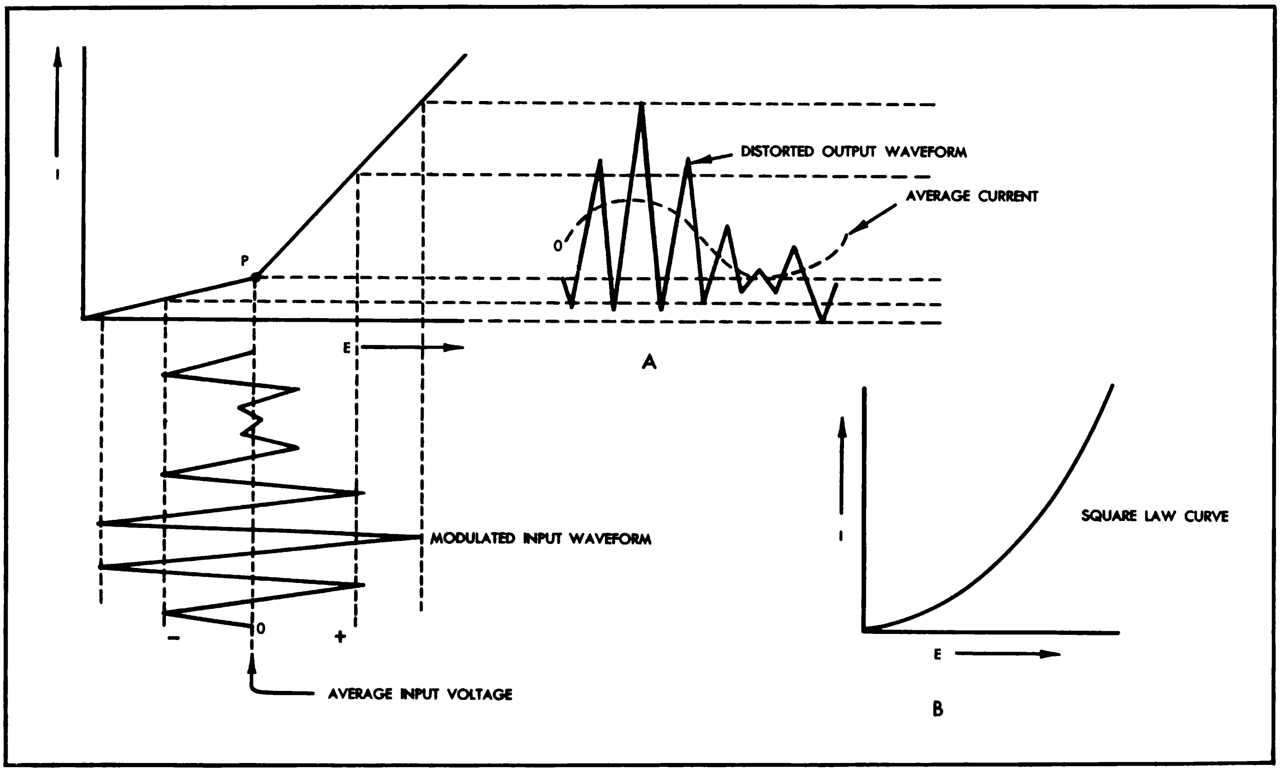


Figure 9–6. Nonlinear Device Used as Detector

which suppresses the radio frequencies, only the original or signal frequency is left.

In some practical detector circuits, the nearest approach to the ideal curve is the square-law curve shown in Figure 9–6B. The output of a device using this curve contains, in addition to all the frequencies which were listed, the harmonics of each of the input frequencies. The harmonics occur because voltage inputs that have a large amplitude are distorted differently from voltage inputs that have a low amplitude. The harmonics of radio frequencies can be filtered out, but harmonics of the signal frequencies, even though they produce an undesirable distortion, have to be tolerated. However, the square-law curve offers an advantage in that the output amplitude varies as the square of the input. In contrast, a linear curve produces an output which varies directly with the input. Therefore, the square-law device produces a greater output for a given input.

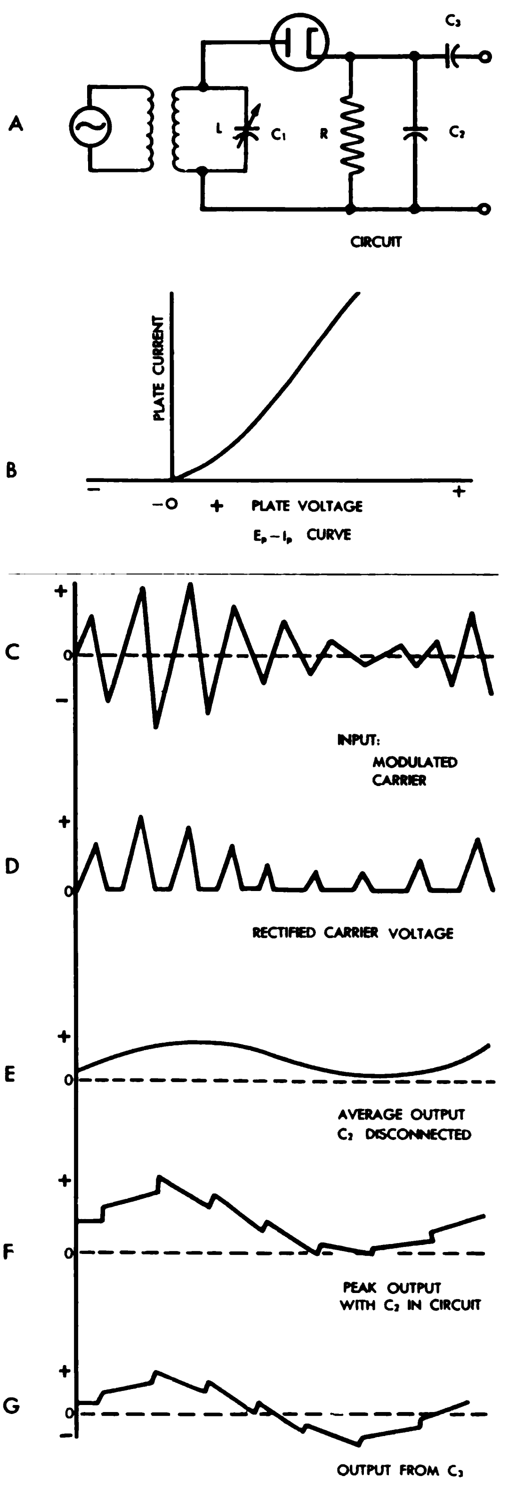


Figure 9–7. Diode Detector

Diode Detector

One of the simplest and most effective types of detectors and one with nearly an ideal nonlinear resistance characteristic is the diode detector. Notice the E_p-I_p curve in Figure 9–7B. This is the type of curve on which the diode detector at A operates. The curved part of the curve is the region of low plate current, and indicates that for small signals the output of the detector will follow the square law. For input signals with large amplitudes, however, the output is essentially linear in the positive direction from the operating point. This type of detector is classed as a power detector since it handles large input amplitudes without much distortion.

The modulated carrier is introduced into the tuned circuit made up of LC_1. This circuit is designed so that the receiver has high selectivity. The waveshape of the input to the diode plate is shown in Figure 9–7C. As a diode conducts only during positive half-cycles, this circuit removes all the negative half-cycles and gives the result shown in Figure 9–7D. The average output is shown at E. Although the average input voltage is zero, the average output voltage across R always varies above zero and has an average voltage of $\frac{1}{2} \times 0.637 \times$ peak voltage for any positive half-cycle.

The low-pass filter, made up of capacitor C_2 and resistor R, removes the r-f (carrier frequency) which, so far as the receiver is concerned, serves no useful purpose. Capacitor C_2 charges rapidly to the peak voltage through the small resistance of the conducting diode, but discharges slowly through the high resistance of R. The sizes of R and C_2 normally form a rather short time constant at the signal (audio) frequency and a very long time constant at the radio frequencies. The resultant output with C_2 in the circuit is a varying voltage that follows the peak variation of the modulated carrier (see Figure 9–7F). The DC component produced by the detector circuit is still in the waveshape but may be removed by capacitor C_3, producing the AC voltage waveshape in Figure 9–7G. In communications receivers the DC component is often used for providing automatic volume (gain) control.

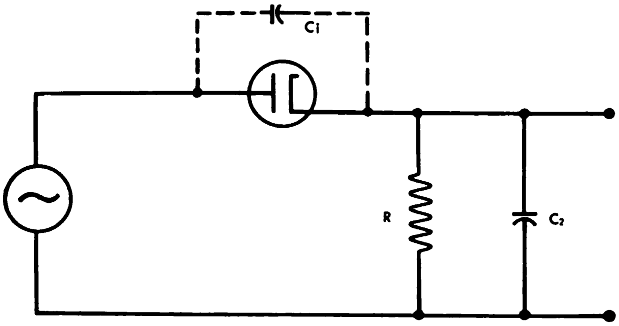
Advantages of diode detectors are as follows:

1. Ability to handle relatively high-power signals. There is no practical limit to the amplitude of the input signal.
2. Low distortion. Distortion decreases as the amplitude increases.
3. High efficiency. When properly designed, 90% efficiency is obtainable.
4. Diode detectors develop a readily usable DC voltage for the automatic gain control circuits.

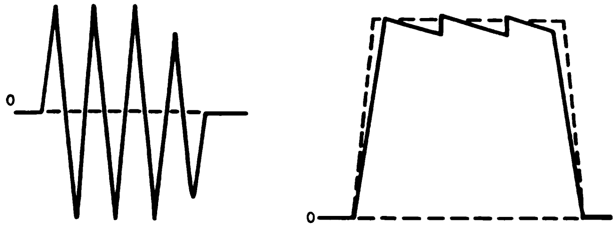
Disadvantages of the diode detectors are:

1. Power is absorbed from the tuned circuit by the diode circuit. This reduces the Q and selectivity of the tuned input circuit.
2. No amplification occurs in a diode detector circuit.

Careful selection of component parts is necessary for obtaining optimum efficiency in diode detector circuits. One very important fact to consider is the value of the time constant RC_2, particularly in the case of pulse modulation. When a carrier modulated by a square pulse (Figure 9–8B) is applied to an ideal diode detector, the waveshape shown in Figure 9–8C is produced. Notice that for clarity the amplitude of the wave at C is exaggerated in comparison to the high-frequency carrier shown at B.

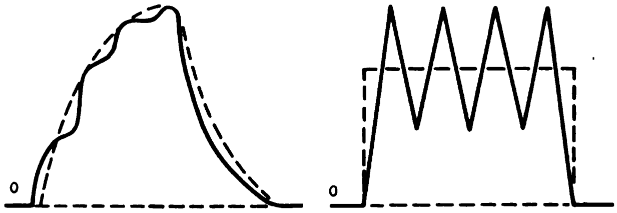


A CIRCUIT



B INPUT TO DETECTOR C IDEAL R C TIME CONSTANT

(DOTTED LINES INDICATE AVERAGE VOLTAGE DURING PULSE)



D RC TIME CONSTANT TOO LONG E RC TIME CONSTANT TOO SHORT

Figure 9–8. Diode Detector Component Considerations

If the time constant of RC_2 is too long, several cycles are required to charge C_2, and the leading edge of the output pulse is sloped as shown in Figure 9–8D. After the pulse passes by, the capacitor charges slowly and the trailing edge is exponential rather than square as desired. If the time constant is too short, both the leading and trailing edges are steep. However, the capacitor discharges considerably between cycles, and this reduces the average amplitude of the pulse, leaving a sizable component of the carrier frequency in the output as shown in Figure 9–8E.

For these reasons the selection of the time constant is a compromise between an output that is large but distorted, and an output that is undistorted but small and only partly filtered. To achieve this compromise, the load resistor R must be large, as the total input voltage is divided across R and the internal resistance of the diode when the tube is conducting. A large value of load resistance insures that the greater part of this voltage will be in the output where it is desired. On the other hand the load resistance must not be so high that capacitor C_2 becomes small enough to approximate the size of C_1, the internal capacitance of the tube. When this occurs, capacitor C_2 will try to discharge through C_1 during the nonconducting periods, which would reduce the amplitude of the detector output.

Grid-Leak Detector

The grid-leak power detector is similar to the diode detector with the advantage that it serves as a stage of amplification. Grid-leak detectors use no fixed bias arrangement in the grid circuit; they depend for their operation on the flow of grid current whenever the grid is positive.

In the typical grid-leak circuit illustrated in Figure 9–9A, the grid, cathode, C_4, R_1, and the tuned circuit form a diode detector circuit. If capacitor C_4 and resistor R_1 are correctly proportioned, the capacitor charges during the period of current flow and discharges slowly through R_1 during periods of no current flow. Since the capacitor maintains an average charge, it places an average negative voltage on the grid with respect to the cathode. (This voltage is represented by the heavy line in the grid voltage curve in Figure 9–9D.)

The r-f voltage at the grid varies around this average value. If the capacitor should discharge slightly, the next positive cycle of the modulated voltage will go higher in the positive direction, recharging the capacitor. As the modulated waveform decreases in amplitude (that is, as the positive peaks become smaller), the capacitor does not fully recharge with each cycle, and the average charge (grid bias) reaches a lower value.

This means that the average grid voltage varies in accordance with the variation of the amplitude of the modulated wave and, in doing so, reproduces the signal voltage. Consequently, the plate current and the plate voltage shown in Figure 9–9E also vary at the signal rate. The low-pass filter in the plate circuit prevents the plate voltage from changing at an r-f rate and thus removes the radio frequencies from the output. The normal gain of the triode stage produces an amplified waveshape in the plate circuit. The time constant of C_4R_1 is governed by the same factors as the filter used in the diode detector.

The voltage applied to a grid-leak detector must not be so high that it causes the average grid voltage to exceed the plate current cutoff voltage for the tube. This characteristic limits the power handling capacity of the grid-leak detector. Like the diode detector, the grid-leak detector produces little distortion when the received signal is strong. Although it produces

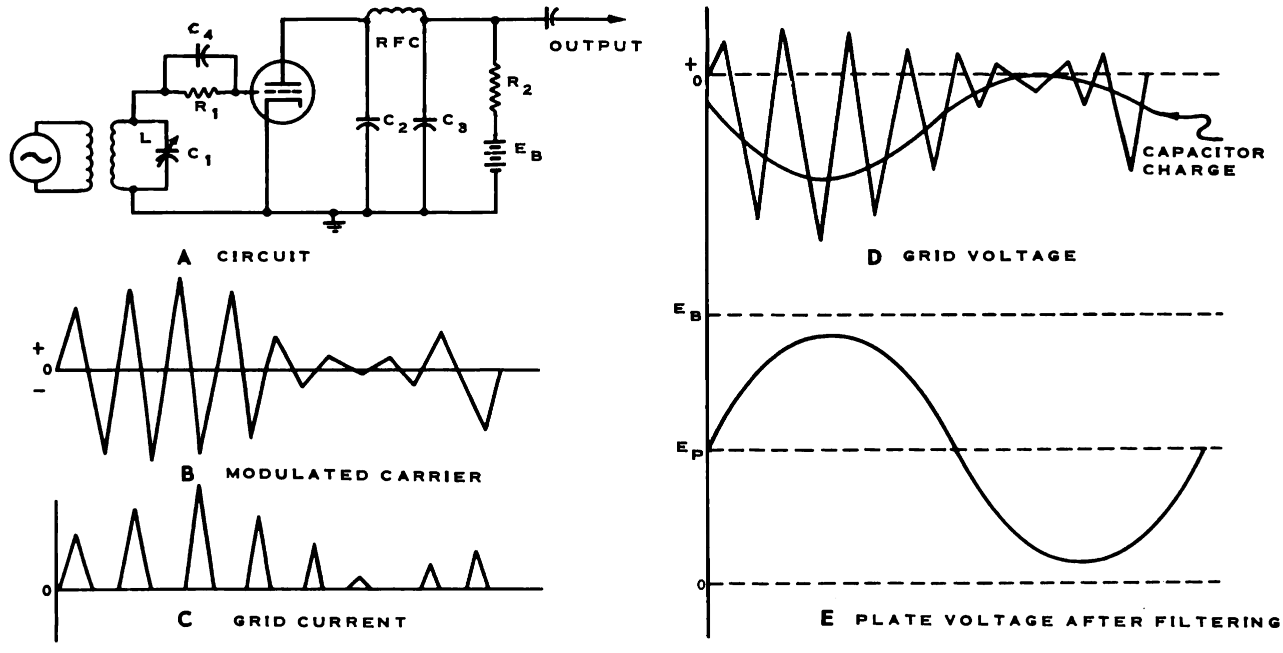


Figure 9–9. Grid-Leak Detector

only a limited amount of DC voltage for AVC, the grid current required for this purpose reduces the Q of the preceding stage. Due to the amplification properties of a grid-leak circuit, its output for a given input is much greater than that of the diode detector.

Plate Detector

The plate detector shown in Figure 9–10B, sometimes called a bias detector or plate detector, is similar to a class A amplifier stage except that it is operated at very near cutoff bias. For ideal operation, this bias may be provided from a fixed DC potential, but for practical purposes, a properly bypassed cathode resistor will bias the tube near enough to cutoff. For this reason R_k, the cathode resistor, shown in Figure 9–10B, is rather large, and the bypass capacitor C_k is large enough to bypass both carrier and modulation frequencies. Because of the extremely high operating bias, the tube characteristics are very nonlinear as you can see by examining the E_g-I_p curve at A. Plate current flows only during positive halves of the input signal and a small portion of the negative halves. The grid does not draw current as long as the signal does not drive the grid into the positive region of the E_g-I_p curve. The average plate current, shown at A by dashed lines, varies as the modulation varies.

The removal of the r-f component from the plate circuit is accomplished by the low-pass filter, formed by the choke RFC and capacitor C_1 and C_2. The variation remaining after removal constitutes the original modulation. Because of the amplification factor of the tube, the voltage representing the modulation which appears across the load resistor is greater than the input (grid) voltage. All other AC voltages are effectively placed at ground potential by capacitor C_1.

When a pulse-modulated signal is applied to the detector, a pulse appears as a positive pulse at the grid and as a negative pulse in the plate circuit. In some applications positive output pulses are desired. This is done by changing the circuit so that R_L is located in the cathode-to-grid circuit instead of in the plate circuit. By this arrangement the output has the correct polarity and may be taken across R_L. The other ground connections are removed, as you can see by examining the inverted output circuit for pulse type modulation in Figure 9–10C.

The advantage of the plate detector is that since the grid does not draw current, the Q of the input circuit remains the same and amplification occurs in the circuit. Its disadvantages are higher distortion and lower power handling ability than the diode detector.

Infinite-Impedance Detector

The infinite-impedance detector is a modified type of plate detector. It has the same ability

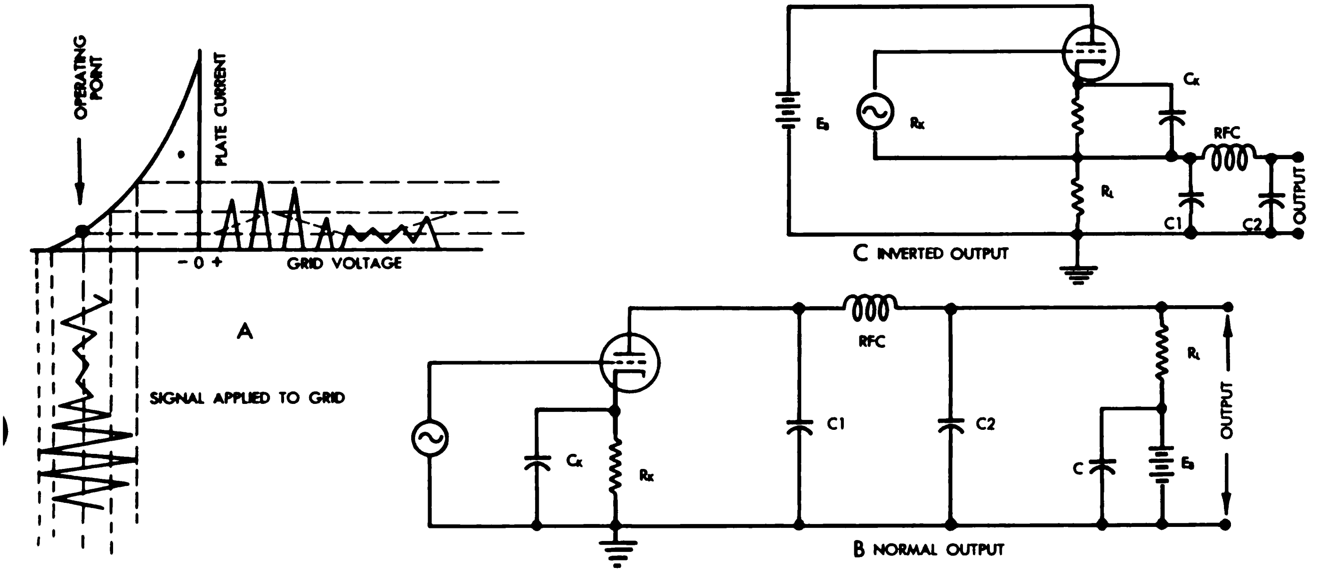


Figure 9–10. Plate Detector

as the diode detector to handle large input amplitudes with little distortion but without drawing current from the circuit driving it. Drawing current by any type of detector causes reduction in the Q of the tuned circuit and a decrease in the selectivity.

In Figure 9–11, which shows the infinite impedance detector, E is the modulated carrier voltage. LC_1 is a tuned circuit that contributes to the selectivity of the receiver. The plate of the triode is connected directly to the power supply. Capacitor C_3 is a virtual short circuit to ground for AC voltages. The load resistance R_1 is in series with the cathode. Capacitor C_2, in conjunction with R_1, forms a long RC to the carrier frequency but a short RC to signal frequencies.

Under no-signal conditions the plate current through R_1 provides a bias that keeps the grid very near cutoff. Under signal conditions when an r-f voltage is applied, the positive peak of the modulated voltage charges capacitor C_2 to a higher voltage. This capacitor holds its charge between peaks because of the long time constant of R_1C_2. No grid current flows during the positive peaks because the cathode bias changes at the same time that grid voltage changes, and the cathode-grid voltage difference is never great enough to exceed the cutoff value. When the crest of the modulation peak has passed, the amplitude of the peak decreases and the capacitor discharges at the same rate. Therefore, the capacitor charge follows the modulation peaks in the same manner as in the diode detector.

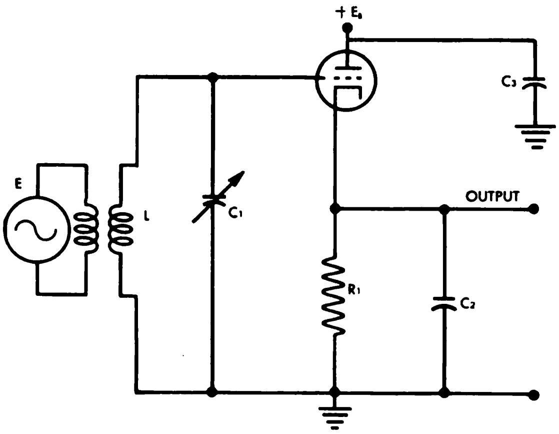


Figure 9–11. Infinite Impedance Detector

The infinite-impedance detector employs a very high input impedance for two reasons. First, the grid does not draw current. Second, because of the capacitance in the plate circuit, the impedance reflected into the grid circuit is purely resistive.

Another characteristic of this detector is that, like the cathode follower, negative feedback exists because of the common input and output impedance at signal frequencies. This negative feedback is responsible for reducing the distortion due to characteristics of the vacuum tube.

Distortion is also minimized by the employment of high input voltages. This input voltage can be as large as half the DC voltage on the plate without overdriving the tube. The ability of the tube to handle large signals is due largely to the fact that the bias increases as the signal input increases.

Crystal Detector

A small piece of silicon, galena, iron pyrite or carborundum is capable of providing the nonlinear characteristic required for detection. When a crystal of one of these elements is in contact with a sharp pointed wire, the arrangement offers a different resistance to current flow from the crystal to the wire than it does from the wire to the crystal. Such a device is called a crystal rectifier or a crystal detector. (Do not confuse this type of crystal with the piezoelectric crystal used to control the frequency of oscillators.)

For use in radar sets, the crystal is silicon and the wire is tungsten. Figure 9–12D shows the construction. Since some spots on a crystal are more sensitive than others, the point of the tungsten wire is placed in contact with the most sensitive part, and the contact is made permanent by using a sealing compound and mounting the assembly in a watertight, standard-sized cartridge.

Crystals are most frequently used in the mixer stage in the superheterodyne receivers of radar sets. Seldom are they used as detectors to demodulate the modulated carrier. Their use as mixers requires the same nonlinear characteristics as are required in a detector circuit.

Mixer crystals are designated according to use and frequency. The crystal suitable for 10-cm waves comes in at least three types, as does the crystal for 3-cm waves. Three types for each

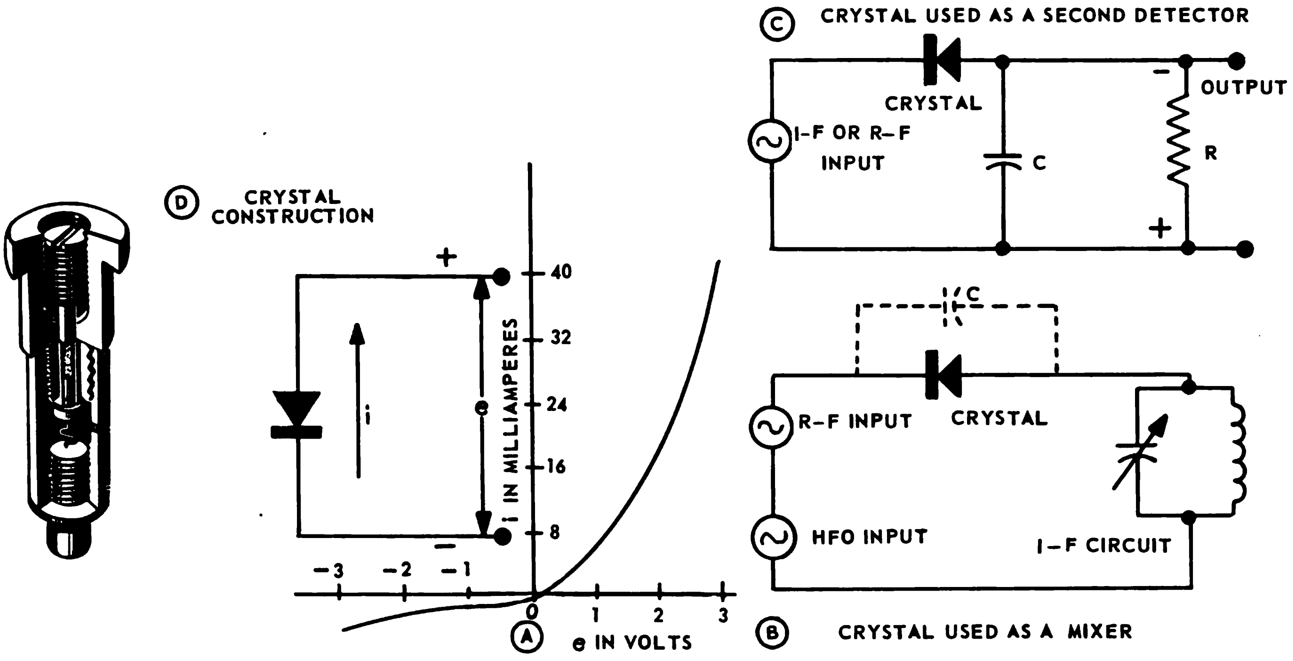


Figure 9–12. Crystal Detector

frequency are characterized by poor, medium, and high-frequency sensitivity. Another type of crystal is used for detector circuits.

As shown in the crystal mixer circuit in Figure 9–12B, the radio-frequency and the local oscillator signals are applied to the resonant circuit through the crystal. The nonlinear characteristics of the crystal cause the voltage across the resonant circuit to contain the sum and difference frequencies and, in addition, the original frequencies. The coil-capacitor circuit resonates to the difference frequency. The other frequencies are rejected and the difference frequency is applied to the i-f amplifiers.

The second detector circuit in Figure 9–12C using the crystal operates similarly in all respects to the diode detector.

Notice in the DC voltage-current characteristic curve of a crystal in Figure 9–12A that the current flow in the crystal, caused by negative voltages, is very small.

You can see from the curve in Figure 9–12A that with two volts negative applied, the current is approximately 2 ma in the reverse direction. Computing the resistance of the crystal we find it to be approximately 1000 ohms. With two volts positive, the curve shows the resistance is about 100 ohms. The back-to-front ratio, or the ratio of the resistance to current flow in a forward direction, to the resistance in the opposite direction, is 10 to 1 for this crystal.

When the crystal is used as a rectifier at 3000 mc, the capacity between the point and the crystal is about 1 mmf, making the capacitive reactance about 50 ohms. This reactance is in parallel with the 1000-ohm DC resistance, making the back impedance about 47 ohms.

Measuring the resistance of a crystal in each direction with an ohmmeter is a rough test of crystal sensitivity. When you know that the back (high) resistance of a crystal has dropped to one-half the original value, the sensitivity of the crystal is too low for that crystal to be used. The only solution is to replace the crystal with a known good crystal.

An ohmmeter test is not conclusive because of the change in back impedance at high frequencies. Since the factory test equipment is not available in the field, you may test the crystal at high frequencies by comparing the signal-to-noise ratio of the receiver with a questionable crystal installed against this ratio with a good crystal installed.

Even though the contact point causes reduced sensitivity, the crystal is superior to vacuum tube mixers at 3000 and 10,000 mc since it introduces less noise into the receiver system. This means a better signal-to-noise ratio. In addition, there are no transit time difficulties involved in crystals. Disadvantages of crystals are that they damage easily, even while installed in the radar set, and have a gain of less than one. Damage in the radar set usually results from inadequate protection from the transmitter pulse. As a second detector, the crystal has the advantage of being smaller in size than vacuum tube detector circuits and requiring no heater voltage.

Superregenerative Detector

The superregenerative detector is an extremely sensitive detector. It has a gain comparable to a complete superheterodyne receiver. On examining the circuit of the superregenerative detector in Figure 9–13, you can see that it consists of an r-f oscillator and a tuned circuit that is resonant to the frequency being detected. The superregenerative detector also includes a special circuit that allows the detector to break into oscillation, then quench, or stop the oscillations at regular intervals. This action is accomplished by variations in the detector plate voltage. When the plate voltage is above a certain value, the circuit oscillates; below this value, it ceases oscillations.

The voltage supplied by the special circuit is called the quench voltage. It varies the plate voltage above and below the value just mentioned at a rate above the audible range as shown in Figure 9–13C. As the plate voltage gets near what is called the critical value, small irregular current changes in the cathode emission cause the plate voltage to rise above the critical point, and the circuit oscillates until the plate voltage drops below the critical value. When the plate voltage comes back up again, the process is repeated. When a radio signal is applied to the tuned circuit, the signal adds enough voltage to the grid circuit to cause oscillations at a somewhat lower value of plate voltage. This causes oscillation to start sooner, and to last until the plate voltage drops back to the critical value.

Under signal conditions, the duration of the oscillation cycle is longer than under no signal conditions. Within limits this time varies directly with the signal strength. Short periods of oscillation build up small charges on the grid leak capacitor, while long periods of oscillation cause

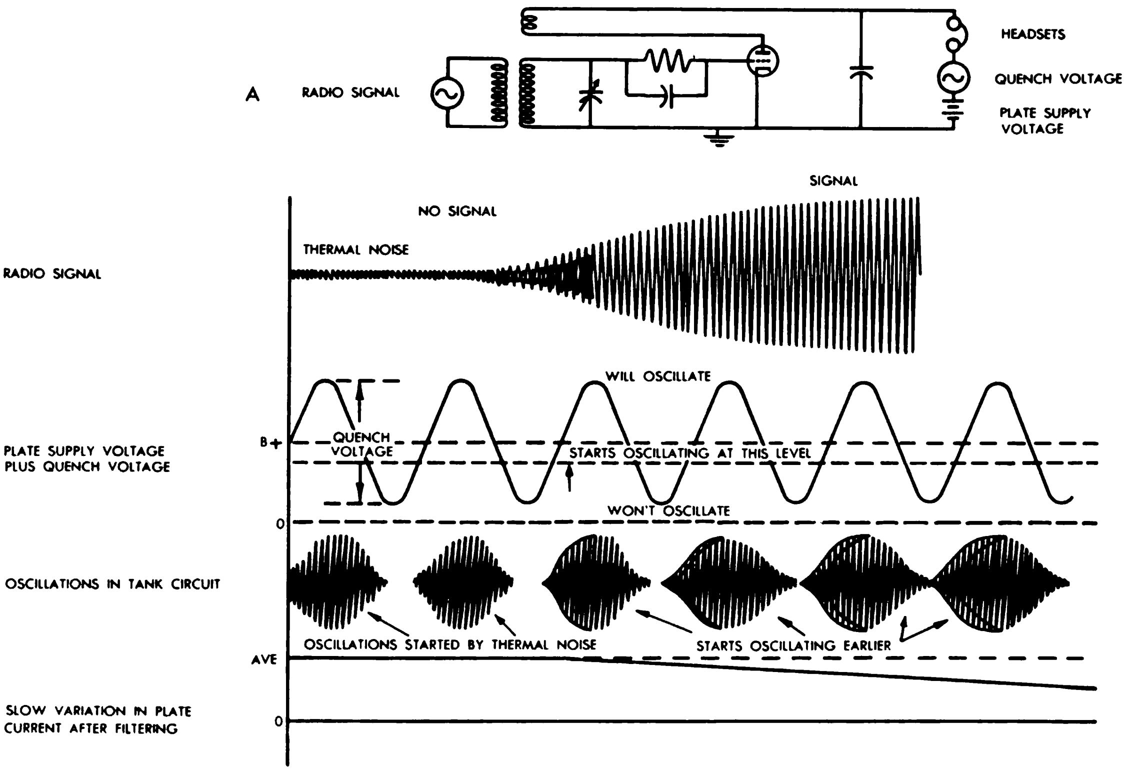

A
RADIO SIGNAL
HEADSETS
QUENCH VOLTAGE
PLATE SUPPLY VOLTAGE
SIGNAL
NO SIGNAL
THERMAL NOISE
RADIO SIGNAL
WILL OSCILLATE
B+
QUENCH VOLTAGE
STARTS OSCILLATING AT THIS LEVEL
PLATE SUPPLY VOLTAGE PLUS QUENCH VOLTAGE
WON'T OSCILLATE
0
OSCILLATIONS IN TANK CIRCUIT
OSCILLATIONS STARTED BY THERMAL NOISE
STARTS OSCILLATING EARLIER
AVE
SLOW VARIATION IN PLATE CURRENT AFTER FILTERING
0

Figure 9–13. Superregenerative Detector

grid current to flow longer and consequently build up greater charges. The charge, in turn, determines the average plate current and it, in turn, varies with the signal strength. If the signal is amplitude-modulated, the variation in plate current will roughly resemble the modulation voltage.

Since the incoming radio signal only initiates the oscillation while the circuit itself determines the amplitude, the circuit produces a tremendous gain when properly adjusted. In fact, any signals stronger than the thermal noise level are audible. Since this is the limit of amplification in a superheterodyne receiver, the sensitivity of a superregenerative detector is near that of the superheterodyne receiver.

The superregenerative detector has the advantage of high gain and light-weight components. It finds its greatest use at very high frequencies where distortions and noise are compensated for by the high gain, which is not obtainable with conventional circuits. Another advantage is that the output voltage is about the same for weak or strong signals. Weak signals bring it to full oscillation equally as well as strong signals do. (For this reason any noise voltages stronger than the signals are reducible to the level of the signal. Suppressing ignition noises and other types of interference further reduces the annoyance to the listener.)

Its disadvantages are excessive distortion, poor selectivity, and a characteristic hiss when no signal is being received. The poor selectivity is because the detector responds to signals off frequency as well as to those on frequency, even though the amplitude of the off-frequncy signal is less. The hiss, when no signal is being received, is from the noise produced by the random thermal voltages in the tube that start oscillations at irregular times. The hiss disappears when a signal is present, since the signal has a definite amplitude and since the oscillations start at a definite time. The distortion in the detector results from grid current clipping the positive peaks and from accentuation of the negative modulation peaks.

FREQUENCY CONVERSION

Many stages of amplification are required in a radar receiver to make a very weak echo voltage at the antenna give a good indication on the radar indicator screen. In the simplest type receiver, there are several r-f stages followed by a detector and by a video amplifier. In spite of this, the gain (amplification) and selectivity of an r-f amplifier decreases as the operating frequency increases. Therefore, it is desirable to convert the high radio frequency to a lower radio frequency, and then to amplify the lower frequency. Furthermore, it is easier to design amplifiers with a higher gain when the frequency is constant. The modern receiver employs a high gain and highly selective amplifier in the intermediate-frequency stage, designed to operate at the one frequency to which all received frequencies are converted.

Frequency conversion in a superheterodyne receiver is accomplished by mixing the r-f frequency with the local oscillator frequency (Figure 9–14). This mixing produces a frequency equal to the difference of the two frequencies. If the two frequencies are nearly the same, the difference frequency will be low, but not as low as the audio frequencies. This frequency lies between

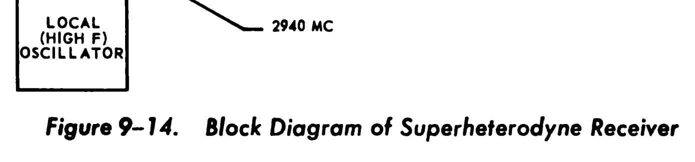


Figure 9–14. Block Diagram of Superheterodyne Receiver

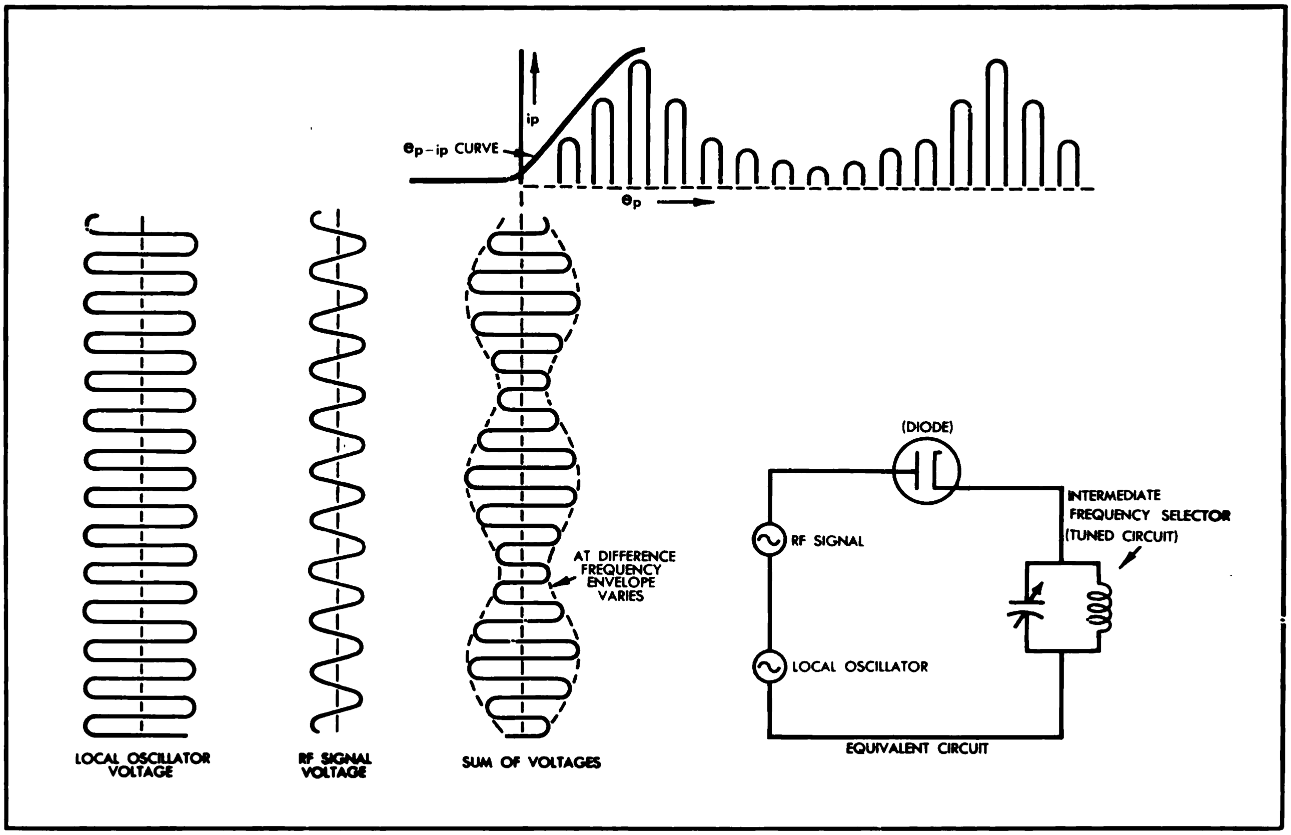


Figure 9–15. Mixer Action

the carrier and signal frequencies, and is called the *intermediate frequency*. If the carrier frequency is below 1000 mc, it is usually amplified by an r-f amplifier before being fed to the mixer. If higher, the r-f signal is introduced directly into the mixer. After mixing, the intermediate frequency is amplified by an i-f amplifier and then is detected in the same manner as any other signal.

Production of a difference frequency in the mixer stage requires the use of a nonlinear electrical circuit for the generation of the new frequency. The mixer, as this circuit is called, produces the new frequency in the same way as a modulator or a detector. For this reason this stage was often called the *first detector.*

When two slightly different frequencies are added, as shown in Figure 9–15, the waveshape representing the combination of the local oscillator frequency and the r-f signal contains only these two frequencies. After distortion by the mixer the waveshape contains not only these two frequencies, but a frequency equal to the sum of the two frequencies and another equal to the difference between the two frequencies. Since it is the difference frequency that is desired, the tuned circuits in the i-f amplifier select this frequency and the stage amplifies it. If the r-f signal is modulated, the difference frequency (i-f) will also be modulated in the same manner.

Any of the detector circuits previously discussed will work in this stage. Grid-leak and plate detectors are used in the mixer stages of communications receivers in order to utilize the gain of the triode or pentode used. In low-frequency radar sets, the diode is sometimes used. In high-frequency sets, however, the transit time in the tube and the thermal noise it introduces make the vacuum tube inferior to the crystal.

Most 10-cm and all 3-cm sets use the silicon crystal for the nonlinear element in the mixer stage. In the simplified crystal mixer circuit shown in Figure 9–16, currents from both the r-f signal and local oscillator signal flow through the crystal. The difference frequency is selected by the tuned circuits which are connected to the grid of the first amplifier tube. Any nonlinear device produces a unidirectional current that may

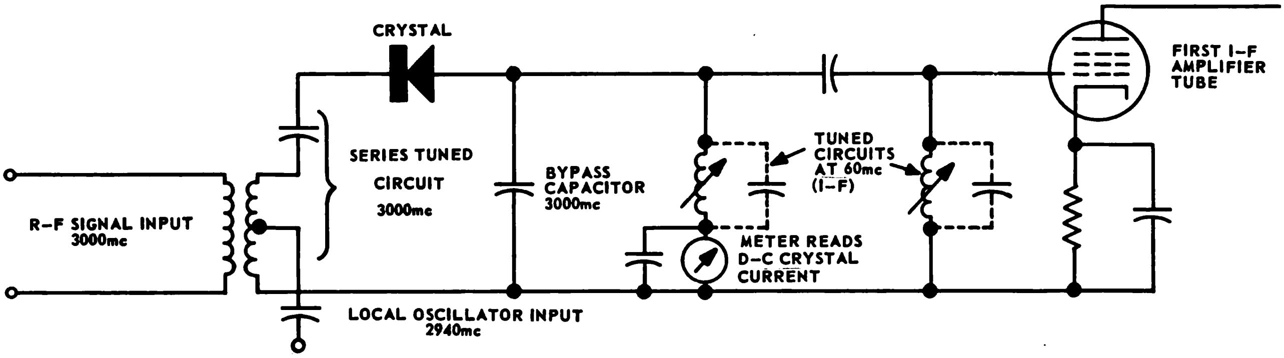

CRYSTAL
FIRST I-F AMPLIFIER TUBE
SERIES TUNED CIRCUIT 3000mc
BYPASS CAPACITOR 3000mc
TUNED CIRCUITS AT 60mc (I-F)
R-F SIGNAL INPUT 3000mc
METER READS D-C CRYSTAL CURRENT
LOCAL OSCILLATOR INPUT 2940mc

Figure 9–16. Crystal Mixer

be read on a meter. The adjustment of the oscillator for maximum output is determined by observing the meter reading, which varies directly with the amplitude of the oscillator signal.

DISCRIMINATOR CIRCUIT

Even though the design and construction of radar equipment is precision work, the frequency of a transmitter may shift. This causes considerable trouble at the receiver. Since the transmitter is expected to operate on just one frequency, the local oscillator in the receiver is set on the frequency which, when combined with the input signal (same frequency as the transmitter), will give the desired intermediate frequency. When the transmitter shifts frequency, the difference between this new frequency and the local oscillator frequency will no longer give the preset i-f.

Signals which supply weak returns may no longer be visible on the indicator screen. Any change of the transmitter frequency, therefore, requires either continued readjustment of the receiver tuning by the operator or the use of an automatic tuning device. Constant frequency checking by the operator not only is time consuming, but certainly is not in the interest of highest efficiency. Therefore, most radar receivers have an automatic frequency control circuit in which changes of frequency are detected by a discriminator circuit.

Circuit Operation

In a discriminator, a transformer, due to special connections, distorts the frequency change into a voltage of varying amplitude. After distortion, this voltage of varying amplitude is fed to an amplitude modulation detector. Figure 9–17 shows the circuit of a typical discriminator circuit.

The two coils L_1 and L_2 are the primary and the secondary, respectively, of the i-f transformers. The secondary coil is tuned to the correct transmitter frequency by capacitor C. Tubes V_1 and V_2 are diode detector tubes. The filters R_1C_1 and R_2C_2 serve the same purpose as the filters in the diode detector, that is, the removal of the r-f component from the circuit.

The presence of L_3 and its connection to the primary at the top and to the secondary at the center presents an unusual feature in the discriminator circuit. L_3 is an r-f choke which has a high reactance to the radio frequency. Since it is connected across the primary, the primary voltage appears across it at all times. The connection to each diode from this choke causes the primary voltage (E_p) to appear at the plates of the diode with the same phase shift in each case. This voltage causes currents to flow in opposite directions in resistors R_1 and R_2, resulting in

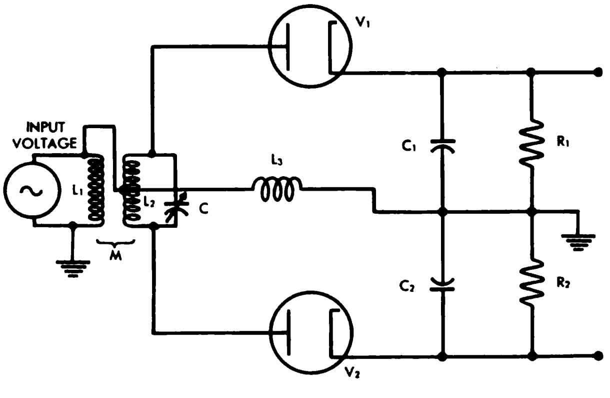


Figure 9–17. Discriminator Circuit

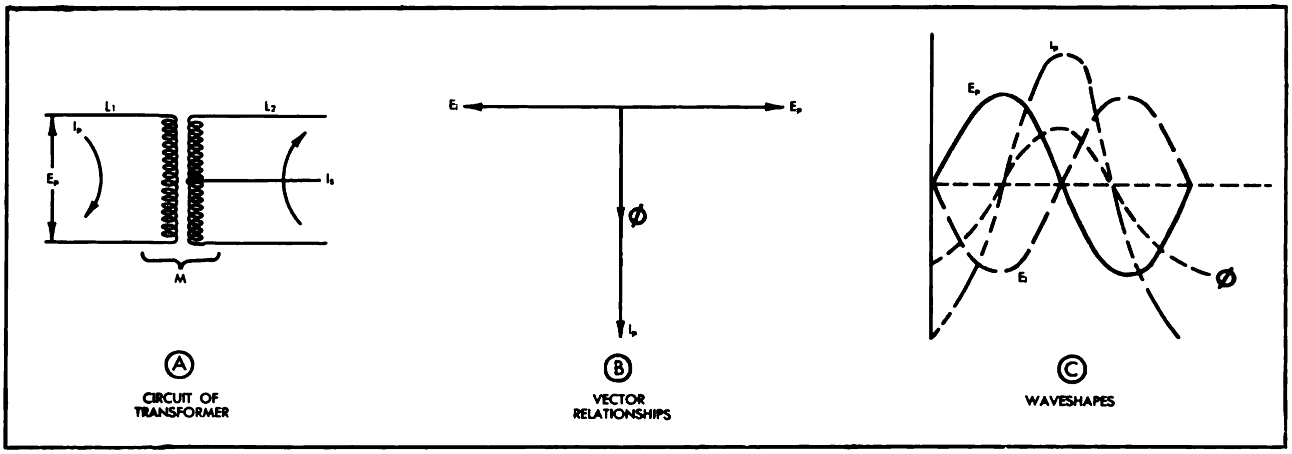


Figure 9–18. Transformer Vector Relationships and Waveshapes

zero output. Therefore, the discriminator is not affected when the amplitude of the applied voltage changes.

The phase relationship of the voltage across the r-f choke to the voltage induced in the secondary is the key to the operation of the discriminator circuit. For this reason you will find it is desirable to analyze its operation in detail. To make the analysis, refer to Figure 9–18 showing the discriminator transformer at A and the vector relationships of the current and voltages in the transformer and their waveshapes.

At the desired frequency the transformer vector relationships are as follows: The primary voltage E_p is the reference vector. Since the mutual inductance (coupling) is small, the primary is inductive. The primary current I_p lags E_p by 90°. The magnetic field which affects the secondary (called the *flux* and shown by the symbol ϕ) is in phase with the primary current. Due to normal transformer action, the voltage induced in the secondary is 90° behind the flux.

This induced voltage, labeled E_i, is simulated by a generator in the equivalent circuit of the secondary shown in Figure 9–19A. The generator is inserted at the center of the transformer secondary where the voltage across the secondary is divided between the two tubes. A current (I_s) flows around the loop composed of the secondary windings and capacitor C. This loop is a series-resonant circuit insofar as the generator sees it. Since the circuit is at resonance, the current I_s is in phase with the induced voltage.

Each half of the secondary has considerable inductive reactance at the radio frequency. Therefore, there is a voltage drop across this

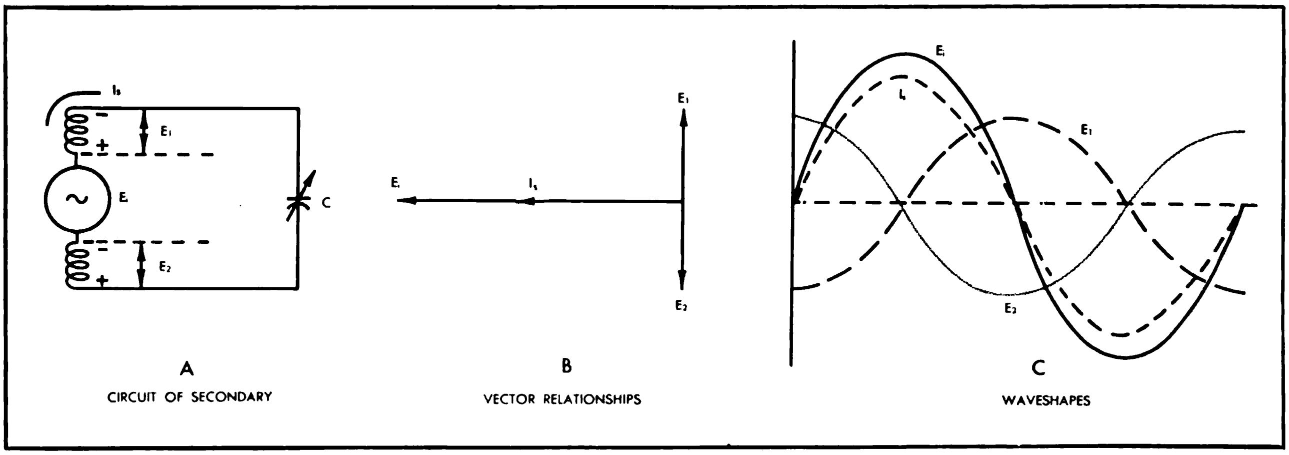


Figure 9–19. Secondary Voltages at Resonance

reactance caused by the current I_s. These voltages are shown in Figure 9–19A in the secondary circuit as E_1 and E_2. Since voltage leads current by 90° in an inductor, E_1 and E_2 are 90° out of phase with I_s. The voltage E_2 actually leads I_s as it should, but the voltage E_1 is 180° out of phase; that is, its polarity is reversed, due to the manner in which it is connected to the tube. Current flows in the same direction through both coils, but the vector voltages are both measured with respect to the center. This causes opposite polarities to exist.

Note in Figure 9–20A, the equivalent circuit of the discriminator in which the voltages are replaced by generators, that the plate voltage on V_1 is the sum of the voltages E_p and E_1, and the plate voltage on V_2 is the sum of E_p and E_2. The vector diagram at B shows E_p and E_1 added vectorially to produce a vector sum E_{V1}. E_{V1} is the actual plate voltage on V_1. The two sine wave voltages E_p and E_1 are also shown. They add to produce a third sine wave called E_{V1}. When plate current flows, C_1 charges to near the peak voltage of E_{V1}, producing a constant DC voltage almost equal to E_{V1} across resistor R_1. In the same manner, E_p and E_2 add, as E_p and and E_1 did previously, resulting in E_{V2}, the plate voltage on the other diode. Consequently, C_2 charges to E_{V2} and a constant voltage almost equal to E_{V2} appears across R_2 as shown in diagram C. The vector and sine waves are shown at C.

Observe that the voltage across the filters R_1C_1 and R_2C_2 are DC voltages. Obviously, phase relationships cannot exist there. In this case only amplitude relationships are important. Since E_p is common to both, and E_1 equals E_2, the vector sums of E_{V1} and E_{V2} are equal. The sine-wave illustrations show that the resultants of the sine waves, although of different phase, have equal amplitudes. The DC voltages across each filter are alike, but have opposite polarities. Therefore, the sum of these voltages will be zero between X and Y. This fact indicates that the output at resonance is zero.

Output at Frequencies Below Resonance

When the frequency is lower than the center, or resonant, frequency, the output is no longer zero. With the primary voltage as a reference, the voltage induced in the secondary is still 180° out of phase, as shown by the vector diagrams in Figure 9–21A. Since inductive reactance decreases as frequency decreases, while capacitive reactance increases, the secondary circuit of the transformer in Figure 9–19A will be capacitive at frequencies below resonance. The current in a

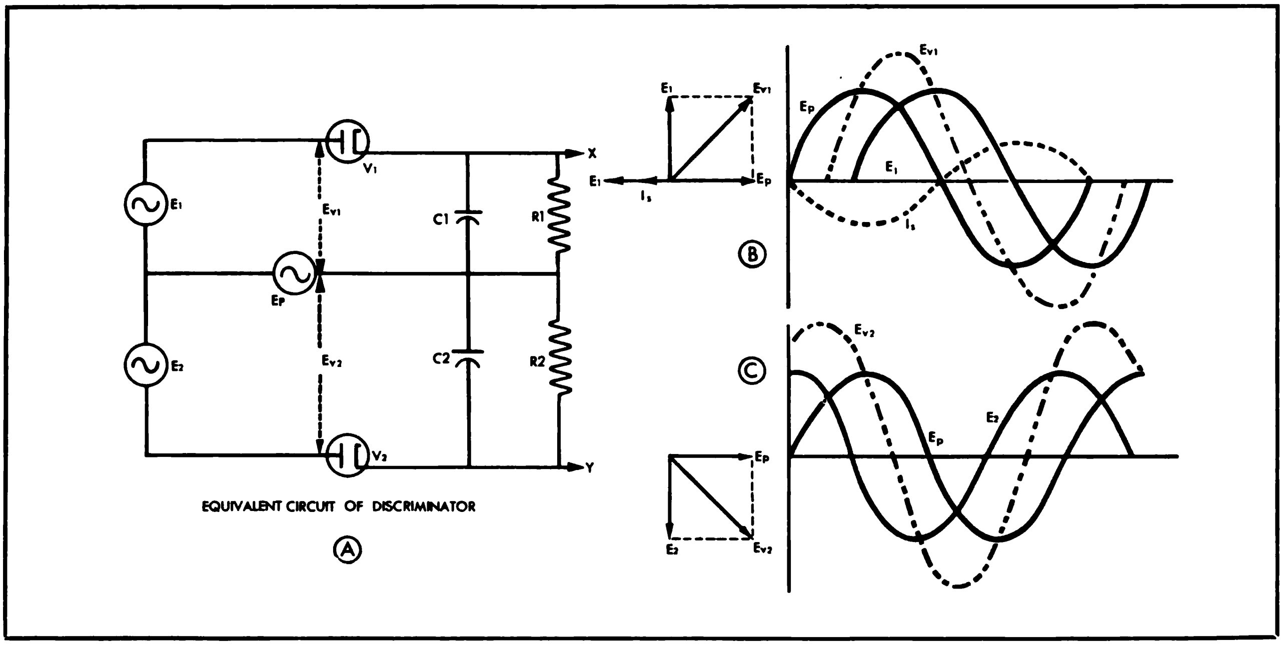


Figure 9–20. Resultant Plate Voltages at Resonance

capacitive circuit leads the applied voltage. Here, the applied voltage is E_i, and I_s will lead.

An arbitrary amount of lead is used in the vector diagram in Figure 9–21A. The voltage drops E_1 and E_2 will still be 90° out of phase with the current since this is not a function of frequency. Since under these conditions voltage E_1 is more than 90° away from E_p when added vectorially, their sum will be less than at resonance even though the magnitude of the components E_p and E_1 has not changed. At the same time, E_2 is less than 90° away from E_p — or they are more nearly in phase — so the magnitude of the resultant E_{V2} is greater than at resonance. Again, since the resistor voltages are proportional to the length of the resultant vectors, there will be a high voltage across R_2 and a lower voltage across R_1. Therefore, the output voltage will be equal to the R_2 voltage minus the R_1 voltage and the output point X, Figure 9–20, will be negative with respect to Y which may be grounded.

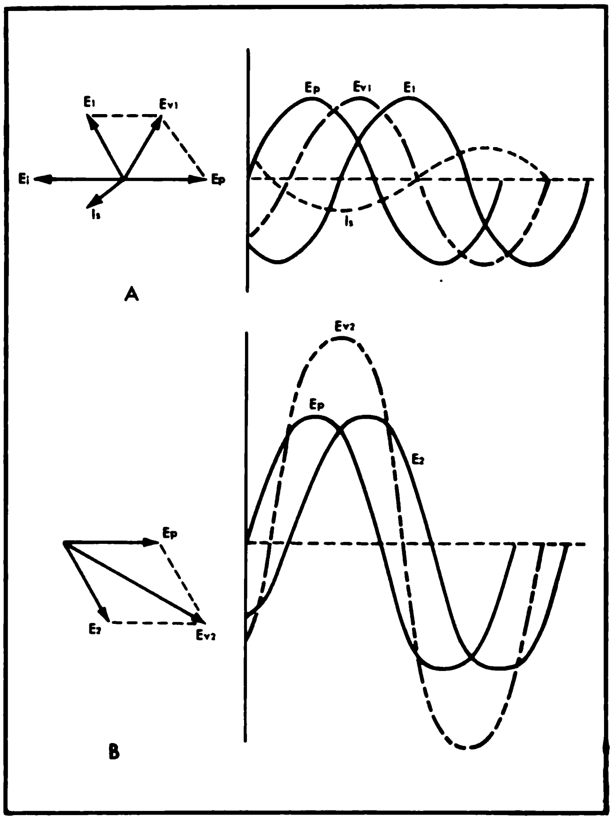


Figure 9–21. Voltages at Frequencies Below Resonance

The waveshape shows the same thing. I_s is shifted in phase; voltage E_1 is further out of phase with E_p, and E_2 is more nearly in phase with E_p. The sum of the more out-of-phase components is small; but the resultant waveshape of the two nearly in-phase components is large.

Studying the vector diagrams, you can see that the greater the deviation from resonance, the greater the DC output voltage.

Output at Frequencies Above Resonance

When the input frequency is greater than the center frequency, the circuit L_2C is inductive, and the current I_s lags the induced voltage. Figure 9–22 shows the vector relationship of voltages at frequencies above resonance. Note that at frequencies above resonance, E_1 is more nearly in-phase, and E_2 is more out of phase than at the center frequency. Also notice that E_{V1} has a greater magnitude than E_{V2}. Therefore, the DC voltage across R_1 is greater than the R_2 voltage, and consequently the output voltage is positive.

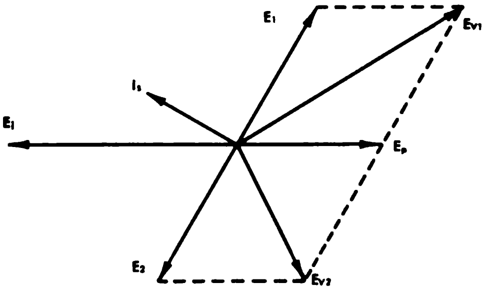


Figure 9–22. Voltages at Frequencies Above Resonance

Remember that a frequency above resonance causes a positive output at X with respect to Y (Figure 9–20). At resonance, the output is zero, and below resonance the output is negative. These polarities may be reversed by measuring the voltage at Y with respect to X (Figure 9–20).

Use

The discriminator circuit is used as a part of the automatic tuning circuit in a receiver. Part of the i-f signal is diverted to the input transformer of the discriminator. The output of the discriminator is connected to the local oscillator of the receiver in such a way that the DC voltage affects the frequency of the oscillator. If the intermediate frequency is correct, the receiver is operating at peak efficiency, the discriminator has a zero output, and the oscillator frequency is not affected. If the i-f changes, the receiver is said to be detuned, and the i-f section operates at lower gain. However, the discriminator develops a DC voltage from this incorrect frequency and this DC voltage changes the frequency of the oscillator. By proper circuit design, this change can be made to occur in the direction that will correct the intermediate frequency.

The oscillator, of course, beats with the incoming r-f signal to form the i-f signal. Therefore, a change of oscillator frequency, or a change of input frequency, changes the intermediate frequency. The discriminator and its associated circuit corrects either change. This system is known as *automatic frequency control,* and is usually abbreviated as AFC.

AUTOMATIC FREQUENCY CONTROL

The output of a discriminator is normally applied to a loudspeaker or to some indicating device. However, a discriminator used in conjunction with a reactance tube and oscillator may be used in a circuit to achieve automatic frequency control. Automatic frequency control used in AM or FM receivers is usually referred to as AFC, but in radar circuitry the same device may be referred to as a *phase detector* or *resolver.* Circuit operation in either case is identical but their applications may vary.

In the block diagram of Figure 9–23, the sine-wave oscillator, which is operating at or near the desired constant output frequency, delivers an output in two directions: one is called the constant frequency output; the other, a sample signal, is applied to the discriminator where it is compared with the incoming carrier frequency. Slight variations in frequency or phase between the carrier frequency and the sine-wave output of the oscillator produce a voltage output from the discriminator that indicates the magnitude of phase difference between the two signals.

The balanced diode discriminator circuit produces a change in rectified output when the frequency or phase of the carrier frequency changes. Therefore, any difference in frequency between the carrier frequency and the sine-wave oscillator frequency is measured by the discriminator. The discriminator output can then be used to adjust the sine-wave oscillator frequency with the reactance tube, bringing the two signals back into phase and frequency to produce the desired constant frequency output (with respect to the input carrier).

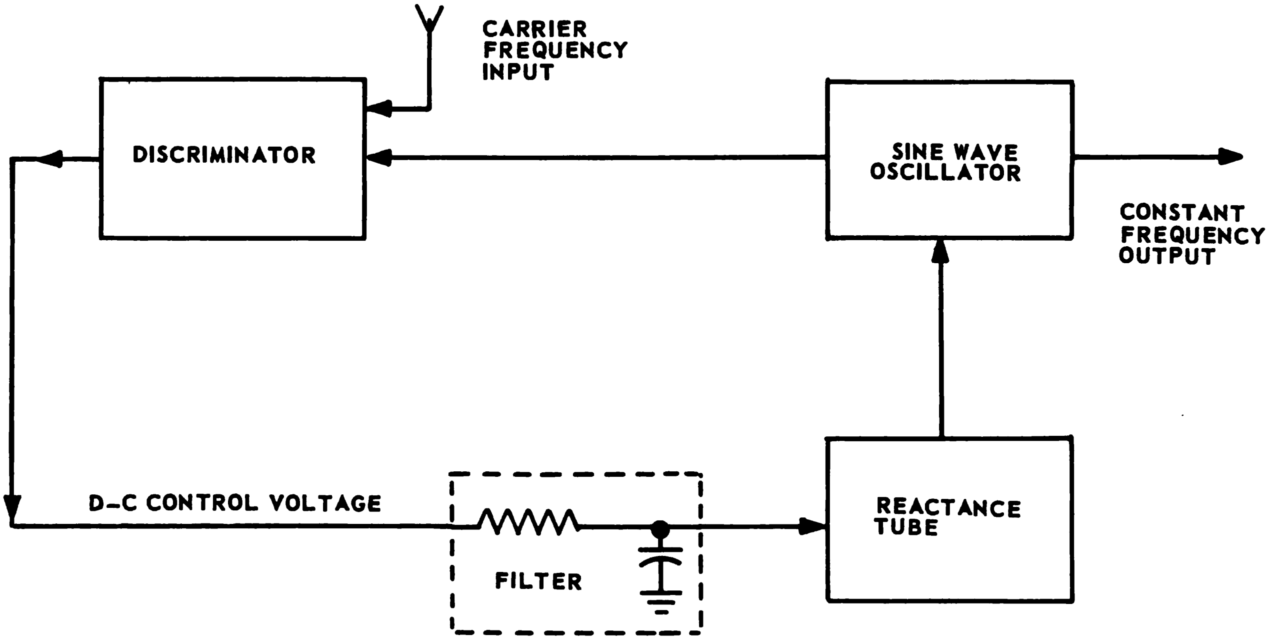


Figure 9–23. Automatic Frequency Control Circuit Block Diagram

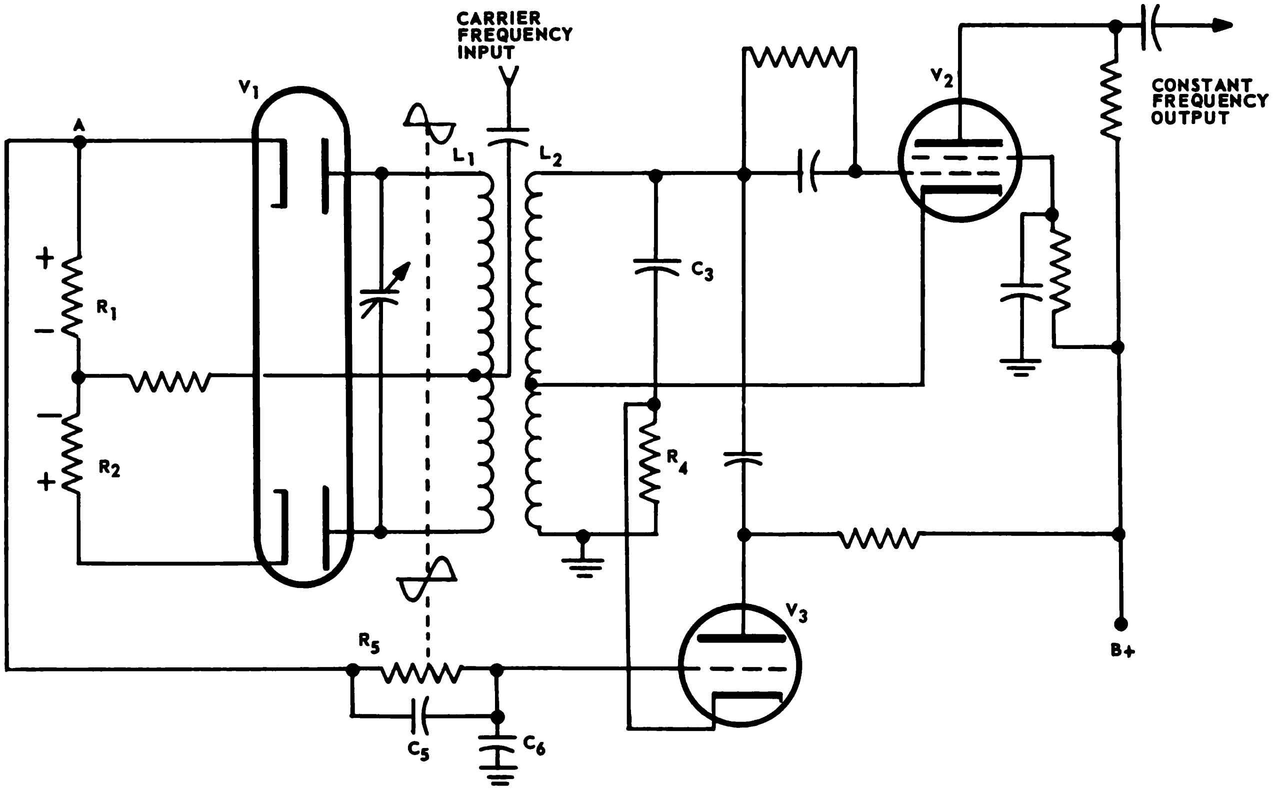


Figure 9–24. Schematic Diagram of an Automatic Frequency Controlled Circuit

Figure 9–24 shows an electron-coupled Hartley oscillator circuit (V_2), with the screen grid acting as the oscillator anode. The oscillator coil L_2 is closely coupled to the center-tapped discriminator coil L_1, applying equal voltages in opposite phase to the diode plates of the discriminator. This creates a zero difference voltage (DC control voltage) across R_1 and R_2 which is applied to the grid of the reactance tube (V_3). C_3 and R_4 provide the required quadrature network, with the voltage across resistor R_4 coupled to the cathode circuit instead of the control grid so that the DC control voltage from the discriminator can be coupled to the control grid of V_3. Rapid changes in the DC level of the discriminator output that might be produced by interfering noise signals are attenuated by the RC filter network (R_5, C_5, and C_6).

If the carrier frequency is above its correct value (the resonant frequency of the discriminator circuit), the resulting DC control voltage between point A and ground is positive. This positive control voltage is applied to the control grid of the reactance tube (V_3) and causes the oscillator frequency to increase to its proper value. When the oscillator is again operating at the proper frequency, the discriminator output voltage is zero, and therefore the reactance tube causes no further change in frequency. If the carrier frequency is below its correct value, the resulting DC control voltage between point A and ground is negative. This negative control voltage, which is applied to the grid of the reactance tube, causes the oscillator frequency to decrease to its proper value. A similar action occurs when the sine-wave oscillator tends to fall above or below the carrier frequency.

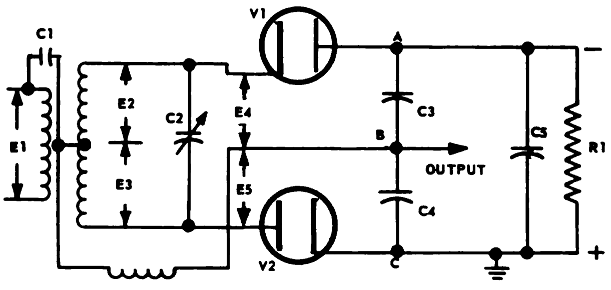


Figure 9–25. Ratio Detector

RATIO DETECTOR

The output of nearly all frequency modulated oscillators also varies in amplitude to some degree. The input to the discriminators discussed so far should be of constant amplitude, varying only in frequency. Otherwise, the output of the discriminator will be distorted; it will not vary exactly in accordance with the modulating signal that produced the frequency variations. A limiter circuit must be used to eliminate the amplitude modulation before the signal is applied to these discriminators.

The ratio detector, shown in Figure 9–25, is a circuit designed to respond only to frequency changes of the input signal. Amplitude changes in the input have no effect upon the output. The input circuit of the ratio detector is identical to that of the discriminator circuits previously covered in this chapter. It uses a center-tapped transformer secondary as they did.

The voltages appearing across V_1 (E_4) and V_2 (E_5) when the input is below, at, and above the center frequency are shown in Figure 9–26. The input circuit is the only portion of the ratio detector that operates in the same way as the discriminators already discussed. V_1, R_1, and V_2 form a series circuit that is fed by the tuned circuit of the transformer secondary. Since the two diodes are connected in series, they conduct on the same half-cycle of the input r-f.

The rectified current flowing through V_1 and V_2 produces a voltage drop across R_1 with a negative polarity at the plate of V_1 as shown in Figure 9–25. C_5 is a very large capacitor connected in parallel with R_1. Its function is to keep the voltage drop across R_1 constant even for the slowest frequency deviations of the input signal. Thus, the voltage between points A and C, Figure 9–25, remains constant. This is made possible by the large value of R_1 and C_5. This is the reason amplitude variations in the input voltage have no effect on the output.

The rectified voltage across C_3 is proportional to E_4, the voltage appearing across V_1. The rectified voltage across C_4 is proportional to E_5, the voltage appearing across V_2. It has been pointed out that E_4 and E_5 vary in magnitude in accordance with the frequency variations of the input signal. Thus, the voltage across C_3 and that across C_4 vary in the same manner. The sum of the voltages across the two (between points A and C) remains constant. However, the output taken at point B varies with respect to points A and C. If the potential across C_3 increases, that across C_4 decreases by the same amount, since their sum is fixed by the constant voltage across C_5. This circuit gets the name ratio detector because the ratio of the voltage across C_3 to that across C_4 may be continually varying, while the sum of the two voltages remains constant.

With the circuit connected as shown in Figure 9–25 the output is always taken with respect to ground and varies about an average negative potential. The output voltage, as the frequency

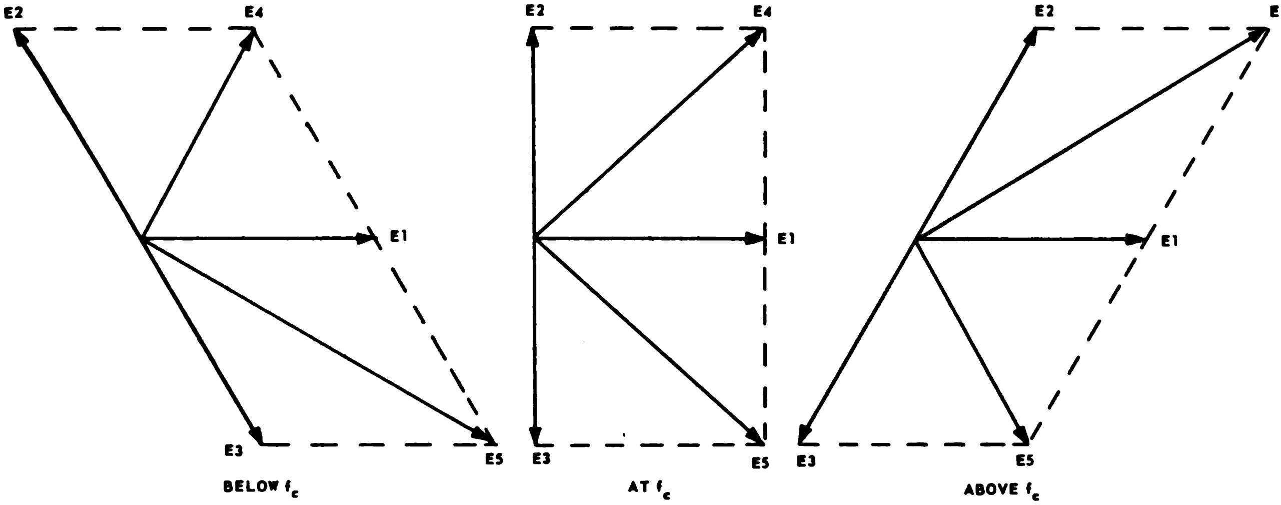


Figure 9–26. Voltage Relationships at Input of Ratio Detector

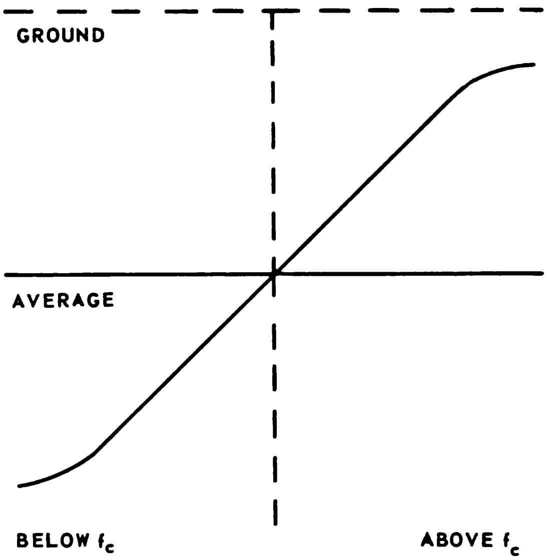


Figure 9–27. Output of Ratio Detector

varies, can be determined from Figure 9–26. Remember, that the voltage across C_3 is proportional to E_4 and that across C_4 is proportional to E_5. When the input frequency is below f_c, E_5 is greater than E_4, producing a voltage across C_4 that is greater than that across C_3. At this time, then, the output is maximum negative with respect to ground, as shown in Figure 9–27. When the input is at f_c, the voltages across C_3 and C_4 are equal, since E_4 equals E_5. At this time the output is at the average negative potential. When the input frequency is above f_c, the voltage across C_3 is greater than that across C_4, since E_4 exceeds E_5. The output at this time is the least negative, as shown in Figure 9–27.

The ratio detector is becoming more and more popular, since its use eliminates the necessity of a limiter stage in FM receivers.

SINGLE–SIDEBAND TRANSMISSION

The basic concept of single-sideband (SSB) communications was understood as early as 1914. It was first realized through mathematical analysis of an amplitude-modulated r-f carrier. It can be shown that a carrier modulated by a single sine wave of voltage consists of three different frequencies. These are: (1) the original carrier with amplitude unchanged; (2) a frequency equal to the difference between the carrier and the modulating frequencies, with an amplitude equal to one-half that of the modulating signal; and (3) a frequency equal to the sum of the carrier and the modulating frequencies, with an amplitude equal to one-half that of the modulating signal. The two new frequencies are called *sidebands.*

Upon recognition of the fact that sidebands existed, further studies were made. These investigations and experiments proved that after the carrier and one of the side bands were eliminated, the other sideband could be used to transmit the intelligence. (Since the carrier amplitude is unchanged, there is no intelligence contained in the carrier.) Further experiments proved that both sidebands could be transmitted, each containing different intelligence, with a suppressed or completely eliminated carrier.

By 1923 the first patent for this system had been granted, and a successful SSB communications system was established between the United States and England. Today SSB communications play a vital role in providing dependable and efficient communications between the United States and its Armed Forces scattered throughout the world.

To gain a thorough understanding of single sideband communications, you must first understand the principles of amplitude modulation. When two different signals are applied to a nonlinear device, heterodyning action occurs. The output of this device will contain the upper sideband (USB), the lower sideband (LSB) and the carrier frequencies.

Let us look at the composite AM waveform from a frequency spectrum standpoint. In Figure 9–28, one represents the carriers, two the LSB, and three the USB. Assume that the constant amplitude 100-kc r-f carrier is modulated by a voice frequency signal that is varying in frequency from 100 to 6000 cycles and is changing in amplitude. The output of this device will contain difference frequencies ranging from 94 to 99.9 kc (LSB) and sum frequencies ranging from 100.1 to 106 kc (USB). Notice that each sideband has the same frequency separation from the carrier. They contain the same intelligence, which can be extracted from either sideband at the receiver.

The sidebands vary in a horizontal direction (left-right, Figure 9–28B) from the carrier. As the modulating frequency varies, they move closer to, or farther away from the carrier, the

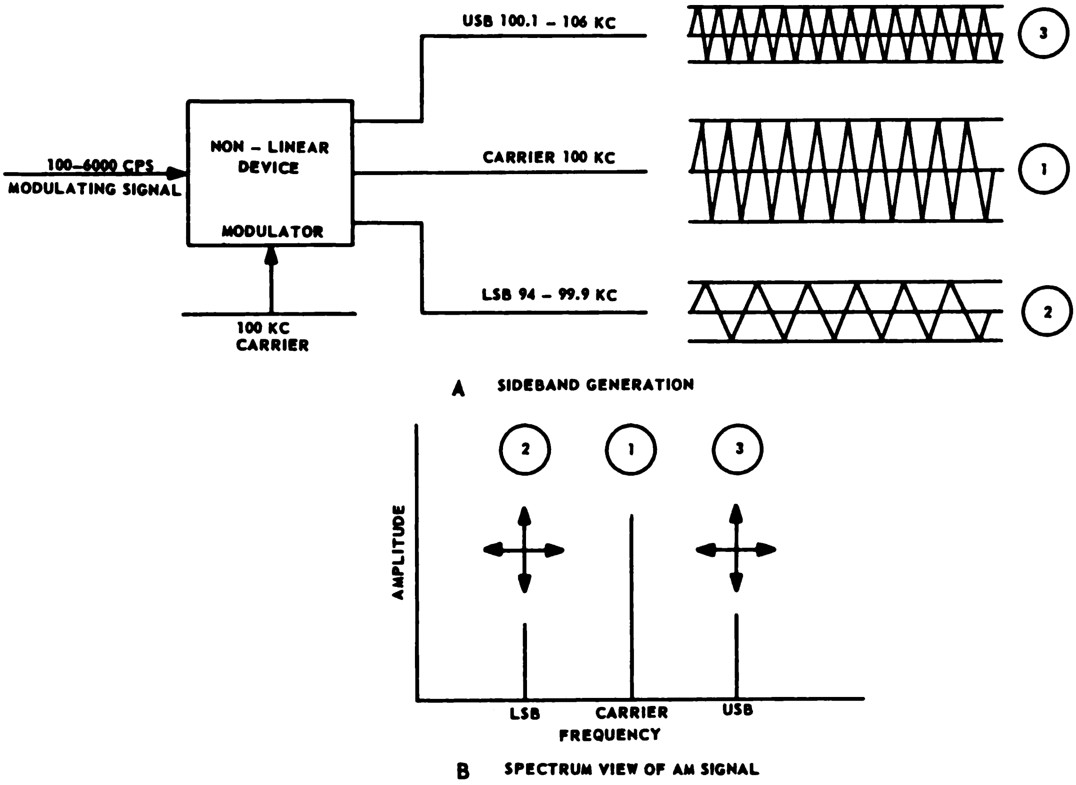


Figure 9–28. Amplitude Modulation

amount depending upon the low and high frequencies of the modulating signal. The amplitude of the sidebands varies, depending upon the volume level of the modulating signal. In voice modulation, speaking in a loud tone increases the amplitude of the sidebands; soft tones decrease the amplitude. From this you can see that the sidebands are continuously moving left and right (frequency) and up and down (amplitude), as shown in Figure 9–28B.

These sideband frequencies are generated as a result of the mixing of the r-f carrier and the modulating signal in a nonlinear device. The radiated signal consists of three frequency components (carrier, USB and LSB) of a definite amplitude and frequency relationship.

When we view the radiated signal on an oscilloscope, we see a pattern that is referred to as a composite AM waveform. The outline of this pattern is called the *modulation envelope*. It will always resemble the original modulating signal, if no distortion is introduced. A 100-kc carrier modulated by a 6-kc tone will appear as shown in Figure 9–29. The composite AM waveform appears to be a radio-frequency carrier varying in amplitude. It is the pattern of the instantaneous frequency-voltage relationship of the carrier and the sideband frequencies, or the vector sum of their three frequency components. It can also be said to be the over-all resultant signal voltage of the phase relationship of the three frequency components at one given instant. The carrier, upper sideband, and lower sideband power and frequency are distributed throughout this waveform. That is, there is as much carrier, upper, and lower sideband power and frequency at point X as there is at points Y and Z.

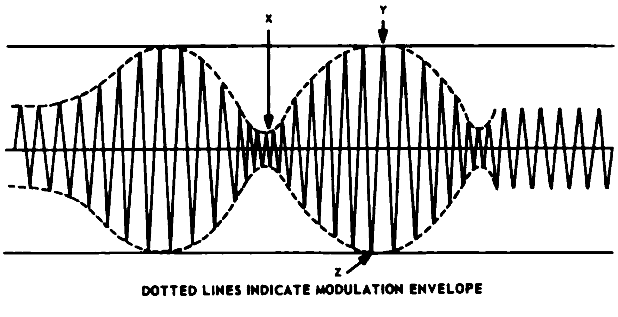
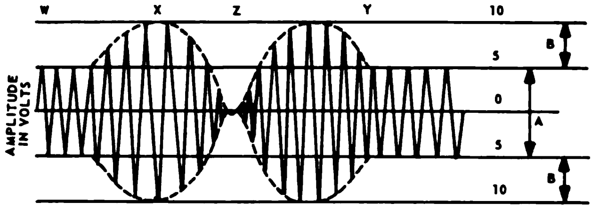


Figure 9–29. Composite AM Waveform

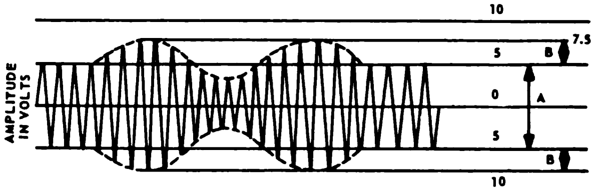
Percentage of Modulation

The amplitude of the sidebands generated is always proportional to the amplitude of the audio or modulating signal. The amplitude of this modulating signal will also determine the percentage of modulation. This term is used to describe the power distribution of the carrier and the sidebands; it will determine the usability of the transmitted signal. One-hundred percent modulation takes place when the system is delivering maximum undistorted power output. With less than 100% modulation, the intelligence in the envelope is weaker. With more than 100% modulation, excessive distortion results. The percentage of modulation, therefore, is quite important. Just as sideband generation depends upon the audio or modulating signal, so does the percentage of modulation.

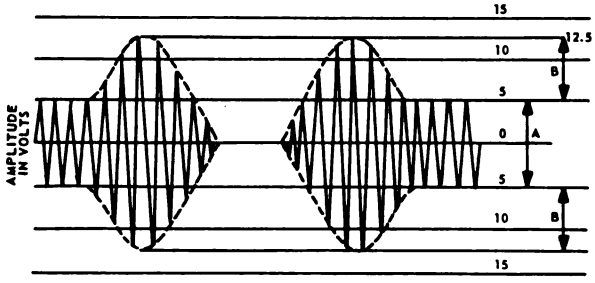
When two equal voltages of different frequencies are applied to a nonlinear device, the voltage at the output, or plate, of the stage will vary from twice its normal value to zero, depending upon whether the two voltages are in phase or out of phase. Because of the frequency difference between the two signals, it will be impossible for them to be completely in or out of phase at all times. This explains the oscilloscope presentation of the composite AM waveform, where the carrier *appears* to be varying at an audio rate. The sidebands that are generated form a definite phase relationship with respect to the carrier. Once this relationship is formed, it must be maintained all the way to the detector in the receiver. Otherwise, severe distortion will occur, and in some cases, complete loss of carrier or signal intelligence.



A 100% MODULATED AM WAVEFORM



B 50% MODULATED AM WAVEFORM (UNDER MODULATED)



C OVERMODULATED AM WAVEFORM

A = CARRIER VOLTAGE
B = MODULATING VOLTAGE

Figure 9–30. Percentage of Modulation

Figure 9–30 shows examples of modulated AM signals. Figure 9–30A represents a 100% modulated wave. Notice that the section labeled W is of constant amplitude. This is the carrier before it has been modulated. Since the carrier and modulating voltages are of equal amplitude, the carrier appears to increase in amplitude until at point X the amplitude of the composite waveshape is twice the unmodulated carrier. At Z the amplitude has decreased to zero. After one and one-half cycles of the modulating signal, the carrier is again unmodulated and of constant amplitude.

In Figure 9–30B, the modulating voltage amplitude is less than that of the carrier. The composite waveshape does not increase as much as in the first example, nor does the amplitude ever drop to zero. Figure 9–30C shows a carrier modulated by a signal of greater amplitude. The peak voltage is greater than twice the unmodulated carrier, while the output drops to zero at an earlier point than in example A and remains there for a period of time. The envelope in Figure 9–30C does not resemble the original modulating signal.

The percentage of modulation can be figured by using the following formula.

$$\text{Percentage of modulation} = \frac{E_{max} - E_{min}}{E_{max} + E_{min}} \times 100 \qquad (1)$$

where E_{max} is the maximum voltage of the modulated wave and

where E_{min} is the minimum voltage of the modulated wave.

Another way of finding the percentage of modulation is,

$$\text{Percentage of Modulation} = \frac{\text{Peak amplitude of modulating voltage}}{\text{Peak amplitude of carrier voltage}} \times 100 \qquad (2)$$

Examples

Find the percentage of modulation for the waveshapes in Figure 9–30, using both formulas where applicable.

For those in Figure 9–30A:

$$\frac{E_{max} - E_{min}}{E_{max} + E_{min}} \times 100 \qquad (1)$$

$$\frac{10 - 0}{10 + 0} \times 100 = 100\%$$

$$\text{or} \quad \frac{\text{Peak amplitude of modulating voltage}}{\text{Peak amplitude of carrier voltage}} \times 100 \qquad (2)$$

$$\frac{5}{5} \times 100 = 100\%$$

For those in Figure 9–30B:

Using formula (1),

$$\frac{7.5 - 2.5}{7.5 + 2.5} \times 100 =$$

$$\frac{5}{10} \times 100 = 50\%$$

Using formula (2),

$$\frac{2.5}{5.0} \times 100 = 50\%$$

For those in Figure 9–30C:

Formula (1) cannot be used for overmodulation (greater than 100% modulation).

Using formula (2),

$$\frac{7.5}{5.0} \times 100 = 150\%$$

Power Distribution

In the composite AM waveform, there are two values of power to be considered: the carrier power, and the peak power. Carrier power is the average power in the unmodulated carrier. This is point W in Figure 9–30. Peak power is the power developed when the carrier and both sidebands are in phase. If we know the voltages, we can calculate the power in any of the components of the AM waveform.

If we have a carrier modulated 100% by a single sine wave of voltage of constant amplitude, the computations are not too difficult. Assume that the rms voltage of the unmodulated carrier is 100 volts and the radiation resistance is 50 ohms. The power in the unmodulated carrier can be found from the power formula,

$$P = \frac{E^2}{R}$$

$$P = \frac{(100)^2}{50} = \frac{10000}{50} = 200 \text{ watts.}$$

Since the amplitude of the carrier remains constant, modulated or unmodulated, the power in the carrier is 200 watts at all times.

For 100% modulation, the modulating signal voltage is equal to the carrier voltage, in this case 100 volts. Then the rms value of the sideband voltage is 50 volts. The power in either sideband can be found from the power formula,

$$P = \frac{(50)^2}{50} = 50 \text{ watts.}$$

As you can see, the power in each sideband is one-fourth the power in the carrier at 100% modulation. The total sideband power is 100 watts. With a carrier power of 200 watts, this gives a total power of 300 watts. This means that to attain 100% modulation of an r-f carrier with a sine wave modulating signal, a modulating power equal to one-half the carrier power is required. The additional modulating power is divided equally between the upper and lower sidebands.

The peak power for this 100% modulated wave can be found as follows. The rms value of E_{max} is 200 volts. Using the power formula,

$$P = \frac{(200)^2}{50} = \frac{40000}{50} = 800 \text{ watts.}$$

While all of the intelligence is contained in the sidebands, two-thirds of the total power is in the carrier. It would appear that a great amount of power is wasted during transmission. The basic principle of single sideband transmission is to eliminate or greatly suppress the high energy r-f carrier. This can be accomplished without affecting the fidelity of the emitted intelligence, since the carrier contains no intelligence.

If a means of suppressing or completely eliminating the carrier is devised, the power that was used for the carrier can be converted into useful power to transmit the intelligence in the sidebands. Since both upper and lower sidebands contain the same intelligence, one of these could also be eliminated, thereby adding more power for transmission of the other intelligence-bearing sideband. This would result in more power in the radiated signal and a greatly extended range of transmission.

Power Comparison

The total power output of a conventional AM transmitter is equal to the carrier power plus the power output of the modulator. Conventional AM transmitters are rated in carrier power output. In single sideband operation, there is either a greatly attenuated carrier or a completely eliminated carrier. For this reason, the SSB transmitter is rated in peak envelope power. In the example of the 200-watt transmitter previously used, the peak power was 800 watts. If this same transmitter were converted to SSB operation, it would be rated as an 800-watt single sideband transmitter, and a 6-db gain in power would be realized. (Any power doubled equals a 3-db gain.) This gain is due to the elimination of the carrier and one of the sidebands.

The bandwidth of the 800-watt SSB transmitter would be only one-half that required for a conventional AM transmitter. This is an advantage that results in 3-db gain at the receiver. This gain is due to the better signal-to-noise ratio obtained. Another advantage realized is that the sideband signal is less subject to the effects of selective fading, which is caused by the out-of-phase relationship between the carrier and sidebands. It commonly occurs in conventional AM transmission. In this type of single sideband system, a theoretical 9-db gain is realized over the conventional AM system.

Types of Sideband Transmission

Several types of single sideband systems have been developed. Therefore, it is necessary to clarify the various applications of the transmission principle involved. The term *single sideband modulation* is defined as a form of amplitude modulation in which one sideband and the carrier are suppressed.

In one system, the carrier and one of the sidebands are eliminated at the transmitter; only one sideband is emitted. This is called *single sideband suppressed carrier* (SSSC); it is quite popular with amateur radio operators. The chief advantage of this system is maximum transmitted signal range with minimum transmitter power.

Another system eliminates one sideband and suppresses the carrier to a desired level. The suppressed carrier is then used at the receiver for a reference, AVC, AFC, and in some cases, demodulation of the intelligence-bearing sideband.

The type of system most commonly used in Air Force communications is referred to as *twin sideband suppressed carrier*, or *independent sideband* transmission. This system involves the transmission of two independent sidebands, each containing different intelligence, with the carrier suppressed to a desired level.

Advantages of SSB Transmission

The most important advantage of the independent sideband system is a more effective utilization of the available frequency spectrum. In the same bandwidth required for the transmission of one conventional AM signal, with present day multiplexing techniques, many channels of different intelligence can be simultaneously transmitted by one SSB transmitter within one assigned operating frequency. This makes this type of communications especially adaptable to the already overcrowded high-frequency spectrum.

A second advantage of this system is that it is less subject to the effects of selective fading. This is because there is no definite phase relationship between the upper and lower sidebands and the carrier as there is in conventional AM. In the propagation of conventional AM transmissions, if the upper sideband frequency strikes the ionosphere and is refracted back to earth at a different angle from that of the carrier and lower sideband frequencies, distortion is introduced at the receiver. Under extremely bad conditions, complete signal cancellation may result, which means a complete loss of the intelligence.

Any change in the phase relationship between the sidebands and the carrier during the transmission of the conventional AM signal will result in distortion at the receiver during demodulation. If the upper sideband frequency of an independent sideband transmission is refracted back to the earth at a different angle from that of the carrier and lower sideband, it will cause the signal strength for that sideband to decrease. There will be no effect on the other sideband or the suppressed carrier whatsoever; thus, only a partial loss (in the USB) of the entire intelligence transmitted occurs.

Another advantage realized by all types of SSB systems is that a higher percentage of power is in the radiated intelligence. The independent sideband system has an estimated over-all 6-db gain, while the single sideband suppressed carrier system (SSSC) has an over-all 9-db gain, 6 db at the transmitter and 3 db at the receiver. In the example in the text, the 200-watt conventional AM transmitter was converted to a SSSC SSB transmitter, and an over-all 9-db gain was realized. If this same transmitter were used for independent sideband operation, we would realize only an over-all gain of 6 db (3 db at the transmitter and 3 db at the receiver). The power would be equally divided between the upper and lower sidebands. That is, there would be approximately 400 watts in each sideband. This, then, represents twice the value of our original 200-watt reference, or a 3-db increase at the transmitter.

Receiver Considerations

When considering the gain of any system, the receiver must be taken into consideration. In the conventional AM system, the signal applied to the detector consists of the upper and lower sidebands with a full-power carrier. However, due to the rectification by the diode detector, only the positive portion of the signal is used to develop the output. Therefore, we have wasted half the available power in the signal.

In most SSB systems used by the Air Force, each full sideband is detected or demodulated by its own full-wave demodulator. Since all of the power in each sideband signal is applied to a different demodulator, we realize a 3-db gain at the receiver. Another advantage contributing to the 3-db gain of the receiver is the narrow bandwidth. Since the bandwidth required for each sideband is one-half that required for conventional AM, a higher signal-to-noise ratio is possible. As propagation conditions become worse because of fading or other atmospheric interferences, the signal-to-noise ratio improvement of the SSB system over an equivalent AM system becomes increasingly greater.

Disadvantages of SSB

One of the disadvantages of SSB equipment is its high cost. This results from the complex circuitry and type of components used. Special emphasis is placed upon voltage regulation, stability, and reliability. The oscillator frequencies involved are very critical and must be stabilized; otherwise, distortion will occur. In SSB systems every precaution is taken to prevent frequency drift, because when the carrier drifts, the sidebands will drift proportionately. If corrections are not made, the intelligence will be distorted upon demodulation.

The suppressed carrier is amplified and filtered for use in special circuits such as AVC and AFC at the receiver. In some cases this *reconditioned carrier* is also used to demodulate the sidebands of intelligence. By using these special circuits, any frequency drift of the carrier will be corrected by the AFC; variations of the input signal level will be compensated for by the AVC, thereby maintaining a constant output to overcome the effects of fading. The reconditioned carrier may be used to demodulate the sidebands; however, due to propagation conditions, this signal is usually varying in both frequency and amplitude, and therefore is unsuitable for this purpose.

In some systems the reconditioned carrier must be used in the demodulation process; however, in normal communications systems, a highly stable local oscillator is used for demodulation. This highly stable reference oscillator is used at the receiver to generate the conversion frequencies, which will provide distortionless demodulation of the signal intelligence.

The importance of AVC, AFC and the reconditioned and local carriers cannot be overemphasized. When demodulating the sideband signal in the receiver, the final conversion frequency must be within 15 cycles of the transmitter carrier for good quality voice reproduction. It must be within 5 cycles for peak performance of frequency-shift tone teletype signals, and within 1 cycle for the highly sophisticated multitone systems.

Sideband Generation

The techniques of generating sidebands will vary with different equipment designs. In the conventional AM systems, the sidebands are generated at the final PA stage. In an SSB system, they are generated preceding the final stage. Also in the AM system the fundamental or carrier frequency is produced by an oscillator and then applied to a series of multiplier stages until the final operating frequency is obtained. At this point, the r-f carrier is modulated by the audio-frequency intelligence and applied to the antenna.

The basic principle of SSB requires the suppression or elimination of the r-f carrier and one of the sidebands. Therefore, specially designed circuits are used to perform these functions at a low-power level. Modulation at the low level necessitates the use of linear power amplifiers rather than frequency multipliers in order to keep distortion to a minimum. The most common method used to generate sidebands is frequency conversion coupled with selective filtering.

Since the primary objective is to convert the audio intelligence into a sideband frequency, it is necessary that a means be devised to eliminate the carrier and one of the sidebands. This is easily accomplished by using a *balanced modulator circuit.* The main characteristic of a balanced modulator is that the carrier or conversion frequency is canceled in the output of the modulator. Therefore, the output contains only the upper and lower sidebands of intelligence. The undesired sideband may be eliminated by using a selective filter, leaving only the desired sideband for transmission. This signal is applied to linear amplifiers and further converted until the final operating frequency is obtained.

Balanced Modulators

Before single sideband, suppressed-carrier transmission can occur, two things must be accomplished. First, the carrier must be eliminated, or suppressed; and second, one of the two sidebands must be filtered out. Once this has been achieved, the selected sideband can be applied to the final power amplifier for transmission. A *balanced modulator* circuit can be used to pass only the two sidebands. Balanced modulation is a system of adding intelligence to a carrier wave whereby only the sidebands are produced; the carrier wave is eliminated.

The balanced modulator resembles the conventional push-pull amplifier in circuitry but not in operation. Figure 9–31 shows a conventional push-pull amplifier. For proper operation as an amplifier the signals applied to the grids of V_1 and V_2 must be 180° out of phase. When the positive cycle is applied to the grid of V_1, plate current flowing through T_2 will produce the positive portion of the output voltage. When the grid of V_1 goes negative, the grid of V_2 goes positive and plate current from V_2 flows in the opposite direction through the primary of T_2, producing the negative-going alternation of the output waveshape.

By slightly modifying the push-pull amplifier, as shown in Figure 9–32, no output signal will be obtained when an input signal is applied. Notice that the input signal applied to the grids of V_1 and V_2 in Figure 9–32 has the same polarity. Both grids are positive at the same time. A positive-going grid in each tube causes the plate current to increase simultaneously in each tube. Since the plate current of V_1 is flowing down through the primary of T_2 at the time the plate

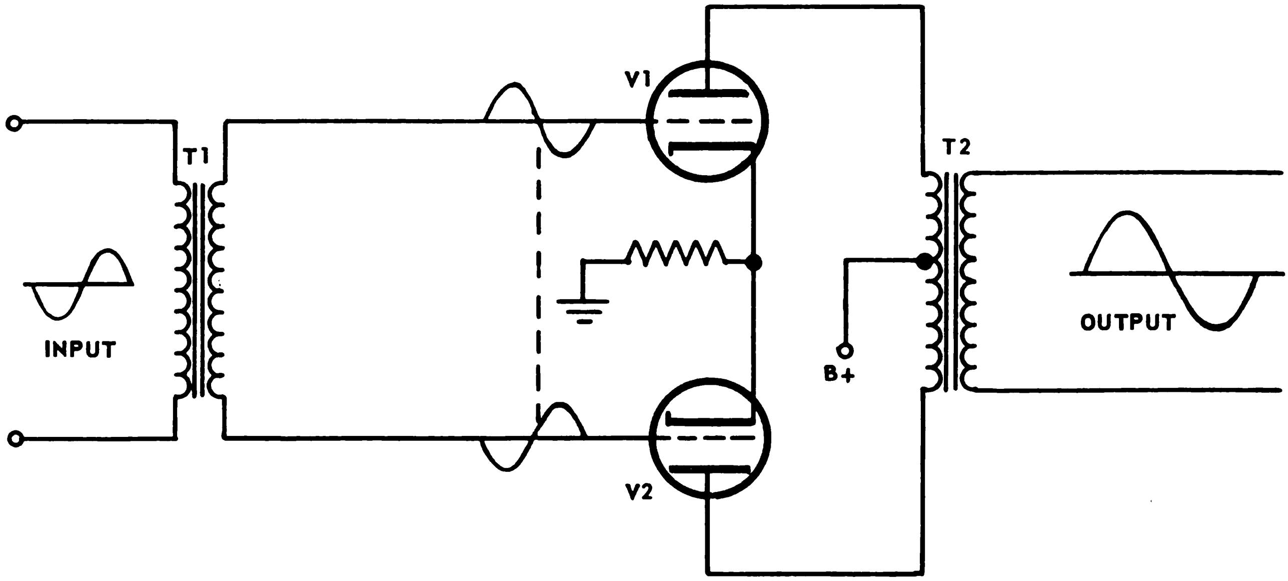


Figure 9–31. Conventional Push-Pull Amplifier

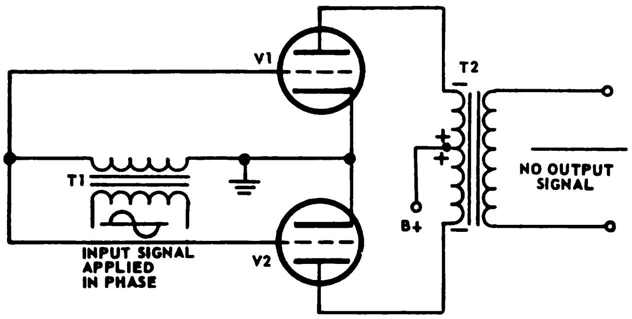


Figure 9–32. Push-Pull Amplifier with Signal Applied in Phase

current of V_2 is flowing up, the two magnetic fields produced around T_2 will effectively cancel. Thus, no output signal will be developed. At first glance, this particular circuit is not performing any useful function. On the other hand, assume the input signal to be the carrier frequency. When the carrier frequency is applied in phase, no output signal will be developed in the output; thus, the carrier frequency has been suppressed, or eliminated, in the output.

Again modify the push-pull amplifier circuit and get the circuit shown in Figure 9–33 (essentially a combination of Figures 9–31 and 9–32). In this simplified push-pull balanced modulator, the carrier input is applied to the grids of both V_1 and V_2 in phase; when the r-f voltage is going positive, both grids are driven positive. The modulating voltage is applied to the two grids in the conventional push-pull manner; when the modulating signal drives the grid of V_1 in a positive direction, the grid of V_2 is driven in a negative direction.

Now consider the effect of the r-f voltage on the stage. Since the carrier frequency causes both grids to be driven positive at the same time, the plate current of both V_1 and V_2 increases at the same time. Consequently, the magnetic fields caused by the two equal plate currents are effectively cancelled in the output transformer T_2, which is center tapped to B+. On the negative swing of the r-f voltage, the plate current of both tubes decreases at the same time. Again, the magnetic fields cancel. The carrier frequency, therefore, does not appear in the output.

The effect of the modulating voltage is different. When the positive swing of the modulating voltage is applied, the grid of V_2 is driven negative. On the negative alternation of the modulating voltage, the grid of V_1 is driven negative, but the grid of V_2 is driven positive. Because of this push-pull arrangement, plate current flows on both the positive and the negative swings, first through one tube and then through the other. There is no cancellation in the output.

When the modulating and r-f voltages are applied at the same time, first one tube conducts the most and then the other, depending upon which grid is driven positive by the modulating voltage. The modulating voltage acts as a varying bias. When the grid of V_1 is positive, r-f currents at the carrier frequency, the modulat-

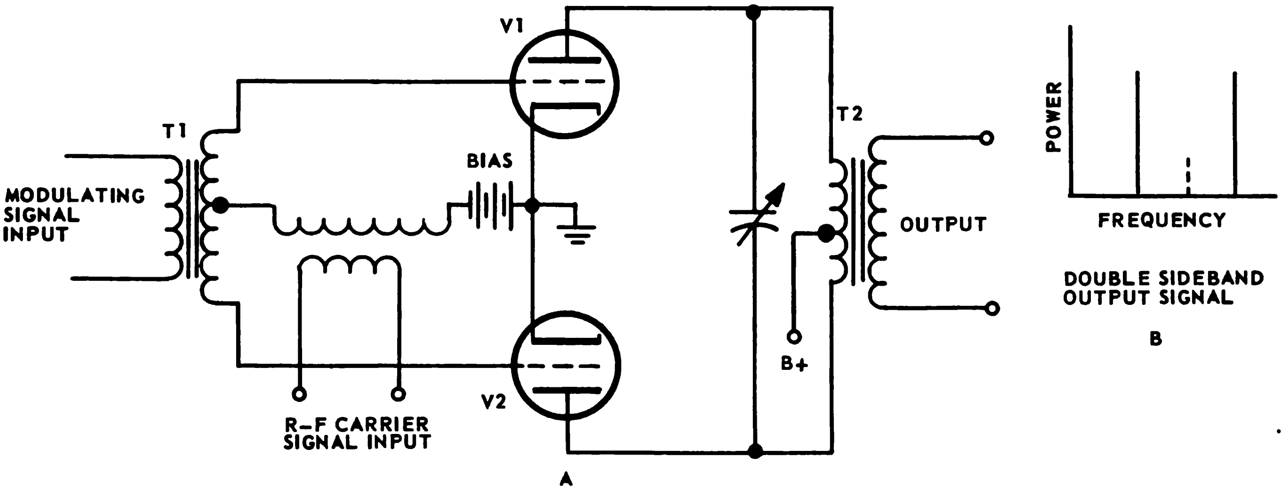


Figure 9–33. Simplified Push-Pull Balanced Modulator

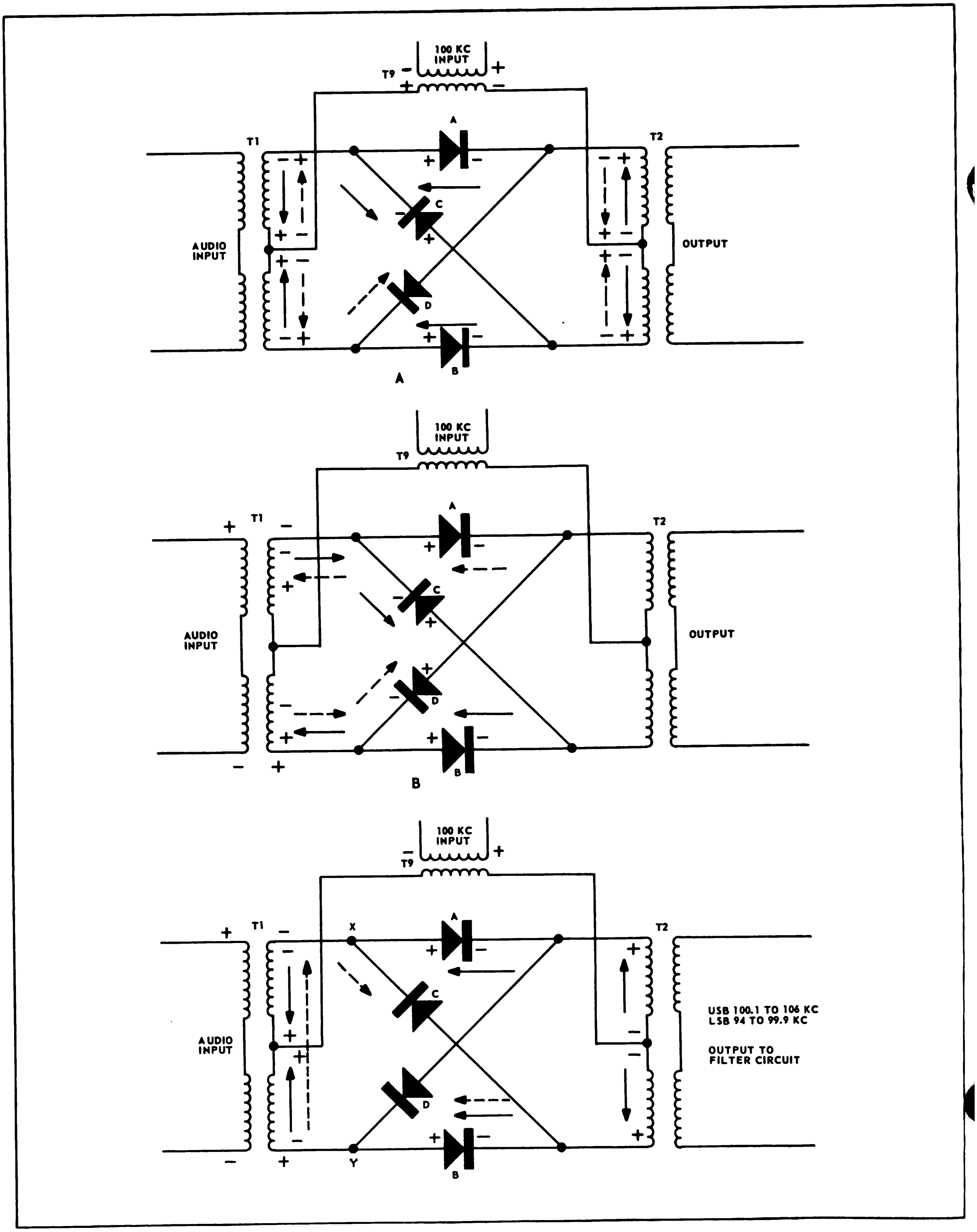


Figure 9–34. Balanced Ring Modulator

ing frequency, and the sum and difference frequencies flow through V_1. The carrier frequency currents are effectively cancelled by the carrier frequency currents from V_2 in the common plate load as explained before. The sum and difference frequencies cause induced voltages in the secondary of transformer T_2, and the modulating frequency is so low that it has no appreciable effect on the plate coil.

The output signal of the balanced modulator consists of the upper and lower sidebands, the carrier frequency has been eliminated. This is shown in Figure 9–33B. Now that the carrier has been eliminated, all that remains is to suppress one sideband. This can be accomplished by either the filter method or the phasing method.

The balanced ring modulator shown in Figure 9–34 is also commonly used in Air Force SSB equipment. This modulator consists of four copper oxide varistors, which exhibit the property of variable resistance. The amount of resistance depends upon the magnitude and polarity of the voltage across the varistor. Electrons flow from the copper (bar) to the copper oxide (arrow) when the bar is negative with respect to the arrow; the varistor is said to be forward biased. When the bar is positive with respect to the arrow, there is no electron flow; the varistor is said to be back biased. The operation is that of the copper oxide rectifier which was explained in Chapter 4. The conduction through the varistors, therefore, is unidirectional with the magnitude of conduction depending upon the potential difference between the copper and the oxide.

To understand the operation of the ring-type balanced modulator, assume that a 100-kc r-f signal is applied to the circuit as in Figure 9–34A, with polarity across T9 as indicated. Also assume that there is no audio signal input. At this time the voltage across varistors A and B will allow conduction, while the voltage across varistors C and D prevents conduction. C and D act as an open circuit. Current flow is indicated by the solid arrows. The polarity of the r-f voltage developed across the secondary winding of T1 and the primary winding of T2 is indicated by the negative and positive signs at the ends of the solid arrows. The voltages developed across each section (center tap to either end) of the secondary of T1 are equal in amplitude and 180° out of phase. This also holds true for the r-f voltage developed across each section (center tap to either end) of the primary winding of T2.

Since voltages of equal amplitude and opposite polarity cancel, there will be no output voltage developed across the secondary winding of T2. When the 100-kc input signal reverses polarity, the current will be in the direction indicated by the dotted arrows. The polarity of the voltage developed across the secondary winding of T1 and the primary winding of T2 is indicated by the signs at the ends of the dotted arrows. Now varistors C and D conduct and A and B do not conduct. The voltages developed across each section of the primary of T2 are again equal in amplitude and 180° out of phase. Therefore, with only a 100-kc r-f input, the circuit is balanced, and there is no output.

Figure 9–34B shows the same modulator with an audio-frequency input, but no 100-kc r-f input. When the polarity of the audio signal across the secondary winding is negative at the top, the voltages across the varistors are such that C and B conduct, while A and D do not conduct. The current path is indicated by the solid arrows. Current does not flow in the top half of the primary winding of T2 since A is effectively an open circuit. Varistors C and B are in series across the secondary of T1. The secondary of T9 effectively connects the center taps of the T1 secondary and the T2 primary. Thus the bottom half of the T2 primary is connected between the center tap on the T1 secondary and the connection between varistors C and B. The ends of this section of the T2 primary are at the same potential in this network; therefore, no current flows in T2.

When the polarity of the audio-frequency input reverses, the current path will be as indicated by the dotted arrows. Under these conditions, varistors A and D conduct, while C and B do not conduct. Current does not flow in the primary of T2 under these conditions. Thus, there is no output from the circuit when only an audio-frequency voltage is applied through transformer T1.

Figure 9–34C shows the balanced ring modulator with both a-f and r-f inputs. Assume that the 100-kc r-f signal has the polarity indicated across T9 and that the audio signal (100 to 6000 cycles) has the polarity indicated across T1. The amplitude of the r-f voltage is much greater than that of the audio voltage. With the polarity indicated for the r-f voltage, varistors A and B conduct in the direction shown by the solid arrows. The dotted arrows indicate the current due to the a-f voltage alone. Remember that the relative amplitudes and polarities of the two input signals are an important consideration in the analysis of the operation of this modulator. The actual voltages at points X and Y are the vector sums of the r-f and a-f voltages at the point concerned. At point X at the instant shown, the two voltages are opposing, while at point Y they are aiding.

The actual current flow through varistors A and B depends upon the summation of the applied voltages. The voltage developed across the top section of the secondary winding of T1 is the difference between the a-f and r-f voltages; the voltage developed across the bottom half of the T1 secondary is the sum of the two signal voltages. The voltage developed across the top section of the T2 primary is the difference between the two signal voltages, while that developed across the bottom section is the sum of the two voltages. The output voltage developed across the secondary of T2 is the resultant of the two voltages developed across the two sections of the primary of T2.

When the r-f voltage reverses polarity, varistors C and D conduct. The voltages at points X and Y are the resultants of the a-f and r-f voltages. The voltage developed across the top section of the T1 secondary is the sum of the two signal voltages, while that across the bottom section is the difference between the two signals. The voltages across the two sections of the T2 primary are the reverse of the voltages on the first r-f alternation. The output voltage across the secondary of T2 is the resultant of the voltages developed across the two sections of the T2 primary.

During the next alternation of the audio-frequency voltage, the top of T1 will be positive and the dotted arrows will be reversed. When the polarity of the r-f voltage is as shown across T9 in Figure 9–34C, the solid arrows again indicate conduction through varistors A and B. The voltage developed across the top half of the T1 secondary is now the sum of the two voltages, and the voltage across the bottom section of the T1 secondary is the difference between the a-f and r-f voltages. The voltage across the top section of the T2 primary is the sum of the two voltages, and the voltage across the bottom section is the difference between the two signal voltages. Once again the output voltage across the secondary of T2 is the resultant of the voltages developed across the two sections of the T2 primary.

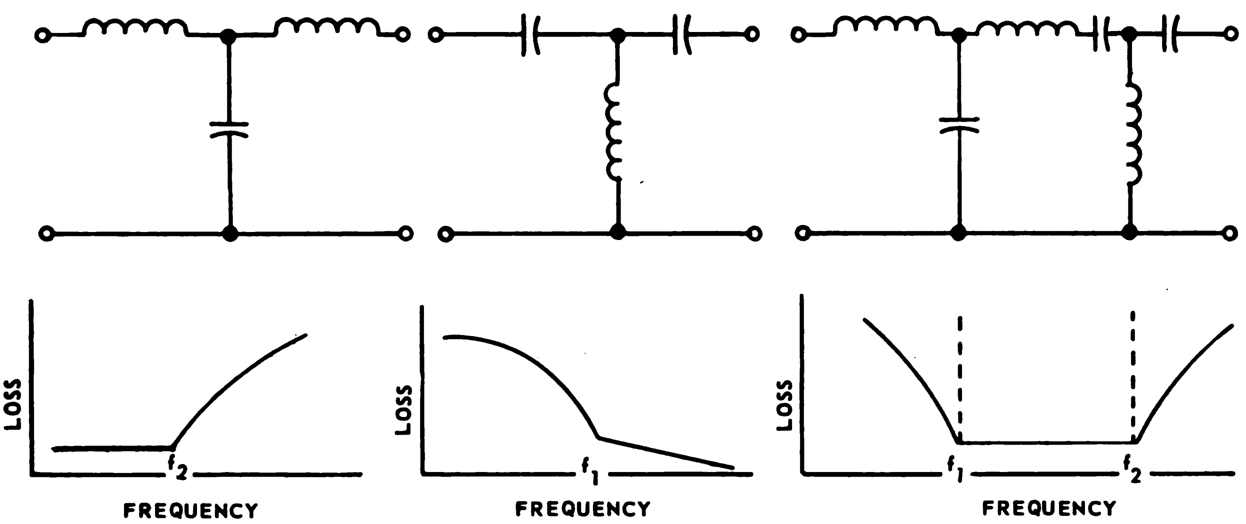
When the r-f voltage reverses polarity, varistors C and D conduct; the voltages subtract in the top section of the T1 secondary and add in the bottom section. This means that the difference voltage appears across the top section of the T2 primary, and the sum voltage across the bottom section. The output voltage is the resultant of the voltages developed across the two sections of the T2 primary.

Because the varistors are nonlinear devices, mixing of the two input signals in these elements produces the upper and lower sidebands. Since the ring-type modulator circuit is not balanced for these sideband frequencies, they will appear across the output transformer. Therefore, the output consists of the voltages developed from the sideband frequencies. The upper sideband frequencies extend from 100.1 kc to 106 kc, and the lower sideband frequencies extend from 94 kc to 99.9 kc. For all practical purposes, the 100-kc carrier signal and the audio-frequency modulating signal are balanced out in the modulator and do not appear in the output. Only the sideband frequencies containing the intelligence will appear in the output.

Both SSB transmitters and receivers require extremely sensitive bandpass filters in the region of 100 to 500 kc. In receivers a high order of adjacent channel rejection is required if channels are to be closely spaced in order to conserve spectrum space. In SSB transmitters the signal bandwidth must be limited sharply in order to pass the desired sideband and reject the other sideband. The filter used, therefore, must have very steep skirt characteristics (fast cutoff) and a flat bandpass characteristic in order to pass all frequencies in the band equally well. These filter requirements are met by crystal filters, LC filters, and mechanical filters.

Simple Filters

Filters are designed to produce high attenuation at particular frequencies. Filters are made up of inductors and capacitors, arranged to offer a high loss to currents of undesired frequencies and a low loss to other frequencies. There are



A – LOW PASS FILTER

B – HIGH PASS FILTER

C – SIMPLE BANDPASS FILTER

Figure 9–35. Basic Filters

three types of filters: the low-pass filter, the high-pass filter, and the bandpass filter.

A simple low-pass filter is shown in Figure 9–35A. When low-frequency currents are applied to a low-pass filter, the filter causes very little loss. At low frequencies the reactance of the series inductors is low and the reactance of the shunt capacitor is high. However, when high-frequency currents are applied to a low-pass filter, the reverse is true; the reactance of the series inductors is high, and the reactance of the shunt capacitor is low. The frequency response of a low-pass filter is illustrated below the schematic diagram; f_2 is the highest frequency passed.

In a high-pass filter the capacitors are in the series arm and the inductor is in the shunt arm. A high-pass filter offers a high loss to low-frequency and a low loss to high-frequency currents. The relation between loss and frequency is shown in Figure 9–35B. In this illustration, f_1 is the lowest frequency passed.

A simple bandpass filter can be made from a combination of these two low- and high-pass filters, as shown in Figure 9–35C. Assume that the low pass section (coils in the series arm) is designed to pass a frequency range from zero to 2000 cycles and to suppress frequencies above that point. In addition, assume that the high pass section (series capacitors) is designed to pass a frequency range from 1000 cycles up. These two filters in series, then, will pass only frequencies between 1000 and 2000 cycles. These frequencies mark the lower and upper limits of the bandpass. This is shown in the frequency response curve of the simple bandpass filter in Figure 9–35C. None of the three filter circuits shown would actually eliminate the unwanted sideband, although the simple bandpass filter comes the nearest to doing so.

Bandstop Filter

From your study of resonant circuits you know that a series resonant circuit offers minimum opposition to the resonant frequency. A circuit such as that shown in Figure 9–36A would effectively short out the frequency to which the series circuit was resonant. All other frequencies would be passed down the line. A parallel resonant circuit offers maximum opposition to the frequency of resonance. As shown in Figure 9–36B, the parallel resonant circuit in the line offers high opposition to the frequency to which the tank is resonant, and low impedance to other frequencies.

A filter made up of both series and parallel resonant circuits, such as that shown in Figure 9–36C, would prove effective in eliminating one sideband. Let us assume that both sidebands are applied to the input of this bandstop filter, one of 795 kc and the other of 805 kc. The

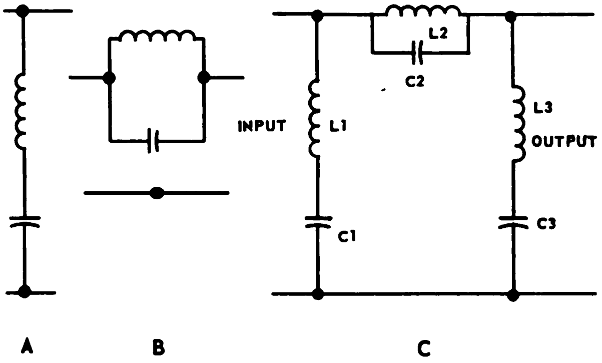


Figure 9–36. Bandstop Filter

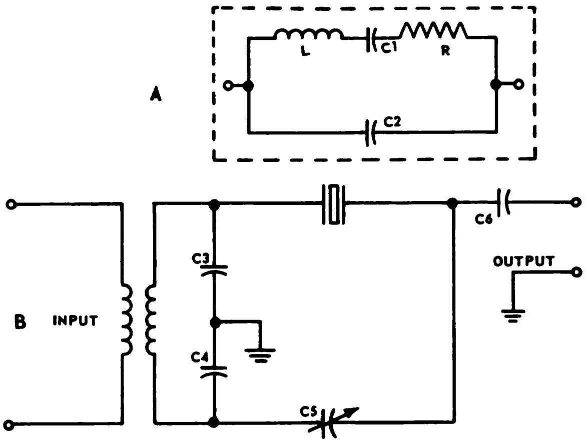


Figure 9–37. Crystal Filter Circuit

balanced modulator has already eliminated the carrier frequency of 800 kc. We wish to eliminate the upper sideband (805 kc). L_1C_1 and L_3C_3 form two series resonant circuits whose resonant frequency is 805 kc. L_2C_2 is a parallel resonant circuit whose frequency of resonance is also 805 kc. When the two sideband frequencies are applied to this filter, L_1C_1 effectively bypasses the 805-kc signal to ground, and develops a high voltage for the 795-kc signal. The parallel resonant circuit of L_2C_2 in the line offers maximum opposition to 805 kc, effectively blocking it.

Of course, it is extremely difficult to construct a resonant circuit with sharp cutoff frequency characteristics, so that a small amount of the 805-kc signal will not be eliminated. However, any of the 805-kc signal passing the parallel resonant circuit will be eliminated by the series resonant combination of L_3C_3, which offers minimum impedance to that frequency. Now, only the lower sideband, 795 kc, will be developed across this filter circuit; the upper sideband has been attenuated to the point where it is no longer of any effect.

Crystal Filter

The crystal filter may also be used in single sideband systems to attenuate the unwanted sideband. This is possible since the crystal is a series resonant circuit. Because of its very high Q, the crystal filter passes a much narrower band of frequencies than the best LC filter. Because of its high selectivity it reduces the number of undesired frequencies in the output.

The equivalent circuit of the crystal and crystal holder is illustrated in Figure 9–37A. The components L, C_1 and R represent the series resonant circuit of the crystal itself. C_2 represents the parallel capacitance of the crystal holder. The crystal offers a very low impedance path to the frequency to which it is resonant and a high impedance path to other frequencies. However, the crystal holder capacitance, C_2, shunts the crystal and offers a path to other frequencies. For the crystal to operate as a bandpass filter, some means must be provided to counteract the shunting effect of the crystal holder. This is accomplished by placing an external variable capacitor in the circuit.

In Figure 9–37B variable capacitor C_5, called the *phasing capacitor*, counteracts holder capacitance C_2. C_5 can be adjusted so that its capacitance equals the capacitance of C_2. Then both C_2 and C_5 pass undesired frequencies equally well. Because of the circuit arrangement, the voltages across C_2 and C_5 due to undesired frequencies are equal and 180° out of phase. Therefore, undesirable voltages cancel, and undesired signals will not appear in the output. Actually, there is only one frequency in the filter's passband at which total cancellation can be obtained. C_5 can be adjusted to cause exact cancellation at any frequency within the passband except for frequencies very close to the crystal's resonant frequency. This cancellation effect is called the *rejection notch.*

To understand why the rejection notch may be positioned as desired by varying C_5, the effective impedance of the holder-crystal combination must be considered. At the crystal's series-resonant frequency, its capacitive and inductive reactances are equal and effectively cancel. As the frequency increases, the inductive reactance increases while the capacitive reactance decreases so that the crystal's impedance becomes inductive. At some frequency slightly higher than the crystal's series-resonant frequency, this net inductive reactance of the crystal equals the capacitive reactance of C_2. This is the parallel resonant frequency of the holder-crystal combination. At still higher frequencies, the net inductive reactance of the crystal exceeds the capacitive reactance of C_2 so that the impedance of the holder-crystal combination (parallel circuit) becomes capacitive. C_5 can be adjusted to be equal to this effective capacitance causing cancellation at the output.

Thus, the rejection notch can be positioned at frequencies above the holder-crystal combination's parallel-resonant frequency. At frequencies below series resonance, the crystal's capacitive reactance exceeds its inductive reactance and the net crystal impedance is capacitive. This capacitance, paralleled with C_2, causes the holder combination's impedance to be capacitive. C_5 can be adjusted to equal this value of capacitance and cause cancellation at the output.

The effective impedance of the holder-crystal combination is capacitive at frequencies both below the crystal's series-resonant frequency and above the holder-crystal combination's parallel-resonant frequencies. C_5 can be adjusted, then, to equal the effective capacitance of the holder-crystal combination and to position the rejection notch at the desired point.

For circuit operation, assume that a lower sideband frequency of 795 kc and an upper sideband frequency of 805 kc are applied to the input of the crystal filter. As in the previous example, assume that the upper sideband is the unwanted sideband. By selecting a crystal that will provide a low impedance path at 795 kc, the lower sideband frequency will appear in the output. The upper sideband, as well as all other frequencies, will have been attenuated by the crystal filter.

Mechanical Filters

Mechanical filters are now used in single sideband equipment because of recent advancements in the development of this type of filter. Some of the advantages of mechanical filters are their excellent rejection characteristics, extreme ruggedness, size small enough to be compatible with the miniaturization of equipment, and a Q in the order of 10,000, which is about 100 times that obtainable with electrical filters.

The basic principles of mechanical filters are well established, although their commercial use is relatively new. The mechanical filter is a device that is mechanically resonant; it receives electrical energy, converts it to mechanical vibration, then converts this mechanical energy back into electrical energy at the output. There are four elements constituting a mechanical filter: (1) an input transducer that converts the electrical energy at the input into mechanical vibrations, (2) metal disks that are manufactured to be mechanically resonant at the desired frequency, (3) rods that couple the metal disks, and (4) an output transducer that converts the mechanical vibrations back into electrical energy.

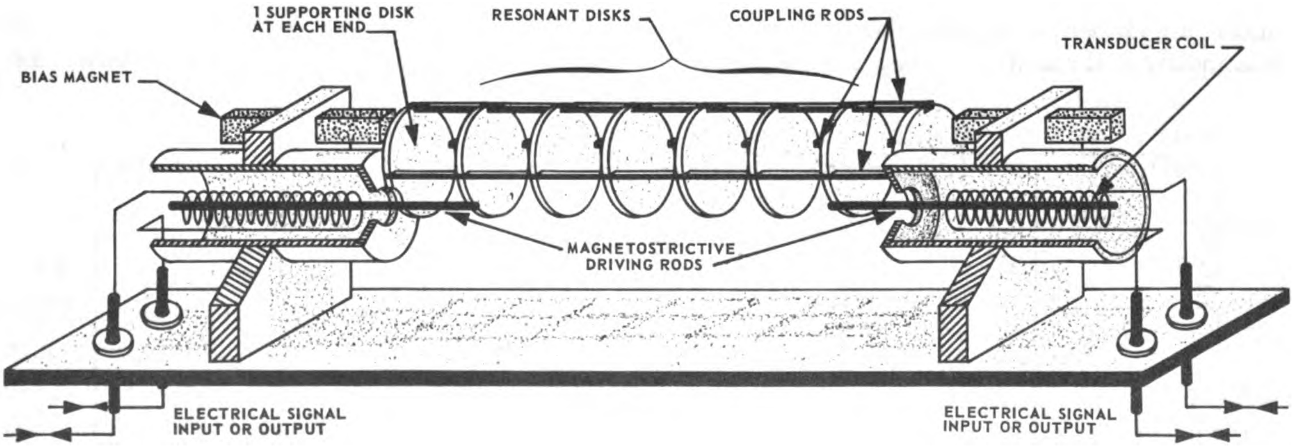

1 SUPPORTING DISK AT EACH END
RESONANT DISKS
COUPLING RODS
TRANSDUCER COIL
BIAS MAGNET
MAGNETOSTRICTIVE DRIVING RODS
ELECTRICAL SIGNAL INPUT OR OUTPUT
ELECTRICAL SIGNAL INPUT OR OUTPUT

Figure 9–38. Mechanical Filter

Figure 9–38 shows the elements of a mechanical filter. Not all of the disks are drawn in the illustration. The shields around the transducer coils have been cut away to show the coil and magnetostrictive driving rods. As you can see, either end of the filter may be used as the input.

Figure 9–39 is the electrical equivalent of the mechanical filter. The disks of the mechanical filter are represented by the series resonant circuits L_1C_1 while C_2 represents the coupling rods. R in both input and output represents the matching mechanical loads.

TRANSDUCERS. The transducer converts mechanical energy into electrical energy or electrical energy into mechanical energy. Both magnetostrictive and electrostrictive devices are transducers. Electrostriction is defined as the deformation produced by electric stress when a dielectric is charged. Since electrostriction in dielectrics applies to any interaction between an electric field and the corresponding deformation, electrostriction includes the piezoelectric effect. An electrostrictive transducer such as a piezoelectric crystal will compress when subjected to an electric current or voltage.

The magnetostrictive transducer depends on the principle that certain materials can be made to lengthen or shorten when in a magnetic field. If an electrical signal is sent through a coil whose core is made of a magnetostrictive material, the electrical variations will be converted into mechanical changes. The mechanical oscillations can be used to drive the mechanical elements of a filter, which are the disks.

Both types of transducers have a natural resonant frequency. They are manufactured or tuned to be resonant at a specific frequency. Transducers are chosen for a particular application, in this instance, the ability to pass the frequencies of one sideband and to reject all other frequencies.

OPERATION. Let us assume that the mechanical filter of Figure 9–38 has disks tuned to pass the frequencies of the desired sideband. The input to the filter contains both sidebands, and the transducer driving rod applies both sidebands to the first disk. The vibration of the disk will be greater at the frequency to which it is tuned (resonant frequency), which is the desired sideband, than at the undesired sideband frequency. The mechanical vibration of the first disk is transferred to the second disk, but a smaller percentage of the unwanted sideband frequency is transferred. Each time the vibrations are transferred from one disk to the next, there is a smaller amount of the unwanted sideband. At the end of the filter there is practically none of the undesired sideband left. The desired sideband frequencies are taken off the transducer coil at the output end of the filter.

Varying the size of C_2 in the equivalent circuit in Figure 9–39 varies the bandwidth of the filter. Similarly, by varying the mechanical coupling between the disks (Figure 9–38), that is, by making the coupling rods either larger or smaller, the bandwidth of the mechanical filter is varied. Because the bandwidth varies approximately as the total area of the coupling rods, the bandwidth of the mechanical filter can be increased by using either larger coupling rods or more coupling rods. Mechanical filters with bandwidths as narrow as 0.5-kc and as wide as 35 kc are practical in the 100- to 500-kc range.

Single-Sideband Transmitters

Figure 9–40 is a block diagram of a single sideband suppressed carrier transmitter using me-

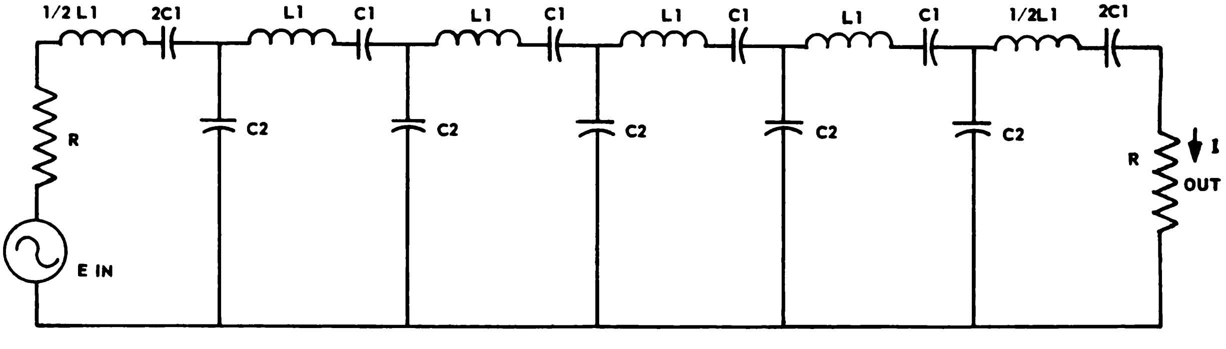


Figure 9–39. Electrical Analogy of a Mechanical Filter

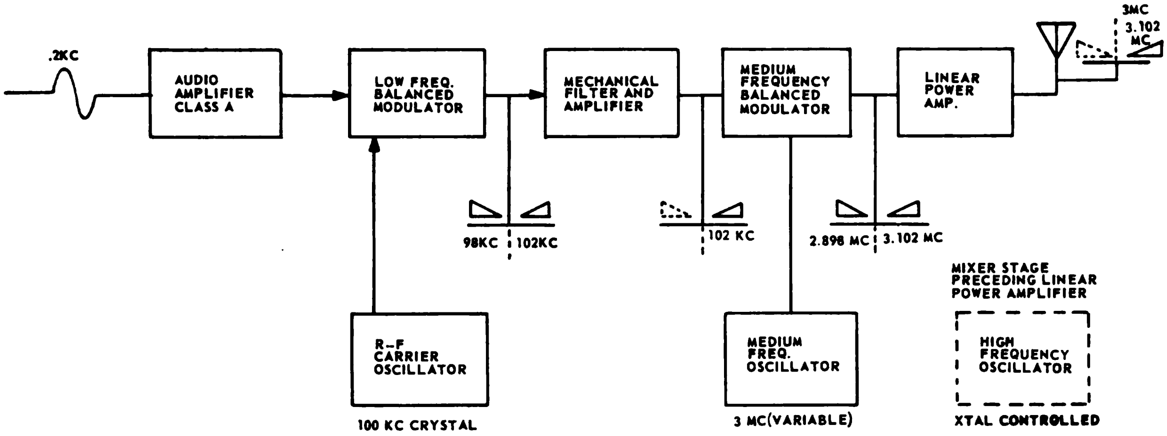


Figure 9–40. SSB Transmitter Block Diagram

chanical filters and a balanced modulator. For illustrative purposes, a single tone, 2000-cycle audio signal is used, but in actual practice a complex audio signal would be applied.

The 2-kc signal is amplified and mixed with a 100-kc carrier or conversion frequency in the balanced modulator. Remember, neither the carrier nor the audio frequency appear in the output of the balanced modulator; but the sum and difference frequencies (98 kc and 102 kc) do appear in the output. As illustrated in Figure 9–40, two sidebands are applied to the mechanical filter. Only the desired upper sideband is passed. The dotted lines show where the carrier and lower sideband have been removed.

Because the remaining sideband (containing the intelligence) is too low in frequency to transmit efficiently, it must be mixed again with a new conversion frequency to raise it to the desired transmitter frequency. The 3-mc oscillator applies a signal to a balanced modulator. Again, the balanced modulator, after mixing the two inputs to get two new sidebands, removes the two fundamentals and applies the two new sidebands (3102 kc and 2898 kc) to a tunable linear power amplifier.

The input and output circuits of the linear power amplifier are tuned to reject one sideband and pass the other to the antenna for transmission. The high frequency oscillator is variable so that the transmitter output frequency can be varied over a range of operating frequencies. Since both the carrier and one sideband have been eliminated, all of the transmitted energy is in the single sideband.

Under certain applications another medium-frequency oscillator and modulator may be inserted for increased stability and a wider operating band.

The independent sideband type of SSB transmitter generates both sidebands, each containing different intelligence, with the carrier suppressed to a desired level. Basically, this is accomplished with a triple conversion process that translates the two audio frequency inputs (100 to 6000 cps) into two r-f sideband signals centered around a reinserted pilot carrier. The pilot carrier is used as a reference and to control receiving equipment auxiliary circuits such as AVC, AFC, and so forth. Figure 9–41 illustrates a typical block diagram for an independent sideband transmitter.

The first conversion step converts the audio inputs into a low frequency (LF) signal of 100 ± 6 kc. Each audio input is applied to a separate balanced modulator at the same time as a 100-kc conversion frequency. The output of each modulator will contain the upper sideband frequency of 100.1 to 106 kc and the lower sideband frequency of 94 to 99.9 kc. The 100-kc conversion frequency and the audio inputs will not appear in the modulator output.

Associated with each modulator is a lattice type crystal filter. The Group A signal is fed to the A crystal filter, which passes only the upper sideband (sum frequency) output from LF Mod 1. The Group B signal is fed to the B filter, which passes only the lower sideband (difference frequency) output from LF Mod 2. At this point we have succeeded in generating, or developing, the independent sidebands.

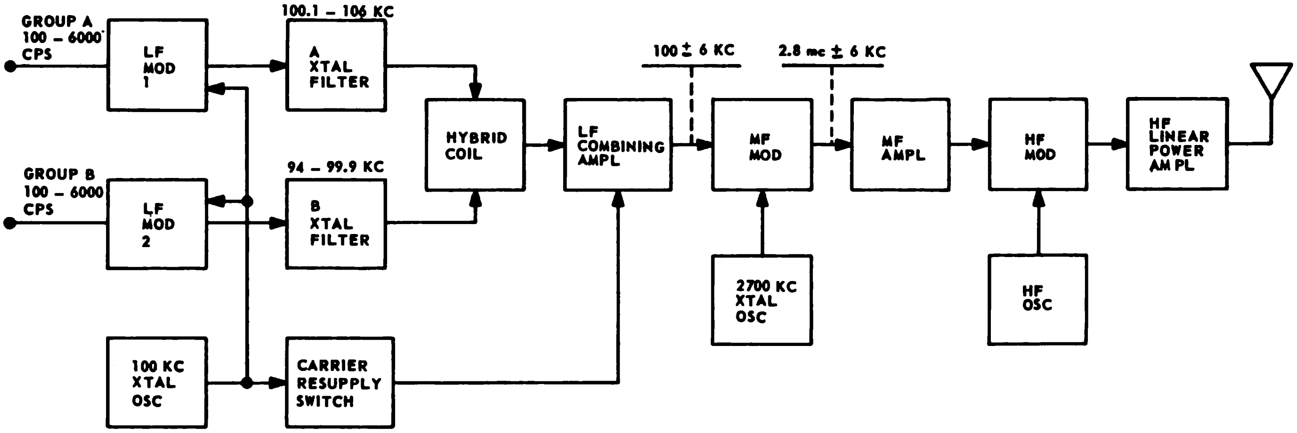


Figure 9–41. Independent Sideband SSB Transmitter Block Diagram

This brings up another problem. How can these two signals be transmitted? To accomplish this task, we use what is termed a *hybrid coil.* This is a specially designed transformer that combines two signals without mixing or interaction. The hybrid coil is a linear device which combines the two sideband signals into one composite signal voltage that represents the two bands of frequencies of 94 to 99.9 kc and 100.1 to 106 kc. The output of the hybrid coil is then applied to the LF combining amplifier.

The 100-kc crystal-controlled oscillator supplies the conversion frequency for the LF modulators and the signal that will be used as the reinserted carrier. This signal is applied to the carrier resupply switch, which is adjusted for any desired level of carrier suppression. Normal independent sideband operation requires that the carrier be suppressed 20 db with respect to the level of the sidebands of intelligence. When the carrier of a transmitter rated at 4000 watts peak envelope power is suppressed 20 db, there will be approximately 40 watts of power in the carrier. The remaining 3960 watts of power is contained in the upper and lower sidebands.

The LF combining amplifier in Figure 9–41 is a Class A linear amplifier. Its function is to combine the sidebands with the reinserted carrier. The output of this amplifier is applied to the medium-frequency (MF) modulator, where the second frequency conversion takes place. The output of the MF modulator contains the 2600 ± 6 kc lower sideband and the 2800 ± 6 kc upper sideband. The LF input signal and 2700-kc conversion frequencies are balanced out in the modulator circuit. The undesired modulation products are passed to ground by resonant filters, while the sum frequency is passed by a filter in the MF amplifier input. The 2800 ± 6-kc signal is amplified by the MF amplifier and then applied to the next conversion stage.

The final frequency conversion takes place in the HF modulator when the MF signal is mixed with the conversion frequency generated by the HF oscillator, which determines the operating frequency of the transmitter. This oscillator makes possible an output over a wide range of frequencies in the high-frequency spectrum. Assuming that the transmitter is to operate on 5 mc, the HF oscillator must produce an output of 7.8 mc or 2.2 mc, depending on whether the HF linear power amplifier is tuned to the upper or the lower sideband output of the HF modulator circuit. To operate on 25 mc, the HF oscillator must produce an output of either 22.2 mc or 27.8 mc, depending on how the linear power amplifier is tuned. The HF linear amplifier is always tuned to the operating frequency, which will be either the upper sideband (sum) or the lower sideband (difference) output of the HF modulator.

Single-Sideband Receivers

To see the relationship of the parts in a single sideband receiver, observe the block diagram in Figure 9–42. Basically, the receiver is similar to an ordinary AM superheterodyne; that is, it has r-f and i-f amplifiers, mixer, detector, and audio amplifiers. However, to permit satisfactory SSB reception, an additional mixer (demodulator) and oscillator must replace the conventional diode second detector.

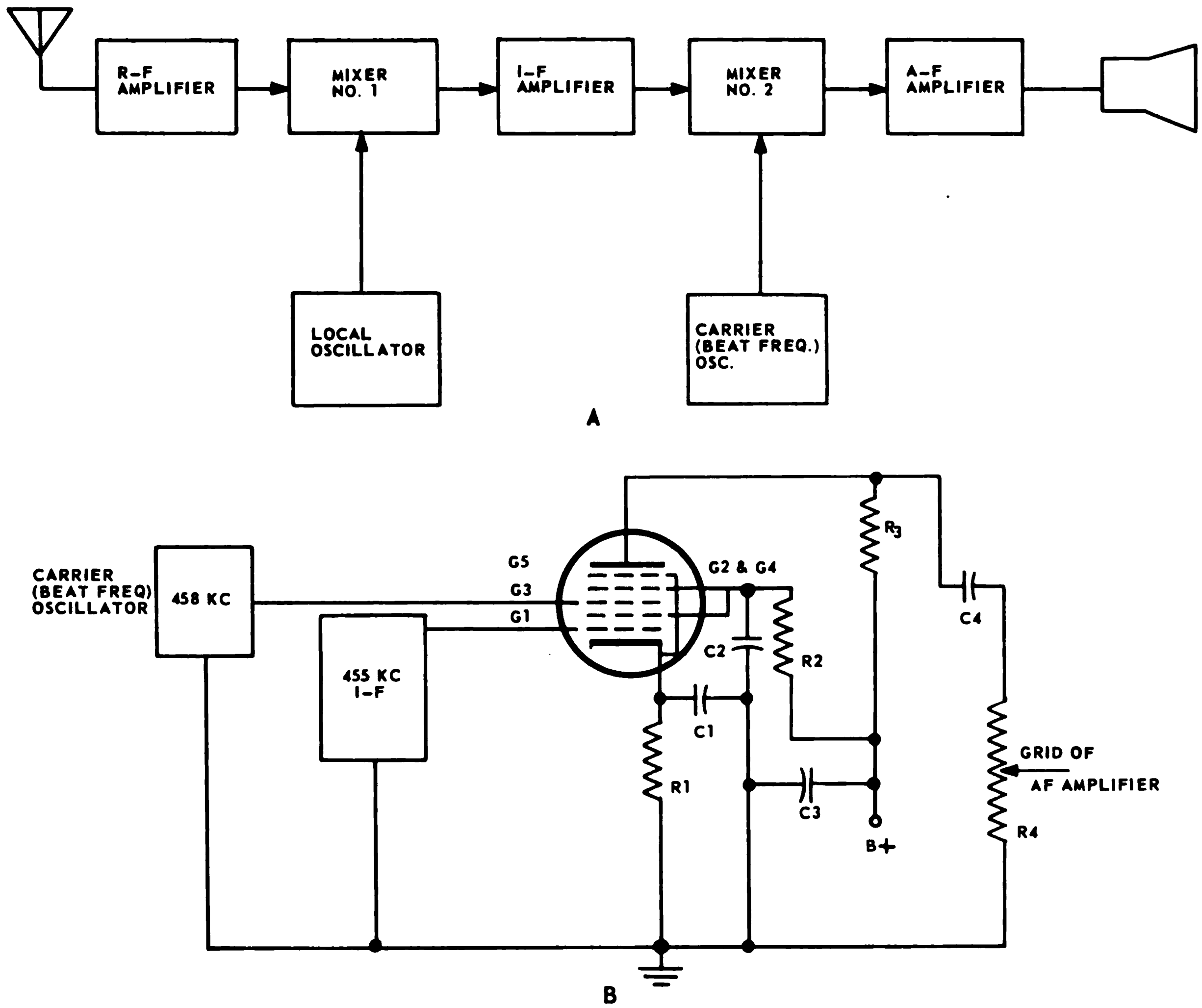


Figure 9–42. Basic Single Sideband Communications

As shown before, the carrier frequency was suppressed at the transmitter; thus, for a sideband to be received properly, a carrier must be inserted by the receiver. The receiver illustrated in Figure 9–42 inserts a carrier frequency preceding the detector, although the carrier frequency may be inserted at any point in the receiver before demodulation of the signal.

When the SSSC signal is received at the antenna, it is immediately amplified by the r-f amplifier and applied to the first mixer. By mixing the output of the local oscillator with the input signal (heterodyning), a difference frequency, or i-f, is obtained. The i-f is then amplified by one or more stages of amplification. Of course, this is dependent upon the type of receiver. Up to this point, an AM superheterodyne receiver is exactly the same. Ordinarily, the signal would now be applied to the detector for demodulation of the intelligence.

Since the sideband frequency is the only frequency present, the carrier must now be inserted. This is accomplished by the second mixer (demodulator) and carrier generator. This latter stage is a variable frequency oscillator that can be adjusted to simulate the carrier frequency, which was suppressed at the transmitter. Now the original intelligence can be recovered by the second mixer stage. In the case of the audio intelligence, the output of the second mixer is applied to a stage of audio amplification and then to the speaker.

The second mixer is identical to the first mixer except for the frequencies at the input and output. Consider an SSB of 2 mc to be transmitted. This signal is generated by combining a carrier of 1997 kc with an audio frequency of 3 kc in a balanced modulator and selecting the upper sideband, 1997 kc + 3 kc = 2 mc (2000 kc). When the receiver is tuned to this frequency, the 2-mc signal is converted to a 455-kc signal, amplified and applied to the second mixer. When the carrier oscillator is adjusted to 458 kc, a 3-kc signal (458 kc — 455 kc) is produced in the output of the mixer.

Tuning the sideband receiver is somewhat more difficult than a regular AM receiver. The carrier (beat frequency) oscillator must be precisely adjusted to simulate the carrier frequency at all times. Any tendency of drift within the oscillator will cause the output intelligence to be distorted.

In order to separate the sidebands when both are transmitted, a selective crystal filter is placed in the i-f amplifier strip. This filter is capable of shifting the passband slightly above or below the normal i-f frequency for reception of the upper or lower sideband respectively.

The independent sideband type of SSB receiver (Figure 9–43) is basically the same as the SSSC type except that this receiver must have two audio outputs. This is necessary to reproduce the two audio signals originally applied to the transmitter. The basic principles involved in this type of reception are the same as those previously explained for the transmitter.

The first and second conversion stages of the independent sideband receiver are almost identical to those used by the normal military communications receiver operating on the superheterodyne principle. The main difference in this receiver is the use of a hybrid coil that separates the 100 ± 6 kc signal into two signals before application to their respective filters and demodulators. The A crystal filter passes frequencies from 100.1 to 106 kc, while the B filter passes the lower sideband frequencies from 94 to 99.9 kc.

The signals are each demodulated by their own separate balanced demodulator when mixed with the 100-kc conversion frequency. These demodulators are identical to the balanced ring modulator. The two 100- to 6000-cycle audio signals are amplified by their respective amplifiers and then applied to the external terminal equipment. The signal that is taken from the LF amplifier is used to operate the special circuits required for AVC, AFC, and reconditioned carrier applications.

Pre-Emphasis and De-Emphasis Networks

Audio frequency pre-emphasis and de-emphasis networks are also known as audio frequency accentuation and deaccentuation circuits, respectively. Where the former network is used in the FM transmitter, the latter network will be used in the FM receiver.

The amplitudes of the higher audio frequencies in the audio input to a transmitter are comparatively low when considering the whole audio frequency spectrum. This is not due to any transmitter characteristic, but to the natural distribution of sound in radio program material and communications as used by the military. Noise is unavoidably added as the transmission

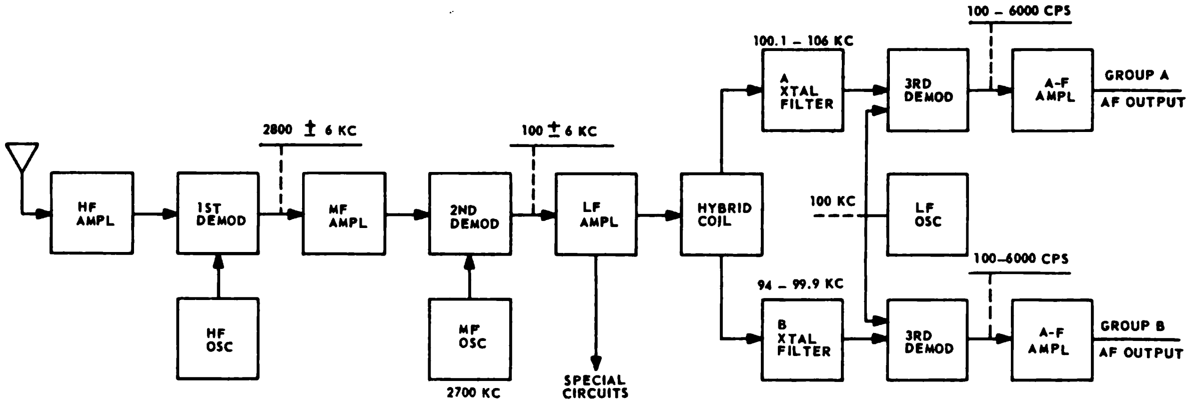


Figure 9–43. Independent Sideband SSB Receiver Block Diagram

passes through the transmitter and then on through the receiver.

The useful relative amplitudes of the desired audio frequency signals are generally small in the upper audio frequency range. Since noise is predominantly high-frequency audio, this condition tends to produce an undesirable signal-to-noise ratio at the high audio frequencies. The signal is relatively weak and the noise is relatively strong. Pre-emphasis circuits are used in the transmitter to overcome this condition.

By definition, pre-emphasis is the increasing of the relative strength of the high-frequency components of the audio signal before it is fed to the modulator. Stated differently, pre-emphasis provides in the upper audio-frequency range the desired signal currents, with respect to desired signal currents in the lower audio-frequency range. Thus, the undesirable relationship between the high-frequency signal and the high-frequency noise is altered. While the noise remains the same, the signal strength is increased. There is a disadvantage, however, in the change in the natural balance between high- and low-frequency tones.

A de-emphasis circuit in the receiver, however, corrects this defect, as it reduces the high-frequency audio the same amount as the pre-emphasis circuit increased it, thus regaining the original tonal balance. In addition, the de-emphasis network operates on both the high-frequency signal and the high-frequency noise; therefore, there is no change in the improved signal-to-noise ratio.

The main reason for the pre-emphasis network then, is to prevent the high-frequency components of the transmitted intelligence from being blocked out by the high-frequency noise characteristics inherent in the transmitter. In the intelligence this same pre-emphasis characteristic will be picked up by the receiver.

The de-emphasis network is normally inserted between the detector and the audio amplifier. This makes certain that the audio frequencies are returned to their original relative level before amplification. The pre-emphasis characteristic curve is flat up to 500 cycles. From 500 to 15,000 cycles there is a sharp increase in gain up to approximately 17 db. The gain at these frequencies is necessary to maintain the high signal-to-noise ratio at high audio frequencies. The frequency characteristic of the de-emphasis network is directly opposite to that of the pre-emphasis network. The high-frequency response decreases in proportion to its increase in the pre-emphasis network. The characteristic curve of the de-emphasis circuit should be a mirror image of the pre-emphasis characteristic curve.

If the two characteristic curves are mirror images of each other, the time constant of both networks should be the same. For commercial broadcasting a time constant of 75 microseconds has been set as a standard. For military pur-

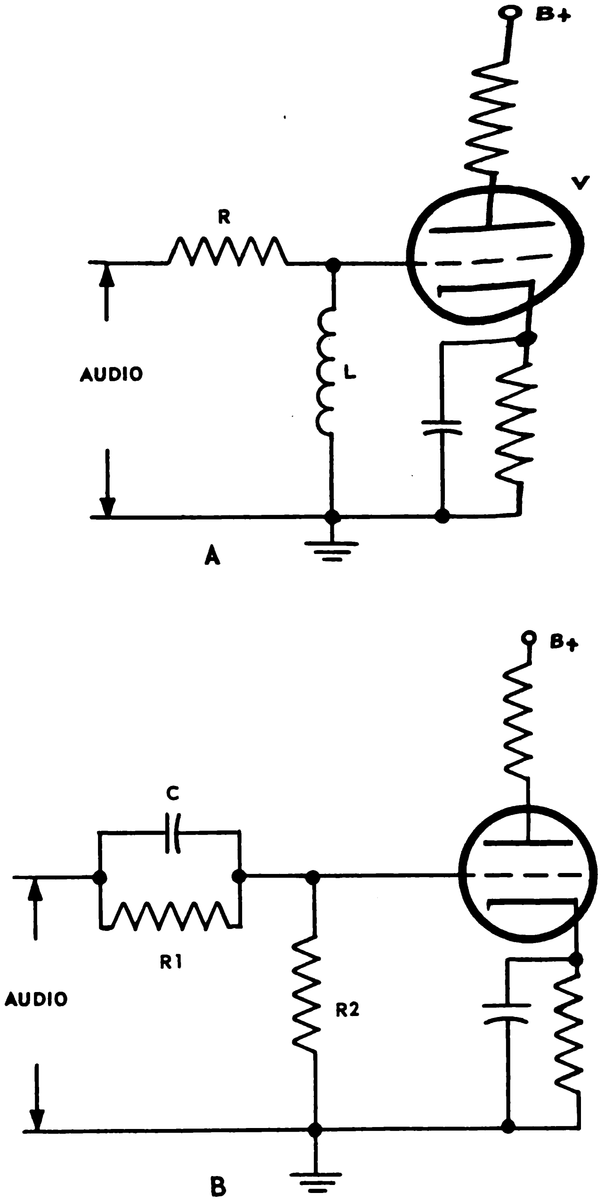


Figure 9—44. Typical Pre-Emphasis Circuits

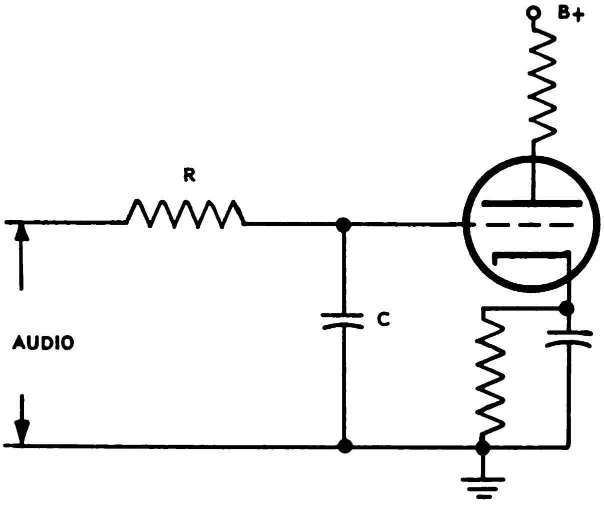


Figure 9–45. De-Emphasis Circuit

poses design specifications will determine the time constant.

Two typical pre-emphasis networks are illustrated in Figure 9–44. In Figure 9–44A the series circuit of R and L forms the total impedance to the audio. Since $X_L = 2\pi fl$, if f (frequency) increases, then X_L increases. This condition being true, an increase in frequency will produce an increase in voltage across L. This is the input to the grid of the tube. In Figure 9–44B the impedance to the audio voltage is that of the parallel circuit of C and R_1, as the ohmic value of R_2 is small in comparison to that of either C or R_1. $X_c = \frac{1}{2\pi fC}$. As the audio frequency increases, the reactance of C decreases. This decrease of X_c provides an easier path for high frequencies as compared to R. Thus, with an increase of audio frequency, there is a like increase in signal current. The result is a larger voltage drop across R_2 at the higher frequencies.

Figure 9–45 depicts a typical de-emphasis network. Note the physical position of R and C in relation to the grid of the tube. As the frequency of the audio signal increases, the reactance of capacitor C decreases. The voltage division between R and C now provides a smaller drop across C. The audio voltage applied to the grid of the tube decreases; therefore, a reverse of the pre-emphasis circuit is accomplished. For the signal to be exactly the same as before pre-emphasis and de-emphasis, the time constants of the two circuits should be equal to each other.

CHAPTER 10

Transmission Lines

A transmission line is a conductor or a group of conductors used singly or in multiple to transmit electrical energy from a source to a load. Transmission lines are necessary because the load is normally located at a distance from the source of the energy. The primary purpose of a transmission line is to carry electrical energy from the source to the load with minimum loss.

This chapter is to acquaint you with the principles of transmission lines. It discusses their theory, uses, and construction. It also explains the types of losses to which radio frequency lines are subject, methods of making measurements on them, and how the principles of radio frequency lines are used in constructing artificial lines.

DEFINITION

A radio frequency line is a special form of transmission line. A transmission line is a device, usually a pair of electrical conductors, that transfers electrical energy from a power source to a distant electrical circuit requiring power. The using device is called the *load.*

In transferring DC power the sole requirement of a transmission line is that it offers low DC resistance. In transferring AC power at frequencies as low as 60 or 400 cycles per second (commercial and aircraft power frequencies), the reactance of a transmission line is negligible compared to its resistance. As frequency increases, however, the effects of capacity and inductance increase, until at radio frequencies (above 50 kc) the reactive components retard the transfer of energy to such an extent that a complete cycle may be generated at the source before the voltage at the start of the cycle reaches the load. Whenever this occurs, the line is said to be *electrically long.* At the frequency of 400 cps it takes a line about 400 miles long to cause this phenomenon. The longest line carrying 400-cycle frequencies possible in an aircraft is about 200 feet. Such a line is considered very short electrically.

In such cases it is possible to disregard the reactive effects. But at a frequency of 10,000 megacycles (a frequency commonly used in radar), a distance of only 2 inches in a line causes this effect. The 2-inch line is electrically long. A transmission line is considered to be electrically short when its physical length is small compared to a quarter-wavelength of the energy it is to carry. A line is said to be electrically long when its length is great in comparison to a quarter-wavelength. This means that at these high frequencies used in radar you must consider the length of the conductor from one vacuum tube to another as electrically long. Whenever a line is electrically long it is considered a *radio-frequency line,* and as such there are special considerations in design and adjustment.

FUNCTION

In radar equipment r-f lines transfer power from the transmitter to the antenna and from the antenna to the receiver. Since the r-f line displays reactive characteristics, short sections of r-f lines are used as tuned oscillator circuits, as inductors or capacitors, and as delay networks. The capacity in an r-f line can be charged, and upon discharging, produces a square wave rather than an exponential one. The r-f line is also used to produce a square radar transmitter pulse.

R-F LINE THEORY

By resolving an r-f line into a simple equivalent circuit and analyzing this circuit, you will be able to understand the characteristics of the r-f line. This equivalent circuit is made by lumping all the resistances into a single large resistor, all the inductances into a single large inductor, and so on. Such a circuit is an r-f line with lumped constants. To evaluate the effect of the r-f line shown in Figure 10–1A, it is necessary to put only the constants of a small section of a line into the equivalent circuit and to multiply the effect of this circuit by the number of these sections.

The first constant to consider is the DC resistance of the conductors. In the equivalent circuit this constant is a series-connected element. It is represented by R in each line of the equivalent circuit shown in Figure 10–1E.

The second constant is inductance. Previously you learn that whenever electrons move in a wire, a magnetic field is produced about the wire which, because of self-induction, retards the acceleration of the electrons as it builds up. In an r-f line there is a similar self-inductive action.

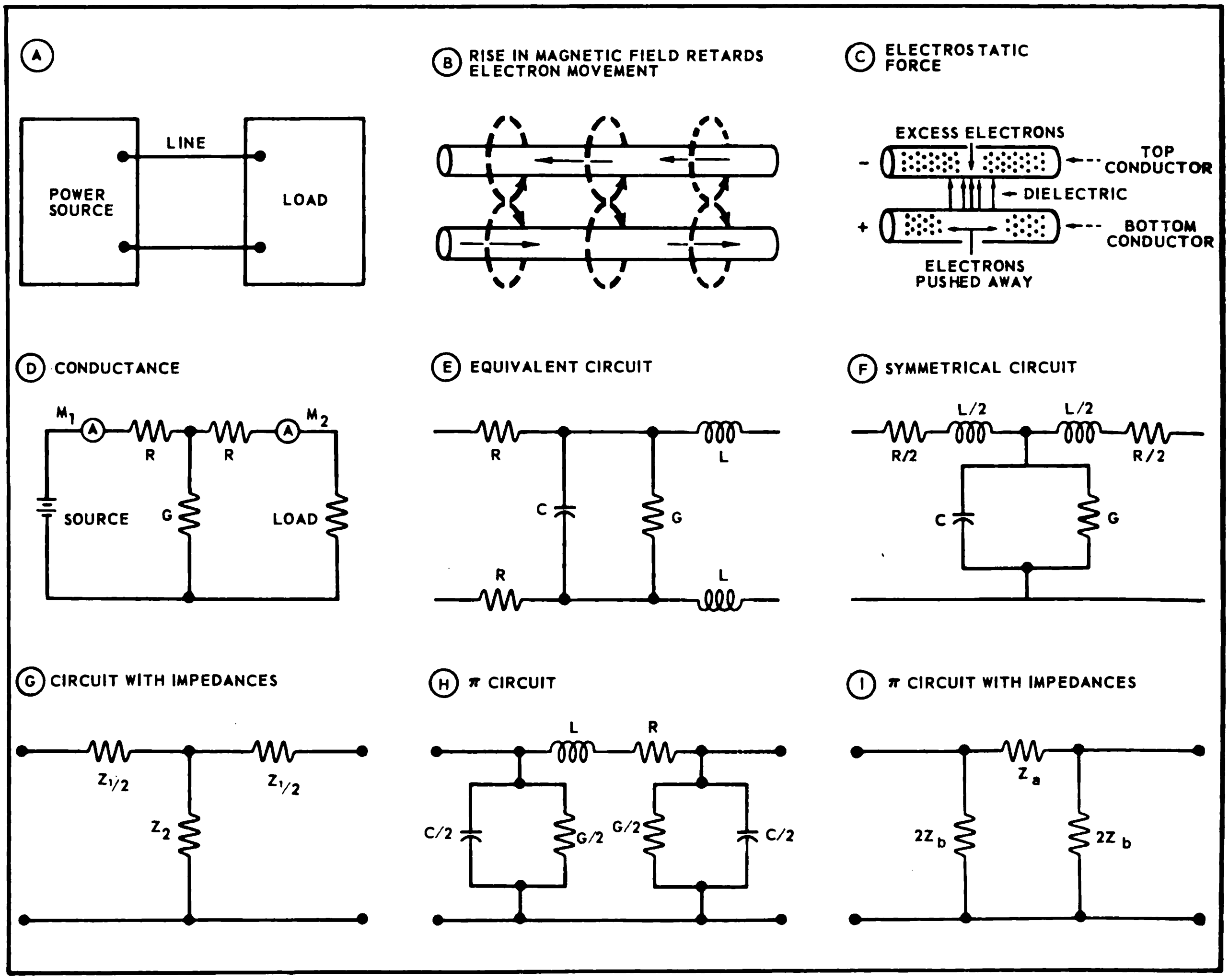


Figure 10–1. Equivalent Circuit of R-F Transmission Line

This action is represented in the equivalent circuit by inductors in each line, as shown in Figure 10–1.

Another constant to consider is capacitance. To understand the effect of capacitance in r-f lines, recall that the term *voltage difference* means that an excessive number of electrons exist at a certain point in an electrical circuit. In an r-f line, voltage difference means that there are more electrons on one conductor than on the other. The point of excess electrons exerts a force through the dielectric material between the two conductors and pushes the electrons away from this point as indicated in Figure 10–1C. This action is identical to the action that takes place between the two plates of a capacitor, and indicates that there is a capacitance between the two conductors. The capacitor C in the equivalent circuit at E represents the capacitance of the r-f line.

The fourth constant in an r-f line is the leakage that takes place from one wire to the other through the insulation between the wires. There is no such thing as a perfect insulator. Any insulator, therefore, will act as high resistance rather than an open circuit. The insulation conductance between wires is represented as G in Figure 10–1D. In this case note that with G removed from the equivalent circuit, the current on meter M_1 at the source will read the same as meter M_2 at the load. When you replace the shunt conductance (G) in the circuit, more current will flow at the source. Thus, you see that without changing the load resistance or the source voltage, you can increase the current.

This completes the equivalent circuit and it appears as shown in Figure 10–1E. But it will not perform exactly like the actual line itself. An actual line is uniformly constructed so that energy going either way will encounter the same effects. This is not the case in the equivalent circuit, for if you look into the line shown at E from the left end, you see two R's and one C connected in series, and if you look into the right end, you see two L's and one G in series. This is not the same circuit.

To balance and simplify the circuit, it is changed as you can see in Figure 10–1F so that all the series resistance is put into the top line, giving the same electrical effect. The series resistance is then divided into halves and each half is located as shown at F. The same is done with the inductance. The net result is that the circuit looks the same from either end.

You can further reduce the circuit to the simple impedance network as shown in Figure 10–1G. In this case all the series impedances (represented by Z_1) are symmetrically spaced about the parallel impedance (represented by Z_2). When the circuit is reduced to this form, it is possible to develop certain general equations in terms of Z_1 and Z_2. These equations, which are discussed later, apply to any circuit regardless of the resistance, the inductance, or the capacitance in Z_1 and Z_2.

Figure 10–1H shows another form into which the r-f line constants can be lumped. In this circuit the shunt constants are symmetrically placed about the series constants. This is known as a Pi section and is shown in terms of impedances in Figure 10–1I.

The Characteristic Impedance of a Lossless Line

In an r-f line there is an inherent impedance that is constant regardless of the length or the applied voltage. In the equivalent circuit shown in Figure 10–2A, the r-f line is assumed to be without losses. Remember that inductance or capacitance will store energy, then return it to a circuit, while resistance will dissipate energy in the form of heat. Since no resistors appear in the circuit at A, no energy will be lost. A line can be constructed with extremely small losses; this approximates the ideal situation. Such a line is assumed to be infinitely long. Although this, too, is a theoretical situation, cross-country telephone lines approach the condition described.

Notice what happens when you apply a DC voltage to this circuit. At the time you close the switch, the full voltage E will appear across the line terminals A–B. This end is referred to as the *sending end*. The end to which energy travels is called the *receiving end*. The first capacitor C_1 starts to charge. But its charge is not instantaneous, since the charging current is impeded by the inductance of L_1. After a short time, however, C_1 will approach full charge, and its charging current will decrease. C_2 will start to charge through L_2, and the resultant current through the ammeter (A) in the circuit will remain constant.

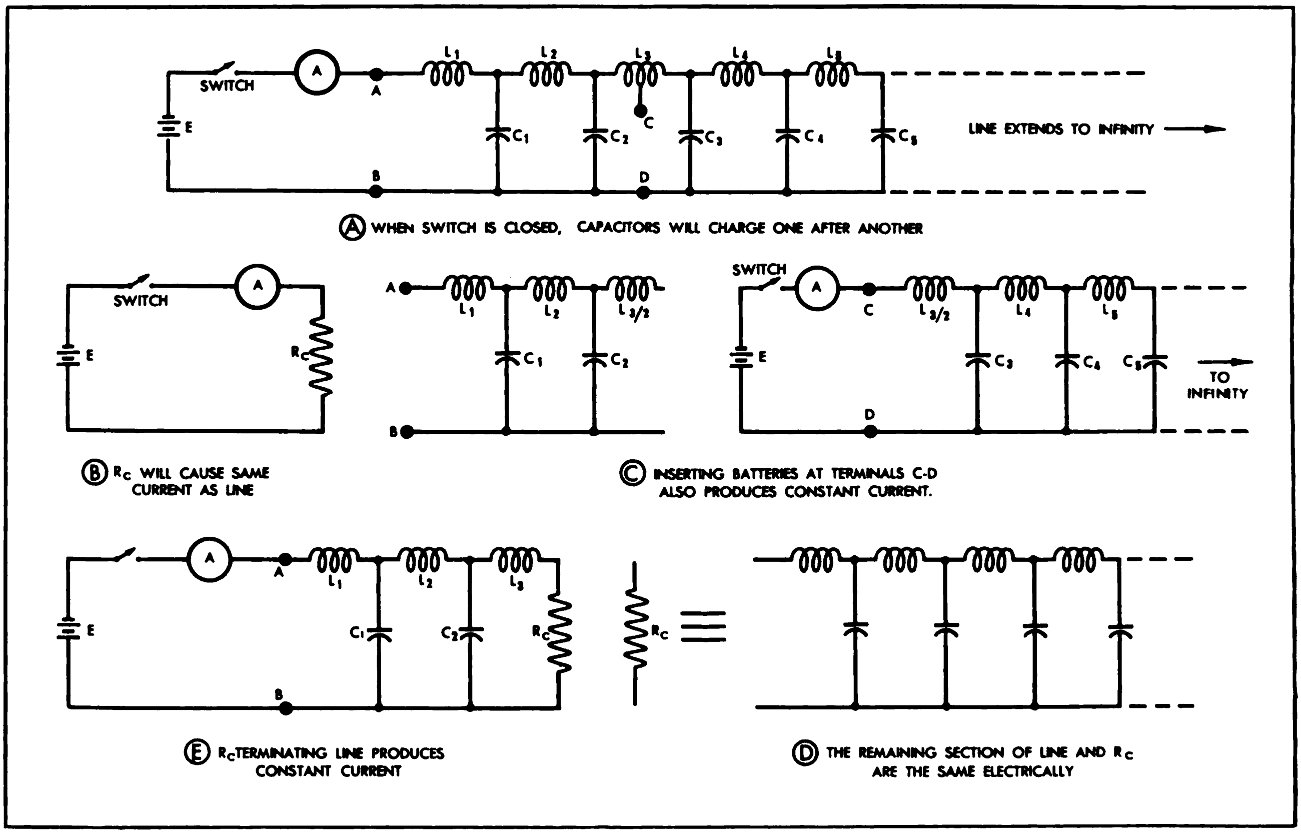

SWITCH
A
E
B
L_1
L_2
L_3
L_4
L_5
C_1
C_2
C
D
C_3
C_4
C_5
LINE EXTENDS TO INFINITY
Ⓐ WHEN SWITCH IS CLOSED, CAPACITORS WILL CHARGE ONE AFTER ANOTHER
SWITCH
A
E
R_C
Ⓑ R_C WILL CAUSE SAME CURRENT AS LINE
A
B
L_1
L_2
$L_{3/2}$
C_1
C_2
SWITCH
A
E
C
D
$L_{3/2}$
L_4
L_5
C_3
C_4
C_5
TO INFINITY
Ⓒ INSERTING BATTERIES AT TERMINALS C-D ALSO PRODUCES CONSTANT CURRENT.
A
E
B
L_1
L_2
L_3
C_1
C_2
R_C
R_C
≡
Ⓔ R_C TERMINATING LINE PRODUCES CONSTANT CURRENT
Ⓓ THE REMAINING SECTION OF LINE AND R_C ARE THE SAME ELECTRICALLY

Figure 10–2. Developing Characteristic Impedance

A current will flow through the inductors to charge the next capacitor. Each capacitor down the line will add to the charging current as the current through the preceding capacitor starts to decrease. Since the line is infinitely long, there will always be more capacitors to be charged, and current will not stop flowing until they are all charged. The line has the same effect upon the battery as would a fixed resistor, with a value equal to the characteristic impedance of the line.

Another circuit that will display identical characteristics is shown in Figure 10–2B. The resistor R in this circuit will cause a certain current to flow indefinitely when voltage E is applied. (To determine the value of R, use Ohm's law formula, $R = E/I$.) Since the voltage E is known and a constant current I is indicated by the meter, the r-f line displays a resistance equal to the value E/I. This resistance is the characteristic resistance of the r-f line, and may be called R_c. Mathematically, it is

$$R_c = \frac{E}{I}.$$

If you broke the line at the center of L_3 and connected the battery to terminals C–D as shown in Figure 10–2C, again the same current would flow, for the same constants are displayed and the receiving section is still infinitely long.

If a resistance equal to the characteristic resistance of the section removed is placed across the receiving end-terminals of the short section and the battery is again connected across terminals A–B as shown in Figure 10–2E, the same current will flow again. Therefore, whether the line is infinitely long or fixed in length and terminated in a resistance equal to its characteristic resistance, the effect at the battery end is the same. The display of constant resistance that you see at the sending end is equally important, for it is this resistance that must be matched by the generator for maximum power transfer. When AC voltages are applied to the line, the characteristics of the line may be resistive with a reactive component — either capacitive or inductive. Therefore, the characteristic of a line generally is described as the characteristic impedance, for which the symbol is Z_0.

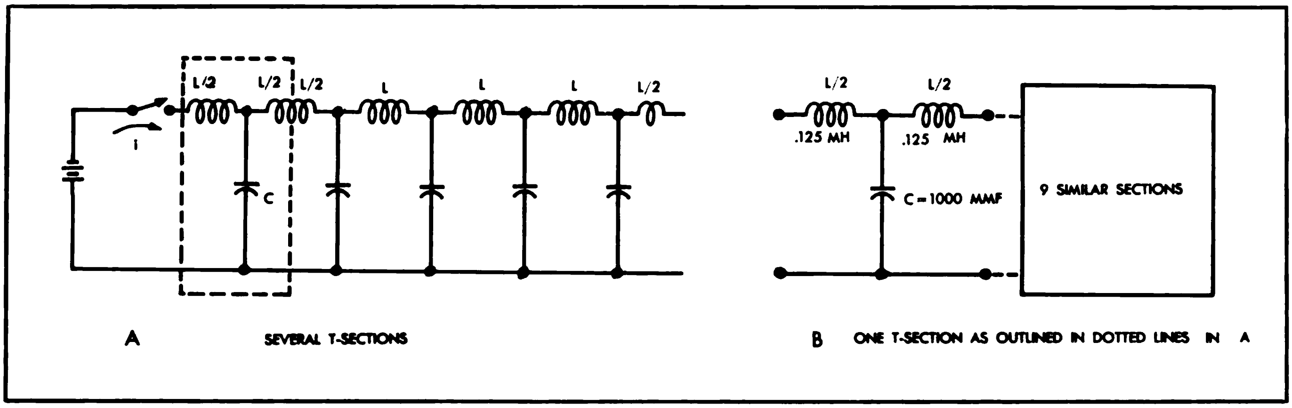


Figure 10–3. Circuit for Calculating Time for Wave to Travel Down the Line

Voltage Changes Moving Along a Line

Previously it was stated that a voltage suddenly applied will appear at a point some distance from the sending end at some later time. This is true regardless of whether the voltage change is a jump from zero to some value, a drop from some value to zero, or for that matter, any other change in voltage. Whatever the change, the voltage will be conducted down the line at a constant rate.

If you know the inductance and capacitance of an r-f line, you can easily compute the time required by any waveform to traverse the length of the line. The circuit in Figure 10–3A is the equivalent circuit of a line reduced to T-sections and assumed to be lossless. The inductance per section is represented by L; the capacitance per section, by C. In this connection recall a few fundamental relationships, such as $Q = IT$, which means that the quantity of electricity in coulombs is equal to the current multiplied by the time the current flows, and $Q = CE$, which means that the quantity of electricity in a capacitor is equal to the capacitance times the voltage across it. In the case of the transmission line shown at A, the Q of electricity leaving the battery is also shown by the equation,

$$Q = IT \qquad (1)$$

where I is the current and T the time the current flowed.

The electricity that leaves the battery goes into the line where it is deposited in the capacitors as a charge (Q). This charge is shown by the equation,

$$Q = CE \qquad (2)$$

where C is the capacitance per section and E the voltage applied.

The quantity of electricity that left the battery is equal to the quantity in the line; that is, $Q = Q$. Therefore, by equating (1) and (2),

$$IT = CE \qquad (3)$$

As each capacitor charges to CE, the voltage at the end of the inductor must change by E with respect to the other end. This occurs because the first capacitor charges to E voltage, while the second capacitor is still at zero voltage. This makes the voltage E appear across the inductor. In an inductor these circuit components are related mathematically as,

$$E = L\frac{\Delta I}{\Delta T} \qquad (4)$$

The voltage across the inductor is directly proportional to the inductance and the change in current but increases as the time becomes shorter. This is expressed mathematically by the equation,

$$E\Delta T = L\Delta I \qquad (5)$$

which is the result of multiplying the preceding equation by ΔT.

Since current and time start from zero, the change in time (ΔT) and the change in current (ΔI) are equal to the final time T and final current I. For this case, equation (5) becomes,

$$ET = LI \qquad (6)$$

If voltage E is applied for time T across inductor L, the final current I will flow. The following equations contain all three terms (T, L, and C).

$$IT = CE \qquad (3)$$
$$ET = LI \qquad (6)$$

Another way of using them is to get an equation for T in terms of L and C. This is done in the following manner: Multiply the left and right members of each equation as follows:

$$(IT)(ET) = (CE)(LI) \qquad (7)$$

$$EIT^2 = LCEI$$

Cancelling the EI terms, $T^2 = LC$ (8)

and $T = \sqrt{LC}$ (9)

Equation (9) is the equation for the time required for a voltage change to travel along a section composed of L and C.

Since the characteristic impedance is resistive and equal to $R = E/I$, it is equal to the voltage-to-current ratio. Since E and I appear in the preceding equations, these equations can be solved for the E/I ratio. This is done as follows:

Divide equation (6) by equation (3):

$$\frac{ET}{IT} = \frac{LI}{CE} \qquad (10)$$

Multiply both sides by $\frac{E}{I}$:

$$\frac{E^2T}{I^2T} = \frac{L}{C} \qquad (11)$$

Cancel the T's: $\frac{E^2}{I^2} = \frac{L}{C}$ (12).

Take square root: $\frac{E}{I} = \sqrt{\frac{L}{C}}$ (13)

Since E/I is the Z_o of the line, $Z_o = \sqrt{\frac{L}{C}}$ (14)

Example

Problem

Assume that the line illustrated in Figure 10–3A is 1000 feet long and that a 100-foot section is measured and found to have an inductance of ¼ millihenry and a capacitance of 1000 mmf as shown at B. Find the characteristic impedance of the line and the time required for a voltage to travel the length of one section.

Solution

$$Z_o = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.25 \times 10^{-3}}{10^{-9}}} = \sqrt{0.25 \times 10^{6}} = 0.5 \times 10^{3}$$

$Z_o =$ 500 ohms

The time required for a voltage to travel the length of one section is equal to,

$$T = \sqrt{LC} = \sqrt{0.25 \times 10^{-3} \times 10^{-9}} = \sqrt{0.25 \times 10^{-12}}$$

$$= 0.5 \times 10^{-6} \text{ sec.}$$

T = 0.5 microsecond

The time required for voltage to travel over ten sections (1000 feet) is equal to $0.5 \times 10 = 5$ microseconds.

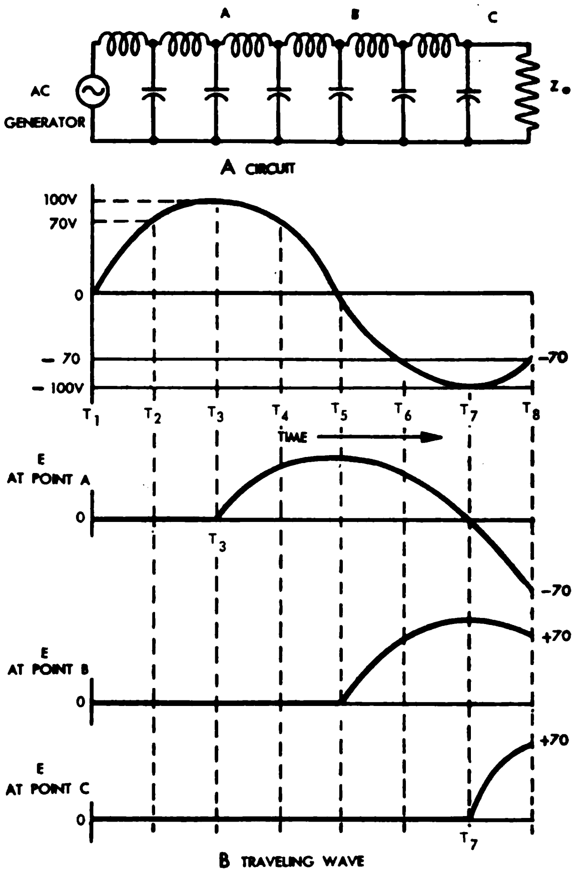


Figure 10–4. Sinusoidal Voltage Travels Down Line

This example was based on a DC change. An r-f line displays similar characteristics when an AC voltage is applied to its sending end terminals. Assume that an AC voltage is applied to the line simulated by the circuit shown in Figure 10–4A. The generator voltage starts from zero, and produces the voltage indicated. As soon as a small voltage change is produced, it starts its journey down the line, and the generator in the meantime continues to produce new voltages along a sine curve. At time T_2, the generator voltage is 70 volts. The voltages still move along the line. Finally, at time T_3, the first small change arrives at point A, and the voltage at that point starts increasing. At time T_5 the same voltage arrives at point B on the line, and finally at time T_7 the first small change arrives at the receiving end of the line. Meanwhile all the changes in the sine wave pass each point in turn. The time required for the changes to travel the length of the line is the same time required for a DC voltage to travel the distance.

At time T_7, the voltages at the various points on the line are as follows:

At generator:	–100v
At Point A:	0v
At Point B:	+100v
At Point C:	0v

If you plot these voltages along the length of the line, the resulting curve is like the curve shown in Figure

10–5A. Note that such a curve of instantaneous voltages resembles a sine wave.

At time T_8 in Figure 10–4, the voltages line up in the following manner:

At generator:	Drop from −100 to −70v
At Point A:	Rise from zero to −70v
At Point B:	Drop from 100 to 70v
At Point C:	Rise from zero to 70v

A plot of those voltages produces the solid curve shown in Figure 10–5B. For reference, the curve from time T_7 is drawn as a dotted line. The solid curve has exactly the same shape as the dotted curve, but it has moved bodily down the line the distance X. Another plot at T_9 would show a new curve similar to the one at T_8 but moved to the right by the distance X.

Summary

By analyzing the points just discussed, you will find that the actions associated with voltage changes along an r-f line are as follows:

1. All parts of the sine wave produced by the generator travel down the line in the order produced.
2. At any point you will obtain a sine wave if you plot all the instantaneous voltages passing the point during eight of the time intervals previously shown. The oscilloscope will plot all values instantaneously.
3. The oscilloscope pictures (instantaneous voltages) will be the same in all cases except that a phase difference will exist in each picture taken down the line. The phase will change continually with respect to the generator until over a certain length of line the change will be 360°.
4. Since all parts of a sine wave pass every point along the line, a plot of the readings of an AC meter (which reads the magnitude of the voltage over a period of time) will show that the voltage is constant at all points as shown in Figure 10–5C.
5. Since the line is terminated with Z_0, the energy arriving at the end is absorbed by the resistance.

The distance traveled by the start of one cycle, during the time required to generate the entire cycle, is one wavelength. In Figure 10–6 the voltage shown by the solid line arrives at the receiving end at the same time a similar voltage is generated in the next cycle. This line is exactly one electrical wavelength long and contains a full wave of voltage at one time.

The electrical length of a line is a function of the frequency of applied voltage and the velocity of energy travel. The time required for travel per section is equal to,

$$T = \sqrt{LC} \qquad (9)$$

Velocity is the distance per unit of time and distance D is the length of one section, thus,

$$V_g = \frac{D}{T} \qquad (15)$$

The subscript of g indicates velocity of waves on a line as distinguished from velocity of waves in space. In terms of L and C, equation (15) may be expressed,

$$V_g = \frac{D}{\sqrt{LC}} \qquad (16)$$

If, for example, $\sqrt{LC}$, or T, is 0.5 microsecond, and the section is 100 feet long, the velocity of

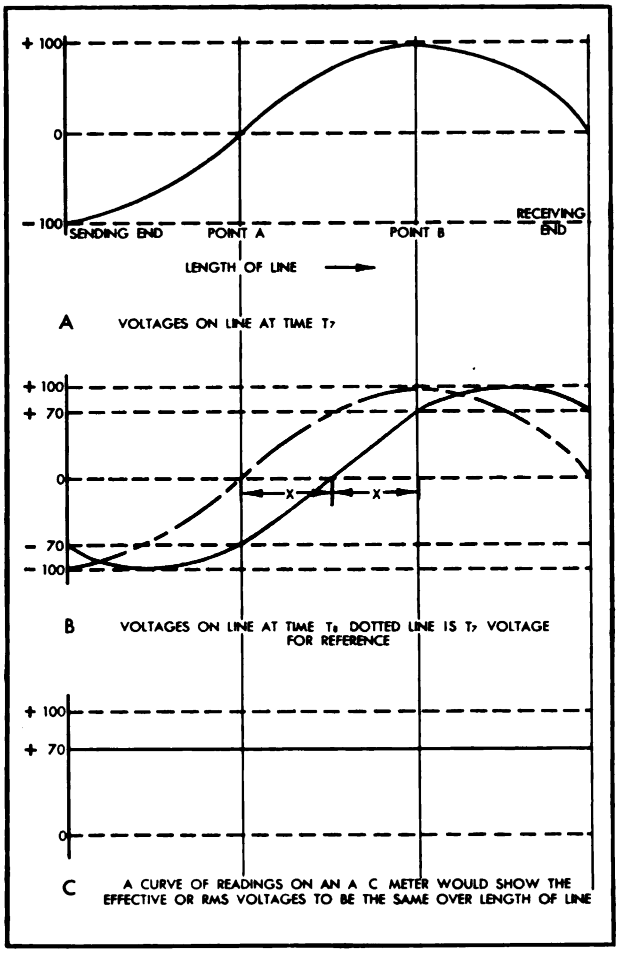


Figure 10–5. Instantaneous Voltages are Different Along Length of Line Voltages for Moving Sine Wave

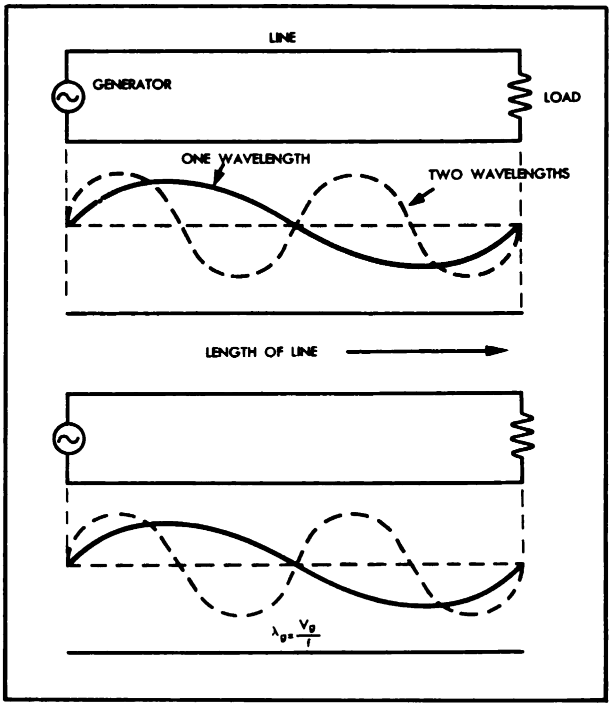


Figure 10–6. Meaning of Wavelength

the wavefront, V_g, is found as follows:

$$V_g = \frac{D}{\sqrt{LC}} = \frac{100}{0.5 \times 10^{-6}} = \frac{100 \times 10^6}{0.5}$$

$V_g = 200 \times 10^6$ or 200 million feet per second or approximately 39,000 miles per second.

The wavelength (of the group) of voltages on the line is found by the equation,

$$\lambda_g = \frac{V_g}{f} \qquad (17)$$

where f is the frequency in cycles per second. λ_g is wavelength in feet if V_g is velocity in feet per second.

Example

Problem

Assuming that the generator frequency in the preceding example is 200 kc, find λ_g.

Solution

$$\lambda_g = \frac{V_g}{f} = \frac{200 \times 10^6}{200 \times 10^3} = \frac{10^6}{10^3} = 10^3$$

$\lambda_g = 1000$ feet

One wavelength will occupy 1000 feet of the line. Notice that if you double the frequency, the wavelength is only one-half of the former value. A 1000-foot line will have two full cycles of 400-kc voltage on it at any one time. This is shown by dotted line in Figure 10–6.

Little has been said thus far about the current changes in the r-f lines. The r-f lines described were all assumed to display a resistance to the current, but no reactance to it, and the termination to have the characteristic impedance of the line. Under these conditions, the current is always directly proportional to the voltage, and whenever a voltage appears across a point on the line, a current equal to E/Z_0 will immediately flow at that point as shown in Figure 10–7. As a DC voltage change moves along the line, a current change moves with it. As an AC voltage moves along the line, a current exactly in phase with it is present at all points.

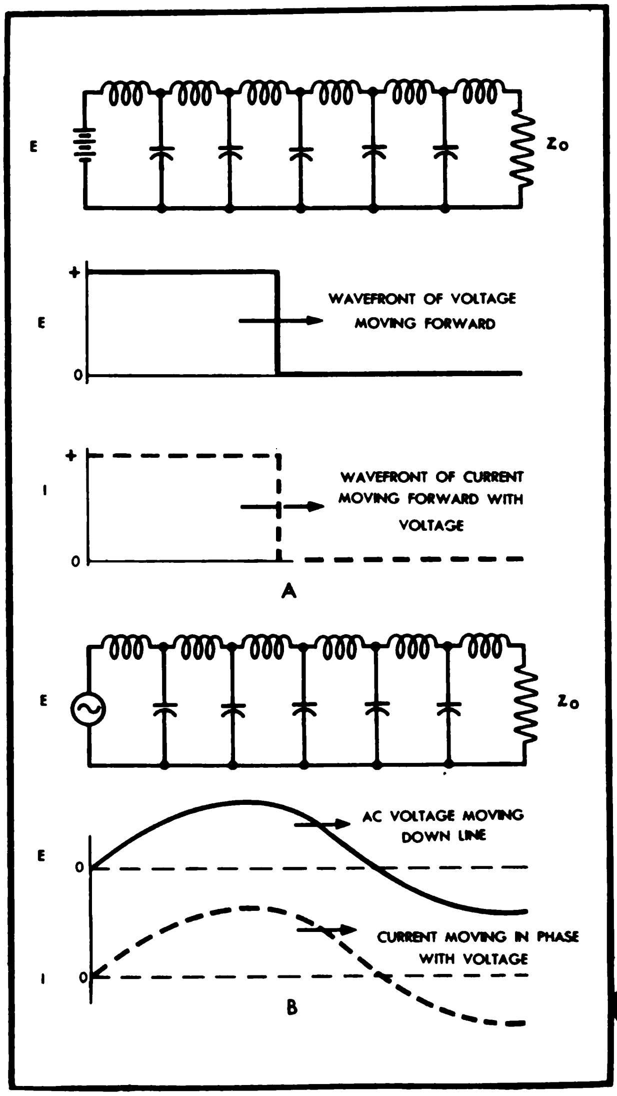

E
Zo
+
E
0
WAVEFRONT OF VOLTAGE MOVING FORWARD
+
I
0
WAVEFRONT OF CURRENT MOVING FORWARD WITH VOLTAGE
A
E
Zo
E
0
AC VOLTAGE MOVING DOWN LINE
I
0
CURRENT MOVING IN PHASE WITH VOLTAGE
B

Figure 10–7. Current Changes Move in Step with Voltage

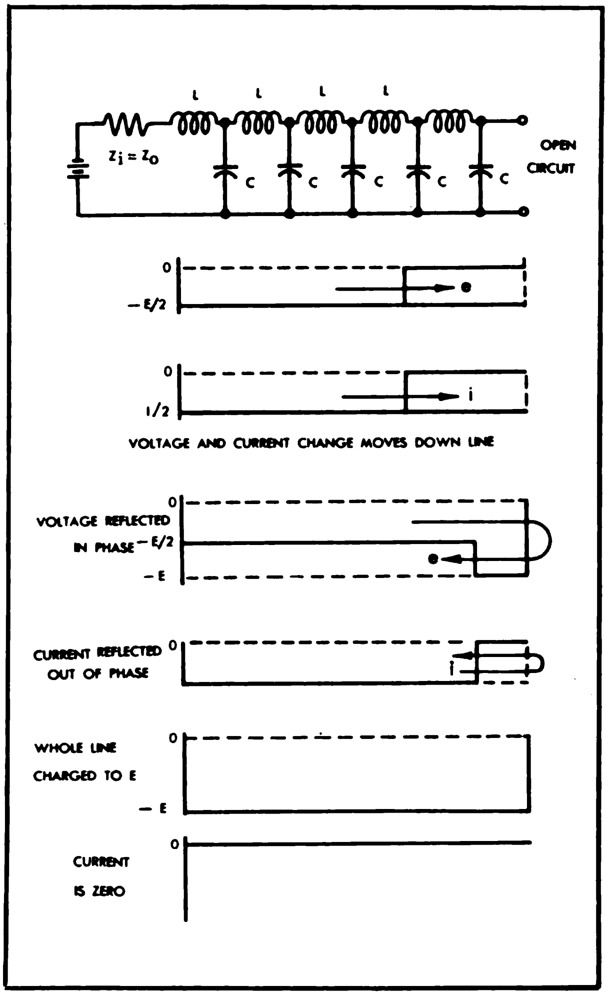


Figure 10–8. Reflection from Open-Ended Line

REFLECTION

It was assumed in the analysis of r-f lines that the terminating resistor absorbed all the energy. However, r-f lines are not always terminated in this manner. In some cases you will find the receiving end open-circuited or short-circuited.

Take the case of the line which has an open circuit at the receiving end and which has DC voltage applied at the sending end. As the switch in the equivalent line shown in Figure 10–8 is closed at the sending end, the negative voltage moves down the line, charging each capacitor through the preceding inductor. When the last capacitor is charged, there is no longer any voltage change. The capacitor at each end of the inductor is charged to ½E (since $Z_i = Z_0$, one-half the voltage drop is across Z_i and one-half across Z_0). As there is no voltage across the inductor to maintain the magnetic field, the field collapses and causes current to flow into the last capacitor. Furthermore, as the direction of current has not changed, the capacitor charges in the same direction.

Since the energy in the magnetic field described equals the energy in the capacitors, the transfer to the capacitor is just enough to double the voltage across the last capacitor. The current in the inductor drops to zero. Since there is no place for current to flow from the next to the last inductor, it's field collapses, doubling the voltage across the next to the last capacitor. This change of voltage (moving backward) is the same as though the voltage on arriving at the end found no place to go and started back in the same polarity. Such action is called *reflection.* A reflection of voltage at an open always occurs in the same polarity. The voltage change moves back to the source, charging each capacitor in turn until the last capacitor is charged to the source voltage and the action stops.

In the meantime the field collapsed around the coil and the current there became zero. As each capacitor is charged, the current in each inductor drops to zero, effectively reflecting the changed current in opposite polarity. The opposite polarity cancels the current at each point and it drops to zero at that point. When the last capacitor is charged, the current from the source stops flowing.

Notice that the line current flowed through the resistor called Z_i. This is the internal resistance of the source. For maximum power transfer to the line, Z_0 of the line must equal Z_i. Because of this resistance, the voltage change delivered to the line is one-half of the source voltage. On the return trip, the additional half charges each capacitor to the source voltage.

The principal facts in the reflection of voltages in open-end lines are:

- Voltage is reflected in phase and without change in amplitude or shape from an *open* end.
- Current is reflected in opposite phase and without change of amplitude or shape from an *open* end.

Reflection from Short Circuit

The effect of a short-circuited line on voltage change is different from that of an open-circuited line. In a short-circuited line the voltage change from zero to E/2 arrives at the last inductor as usual, but in this case the current in the inductor has no capacitor to charge. As you can see in Figure 10–9C, the current in the final inductor causes the indicated polarity to exist. When the field collapses, the inductor becomes a source and the polarity reverses. In the instantaneous equivalent circuit at D showing this action, the inductor acts as a battery and forces current through the capacitor in the opposite direction, causing it to discharge. Since the energy in the magnetic field is the same as that in the capacitor, the capacitor discharges exactly to zero. The second inductor current consequently has no voltage to maintain it; therefore, it will discharge the inductor which is next to the last one.

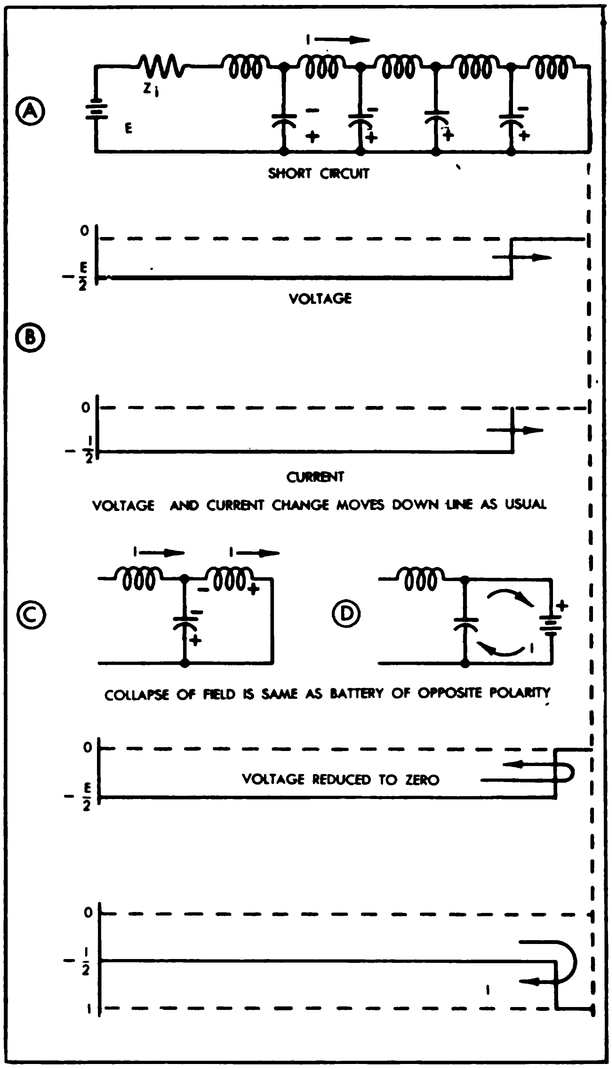


Figure 10–9. Reflection from Short-Circuited Line

The reduction of voltage to zero by the inductor voltage becomes effectively a new voltage that appears at the end of the line. It is equal to E/2, but has the opposite polarity. The collapsing field adds a current which, in turn, adds to the current created by the source. In effect, the current change appears at the end of the line in the same direction as the original current.

An r-f line with a short circuit across it is characterized by:

- Voltage changes that are reflected in opposite polarity but with the same amplitude.
- Current changes that are reflected in the same polarity and with the same amplitude.

Reflection of AC on an Open-Ended R-F Line

In most cases where r-f lines are used, the voltages that are applied to the sending line are AC voltages. When AC voltages are applied,

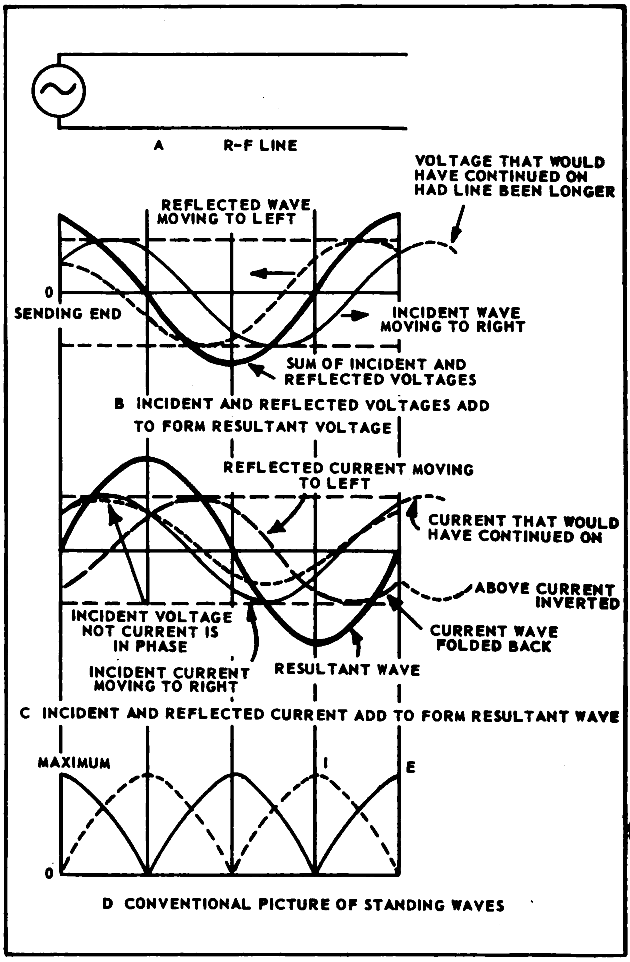

A R-F LINE
VOLTAGE THAT WOULD HAVE CONTINUED ON HAD LINE BEEN LONGER
REFLECTED WAVE MOVING TO LEFT
0
SENDING END
INCIDENT WAVE MOVING TO RIGHT
SUM OF INCIDENT AND REFLECTED VOLTAGES
B INCIDENT AND REFLECTED VOLTAGES ADD TO FORM RESULTANT VOLTAGE
REFLECTED CURRENT MOVING TO LEFT
CURRENT THAT WOULD HAVE CONTINUED ON
ABOVE CURRENT INVERTED
INCIDENT VOLTAGE NOT CURRENT IS IN PHASE
CURRENT WAVE FOLDED BACK
INCIDENT CURRENT MOVING TO RIGHT
RESULTANT WAVE
C INCIDENT AND REFLECTED CURRENT ADD TO FORM RESULTANT WAVE
MAXIMUM
I
E
0
D CONVENTIONAL PICTURE OF STANDING WAVES

Figure 10–10. Formation of Standing Waves

the action at the receiving end of the line is exactly the same as for DC. In the open-ended line shown in Figure 10–10A the generated AC voltage is distributed along the line (as shown at B) in such a way that as it arrives at the end, it is reflected with the same polarity and amplitude. When AC is used, this reflection is in phase. The reflected voltage wave travels back along the line until it reaches the generator. If the generator impedance is the same as the line impedance, energy arriving at the generator is absorbed. There are now two voltages on the line. However, most indicating instruments are unable to indicate these voltages separately. Instead, they read their vector sum.

An oscilloscope is usually used to study the instantaneous voltages on r-f lines. It is used because meters read average values over a period of time. However, you may first add the two voltages algebraically on paper as in Figure 10–10B to see how they will appear on the oscilloscope.

Since there are two waves of voltage moving on the line, it is desirable to distinguish between the two. This is done by giving them names. The voltages moving toward the receiving end are called *incident voltages*, and the whole waveshape is called the *incident wave*. The wave moving back to the sending end after reflection is called the *reflected wave*.

Another step in investigating the open-ended r-f line is to see how the current waves act. Note that the incident current wave is the solid line in Figure 10–10C. The voltage at the moment is represented by the dotted line. The current is in phase with the voltage while traveling toward the receiving end. At the end of the line the current is reflected in opposite polarity; that is, it is shifted 180° in phase, but its amplitude remains the same. The reflected wave of current is shown by dashed lines in Figure 10–10C.

Since a current indicating device would read the sum of the two currents, you must add them to determine what the current indicator will show. The heavy-line curve represents the sum of the two currents. Notice that current is zero at the end of the line. This is reasonable since there can be no current flow through an open circuit. The voltage curve shows that the voltage is maximum at the end of the line, a condition that can easily occur across an open circuit.

Another factor to keep in mind is that when you use an AC meter to determine the current or voltage on an r-f line, it will indicate only the magnitude of the current, not the polarity. If you plot all the readings along the length of the line, you will obtain a curve like the one shown in Figure 10–10D. Notice that all the readings are positive due to current rectification in the meter circuit. The curves in Figure 10–10D represent the conventional method of showing current and voltage on an r-f line.

The voltages and currents shown at B and C in Figure 10–10 are those at a single instant. During the generation of a complete cycle by the generator, there are a large number of these pictures generated. The instantaneous incident and reflected waveshapes are shown in Figure 10–11.

Drawings 1 through 8 in Figure 10–11 show reflection in phase. This is the case with voltage when the line is open-circuited at the end. If you check each picture individually, you will see that the heavy curve is the algebraic sum of the other two curves. In going from picture to picture, you can see that one curve moves to the right and the other moves to the left. The curve moving to the right is the incident wave of voltage; the one moving to the left is the reflected wave of voltage. The composite picture at A in Figure 10–12 shows all resultant curves over a complete cycle. Notice that the voltage varies between zero and maximum in both directions at the center and at both ends as well. But the voltage is always zero at intermediate points, in this case, one-fourth of the distance from each end. The resultant waveshape, which is not traveling like the other waves, is referred to as a *standing wave* of voltage.

The voltage at the center and the ends varies at a sinusoidal rate between the limits shown and is always at zero at the one-quarter and three-quarter points. A continuous series of pictures such as these are difficult to see with conventional test equipment, for this equipment reads the effective or average voltage over a number of cycles. The curve of amplitudes over the length of line for several cycles is shown in Figure 10–12B. A meter would read zero at the points shown and show a maximum voltage at the center no matter how many cycles went by.

The current waveshapes on the open-ended line are shown in the right-hand column in

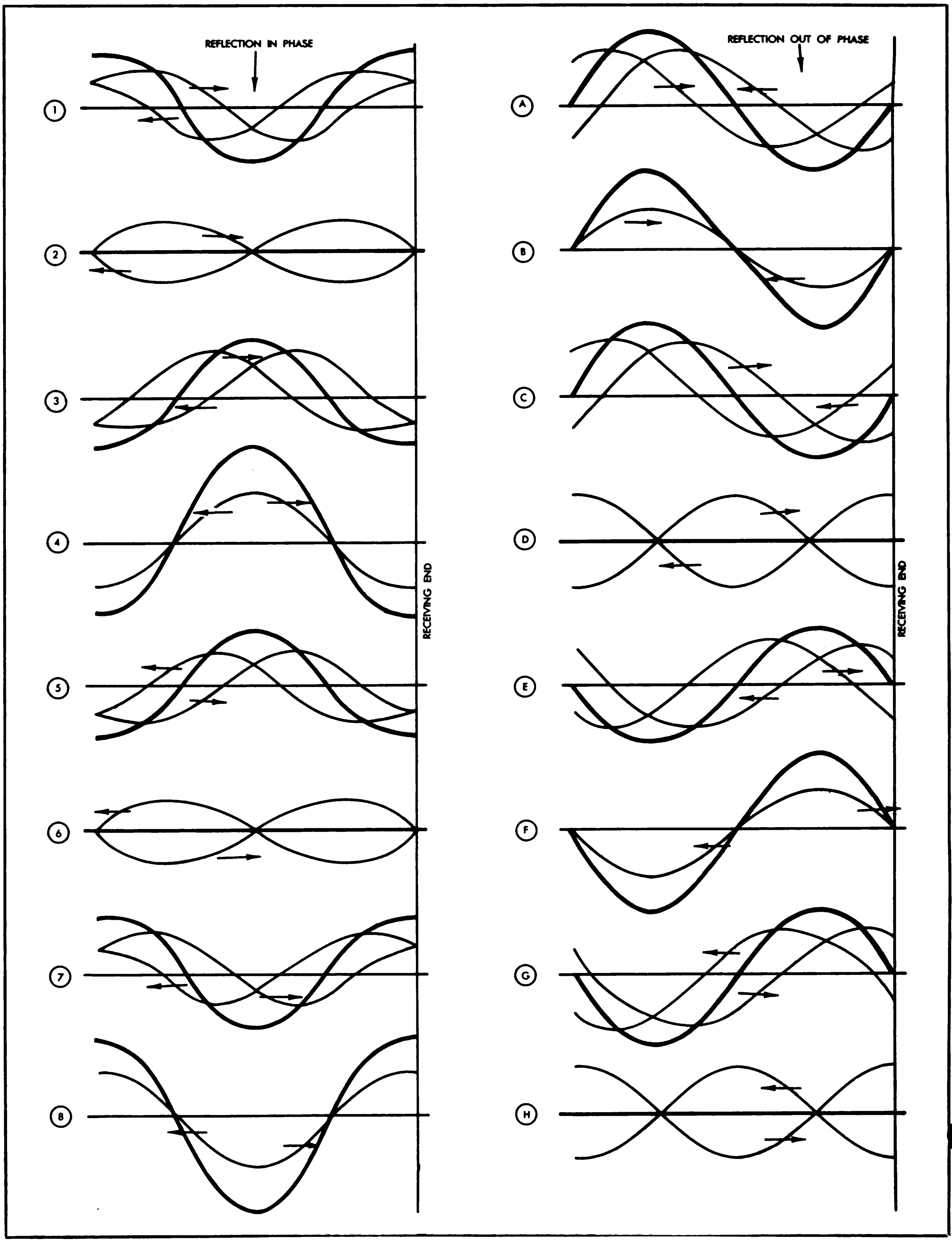
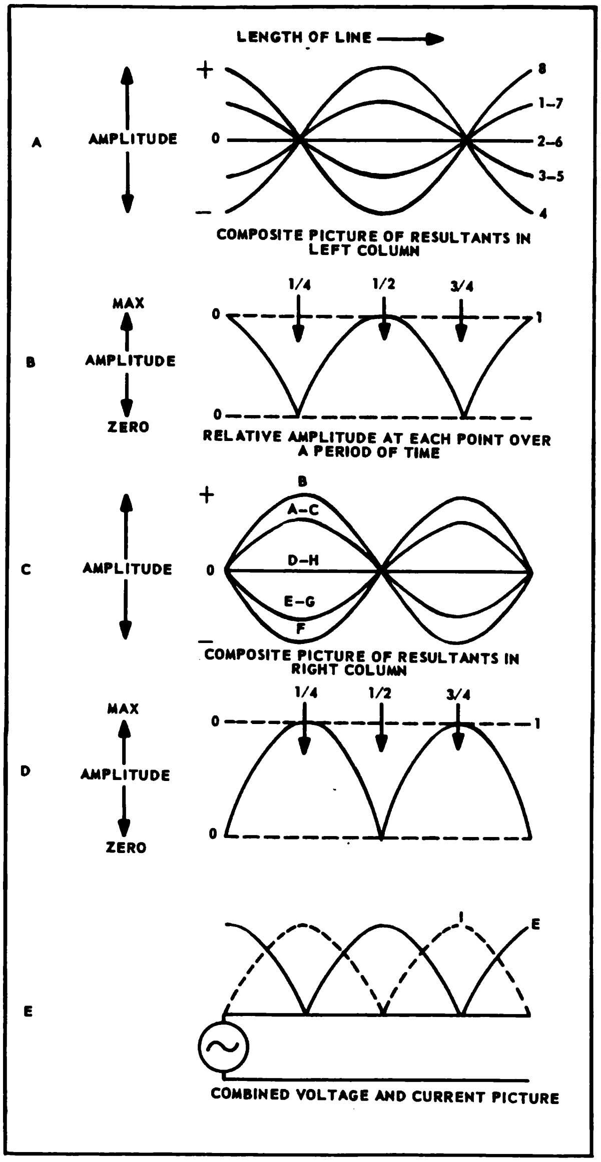


Figure 10–11. Instantaneous Values of Incident and Reflected Waves



LENGTH OF LINE
A
AMPLITUDE
+
0
–
8
1–7
2–6
3–5
4
COMPOSITE PICTURE OF RESULTANTS IN LEFT COLUMN
B
MAX
AMPLITUDE
ZERO
1/4
1/2
3/4
0
1
RELATIVE AMPLITUDE AT EACH POINT OVER A PERIOD OF TIME
C
AMPLITUDE
B
A–C
D–H
E–G
F
COMPOSITE PICTURE OF RESULTANTS IN RIGHT COLUMN
D
MAX
AMPLITUDE
ZERO
E
I
COMBINED VOLTAGE AND CURRENT PICTURE

Figure 10–12. Composite Results of Instantaneous Waves

Figure 10–11. Since the reflection of current is out of phase at an open end, the resultant waveshapes differ from those on the left side. The two out-of-phase components always cancel one another at the end, and thus the resultant is always zero at that point. (See the composite picture of resultants in Figure 10–12C.) Again the voltage is zero at certain points at all times, but maximum at other points. Therefore, it is a standing wave. As shown at D, meters would show the amplitude along the length. In this case, it is zero at the end and center, but maximum at the one-quarter and three-quarter points.

The entire picture of the open-circuited line conditions is usually as shown in Figure 10–12E. The standing waves of voltage and current appear together. Observe that one is maximum when the other is minimum. The current and voltage standing waves are one-quarter cycle, or 90°, out of phase with one another.

Reflection of AC on a Short-Circuited R-F Line

We have stated before that when an r-f line is terminated in a short circuit, reflection is complete but the effect on voltage and current differs from the effect in an open-circuited line. Voltage is reflected in opposite phase, while current is reflected in phase. Again refer to the series of pictures shown in Figure 10–11. The left column represents current since it shows reflection in phase. The right column of pictures shows the voltage changes on the shorted line. The final effect of this is shown in Figure 10–13. It shows the voltage as a solid line in which the voltage is zero at the end and center, and maximum at the one-quarter and three-quarters points.

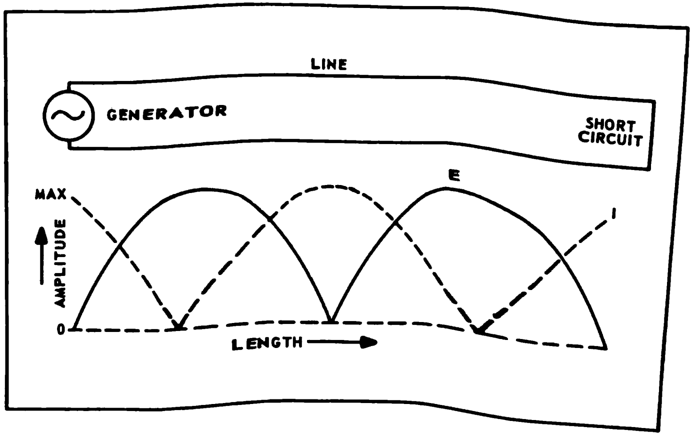


Figure 10–13. Standing Waves on Shorted Line

COMPARISON OF STANDING WAVES WITH VARIOUS TYPES OF TERMINATIONS

There are many different terminations for r-f lines. Each type of termination has a characteristic effect on the standing waves on the line. From the nature of the standing waves, you can determine the type of termination which produced the waves.

Termination in Z_o

Termination in Z_o (characteristic impedance) will cause a constant reading on an AC meter when it is moved along the length of the line. As illustrated in Figure 10–14A, the curve for such a condition, provided there are no losses in the line, will be a straight line. If there are losses in the line, the amplitude of the voltage and current will diminish as they move down the line. The losses are due to DC resistance in the line itself. As the DC losses in radar equipment are often small enough to be disregarded, it is practical to assume no losses.

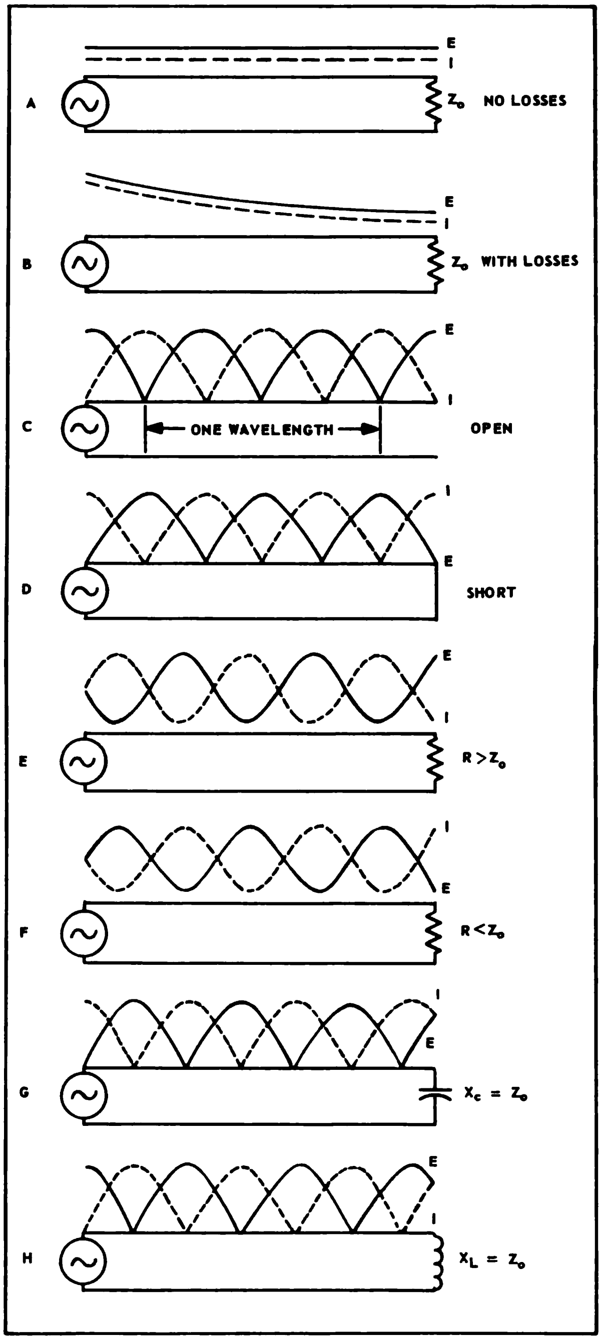


Figure 10–14. Effects of Various Terminations on Standing Waves

Termination in Open Circuit

In an open-circuited r-f line (Figure 10–14C), the voltage is maximum at the end, but the current is minimum. The distance between the two zero points is one-half wavelength, and the distance between alternate zero points is one wavelength. It can be said that the voltage is zero at a distance of one-quarter wave from the end of the line. (This is true at any frequency.) A voltage peak occurs at the end of the line, and also one-half wavelength from the end.

Termination in Short Circuit

On the line terminated in a short circuit (Figure 10–14D) the voltage is zero at the end and maximum one-quarter wavelength from the end.

Termination in a Resistance not Equal to the Characteristic Impedance

Whenever the termination is anything but Z_o, there are reflections. For example, if the terminating element contains resistance, it will absorb some energy, but if the resistive element does not equal the Z_o of the line, some (not all) of the energy will be reflected. The amount of voltage reflected may be found by the equation,

$$E_R = E_I \frac{R_L - Z_o}{R_L + Z_o} \qquad (18)$$

Where E_R is reflected voltage, E_I is incident voltage, R_L is the terminating resistor, and Z_o the characteristic impedance of the line.

If you try different values in the preceding equation, you will find that the reflected voltage is always less than the incident voltage except when R_L is zero or is infinitely high. (E_R and E_I are equal.) A smaller value of reflected voltage causes a standing wave of smaller variation.

When R_L is larger than Z_o, the end of the line is somewhat like an open circuit. Thus, the standing wave of voltage will be greatest at the end of the line, while the current will be at a minimum (but not zero). (See Figure 10–14E.)

When R_L is less than Z_o as shown in Figure 10–14F, the line is terminated approximately as a short circuit. In this case the voltage is minimum at the end of the line (this fact may be verified by the preceding equation), and the value of E_R becomes negative, subtracting from E_I at the end.

The variation of standing waves will show how near the r-f line is to being terminated in Z_o. A wide variation in voltage along the length means a termination far from Z_o. A small variation means termination near Z_o. Thus, the ratio of the maximum standing wave voltage to the minimum is a measure of the perfection of the termination of a line. It is called *standing-wave ratio* and is always expressed in whole numbers. For example, a ratio of 1:1 describes a line terminated in its characteristic impedance (Z_o).

Terminating the Line in Capacitance

When a line is terminated in capacitance, the capacitor does not absorb energy, but returns it all to the circuit. This means 100% reflection. The current and voltage relationships are somewhat more involved than in previous types of termination. For purposes of understanding, assume that the capacitive reactance is equal to the Z_o of the line. The current and voltage will be in phase upon arriving at the end, but in flowing through the capacitor and Z_o connected in series with it, they are shifted in phase relationship.

Thus, current and voltage arrive in phase and leave out of phase, due to the shift introduced by the capacitor. This results in the standing wave configuration shown in Figure 10–14G. The standing wave of voltage is minimum at a distance of exactly one-eighth wavelength from the end. If the capacitive reactance is greater than Z_o (smaller capacitance), the termination looks more like an open circuit; the voltage minimum moves away from the end. If the capacitive reactance is smaller than Z_o, the minimum moves toward the end.

Termination in Inductance

When the line is terminated in inductance, both the current and voltage are shifted in phase upon arriving at the end of the line. When X_L is equal to Z_o, the resulting standing waves are as shown in Figure 10–14H. The current minimum is located one-eighth of a wavelength from the end of the line. When the inductive reactance is increased, the open circuit situation is approached and all standing waves appear closer to the end. When the inductive reactance is decreased, the standing waves move away from the end of the line.

Summary

A line does not have to be of any particular length to produce standing waves, but it cannot be an infinite line. The only requirement for the production of standing waves is that reflection occur; the line must not be terminated in its characteristic impedance. Reflection occurs in lines terminated in opens, shorts, capacitances and inductances, because no energy is absorbed by the termination. Because energy exists across zero resistance which cannot absorb it, the energy has no place to go except back toward the source.

The voltage and current relations for open-end and shorted lines are opposite to each other, as shown in Figures 10–14C and 10–14D. The points of maximum and minimum voltage and current are determined from the output end of the line, because reflection always begins at that end.

SENDING-END IMPEDANCE OF LINES OF VARIOUS LENGTHS AND TERMINATIONS

When a line is terminated in something other than Z_o, the impedance at the generator (sending end) varies widely from the characteristic impedance. An example of the standing waves on a line of random length terminated in a short circuit is shown in Figure 10–16A. The voltage is low and the current is high at the end of the line. Ohm's law shows that the impedance at this point must be low. The voltage and current distribution on the line varies. Where the voltage is high, the current is low; where the current is high, the voltage is low.

The impedance along the line will vary, tending to be large where the voltage is high and small where the voltage is low. At a point a quarter-wavelength from the end of the line, the voltage and current have shifted a quarter-cycle. The voltage here is high and the current is low. At this point the impedance is high. This will also be true of all points that are an odd multiple of quarter-waves from the receiving end of a shorted line.

At a point a half-wavelength from the end of the line both the current and the voltage have shifted one-half cycle. The voltage is low and the current is high at all distances from the receiving end that are even multiples of a quarter-wavelength (multiples of a half-wavelength) from the receiving end. At these points the impedance is low.

If the transmission line were cut at a point a quarter-wave from the receiving end of the line, a generator inserted at this point would see a high impedance. Similarly, if the generator were inserted at the half-wave point, it would see a low impedance. The impedance along a line at any point is the impedance that would be obtained if the transmission line were cut at that point, and the impedance looking toward the receiving end were measured with the proper equipment.

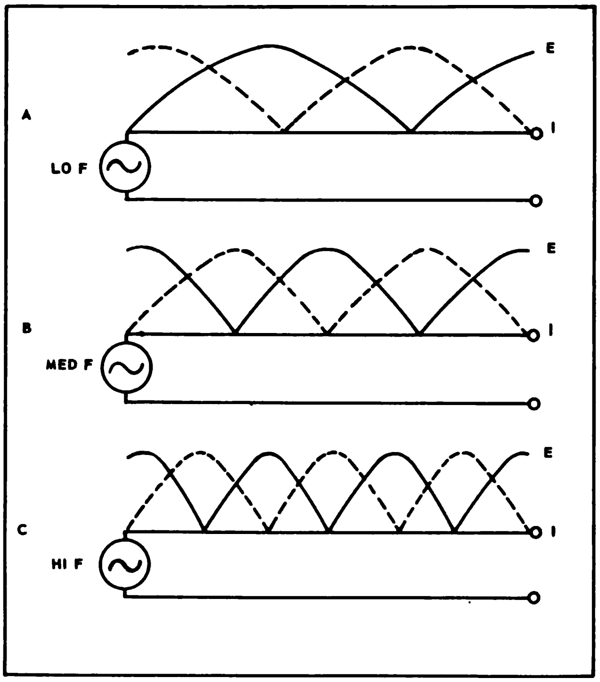


Figure 10–15. Effect of Frequency on Electrical Length of Line

What happens if frequency of a generator working into a transmission line increases or decreases? Figure 10–15B illustrates a generator feeding a full-wave line. If the frequency of the generator is decreased, the lower frequency produced has a longer wavelength. The same physical length of transmission line is now not long enough to have a complete cycle on it. Figure 10–15A shows the same transmission line as used in B. Notice that now, at the lower frequency, the line is approximately three-quarters of a wavelength.

Similarly, the electrical length of the line will change if the frequency of the generator increases. Figure 10–15C again shows the same piece of transmission line with a generator of higher frequency feeding it. A higher frequency will have a shorter wavelength. Therefore, more cycles of the standing waves will appear on the line. The line appears to the high-frequency generator as approximately one and a half wavelengths.

Power Factor of Transmission Lines

The power factor of the impedance of a transmission line also varies. Power factor, as you know, is the cosine of the phase angle between the voltage and the current. Where there is no reflected wave, the power factor of the impedance at the point concerned and the power factor of the characteristic impedance of the line are equal. We have shown that there is no reflected wave when the line is terminated in its characteristic impedance. At the high frequencies used in radio and radar, the power factor of such a line is resistive, that is, equal to unity. A generator inserted at any point in a "flat" line (a line terminated in its Z_0) will always see the characteristic impedance of the line.

When the line is not terminated in its characteristic impedance, there is a reflected wave. The power factor is resistive only at the points of minimum and maximum voltage. On the lines shown in Figures 10–16A and 10–16B this would be at the quarter-, half-, and three-quarter-wave points. Between the points of unity power factor, the power factor is reactive. The line in Figure 10–16A, which is terminated in a short, is inductive from the receiving end to the first voltage maximum. At this point, as we have explained, the power factor is resistive.

Immediately beyond this resistive point, the power factor along the line is capacitive as far as the voltage minimum point, where the power factor is again suddenly resistive. The power factor is alternately inductive and capacitive for

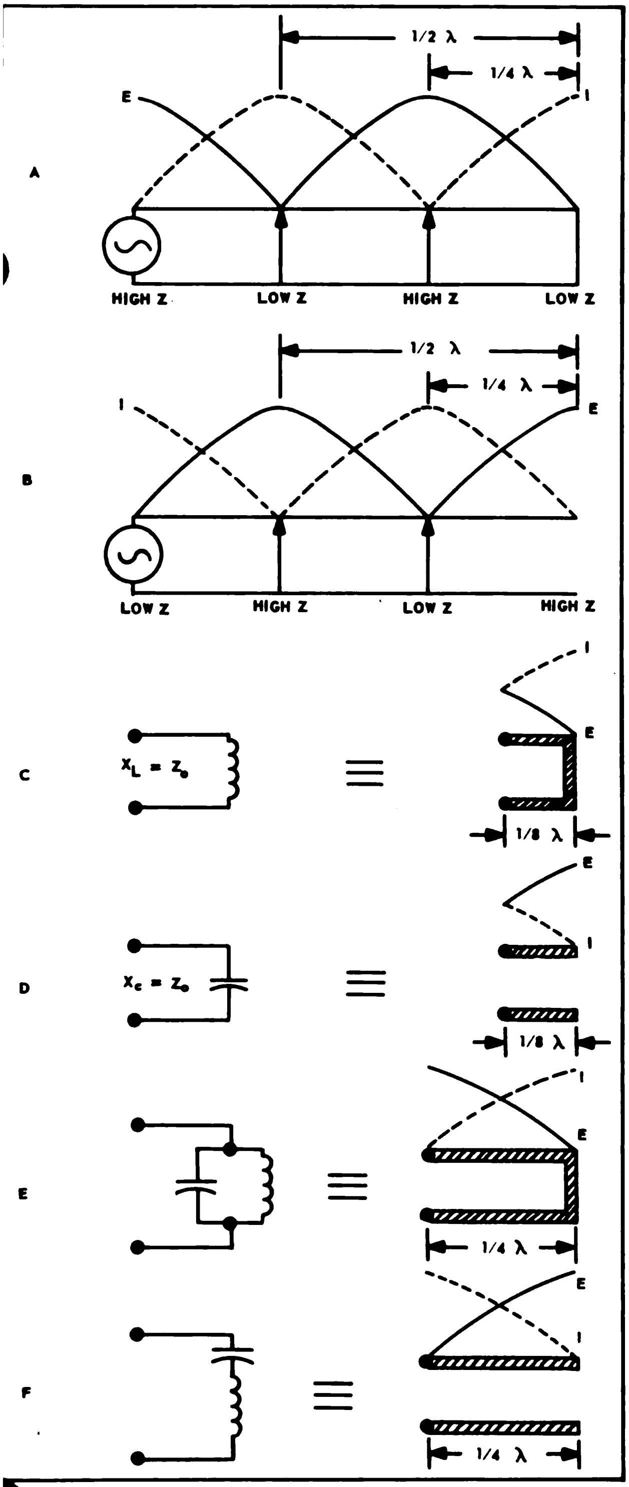


Figure 10–16. Sending End Impedance of Various Lengths and Terminations

quarter-wave intervals. In Figure 10–16B, where the line is open-circuited, the power factor is alternately capacitive and inductive at quarter-wave intervals, exactly as in the case of the shorted line. However, the open line is capacitive over the quarter-wave section nearest the receiving end. These variations in the impedance of a line are shown at the top of Figure 10–20.

Transmission Lines as Resonant Circuits

At the points of voltage maxima and minima on a short-circuited or open-circuited line, the line impedance is resistive. On a short-circuited line, the points an odd number of quarter-wavelengths from the receiving end have a high impedance (Figure 10–16A). If the frequency of the applied voltage to the line is varied, this impedance decreases as the effective length of the line changes. This variation is exactly the same as the change in the impedance of a parallel resonant circuit when the applied frequency is varied.

At the points an even number of quarter-wavelengths from the short circuit, the impedance is extremely low. When the frequency of the voltage applied to the line is varied, the impedance at these points increases in the same way as the impedance of a series resonant circuit varies as the frequency applied to it is changed. The same thing is true for an open-ended line (Figure 10–16B) except that the points of high and low impedance are reversed.

Since the impedance a generator sees at the quarter-wave point in a shorted line is that of a parallel resonant circuit, a shorted quarter-wavelength of line may be used as a parallel-resonant circuit (Figure 10–16E). An open quarter-wavelength of line may be used as a series-resonant circuit (Figure 10–16F). The Q of such a resonant line is much greater than can be obtained with lumped capacitance and inductance.

Transmission Lines as Circuit Elements

A shorted section of line less than a quarter-wavelength appears inductive to a generator. If the line is exactly an eighth-wavelength, the inductive reactance is equal to the Z_0 of the line. This is shown in Figure 10–16C. An open section of line less than a quarter-wavelength appears capacitive to a generator. If the line is an eighth-wavelength, the capacitive reactance is equal to the Z_0 of the line. This section is shown in Figure 10–16D.

Voltage, Current and Impedance Inversion

When the quarter-wave section is open at the end, the voltage at the end is high and the current is low, as you can see in Figure 10–16F. At the sending (generator) end the current is maximum and the voltage is low. Effectively the quarter-wave section has inverted the voltage and the current. The low voltage and high current at the generator indicates a low impedance. The quarter-wave section has inverted the impedance at the receiving end (open) to a short circuit at the sending end. Similarly, a shorted quarter-wave section will invert the short at the receiving end to an open at the sending end.

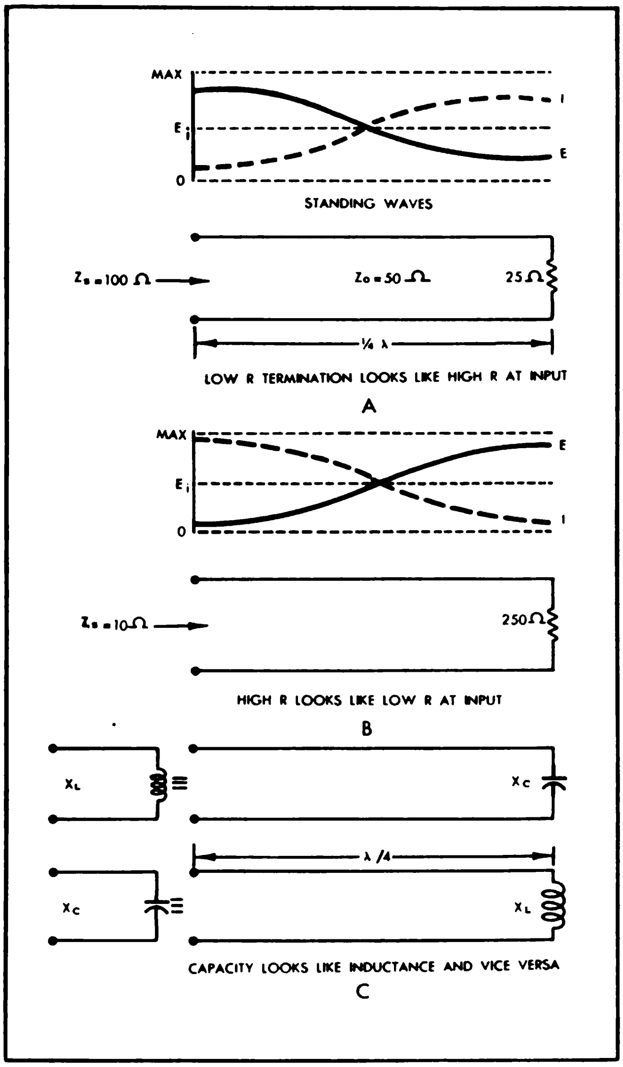


Figure 10–17. Impedance Inversion with Quarter-Wave Section

There are other terminations for the quarter-wave sections. When a quarter-wave section is terminated in a resistance greater than Z_o, the section will invert it to resemble a resistance less than Z_o. Mathematically, the inversion is expressed by the equation,

$$\frac{Z_R}{Z_o} = \frac{Z_o}{Z_s} \qquad (19)$$

where Z_R is the impedance at receiving end, Z_s the impedance at sending end, and Z_o the characteristic impedance of the line.

For example, the impedance Z_s at the sending end of a 50-ohm line, terminated with a 25-ohm resistor, is equal to,

$$Z_s = \frac{Z_o^2}{Z_R} = \frac{50^2}{25} = \frac{2500}{25} = 100 \text{ ohms}$$

From equation (18) you can calculate the reflected voltage from this termination to be one-third of the incident voltage with a negative sign. The standing wave of voltage at the end is equal to two-thirds of the incident voltage. With this information you can determine the standing wave pattern to be as shown in Figure 10–17A. At the sending end the voltage is higher, but the current ratio at the sending end terminals is 100 (ohms).

If the terminating resistance is greater than Z_o, the input impedance will be lower than Z_o. For example, if the same line were terminated with 250 ohms, its input impedance would be equal to,

$$Z_s = \frac{Z_o^2}{Z_R} = \frac{50^2}{250} = \frac{2500}{250} = 10 \text{ ohms}$$

When the 250 ohms is inverted, it will look like 10 ohms, as illustrated in Figure 10–17B.

Effectively the quarter-wave section is an impedance-matching transformer at *one frequency*. This inverting property also holds for reactance. If the section is terminated in capacitive reactance, the input will be inductively reactive. If the section is terminated in inductance, the input will display all the characteristics of a capacitor, as illustrated in Figure 10–17C.

Three-Eighth-Wave Section

When the r-f line is exactly three-eighths wave long, you can determine the input impedance as easily as you can with a quarter-wave section. For example, consider the shorted section shown in Figure 10–18A. This can be reduced, for

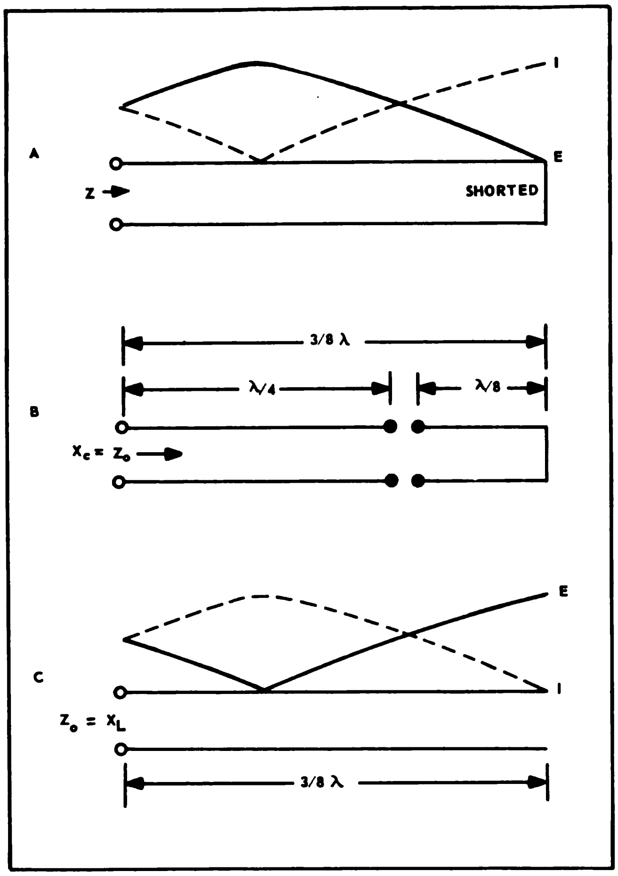


Figure 10–18. Three-Eighth-Wave Section

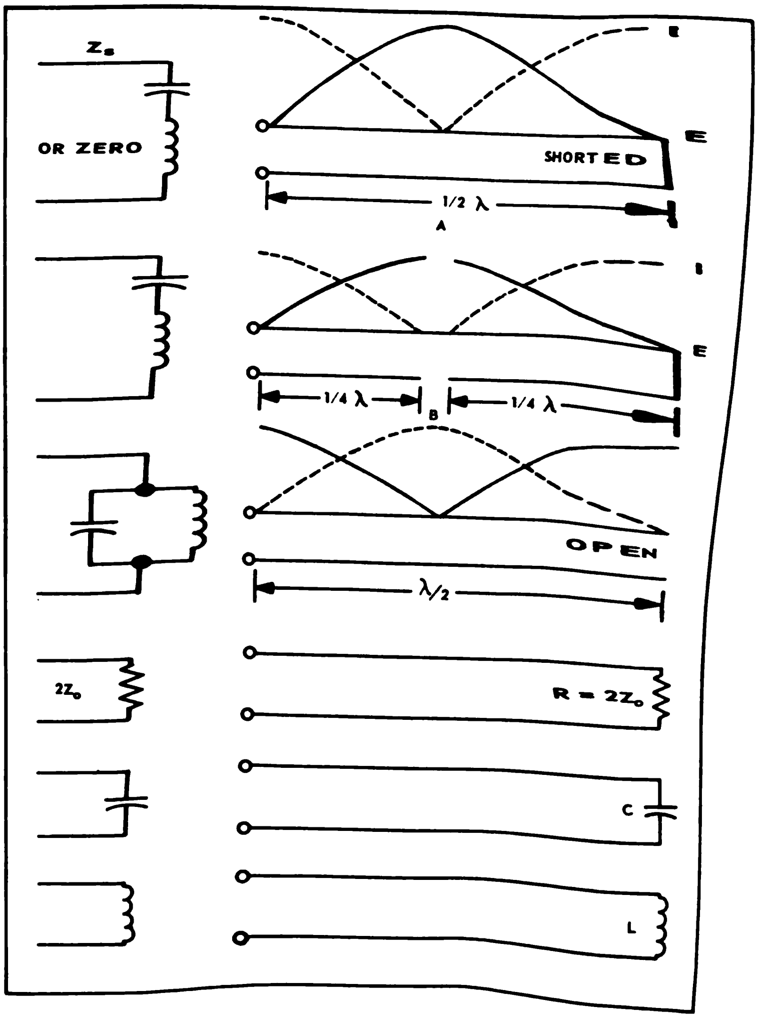


Figure 10–19. Half-Wave Section

simplification, to two sections as shown at B. Here, an eighth-wave section is connected to a quarter-wave section. The eighth-wave shorted section displays inductive reactance to the quarter-wave section. Since a quarter-wave section inverts the inductance to look like capacitance, the three-eighth wave section is capacitive. The capacitive reactance is equal to Z_o. If the end is open as illustrated in Figure 10–18C, similar reasoning will show the line to be inductively reactive at the input terminals. Reference to the standing waves will verify this reasoning, for at the sending end you see the same situation as at the end of a line terminated in reactance — that is, the standing waves are between minimum and maximum.

Half-Wave Section

When the section is a half-wave long, the situation resembles that of two quarter-wave sections connected together, Figure 10–19. The first quarter-wave section inverts the terminal impedance and the second inverts it again as shown in Figure 10–19B, making the input impedance the same as the terminating impedance as shown at A. The input impedance to a shorted half-wave section is zero impedance because the voltage is zero and the current maximum. In addition, the properties of the quarter-wave resonant circuit also are inverted.

For example, in Figure 10–18B, the shorted quarter-wave section is like a parallel resonant circuit. This is inverted, and looks like a series resonant circuit at the input to the half-wave section. In a like manner, the half-wave section, open at the end, has a high impedance at the input terminals and displays reactance to frequencies at which it is not a half-wave, so it is like a parallel resonant circuit. When the half-wave section is terminated in resistance (other than Z_0), in capacitance, or in inductance, it will always repeat this impedance at the sending end.

Lengths of line which are not exact multiples of those discussed will display reactance if terminated in a short circuit or an open circuit, but by varying the length and termination, they can be made to display resistance. If the eighth-wave section is shortened, it becomes less than an eighth-wavelength, and it will still be inductively reactive, but by an amount less than Z_0. If it is longer than an eighth-wavelength, it will display an inductive reactance greater than Z_0.

Impedance Chart for Open and Shorted Lines

Figure 10–20 shows the relation of voltage, current, and impedance for various lengths of both open-end and closed-end transmission lines. Directly above the line is shown the impedance which the generator sees for various lengths of line. The curves above the letters of various height indicate the relative values of the impedance presented to the generator as it moves from right to left. The circuit symbols indicate what the equivalent electrical circuits are for the transmission lines at that particular length. The standing waves of voltage E and current I, whose ratio is the impedance Z, are shown without polarity above each line.

TYPES OF R-F LINES

The principal types of r-f lines are the twisted two-wire line, the spaced two-wire line, the shielded two-wire line, the flexible coaxial cable, and the rigid coaxial cable.

Twisted Two-Wire Line

The twisted two-wire line in Figure 10–21 consists of a pair of insulated and twisted conductors. The twisting holds the lines together mechanically and aids in balancing each conductor to the influence of nearby magnetic and electrostatic fields. Twisted wires usually have a characteristic impedance of 70 to 100 ohms. Although the

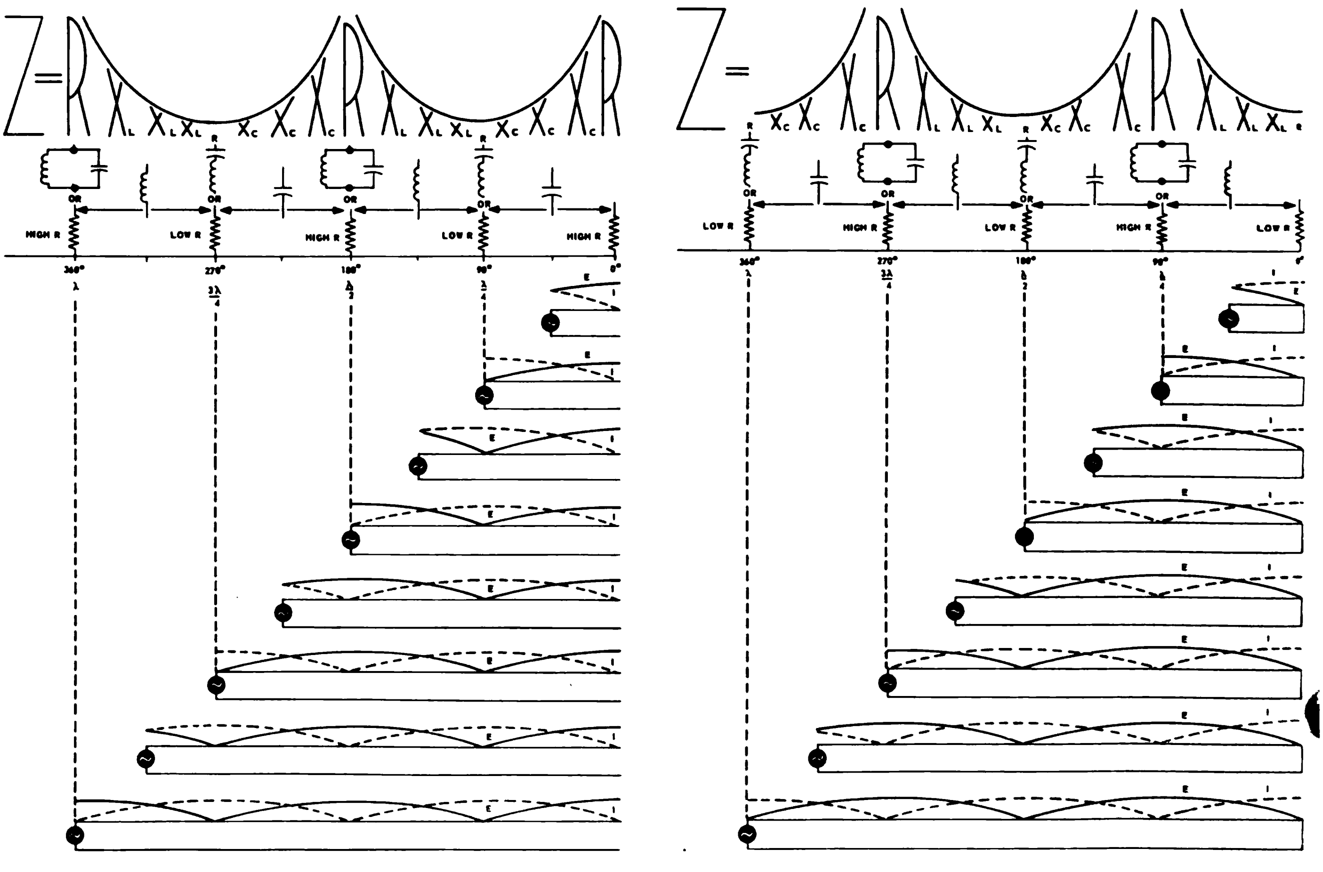

Z=R X_L X_L X_L R X_C X_C X_C R X_L X_L X_L R X_C X_C X_C R
Z= R X_C X_C X_C R X_L X_L X_L R X_C X_C X_C R X_L X_L X_L R
OR
HIGH R
LOW R
360°
270°
180°
90°
0°
λ
3λ/4
λ/2
λ/4
E
I

Figure 10–20. Impedance Charts

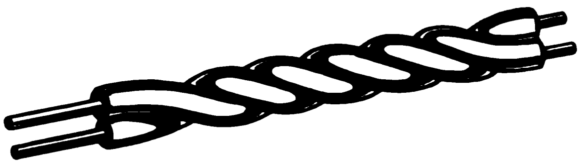


Figure 10–21. Twisted Two-Wire Line

twisted line is flexible, occupies little space, and is simple to manufacture, it is not usable at radar frequencies because of capacitance between the wires, lack of shielding, and changes in characteristics due to moisture on the line.

Spaced Two-Wire Line

The spaced two-wire line (Figure 10–22) consists of two parallel conductors that are maintained a fixed distance apart by insulating spacers or spreaders. This line has two types — the spreader bar, and the newer twin-lead line. The spreader bar type uses ceramic or polystyrene bars for spacing wires. The spacing may be up to six inches or more. The twin-lead line is the result of improved insulating materials and manufacturing techniques. It consists of two wires which are molded into a low-loss plastic called polyethylene. Impedances ranging from 75 to 300 ohms are available in the twin-lead line.

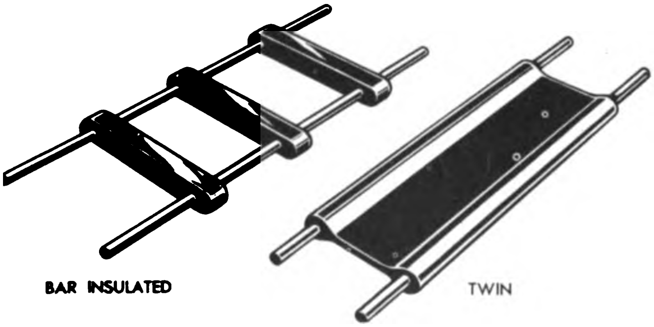


Figure 10–22. Spaced Two-Wire Line

Lines in general, however, may be constructed with any impedance up to 700 ohms by varying the wire diameter and spacing. The twin-lead line has a number of advantages over the spreader bar two-wire line. Wide spacing between the leads in the twin-lead type gives lower shunt capacitance between the leads. Insulation losses are lower in it, since the wires are separated by something other than air, unlike the wires in the spreader bar line.

The twin-lead two-wire line is not perfect. It has two principal disadvantages. In the first place the line acts as an antenna. The magnetic fields extend some distance out from the conducting wires. When these fields collapse at a change in frequency cycle, some of the lines of force do not retract when the change is rapid, and consequently a few lines of force are projected outward in the form of radiation, just like radiated energy on an antenna. This energy is lost. In the second place, nearby objects set up capacitance with the wires and disturb current flow through the line. This obstructs the orderly transfer of energy by the line.

Characteristic Impedance of the Two-Wire Line

The relationship between the characteristic impedance and the dimensions of a two-wire line is expressed mathematically by the equation,

$$Z_o = 276 \log_{10} \frac{2D}{d} \qquad (20)$$

Where D is the distance between wires (center to center), and d the diameter of one wire.

Example

Problem

Find the Z_o of a two-wire line constructed of No. 12 wire (.08 inch in diameter) spaced six inches apart.

Solution

$$\begin{aligned} Z_o &= 276 \log_{10} \frac{2D}{d} \\ &= 276 \log_{10} \frac{2 \times 6}{.08} \\ &= 276 \log_{10} 150 \\ &= 276 \times 2.1761 \end{aligned}$$

The characteristic impedance (Z_o) then, is approximately 600 ohms.

The graphs shown in Figure 10–23 are for plotting characteristic impedances. One is for a two-wire line. The other is for coaxial lines, which are explained later. In Figure 10–23A, dotted lines show how the characteristic impedance can be obtained from the impedance curve.

Shielded Two-Wire Line

A shielded two-wire line is similar to the two-wire spaced line except that it is shielded by a copper braid. Notice the practical form of this line in Figure 10–24. The copper braid gives it

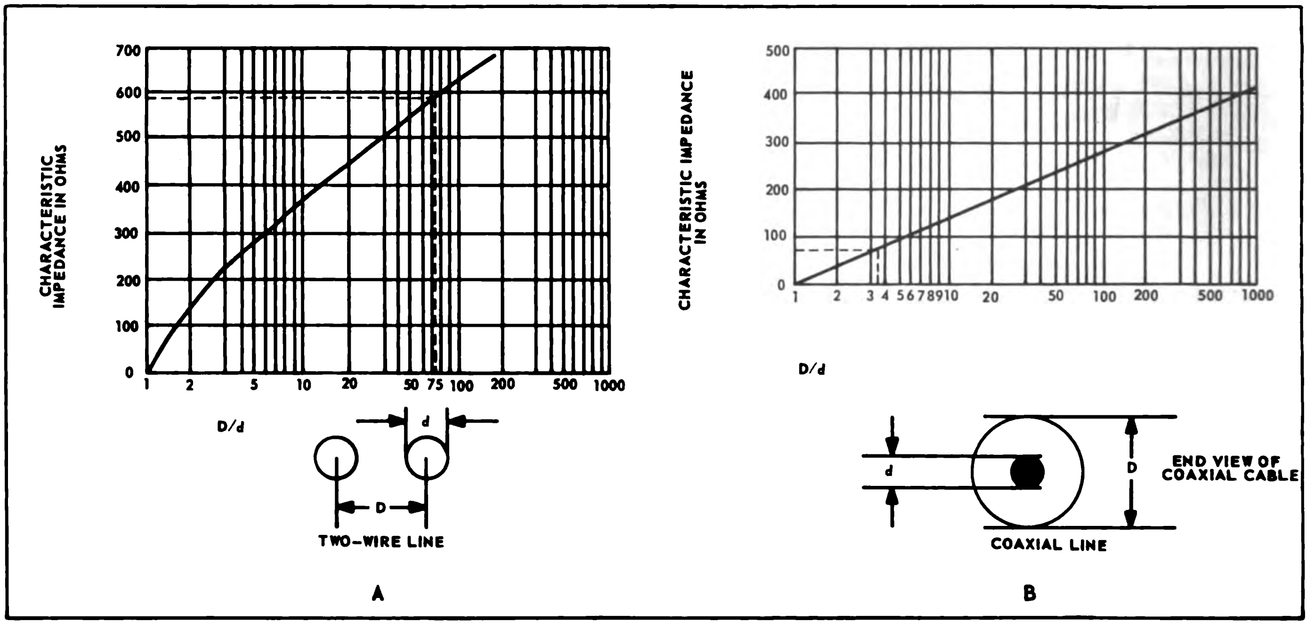


Figure 10–23. Characteristic Impedance Graphs for Two-Wire and Coaxial Lines

flexibility. Flexibility could be obtained by using unbraided tubing. However, the use of braid permits uniform spacing of each conductor during manufacture. This results in each wire being perfectly balanced capacitively to the surrounding conductor. As long as the balance is maintained, certain detrimental effects, such as high capacity to ground when the shield is grounded, are very slight. This line does not radiate energy because of the shield and thereby is not affected by nearby magnetic fields.

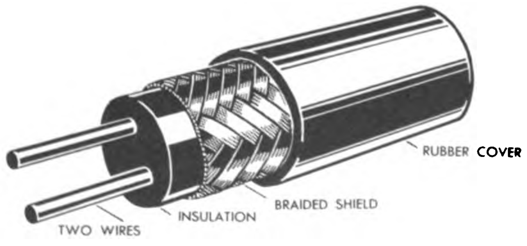


Figure 10–24. Shielded Two-Wire Line

Coaxial Line

The coaxial line is the most universally used type of line for r-f transmission. It consists of two conductors. One of the conductors is hollow; the other conductor is located inside the hollow conductor. The two conductors are made concentric in cross section to maintain uniform characteristics. Early coaxial lines were made of copper tubing in which there was a wire held in the center by ceramic or polystyrene washers or beads. Attempts to build flexible cables led to the use of outer conductors of copper braid spaced by a continuous row of beads or rubber insulators. But rubber insulation caused excessive losses at high frequencies. In addition the bead arrangement permitted air to enter the line, causing moisture to collect. This resulted in high leakage currents and arc-over when high voltages were applied.

Research solved this problem by using polyethylene plastic, a hard, solid material, safely flexible over a temperature range of —40° to +80° C. Polyethylene is unaffected by such fluids as acids, alkalis, aviation gasoline, oil, hydraulic brake fluid, and sea water. Furthermore, there is no known solvent for polyethylene at ordinary temperatures. Its extremely low losses at high frequencies is important for radar. At 200 mc the losses may be only one-hundredth of those of rubber under certain conditions. When carefully manufactured, a coaxial cable insulated with polyethylene will be uniform within .005 inch and will have no air between the conductors. Thus, due to its uniform characteristics, it will operate indefinitely with low leakage and with little danger of arc-over.

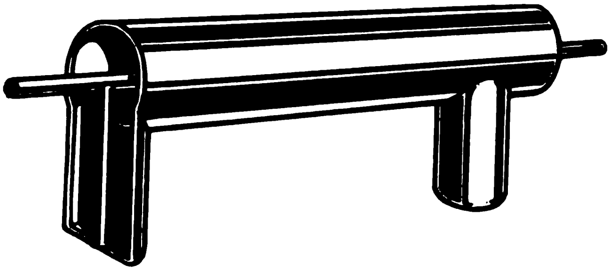


Figure 10–25. Rigid Coaxial Cable with Metallic Insulators

Where installations require still lower losses, such as 3000-mc airborne radar installations, an air-insulated rigid coaxial cable meets these requirements. In this cable (Figure 10–25) the center conductor is supported with metallic insulators which are quarter-wave sections of coaxial line. Since there is air in the interior, the interior of the line is usually pressurized to keep moisture out.

Coaxial lines have a number of advantages. The shielding is perfect for both magnetic and electrostatic fields. If you refer to the two end views of the coaxial cable in Figure 10–26, you can see that the electrostatic field is terminated at the outer conductor so that none of the field is outside of the line. The magnetic field from the inner conductor extends beyond the outer conductor, but an equal amount of current flows in the outer conductor, setting up the field shown, which is outside of the outer conductor. These two fields cancel. Inside the cable these two fields add. Hence, there is neither an electrostatic nor a magnetic field outside of the cable. For the same reasons that a coaxial line does not radiate, it also does not pick up any energy. Therefore, it can be installed anywhere without being influenced by other strong fields. Also, its capacitance is lower than that of a shielded pair line.

Some of the disadvantages of coaxial lines are: they cost more than two-wire lines, they are more difficult to install than flexible cables, and it is more difficult to measure the fields in a coaxial cable than on an open-wire line.

Characteristic Impedance of Coaxial Lines From Dimensions

When you know the dimensions of a coaxial line, you can determine its characteristic impedance by the following equation,

$$Z_o = 138 \log_{10} \frac{D}{d} \qquad (21)$$

where D is the inside diameter of the outer conductor and d is the diameter of the inner conductor.

Example

Problem

If the inner conductor is a quarter-inch rod, and the outer conductor is a 0.875-inch tubing (inside diameter), find Z_o.

Solution

$$\begin{aligned} Z_o &= 138 \log_{10} \frac{D}{d} \\ &= 138 \log_{10} \frac{0.875}{0.25} \\ &= 138 \log_{10} 3.5 \\ &= 138 \times 0.5428 \end{aligned}$$

The characteristic impedance (Z_o) equals 75 ohms.

You can conveniently plot this equation in the impedance vs. dimensions graph shown in Figure 10–23B. The dotted lines show how Z_o for this example is obtained graphically. A popular value for characteristic impedance is 75 ohms since the

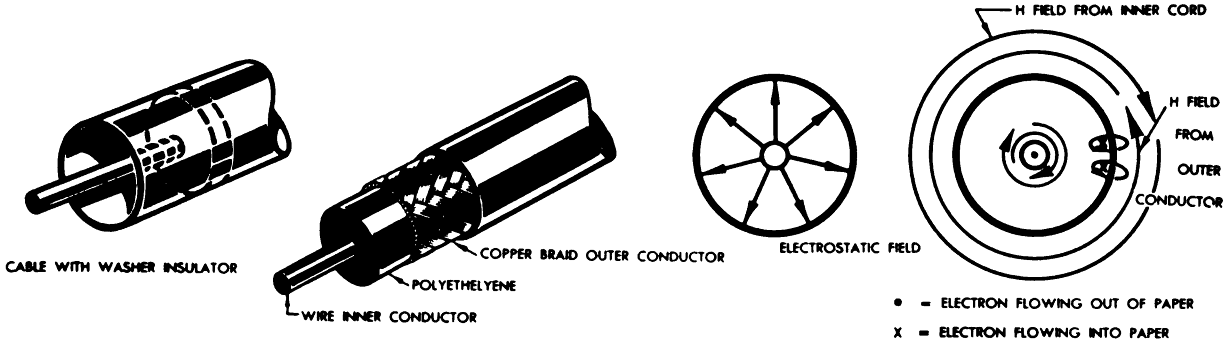

CABLE WITH WASHER INSULATOR
COPPER BRAID OUTER CONDUCTOR
POLYETHELYENE
WIRE INNER CONDUCTOR
ELECTROSTATIC FIELD
H FIELD FROM INNER CORD
H FIELD FROM OUTER CONDUCTOR
• = ELECTRON FLOWING OUT OF PAPER
X = ELECTRON FLOWING INTO PAPER

Figure 10–26. Coaxial Cables

Q of a coaxial line is highest when its dimensions provide for 75 ohms resistance.

LOSSES IN R-F LINES

Although the theory of r-f lines just given dealt with lines assumed to be lossless, actually there are some losses in all lines. Line losses may be of three types — copper losses, dielectric losses, and radiation or induction losses.

Copper Losses

One type of copper loss is I^2R loss. In r-f lines the resistance of the conductors is never equal to zero. Any time current flows through one of these conductors, there is some energy dissipated in the form of heat. This loss of energy is an I^2R, or power loss, since the formula for power is $P = I^2R$. With copper braid, in which the resistance is higher than the resistance in a solid tube, this power loss is accordingly higher.

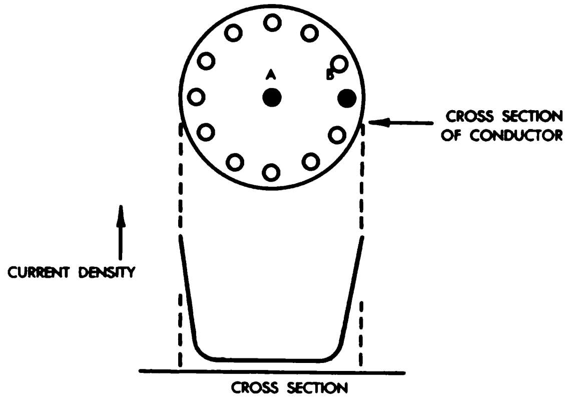

A
B
CROSS SECTION OF CONDUCTOR
CURRENT DENSITY
CROSS SECTION
GRAPH OF CURRENT VS CROSS SECTIONAL AREA

Figure 10–27. Skin Effect Losses in R-F Lines

Another copper loss results from skin effect, which can be explained as follows: When a direct current flows through a conductor, electrons move uniformly through the cross section of the conductor. On the other hand, when an alternating current flows, this is not the case. Expanding and contracting magnetic fields about each electron encircle other electrons and retard their movement. This is the phenomenon of self-induction. From Figure 10–27, you can see that the fields about the electrons moving in area A expand and affect the movement of all the electrons near the surface of the conductor at B. At the same time all the electrons moving at the surface have equal fields. These fields expand and retard the electrons at A.

While a single electron exerts only a small effect on the outer electrons, the outer electrons in combination exert a tremendous effect on the center electron — so much so, that at frequencies of 100 mc and above, electron movement (current flow) is so small at the center that the center of the conductor can be removed without noticeable effect on current flow. The cross section (because current flows only in the outer portion of the conductor) is less and the actual resistance is greater, since the resistance of a conductor varies inversely with the cross section.

In conclusion, skin effect is the tendency of alternating currents to flow near the surface of the conductor, thus being restricted to a small part of the total cross-sectional area. This effect increases the resistance and becomes more marked as the frequency rises.

Because of the flow of current near the surface of a conductor, the conductivity of an r-f line can be increased by plating it with silver. Most of the current will flow in the silver layer. The tubing serves chiefly for mechanical support.

Dielectric Losses

Dielectric losses are due to the heating of the dielectric material (insulation) between conductors. This heating of the dielectric material takes power from the source. The heat itself is dissipated into the surrounding medium. Figure 10–28 shows the mechanics of dielectric losses. When there is no potential difference between two conductors, the atoms in the dielectric material between them are normal; that is, the orbits of the electrons are circular, as shown in Figure 10–28A. But, when a potential difference exists, as shown in Figure 10–28B, the electron orbits change.

The excessive negative charge on one conductor repels electrons in the dielectric toward the positive conductor and thereby distorts their orbits. This change in the path of electrons requires work. The power is furnished from the power source for the r-f line, thus introducing a power loss. The structure of the atoms in some dielectric materials, rubber for example, is harder to distort than the structure of other materials. On the other hand, the atoms of some materials distort easily. Polyethylene is this type of material. It is used as a dielectric because little power is consumed when its electron orbits are distorted.

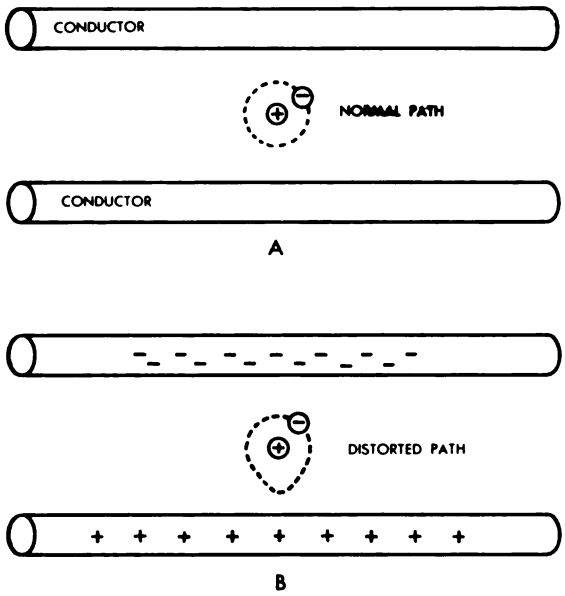


Figure 10–28. Origin of Dielectric Losses

Radiation and Induction Losses

Radiation and induction losses are similar in that they are both due to the fields surrounding conductors. When the field about conductors cuts a nearby metallic object, there is a current induced in the object. This induced current results in power being dissipated by the object. The power thus lost is supplied by transformer action from the source for the r-f line. Radiation losses occur because some lines of force about a conductor do not return to it when the frequency cycle changes. These lines of force are projected into space as radiation, and since they do not return, the energy they use must be supplied by the power source, thus introducing additional power loss.

Importance of Losses

Although in cross-country telephone lines resistance is a major problem from the standpoint of losses, resistance is of little concern in r-f systems, since r-f lines are short and the DC resistance is quite small. However, the other types of losses previously discussed are of more concern. Most copper losses in r-f lines are due to skin effect. These, however, can be minimized by using large diameter conductors with a low resistance surface material.

Dielectric losses, on the other hand, are of considerable concern. To obtain minimum dielectric loss, it is necessary to use *dry air* as the dielectric. When the line is to be flexible, it is necessary to use a low loss dielectric, such as polyethylene. Furthermore, since dielectric losses increase with the length of the line, systems requiring extremely low loss use a line as short as possible.

In two-wire lines, radiation losses can be decreased either by reducing the spacing between wires or by shielding. Both methods increase the capacitance between the wires, which is usually undesirable. For the least losses from this cause, coaxial cables are recommended because of their natural shielding. This type of loss increases as the square of the frequency. Thus, two-wire lines are not usable at 3000 mc and up. Shielded and coaxial cables are used for high-frequency radar systems.

MEASUREMENTS IN R-F LINES

Electrical measurements must be made on the r-f lines to be certain that they are transferring energy with the least possible losses, to find the electrical length, and to obtain other pertinent data.

There are several simple methods for determining the magnitude of voltage or current at any point on an r-f line. In using these methods remember that the magnetic field around a line varies directly with the current, and that the electrostatic field about the line varies directly with the voltage.

One method for observing current at any point on a line is to break the line and insert an ammeter. Figure 10–29A shows an AC ammeter connected in series with the line. Here the meter reads the standing wave of current at the point indicated. Sometimes it is not practical to cut a line. In that case a magnetic loop that is connected to a meter is moved along the line. With this method the meter reads maximum at the point where the greatest current is induced in the loop — that is, at the current loop — and it reads minimum at the nodes, or point of current minimum.

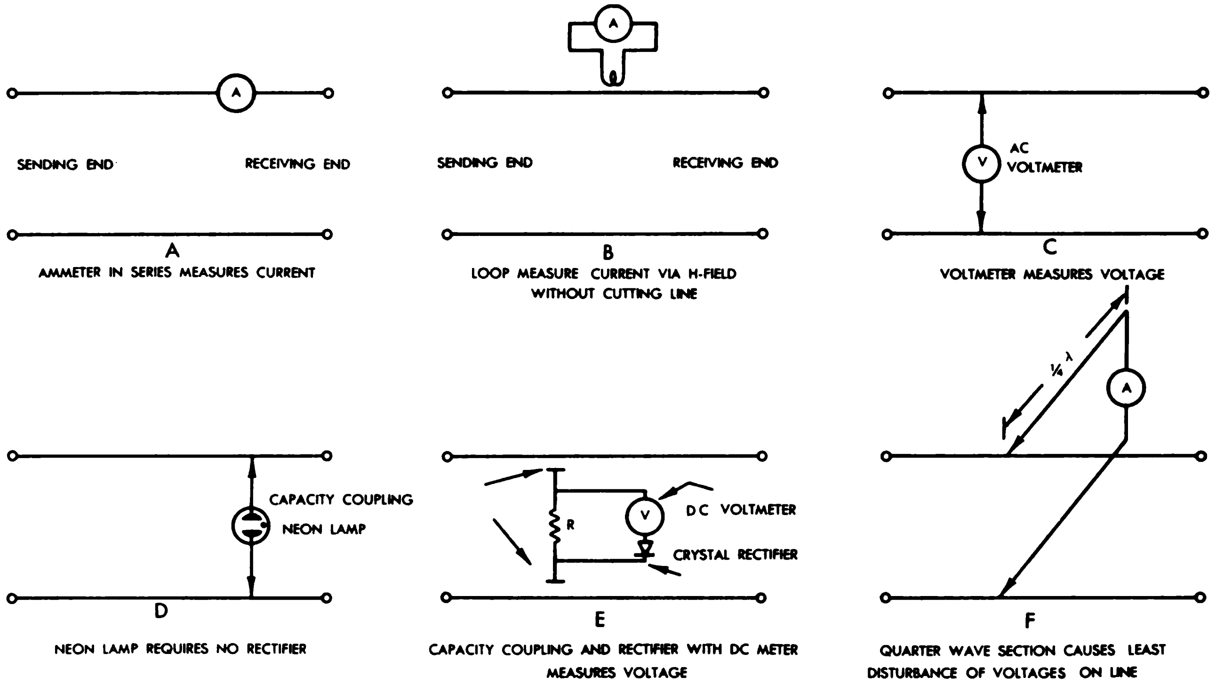

SENDING END
RECEIVING END
A
AMMETER IN SERIES MEASURES CURRENT
SENDING END
RECEIVING END
B
LOOP MEASURE CURRENT VIA H-FIELD WITHOUT CUTTING LINE
AC VOLTMETER
C
VOLTMETER MEASURES VOLTAGE
CAPACITY COUPLING
NEON LAMP
D
NEON LAMP REQUIRES NO RECTIFIER
R
DC VOLTMETER
CRYSTAL RECTIFIER
E
CAPACITY COUPLING AND RECTIFIER WITH DC METER MEASURES VOLTAGE
¼ λ
F
QUARTER WAVE SECTION CAUSES LEAST DISTURBANCE OF VOLTAGES ON LINE

Figure 10–29. Making Measurements on R-F Lines

To read voltage at any point, connect an AC voltmeter across the line at the point you are checking as shown in Figure 10–29C.

Another simple method is to connect a neon lamp across the line. When the lamp is connected across the line, the voltage in the line causes the lamp to glow. The greater the voltage, the brighter the lamp will glow. A strong enough electrostatic field will make a neon lamp glow, even when the lamp is not directly connected to the line. In high-powered installations you need only to place the neon lamp near the line. A more accurate device is the sensitive DC meter with a rectifier. A crystal or diode may be used as the rectifier, as shown in Figure 10–29E. The resistor shown is capacitively coupled to both wires; the voltage drop across the resistor is measured by the meter.

All the preceding methods of measuring have the common disadvantage that ordinary measuring devices take a certain amount of energy from the line being measured. Such use of energy represents a lowered impedance at the point of measurement. Any change in impedance at any point along a line causes reflections. Thus, ordinary measuring instruments load (take energy from) a line and upset its normal characteristics.

A high impedance measuring device suitable for making measurements in a two-wire line is the quarter-wave section shown in Figure 10–29F. This device, which consists of an AC ammeter, forms a short circuit across the section. As the quarter-wave section is shorted at the ammeter end, it presents an extremely high impedance to the line and very little current is needed to energize it.

When coaxial lines are used, as is the case with radar equipment, the magnetic and electrostatic fields are not accessible to loops and contacts. In measuring the field in this type of line it is necessary to use a slotted line section as in Figure 10–30. A probe is inserted in the slot in the line not far enough to touch the center conductor. The probe is a slender rod that acts as an antenna and is excited by the electrostatic field which is parallel to it. It is not affected by the magnetic field. The slot does not reduce the effectiveness of the coaxial line because the currents flow parallel to the slot. With this arrangement the coupling is very slight and therefore very little energy is extracted by the probe.

The energy at the probe is AC at a radio frequency and is detected with some form of detector. A vacuum tube detector can be used, but usually a crystal detector or a bolometer is used.

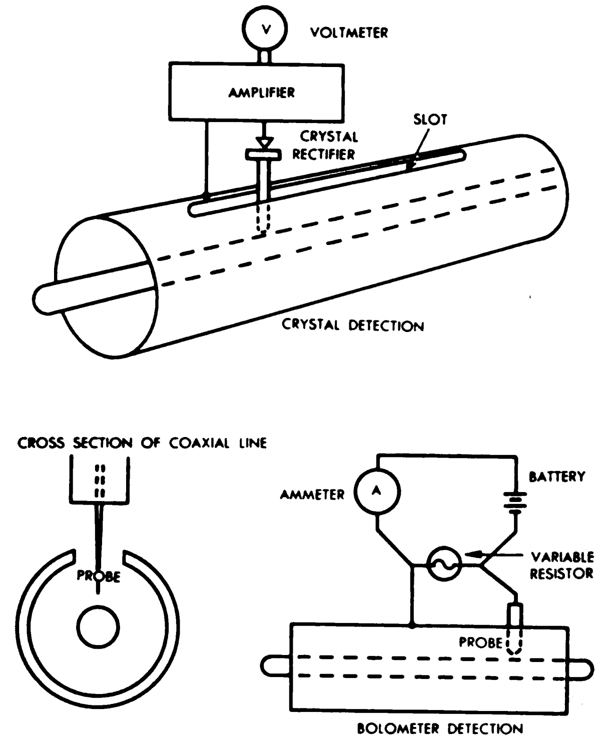


Figure 10–30. Crystal and Bolometer Detection

In the crystal detector, a crystal rectifier changes the r-f to DC, whose magnitude is determined by the amplitude of the AC voltage at the point where the probe is inserted. The DC can be connected to a DC meter. Since the DC is usually quite small, it is first amplified by a vacuum tube amplifier before it is fed to the meter.

A bolometer is a piece of very fine wire in which the resistance varies with temperature. As it is used in Figure 10–30, the r-f energy heats the wire. The heat changes the resistance of the wire to the direct current flowing from the battery. The meter then changes its reading. This change also can be amplified by a vacuum tube amplifier. The resistance element in the bolometer is usually a standard 1/100 ampere fuse.

THE LECHER LINE

Lecher line is the term applied to a two-wire r-f line which is one-quarter to two or three wavelengths long. Such a device is used for either investigating standing wave phenomena, measuring half-wavelengths, or utilizing the principles of an r-f line. In the Lecher line in Figure 10–31A, two wires run parallel for about one wavelength. With this line you can determine several facts by experimentation. For example, when the frequency of a transmitter is unknown, you can use a Lecher line to measure the wavelength of the transmitted frequency. Further, since the velocity at which energy travels on a line of this type is about 97.5% of the velocity in free space (300 million meters per second), you can compute the frequency by the formula,

$$f = \frac{300 \times 10^6 \times 0.975}{2\omega} \qquad (22)$$

where ω is the distance between two minimum or maximum standing wave points, and f the frequency in cycles per second.

Generally, a sliding shorting bar is a part of a Lecher line. As the sliding bar moves toward the generator, the standing waves move with the shorting bar. Results are read by a pickup coil with an indicating meter which is placed near the excitation coil of the Lecher line.

The accompanying illustration explains the use of the shorting bar and the pickup coil in determining current minimums and maximums. The standing current wave shown in Figure 10–31A shows that the standing wave of current is minimum at the location of the excitation coil. Thus, very little current flows through it, and the small magnetic field resulting induces little voltage in the meter circuit. The meter will indicate a current minimum. In Figure 10–31B, which shows the shorting bar moved, the current is maximum at the excitation coil, and the meter pickup coil is excited by the strong magnetic field about the excitation coil. The high current on the meter shows the presence of a current maximum.

A Lecher line one-quarter wavelength long displays the characteristics of a parallel resonant circuit. For this reason, a quarter-wave Lecher line often is used as a tuned circuit in an ultra high-frequency oscillator. At 200 mc a quarter-wave is only about 14 inches, which is small enough for practical use as is shown in Figure 10–31C. (Note the typical Lecher line oscillator at D.) A Lecher line oscillator is impractical at lower frequencies. For example, at a frequency of one mc, a quarter-wave line is 250 feet long. Obviously, this is too long for the ordinary transmitter operating at these frequencies.

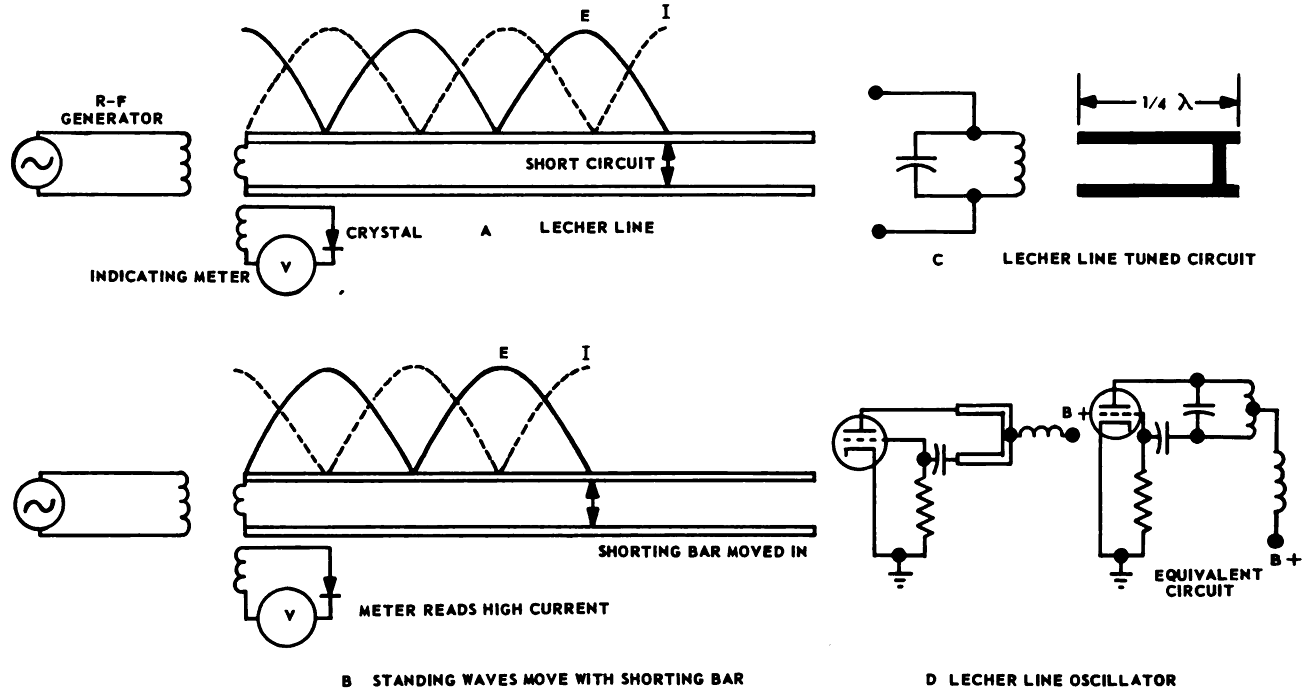


Figure 10–31. Use of Lecher Lines

USE OF R-F LINES

In radar, standing-wave ratios must be as near unity as possible for satisfactory operation. However, even with a near unity ratio, standing waves cause several deficiencies. First, they reduce power handling capacity. The maximum power that a line can handle is limited by the maximum voltage that can be impressed across the line. The maximum voltage, in turn, is limited by the breakdown voltage of the insulation. Standing waves increase the voltage at some points without additional power. Second, the efficiency of the line is lower with standing waves because of the high circulating currents at the current loops. The I^2R losses introduced by this current cannot be tolerated. Third, standing waves change the input impedance of the line. The frequency and power output of the usual radar transmitter varies with load impedance, so the line can change both the transmitter frequency and power output. In practice it is usually possible to get the standing-wave ratio down to less than 1.5:1, a ratio which is considered satisfactory.

When energy is transferred from the antenna to the receiver, it is also desirable to get a low standing-wave ratio, since any losses that occur subtract from the energy received by the antenna system. A low standing-wave ratio also prevents excessive attenuation of the received echoes, which are usually too weak to stand much loss.

Metallic Insulators

In any of the previous uses of r-f lines described, standing waves should not exist because of the unusual phenomena which result. However, there are many uses of r-f lines other than the transfer of energy in which standing waves are desirable. The metallic insulator is an example (Figure 10–32). Since radar sets operate at a single frequency, it is possible to support the center conductor of the coaxial line by a quarter-wave section of r-f line.

The inverting properties of this quarter-wave section make it possible to short the receiving end of the section. This makes the input ends look like an infinitely high resistance. However, because of slight copper losses, the infinite resistance is not quite attained, but the input resistance nevertheless is very high. The metallic insulator has no dielectric losses and is not damaged by arc-overs. Ordinary insulating materials are carbonized by an arc. The carbonized path

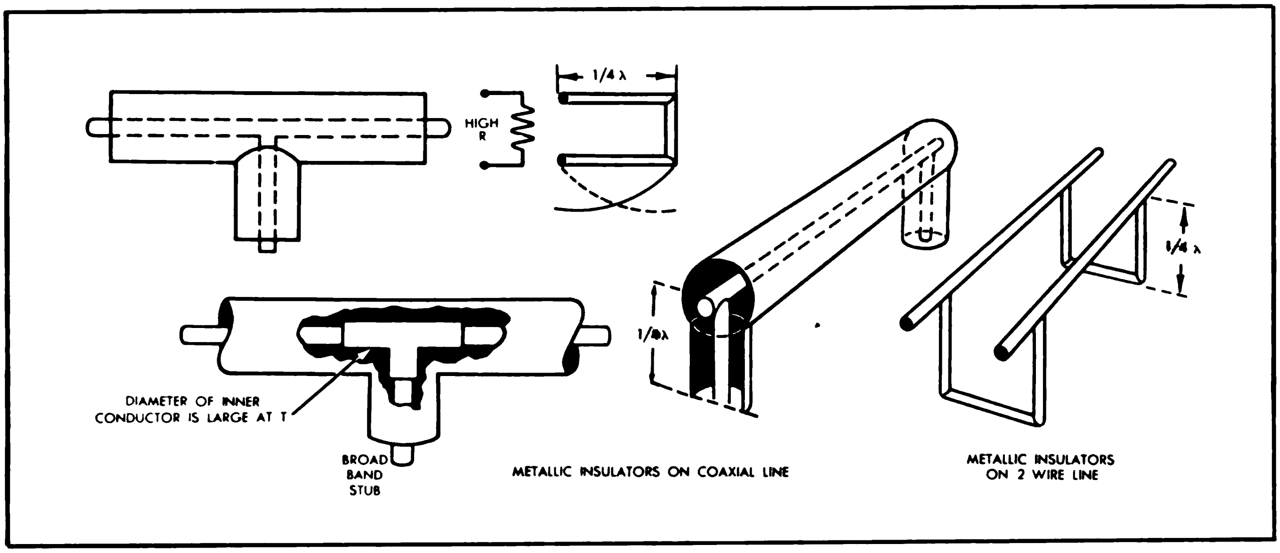

1/4 λ
HIGH R
1/4 λ
1/4 λ
DIAMETER OF INNER CONDUCTOR IS LARGE AT T
BROAD BAND STUB
METALLIC INSULATORS ON COAXIAL LINE
METALLIC INSULATORS ON 2 WIRE LINE

Figure 10–32. R-F Line Used as Metallic Insulator

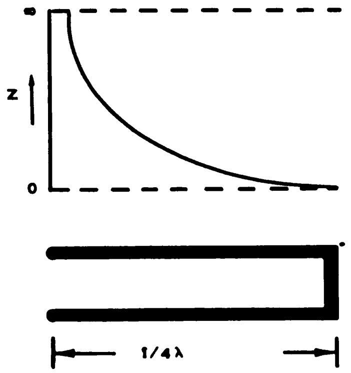
acts as a low resistance conductor at any voltage. Air insulation in quarter-wave sections does not carbonize, and the line is not shorted after the arc.

Metallic insulators are practical only at the higher frequencies, such as 3000 mc, where the quarter-wave "stub" is about two inches long. At lower frequencies the physical length of the insulator would be too great for practical use. A disadvantage of this insulator is that it is a quarter-wave at only one frequency. This limits its use to constant-frequency equipment.

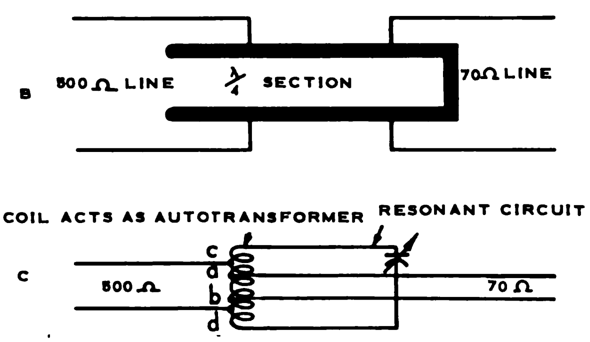
This is the function of the broad band stub. In pulse modulation, many sideband frequencies are present in a radar wave. At frequencies slightly different from the radar frequency a stub is normally reactive. This causes reflections in the main line. These reflections can be minimized by using a broad (frequency) band stub, like the one shown in Figure 10–32. Note that the diameter of the center conductor is enlarged at the junction.

Impedance-Matching Device

The impedance of a shorted r-f line varies widely over its length. Figure 10–33A shows the variation of impedance along the length of a quarter-wave shorted r-f line. The impedance varies from zero at the short to an infinite value at a distance of a quarter-wave from the short. The curve shown is a hyperbola. When this section of r-f line is excited, it is possible to connect devices of any impedance to it with perfect match somewhere along its length.



A VARIATION OF IMPEDANCE ALONG LENGTH OF QUARTER-WAVE R-F LINE



LUMPED CONSTANT ANALOGY

Figure 10–33. R-F Line Used as Impedance-Matching Device

For example, you can match both a 500-ohm line and a 70-ohm line to this section so that there will be no standing waves on either line. Figure 10–33B shows both the 500-ohm and the 70-ohm line connected to a quarter-wave section. Energy from the 500-ohm line sets up standing waves on the section. When the section is excited, the voltage-to-current ratio is correct at the point indicated. This means that the 500-ohm line is terminated in its Z_0. At the shorted end of the section, the voltage-to-current ratio is less. Therefore, at some point near this end of the section the value of E/I must equal 70 ohms. At this point you can properly match the 70-ohm line.

The action involved in matching a 500-ohm line to a 70-ohm line by a quarter-wave section is comparable to the action involved in a tuned autotransformer. Notice the diagram in Figure 10–33C. The action of the coil in this diagram is like that of an autotransformer. When a 70-ohm line is connected to points a and b, the reflected impedance between c and d is equal to 500 ohms. As in transformer action, the direction of energy transfer may be reversed without changing connections. The 70-ohm line can feed the quarter-wave section — a coil in this case — and the 500-ohm line will present the proper impedance to it. Any other value of impedance can be found for either line by moving the connections along the quarter-wave section.

Figure 10–34A shows still another way to get impedance-matching by using a quarter-wave section. Since a quarter-wave section will invert any value of impedance, by selecting a section of the correct impedance, you can invert the 70-ohm impedance of the line used in the preceding example to appear as 500 ohms at the other end. The equation for this relationship is

$$\frac{Z_s}{Z_o} = \frac{Z_o}{Z_r}$$

Solving this equation for Z_o,

$$Z_o = \sqrt{Z_s Z_r} \qquad (23)$$

the impedance of the quarter-wave in Figure 10–34 is

$$Z_o = \sqrt{Z_s Z_r} = \sqrt{500 \times 70} = \sqrt{35{,}000} = 187 \text{ ohms}$$

When you choose conductors of the proper dimensions and space them correctly, you can construct a 187-ohm line. In this arrangement there are standing waves on the quarter-wave section, but none on either line connected to it.

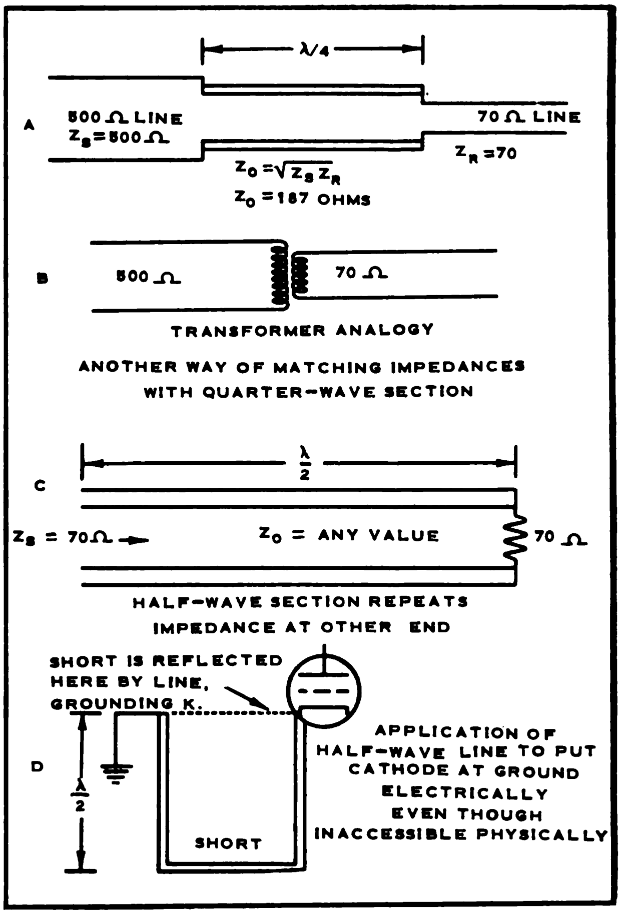

λ/4
A
500 Ω LINE
Z_s = 500 Ω
70 Ω LINE
Z_R = 70
Z_0 = √(Z_s Z_R)
Z_0 = 187 OHMS
B
500 Ω
70 Ω
TRANSFORMER ANALOGY
ANOTHER WAY OF MATCHING IMPEDANCES WITH QUARTER-WAVE SECTION
C
λ/2
Z_s = 70 Ω
Z_0 = ANY VALUE
70 Ω
HALF-WAVE SECTION REPEATS IMPEDANCE AT OTHER END
SHORT IS REFLECTED HERE BY LINE, GROUNDING K.
D
λ/2
SHORT
APPLICATION OF HALF-WAVE LINE TO PUT CATHODE AT GROUND ELECTRICALLY EVEN THOUGH INACCESSIBLE PHYSICALLY

Figure 10–34. R-F Lines as Transformer

In the half-wave section in Figure 10–34C, the output impedance is repeated at the input. For example, a 70-ohm load is inverted over a quarter-wavelength to some other value, depending on the characteristic impedance of the line. It is further reinverted to 70 ohms over the remaining quarter-wave section. The entire action is like that of a one-to-one transformer. This type of inversion is frequently used in radar.

Sometimes a specific impedance is required between two tube elements, and connecting the impedance outside the tube does not provide the desired results. The connecting leads may be electrically long at the operating frequency. A half-wave line using the tube leads as part of the line will effectively place the required impedance right at the tube elements themselves. Figure 10–34D shows a half-wave line being used to put the cathode electrically at ground potential, even though it is not possible to physically place the ground within the tube.

Impedance-matching lines are not used solely with transmission lines. There are several types of devices that can be connected to an impedance-matching line. For example, the input to, or load on, an impedance-matching line could be an r-f device such as a transmitter, an antenna, a receiver input network, or a vacuum tube plate or grid circuit.

Tuned Circuits

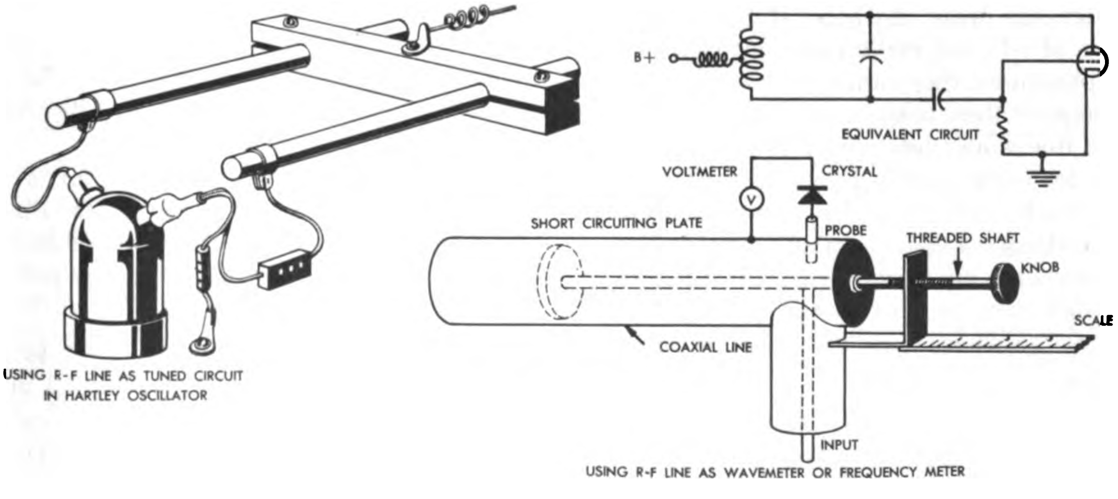
An r-f line having a certain length displays the properties of a resonant circuit. There are many cases in high-frequency equipment where this property is used. For example, quarter-wave and half-wave sections are commonly used as tuned circuits in radar equipment. The Hartley oscillator shown in Figure 10–35 uses a quarter-wave Lecher line for the tuned circuit. Compare this circuit with its equivalent circuit and note how the radar circuits differ from the conventional representation used in the equivalent circuit.

Another use of a section of r-f line is as a wavemeter. Figure 10–35 shows the r-f wavemeter. The meter consists principally of a coaxial line with a movable short-circuiting plate. When this plate is at a distance any multiple of a half-wavelength from the probe in the meter, the voltage at the probe drops to zero. The amount of voltage at the probe can be read on a rectifier type voltmeter. The distance represented by any two consecutive zero voltage readings on this meter represents one wavelength. The knob at the right of the meter provides a means for moving the position of the short-circuiting plate. The distance traveled by this plate can be read from the metric scale, which is calibrated in frequencies.

Phase Inverting or Shifting Networks

Sometimes it is desirable in electronic operations to shift the phase of current or voltage a certain number of degrees, or in some cases to invert the phase completely. Previously it was explained that the phase of voltage changes 360° in one wavelength of an r-f line. Thus it follows that the phase relationship of a sine wave at a distance of a half-wavelength from the sending end will be 180° out of phase with the sine wave at the sending end. This is the same action that occurs in a transformer. An RL circuit as shown in Figure 10–36, however, will shift the phase less than 90°. Lines made up of small parts of a wavelength shift the phase only in small amounts.

An example of a use of phase shifting is antenna excitation. Whenever two antennas are excited out of phase, a certain antenna pattern is produced. One easy way to obtain a 180° phase difference for exciting the antennas is with a half-wave r-f line. Figure 10–36 shows the use of a phase shifting line to provide the correct phase difference in the elements of the antenna array. The antenna on the left is excited by the r-f generator. The generator also applies the same signal to a half-wave line. Due to the phase shifting characteristics of the r-f line, the



USING R-F LINE AS TUNED CIRCUIT IN HARTLEY OSCILLATOR

USING R-F LINE AS WAVEMETER OR FREQUENCY METER

Figure 10–35. Using R-F Lines as Tuned Circuits

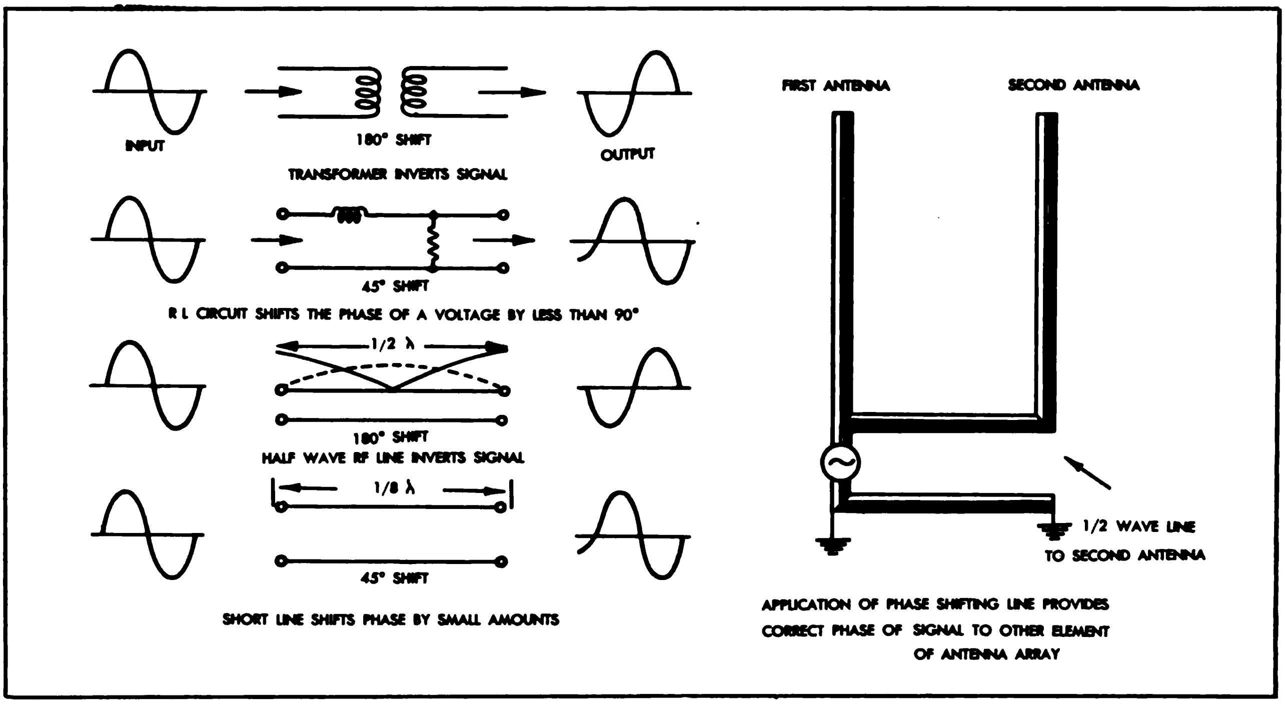

INPUT
180° SHIFT
TRANSFORMER INVERTS SIGNAL
OUTPUT
45° SHIFT
R L CIRCUIT SHIFTS THE PHASE OF A VOLTAGE BY LESS THAN 90°
1/2 λ
180° SHIFT
HALF WAVE RF LINE INVERTS SIGNAL
1/8 λ
45° SHIFT
SHORT LINE SHIFTS PHASE BY SMALL AMOUNTS
FIRST ANTENNA
SECOND ANTENNA
1/2 WAVE LINE
TO SECOND ANTENNA
APPLICATION OF PHASE SHIFTING LINE PROVIDES
CORRECT PHASE OF SIGNAL TO OTHER ELEMENT
OF ANTENNA ARRAY

Figure 10–36. Shifting Phase with R-F Line Sections

antenna at the other end is fed a signal which differs 180° from that fed to the first antenna.

Other antenna directional characteristics can be obtained by varying the length of the r-f line. This variation of line length causes a change in phase which, in turn, gives different antenna radiation characteristics.

ARTIFICIAL LINES

Although the delay characteristics and charging action of r-f lines make them useful in some radar applications, they cannot be used in others, chiefly because they would have to be long. An actual r-f line would have to be several hundred feet long for some uses in radar. Even if a line of such length could be coiled, it would occupy more space than is available in an aircraft. Thus, so far as airborne equipment is concerned, actual transmission lines are out of the question. For this reason artificial lines are used instead of actual lines.

Artificial lines possess all the characteristics of actual lines, but do not have the bulk and physical length of an actual line. They are constructed by first determining the capacitance and inductance of the desired line, and then lumping these variables into equivalent inductors and capacitors. The result is an electrically equivalent, but physically different, line.

Figure 10–37A shows an actual two-wire line. Figure 10–37B shows the equivalent circuit for each section as it is when the section is split into two parts, and its characteristics measured. Since both sections have identical characteristics, the two can be joined together (Figure 10–37C). The final circuit, shown at D, lumps the inductive and capacitive characteristics of each section and simulates the artificial line. As you can see, the artificial line performs just like an actual line, but occupies much less space.

In any line time is required for a voltage change to travel the length of the line. This time characteristic makes it possible to slow down the transfer of a voltage change in its travel from one circuit to another.

The artificial line in Figure 10–38 provides a means of delaying a voltage pulse 10 microseconds. This line is constructed with an inductance of 0.5 millihenry and a capacity of 0.05 mf in each of the two sections. To compute the time for a voltage change to move through one sec-

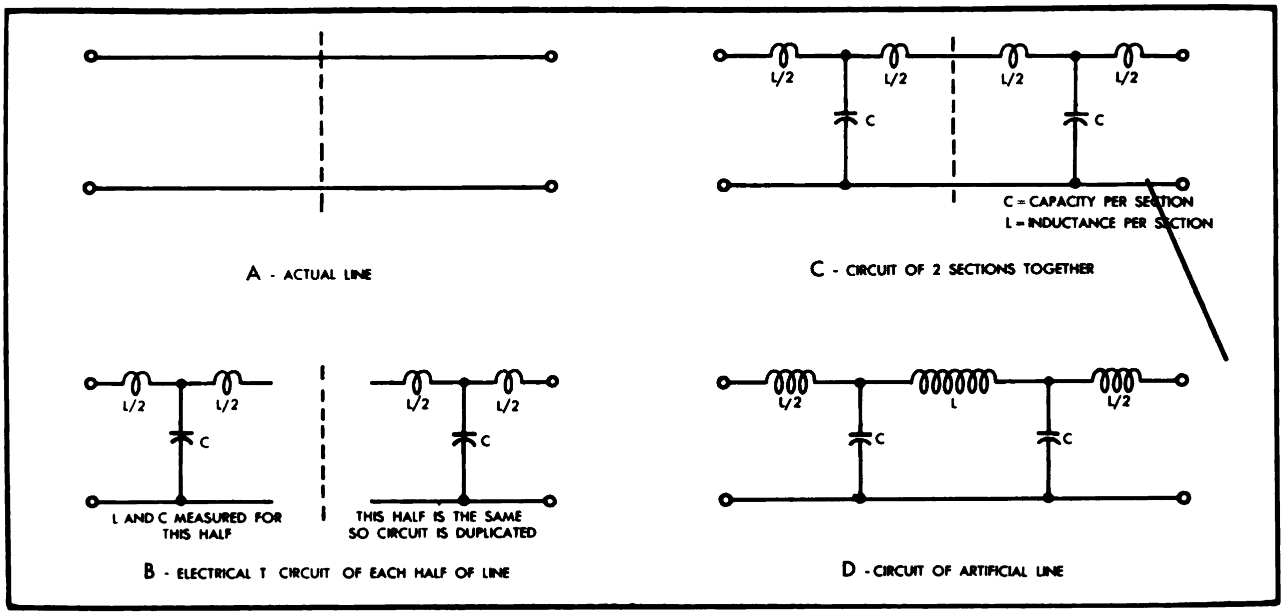

L/2
L/2
L/2
L/2
C
C
C = CAPACITY PER SECTION
L = INDUCTANCE PER SECTION
A - ACTUAL LINE
C - CIRCUIT OF 2 SECTIONS TOGETHER
L/2
L/2
C
L/2
L/2
C
L AND C MEASURED FOR THIS HALF
THIS HALF IS THE SAME SO CIRCUIT IS DUPLICATED
B - ELECTRICAL T CIRCUIT OF EACH HALF OF LINE
L/2
L
L/2
C
C
D - CIRCUIT OF ARTIFICIAL LINE

Figure 10–37. Design of the Artificial R-F Line

tion, use equation (9). The equation for time delay for the entire line is:

$$T_d = N\sqrt{LC} \qquad (24)$$

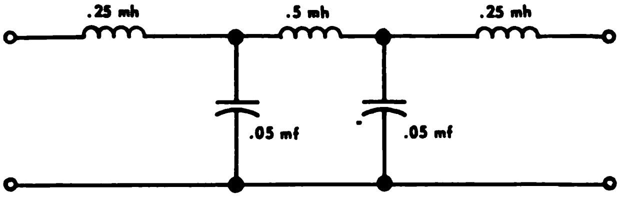
where T_d is the time delay in seconds for the entire line, N, the number of sections, and L and C, the inductance and capacitance per section in henrys and farads respectively.

To calculate the time delay for the entire artificial line shown in Figure 10–38:

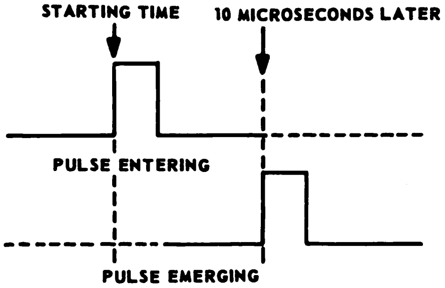
$$\begin{aligned} T_d = N\sqrt{LC} &= 2 \times \sqrt{0.5 \times 10^{-3} \times .05 \times 10^{-6}} \\ &= 2 \times \sqrt{5 \times 10^{-4} \times 5 \times 10^{-8}} \\ &= 2 \times \sqrt{25 \times 10^{-12}} \\ &= 2 \times 5 \times 10^{-6} \\ &= 10 \times 10^{-6} \text{ seconds} \end{aligned}$$

Time delay equals T_d = 10 microseconds

In one radar set the time base is generated in such a way that it does not start until after the first radar echo returns to the receiver. For these signals to be displayed at their correct range mark, they are sent through an artificial r-f delay line which delays the signals just enough so that the very first returning signal does not arrive at the cathode-ray tube (CRT) until the time base has begun to form. Although a large number of video frequencies are present in the signal going through the line, all of them are delayed the same amount.



CONSTRUCTION OF ARTIFICIAL LINE



EFFECT OF LINE ON VOLTAGE CHANGE

Figure 10–38. Artificial Delay Line

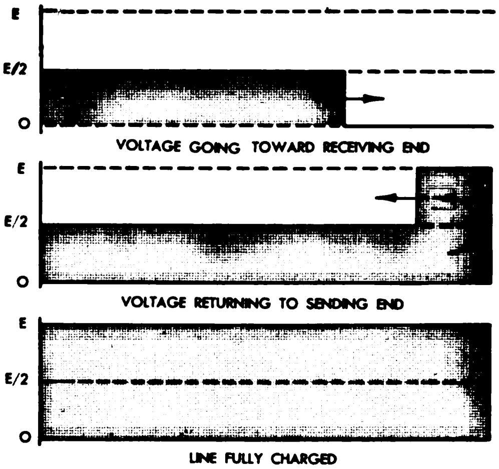
Another phenomenon of r-f lines is the manner in which the lines charge and discharge when a DC voltage is applied to them. When DC is applied to a line in which the source impedance is matched to the line impedance, half the source voltage appears across the line impedance at the time the battery is connected to the line. A voltage change is produced across the line. Figure 10–39, which shows the oscilloscope picture at the sending end, illustrates this condition. This voltage change travels down the line, charging it as it goes. When the voltage change reaches the open end, it is reflected and starts back along the line.

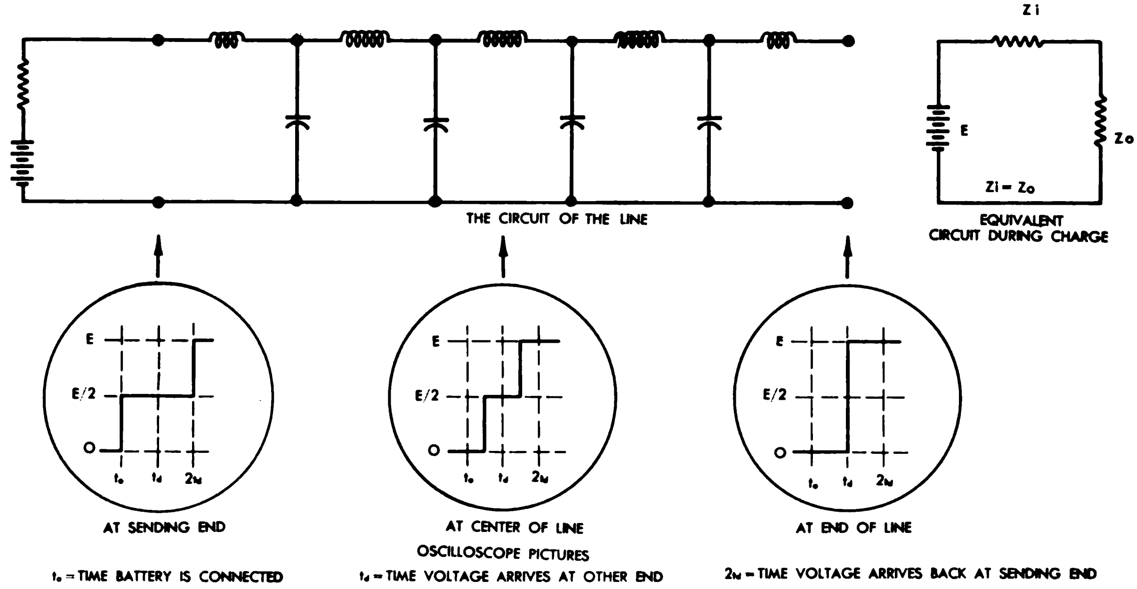
All capacitors charge to half the battery voltage when the voltage change is going down, and to full battery voltage when the voltage rise is coming back. The oscilloscope picture in Figure 10–39, at the center of the line, illustrates this condition. This change in voltage is commonly called a *step* voltage. When all capacitors have been charged to a value equal to the battery voltage, current stops flowing from the battery. By observing the oscilloscope picture which illustrates the end of the line, you will note that the end of the line experiences a continuous rise of voltage which equals the applied voltage.

When this line is discharged into a resistance equal to its characteristic impedance, the result is a square wave that has a constant amplitude and a duration equal to twice the time for the pulse to travel the length of the line. The amplitude is equal to half the charge voltage because the line discharges through its own impedance and the load impedance (Z_R), which are connected in series. This cuts the load voltage by half and produces a drop of voltage that immediately travels down the line and back to the starting place as shown in Figure 10–40. As the end of the line is open, the reflection there is in phase. This produces a voltage across the load impedance (Z_R) equal to half the charge voltage and which lasts for twice the time of travel on the line.

Usually an artificial line is constructed to generate this square wave. When the L and C for each section is known, the time duration of the square pulse, t_p, is equal to

$$t_p = 2N\sqrt{LC}$$


E
E/2
0
VOLTAGE GOING TOWARD RECEIVING END
E
E/2
0
VOLTAGE RETURNING TO SENDING END
E
E/2
0
LINE FULLY CHARGED



t_o = TIME BATTERY IS CONNECTED

t_d = TIME VOLTAGE ARRIVES AT OTHER END

$2t_d$ = TIME VOLTAGE ARRIVES BACK AT SENDING END

Figure 10–39. Mechanism of Charging Artificial Line

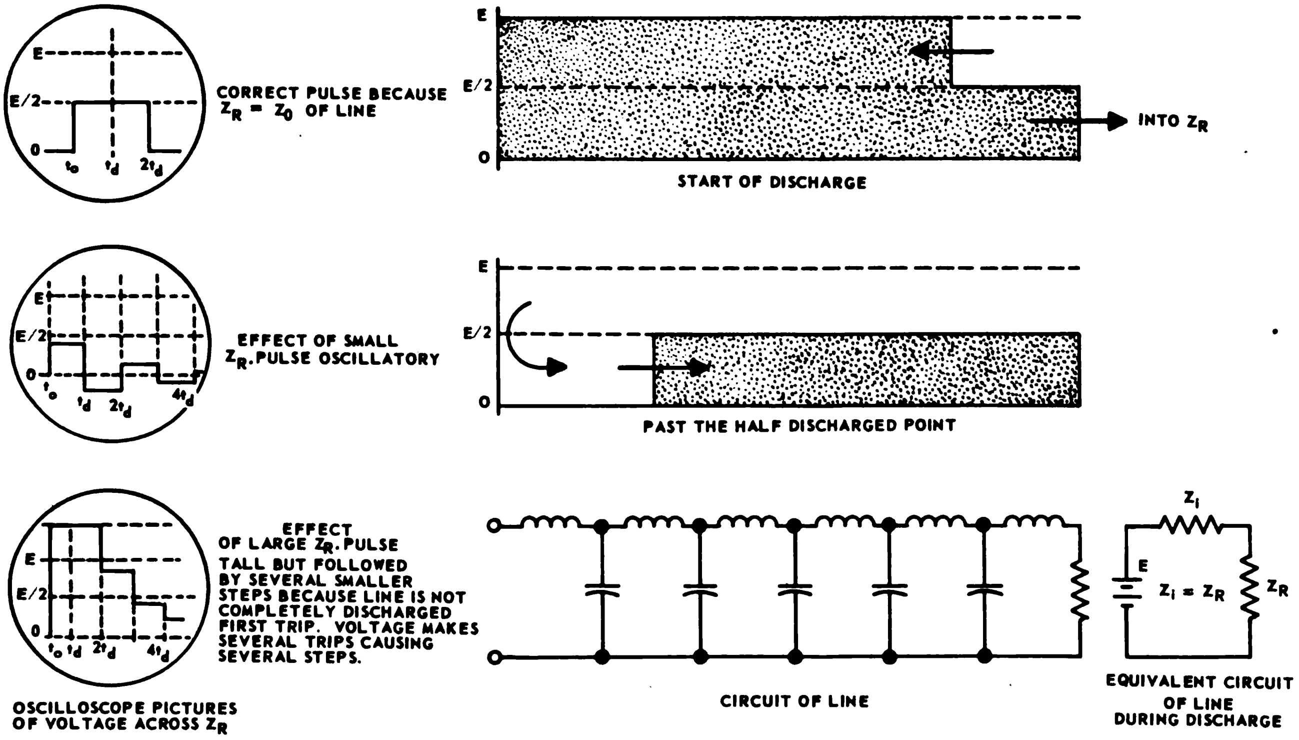


Figure 10–40. Discharging Artificial Line Through Z_R

The following are the calculations for the time duration of the pulse generated in the artificial line in Figure 10-38:

$$L = 0.5 \text{ mh per section}$$

$$C = .05 \text{ mf per section}$$

$$N = 2 \text{ sections}$$

$$t_p = 2N\sqrt{LC} = 2 \times 2 \times \sqrt{0.5 \times 10^{-3} \times .05 \times 10^{-6}}$$

$$= 4 \times \sqrt{25 \times 10^{-12}}$$

$$= 4 \times 5 \times 10^{-6}$$

$$= 20 \times 10^{-6}$$

The time duration is

$$t_p = 20 \text{ microseconds}$$

When an artificial line is not terminated in its characteristic impedance (Z_o), several trips (reflections) are required for complete discharge. This means that discharge cannot be completed in a single round trip.

In Figure 10–40 there are three oscilloscope pictures that show the discharge action of an artificial line across Z_R. When Z_R is large, the pulse is tall. But it is followed by several smaller steps — called tails. These tails exist because the line did not completely discharge during the first trip and more trips are necessary for complete discharge. When Z_R is small, the change that is reflected not only cancels the voltage left on the line, but also recharges the line to a smaller voltage in the opposite direction, producing an oscillatory output waveshape.

To calculate the Z_o of the line, use the formula

$$Z_o = \sqrt{\frac{L}{C}}.$$

The following are the calculations for finding the Z_o of the artificial delay line shown in Figure 10–38:

$$Z_o = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.5 \times 10^{-3}}{5 \times 10^{-8}}}$$

$$= \sqrt{\frac{5 \times 10^{-4}}{5 \times 10^{-8}}}$$

The characteristic impedance is equal to

$$Z_o = \sqrt{10^4} = 10^2 = 100 \text{ ohms}$$

An artificial pulse line is capable of producing a perfectly measured square wave pulse which has an amplitude equal to several thousand volts. As a matter of comparison, a vacuum tube multivibrator and amplifier circuit that could provide a similar output would weigh 100 pounds and occupy several cubic feet of space. On the other hand, a pulse-forming line of equal ca-

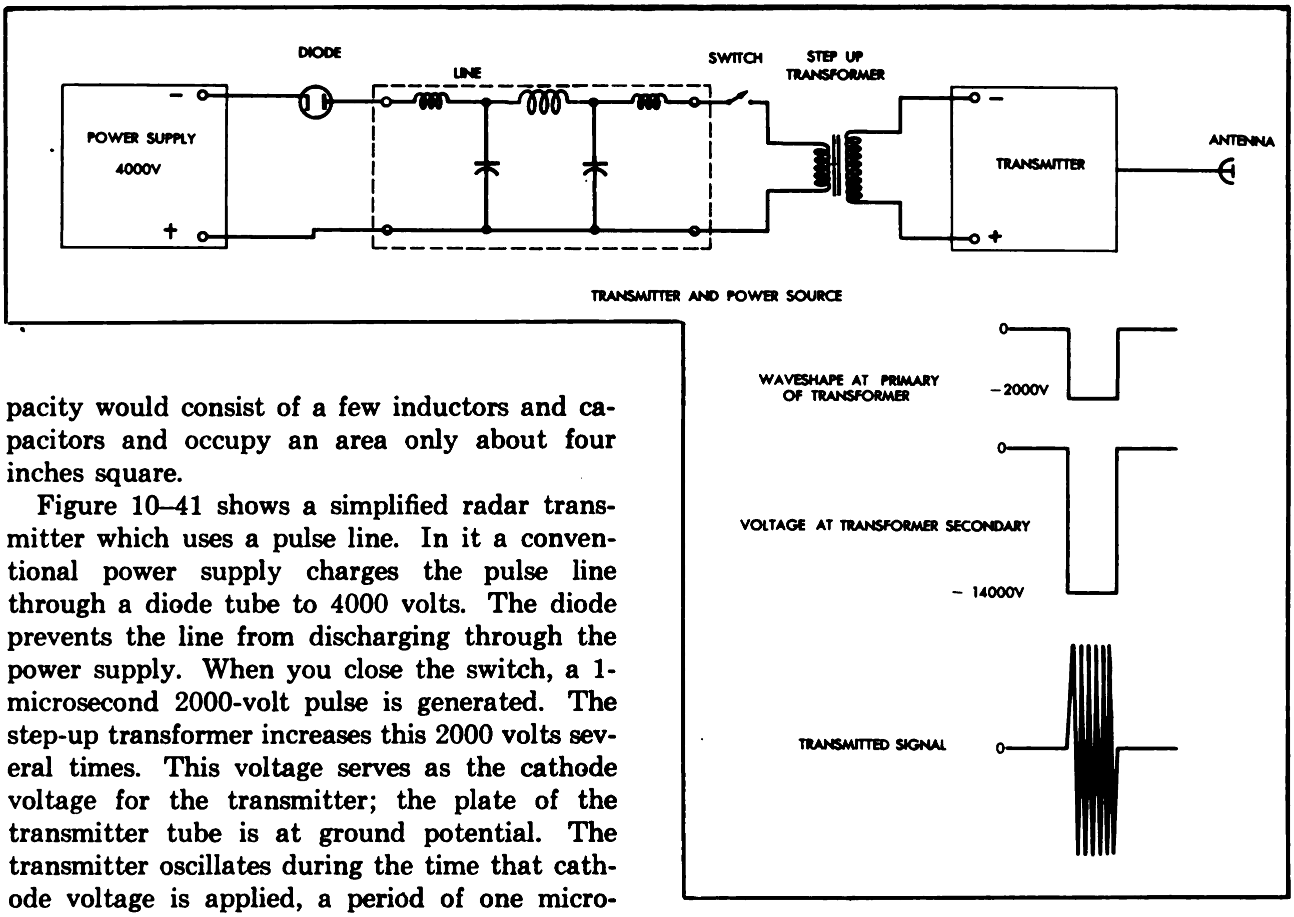


Figure 10–41. Simple Circuit Showing Pulse Line used in Radar Transmitter Circuit

pacity would consist of a few inductors and capacitors and occupy an area only about four inches square.

Figure 10–41 shows a simplified radar transmitter which uses a pulse line. In it a conventional power supply charges the pulse line through a diode tube to 4000 volts. The diode prevents the line from discharging through the power supply. When you close the switch, a 1-microsecond 2000-volt pulse is generated. The step-up transformer increases this 2000 volts several times. This voltage serves as the cathode voltage for the transmitter; the plate of the transmitter tube is at ground potential. The transmitter oscillates during the time that cathode voltage is applied, a period of one microsecond. Thus, a radio-frequency signal of one microsecond duration (or a radar pulse) is radiated by the antenna.

A pulse can also be formed by a short-circuited line. The operation of the line depends upon the energy stored in the magnetic fields around the inductors in the line. The line is charged from a constant-current source and produces a voltage across the load (R_L). Figure 10–42A shows a short-circuited line during charge. At the instant the constant-current source is connected to the line (T1 in Figure 10–42B), a current (I_0) flows. If R_L is equal to the characteristic impedance of the line (Z_0), the current is divided equally between the line and R_L. A wave of voltage whose amplitude is one-half I_0Z_0 travels down the line, and is reflected in *opposite* polarity at the short-circuited end. This reflected wave travels back to the input end.

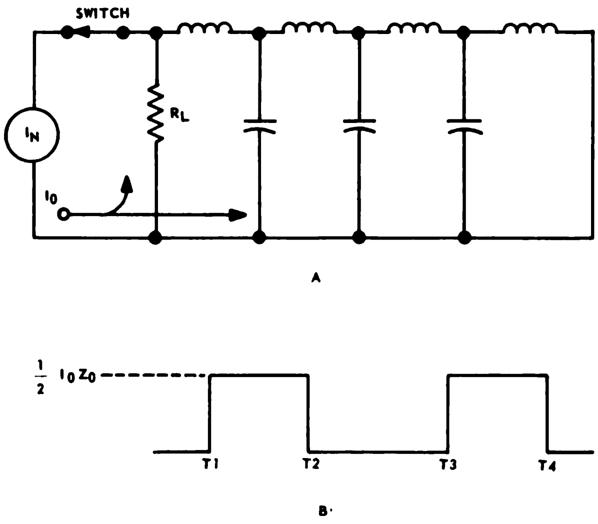


Figure 10–42. Pulse Formation with Shorted Line

The voltage pulse generated at the load during the charging period has an amplitude of $\frac{1}{2}$ I_0R_L. It lasts the length of time it takes the pulse to travel to the end of the line and back.

This is the pulse shown from T1 to T2 in Figure 10–42B.

The voltage across R_L is now zero. (The line is effectively shorting the load.) A constant current I_o is flowing through the inductances of the line, and the line is fully charged with magnetic energy. The current flowing down the line during charge is reflected in phase, and the incident current and reflected current add, doubling the current on the line.

At time T3 the constant-current source is disconnected from the transmission line. A voltage pulse that is exactly similar to the charging voltage pulse is developed across R_L as the magnetic energy stored on the line is discharged. The duration of the pulse is the time it takes the current to travel down the line and back (T3 to T4). At T4 the line is discharged completely.

As the magnetic fields about the inductances collapse, half the current ($\frac{1}{2}$ I_o) flows through the load and half charges the capacitors. As the current tends to decrease through the inductors, the capacitors discharge, maintaining the current constant to the end of the pulse.

SUPERSONIC DELAY LINES

The supersonic delay line has two general purposes: (1) to delay an electrical signal, such as a trigger or a range mark, and (2) to delay a pulse carrier without introducing distortion. In present-day radar systems comparison of video signals with each preceding signal is a necessity for efficient MTI operation. Recent advancement in the development of electronic equipment has indicated an increasing need of triggers whose timing cannot vary more than 0.5 microsecond.

The principal elements of a supersonic delay line are the transmission media and the electromechanical transducers. In the supersonic delay line electrical impulses are changed into sound impulses; after they have traversed a transmission medium, they are converted back into electrical signals. Since sound waves travel slower than electrical waves, delay is introduced. For frequencies in the megacycle range, quartz crystals are the most satisfactory transducers. Liquids (water and mercury) and solids (fused quartz) are used as the transmitting medium.

Liquid Media

Figure 10–43 illustrates a typical mercury delay line. The signal is applied to a quartz crystal which is the electro-mechanical transducer. (A transducer is a device that converts electrical energy into mechanical energy, and vice versa.) The crystal undergoes changes in thickness due to the piezoelectric effect; electrical energy is converted into mechanical energy (vibrations). The vibrations of the crystal are communicated to the mercury as supersonic, or ultrasonic, waves. The waves travel down the tank with the mercury acting as a transmission medium.

On reaching the end of the tank, the waves are reflected by a reflecting plate. The reflectors consist of circular surfaces arranged on the two opposing ends of the tank. They are formed in the walls of the tank by grooves cut around these areas. If the reflecting surfaces make good contact with the mercury, they will tend to absorb the mechanical vibrations. For maximum reflection capabilities, a rough surface is used, with air trapped in the pockets of the rough surface area. This provides a mismatch to the mercury that causes almost perfect reflection. This process is repeated over several paths. The waves finally strike a crystal at the opposite end of the tank and produce electrical impulses. The piezoelectric effect here changes the mechanical energy to electrical energy.

In mounting the crystal care must be taken that it is supported and protected so that it will not bend or break. The crystal must be aligned accurately with the tube axis. Provision must be made to attach electrodes to the crystal surface so that there will be no interference with the crystal's sonic operation. Elimination of unwanted multiple echoes is a requirement for lines that must reproduce the pulse shape accurately. This is accomplished by reducing the time the signal is applied to the delay line so that the remaining wave can be damped out. The thickness vibration of the crystal is usually at the crystal's lowest resonant frequency. Some crystals are backed by steel on the sides not in contact with the mercury. In other crystal mountings the crystal is backed by mercury, which makes unnecessary any plating on that side. The large acoustic impedance of the mercury likewise loads the crystal and widens its bandpass.

In the mercury delay line shown in Figure 10–43, direct mercury-to-the-crystal contact is

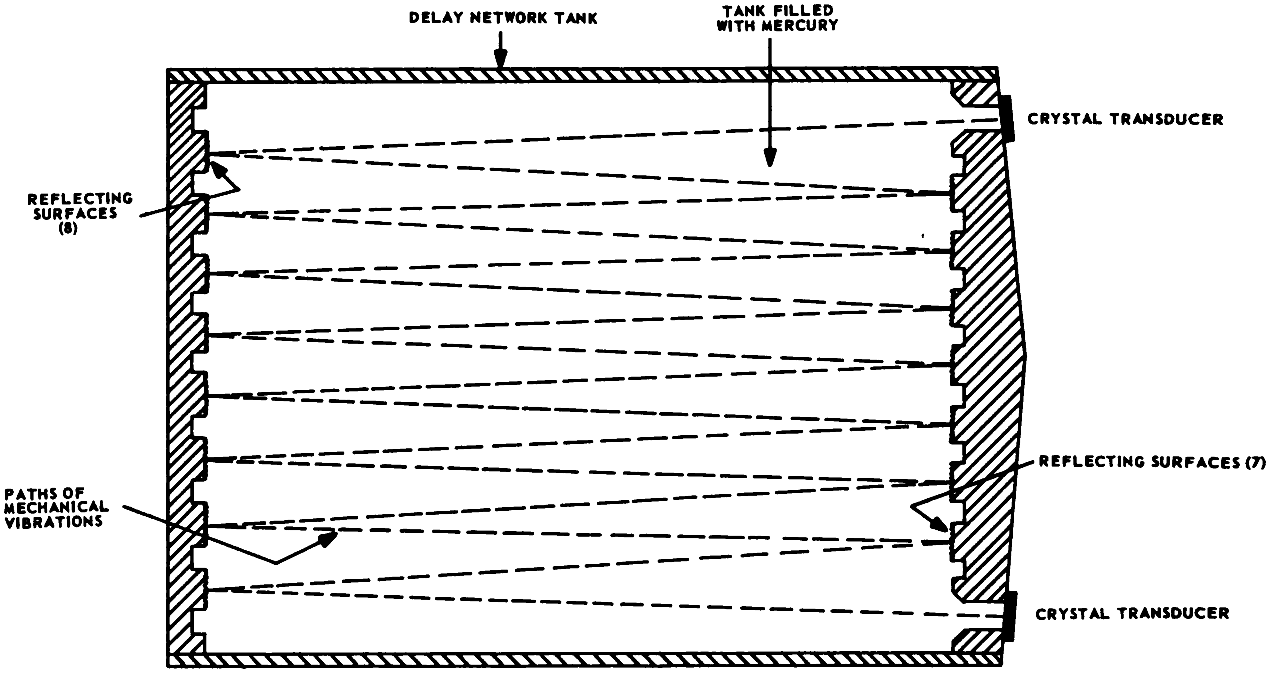


Figure 10–43. Delay Network Details

used at both the input and the output of the delay line. The mercury acts as the ground side of the electrical input and output circuits. Insulated plates on the opposite sides of the crystals provide the other electrical connections for crystal operation.

Solid Media

Fused-quartz delay lines are mechanical delay circuits that depend upon the time it takes mechanical ultrasonic waves to pass through the fused quartz. Normally each quartz delay line is contained in an hermetically sealed shell. Figure 10–44 shows the *multiple symmetry* type of fused-quartz delay line. Transducer crystals with a resonant frequency near the frequency of the carrier signal convert electrical input waves to supersonic mechanical waves.

A polygon-shaped quartz block is used to provide surfaces from which the signals are reflected. The facets, or sides, of this polygon are precision ground at the necessary angles to obtain the desired signal reflection path. Figure 10–44 illustrates the theoretical polygon-shaped quartz crystal. The sides and angles of an actual fused-quartz delay line of 14 sides would vary. In Figure 10–44, the signal is fed in at side 2 and fed out at side 11. Some of the facets of the multiple symmetry quartz delay line reflect the signal as many as three times. The illustrated figure shows only one of the possible paths which the signal may use.

Fused-quartz delay lines of the multiple symmetry type and mercury delay lines are used because of their ability to delay signals faithfully by much longer times than electrical delay lines. In the propagation of acoustical waves there is much less phase and amplitude distortion than in the electromagnetic waves. Such delay lines can also be constructed to withstand severe conditions of temperature and pressure and may be stored indefinitely without any harmful effects. Fused-quartz delay lines may be mounted in any position for operating.

One factor limiting operation of fused-quartz delay lines is that spurious secondary signals are produced in the line because of the signal traveling paths other than the desired path. These secondary signals are fed out of the line at times other than the desired delay time, and must be attenuated to a level at which they can be ignored. Another important factor concerning the use of the fused-quartz delay line is the amount of signal attenuation in the line. Some radar systems have a requirement of less than 60 db signal attenuation.

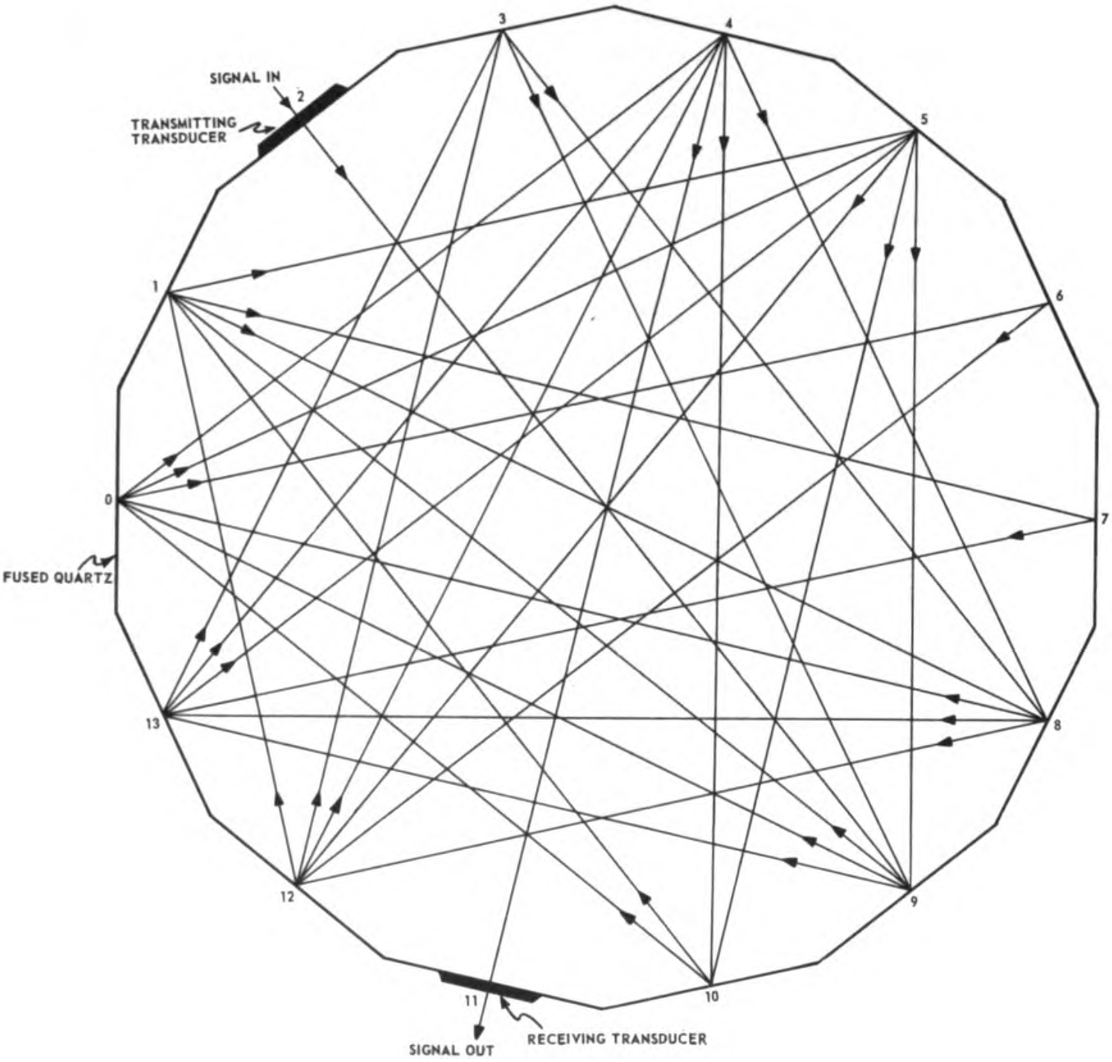


Figure 10–44. Quartz Delay Line

Magnetostriction Delay Line

The basic material used in the construction of a magnetostriction delay line is a nickel ribbon or tube. Kinking should be avoided, as serious reflections occur at kinks or dents. The driving coil is approximately the same as the pickup coil, the pickup coil having more turns than the driver coil. A nonmetallic, nonmagnetic material is used as a coil mount, with the coil centered very accurately on the coil form. The output coil requires a polarizing magnetic field for proper operation. Undesired end reflections are prevented by some means of mechanical dampening, usually a rubber mount.

The main theory of ferromagnetism is based on the assumption that a ferromagnetic crystal is composed of a large number of regions, each of which is spontaneously magnetized to an intensity that is a function of the temperature alone. When the net over-all magnetization is zero, these regions are aligned in equal numbers along the various directions of easy magnetization of the crystal. The effect of a magnetizing field is to alter this uniform distribution so that most of the regions lie in that direction of easy magnetization which most nearly coincides with the field.

The application of a magnetic field increases the stability of the atoms aligned with the field and decreases the stability of other orientations. As the field intensity increases, the atoms with unstable orientation shift suddenly. The corresponding sudden changes in the interatomic force of the face-centered cubes of nickel crystals set up elastic vibrations that act upon adjacent atoms and are thereby transmitted. The sum of a large number of these sudden local shifts in orientation results in the observable magnetostriction effect, which is used in present-day electronic equipment.

The effect may be described as the change in dimensions of a ferromagnetic material when subjected to a longitudinal magnetic field. The maximum variations occur in the length of the material with a smaller change occurring in the volume of the material.

In Figure 10–45, V_1 is a twin triode whose grids are biased beyond cutoff. A positive pulse drives both grids of V_1 positive, causing a short pulse of current to flow in each plate circuit. Each plate circuit of the twin triode contains 3 coils, with a set of relay contacts in parallel with each coil. The driver coils are wound around the magnetostriction line of nickel tubing. The relays are remotely controlled. If any relay contact is opened, the plate current pulse flows through the associated coil and creates a magnetic field along the longitudinal axis of the nickel tubing. This field sets up a shock wave in the tubing because of the magnetostriction effect.

A mechanical stress is felt along the tubing in the vicinity of the driver coils. The shock wave travels from its source toward the end of the line with a velocity of 0.187 inch per microsecond. When this shock wave reaches the end of the line, most of the energy is absorbed in the rubber tubing that is clamped around the end of the line. The driver coils may be spaced so that the time required for the shock wave to travel from one coil to another meets the time requirements of the equipment used.

A permanent horseshoe magnet is mounted near the tubing and provides a magnetic field in the vicinity of the pickup coil. The shock wave, which actually compresses the nickel tubing, travels down the tubing and shifts the lines of flux created by the permanent magnet. As the lines of flux cut the pickup coil, a voltage is induced. Because of the short duration of the shock wave and the direction of the magnetic field, a short negative pulse is applied to the grid of V_2. The magnetostriction delay line has been developed to a satisfactory state as a flexible device for delaying pulses, mixing pulses and generating codes.

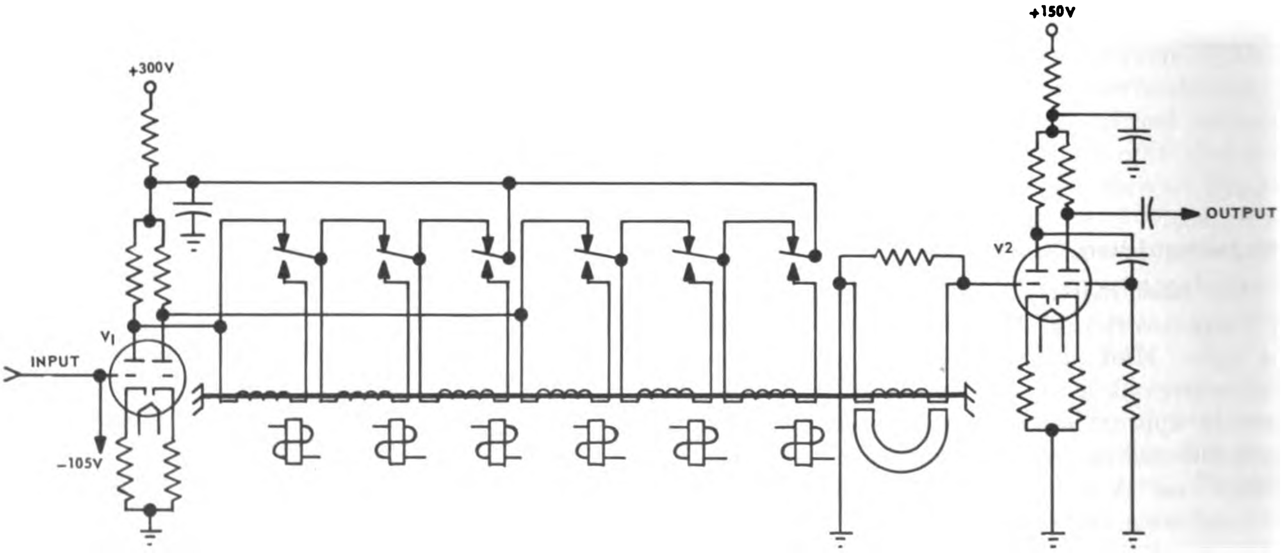


Figure 10–45. Magnetostriction Delay Line

CHAPTER 11

Waveguides, Cavity Resonators, and Duplexers

The Air Force operates some electronic equipment at frequencies above 1000 mc. The very short wavelengths of these frequencies give them the name of *microwaves*. The transmission lines that are used at these frequencies are different both physically and electrically from the ones previously discussed.

Since conductors radiate the energy they carry, and since radiation increases with an increase in frequency, special transmission lines are used at microwave frequencies. If a microwave signal were applied to a two-wire line, the radiation would increase to such an extent that practically no energy would reach the output end.

Even though the inner conductor of a coaxial line radiates energy, the coaxial line can be used for microwave transmission. All of the energy is confined between the outer and inner conductors. Removing the center conductor and transmitting the wave through the remaining conductor results in a transmission line called a cylindrical waveguide. When a hollow rectangular conductor is used, the transmission line is called a rectangular waveguide.

Another device that is used at microwave frequencies is a cavity resonator. The purpose of this chapter is to acquaint you with these devices. It discusses their theory, operation, and use.

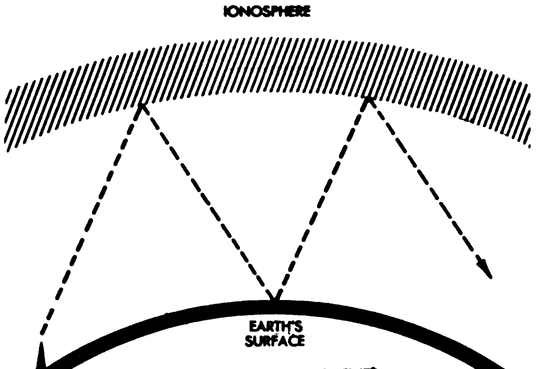
GUIDED AND UNGUIDED ELECTROMAGNETIC WAVES

In general, there are two methods for transferring electrical energy — one is by current flow in wires; the other is by movement of electromagnetic fields in space. Electrical energy can be transferred as current flow in a number of types of transmission lines, for example, two-wire lines and coaxial lines; in space it moves as electromagnetic fields whenever a radio or radar antenna radiates energy.

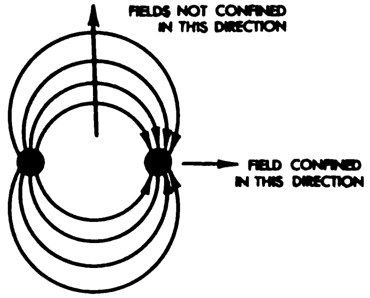
Electromagnetic fields that move in space are confined largely to the area between the earth and the ionosphere as you can see in Figure 11–1A. (Chapter 15 explains the ionosphere.) While the change in the dielectric constant of the ionosphere is sufficient to reflect most electromagnetic fields that strike it, extremely high frequency radiation that strikes it almost perpendicularly is not reflected. An electromagnetic field that does not start out in a direction parallel to the earth and the ionosphere, follows a zigzag path between these two areas and may or may not be reflected back to the earth.

Although the transfer of energy by electromagnetic fields and by currents in wires may seem to be unrelated phenomena, actually the trend on the part of electronic scientists is to look even on two-wire lines as elements that guide electromagnetic fields from one place to another. The currents in the wires are merely considered incidental to the action and the result of the moving fields.

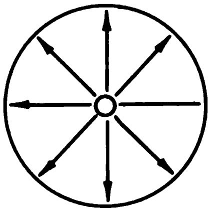
A two-wire line is a poor guide for transferring electromagnetic fields because it does not confine the fields in a direction perpendicular to the plane that contains the wires as shown in Figure 11–1B. This results in some energy escaping in the form of radiation. Electromagnetic



A RADIO ENERGY IS CONFINED BY EARTH AND IONOSPHERE


FIELDS NOT CONFINED IN THIS DIRECTION
FIELD CONFINED IN THIS DIRECTION

B END VIEW OF TWO WIRE LINE
FIELDS CONFINED IN TWO DIRECTIONS BUT NOT IN OTHER TWO.



C END VIEW OF COAXIAL CABLE
FIELDS CONFINED IN ALL DIRECTIONS

Figure 11–1. Guiding Waves

fields may be completely confined in this direction when one conductor is extended around the other to form a coaxial cable as shown in Figure 11–1C. In a coaxial cable, energy transfer is said to take place by electromagnetic fields, rather than by current flow. However, this method is not too efficient at high radio frequencies, since skin effect limits the current-carrying area of a conductor to a thin layer at its surface.

Another disadvantage is that the ability of the fields to form is limited by the amount of current flow associated with them. When the resistance in a conductor is increased, current flow in it is reduced, the reduction depending on the amount of increase. This decreases the magnitude of the fields. On inspecting the cross section of the coaxial cable, you can see that the surface area of the inner conductor is much less than the surface area of the outer conductor. This causes the inner conductor to retard the current considerably more than the outer conductor and results in a reduction in the efficiency of energy transfer. If you could remove the center conductor and retain the fields, energy might be transferred with less loss.

Electromagnetic fields can transfer energy in a line that does not have a center conductor provided the configuration of the fields is changed to compensate for the missing conductor. The area remaining, which might be considered a pipe, is called a *waveguide*. A waveguide does not necessarily have to be circular in cross section. Practical waveguides, for example, are also square, rectangular, or eliptical in construction.

Metallic walls are not necessary to guide electromagnetic fields in a waveguide, for the fields will be reflected whenever they encounter any kind of a substance which has a different dielectric constant from the substance in which they are traveling. For example, fields can be made to travel through a ceramic rod with little loss of energy. When they encounter the air at the surface of the rod, they are reflected back into the rod.

As you previously learned, the three types of losses in r-f lines are copper losses, dielectric losses, and radiation losses. Briefly, these losses are described as follows: Copper loss is an I^2R loss. It becomes appreciable whenever skin effect reduces the conducting area of the lines. Dielectric losses are losses caused by the heating of the insulation between the conductors. Radiation losses are losses due to energy escaping from r-f lines in the form of radiation. Let us consider waveguides from the point of view of these losses.

Copper losses are small in waveguides. Since a two-wire line consists of a pair of small conductors, the surface area of each is likewise small. Although the surface area of the outer conductor of a coaxial cable is large, the inner conductor is small, and it produces considerable copper losses.

On the other hand, a waveguide has a large surface area as it does not have a center conductor. Therefore, whenever current flows, the copper losses in the waveguide are less than those in other types of lines.

Dielectric loss of energy is eliminated in a waveguide, since there is no center conductor requiring a solid dielectric support. One of the major reasons for signal attenuation in a coaxial line is the leakage in the dielectric used to support the inner conductor. Because the dielectric material has some resistance, a part of the energy transmitted down the line is absorbed by the insulator.

The frequency of the transmitted energy is the principal factor governing dielectric loss. As the frequency is increased, the amount of electromagnetic energy absorbed by the dielectric also increases. The dielectric loss is therefore much higher at microwave frequencies than at the lower radio frequencies.

Radiation losses are less in a waveguide than in a two-wire line. In a waveguide, fields are contained wholly within the guide itself just as in a coaxial line. Therefore, only a negligible amount of energy is radiated.

The power handling capacity of a waveguide is greater than that of a coaxial line having an equal size. Power is a function of E^2/Z_0, where E is the maximum voltage in the traveling wave and Z_0 is the characteristic impedance of the line. E is limited by the distance between the conductors. In the coaxial line illustrated in Figure 11–2, this distance is S_1. In the waveguide this distance, which is S_2, is much greater than S_1. Therefore, the waveguide is able to handle greater power before the voltage exceeds the breakdown potential of the insulation.

A waveguide is simpler to construct than a coaxial line. This is because in the waveguide the center conductor is eliminated completely.

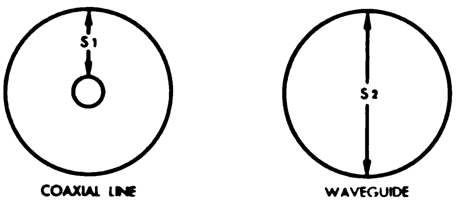


Figure 11–2. Comparison of Spacing in Coaxial Line and Waveguide

The physical stamina of a waveguide is greater than that of a coaxial line. This is due to the fact that, unlike a coaxial line, it has no center conductor or insulators which can be displaced or broken.

In view of these advantages you may wonder why waveguides are not used exclusively for transferring energy. There are, however, two disadvantages which make waveguides impractical to use at any but extremely high frequencies.

In the first place, the cross-sectional dimensions of a waveguide must be in the order of a half-wavelength for it to contain electromagnetic fields properly. A waveguide used at one megacycle, for example, would be about 700 feet wide. At lower radar frequencies, 200 mc for example, this waveguide would have to be about four feet wide, while at higher radar frequencies, such as 10,000 mc, it need be only one inch wide. Therefore, dimensions that waveguides require make them impractical at any frequency lower than about 3000 mc.

In the second place, if the dimension of the guide is a half-wavelength or less, energy will not be propagated through the waveguide. The reason for this is that for any given waveguide there is a cutoff frequency, below which it does not function as a power transfer device. This limits the frequency range of any system using waveguides.

The installation of waveguides presents problems that are different from those encountered with two-wire or coaxial lines. Waveguide installations are often referred to as *electrical plumbing*, because the exact positioning and rigidity required calls for installation techniques well known to plumbers, but less known to electronic or microwave technicians.

Special couplings are required to fasten the lengths of the waveguide together. At the lower radio frequencies adequate transmission characteristics are assured by soldering the conductors together. At higher microwave frequencies precautions to prevent the leakage of any energy must be taken. An ideal situation is to have the waveguide one continuous section between transmitting and receiving points. But the normal situation has the waveguide made in fairly short sections that must be connected together. Figure 11–3 illustrates some of the waveguide sections manufactured for specific applications. Special flexible waveguide sections have been

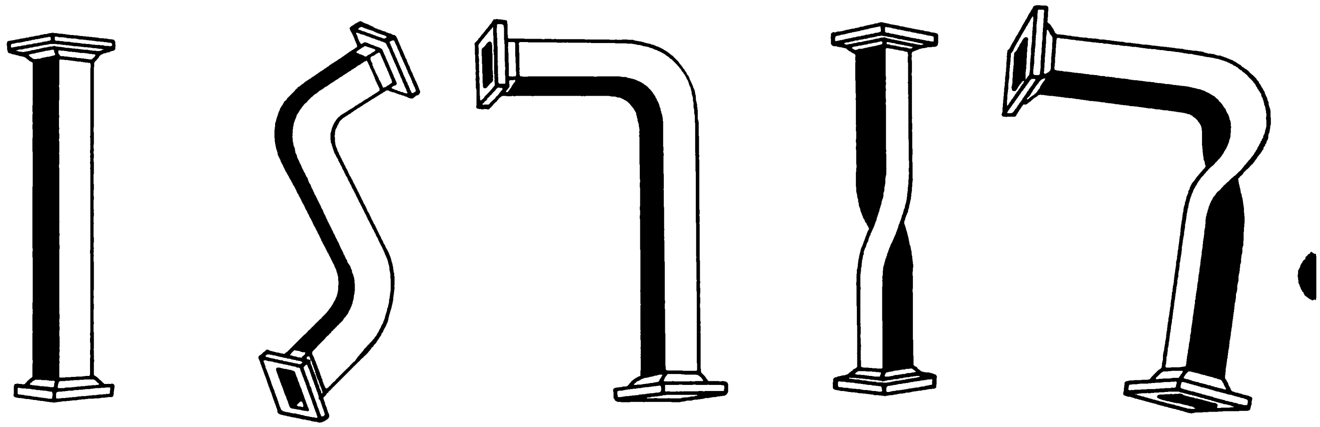


Figure 11–3. Waveguide Sections

developed to be used where the standard, or fixed, curved sections cannot be used.

It is easily understood why each new installation must be carefully planned beforehand. After the initial planning the various sections of waveguide required can be ordered and the whole line can be assembled. Compare this type of installation to that encountered when flexible coaxial cable, which is easily bent around corners or passed through wiring troughs, is used. You can understand that waveguide installations are more complex and more expensive.

Besides installation costs, the price of the waveguide is high. The waveguide must meet rigid mechanical specifications to prevent discontinuities along the transmission path which might introduce energy losses. During the manufacture of the waveguide, special machining is performed to minimize such discontinuities. This special machining also adds to the initial cost. To reduce the skin-effect losses sometimes the inside of the waveguide is silver-plated or even gold-plated. This also results in increased cost.

WAVEGUIDE THEORY

An exact mathematical analysis of the way in which fields exist in a waveguide is quite complicated. But it is possible to obtain an understanding of many of the properties of waveguide propagation by using the following simple analogy, which shows both how the fields are able to exist in a waveguide and how you can handle them.

To understand the action of a waveguide, assume that a waveguide has the form of a two-wire line. In this condition there must be some means of supporting the two wires. Furthermore, the support must be a nonconductor, so that no power will be lost by radiation leakage. An efficient way for both insulating and supporting the two-wire line is shown in Figure 11–4A. This line is spaced, insulated, and supported by porcelain stand-off insulators. At communication frequencies, the absorption of power by the dielectric material (insulators) causes them to look like a low resistance and capacitance.

The equivalent electrical circuit at higher frequencies is shown in Figure 11–4B. For frequencies of 3000 mc and up, a better insulator than nonconducting porcelain insulators must be used. A superior high-frequency insulator for this purpose is a quarter-wave section of r-f line, called a *metallic* insulator, which was discussed in the preceding chapter. Such an insulator is shown in Figure 11–4C. As there are no dielectric losses in a quarter-wave section of r-f line, the impedance at the open end (the junction of the two-wire line) is very high.

A metallic insulator can be placed anywhere along a two-wire line. Figure 11–5A shows several on each side of a two-wire line. A point to note in this line is that the supports are a quarter-wave at only one frequency. This limits the high efficiency of the two-wire line to one frequency only.

The use of several insulators results in the improved conductivity of a two-wire line when

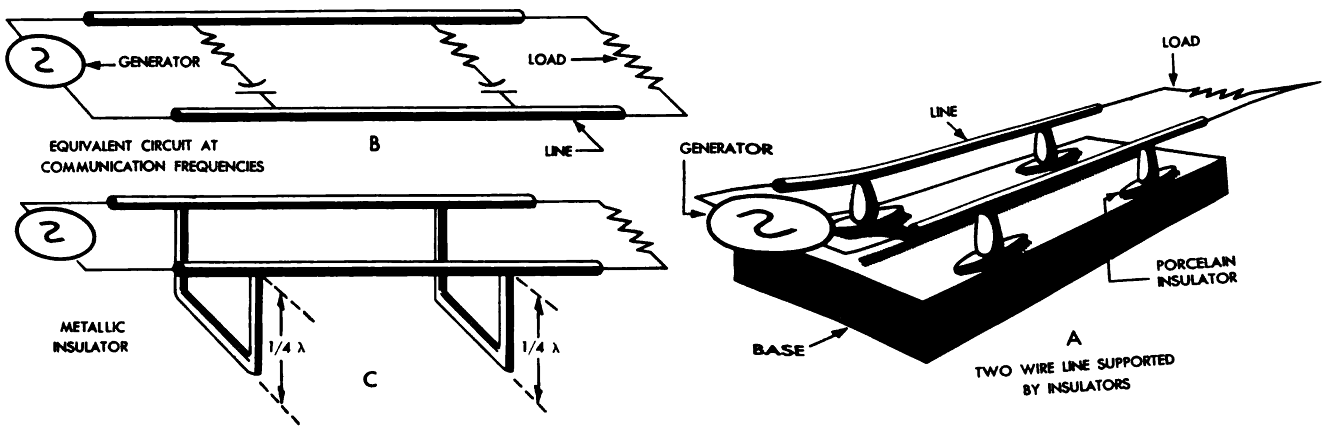


Figure 11–4. Insulating the Two-Wire Line

the sections are connected together. This connection is made between the two adjacent insulators through a switch, as shown in Figure 11–5B. When the switch is open, both quarter-wave sections are excited by the main line. In this condition, there will be standing waves on the quarter-wave sections.

When the switch is connected to the same place on each section, the relative phase relationship of the voltages at the connection will be the same for each section. In this condition the No. 1 section will be excited first by the generator. When the switch is closed, the No. 2 section will be partly excited by the No. 1 section through the switch connection. In this manner, less energy from the main line is required to excite the No. 2 section. The parallel paths shown cause less resistance to exist along a given length of line, and energy is transferred with less copper loss.

When more and more quarter-wave sections are added to the line until each section makes contact with the next, the result is a rectangular box in which the line is at the center, Figure 11–5C. The line itself is actually part of the wall of the box. The rectangular box thus formed is a waveguide.

EFFECT OF DIFFERENT FREQUENCIES ON A WAVEGUIDE

Previously it was stated that a quarter-wave section is limited in operation to a certain frequency. However, when a solid wall of insulators

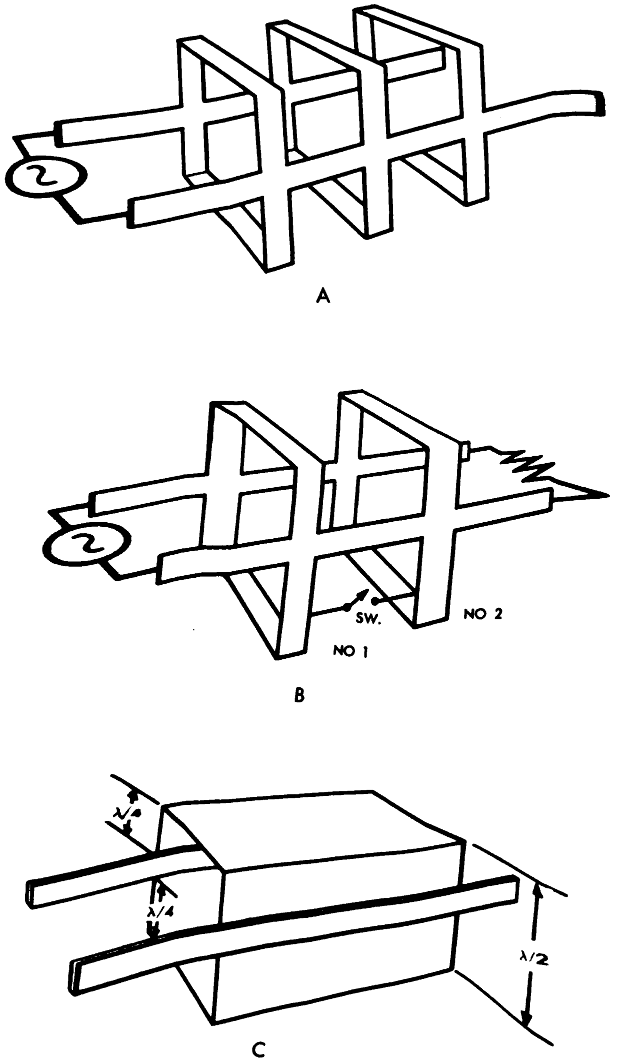


Figure 11–5. Development of Waveguide by Adding Quarter-Wave Sections

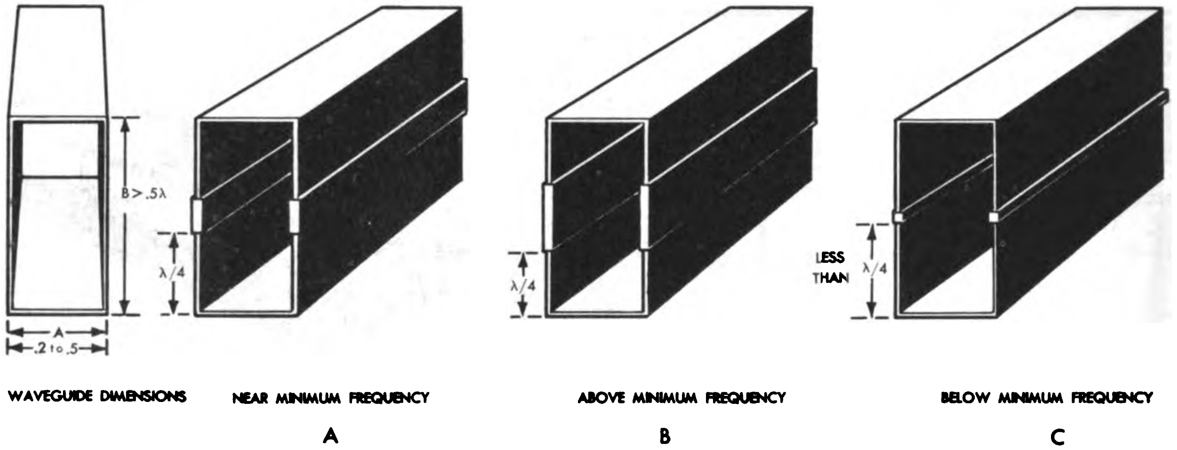


Figure 11–6. Effect of Different Frequencies in the Waveguide

is added, the section will operate at other frequencies. The waveguide shown in Figure 11–6A is the one just discussed. When the frequency being transferred by the guide is made higher, the quarter-wave section must be shorter. These shorter wavelengths are easily accommodated if you assume this two-wire line is made up of a wide bar or strip in each wall of the guide, as shown in Figure 11–6B. The shorter distance remaining is the shorter quarter-wave section.

Thus, the wide bar shown is theoretically well insulated at any frequency higher than the one which creates the near minimum frequency at A. In reality there is a practical upper frequency limit at which this analogy is applicable. For example, when the bar is a half-wavelength across, the waveguide will be 4 quarter-wavelengths across and may act as though there were two bars instead of one, as shown later in the chapter.

The next consideration is a frequency which is lower than the original frequency. Lowering the frequency in a given waveguide will lengthen the sections and narrow the bar. Beyond some lower frequency, this bar does not exist, because the quarter-wave sections meet one another. At a still lower frequency, the sections become less than a quarter-wave as in Figure 11–6C. A sections less than a quarter-wave is inductive. Thus, the impedance across the place where the conducting bars belong is not a high resistance, but an inductance. The inductive reactance will dissipate the energy in the line very swiftly through high currents that flow back and forth in one place, as though the inductance were taking energy during one half-cycle and then returning it to the line during the next half-cycle.

Therefore, we find that in a waveguide there is a low-frequency limit, or *cutoff* frequency, below which the waveguide cannot transfer energy. The width of the waveguide at the cutoff frequency is equal to a half-wavelength, since at this frequency the two quarter-wave sections touch and add to one another. Mathematically, the cutoff wavelength is expressed by the equation,

$$\lambda c = 2B$$

where λc is the cutoff wavelength, and B is the long dimension of the rectangular cross section.

As cutoff occurs when the width of a waveguide is below a half-wavelength, most waveguides are made 0.7 wavelength in the wide dimension to give a margin between the actual size and the size for cutoff. The other dimension is the distance between the conductors, and similarly, as in the two-wire lines, is governed by the voltage breakdown potential of the dielectric, which is usually air. Widths of 0.2- to 0.5-wavelength are common.

ELECTROMAGNETIC FIELDS IN A WAVEGUIDE

A good working knowledge of the fields present in a waveguide is necessary if you want to use them intelligently. Energy in a waveguide is transferred by the electromagnetic fields, while currents and voltages merely aid in forming these

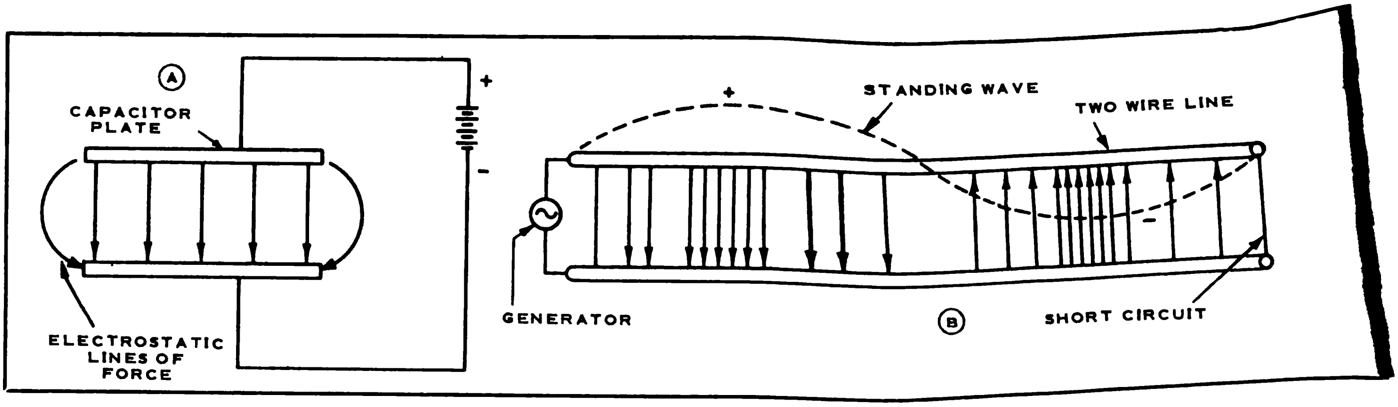


Figure 11–7. Electric Field Between Capacitor Plates and a Full-Wave Section of Two-Wire Line

fields. You should know when a good current path is required and where the voltages will be high. As energy is normally introduced into and removed from the waveguide by the fields, you should know where the fields will exist. In any waveguide two fields — the electromagnetic and the electrostatic field — are always present.

The Electric Field

The existence of an electrostatic (electric) field indicates that there is a difference in the number of electrons between two points. An electric field consists of a stress in the dielectric field and is represented in diagrams by arrows. The simplest form of electrostatic field is the field that forms between the two parallel plates of a capacitor as is shown in Figure 11–7A. When the top plate of the capacitor is made positive by a battery, electrons move from the top plate and deposit themselves on the bottom plate. This immediately sets up a stress in the dielectric between the plates. This stress is represented by arrows, pointing from the more positive voltage point to the more negative voltage point. The amount of stress sometimes is indicated by the length of the arrow. A long arrow represents more stress than a short arrow.

In representing electrostatic fields, the number of arrows indicates the strength of the field. In the case of the capacitor in Figure 11–7A, note that the arrows are evenly spaced across the area between the two plates. As the voltage across the plates is the same at all points, the electrostatic lines between the plates are evenly distributed. This set of lines represents an electrostatic field. This field is usually called the *electric field* and is abbreviated the *E-field*. The lines of stress are called the *E-lines*.

Notice in Figure 11–7B the two-wire transmission line which has an instantaneous standing wave of voltage applied to it. This line is equal to one wavelength. At the same time, part of it is positive, while another part is negative. The instantaneous electrostatic field (E-field) is the same at the negative and the positive points, but the arrows representing each field point in opposite directions. The voltage along the line varies sinusoidally. Therefore, the density of the E-lines varies sinusoidally.

An easy way to show the development of the E-field in a waveguide is from a two-wire line which has quarter-wave insulators. Figure 11–8 shows the two-wire line previously discussed with several double quarter-wave insulators, or half-wave frames used as insulators. The E-field on the main line is the same as that in the transmission line illustrated in Figure 11–7. The half-wave frames located at points of high voltage (strong E-field) will have a strong E-field across them. The half-wave frames located at points of minimum voltage will have no E-field on them.

Frame No. 1 in Figure 11–8 is an example of an insulator which has a strong E-field across it. Each frame is shown separately below the main line for a clearer view. Frame No. 2 is at a zero voltage point, so it will have no field on it. Frame No. 3 also has a strong field, but its polarity is reversed. Frame No. 4 has a weaker field on it due to its being at a lower voltage point on the main line. The picture shown is a build-up to the three dimensional aspect of the full E-field in a waveguide.

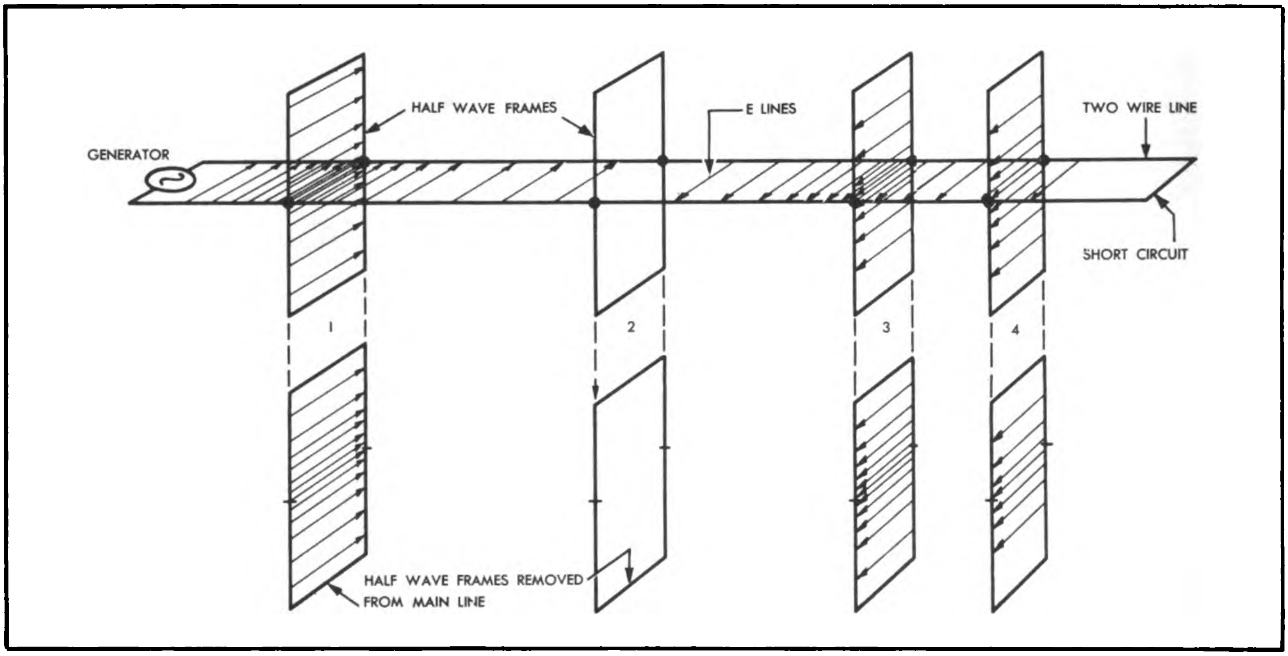


Figure 11–8. Magnitude of Fields on Half-Wave Frames Vary with Strength of Field on Main Line

The illustrations in Figure 11–9 show the E-field in an actual waveguide. This is the field that results when an infinite number of quarter-wave sections are connected to the line to form a rectangular box. The E-field is strong at one-quarter and three-quarter distances from the shorted end, but becomes weaker at the sine rate toward the upper and lower walls and toward the ends and center. Again the phenomenon of wavelength is present, as shown in Figure 11–9A. You should realize, of course, that this is an instantaneous picture taken at the time the

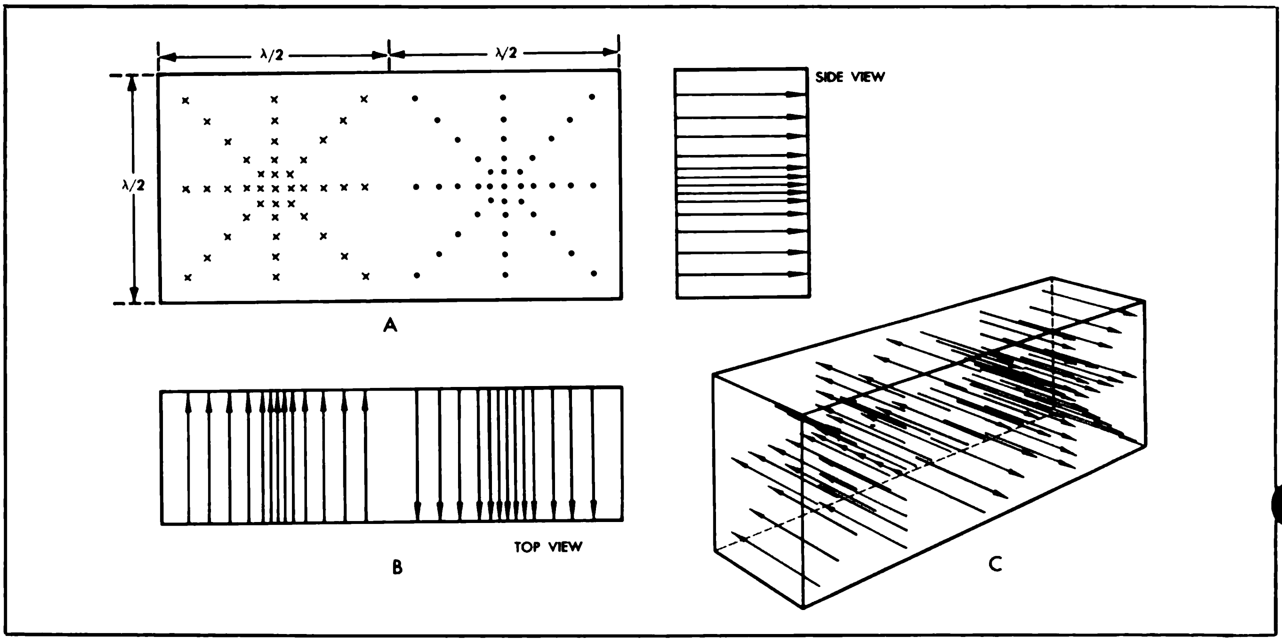


Figure 11–9. E-Field in Actual Waveguide

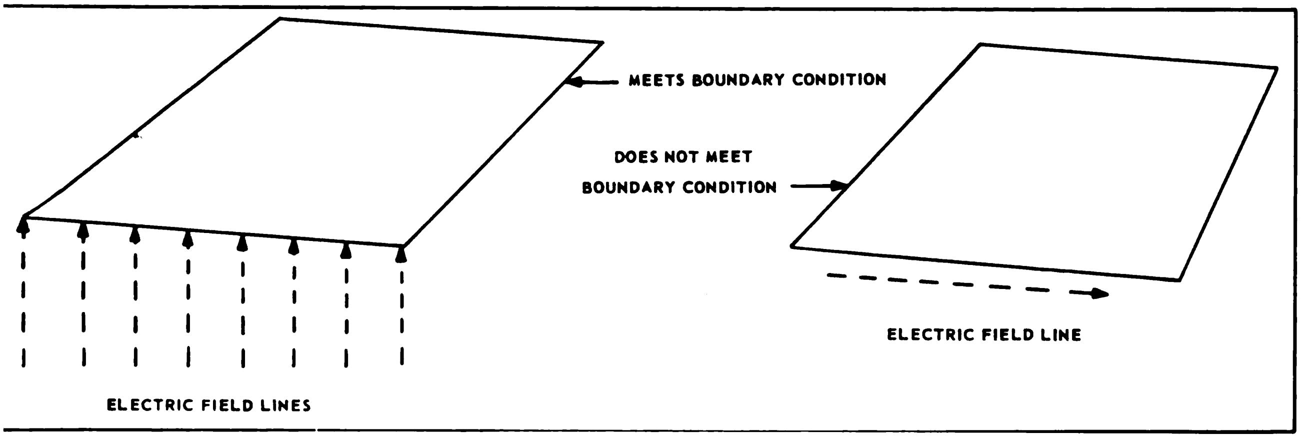


Figure 11–10. Boundary Condition for an Electric Field

standing wave of voltage is at its peak. At other times, the voltage and E-field vary from zero to the peak value, reversing direction every alternation of the applied voltage.

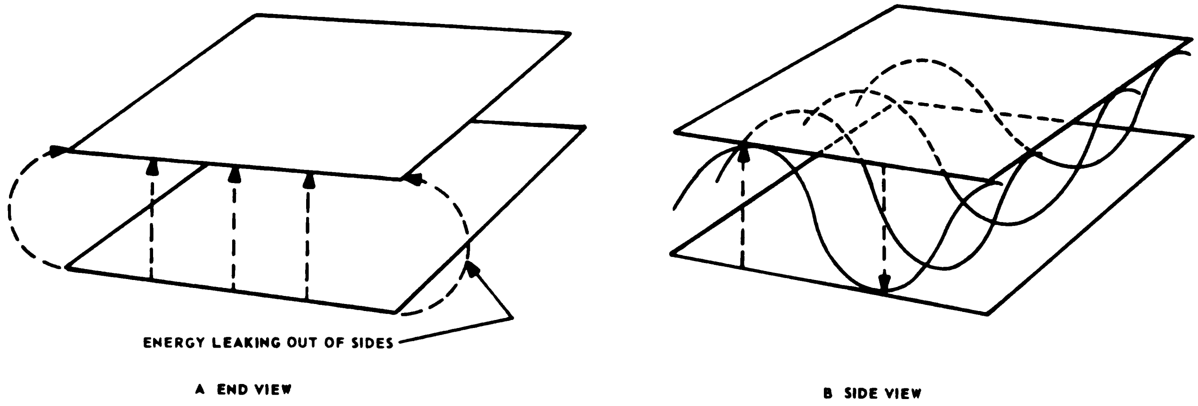
Boundary Conditions

Certain boundary conditions must exist for propagation to occur in a waveguide. The propagation of energy down the waveguide is similar to that of electromagnetic waves transmitted in free space. The exception is that waves in the waveguide are confined to the boundaries of the metallic guide. When a waveguide is being used for transmission, certain *boundary conditions* have to be satisfied; otherwise, transmission will not continue.

There are two boundary conditions that have to be satisfied. The first is that an electric field must be perpendicular to the conductor to exist at the surface of a conductor. The converse of this statement is also true. Assuming a perfect conductor, if an electric field has a component that is parallel to the conductor, an electric field cannot exist at the surface of a conductor. Figure 11–10 illustrates the first boundary condition. The second is that a varying magnetic field must exist in closed loops parallel to the conductors and perpendicular to the electric field.

Normally, if a transmitting system satisfies one boundary condition, it also satisfies the other condition. To check for one condition is all that is normally required to determine whether a wave can exist at the surface of a conductor.

Figure 11–11 illustrates a plane electromagnetic wave that is radiated and is confined between two parallel plate conductors. To satisfy



A END VIEW

B SIDE VIEW

Figure 11–11. Electric Field between Plates

one boundary condition, the electric field is oriented so that the electric field (E) lines are perpendicular to the conductors as shown in Figure 11–11A.

Figure 11–11B shows an E-field that is varying sinusoidally along the distance between the plates. A varying E-field produces a varying magnetic field, which must exist in closed loops and must be parallel to the conductors, the second boundary condition. Figure 11–12 shows the closed loops of the magnetic field that is between the conductors at a given instant of time.

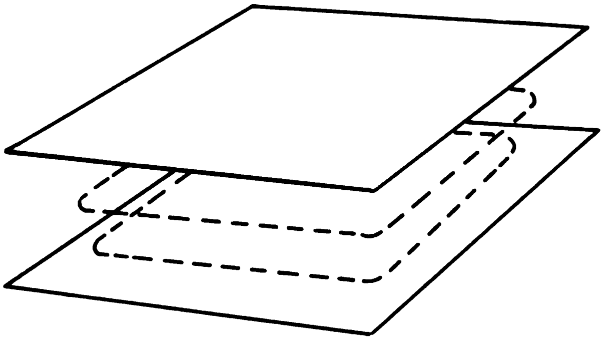


Figure 11–12. Magnetic Loops between Plates

Figure 11–11A also illustrates how some of the transmitted energy might leak or escape through the open sides. To prevent this energy loss, side walls are attached to the parallel plates. A rectangular waveguide is thus formed. However, the plane wave shown cannot exist in a rectangular waveguide because the plane wave has an electric field that is parallel to the surface of the side walls.

The Magnetic Field

The second field that must always exist in a waveguide is the magnetic field. The magnetic lines of force which make up the magnetic field are caused by the *movement* of electrons in the conducting material. All the tiny magnetic forces exerted by the individual moving electrons add together and form a large force around the conductor. The presence of the force is shown by closed loops around the single wire in Figure 11–13A. The line forming the loop is a magnetic line of force or an *H-line*. The H-line must be a continuous closed loop in order to exist.

All the lines associated with current are collectively called a magnetic field or an *H-field*. The strength of the H-field varies directly with the current. Each H-line has a certain direction. You can determine this direction by the left-hand rule. The strength of the H-field is indicated by the number of H-lines in a given area.

Although H-lines encircle a single straight wire, they behave differently when the wire is formed into a coil as is shown in Figure 11–13B. In a coil the individual H-lines tend to form

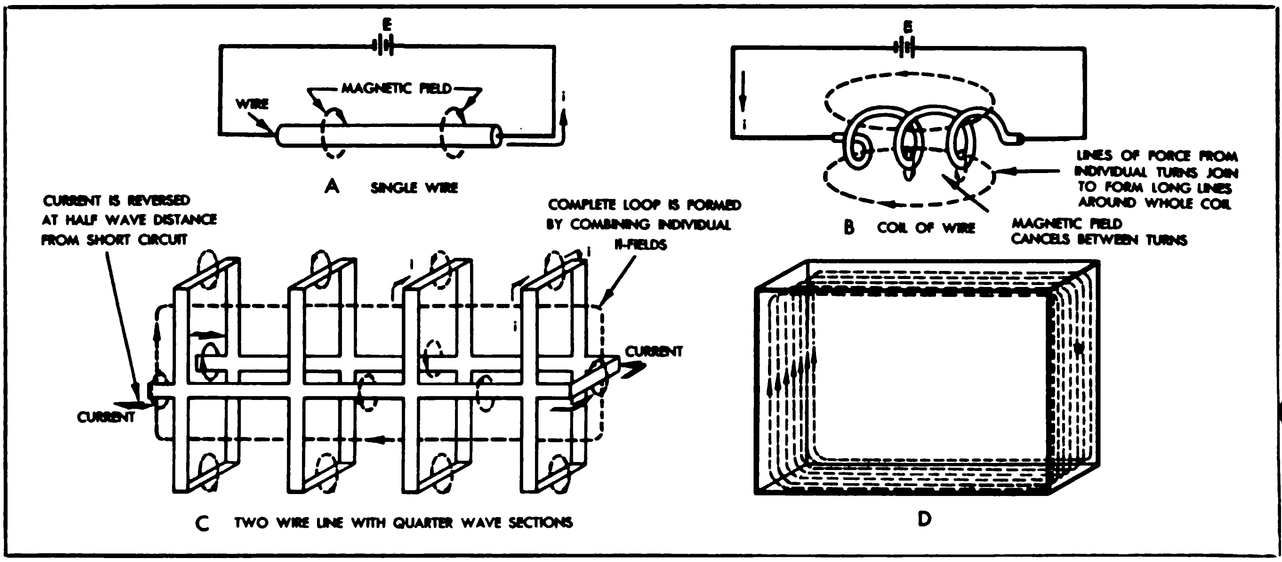

E
WIRE
MAGNETIC FIELD
i
A SINGLE WIRE
E
i
B COIL OF WIRE
LINES OF FORCE FROM INDIVIDUAL TURNS JOIN TO FORM LONG LINES AROUND WHOLE COIL
MAGNETIC FIELD CANCELS BETWEEN TURNS
CURRENT IS REVERSED AT HALF WAVE DISTANCE FROM SHORT CIRCUIT
COMPLETE LOOP IS FORMED BY COMBINING INDIVIDUAL H-FIELDS
i
CURRENT
CURRENT
C TWO WIRE LINE WITH QUARTER WAVE SECTIONS
D

Figure 11–13. Development of Magnetic or H-Field in the Waveguide

around each turn of wire, but in doing so take opposite directions between adjacent turns. This causes cancellations and results in zero field strength between the turns. But inside and outside the coil, the directions are the same for each H-field. Therefore, the fields here join and form a continuous H-line around the entire coil as shown.

Similar action also takes place in a waveguide. In Figure 11–13C a two-wire line with quarter-wave sections is shown. Currents flow in the main line and in the quarter-wave sections. The current direction produces the individual H-lines around each conductor as shown. When a large number of sections exist, the fields cancel between the sections, but their directions are the same inside and outside the waveguide.

At half-wave intervals on the main line, current will flow in opposite directions. This produces H-line loops having opposite directions. In Figure 11–13C, current at the left end is opposite to the current at the right end. The individual loops on the main line are opposite in direction. All around the framework they join so that the long loop shown in Figure 11–13D is formed. Outside of the waveguide the individual loops cannot join to form a continuous loop. Thus, there is no magnetic field outside of a waveguide.

Figure 11–14 shows a conventional presentation of the magnetic field in a waveguide three half-wavelengths long. Note that the field is strongest at the edges of the waveguide. This is where the current is the highest. The current is lowest at the center of each set of loops because there the standing wave of current is zero at all times. In Figure 11–14 the picture shown represents an instantaneous condition. During the peak of the other half-cycle of AC input, all field directions are reversed. An instantaneous picture of this condition would show the fields reversed at half-wave intervals, since the current in the long line is reversed over half-wave distances.

A second boundary condition necessary for electromagnetic fields to transfer power is satisfied by the configuration of the magnetic field. This condition, previously stated, requires that at the surface of the waveguide there be no perpendicular component of the magnetic field. Since all the H-lines are parallel to the surface, this condition is satisfied.

Electric and magnetic fields exist simultaneously in the waveguide. In fact, the H-field causes a current which, in turn, causes a voltage difference. This produces an E-field and it, in turn, causes a current. This current creates an H-field, and so on. One field is dependent on the other, as energy is continually transferred from one field to the other.

Figure 11–15A illustrates both fields in the waveguide. Since this picture is rather complicated, the presence and direction of the field is usually indicated in more simple diagrams, such as those shown in Figures 11–15B, C, and D. In these diagrams the number of E-lines in a given area indicates the strength of the electrostatic field, while the number of H-lines in any given cross section indicates the strength of the magnetic field in that area.

The field configuration shown in Figure 11–15 represents only one of the many ways in which fields are able to exist in a waveguide. Such a

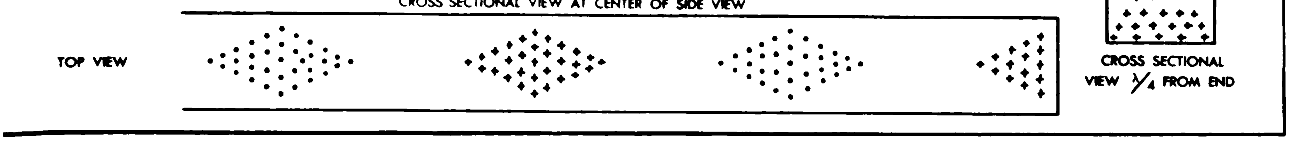

SIDE VIEW
WEAK H-FIELD
STRONG H-FIELD
λ/2
CROSS SECTIONAL VIEW AT CENTER OF SIDE VIEW
TOP VIEW
END VIEW
CROSS SECTIONAL VIEW λ/4 FROM END

Figure 11–14. Magnetic Field in Waveguide Three Half-Wavelengths Long

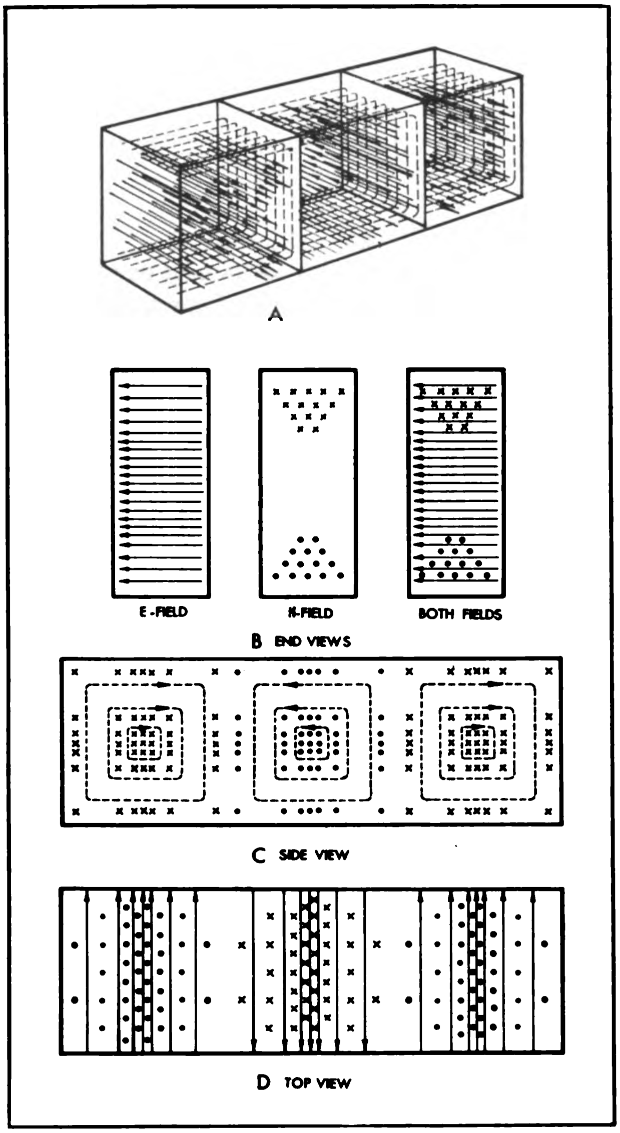


Figure 11–15. Conventional Picture of Both Fields in Waveguide

field configuration is called a *mode* of operation. In the case of the rectangular waveguide illustrated, the configuration is known as the *dominant* mode, since it is the easiest one to produce. Other higher modes — that is, different field configurations — may occur accidentally or may be caused deliberately in a waveguide.

An example of another field configuration is developed in Figure 11–16A. If the size of this waveguide is doubled over that of the waveguide shown in Figure 11–15, the cross section will be a full wave rather than a half-wave. The two-wire conductor can be assumed to be a quarter-wave down from the top (or a quarter-wave up from the bottom). The remaining distance to the bottom is three-quarters of a wavelength. A three-quarter-wave section has the same high impedance input as the quarter-wave section. Thus, the two-wire line is properly insulated and will transfer energy. The field configuration will show a full wave across the wide dimension, as you can see in Figure 11–16B.

This field configuration can be applied to a circular waveguide. The two conductors shown in Figure 11–16C are assumed to be part of the waveguide wall. The remaining part of the wall forms the quarter-wave sections. The quarter-wave section insulates the two conductors. This makes it possible to transfer energy with minimum losses. The resulting field configuration shown at D is the dominant mode for a waveguide with a circular cross section.

ANALOGY OF WAVEGUIDE ACTION BY ELECTRIC WAVES

A somewhat different analogy involving waveguide action deals with the field rather than with current and voltages. This analogy is somewhat more exact than the previous explanation, which dealt with voltages and current in two-wire lines, for power flow in this case is assumed to be in the fields rather than in the conductors.

In a waveguide the fields are the same as those radiated into space by an antenna. Figure 11–17 shows a small portion of a field that is radiated into space from an antenna. In it the electrostatic lines of force, or E-lines, are parallel to the antenna, and the electromagnetic lines of force, or H-lines, are perpendicular to the antenna. They move away from the antenna at the speed of light. At each half-cycle the polarity is reversed. Therefore, at half-wave intervals, the fields are in opposite directions (or polarity). Although only a small part of the total field is shown, the E-lines and H-lines form huge closed loops after they leave the antenna.

As previously mentioned, the energy which moves through a waveguide and the energy which is radiated by an antenna are both the same form of electromagnetic radiation. Nevertheless, the field configuration shown in Figure 11–18 cannot exist in a waveguide because it does not satisfy the required boundary conditions. First, there cannot be any E-lines tangent to the sur-

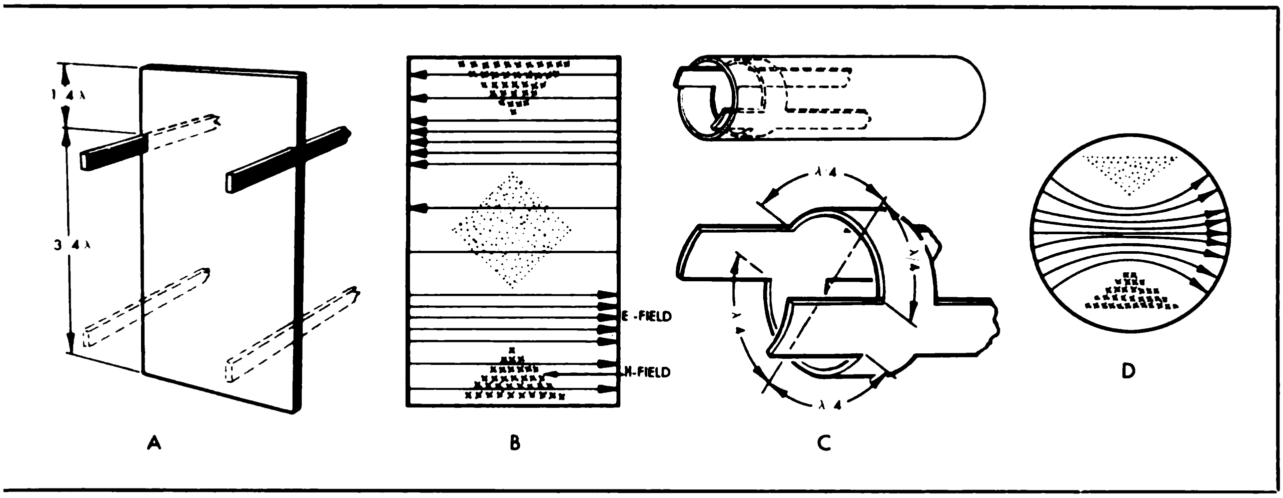


Figure 11–16. Another Field Configuration in the Rectangular Waveguide

face of the walls. Since the E-lines are evenly distributed across the area, some will be across the top and bottom wall. This causes the voltage to short out and this, in turn, causes the E-lines to be removed. Other E-lines which are pushed up to the wall by the repulsion between E-lines likewise short out. This shorting out is accumulative and eventually removes the entire E-field.

The second boundary condition which must be satisfied is that there must be no component of the magnetic field perpendicular to the wall. Note again in Figure 11–18 that the H-lines are parallel to the side walls, which is correct — but are perpendicular to the bottom, which cannot be; so H-lines of this type cannot exist in the waveguide. Furthermore, an H-line cannot exist unless it is a closed loop.

When a small antenna is placed in the waveguide and excited at a radio frequency, both positive and negative half-cycles are radiated as shown in Figure 11–19. The wavefront produced

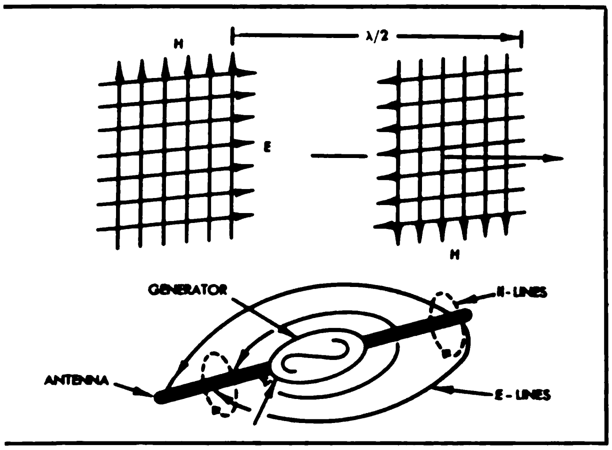


Figure 11–17. Small Portion of Field Radiated into Space by an Antenna

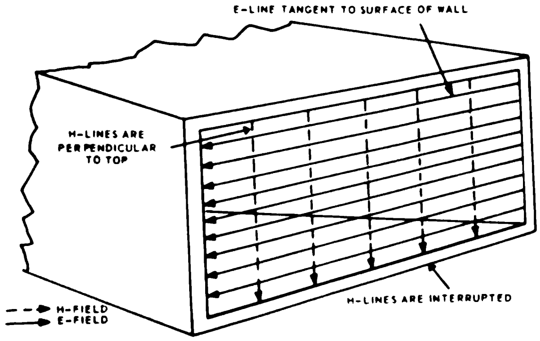

E-LINE TANGENT TO SURFACE OF WALL
H-LINES ARE PERPENDICULAR TO TOP
H-LINES ARE INTERRUPTED
H-FIELD
E-FIELD

Figure 11–18. Fields in a Waveguide Must Satisfy Boundary Conditions to be Radiated

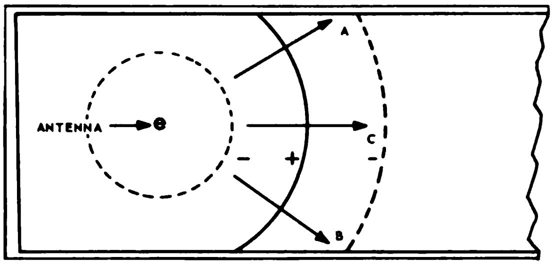


Figure 11–19. How Radiation Fields are Made to Fit a Pipe

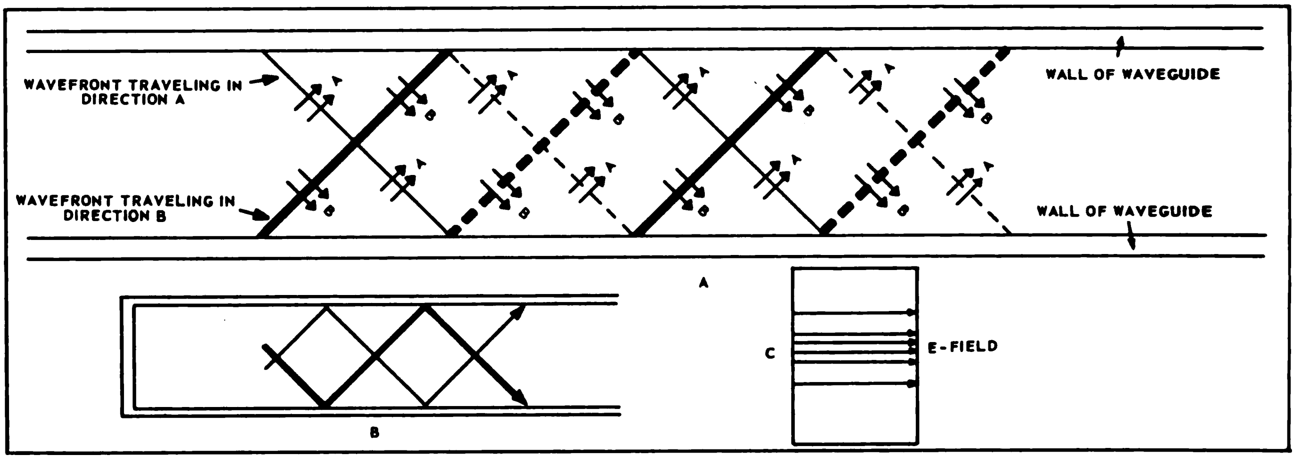

WAVEFRONT TRAVELING IN DIRECTION A
WAVEFRONT TRAVELING IN DIRECTION B
WALL OF WAVEGUIDE
WALL OF WAVEGUIDE
A
B
C
E-FIELD

Figure 11–20. Paths of Wavefronts in Waveguide

is like an expanding circle. The part which travels in the direction of arrow C goes straight down the waveguide and is quickly attenuated, as previously described. However, the part of the wavefront which travels in the direction of arrow A is reflected from the wall. The wall is a short circuit and causes the wavefront to be reflected in reverse phase. Meanwhile, the wavefront that travels in direction B is reflected from the other wall and proceeds in opposite phase. Thus, the radiation fields are contained in the waveguide as illustrated in Figure 11–20B.

Paths of Wavefronts in a Waveguide

Look through the top wall of the waveguide in Figure 11–20A, the light solid and broken lines represent the wavefront going in direction A in Figure 11–19. The heavy lines and dashes represent the wavefront going in direction B. Note that all parts of the wavefront A are traveling upward at an angle across the guide. Wavefront B is traveling at the same angle but downward.

When the wave travels in this fashion in a waveguide, propagation is possible. In understanding what happens, note that the positive wavefront (represented by solid lines) occurs simultaneously throughout the center of the guide. These fronts add and cause a maximum voltage to occur at the center. (The E-field is shown maximum at the center in Figure 11–20C.) The negative wavefront adds in the same manner as the positive wavefront. When the negative wavefront meets the positive wavefront at the walls, the two wavefronts cancel each other, making the total voltage equal to zero. This verifies the E-field condition shown in Figure 11–20C.

Crossing Angle

The angle at which a wavefront crosses a waveguide is a function of the wavelength and the cross-sectional dimension of the waveguide. At some intermediate frequency the reflection is as shown in Figure 11–21B. But as the frequency increases, the angle of incidence becomes less and the signal travels farther before it reaches the other side (see Figure 11–21A). At lower frequencies the wavefront crosses the guide at more nearly right angles to the walls. At some frequency the angle will be 90°. At this point the wave travels back and forth across the guide until the energy is dissipated by the resistance of the walls of the guide. At this frequency the distance from side to side is a half-wavelength for the waveguide. At the cutoff frequency the attenuation is a linear function of length and is very high.

The velocity of propagation of a wave along a two-wire line is less than its velocity in air. The same is true in a waveguide. Movement of a wave along a two-wire line is slower than its movement in air because of the retarding effect of the DC resistance of the conductors and the conductance of the insulation. In the waveguide the lower velocity is due to the way the field travels. As shown in Figure 11–21C, the path of a wavefront at a relatively low frequency is along the

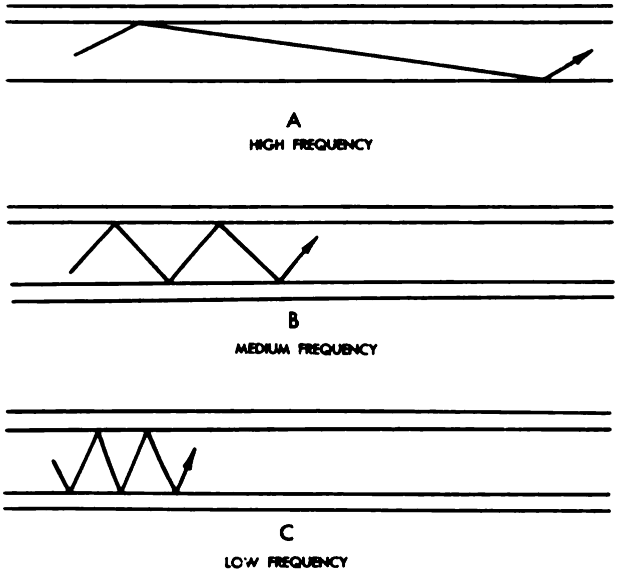


Figure 11–21. Angle at which Fields Cross Waveguide Varies with Frequency

zig-zag arrow at the velocity of light. But because of the long path, the wavefront actually travels very slowly along the waveguide. In Figure 11–21A the frequency is higher, and the wavefront or the group of waves actually travels a given distance in less time than those at C.

The axial velocity of a wavefront or a group of waves is called the *group velocity.* The relationship of the group velocity to diagonal velocity causes an unusual phenomenon. The velocity of propagation appears to be greater than the speed of light. As you can see in Figure 11–22, during a given time, a wavefront will move from point one to point two, or a distance L at the velocity of light (V_c). Due to this diagonal movement (direction of the arrow), the wavefront has actually moved down the guide only the distance G, which is necessarily a lower velocity. This is called group velocity (V_g).

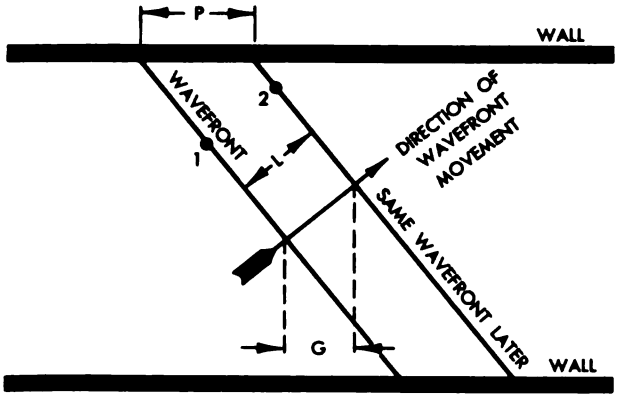


Figure 11–22. Relation of Phase, Group, and Wavefront Velocity

But if an instrument were used to detect the two positions at the wall, they would be the distance P apart. This is greater than the distance L or G. The movement of the contact point between the wave and the wall is at a greater velocity. Since the phase of the r-f has changed over the distance P, this velocity is called the phase velocity (V_p). The mathematical relationship between the three velocities is stated by the equation,

$$V_c = \sqrt{V_p V_g}$$

where

V_c = Velocity of light = 3×10^8 meters/second

V_p = Phase Velocity

V_g = Group Velocity.

This equation indicates that it is possible for the phase velocity to be greater than the velocity of light. As the frequency decreases, the angle of crossing is more of a right angle. In this condition the phase velocity increases. For measuring standing waves in a waveguide, it is the phase velocity which determines the distance between voltage maximum and minimum. For this reason, the wavelength measured in the guide will actually be greater than the wavelength in free space.

From a practical standpoint, the different velocities are related in the following manner: If the radio frequency being propagated is sine wave modulated, the modulation envelope will move forward through the waveguide at the group velocity, while the individual cycles of r-f energy will move forward through the modulation envelope at the phase velocity. If the modulation is a square wave, as in radar transmissions, again the square wave will travel at group velocity, while the r-f waveshape will move forward within the envelope.

Since the standing wave measuring equipment is affected by each r-f cycle, the wavelength will be governed by the rapid movement of the changes in r-f voltage. Since intelligence is conveyed by the modulation, the transfer of intelligence through the waveguide will be slower than the speed of light, as is the case in other types of r-f lines.

Because of the way the fields are assumed to move across the waveguide, it is possible to establish a number of trigonometric relationships between certain factors. As shown in Figure 11–23, the angle that the wavefront makes with the wall (angle θ) is related to the wavelength and dimension of the guide and is,

$$\text{Cos}\,\theta = \frac{\lambda}{2B}$$

where λ is the wavelength in free space of the signal in the guide, and B is the inside wide dimension of the guide. The group velocity (V_g) is related to the velocity of light (V_c) as follows:

$$\frac{V_g}{V_c} = \sin\theta = \sqrt{1 - \left(\frac{\lambda}{2B}\right)^2}.$$

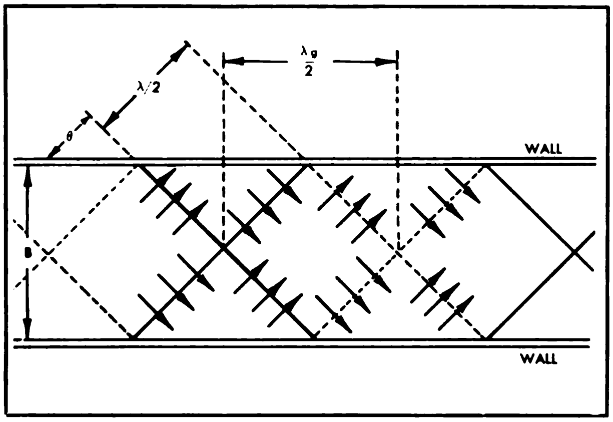


Figure 11–23. Trigonometric Relations Exist between Factors Indicated

Further, since it is possible to measure the wavelength in the guide (λg), the wavelength in space is equal to,

$$\frac{\lambda g}{\lambda} = \frac{1}{\sin\theta} = \frac{1}{\sqrt{1 - \left(\frac{\lambda}{2B}\right)^2}}.$$

Solving this for λ, the equation becomes,

$$\lambda = \frac{2B\lambda g}{\sqrt{\lambda g^2 + 4B^2}}$$

After measuring the wavelength and the inside dimension of the waveguide, it is possible to calculate most other quantities associated with the waveguide (Figure 11–23).

Numbering System of the Modes

The normal configuration of the electromagnetic field within a waveguide is called the *dominant* mode of operation. The mode developed for the rectangular waveguide, as was explained before, is the dominant mode of operation. The dominant mode for the circular waveguide was also shown in Figure 11–16. A wide variety of higher modes is possible in either type of waveguide. The higher modes in the rectangular waveguide are seldom used in radar, but some of the higher modes in the circular waveguide are useful.

For ease in identifying modes, any field configuration can be classified as either a transverse electric mode or a transverse magnetic mode. These modes are abbreviated TE or TM, respectively.

In a transverse electric mode, all parts of the electric field are perpendicular to the length of the guide and no E-line is parallel to the direction of propagation.

In a transverse magnetic mode the plane of the H-field is perpendicular to the length of the waveguide. No H-line is parallel to direction of propagation.

It is interesting to note from these definitions that the wavefront in free space or in a coaxial line is a TEM mode, since both fields are perpendicular to the direction of propagation. This mode cannot exist in a waveguide.

In addition to the letters TE or TM, subscript numbers are used to complete the description of

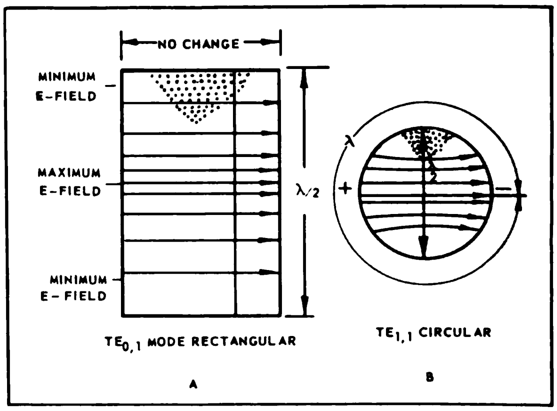


Figure 11–24. How to Count Wavelengths for Numbering Modes

the field pattern. In describing field configurations in rectangular guides, the first small number indicates the number of half-wave patterns of the transverse lines that exist along the short dimension of the guide through the center of the cross section. The second small number indicates the number of transverse half-wave patterns that exist along the long dimension of the guide through the center of the cross section. For circular waveguides the first number indicates the number of full waves of the transverse field encountered around the circumference of the guide. The second number indicates the number of half-wave patterns that exist across the diameter.

Counting Wavelengths for Measuring Modes

In the rectangular mode illustrated in Figure 11–24A, note that all the electric lines are perpendicular to the direction of movement. This makes it a TE mode. In the direction across the narrow dimension of the guide parallel to the E-line, the intensity change is zero. Across the guide along the wide dimension, the E-field varies from zero at the top through maximum at the center to zero on the bottom. Since this is one-half wave, the second subscript is one. Thus, the complete description of this mode is $TE_{0,1}$.

In the circular waveguide in Figure 11–24B, the E-field is transverse and the letters which describe it are TE. Moving around the circumference starting at the top, the fields go from zero, through maximum positive (tail of arrows), through zero, through maximum negative (head of arrows), to zero. This is one full wave, so the number is one. Going through the diameter, the start is from zero at the top wall, through maximum in the center to zero at the bottom, one-half wave. The second subscript is one. The complete designation for the circular mode becomes $TE_{1,1}$.

Several circular and rectangular modes are possible. On each diagram illustrated in Figure 11–25 you can verify the numbering system. Note that the magnetic and electric fields are maximum in intensity in the same area. This indicates that the current and voltage are in phase. This is the condition which exists when there are no reflections to cause standing waves. In previous examples in which fields were developed, the fields were out of phase because of a short circuit at the end of the two-wire line.

INTRODUCING FIELDS INTO A WAVEGUIDE

A waveguide, as was explained before, is a single conductor. Therefore, it does not have the two connections which ordinary r-f lines have, and it is necessary to use special devices to put energy into a waveguide at one end and to remove it from the other. In a waveguide, as with many other electrical networks, reciprocity

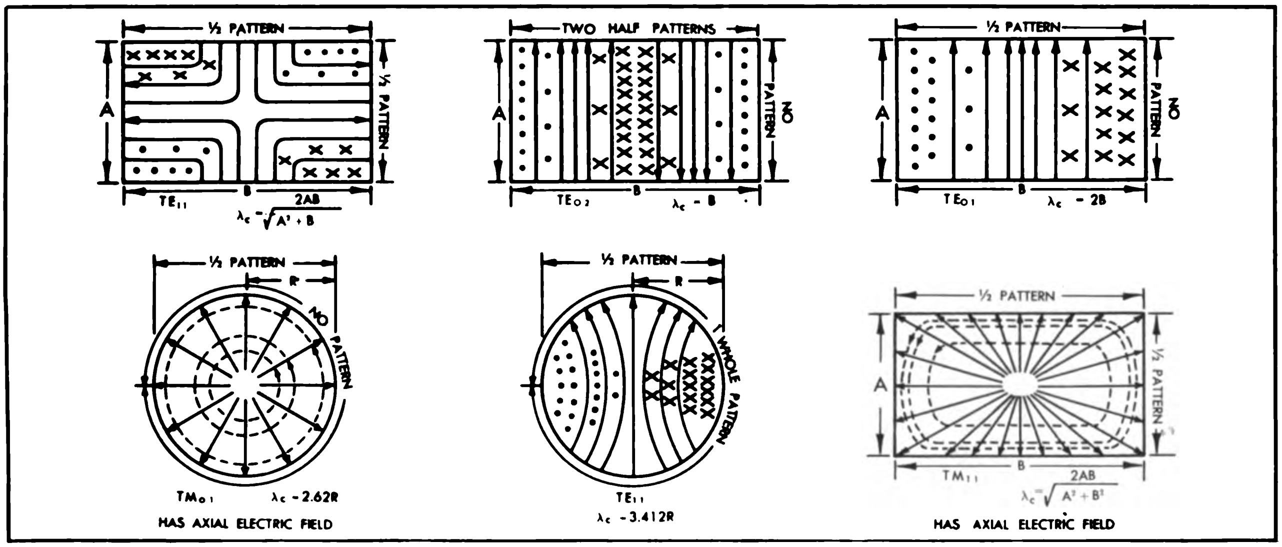


Figure 11–25. Various Modes in Waveguides

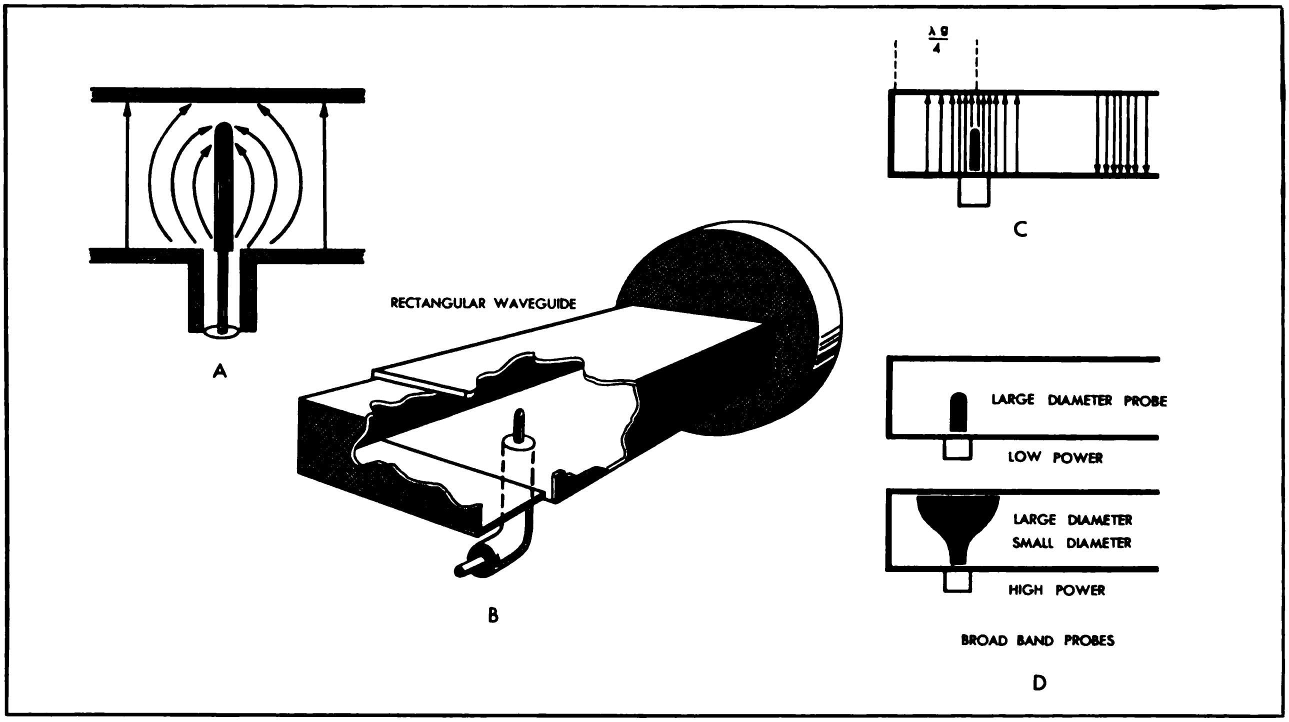


Figure 11–26. Exciting the Waveguide with Electric Field

exists in an excitation system — that is, energy may be transferred either to the waveguide or from the waveguide with the same efficiency.

Waveguides may be excited by three principal methods, namely, electric fields, magnetic fields, and electromagnetic fields.

Exciting with Electric Fields

When a small probe or antenna is placed in a waveguide and fed with an r-f signal, current will flow in the probe and set up an electrostatic field such as shown in Figure 11–26A. This causes the E-lines to detach themselves from the probe and to form in the waveguide. When the probe is located in the right place, a field having considerable intensity will be set up. The best place to locate the probe is in the center, parallel to the narrow dimension and one-quarter-wavelength away from the shorted end of the guide, as shown in Figure 11–26C.

Note here that the field is strongest at the quarter-wave point. This is the point of maximum coupling between the probe and the field. Of course the probe will work equally well in the center of any unidirectional field. For example, a three-quarter-wave distance from the shorted end will also be a good spot to place the probe. Usually, the probe is fed with a coaxial cable. In comparison with the waveguide, this cable is extremely short, which insures that the greatest benefit will be derived from the waveguide.

The degree of excitation can be reduced by reducing the length of the probe, by moving it out of the center of the E-field, or by shielding it. Where it is necessary to vary the degree of excitation frequently, the probe is made retractable and the end of the waveguide fitted with a movable plunger. In airborne radar systems the position of the probe and the end piece is often predetermined by the factory and fixed permanently.

In pulse-modulated radar systems there are wide sidebands on each side of the carrier. In order that a probe feeding system does not discriminate too sharply against frequencies that differ from the carrier frequencies, *wide-band* probes are often used. This probe is large in diameter and is conical or door knob in shape. A conical probe is capable of handling high-powered signals.

The same kind of probe is used to take energy out of the guide and deliver it to the coaxial cable.

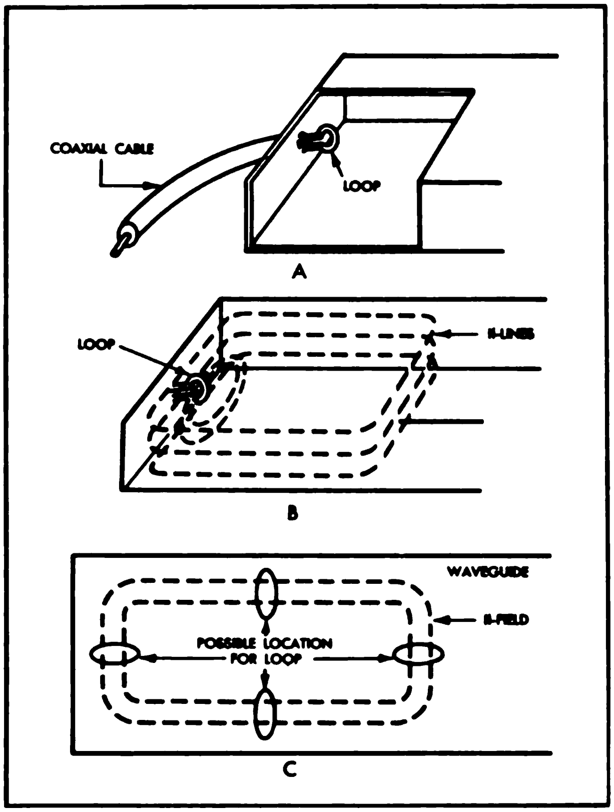


Figure 11–27. Excitation with Magnetic Fields

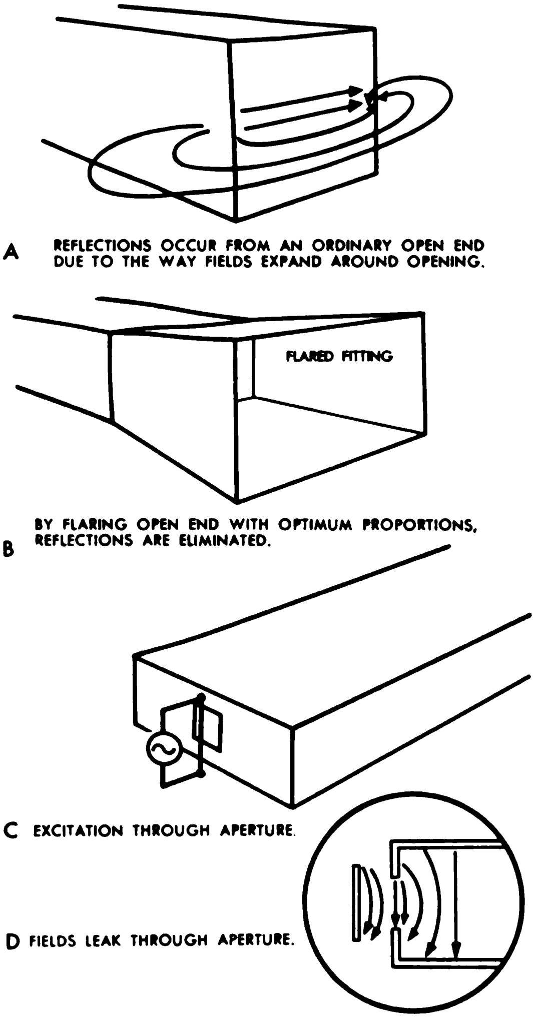
Excitation with a Magnetic Field

Another way of exciting a waveguide is by setting up a magnetic (H) field in the waveguide. This can be accomplished by placing a small loop which carries a high current in the waveguide. A magnetic field builds up around the loop. The field expands and fits the guide. If the frequency of the current is correct, energy will be transferred from the loop to the waveguide. A loop for transferring energy into a guide is shown at A and B in Figure 11–27. Notice that the loop is fed by a coaxial cable. The location of the loop for optimum coupling to the guide is at the place where the magnetic field that is to be set up will be of greatest strength. There are a series of places where this is true. Several are shown in Figure 11–27C.

When less coupling is desired, you can rotate or move the loop until it encircles a smaller number of lines of force.

When an excitation loop is used in radar equipment, its proper location is often predetermined and fixed during either construction or final tuning at the factory. In test or laboratory equipment the loop is often made adjustable.

When a loop is introduced in a guide in which an H-field is present, a current will be induced in the loop itself. When this condition exists, the loop will take energy out of the waveguide as well as put energy into it.



A REFLECTIONS OCCUR FROM AN ORDINARY OPEN END DUE TO THE WAY FIELDS EXPAND AROUND OPENING.

B BY FLARING OPEN END WITH OPTIMUM PROPORTIONS, REFLECTIONS ARE ELIMINATED.

C EXCITATION THROUGH APERTURE.

D FIELDS LEAK THROUGH APERTURE.

Figure 11–28. Excitation with Electromagnetic Fields

Excitation with Electromagnetic Fields

After learning what fields are like, you might think that a good way to either excite the waveguide or to let energy out of it is simply to leave the end open. But this is not the case, for when energy leaves a guide, fields form around the end of the guide and cause an impedance mismatch as shown in Figure 11–28A. In other words, reflections and standing waves would result if the end were left open. Thus, simply leaving the end open is not an efficient way of letting energy out of the waveguide.

In order for energy to move smoothly in or out of a guide, the opening of the guide may be flared like a funnel as shown in Figure 11–28B. This makes the guide similar to a V-type antenna. The *funnel*, in effect, eliminates reflection by matching the impedance of free space to the impedance of the waveguide. When the mouth of the funnel is exposed to electromagnetic fields, they enter the funnel where they are gradually shaped to fit the waveguide. The funnel is directional in characteristic. It sends or receives the greatest amount of energy from in front of the opening.

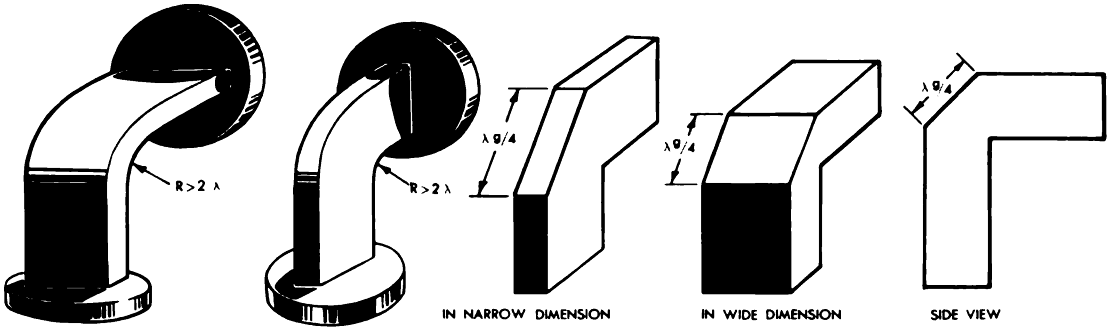
Another method for either putting energy into or removing it from waveguides is through slots or openings. This method is sometimes used when very loose coupling is desired. In this method energy enters the guide through a small aperture, as you can see in Figure 11–28C. Any device which will generate an E-field may be placed near the aperture and the E-field will expand into the waveguide. A single wire is shown in Figure 11–28D. On it E-lines are set up parallel to the wire due to the voltage difference between different parts of the wire. The E-lines, in expanding, will exist first across the aperture, then across the interior of the waveguide. If the frequency is correct and the size of the aperture properly proportioned, energy will be transferred to the waveguide with a minimum of reflection.

BENDS, TWISTS, JOINTS, AND TERMINATIONS

For energy to move from one end of a waveguide to the other without reflections, the size, shape, and dielectric material of the waveguide must be constant throughout its entire length. Any abrupt change in its size or shape results in reflections. Therefore, if no reflections are desired, any change in the direction or the size of the waveguide must be gradual. When it is necessary that the change in direction or size be abrupt, then special devices, such as bends, twists, joints, or terminations, must be used.

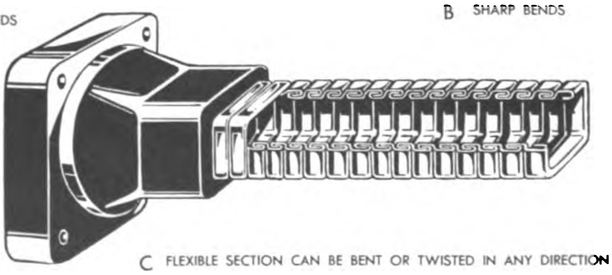
Bends

Waveguides may be bent in several ways to avoid reflections (Figure 11–29). One is to make the bend gradual. It must have a radius of bend greater than two wavelengths in order to minimize any reflection. Some bends may be 90°.



A GRADUAL BENDS

B SHARP BENDS



C FLEXIBLE SECTION CAN BE BENT OR TWISTED IN ANY DIRECTION

Figure 11–29. Types of Bends

Other bends may be greater or less than 90°, depending upon the requirements of the system. Still another type of bend is the sharp bend.

A bend can be made in either the narrow or the wide dimension of a guide without changing the mode of operation. In a sharp 90° bend, reflections will normally occur. To avoid this, the guide has two 45° bends, one quarter-wave apart. The direct reflection at one bend and the inverted reflection from the other bend will cancel and leave the fields as though no reflection had occurred.

To permit using any special bend which an installation might require, sections of a waveguide are often made flexible (Figure 11–29C). These sections can be bent or twisted in any desired direction. They consist of a spiral wound ribbon of brass. In cross section the winding is exactly the same size as a waveguide. The entire assembly is like a spiral spring in that it can be bent or twisted into any desired shape. As skin effect keeps the current at the inner surface of the waveguide, the inside surfaces of the flexible section are chromium-plated. This provides for maximum current conductivity. The outside of the section is covered with rubber. This gives the section flexibility and at the same time makes it both airtight and watertight.

Rotating the Field

Sometimes it is desired to rotate the electromagnetic fields so that they are in the proper direction for matching. This may be accomplished by twisting the waveguide as shown in Figure 11–30. The twist should be gradual and extend over two wavelengths or more to prevent excessive reflections. Flexible sections also are used to rotate fields.

Joints

Since it is impossible to mold an entire waveguide system in a radar set into one piece, it is necessary to construct it in sections and then to connect the sections together by joints. There are three main types of joints: these are the permanent, the semipermanent, and the rotating joints.

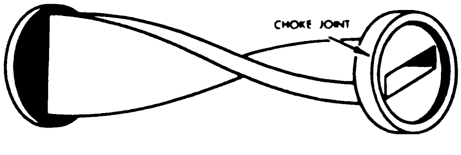


Figure 11–30. Twisted Section of Waveguide Rotates the Field with Minimum Reflection

Joining two waveguide sections together requires more than that the sections be the same size and fit tightly at the joint. Irregularities at the joints set up standing waves and allow energy to escape. The permanent joint affords a good connection between the parts of a waveguide and has very little effect on the fields. This joint is made at the factory. When it is used, the waveguide sections are machined within a few thousandths of an inch and then welded together. The result is a hermetically-sealed and mirror-smooth joint.

Where it is necessary that sections be taken apart for normal maintenance and repair it is impractical to use a permanent joint. To permit portions of the waveguide to be taken apart, they are commonly connected together with semipermanent joints. The choke joint is the most common type of semipermanent joint. A cross-sectional view of a choke joint is shown in Figure 11–31A. It consists of two flanges which are connected to the waveguide at the center. The right-hand flange is flat, and the one at the left is slotted a quarter-wave deep at a distance a quarter-wave from the point where the walls of the guide are joined. The quarter-wave slot is shorted at the end. The two quarter-waves together become a half-wave and reflect a short circuit at the place where the walls are joined together.

Electrically, this creates a short circuit at the junction of the two waveguides. The two sections actually can be separated as much as a tenth of a wavelength without excessive loss of energy at the joint. This separation allows room to seal the interior of the waveguide with a rubber gasket for pressurization. The quarter-wave distance from the walls to the slot is modified slightly to compensate for the slight reactance introduced by the short space and the open circuit from the slot to the periphery of the flange.

The name choke joint is said to come from the similarity between the action of this joint on r-f fields and the action of an r-f choke in a power supply lead. An r-f choke keeps r-f in the circuit where it belongs. Similarly, the choke joint keeps the electromagnetic fields in the waveguide where they belong. The loss introduced by the

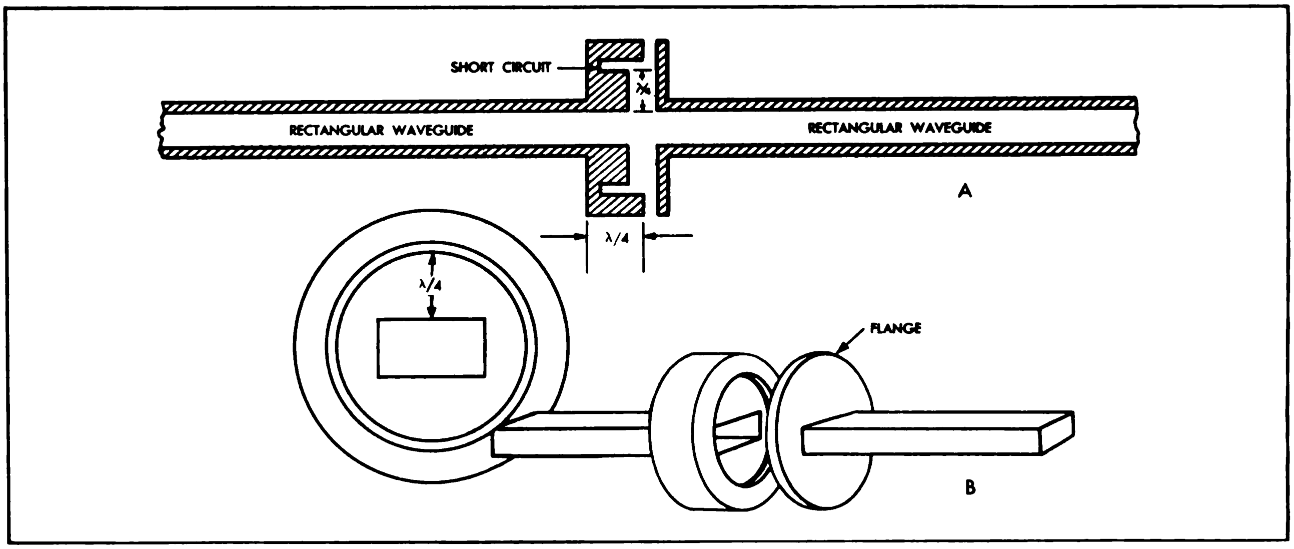


Figure 11–31. Choke Joint Keeps R-F Fields Inside Waveguide

well designed choke is less than .03 db, while an unsoldered permanent joint, well machined, has a loss of .05 db or more.

Rotating joints are usually required in a radar system where the transmitter is stationary and the antenna is rotatable. A simple method for rotating part of a waveguide system is by using a mode of operation that is symmetrical about the axis, as shown in Figure 11–32. This requirement is met by using a circular waveguide and a mode such as $TM_{0,1}$. In this method a choke joint may be used to separate the sections mechanically and to join them electrically. The waveguide rotates, but not the field. The continuous field produced by the fixed field minimizes reflections. This is basically the method used for all rotating joints. But since most radars use rectangular waveguide, the circular rotating joint must be inserted between two rectangular sections. A typical rotating joint is shown in Figure 11–33. The joint consists of two sections of circular guide, one fixed, the other rotating. At the ends of each section of circular guide is a transition from the circular to the rectangular waveguide.

Note that the rectangular guide is operating in the $TE_{0,1}$ mode. (The lines in the illustration of the guide represent the E-lines.) The E-lines

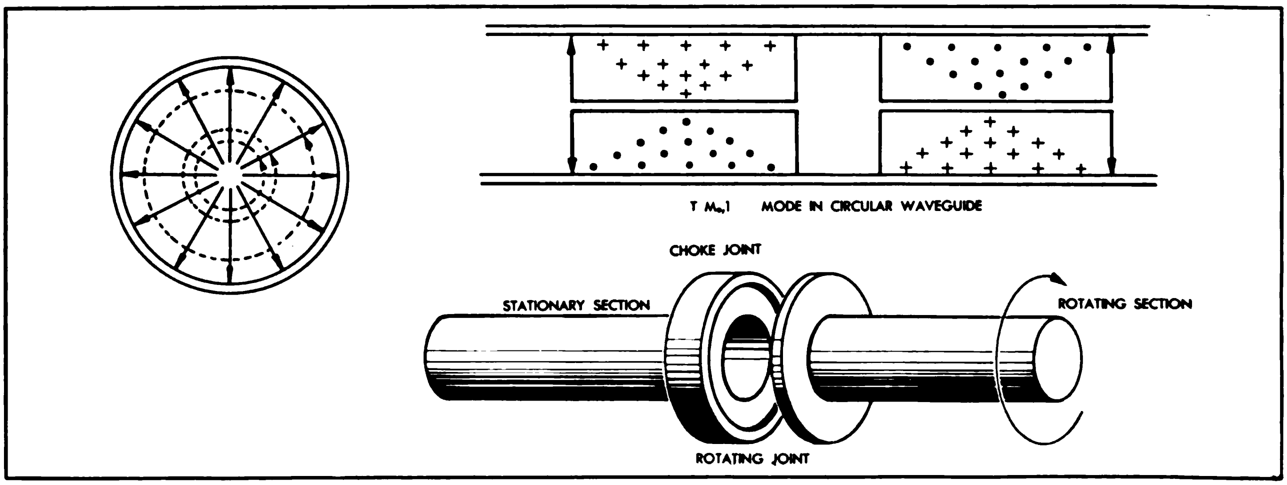


Figure 11–32. Rotating Joint and $TM_{0,1}$ Mode in Circular Waveguide

couple from the rectangular guide into the circular guide and excite the circular guide in the $TM_{0,1}$ mode. This is the mode that provides the required axial symmetry for rotating joints. At the top of the joint, the E-lines couple the energy into the rectangular guide that leads to the antenna, where the guide is operating in the $TE_{0,1}$ mode.

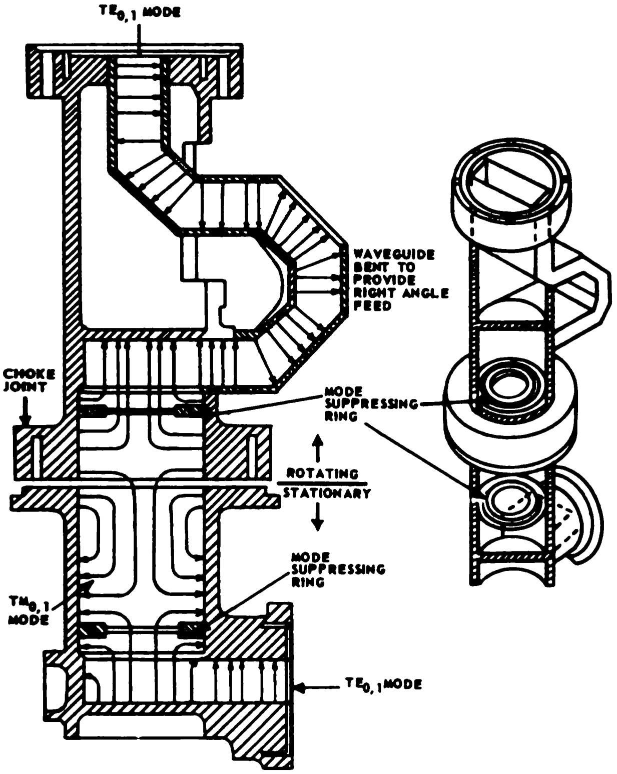


Figure 11–33. Rotating Joint with Rectangular Waveguide

Any circular guide that will carry the $TM_{0,1}$ mode can also be excited in the $TE_{1,1}$ mode. This mode is undesirable for a rotating joint. If you refer to Figure 11–25, you can see that the $TE_{1,1}$ mode is not symmetrical with respect to the center of the guide. Thus, this mode would produce changes in the standing-wave ratio at the joint during rotation. For this reason, mode-suppressing rings are provided in the circular guide to suppress the $TE_{1,1}$ mode. In addition, a teflon matching ring is used at each end of the joint to improve the impedance match between the circular and the rectangular waveguide sections.

T-Junctions

Sometimes it is desirable to connect a section of waveguide into the side of another waveguide. This type of connection forms a T-junction. It may be connected either in the narrow side, as shown in Figure 11–34A, or in the wide side of the waveguide, as shown in Figure 11–34D. When the T-junction is in the plane of an H-field of a $TE_{0,1}$ mode, it is called an *H-type* junction, and when the junction is in the plane of the E-lines, it is called an *E-type* junction.

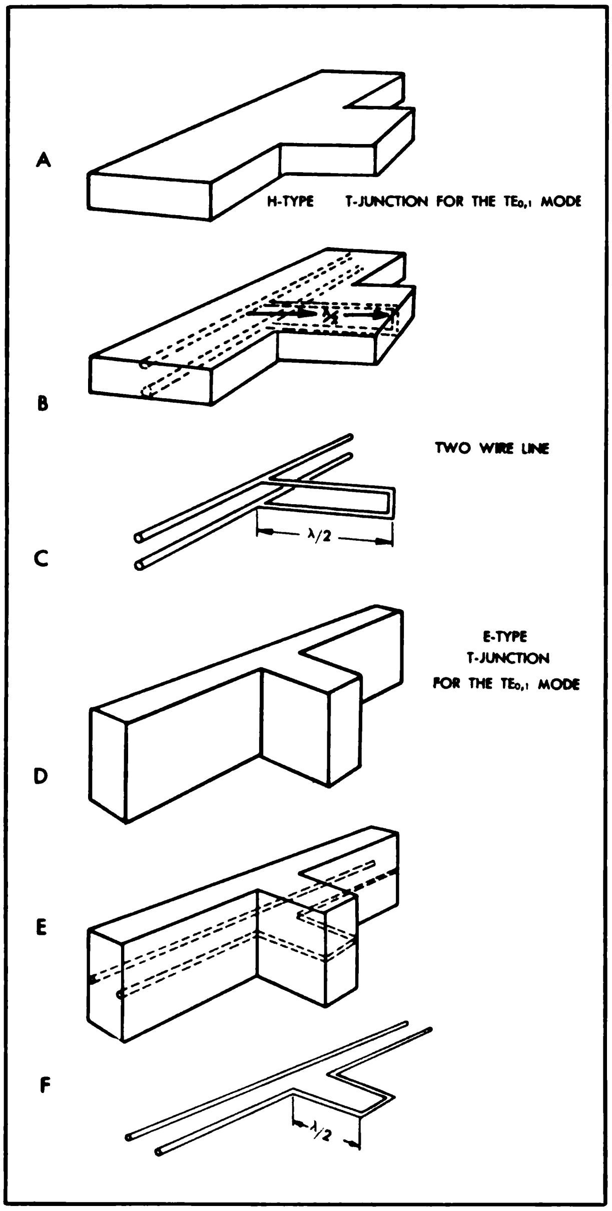

A
H-TYPE T-JUNCTION FOR THE $TE_{0,1}$ MODE
B
$\lambda/2$
TWO WIRE LINE
C
$\lambda/2$
E-TYPE T-JUNCTION FOR THE $TE_{0,1}$ MODE
D
E
F
$\lambda/2$

Figure 11–34. T-Junctions

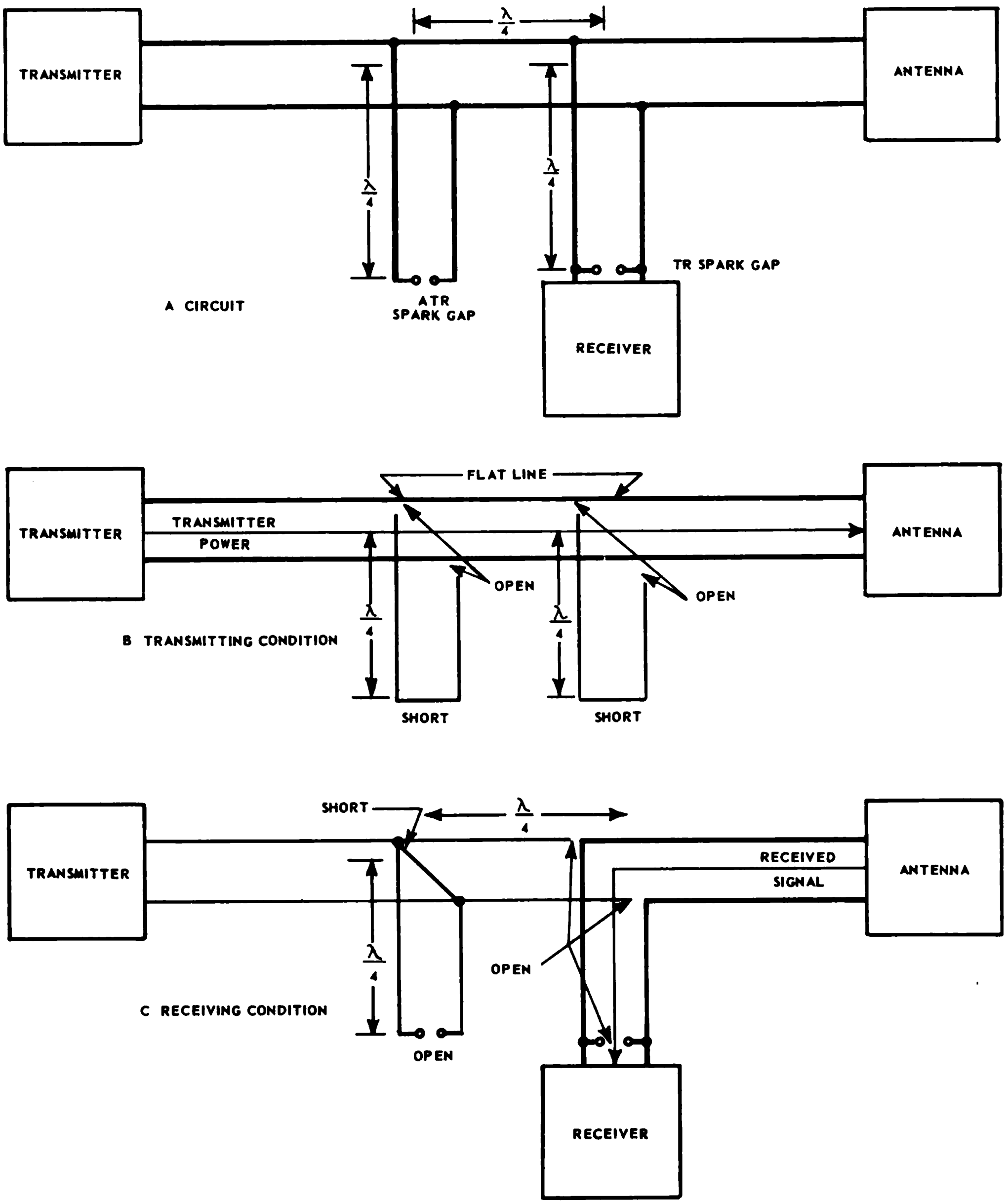


Figure 11–35. TR and ATR System

The H-type junction is effectively a parallel connection with the main line. For example, when the end of the T-joint shown in Figure 11–34B is short-circuited at a distance of a half-wave from the *center* of the waveguide, the result is the equivalent parallel circuit shown at C. This is a half-wave section that is connected to a wire line. This section will reflect a short circuit at the line and will not allow any energy to pass. Similarly, in the waveguide itself a short circuit is reflected to the center, where the E-lines are supposed to be. Since an E-line cannot exist at a short circuit, no energy will pass that point. If the shorted end of the T-section were only a quarter-wave from the center of the waveguide, an open section would be reflected there and the passage of energy would be unaffected.

The E-type joint effectively is a series connection with one side of the main line, as you can see in Figure 11–34E. Its two-wire counterpart is shown at F. In this case, if the section added to the waveguide is a half-wave long, it will act as a short at the junction, but will allow the energy to pass. The length of the added section will have to be a quarter-wave in order to open the circuit at the junction and to stop the ordinary flow of energy past it.

SIMULTANEOUS RECEPTION AND TRANSMISSION WITH COMMON ANTENNA

Two separate antennas and transmission lines are sometimes used for transmitting and receiving r-f signals, one for each function. By using the same transmission line and antenna for both functions, the equipment and installation of equipment can be simplified to a large degree.

By modifying the transmission line, simultaneous reception and transmission using one antenna system can be accomplished. An important point to consider when the transmitter and receiver both operate on the same frequency, is keeping the transmitted signal out of the receiver. The high transmitter power could damage the sensitive receiver to a point where it could not receive at all.

Parallel Duplexing Systems

In radar where the receiver and transmitter use the same frequency, a duplexer system effects single antenna operation. The duplexer system contains TR and ATR circuits. The purpose of the TR circuit is to prevent the transmitter power from reaching the receiver. The purpose of the ATR circuit is to disconnect the transmitter during the period that the antenna is receiving.

Figure 11–35A shows a transmission line drawn as a two-wire line including the duplexer components just mentioned. The transmission line may be either a coaxial line or a waveguide; all operate on the same principle.

In Figure 11–35 the receiver transmission line is in parallel with the transmitter transmission line. A spark gap is placed in the receiver line a quarter-wavelength from the junction. This is the TR spark gap. One quarter-wavelength from the receiver line junction, toward the transmitter, an additional quarter-wavelength line is placed in parallel with the transmitter. This line is also terminated in a spark gap (ATR spark gap).

During transmission both spark gaps fire (Figure 11–35B) and cause the TR and ATR circuits to act as shorted quarter-wave transmission lines. An open circuit, maximum impedance, is reflected at both junctions to the main transmission line. The transmitted power is conducted to the antenna without loss and none enters the receiver line.

During reception the amplitude of the received power is not sufficient to fire either gap. Under this condition the ATR circuit is now a quarter-wave transmission line terminated in an open. A short circuit is reflected back at the ATR junction with the transmission line, Figure 11–35C. This short circuit appears as an open circuit across the receiver line junction with the transmission line. The received signal, looking toward the transmitter, sees an open circuit and therefore enters the receiver circuit without loss.

Figure 11–36B illustrates the same circuit as Figure 11–35. The TR and ATR tubes are placed an odd number of quarter-wavelengths apart and a quarter-wavelength from the waveguide. When the transmitter fires, both tubes fire. At A and B in the waveguide sections, Figure 11–36B, an open circuit is reflected. This permits the transmitted power to be conducted to the antenna without loss. Since a short is across the TR tube, the transmitted power does not enter the receiver.

During reception neither the TR nor the ATR tube fires. The ATR section of waveguide is now an open quarter-wave stub, which has the same effect as a shorted half-wave section. Therefore, it reflects a short to point A. Since the distance between points A and B is a quarter-wave, the short at point A is reflected as a high impedance at point B. This effectively breaks the line from the receiver junction toward the transmitter, and prevents received energy from entering the transmitter and being dissipated. Therefore, the received signal passes into the receiver.

Series-Parallel Duplexer

The action of the series-parallel duplexer is similar to the parallel duplexer system with the exception that the ATR is placed in series with the transmitter while the TR is placed in parallel with the transmitter. Figure 11–37 illustrates the series-parallel duplexer.

The ATR circuit, explained under ATR Devices, is in series with the transmitter, and is placed one wavelength from the junction of the

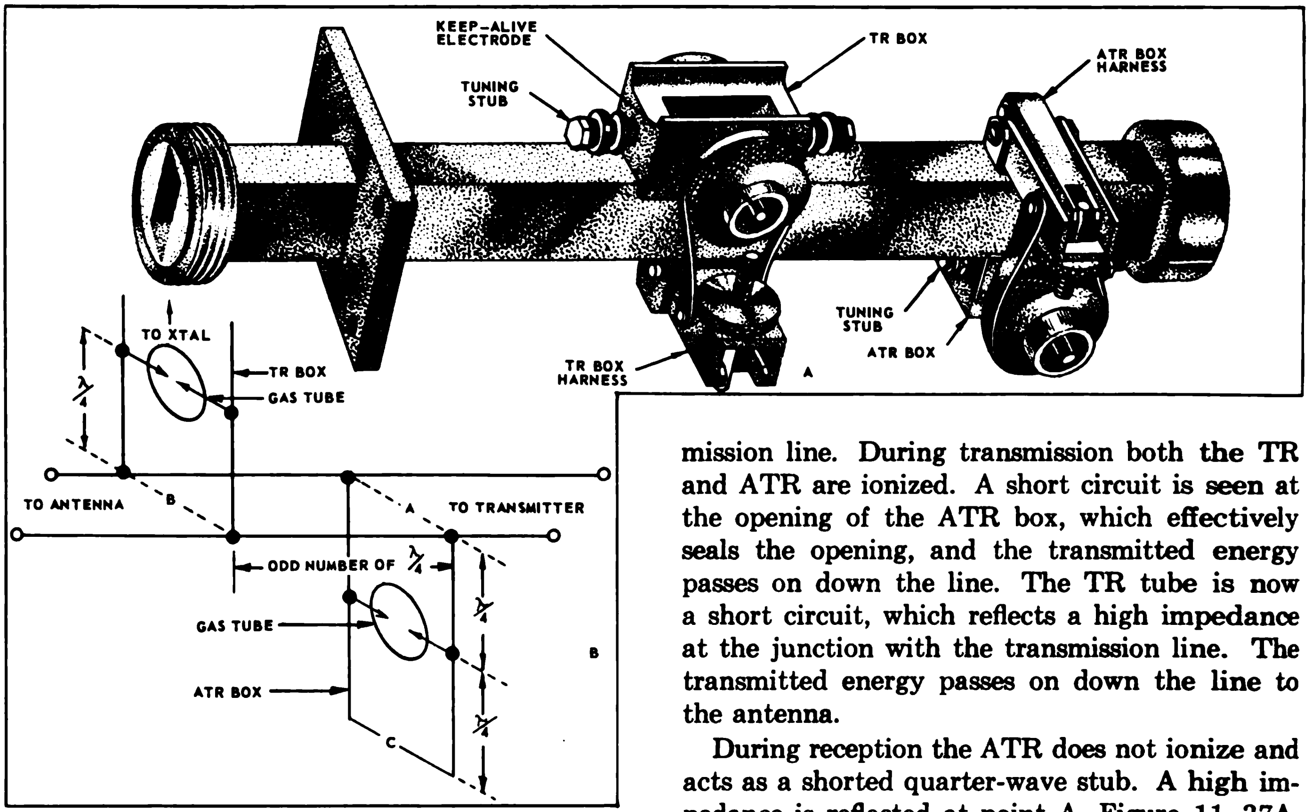


Figure 11–36. TR and ATR System in Waveguide

TR circuit. The TR tube, as before, is a quarter-wavelength from the junction with the transmission line. During transmission both the TR and ATR are ionized. A short circuit is seen at the opening of the ATR box, which effectively seals the opening, and the transmitted energy passes on down the line. The TR tube is now a short circuit, which reflects a high impedance at the junction with the transmission line. The transmitted energy passes on down the line to the antenna.

During reception the ATR does not ionize and acts as a shorted quarter-wave stub. A high impedance is reflected at point A, Figure 11–37A. As the distance from point A to point B is a full wave, a high impedance is reflected at point B. The received energy is prevented from entering the transmitter section and therefore passes into the receiver section.

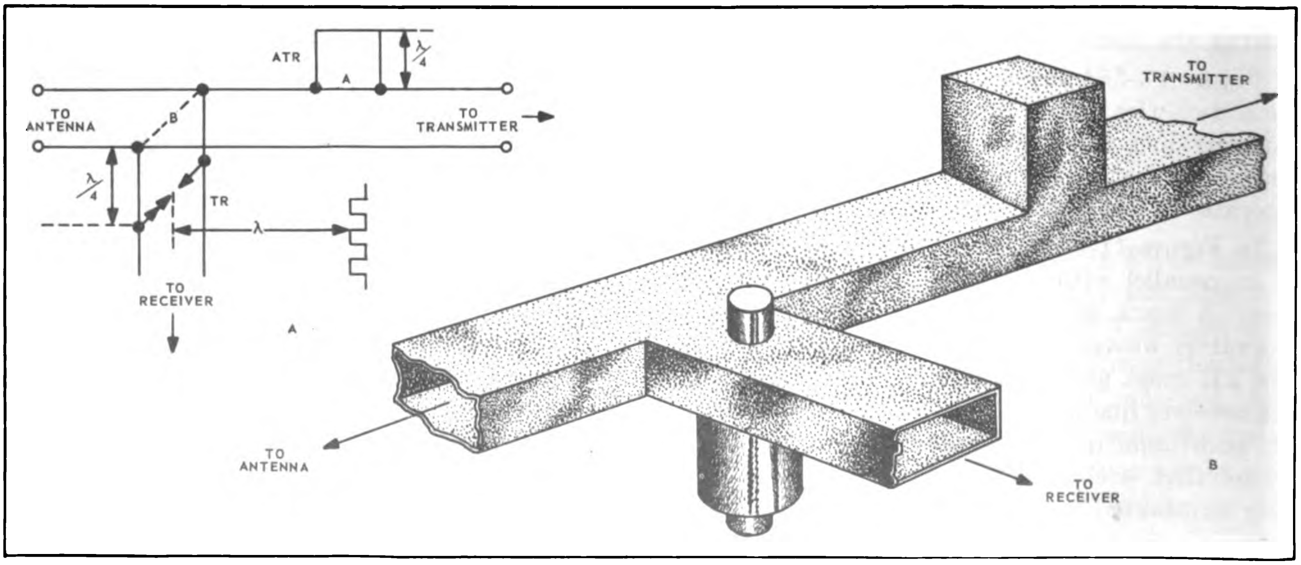


Figure 11–37. Series-Parallel Duplexer

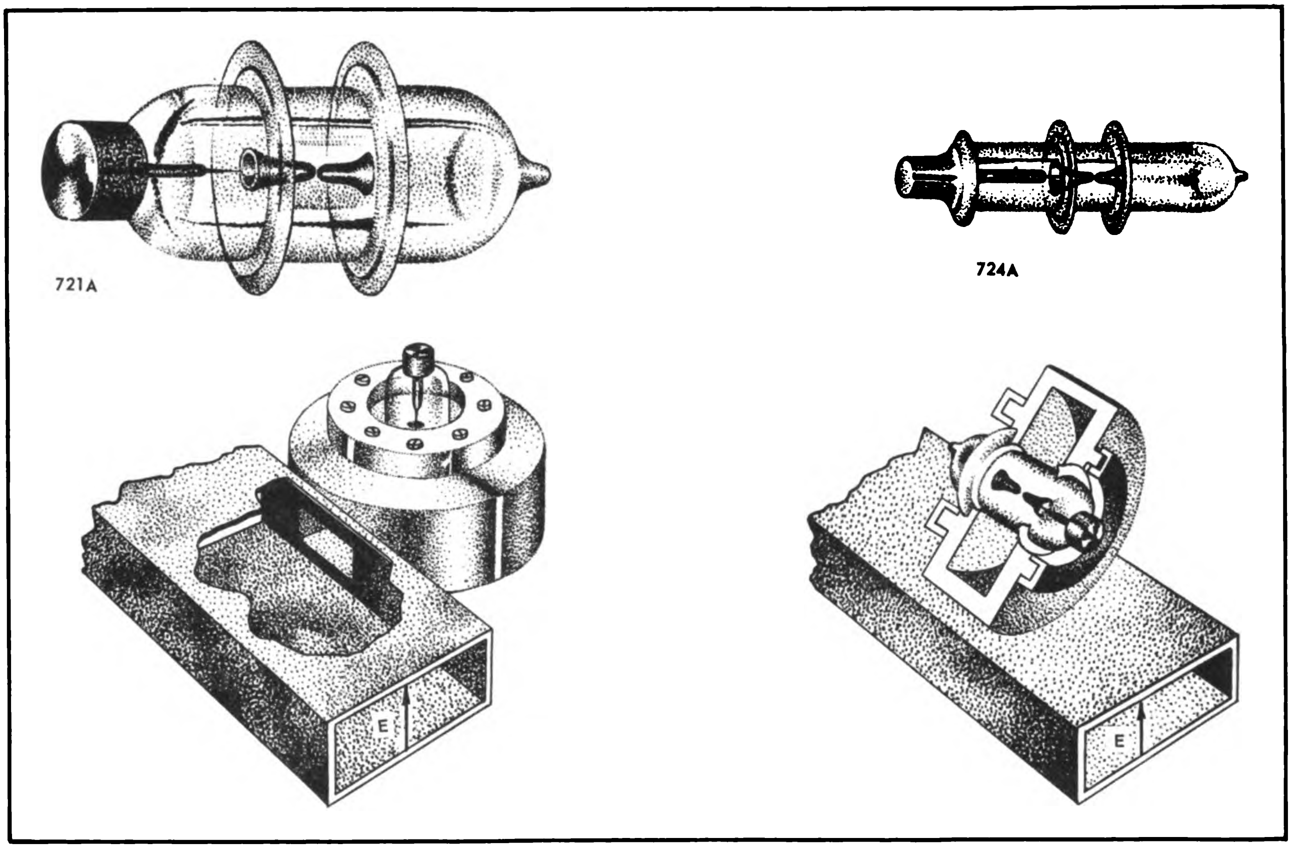


Figure 11–38. TR Tubes

TR Tubes

A spark gap is not ideal in its operation. It acts as a high impedance when not fired; when fired, it acts as a nonlinear resistance with a low voltage across itself. The characteristics of a spark gap change with use. To increase its efficiency, the spark gap is enclosed in a partially evacuated glass envelope and is connected to the transmission line through a resonant cavity. Figure 11–38 illustrates the TR tube and the connection to the waveguide.

The purpose of the TR tube is to prevent the transmitted energy from entering the receiver. If the transmitted energy entered the receiver, the crystal mixer of the receiver would be damaged severely or would be completely burned out. To protect the crystal mixer, the TR tube must fire, or ionize, immediately after the transmitter fires.

The majority of TR tubes are filled with a mixture of argon gas and water vapor. To aid them in ionization, a negative voltage is applied to an electrode that is placed near one of the gap electrodes. A TR tube showing the keep-alive electrode is illustrated in Figure 11–39.

The negative voltage maintains a steady glow discharge that provides a continuous supply of ions and free electrons. When r-f energy from the transmitter is applied to the electrodes, the

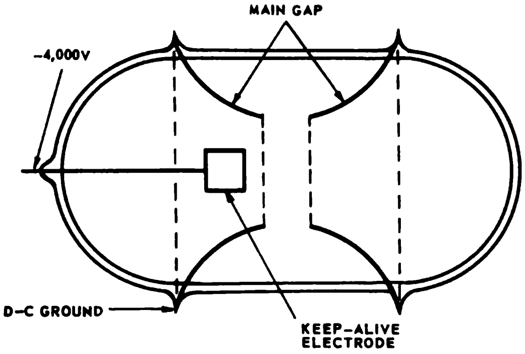


Figure 11–39. TR Tube Showing Keep-Alive Electrode

tubes fire more quickly. In this manner the keep-alive electrode acts as a cathode, while the back of one of the other electrodes acts as the anode. There are different requirements for different radars, but a typical example is that the TR must fire or ionize within .01 microsecond after application of the transmitted pulse. By the same token the tube must deionize 6 microseconds after termination of the transmitted pulse. This interval is called *recovery* time.

To prevent the crystals in the receiver from being damaged when the radar set is turned off, some radar sets are provided with a crystal shutter or gate. This normally consists of a metal vane across the waveguide, which is automatically positioned when the set is turned off.

ATR Devices

When an ATR tube is used in parallel with the transmitter, it is generally of the same construction as the TR tube, but it has no keep-alive voltage applied. When the ATR device is used in series with the transmitter, it is of a different construction.

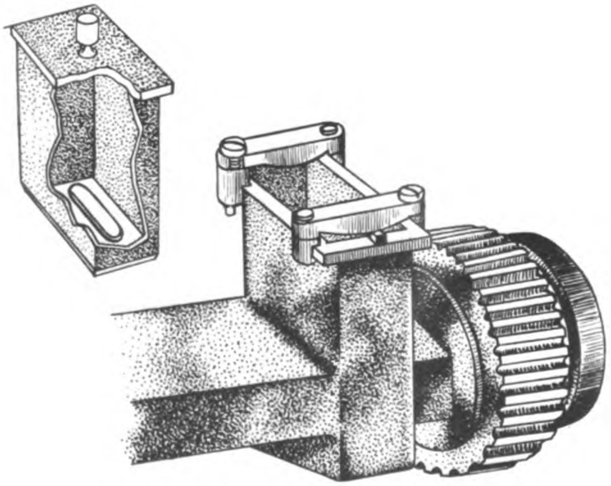


Figure 11–40. IB35 ATR Box and Mounting

Figure 11–40 illustrates the IB35 ATR box, which actually replaces a portion of the broad side of the waveguide. The ATR box consists of a cavity that is filled with argon gas and water vapor. During transmission the gas within the box ionizes and in effect completes the section of the waveguide that it replaces.

Polarization-Shifting Duplexer

One type of duplexer used on a number of radar equipments is termed a polarization-shifting duplexer (see Figure 11–41). In this particular type of duplexer the rectangular waveguide at the antenna end of the duplexer is oriented with its transverse dimensions 90° from those at the end of the duplexer nearest the transmitter. The receiver arm is oriented with its electric vector parallel to the electric vector in the waveguide through which the received signal arrives at the antenna.

Both transmitted and received signals are carried in rectangular waveguide and arrive at the duplexer with a $TE_{0,1}$ mode of transmission, that is, with the electric vector in the rectangular waveguide at all times parallel to the short dimension of the waveguide cross section. Energy being propagated toward the antenna on transmission has its polarization shifted 90° in passing through the duplexer and readily continues in the $TE_{0,1}$ mode through the waveguide to the antenna. It does not couple into the receiver arm, except to a limited degree, because the polarization of electric vectors in the transmitted wave and the receiver arm differ by 90°. The energy of the received wave is coupled into the receiver arm but cannot pass through to the transmitter because of the transverse orientation of the waveguide at that end and because of the ATR device.

The duplexer consists of a section of circular waveguide which is converted at each end to the rectangular section waveguide by transition sections. Within the duplexer (that is, the cylindrical section) 20 gas-filled quartz tubes are placed at regular intervals along the axis of the cylinder. Each is positioned on a diameter of the cylinder. Four of these tubes (the first four encountered from the transmitted signal input end) are all parallel to the long dimension of the rectangular section waveguide at that end. These are vertical in Figure 11–41. The fifth quartz tube is placed at an angle of 5.63° counterclockwise from the first four (viewed from the transmitter end). Each of the remaining 15 tubes is turned an additional 5.63° CCW, so that the last tube is at right angles to the first four.

The transmitted wave enters the duplexer transition in the $TE_{0,1}$ mode; the electric field is parallel to the short dimension of the waveguide. Since the duplexer is circular in cross section, the wave proceeds in the $TE_{1,1}$ mode, but the orientation of the transmitted energy is maintained, assisted by transverse shorting bars inside the transition section, near the end of the duplexer. Just beyond the branch arm for the received signal the transmitted energy strikes the first of the gas-filled quartz tubes. These tubes are filled with argon, generally at a pressure of about 20 mm. The high-power r-f energy ionizes the argon molecules, breaking down the initial resistance of the tube and causing it to conduct. This forms a very low impedance path across the cylindrical duplexer, and the first four "in-line" tubes do effectively the same thing as the metal shorting rods. They tend to maintain the existing polarization of the transmitter wave.

The high-power wave now meets the fifth tube, which has an angular displacement of 5.63° from the first four. The direction of the E vectors is shifted by this amount, and the rotation will continue through the 16 tubes until the transmitted wave arrives at the twentieth tube. Polarization has been shifted through a full 90°, or is at right angles to that of the energy entering the duplexer. The net steady-state result of field interaction within the "fired" duplexer is a nearly uniform axial conducting surface twisting through 90°. The purpose of the matching slug

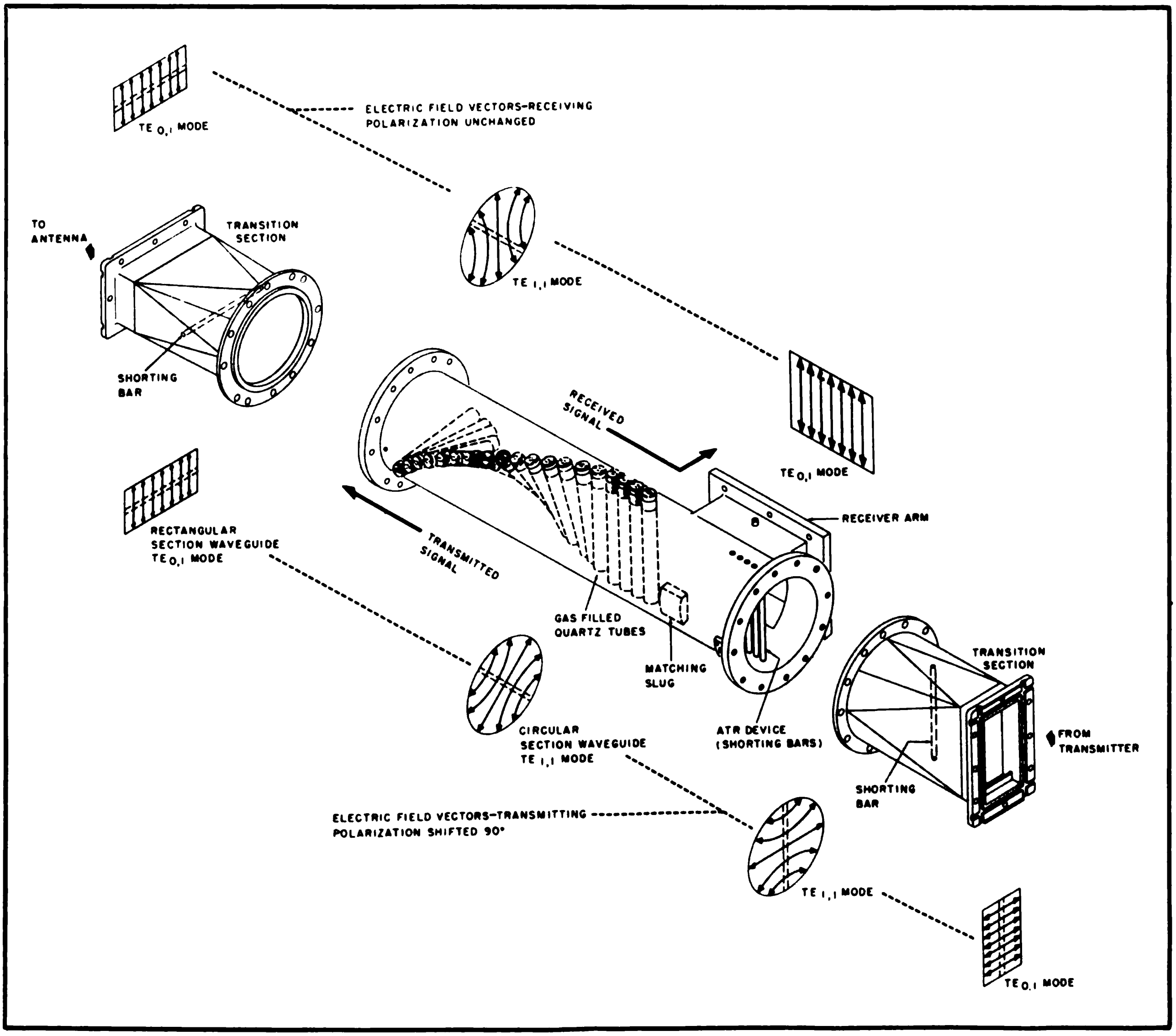


Figure 11–41. Polarization-Shifting Duplexer

on the side of the duplexer opposite the opening into the receiver arm is to compensate for the effect upon the transmitted wave of the discontinuity in the walls of the duplexer caused by the opening into the receiver arm.

The received signal from the antenna arrives at the duplexer with a polarization determined by the orientation of the rectangular waveguide through which it has passed. It is too weak to fire the gas-filled quartz tubes placed spirally across the duplexer. They do not function as in transmission and the received wave passes through with its polarization unchanged.

For two reasons it is coupled into the receiver arm. First, the orientation of the rectangular waveguide of the receiver arm is aligned with the polarization of the incoming wave, and the latter can therefore proceed into the receiver arm without serious attenuation. Second, the received wave is prevented from passing directly through the duplexer toward the transmitter by the five shorting bars across the duplexer just beyond the receiver arm. These serve to reflect the received signal and to set up standing waves with a current minimum at the receiver branch leading to the TR cavity. This provides maximum coupling of the received signal into the TR cavity. These shorting rods are, therefore, an ATR device. Figure 11–41 is of a typical polarization-shifting duplexer.

Matching Devices

Some devices which are used in radar introduce inductance or capacitance. Sometimes these reactances are deliberately introduced. When they are present and not desired, they can be tuned out with small fins or plates in a waveguide.

When you couple energy into or out of a waveguide, you should effect a proper impedance match between the generator, the coupling elements, the waveguide, and the load. In this respect, the waveguide acts exactly like a two-wire line and follows the same principles outlined previously. In other words, unless you terminate the waveguide in its characteristic impedance, standing waves are created.

You have used appropriate matching elements such as impedance-matching transformers and reactive elements to effect maximum transfer of energy. While the same principles are used in matching that were described earlier, the physical appearance of the matching elements is somewhat different in waveguides.

Figure 11–42 shows a number of reactive plates that are used deliberately to introduce capacitance or inductance in a waveguide. When these plates are used as shown in Figure 11–42A, they set up oscillations in the higher modes. Since a waveguide is too small for higher modes at the same frequency, these frequencies are not

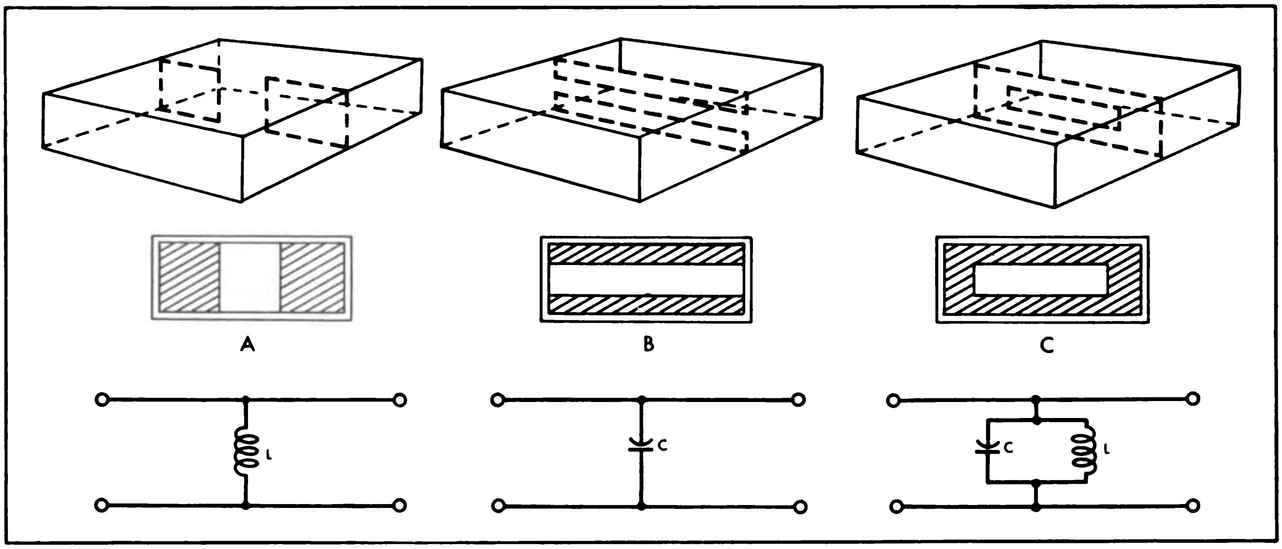


Figure 11–42. Reactive Plates in Waveguide

propagated, but remain in the vicinity of the plates. If the edges of the plates are vertical with respect to the plane of the H-field, the modes produced are the TM type. The effect of this on power flow is that of inductance across the two-wire line. This causes reflections and a shift in the standing wave pattern. The wider the space between the plates, the greater the inductive reactance.

When the partitions are arranged perpendicularly to the E-field as in Figure 11–42B, a local E-field and the higher modes of oscillation are set up between the edges of the plates. These oscillations cannot be propagated but do change the dominant mode to a TE mode and introduce capacitive reactance. As with the TM mode, the wider the opening, the greater the reactance.

From these facts it would seem that combining both types of plates and leaving a small opening in a large guide as in Figure 11–42C would produce a resonant circuit. This is approximately true, if the dimensions are correct. At resonance a resonant circuit acts as a high resistance. In this condition a small opening would introduce a high shunt resistance, and the guide would in effect have connected across it a resonant circuit.

Post or Tuning Screw. Inserting a metallic element which is either cylindrical in shape or in the form of a strip will produce a shunt reactance across a waveguide. When the element extends fully across the narrow part of the waveguide, it is called an inductive post (Figure 11–43A). When the post is in the form of a screw so that the insertion depth can be varied, as in Figure 11–43B, it is called a tuning screw.

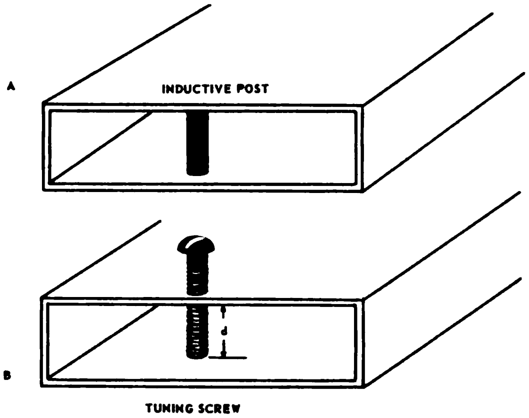


Figure 11–43. Waveguide Reactances

The reactance obtained by the tuning screw varies with the depth of insertion *d*. When *d* is small, the screw will act as a shunting capacitance. When *d* is increased, a point will be reached where *d* becomes equal to a quarter-wavelength. Resonance occurs at this point and the reactance is zero. If *d* is increased beyond this point, the screw becomes inductive. Since the short length of the guide is usually greater than a quarter-wavelength, the element is inductive when it extends fully across the waveguide. In this instance the element is referred to as an inductive post (Figure 11–43A).

Stub Reactance. Open or short-circuited stubs are also used to obtain reactances in waveguides. Shorted stubs are generally used to prevent undesired radiation of energy. By placing the stub as shown in Figure 11–44A, it acts as

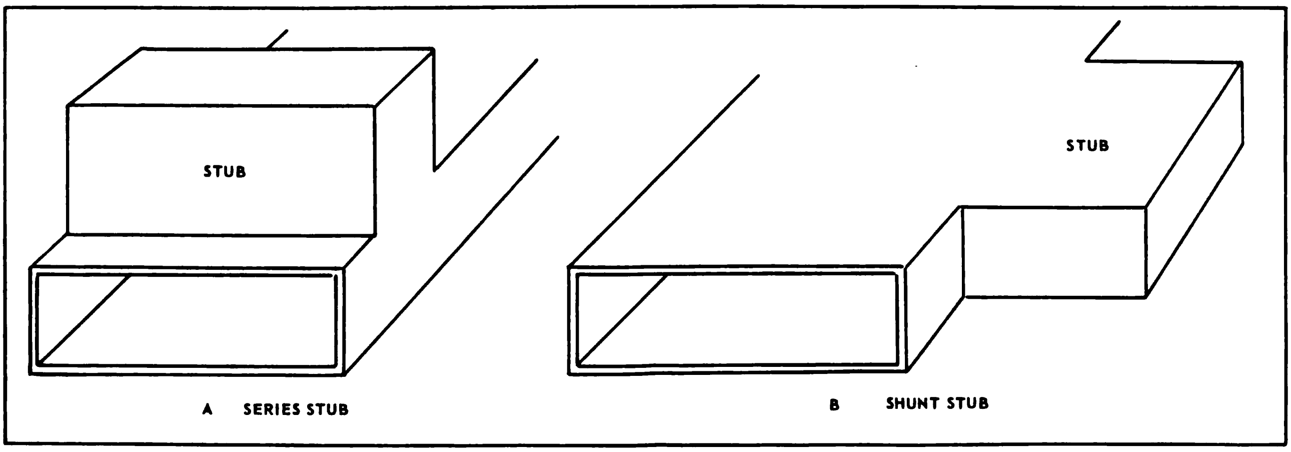


Figure 11–44. Waveguide Stubs

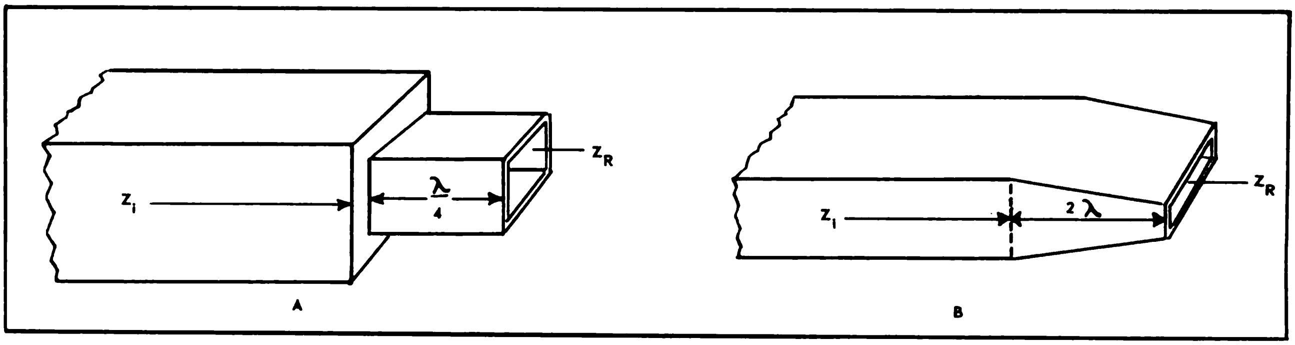


Figure 11–45. Waveguide Transformers

an impedance in series with the line and is therefore called a series stub. Figure 11–44B shows a stub acting as a shunt impedance across the waveguide. For this reason it is called a shunt stub.

IMPEDANCE TRANSFORMERS. You know that an impedance transformer is used to change one value of resistance to another in order to obtain maximum transfer of power. In a waveguide the transformer is placed at a point where the impedance is pure resistance but not equal to Z_0. Placing the transformer at this point transforms this value of resistance to Z_0 and thus matches the waveguide to the load. A *quarter-wave transformer* can be constructed by using a section of waveguide whose dimensions are either greater or smaller than those of the main waveguide. This is illustrated in Figure 11–45A.

Figure 11–45B illustrates another type of transformer, the *tapered-line* transformer. In this type of transformer the dimensions of the guide are varied very gradually, thus changing its characteristic impedance from that of the main line to that of the load. When this change in impedance is made very gradually, the impedance transformation is effective over a very wide band of frequencies. Note that the tapered section is longer than two wavelengths. In comparison, the quarter-wave transformer is effective only at certain frequencies, those at which the quarter-wave transformer is of the proper length.

Terminating a Waveguide

Since a waveguide is a single conductor, it is not as easy to define its characteristic impedance (Z_0) as it is for a coaxial line. Nevertheless, you can think of the characteristic impedance of a waveguide as being approximately equal to the ratio of the strength of the electric field to the strength of the magnetic field for energy traveling in one direction. This ratio is equivalent to the voltage-to-current ratio in coaxial lines on which there are no standing waves.

The lowest characteristic impedance of a circular waveguide is about 350 ohms. In a rectangular waveguide, Z_0 may be any value, depending on the dimensions of the waveguide and the frequency of the electrical energy. In the waveguide, Z_0 is directly proportional to the narrow dimension when the other dimension and the frequency are fixed and may vary from approximately zero to 465 ohms.

On a waveguide there is no place to connect a fixed resistor to terminate it in its characteristic impedance as there is on a coaxial cable. But there are a number of special arrangements that accomplish the same thing. One consists of filling the end of the waveguide with graphited-sand as shown in Figure 11–46A. As the fields enter the sand, currents flow in it. These currents create heat, which is instrumental in dissipating energy. None of the energy thus dissipated as heat is reflected back into the guide. Another arrangement, Figure 11–46B, uses a high resistance rod, which is placed at the center of the E-field. The E-field (voltage) causes current to flow through the rod. The high resistance of the rod dissipates the energy as an I^2R loss.

Still another method for terminating a waveguide is to use a wedge of high resistance material, Figure 11–46C. The plane of the wedge is placed perpendicular to the magnetic lines of force. When the H-lines cut the wedge, a voltage is induced in it. The current produced by the

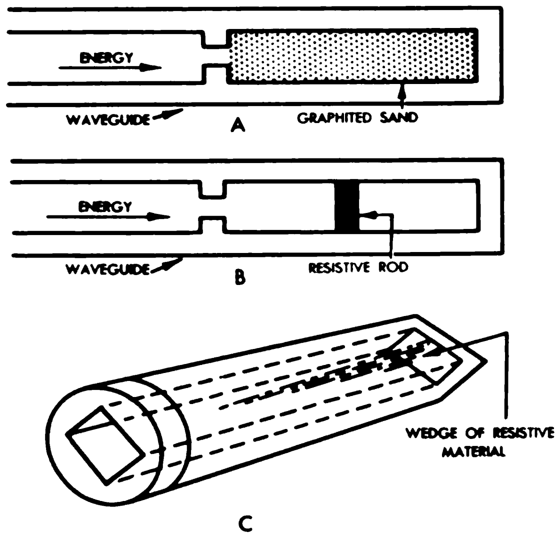


Figure 11–46. Termination for Minimum Reflections

induced voltage flowing through the high resistance of the wedge creates an I^2R loss. This loss is dissipated in the form of heat. This permits very little energy to reach the closed end to be reflected.

Each of the preceding terminations is designed to match the impedance of the guide in order to insure a minimum of reflection. On the other hand, there are many instances where it is desirable for all the energy to be reflected from the end of the waveguide. The best way to accomplish this is to permanently weld a metal plate at the end of the waveguide as shown in Figure 11–47A.

When it is necessary that the end be removable, a removable end plate is attached to the end of the guide. For this method to be satisfactory, the contact between the guide and the end plate must be exceptionally good in order that the H-field will not be attenuated when current flows. Perfect contact is not required when the connection is made at a point of minimum current. This point is located at a quarter-wave from the end. When connection is made at this point, a cup is used instead of the end plate, Figure 11–47B. This cup is a quarter-wave long and large enough to fit over the end of the guide. The voltage between opposite sides of the cup opening is high, but as the reflected H-field cancels the incident H-field, the resulting current is very small and reflection is at a minimum.

When the end must be adjustable, the contact must be nearly perfect. However, it is impossible to get a perfect contact. The best arrangement, one which is similar to the choke joint previously explained, consists principally of an adjustable plunger that fits into the guide as shown in Figure 11–47C. The walls of the waveguide and the plunger form a half-wave channel. The half-wave channel is closed at the end and reflects a short circuit across the other end, where a perfect connection is supposed to exist between the wall and the plunger. The actual physical contact is made at a quarter-wave distance from the short circuit, where the current is minimum due to the standing waves. This makes it possible for the plunger to slide loosely in the guide at a point where the contact resistance to current flow is very low.

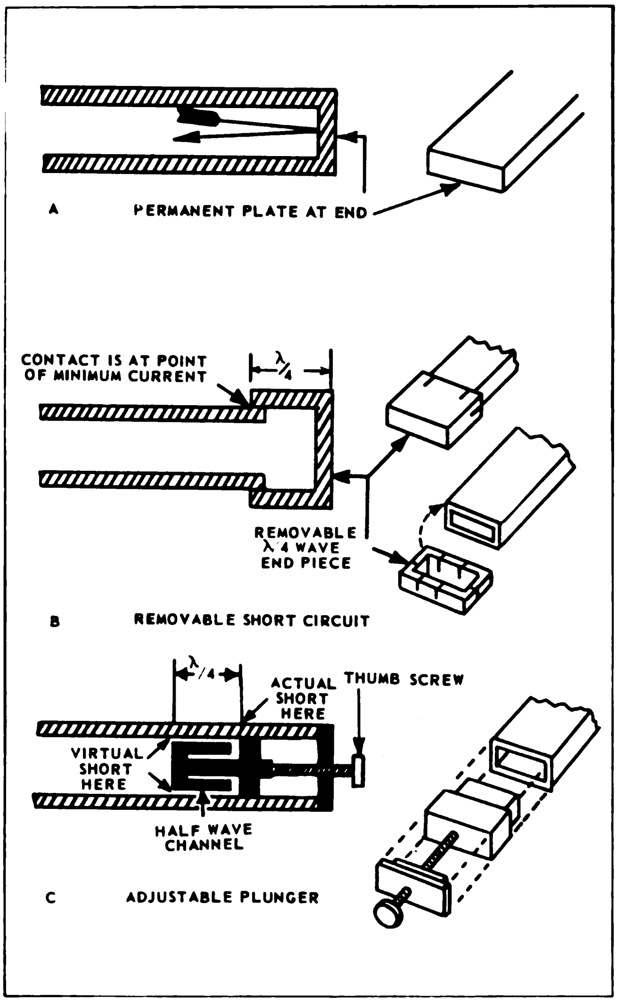


Figure 11–47. Termination for Maximum Reflection

CAVITY RESONATORS

In ordinary radio work the conventional low-frequency resonant circuit consists of a coil and capacitor that are connected either in series or in parallel as shown in Figure 11–48A. To increase the resonant frequency, it is necessary to decrease either the capacitance or the inductance, or both. However, a frequency is reached where the inductance is a half-turn coil and where the capacitance consists only of the stray capacitance in the coil. In this circuit the current handling capacity and breakdown voltage for the spacing would be low.

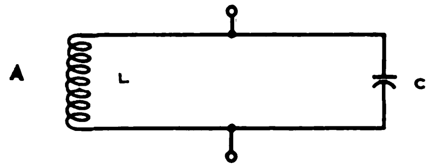
The current carrying ability of a resonant circuit may be increased by adding half-turn loops in parallel. This does not change the resonant frequency appreciably because it adds capacitance in parallel — which lowers the frequency, and inductance in parallel — which increases the frequency. As the effects of one cancel the effects of the other, the frequency remains about the same.

In Figure 11–48C several half-turn loops are added in parallel. D shows several parallel quarter-wave Lecher lines, which you recall are resonant when they are near a quarter-wavelength. When more and more loops are added in parallel, the assembly eventually becomes a closed resonant box as shown at E. This box, called a *resonant cavity*, is a quarter-wave in radius or a half-wave in diameter.

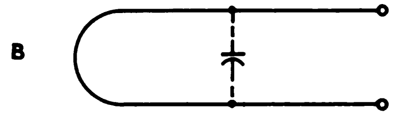
A resonant cavity displays the same resonant characteristics as a tuned circuit composed of a coil and capacitor. In it there are a large number of current paths. This means that the resistance of the box to current flow is very low and that the Q of the resonant circuit is very high. While it is difficult to attain a Q of several hundred in a coil of wire, it is fairly easy to construct a resonant cavity with a Q of many thousands. Although a cavity is as efficient at low as at high frequencies, the large size required at low frequencies prohibits its use.

For example, at one megacycle a resonant cavity would be a cylinder about 500 feet in diameter. When the frequency is in the vicinity of 10,000 megacycles, the diameter of the cavity is only 0.6 inch. This makes the cavity smaller than a conventional tuned circuit. Therefore, equipment which operates at a frequency of 3000 mc or above usually uses resonant cavities as resonant circuits.

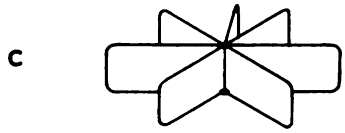
If there were no losses in the cavity, the waves would bounce back and forth between the end walls indefinitely. However, just as in any resonant circuit, some energy is lost during each cycle. Usually, a source of microwave energy feeds the cavity continuously so as to overcome the losses.



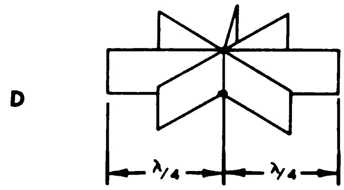
CONVENTIONAL LOW FREQUENCY CIRCUIT



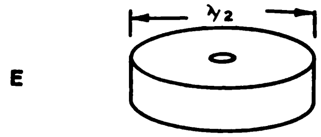
COIL–CAPACITOR COMBINATION REDUCED FOR HIGHEST RESONANT FREQUENCY



HALF TURN LOOP IN PARALLEL



QUARTER WAVE SECTIONS

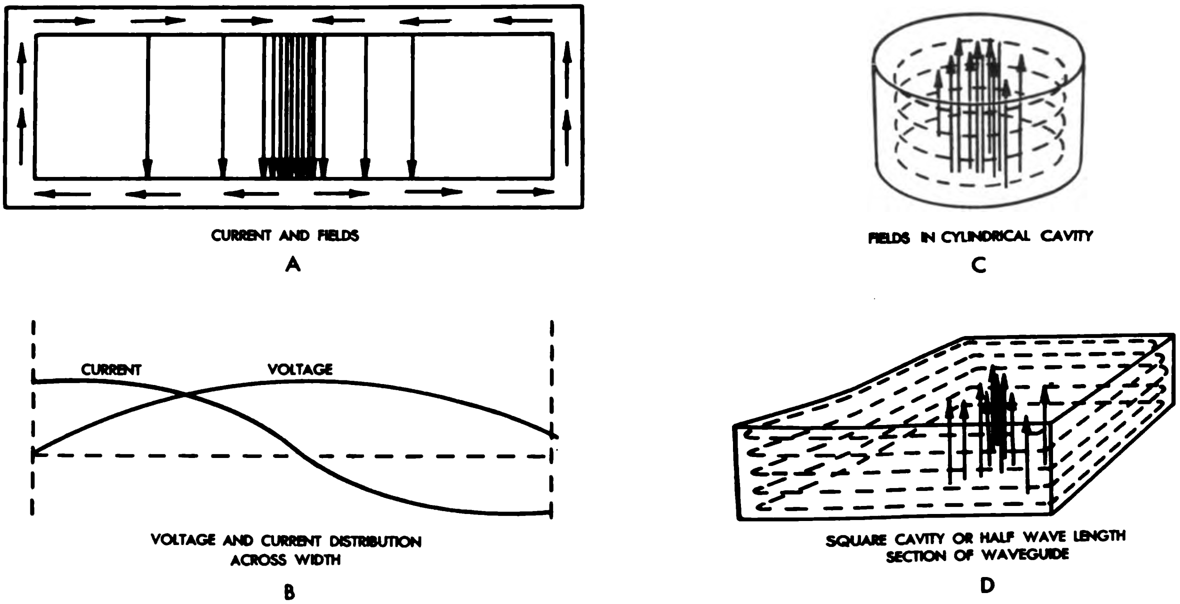


CLOSED METAL CONTAINER

Figure 11–48. Development of Cavity from λ/4 Sections

The Fields in a Cavity

A resonant cavity may be compared to a waveguide, since its operation is best described in terms of the fields rather than in terms of the currents and voltages present. As in waveguides, the different field configurations in cavities are called *modes*. In Figure 11–49 which shows the dominant mode of the cylindrical cavity, note



CURRENT AND FIELDS
A

VOLTAGE AND CURRENT DISTRIBUTION ACROSS WIDTH
B

FIELDS IN CYLINDRICAL CAVITY
C

SQUARE CAVITY OR HALF WAVE LENGTH SECTION OF WAVEGUIDE
D

Figure 11–49. Voltages, Currents and Associated Fields for Simple Mode in a Cavity Resonator

that in A the voltage is represented by E-lines between the top and the bottom of the cavity. The current flows in a thin layer on the surface of the cavity, due to skin effect. The strength of the current is indicated by graduated arrows.

The magnetic field is strong where the current is high. The strongest H-field is at the vertical walls of the cylinder and diminishes toward the center where the current is zero. This is due to the standing waves on the quarter-wave section. The E-field is maximum at the center and decreases to zero at the edge, where the vertical wall is a short circuit to the voltage. The curves of E-field and H-field density are shown in Figure 11–49B. Figures C and D show two types of cavities with their fields represented.

The modes in a cavity are identified by the same numbering system that is used with waveguides, except that a third subscript is used to indicate the number of patterns of the transverse field along the axis of the cavity (perpendicular to the transverse field). For example, the cylindrical cavity shown in Figure 11–49C is a form of circular waveguide. The axis is the center of the circle. The transverse field is the magnetic field. Therefore, it is marked TM. Around the circumference there is a constant magnetic field. (The H-lines are parallel to the circumference.) Therefore, the first subscript is zero. The distance across the diameter is one half-wave. Thus the second subscript is one. Through the center along the axis the H-field strength is a constant zero. This makes the third subscript zero. Therefore, the complete description of the mode is TM$_{0,1,0}$.

When a half-wave section of waveguide is closed on both ends in the form of a rectangular cavity, as shown in Figure 11–49D, standing waves are set up and resonance occurs. The simple mode in this cavity is the same as the dominant mode of a rectangular waveguide, that is, TE$_{0,1}$. The third subscript of the mode, which is determined by the plane of the E-field, is 1. Thus, the complete description of the simple mode in the rectangular cavity at D is TE$_{0,1,1}$.

Cavities may have various physical shapes, for any chamber enclosed in conducting walls resonates at several frequencies and produces a number of modes. Note in Figure 11–50 examples of several types of cavities. The Q of each cavity is indicated. Of those shown, the cylindrical cavity is useful in wavemeters or in frequency-measuring devices. The cylindrical ring type is used in super-high-frequency oscillators as the frequency determining element. The section of waveguide, which is shown diagrammatically, is used in some radar systems as a mixing chamber for combining signals from two sources.

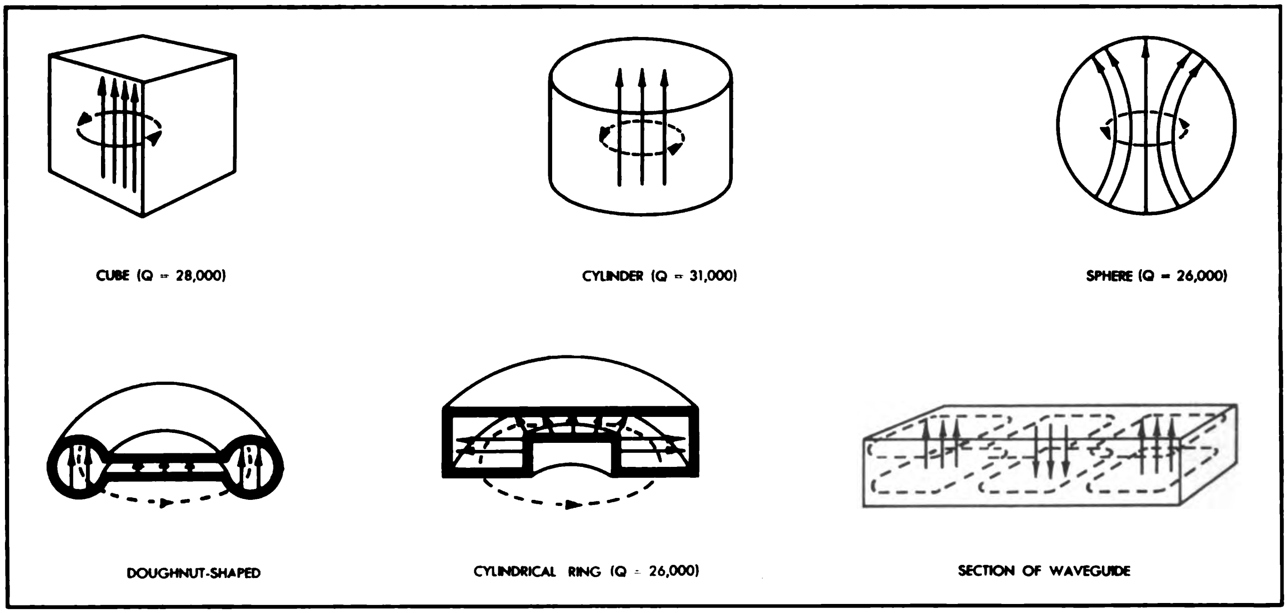


Figure 11–50. Several Types of Cavities

Exciting the Cavity

There are three principal ways in which energy can be inserted into, and removed from, a cavity resonator, Figure 11–51. The first is by inserting a probe into the cavity. The current flowing in the probe sets up E-lines parallel to it and they, in turn, start oscillations. Another method uses a magnetic loop. The loop is placed in the region where the magnetic field will be located. The currents in the loop start an H-field in the cavity. Either of these methods can be used to remove energy from, or to put energy into, the cavity.

A third method uses a cylindrical ring-type cavity. In this method the energy is placed into the cavity by clouds of electrons which are virtually shot through the holes in the center of a perforated plate. As each cloud goes through, it creates a disturbance in the space inside the cavity until a field is set up. In terms of current,

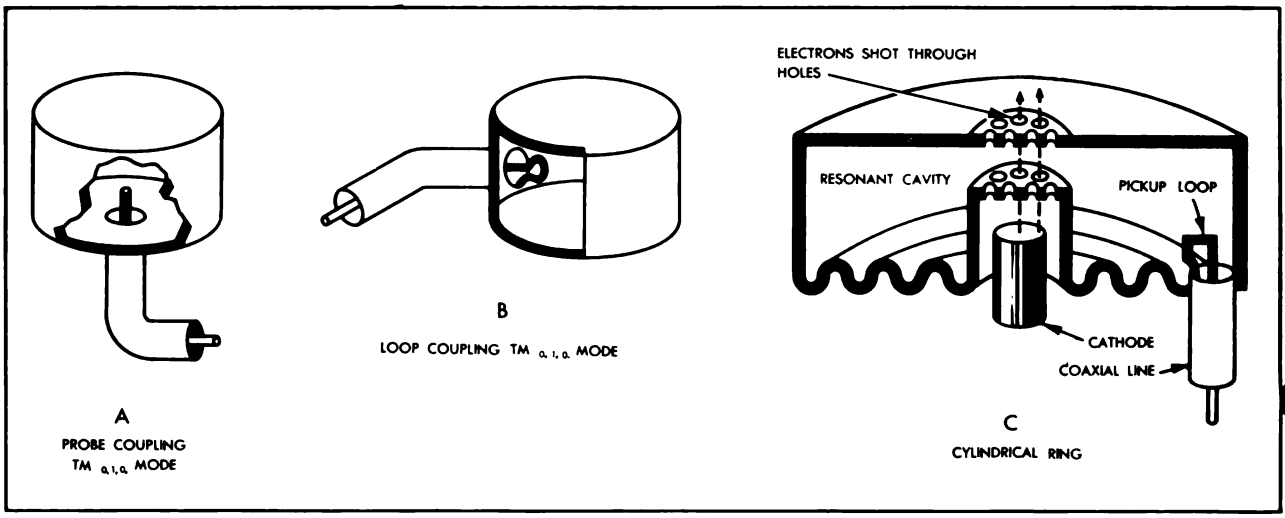


Figure 11–51. Methods of Exciting the Cavity

the approaching cloud of electrons makes the perforated plate positive by repelling the electrons away from it. This current starts an H-field. Energy may be removed from the cavity by placing a loop at the outside edge.

Other methods of feeding the cavity are discussed later in the analysis of systems using cavities.

Varying the Resonant Frequency of the Cavity

Three methods for setting the resonant frequency in a cavity are shown in Figure 11–52. One method uses a cylindrical cavity with an adjustable disc. When a $TE_{0,1,1}$ mode is used, the size of the cylinder may be changed along the axis to change the resonant frequency. The smaller the volume of the cavity, the higher the resonant frequency. The movement of the disc may be calibrated in terms of frequency. In the case of high-frequency equipment, a micrometer scale is usually used to indicate the position of the disc, and a calibration chart is used to determine the frequency.

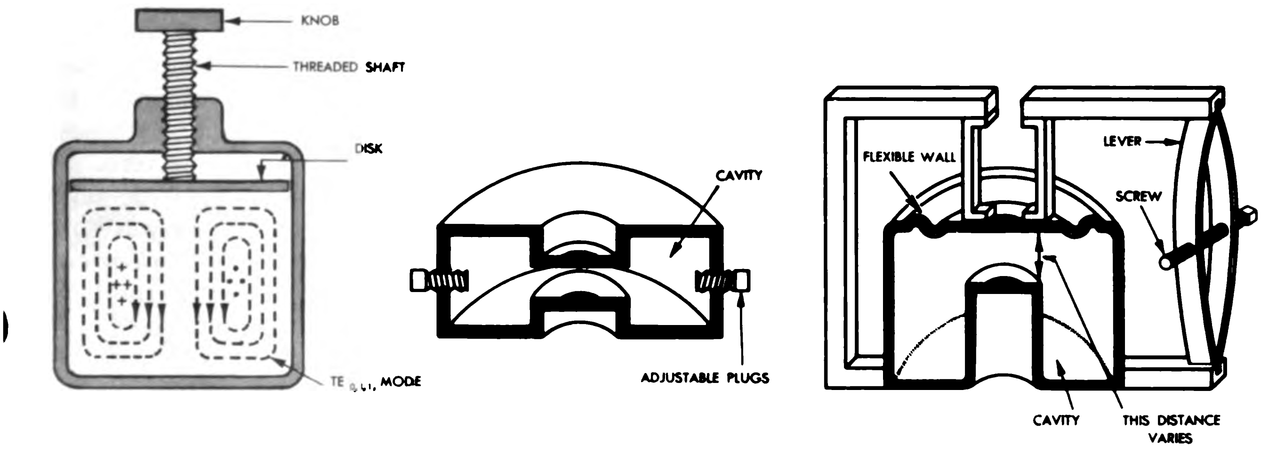
A second method for varying the frequency uses threaded plugs that are inserted in the cavity. The plug reduces the strength of the magnetic field in the cavity in a manner similar to reducing the inductance of the tuned circuit. The deeper the plug extends into the cavity, the higher the frequency.

In a third method the interior of the cavity, which is part of the interior of a vacuum tube, is sealed and evacuated. In this method a frequency change occurs whenever the top and bottom of the cavity are moved away from, or toward, each other. This is accomplished by turning a screw at the end of a lever. As the screw is turned, the two pieces of metal to which the lever is attached are either pulled together or allowed to spring apart. This moves the top of the cavity up and down with respect to the bottom. As the distance varies, the volume and the capacity between the top and bottom of the cavity are changed. As the change in capacity is the chief result of the change in the distance between the plates, the resonant frequency is inversely proportional to the distance from the top to the bottom.

Another way of changing the frequency is by changing the method of exciting the circuit. This can be done by tuning the exciting loop either capacitively or inductively. This can occur either accidentally, due to improperly tuned circuits, or deliberately, as a means of tuning the cavity.

Uses of Cavities

There are many ways in which resonant cavities are used in radar. One important use is as a tuned circuit in 10,000-mc oscillator circuits. For example, the klystron shown in Figure 11–53A, which can be used as the local oscillator in a radar receiver, is tuned by changing the resonant cavity. In the klystron the cathode emits electrons that pass through the grids toward the plate. When they pass through this



CYLINDRICAL CAVITY WITH ADJUSTABLE DISC

CAVITY WITH ADJUSTABLE PLUGS

CHANGING THE SIZE OF CAVITY

Figure 11–52. Method of Changing the Frequency of a Cavity

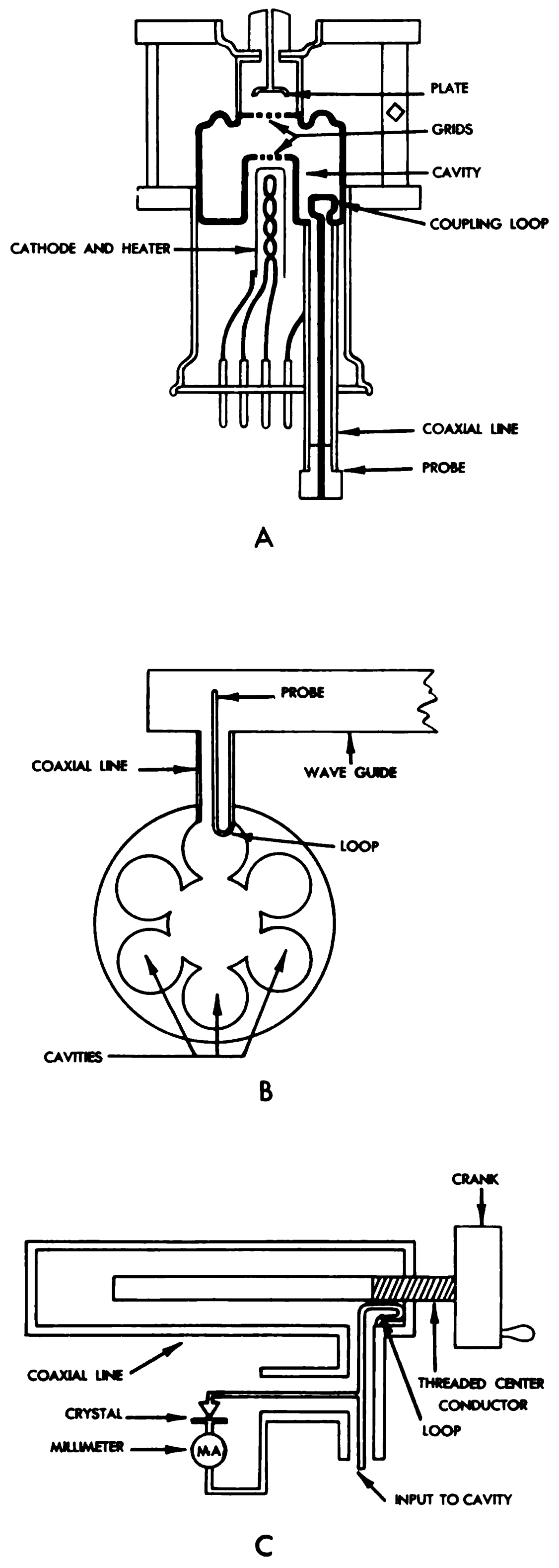


Figure 11–53. Uses of Cavities

area, they disturb the field and excite the cavity. The resonant frequency is changed by the spacing of the grids.

Another use of resonant cavities is the series of cavities arranged around a circle in the magnetron oscillator as shown in Figure 11–53B. These cavities, which are coupled one to another through capacitance at the openings, are excited by moving electrons. The energy from the electrons is passed around the ring from all cavities to the one with a loop. This one serves to transfer the energy to the waveguide. From the waveguide, the energy goes to the antenna where it is radiated into space.

Since the frequency of the transmitter is fixed, there are no frequency adjusting devices on a waveguide. However, the different speeds at which the electrons travel and the variable impedances in the waveguide cause either a change in frequency, or oscillations in a mode which differs from the one intended.

Cavities also may be used as wavemeters — devices for measuring frequency. One type of wavemeter consists of a cylindrical cavity with an adjustable disc. Another type operates essentially as the first one but uses a coaxial cavity and operates in a different mode, as shown in Figure 11–53C. In the second type the signal is introduced into the cavity by a loop located in one end of the cavity. The signal is strongest when the loop is at a high current point in the standing wave.

The standing wave of current is maximum at odd multiples of quarter-wave distances from the open end of the guide. When the length of the threaded center conductor is varied by the crank, the distance from the open end to the loop can be made equal to an odd multiple of a quarter-wave. This causes the current maximum to occur at the loop, and this, in turn, causes the input impedance at the loop to be zero. When the distance from the open end of the guide to the short (closed end) is a quarter-wave or any odd multiple of a quarter-wave, the current introduced into the guide will be at the location of parallel resonance.

The threaded shaft shown in Figure 11–53C can be calibrated either in wavelength or in frequency. The input impedance can be indicated by any suitable indicator, such as a crystal rectifier and a DC milliammeter. In operation, when the input impedance is zero, the signal is shorted out. In this case the meter will read zero. To check the frequency for a zero impedance reading, turn the crank until a dip occurs on the meter and read the frequency on the threaded shaft.

A cavity can be used as a mixing chamber. When two or more signals are put into a cavity, it is possible to remove the combination of their signal voltages. An example of this use is discussed later in this chapter.

Another use of a cavity is as an impedance-matching device such as shown in Figure 11–54; this illustrates a small waveguide connected to a large waveguide. Normally, when you connect the two waveguides together, there will be reflections in the waveguides. However, a small section of waveguide or an intermediate size can be matched without reflections, provided that plates are used at the junction to cancel the reactive effects resulting from the different sizes of the waveguides. Thus, although the cavity itself has standing waves in it, there are none on either waveguide.

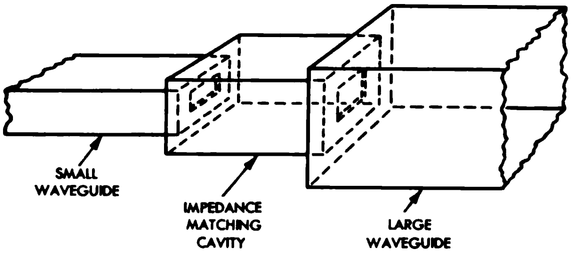


Figure 11–54. Cavity Used for Matching an Impedance Device to One of Different Impedance

Another use of a cavity is a *ringing circuit* in a resonant circuit. It is called ringing circuit because it oscillates for many cycles after being started by an external circuit. In this use the cavity is resonant; once started, it will continue oscillating for a few microseconds after the source of voltage has been removed. This action is analogous to a bell that continues its tonal vibrations for many seconds after being struck.

In the 3-cm (10,000-mc) ringing circuit shown in Figure 11–55, the cavity itself is a cylinder which is 3 inches in diameter and 10 inches long. A movable piston in the cylinder determines the volume of the cavity. Because the position of the piston is calibrated in megacycles, it serves as a frequency meter. The cavity is energized by a signal that enters from the small waveguide through a small aperture. The energy is put into the short waveguide by a probe which, in turn, is fed by a coaxial cable. A good method to put the signal into the coaxial cable is to connect a small antenna to the end of the cable.

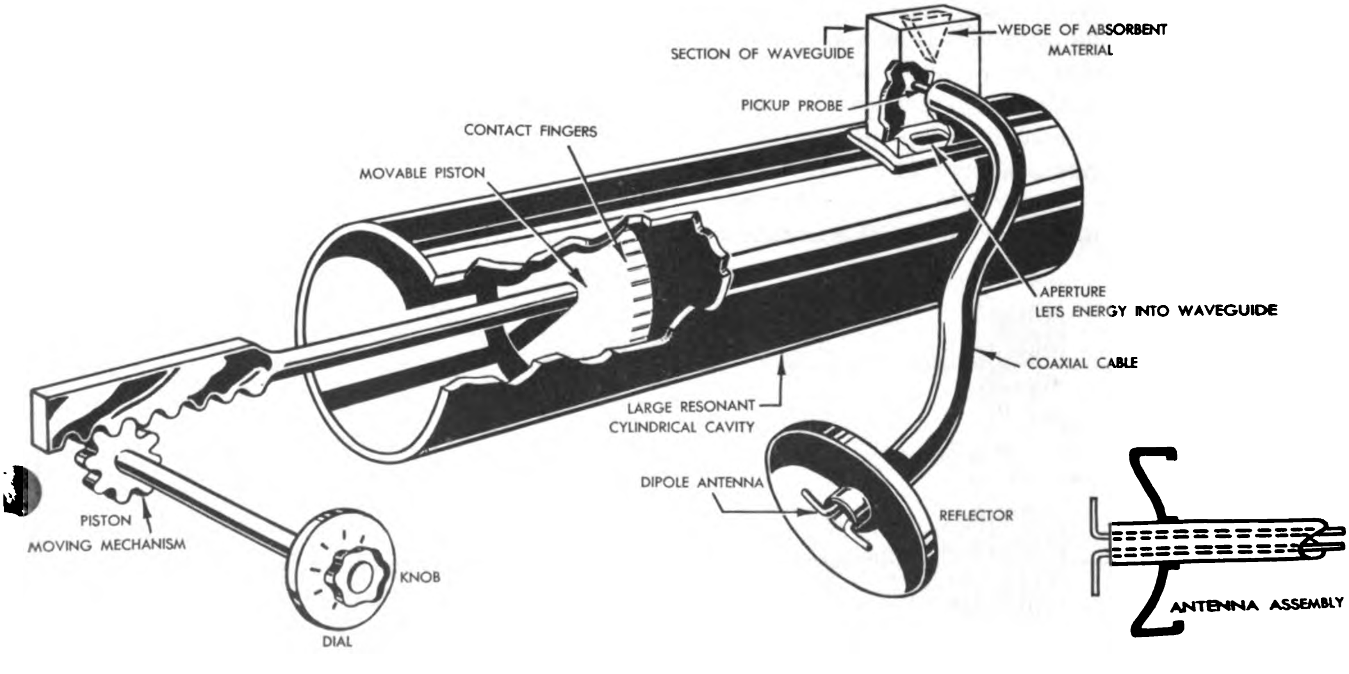


Figure 11–55. 3-cm Ringing Circuit

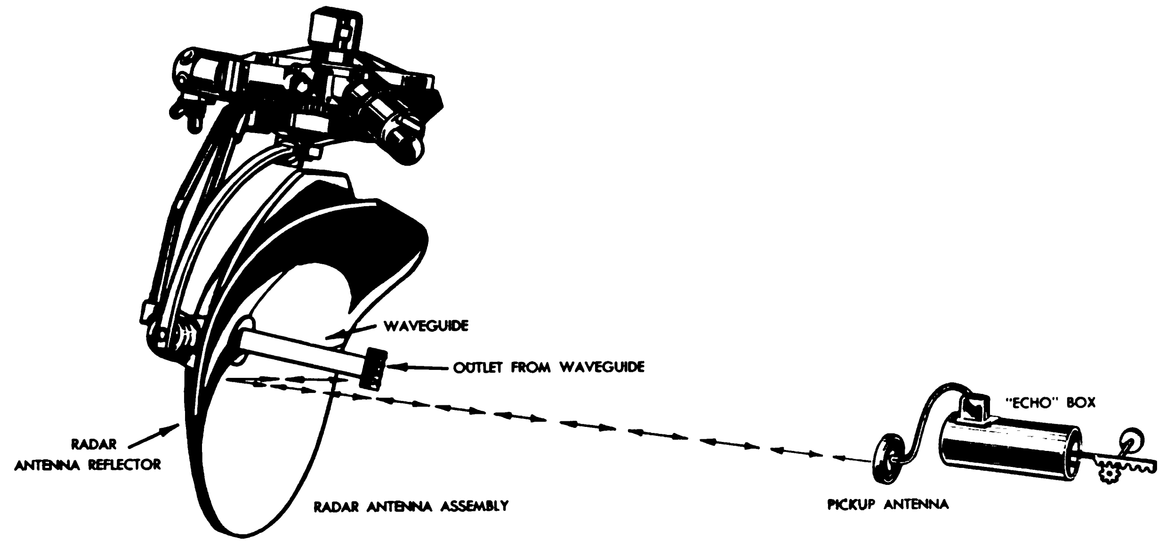


Figure 11–56. Cavity as an Echo Box

The resonant cavity shown in Figure 11–55 is used as an echo box as illustrated in Figure 11–56. Echo boxes are resonant cavities having a very high Q. They are used with microwave radar sets to provide an artificial or "phantom" target, which may be used to tune the receiver to the transmitter when no real targets are available.

This is how the echo box works. When the radar transmitter is turned on, energy is fed into the echo box cavity. There it sets up violent oscillations, and the cavity itself acts as a transmitter for a period of 20 microseconds after the pulse generated by the radar transmitter ends. (When you view the oscillations on an oscilloscope, their waveshapes appear as shown in Figure 11–57.) The energy which the echo box radiates is picked up by the small pickup antenna located in front of the radar antenna assembly and is fed into the radar receiver. The receiver displays it on the indicator screen. Since this signal simulates a radar echo, the cavity which transmitted it is appropriately called an echo box. It is thus a means of supplying a signal for tuning the receiver when there is no real target available.

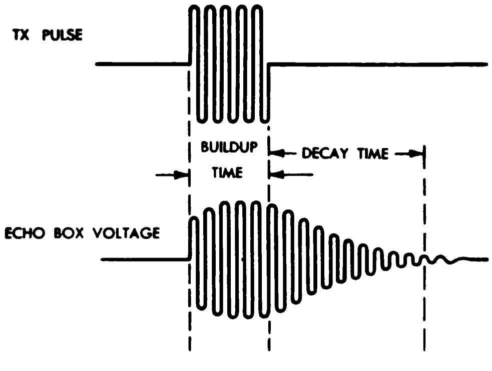


Figure 11–57. Oscillations in Echo Box

MICROWAVE FERRITES

General

Recent developments in microwave techniques have produced a series of waveguide fittings that use the unusual properties of a new type of magnetic material. These materials are called ferrites; they are ceramic materials containing iron and have unusual magnetic and electrical properties. Ferrites have wide use in waveguide applications. They are used as isolators between transmitter and antenna, as waveguide switches, as unidirectional attenuators, and as commutators (Faraday rotation).

Ferrites are compounds of iron, zinc, manganese, magnesium, cobalt, aluminum and nickel oxides. They are manufactured by pressing into shape the required mixture of the finely divided metallic oxide powders and then firing the shaped mixture at about 2000°F. The product is a ceramic with high electrical resistance.

Ferrites have been used commercially for several years. For instance, there are nickel-zinc ferrites in the deflection coils of flyback transformers in television sets; a powdered ferrite is used in magnetic recording tape. Ferrites behave as iron alloys at low frequencies, but at high frequencies their high electrical resistance prevents eddy currents and resonance takes place within the iron atoms themselves. These unusual effects make it possible to use ferrites for special applications in microwave circuits.

Fundamental Theory of Ferrites

A fundamental property of atoms is that both electrons and protons spin on their own axes, which was explained in Chapter 2. In addition, of course, the electron revolves around the nucleus. An analogy is the solar system, where the earth rotates on its axis as it revolves around the sun. As the electron spins, it creates a magnetic moment, or field, along its spin axis. This spinning charge appears as a current flowing around a loop. The atoms with more electron spins in one direction than another act as small magnets. The mutual action of all these atoms explains the magnetic properties of magnetic materials.

If a spinning electron is placed in a static magnetic field, the electron's magnetic moment becomes aligned with the static field. The magnetic moment and its alignment with a DC magnetic field is shown in Figure 11–58.

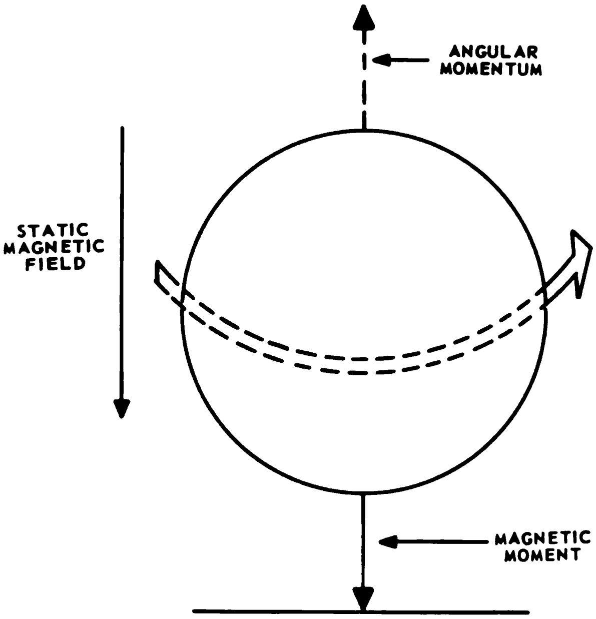


Figure 11–58. Electrons in DC Magnetic Field

Gyroscopic Action

Electrons, because of their spinning motion, behave like very small gyroscopes. When a force is applied to the spin axis of an electron that would cause it to tilt, the electron will behave like any other gyroscope. It will precess, or wobble. Precession is defined as a movement of the axis of rotation at right angles to its original axis. Figure 11–59 shows a gyroscope mounted to a stick so that the stick can pivot freely. Even with the gyroscope spinning, the stick will hang straight down, due to gravity. However, if you try to move the stick from side to side, the gyroscope will force the stick to move around in a circle, or precess.

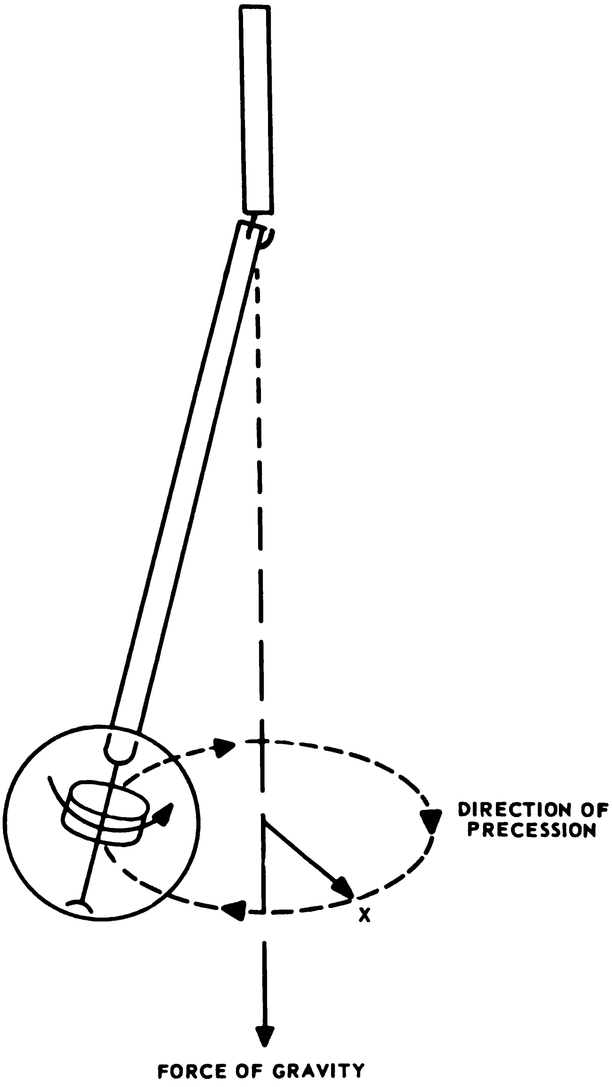


Figure 11–59. Precession of a Gyroscope

The direction is determined by the direction of rotation of the gyro rotor, and the frequency by the gravitational force and the momentum of the gyroscope. This is shown in Figure 11–59. The natural precession frequency could be increased by increasing the force of gravity. A rotational force applied to the stick at the natural precession frequency will displace it from the vertical by a large amount. (The precession path shown in Figure 11–59 will have a greater diameter.) Thus, there will be a large amplitude of moment. A rotational force applied at any other frequency will produce a much lower displacement. This is similar to the feedback in an oscillator. With feedback at the right frequency, the amplitude of oscillation is larger than when the feedback is off frequency.

Electron Action

Electrons behave much like gyroscopes, but gravity has little effect on them. Instead, a steady magnetic field is applied to line up the axes of the spinning electrons. This field causes any precession to die out quickly. Now when an alternating field is applied at right angles to the DC field, the electrons precess, or wobble, just the way the gyroscope and stick did when a sideways force was applied.

The natural precession frequency of an electron in a DC field is between 3000 and 9000 megacycles, depending on the field strength. If we apply an AC magnetic field at the natural frequency, the precessional motion will build up. This increases the frictional damping effects because the entire iron atom is vibrating. The ferrites dissipate as heat the energy extracted from the AC field.

Applications

ATTENUATOR. One application of ferrites is as an attenuator. Figure 11–60 shows a piece of ferrite placed in the center of a waveguide; a steady magnetic field is applied as shown. This arrangement will attenuate frequencies at the

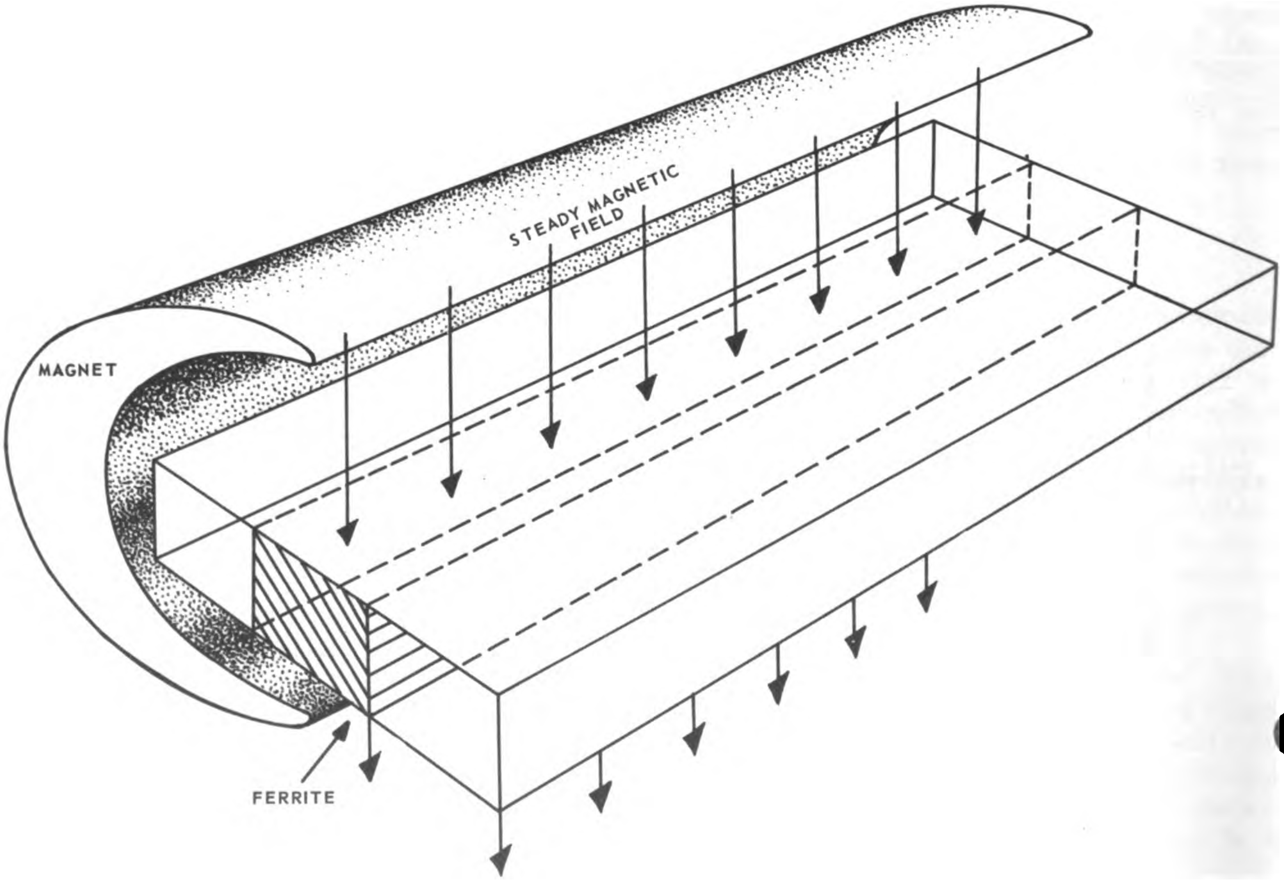


Figure 11–60. Ferrite Slab Mounted in Waveguide

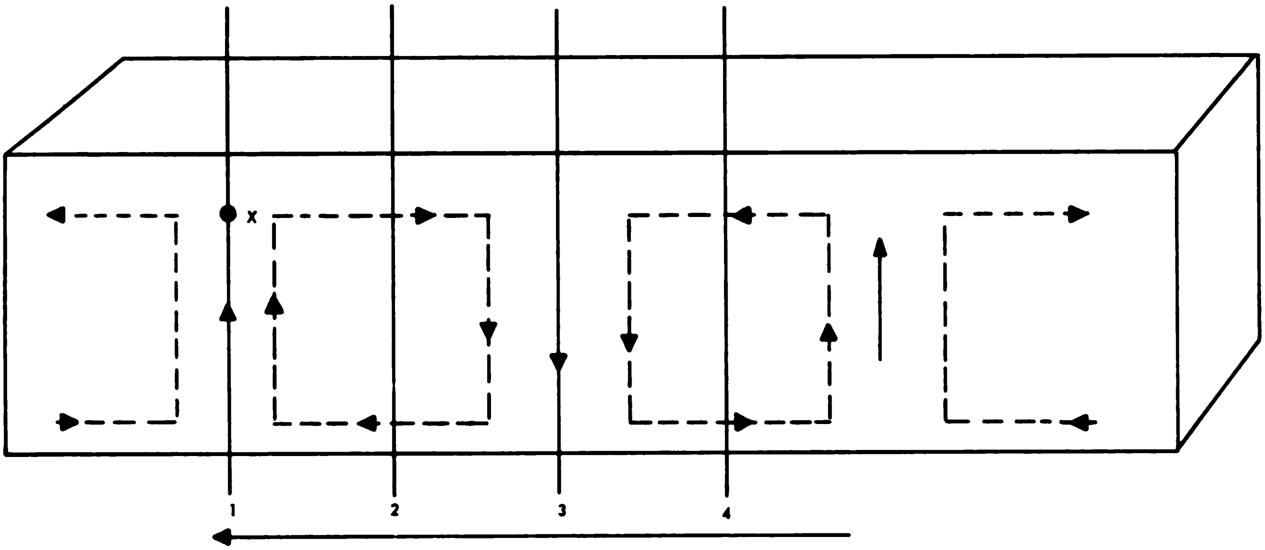


Figure 11–61. Effective Clockwise Rotation of a Magnetic Field

resonant frequency of the electrons in the ferrite, whereas other frequencies will be attenuated very slightly. Changing the strength of the DC field produces a change in the frequency that will be attenuated, although this occurs over only a limited range.

The DC field is produced by current flowing through a coil wound around the waveguide. The strength of the field, which depends upon the current flowing through the coil, determines the frequency of precession. Usually the ferrite attenuator is in the form of an adjustable vane of ferrite extending into the waveguide. The farther the vane extends into the waveguide, the greater the attenuation, since more of the r-f energy must travel through the ferrite.

ISOLATORS. Another application of ferrites is that of an isolator. When used as an isolator, the ferrite allows energy to travel in one direction, but absorbs energy traveling in the opposite direction. Figure 11–61 depicts an electromagnetic wave traveling from right to left. It illustrates how the wave, at a point off the center line of the guide, will appear as a rotating magnetic field. At one instant shown in Figure 11–61, the magnetic field at Point X is pointed up. When the magnetic field at Point 2 reaches Point X, the magnetic field will be directed to the right. When Point 3 reaches X, the magnetic field is downward, and when Point 4 on the wave arrives at X, the field is directed to the left.

Thus, as the wave passes Point X, the magnetic field appears to rotate in a clockwise direction. At any point off the center of the waveguide, the magnetic field will appear to rotate as the electromagnetic wave passes. This same analysis can be used to show that with a wave traveling from left to right the magnetic field appears to rotate counterclockwise at Point X.

Now let us place a section of ferrite in the waveguide at X. This is shown in Figure 11–62, which illustrates a simple isolator consisting of a piece of waveguide, a permanent magnet and a section of ferrite. The ferrite's electron resonant frequency and the microwave frequency are made the same by changing either the magnetic field strength or the microwave frequency. When the frequencies are the same, a wave traveling from left to right in the waveguide produces a rotating force in the direction of the natural precession of the electrons in the ferrite. The amplitude of precession increases, taking power from the electromagnetic wave. This power is dissipated as heat in the ferrite.

A wave that is traveling from right to left in this waveguide acts as a rotating force on the electrons to oppose the natural precession. This will not increase the amplitude of the precession and energy is not absorbed from the electromagnetic field. About 0.4 db attenuation takes place in a wave traveling from right to left, but as much as 10 db occurs in a wave traveling from left to right.

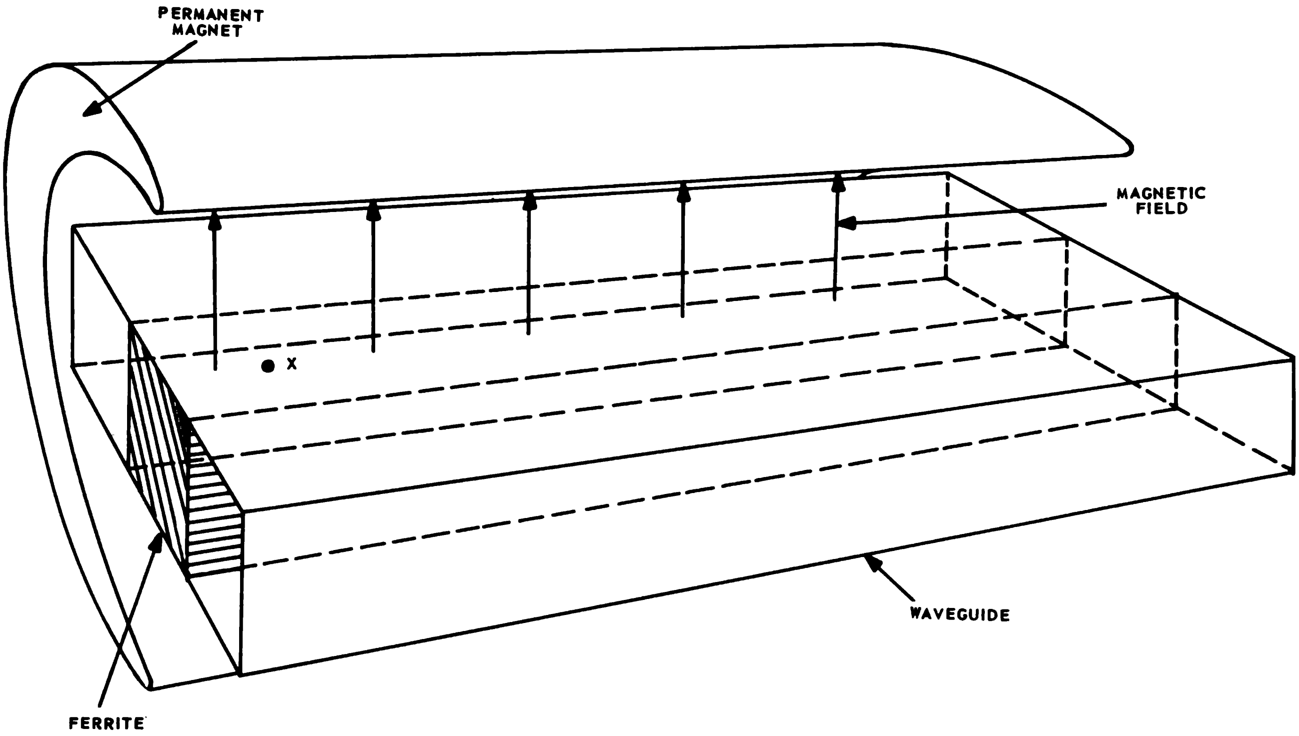


Figure 11–62. Simple Isolator

Faraday Rotation. Another effect takes place when microwaves are passed through a piece of ferrite in a magnetic field. The plane of polarization of the wave is rotated if the frequency of the microwave is above the resonant frequency of the ferrite electrons. This is known as the Faraday rotation effect. When r-f energy enters the ferrite material, the magnetic moment of the electron precesses as usual but at a different frequency from the r-f. The H-lines within the ferrite now are the resultant produced by vectorial addition of the rotating magnetic moment and the r-f field. A new r-f field, which is rotated from the original r-f field, results. The amount of rotation is determined by the DC magnetic field and the length of the ferrite.

Figure 11–63 shows a ferrite rod that is placed lengthwise in the waveguide. The DC magnetic field is set up by a coil. Now assume that a wave that is vertically polarized enters the left end of the waveguide. As it enters the ferrite section, it will set up limited precession motion of the electrons. The interaction between the magnetic fields of the wave and the precessing electrons rotates the polarization of the wave. With the correct dimensions of the ferrite rod, the wave is polarized at a 45° angle from the original. Different dimensions of the rod and magnetic field strengths produce other shifts in polarization.

Duplexer and Diplexer. The duplexer is another application of the ferrite. As you know, a duplexer is used in radar to allow one antenna to both transmit and receive. The ferrite duplexer consists of a folded hybrid, a phase-shift section (ferrite), a short slot hybrid and two output arms. Power from the transmitter is fed into the folded hybrid. Here it is split into two parts of equal size and with the same phase relationship. These two, however, will be 180° out of phase to the receiver and will cancel.

After splitting into two parts, the transmitter energy goes to the ferrite section in phase. Here one channel, let's say the lower, shifts the energy 90° in phase, while the energy in the other channel (upper) passes through unchanged. Next, the energy goes to the short slot hybrid. Here the energy in the lower channel passes through unchanged, while that in the upper channel is shifted 90°. Now the energy in the two channels is back in phase and is radiated by the antenna.

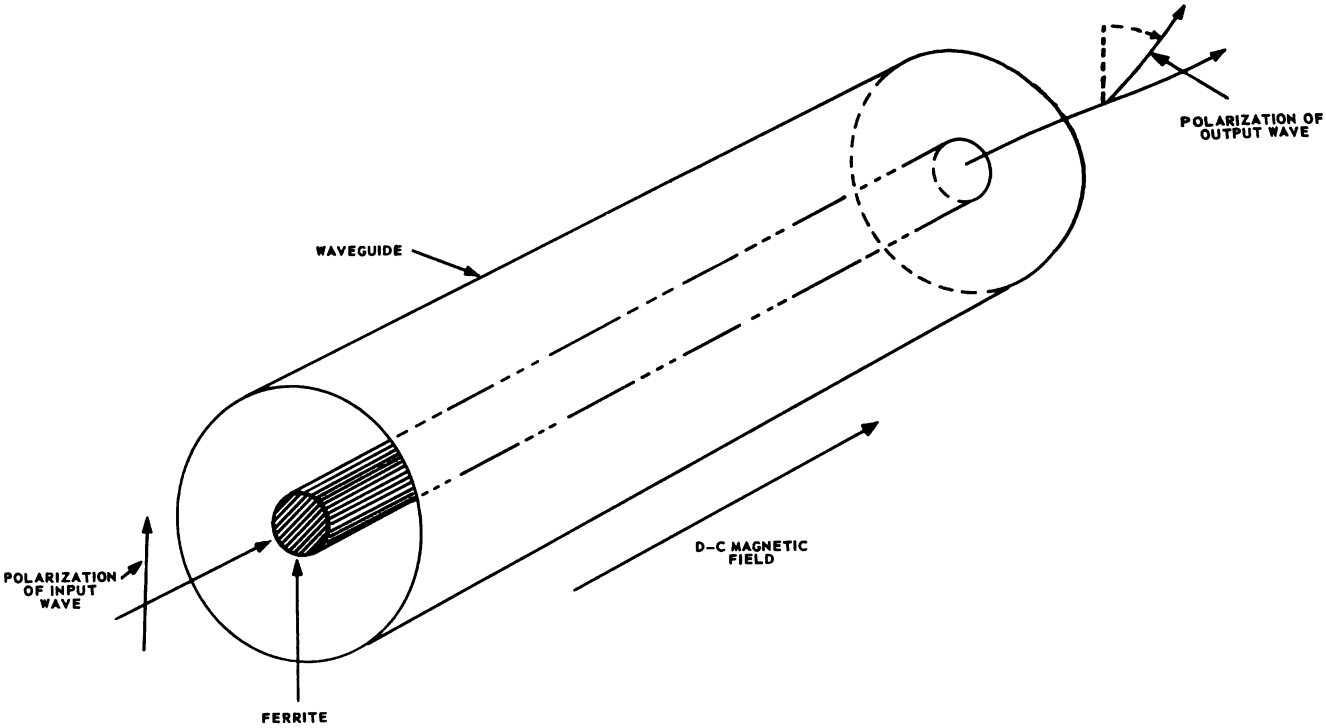


Figure 11–63. Faraday Rotation

When energy is received by the antenna it is divided into two channels when it enters the short slot hybrid. The energy in the lower channel passes through the ferrite section and enters the folded hybrid still in the same phase. There is no phase shift. But the energy in the upper channel is shifted 90° in the short slot hybrid and another 90° in the same direction in the ferrite section. The energy in the upper channel has been shifted 180°. Since the energy in the two channels in the folded hybrid is 180° out of phase, the received energy enters the receiver waveguide section. However, the received energy would leave the folded hybrid for the transmitter section 180° out of phase and here would cancel. This prevents any energy from being lost in the transmitter section.

A *diplexer* is used in radar to allow two different transmitter frequencies to be fed into the same waveguide without interaction between the two transmitters. The diplexer section itself consists of two resonant cavities, which are tunable to different frequencies, feeding a single waveguide section. The two different transmitter channels each feed into one of the resonant cavities. The energy from one resonant cavity cannot feed back into the other transmitter channel, due to the resonant cavity being tuned to the other frequency. The diplexer prevents feedback from one transmitter to the other. The energy from the diplexer is fed into the folded hybrid section of the duplexer discussed in the preceding paragraphs.

Ferrite Cores

The ferrite core is used in the memory units of computers. In this application the magnetizing characteristic of the ferrite is used. The electrons in the circular core of the ferrite material align themselves with a magnetic field. The magnetic field can be produced by current passing through wire looped around the core or passed through the center of the core. The direction of current determines the direction of magnetization.

Ferrites in microwave applications are becoming familiar devices. Typical applications include: load isolation between a microwave source and its load to prevent reflections from affecting the generator; modulation of microwaves with pure amplitude modulation; and fast action radar duplexers.

A COMPLETE WAVEGUIDE R-F SYSTEM

In a complete waveguide r-f system the signal source is the transmitter unit. The antenna that transmits the output is located as near as possible to the radar transmitter. A principal requirement in an efficient radar system is that energy must be transferred from the transmitter to the antenna with minimum losses, for any losses that occur will shorten the maximum range and thereby decrease the efficiency of the entire radar system. Since the transmitting antenna is pointed at the *target* at the time the transmitter pulse leaves, and since it is conveniently located, it is preferable to use it as the receiving antenna, too. Similarly, it is possible to use the same waveguide system for transferring the received energy to the radar receiver.

When the same elements are used for both transmitting and receiving, it is necessary to use a high-speed automatic switch to prevent the powerful transmitter pulse from going into the receiver circuit and causing serious damage. The TR and ATR boxes discussed earlier perform this function in many radar sets.

The highly directional antenna is made rotatable in order to cover all areas. To make rotation possible, signals are coupled to the antenna through vertical and horizontal rotating joints.

Since airborne equipment is flown at high altitudes, temperature changes will cause moisture condensation inside the waveguide. As water causes extremely high losses, the interior of the waveguide must be maintained at a higher pressure than the outside to drive the moisture out and keep it out. Furthermore, all joints have to be airtight to maintain this pressure.

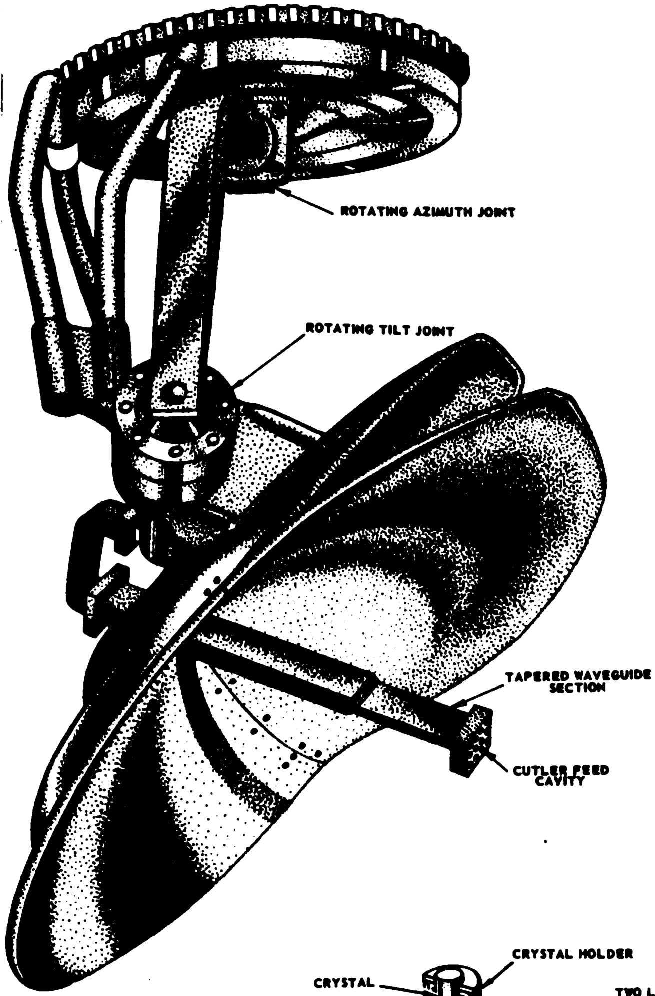
Figure 11–64 shows a waveguide system that uses the components just discussed. The source of the r-f energy is the magnetron tube which is shown at the right in the lower part of Figure 11–64. A probe couples it to the waveguide, which operates in the $TE_{0,2}$ mode. A very short section of waveguide is fixed permanently to the magnetron output. When the magnetron is installed, its waveguide section is connected to the end of the main waveguide. In operating conditions the transmitter energy travels down the main waveguide where it enters a section which has two T-junctions — one called the ATR box, and the other the TR box and mixing chamber.

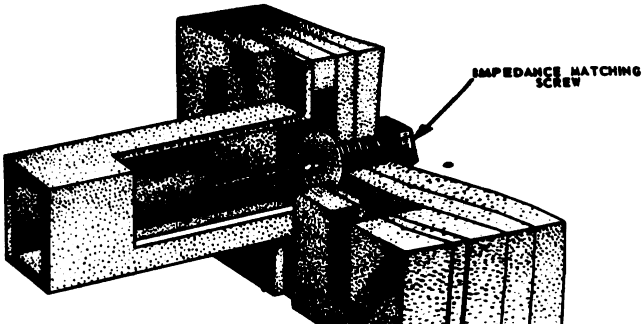
In each T-junction, there is a spark gap at a distance of a quarter-wave from the center of the main guide. Both these gaps arc over when the transmitter is on. Thus, no energy goes past either spark gap tube into the T-junction waveguides. The energy flow is continuous down the main guide to the antenna. Each section is connected to the next with a choke joint.

In going to the antenna, the signal passes through a special section that contains a pair of probes spaced a quarter-wave apart. Test equipment can be inserted at each probe to measure the signal strength at the probe. The ratio of the two signal strengths is the standing-wave ratio. In going through the stationary transmitter unit to the moving antenna, the signal goes through a flexible section of waveguide that insulates the transmitter unit from the mechanical vibration of the antenna.

The antenna is designed to rotate horizontally and to tilt vertically. Thus, the signal is transferred to a rotating coaxial joint twice before being radiated, and each time it is immediately transferred back to a waveguide again. The antenna itself is a resonant cavity on which there are a pair of slots one half-wave apart. As you can see in the enlargement, the waveguide is tapered in the narrow dimension to fit between the slots. The effect of the cavity is as though the waveguide were split in half and each side bent through 180°. Tapering the waveguide prevents the reflections that would occur if the size were changed abruptly. Tapering in the narrow dimension does not affect the operation of the waveguide, since this dimension is not critical. The actual impedance match is accomplished by adjusting the matching screw. This adjustment is made at the factory and soldered in place.

The energy comes out of the back of the cavity and goes to the large reflector. The reflector sends the energy out forward in a narrow beam. The energy which returns from the radar target is reflected by the reflector into the slots. It passes through the slots into the waveguide and travels toward the magnetron, where it stops at the ATR box. The length of the T-junction is a half-wave from the center of the waveguide, so the closed end of the section reflects a short circuit at the outer center of the main waveguide. This reflects the signal back along the guide to the mixing chamber. Of course, neither





CLOSEUP OF CUTLER FEED CAVITY

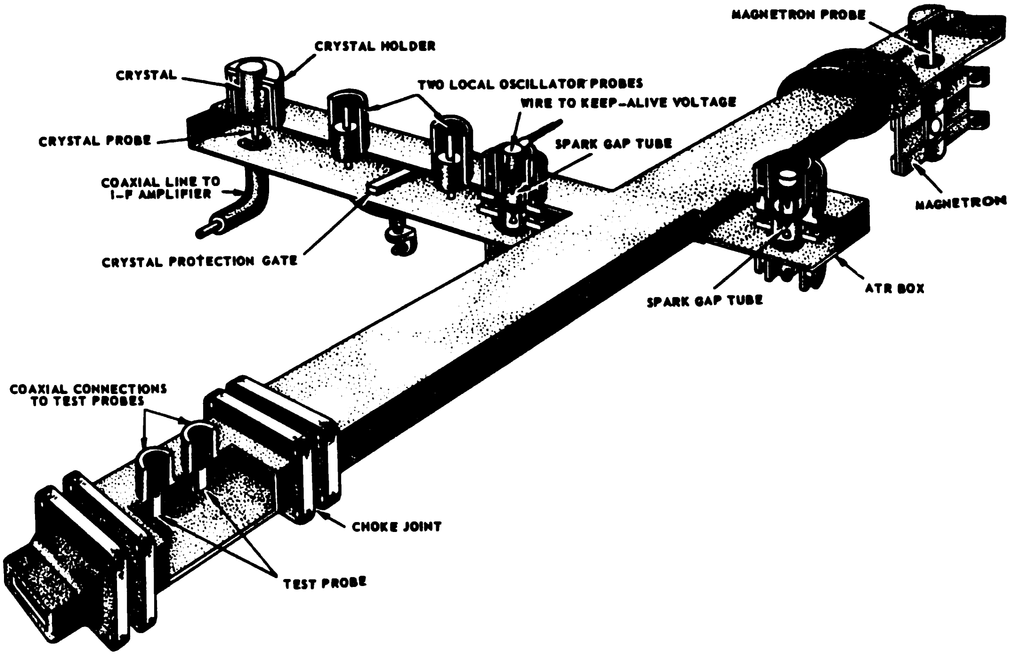


Figure 11–64. Complete Waveguide System

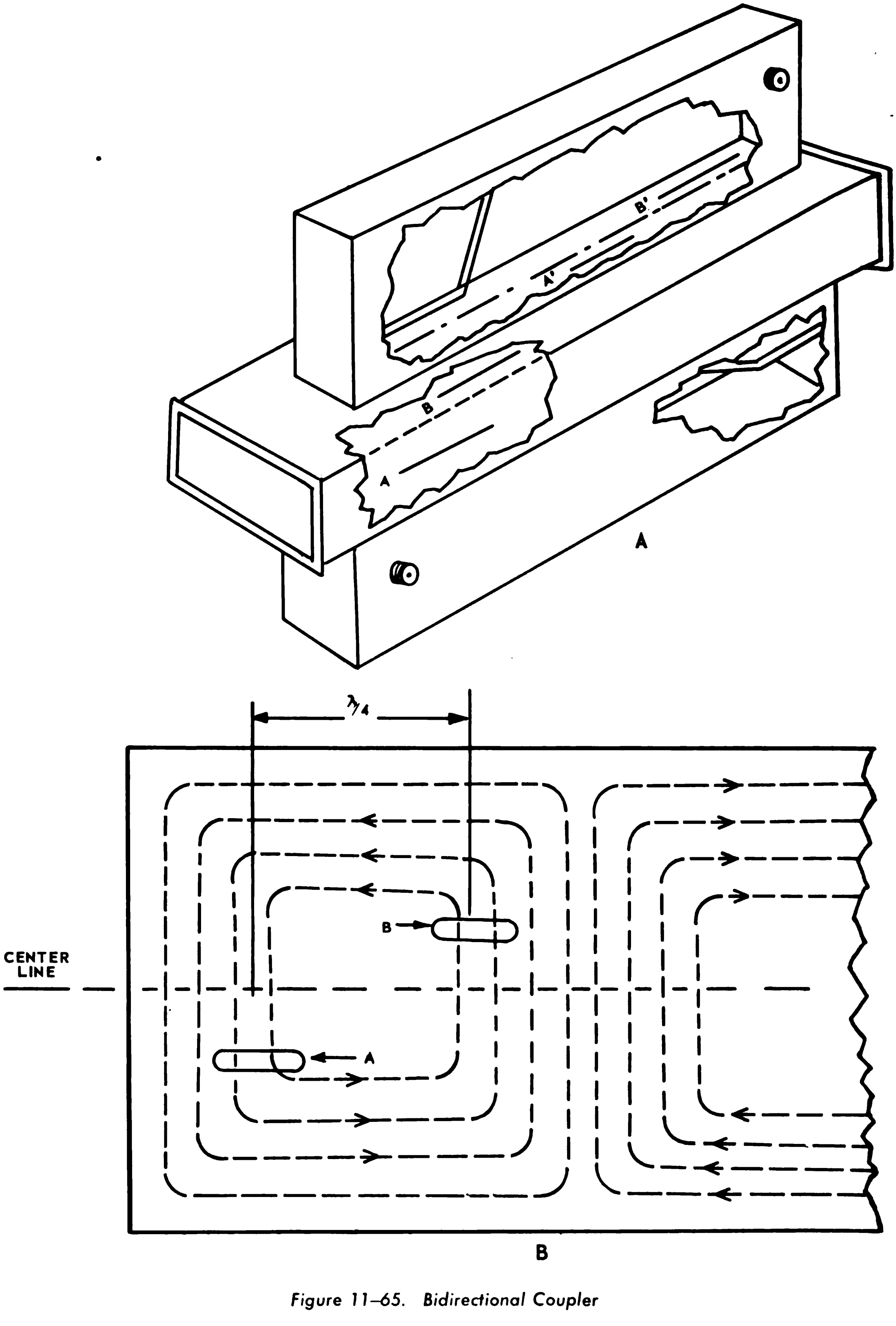


Figure 11–65. Bidirectional Coupler

spark gap is fired because the signal is too weak to strike an arc.

The signal enters the mixing chamber, where it strikes the end wall and reflections are again set up. The chamber becomes a cavity resonator. An additional signal is introduced from another oscillator — the heterodyne oscillator for the receiver. The signal is injected with a probe at the center of the E-field in the cavity. Both signals cause current flow through a pickup probe at a quarter-wave from the end of the cavity. This probe is connected to a crystal mixer. The second oscillator is set at a different frequency and produces the correct intermediate frequency when the returning signal is from a radar beacon transmitter.

As the crystal is easily damaged by high-powered signals, the TR box spark gap protection must be certain. To insure that it will arc over at once when the transmitter signal appears in the mixing chamber or when other nearby radar transmitters accidentally send in a weaker — but still damaging — signal, a *keep-alive* voltage is provided. This is a high DC voltage that causes the spark gap to be partially ionized. Then the additional voltage of any signal that is stronger than a radar echo will cause complete ionization or an arc between the electrodes. When the set is turned off, no keep-alive voltage is present, and the crystal is vulnerable to signals from other nearby radar transmitters. A spring-operated gate closes the waveguide when the set is turned off. No signal can get to the crystal from the antenna. When the set is turned on, an electrically operated relay opens the gate.

The entire system is assembled with rubber gaskets at the joints, or otherwise sealed to permit pressurizing the interior of the waveguide. The small windows or slots in the antenna cavity are covered with mica. It lets r-f energy through but keeps moisture out.

DIRECTIONAL COUPLER

To measure the power from a radar transmitter, to measure the standing-wave ratio of a waveguide, or to feed a signal to a radar receiver, it is necessary to tap into the waveguide. The device used for this purpose is a directional coupler.

A typical coupler is shown in Figure 11–65A. This is a bidirectional coupler of the reverse type. It consists of two couplers mounted on opposite voltage walls of the waveguide. The lower coupler taps off incident power; the upper taps off reflected power. Each coupler consists of an auxiliary waveguide section that receives energy from the main waveguide through two coupling slots. At one end of the auxiliary guide is a probe to couple energy to test equipment. At the other end is a load to dissipate unwanted power.

Note that the coupling slots are located on opposite sides of the center line of the waveguide. In addition, slot A is a quarter-wavelength farther along the guide than slot B. The magnetic field on one side of the center line of the waveguide is 180° out of phase with that on the opposite side. This is illustrated by Figure 15–65B. Since the slots are on opposite sides of the center line of the waveguide, they interrupt currents traveling in opposite directions, as shown by the arrows. For this reason, energy coupled into slot A is 180° out of phase with energy coupled into slot B.

Figure 11–66A illustrates the functioning of the coupler when it is used to tap off incident power. A_0 is a point of reference, and is a quarter-wavelength from point B. The two parallel lines *do not* indicate parallel paths of power; rather, they are used to indicate energy in opposite halves of the waveguide. Keep in mind that the distance AD is the same as distance BC ($\lambda/4$); their relative lengths appear different on the illustration because it is a two-dimensional drawing. A small amount of the energy traveling down the transmission line toward the antenna couples through slot A and travels a quarter-wavelength to point D. One quarter-wavelength farther down the line, energy couples through slot B. This energy travels path BCD to reach point D.

Energy reaching point D from slot A travels a quarter-wavelength (equivalent to 90°). Energy reaching point D from slot B travels the distance from A_0 to B ($\lambda/4$), plus path BCD ($\lambda/2$), the sum of which is equivalent to 270°. This results in a phase difference of 180° between the two signals. Add to this the 180° phase difference due to the energies being coupled from opposite sides of the center line of the waveguide, and the total phase difference is 360°. Since this represents a full cycle, the fields at point D are in phase, and power is available at the probe. Energy from slot A can also travel the path ADC

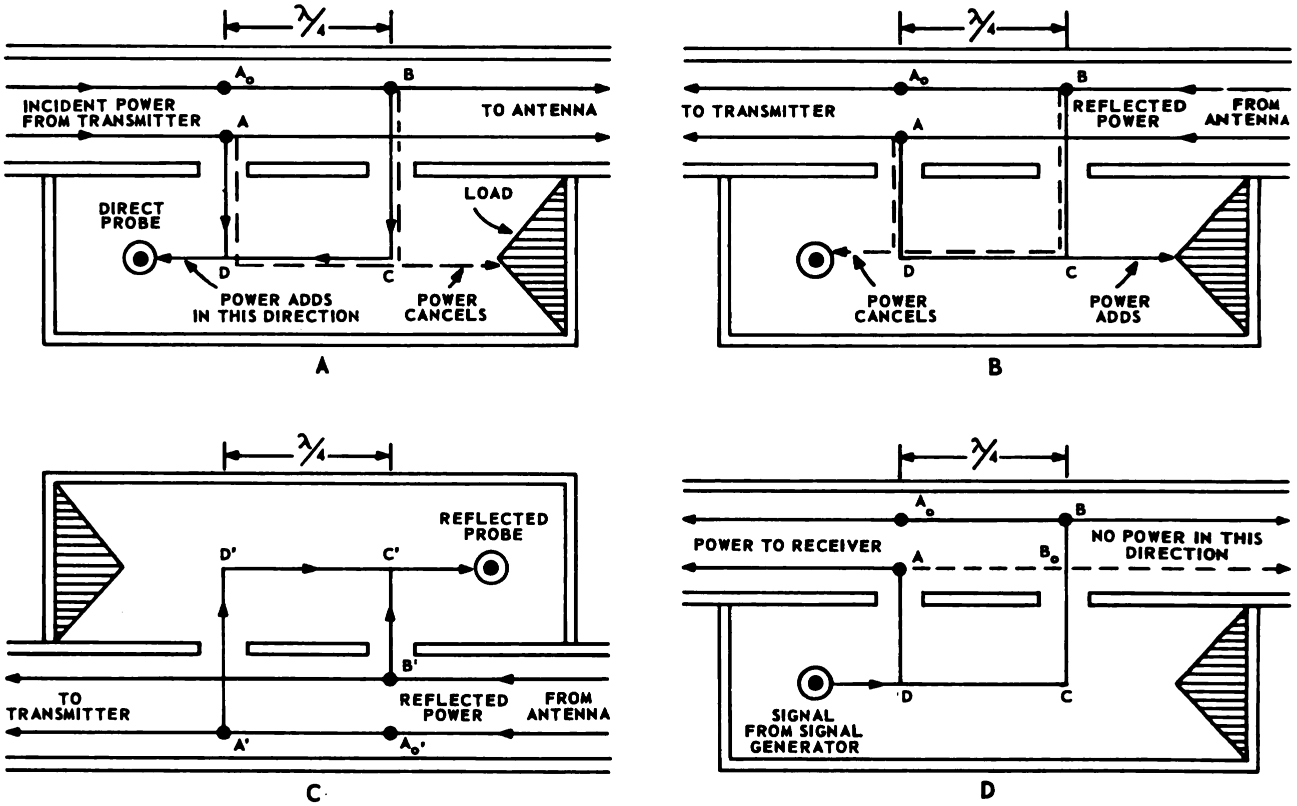


Figure 11–66. Principle of Operation of Bidirectional Coupler

($\lambda/2$), and power from slot B can travel the path A_0BC ($\lambda/2$). Since the paths are equal, and since the energy coupled through slot B is 180° out of phase with that from A, the fields cancel at point C.

Figure 11–66B illustrates the action of the coupler when reflected power is traveling along the waveguide in the opposite direction. In this case, the power components add toward the load in the coupler and are dissipated. Power cancels toward the probe; hence, no reflected power is measured. This illustrates the meaning of *directional coupler*, a device that is designed to measure power flowing in only one direction in the waveguide. With this coupler, therefore, it is possible to measure the incident power without its being affected by the standing-wave ratio. If the waveguide has a low standing-wave ratio, the incident power is a good measure of the power delivered by the transmitter.

In order to measure standing-wave ratio, it is necessary to measure both incident and reflected power. To accomplish this, a second coupler must be used. It is placed on the opposite side of the waveguide and turned in the opposite direction. Note that this coupler is identical to the one just discussed, with the exception that the probe is in the opposite end of the coupler. (See Figure 11–66C.) The power coupled through slot B′ is 180° out of phase with the power coupled through slot A′. However, power entering at slot A′ must travel a half-wavelength farther, so it is shifted 180° by the time it reaches point C′. Therefore, the two fields arrive at C′ in phase and add. As a result, reflected power is coupled to the probe.

Incident power will also enter these slots, but will cancel toward the probe and have no effect.

Figure 11–66D illustrates the feeding of a signal to the main waveguide through the lower coupler. The signal entering the coupler at the probe travels the paths shown to coupling slots A and B. At point A the energy coupled through slot A has traveled a quarter-wavelength (AD). The energy is coupled into one side of the waveguide. By the time the energy coupled through

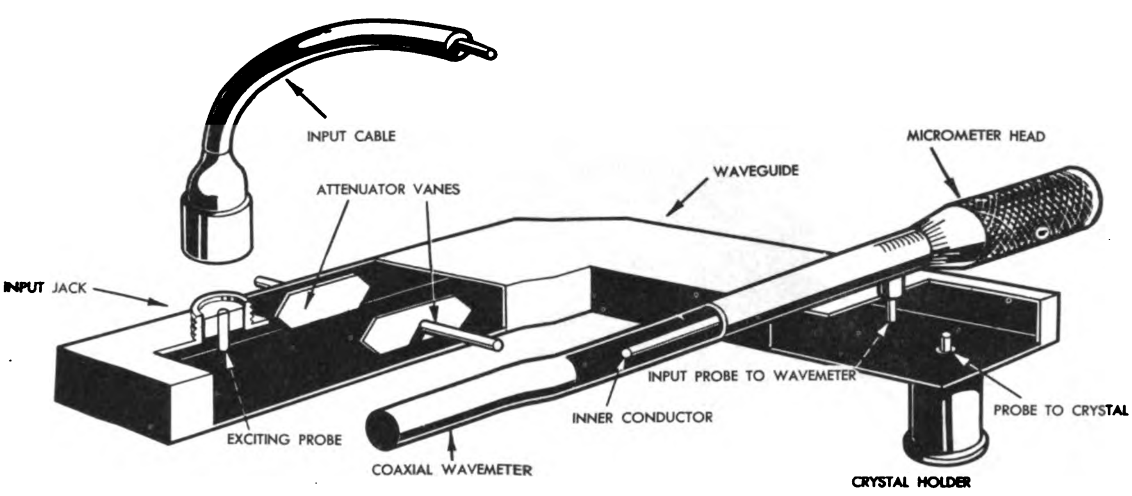


Figure 11–67. Accurate 3-cm Frequency Meter using a Waveguide

slot B reaches point A_0, it has traveled three-quarters of a wavelength ($DCBA_0$). This is a half-wavelength farther than the energy coupled through slot A. It is, therefore, 180° out of phase with the energy coupled through slot A. But this energy is coupled into the opposite side of the waveguide, and hence it is in the correct phase relationship to excite the guide in its dominant mode. Therefore, the power will travel down the guide to the receiver.

The signal coupled through slots A and B will also attempt to travel toward the antenna to be lost as radiation. But no power can travel in this direction. Signal paths DAB_0 and DCB are equal, hence power coupled into one side of the center line of the waveguide (through slot A) is in phase with that coupled into the opposite side through slot B. Thus, the waveguide cannot be excited in the dominant mode in this direction and no energy is lost through radiation.

TEST EQUIPMENT

A resonant cavity can be used to measure the frequency of the signal in a waveguide. When approximate measurements are to be made, either the ring box or echo box can be used. When more accurate measurement is desired, a frequency meter of the type shown in Figure 11–67 is used. In this meter a coaxial cable brings the signal from a test jack in the waveguide of the radar set to the input jack of the wavemeter waveguide. A probe sends the signal down the waveguide through attenuator vanes. When these vanes are at the walls of the waveguide they have no effect, but when they are moved together, they reduce the wide dimension until it is less than the cutoff wavelength for the frequency involved. The signal bounces back and forth between the vanes, with very little of it getting out of the waveguide proper. It then goes to a coaxial cavity.

This cavity is similar to that previously illustrated in Figure 11–53C, except that it has a micrometer head to insure maximum accuracy of reading. The coaxial line is resonant when it is 15 quarter-waves long in this test set. Resonance is indicated by the rectified output from the crystal, which is used to swing a meter; or if pulsed signal is used, the amplitude of the pulse can be observed on a test oscilloscope.

You can measure the standing waves on any waveguide system by inserting a slotted section into the waveguide as shown in Figure 11–68, and then using a travel-probe to measure the electric field strength. A special assembly fits on the slotted section onto the waveguide. This assembly picks up energy by a probe and transmits it through a short waveguide to a crystal, where it is rectified and then carried by a coaxial cable to an indicator, which is similar to that described earlier in the slotted coaxial line.

ADJUSTMENTS

Although there are many adjustments in a waveguide system in any radar set, most of

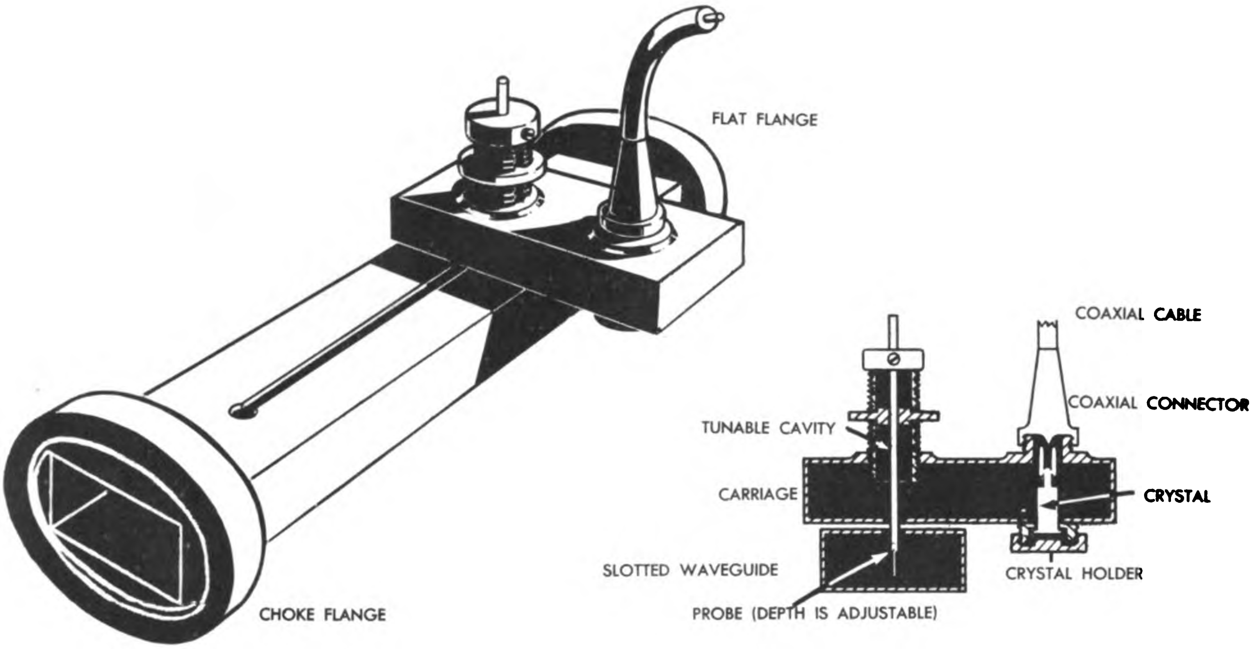


Figure 11–68. Slotted Waveguide Section Permits Access to Fields in Waveguide System

them are made at the factory. For example, almost all of the probes which a waveguide system uses cause an impedance mismatch. This mismatch must be tuned out. The plugs of the type shown in Figure 11–69 are provided to get rid of the mismatch, but adjustment is made at the factory where possible and the adjustment is fixed by soldering.

On the other hand, an adjustment which the radar mechanic does make deals with the amount of signal introduced by the local oscillator. This can be varied by making the probe in the mixing chamber longer or shorter. This probe, which is permanently fixed to the oscillator, moves the entire oscillator up and down with respect to the waveguide.

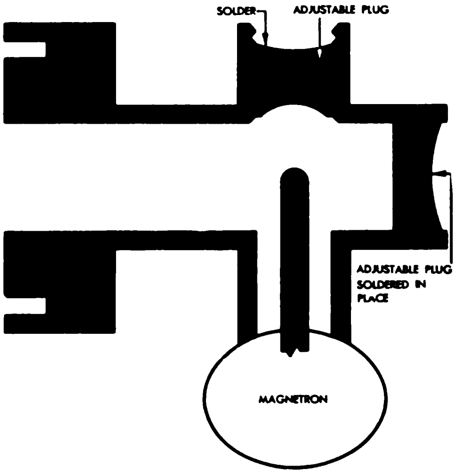


Figure 11–69. Mismatch and Reflections Introduced by Magnetron Probe are Tuned Out by Plugs

CHAPTER 12

UHF Oscillators

A radar transmitter consists of an oscillator and its associated pulsing circuits. The theory of pulsing circuits has been discussed to some extent in previous chapters. One purpose of this chapter is to acquaint you with oscillators and oscillatory circuits employed in radar transmitters; another is to give you additional information about pulsing circuits in transmitters and to acquaint you with local oscillators used in radar receivers. Specifically, it discusses problems involved in generating high-frequency signals, explains the theory and operation of a number of oscillators employed in radar equipment, and describes various methods for pulsing oscillators.

ADVANTAGES OF HIGHER FREQUENCIES

Radar oscillators are operated at higher frequencies for three reasons: higher frequencies produce the best echoes, they make it possible to detect smaller targets, and they permit the use of small antennas.

As mentioned previously, radar transmitters send out short pulses at regular intervals. Usually, the duration of these pulses is a microsecond or less to prevent the echo from any particular pulse from being returned by nearby targets before that pulse ends at the transmitter. Furthermore, it is proved by experiments that satisfactory echoes require that the transmitted pulse consist of 200 or more oscillations. Obviously, then, the only way for a narrow pulse to contain 200 oscillations is to operate the transmitter oscillator at ultra-high or microwave frequencies; for example, 200 million oscillations per second for a 1-microsecond pulse.

The size of the target that radar can detect depends mostly on the frequency. In general the higher the frequency, the smaller the target that the radar set can detect. A target gives best echoes when it is about a half-wave, or some multiple, of the frequency being transmitted. If the target is much smaller than a half-wave, the intensity of pulse that it reflects is low. However, as the operating frequency is increased, the half-wave becomes smaller and makes it possible to detect smaller targets.

Perhaps the main reason for using higher frequencies is that it is possible to construct directive antennas in a smaller space than when lower frequencies are used. Small directive antennas are an especially important consideration in airborne equipment where space and weight are major problems.

FREQUENCY LIMITATIONS OF USUAL OSCILLATORS

The principal factors which limit the frequency of the ordinary type oscillator are the construction of the tube, and the external circuits that connect to the tube.

Tube Construction

The tube construction factors that limit the operating frequency of the usual, or conventional oscillator are the interelectrode capacitances in the tube, the inductances of the leads, and the transit time.

INTERELECTRODE CAPACITANCES. At ordinary radio frequencies the interelectrode capacitances in a vacuum tube have reactances which are so large that they do not cause any serious trouble. However, as frequencies increase, the reactance of these capacitances become small enough to materially affect the performance of a circuit. A 1-mmf capacitor, for example, has a reactance of 159,000 ohms at 1 mc. If this capacitor is the plate-to-grid capacitance and the r-f voltage between these electrodes is 500 volts, there will be an interelectrode capacitance current flow of 500/159000 or .00315 ampere (3.15 ma), an amount of current that will not seriously disturb circuit operation. On the other hand, at 100 mc the reactance of this capacitor becomes 1590 ohms and the current flow is 500/1590 or 0.315 ampere, an amount which will seriously affect circuit performance.

A good point to remember is that the higher the frequency or the larger the interelectrode capacitance, the higher the current flow through this capacitance. In most UHF oscillators, interelectrode capacitance currents are much greater than the power currents supplied by the tube. These higher currents cause power losses in the resistance in the oscillatory circuit.

Since interelectrode capacitances are effectively in parallel with the tuned circuit, they affect the frequency at which the tuned circuit resonates. As you can see in Figure 12–1, the plate-to-cathode capacitance is in parallel with the series combination of the plate-to-grid capacitance and the grid-to-cathode capacitance. All these capacitances together form a part of the total capacity of the tuned circuit. When the tube is operating, the effective capacitance varies due to the Miller effect.

Interelectrode capacitance limits the frequency by establishing a minimum capacitance below which it is impossible to go. In addition, interelectrode capacitance varies with the applied voltages and with the loading of the oscillator. This causes frequency instability, particularly when the interelectrode capacitance forms a large part of the tuning capacitance.

INDUCTANCE OF LEADS. Another frequency-limiting factor within a tube is the inductance of the leads to each tube element. While these inductances do not necessarily impair the efficiency of the oscillator, they may represent a major portion of the inductance of the tuned circuit and limit the frequency by setting a minimum limit on the inductance. Furthermore, since the cathode lead is common to both plate and grid circuits, feedback takes place through it and produces an additional loss of efficiency.

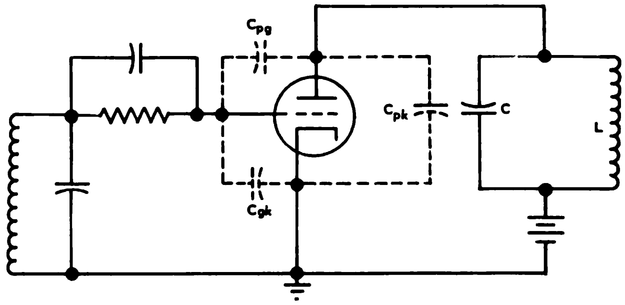


Figure 12–1. Effect of Interelectrode Capacitance on Frequency of TPTG Oscillator

TRANSIT TIME. A third limitation imposed by tube construction is *transit time.* Transit time is the time required for electrons to travel from cathode to plate. At lower frequencies transit time is negligible, since it occupies only a comparatively small portion of an oscillatory period. But as the frequency becomes higher, transit time occupies an appreciable portion of this period and produces undesirable effects in tube operation. The effect of transit time is of special concern in connection with the input impedance of the tube.

Part of the current that flows in the grid circuit is the current which charges the grid-to-plate capacitance, C_{gp}. The voltage that produces this current is the vector sum of the input voltage (grid-to-cathode), and the output voltage across the plate load. At lower frequencies, with a resistive load, these two voltages are 180° out of phase and add algebraically to determine the charging current. This current is 90° out of phase with the input voltage. But at higher frequencies where transit time is an important factor, the plate current begins to lag the input voltage. This causes the plate voltage to be less than 180° out of phase with the input voltage, and the voltage across the capacitor to lag the input voltage slightly. As a result, the charging current will no longer be 90° out of phase but will have an in-phase component. This means that power is consumed in the grid circuit. This consumption of power is effectively the same as adding a high resistance in the input impedance. This resistance decreases as the frequency increases.

OVERCOMING TUBE LIMITATIONS. There are several ways to reduce the effects of interelectrode capacitances in vacuum tubes. None are completely satisfactory. One is to move the electrodes farther apart. However, this is hardly desirable because it increases the transit time. Another method is to reduce the size of the tube and the electrodes. This is satisfactory except for one ill effect. It decreases the power handling ability of the tube directly with the square of the factor by which the electrodes are reduced. Another method is to separate the leads and to bring them out of the envelope at the nearest point. This results in a slight decrease in the capacitances.

Similarly, there are several ways to reduce the inductances of the leads. As just mentioned, bringing out electrode leads through the envelope at the nearest point produces a slight decrease in the electrode capacitance in a tube. This also decreases the inductances of the leads. Another method is to make double connections to the electrode. This makes two parallel inductances which cuts the lead inductance in half. Another method is to arrange the leads as extensions of external transmission lines.

For reducing transit time, closer spacing of electrodes and higher plate voltages are used. Closer spacing of electrodes, however, produces higher interelectrode capacitances. Therefore, this method can be used only when the electrodes are made smaller. For this reason radar oscillators operate at very high plate voltages. The fact that a radar transmitter does not operate continuously is an advantage in that high enough voltages may be used to reduce transit time without exceeding the maximum power rating of the tube. Operating with high voltage does, however, necessitate some precautionary measures, such as separating the seals of the leads to avoid the presence of excessive voltage gradients in or on the surface of the glass envelope, and avoiding sharp projections on the high-voltage leads or electrodes to prevent arcing.

Limitations Due to External Circuits

Increases in frequency cause a quite rapid increase in the power losses in the external oscillatory circuit. These power losses are due principally to skin effect and radiation.

SKIN EFFECT. Skin effect causes a considerable increase in the resistance in a vacuum tube circuit. This results in a lower Q and increased I^2R losses. To prevent skin effect losses, conductors are made large in size and tubular in shape since current flows only in the surface. In addition, they are plated with silver, since it has a higher conductivity than copper, the material out of which most conductors are made.

RADIATION. Radiation, the other cause of power loss, is due to incomplete cancellation of electromagnetic fields in the region surrounding the circuit. When the frequency is low enough so that the spacing between two parallel conductors equals only a very small fraction of a half-wavelength, there is almost complete cancellation of fields in all directions. At higher frequencies, however, an identical spacing would represent a larger fraction of a half-wavelength. This means less cancellation.

In extreme cases, such as where the spacing is a half-wavelength, the fields add in the direction of the plane containing the two conductors. This causes the tuned circuit to radiate energy as does an antenna. Therefore, as the frequency increases, it is necessary to reduce the spacing between the parallel elements. But there is a limit on how far you can reduce the spacing. Too close spacing, for example, causes arcing, which materially increases the r-f resistance of the tank circuit. Another means of eliminating radiation is using concentric lines instead of open wire lines. This eliminates radiation entirely, since the outer conductor acts as a shield, preventing the electromagnetic field from expanding into space.

LIMIT OF INDUCTANCE AND CAPACITANCE. At UHF frequencies it is necessary that the inductances and the capacitances in the oscillatory circuit be very small. The limit of capacitance is the sum of the interelectrode capacitances and the distributed capacitance of the leads. The limit of inductance is the lead inductance plus the inductance necessary to connect the tube electrodes externally. UHF oscillators approach both these limits, the only capacitance in the tank circuit being a small trimmer capacitor for fine tuning adjustments, and the inductance being a short-circuited transmission line less than a quarter-wavelength in length.

Figure 12–2 illustrates a typical parallel line tank circuit. The expression for the input reactance (looking at the circuit from the tube) of the tank circuit is

$$Z = jR_c \tan B_e$$

where R_c is the characteristic resistance of the line and B_e the electrical length.

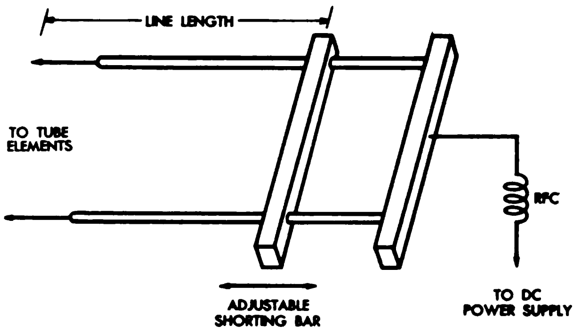


Figure 12–2. Typical Parallel-Wire Resonant Circuit

As you can see in the expression, if B_e is less than 90° (λ/4), then tan B_e is positive and the reactance is inductive. For smaller values of inductance the length of the line is decreased. Further, this expression is true only when the resistance losses are low enough to be neglected. Frequently the tank circuit is located near a flat surface and parallel to it. The flat surface acts as a shield and, while it does not affect the electrical length of the line, it does lower the characteristic impedance of the line.

TRIODE OSCILLATORS AS TRANSMITTERS

A variety of oscillators are used in radio and radar transmitter equipment. While this discussion deals primarily with their use in the transmitting circuits of this equipment, it is also applicable to local oscillators in receivers. The main difference between the two kinds of oscillators is that the radar transmitter oscillator is pulsed and produces considerable power during each pulse, while the local oscillator operates continuously at a lower power output than the transmitter oscillator.

Ultraudion Oscillator

The typical ultraudion oscillator in Figure 12–3 is used as a local oscillator in some radar receivers, and as a transmitter in certain radar sets. The circuit shown is incapable of generating enough power to be useful for high-power long-range sets.

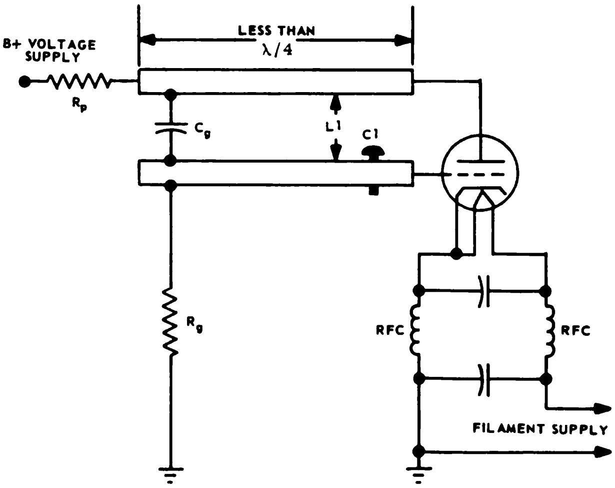


Figure 12–3. Ultraudion Oscillator

The DC circuits consist of the components, R_p, C_g, R_g, and the small DC resistances in the tank circuit and leads. The components, C_g and R_g, form the grid-leak bias circuit. The charge built up on C_g biases the tube to class C operation.

To understand the operation of the ultraudion oscillator, consider its equivalent circuits shown in Figure 12–4. So far as r-f is concerned, in the equivalent circuit the chokes in the cathode are open circuits. The tank circuit inductance L consists of the plate and grid lead inductances connected in series with the parallel combination of L_1 and C_1.

The simplified equivalent shows that this tank circuit inductance forms a resonant circuit with the combined interelectrode capacitance, C, which is given by the equation,

$$C = C_{gp} + \frac{C_{gk} \times C_{pk}}{C_{gk} + C_{pk}}$$

The expression for the frequency of this resonant circuit is,

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

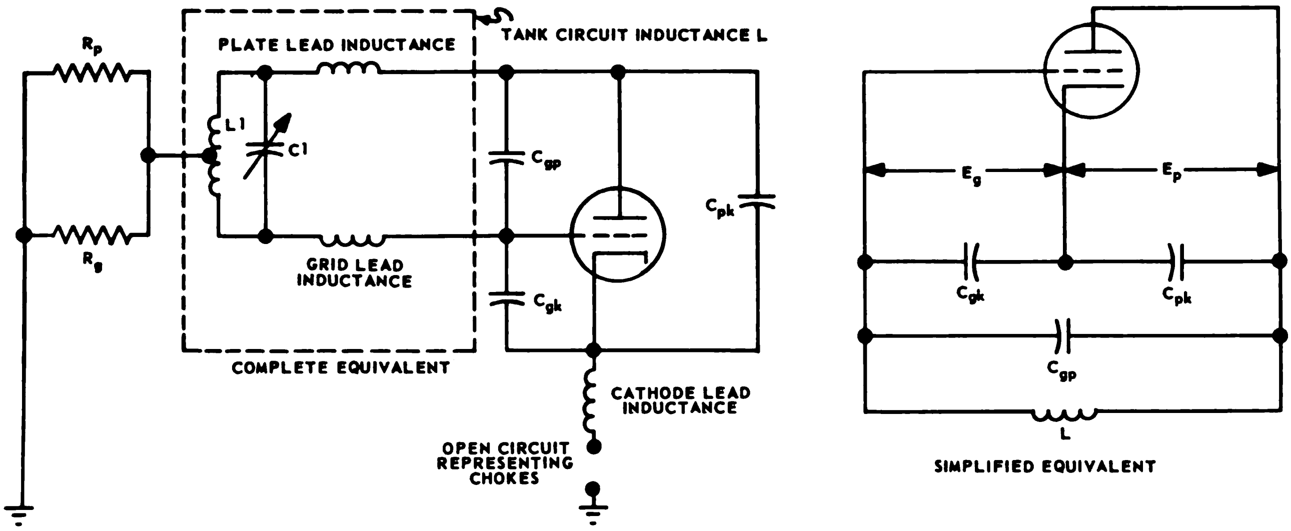


Figure 12–4. Equivalent Circuits of Ultraudion Oscillator for R-F Currents

Basically, the circuit of the ultraudion oscillator (see the simplified equivalent) is that of a Colpitts oscillator. The capacitances C_{gk} and C_{pk} form a voltage divider circuit. The voltage E_g developed across C_{gk} is in the right phase and high enough to sustain oscillations. The minimum amplification required to sustain oscillations is given by the formula $A = C_{gk}/C_{pk}$. If the initial gain is greater than this value, oscillations build up to such an amplitude that the gain is reduced to the value given by this formula.

Lighthouse Tube Oscillator

The lighthouse tube oscillator like the ultraudion oscillator is a single tube oscillator. It is used in a few low-powered radar sets such as short-range gun-laying equipment and in some types of transportable beacon equipment, and also as a local oscillator in ground radar equipment. In a typical application the lighthouse tube, which gets its name from its peculiar construction, operates at a frequency of 2500 mc.

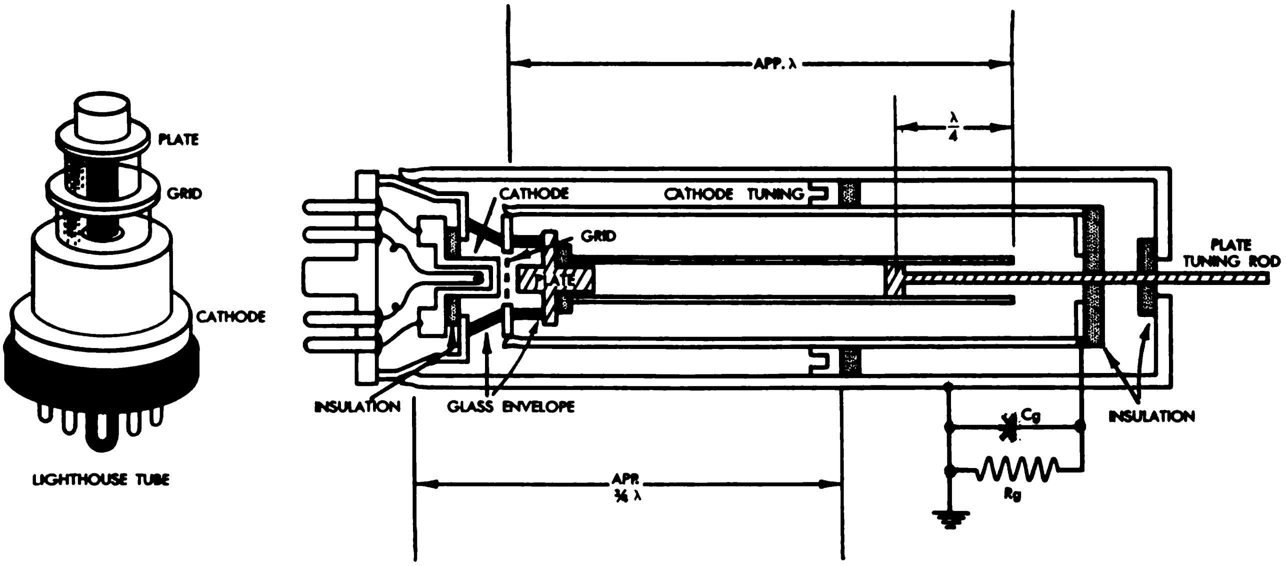


Figure 12–5. Lighthouse Tube Oscillator

Its plate, grid and cathode connections are arranged so that it may be mounted directly in the tuning assembly, as illustrated in Figure 12–5.

The component called the tuner consists of three coaxial cylindrical conductors. The inner cylinder makes contact with the plate, the next with the grid, and the outer conductor with the shell of the tube. The outer conductor provides an r-f connection to the cathode through capacitance. The space between the cathode and the grid conductors forms a coaxial cathode line which is shorted by a plunger. This plunger does not touch the grid conductor since this would form a DC connection from grid to cathode. The capacitance between the plunger and the grid conductor is great enough to form a virtual short circuit for r-f. The DC grid-to-cathode path, consisting of R_g and C_g, forms the grid-leak bias circuit.

In the plate circuit, the plate line consists of the grid and plate conductors. It has an open circuit at the end away from the tube. The plate voltage is applied to the inside of the plate line through the plate tuning rod at a point which is a quarter-wavelength from the end. This quarter-wave section, which is shorted at the point where B+ is applied, presents a high impedance to r-f from the open-ended plate line and, therefore, prevents r-f from flowing into the power supply.

Equivalent Circuits. The following equivalent circuits, Figure 12–6, show the evolution of the lighthouse tube oscillator. In the first equivalent circuit the components, L_k and L_p, represent the inductances of the shorted cathode line and the open-circuited plate line respectively. This is a true representation of the actual circuit, provided that the cathode line is less than three-quarters of a wavelength long and provided that the plate line is less than a full wavelength long.

The three-quarter wavelength and full wavelength were chosen for this circuit rather than the one-quarter and one-half wavelength because at the frequency of operation the smaller fractions of a wavelength are inconveniently short. However, keep in mind that if the cathode line is slightly less than a quarter-wavelength, it appears as an inductance and that if the plate line is slightly less than a half-wavelength, it appears as an inductance. There is no lead inductance shown in the circuit since the lines connect directly to the electrodes. The interelectrode capacitances complete the equivalent circuit.

In the equivalent circuit, Figure 12–6B, the parallel combination of L_p and C_{gp} are represented by X_2 and the parallel combination of L_k and C_{gk} by X_1. Obviously the reactances X_2 and X_1 along with C_{pk}, which you can consider unknown reactances, must form a resonant circuit. The reactance X_1 forms a voltage divider circuit with C_{pk}. Since the voltage across X_1 is the voltage fed back to the grid, it must be 180° out of phase with the plate voltage, which can happen only if X_1 is a capacitive reactance. This condition leads to the third equivalent circuit, Figure 12–6C, in which the reactance X_1 is represented by a capacitor.

An oscillatory circuit requires at least one inductance; therefore, X_2 must be an inductive reactance. Hence in the third equivalent circuit (C) the reactance X_2 is shown as the inductance L. This circuit is representative of the oscillator circuit only at the resonant frequency. Examine this circuit and note that basically it is a Colpitts

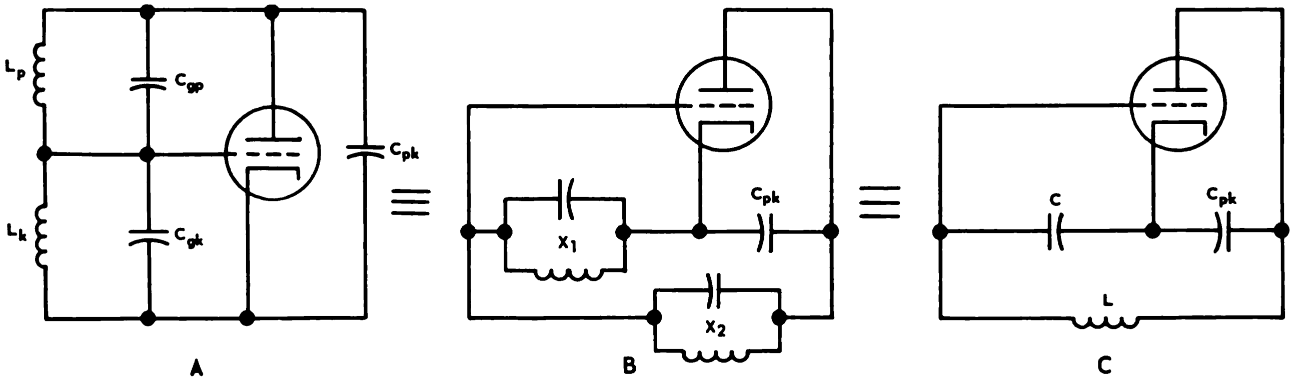


Figure 12–6. Evolution of Equivalent Circuit of Lighthouse Tube Oscillator

oscillator. Now, if X_1 is capacitive, C_{gk} must conduct more heavily than L_k; therefore the oscillator frequency must be above the resonant frequency of L_k and C_{gk}. Likewise, for X_2 to be inductive, the oscillator frequency must be below the resonant frequency of L_p and C_{gp}. Thus, you see that the oscillator frequency is between the resonant frequencies of the plate circuit and the cathode circuit.

FEEDBACK. In this oscillator the amount of feedback depends upon the size of C in relation to C_{pk}. This is, in turn, a function of the tuning of the cathode line; hence, the cathode tuning serves principally for controlling the amount of feedback. If feedback is too small, oscillations will be weak and may cease completely. If feedback is too great, power consumption in the grid circuit will become too large and will result in a decrease in output of the oscillator. The value of C, hence of the cathode tuning, has some effect on the frequency of oscillations, but since C and C_{pk} are in series and C is usually much larger than C_{pk}, C must be changed considerably to affect the frequency appreciably.

PLATE TUNING. Since the tuning of the plate line determines the resonant frequency of the plate circuit which, in turn, determines the amount of inductance represented by the plate circuit at the oscillator frequency, the plate tuning is the chief control of the oscillator frequency. It has a secondary effect of feedback, since C is determined by the relative frequencies of the cathode circuit and of the oscillator. You can see from the foregoing discussion that the plate line is tuned to give the correct frequency, and the cathode line to adjust feedback for maximum output from the oscillator. Since there is some interaction, however, it will be necessary to readjust each line for maximum accuracy.

Push-Pull Oscillators

To obtain higher power outputs, two tubes in push-pull circuits, or four or more tubes in a ring oscillator are sometimes used. In push-pull circuits, the tuned circuits are the parallel line (Lecher line) type. This arrangement forms a balanced-to-ground system.

The oscillator circuit discussed first is the push-pull tuned-plate tuned-grid oscillator. Only the oscillatory action is described here. Its pulsing circuits are taken up later in this chapter.

TUNED-PLATE TUNED-GRID OSCILLATOR. In the tuned-plate tuned-grid oscillator, Figure 12–7, the plate and the grid lines are slightly under $\lambda/4$ in length. They are indicated in the circuit of the tuned-plate tuned-grid oscillator as inductances L_p and L_g. The length of the plate line in the circuit may be varied by the shorting bar. While the grid line is of fixed length in this particular arrangement, a little greater flexibility would result if it, too, were adjustable in length.

The inductances L and capacitors C_1, C_2, C_3, and C_4 in each of the cathode circuits form filters to prevent r-f from reaching the filament transformer. The grid-leak bias circuit includes R_g and C_g. The component C_g is shown as a dotted portion since it is actually the distributed capacitance of the grid line to ground. B+ voltage is applied to the plate lines through R_s and L_s. L_s serves the double purpose of preventing r-f from getting into the power supply and of suppressing parasitic oscillations.

In the equivalent circuit of Figure 12–8A, both tubes of the oscillator are shown. L_p is the inductance representing both the plate line and the plate lead inductances. L_g represents the inductance of the grid line and grid leads. L_k is the cathode lead inductance. With proper adjustment of C_k, L_k forms a series resonant circuit that effectively grounds the cathode within the tube.

The second equivalent circuit, Figure 12–8B, shows only one tube since both circuits are identical. L_{pk} is one-half L_p and with C_{pk} forms a circuit that resonates at a frequency above that at which the entire circuit oscillates.

In the equivalent circuit, Figure 12–8C, the parallel combination of L_{pk} and C_{pk} is represented as an inductance L_2. Likewise L_{gk}, which is one-half L_g, with C_{gk} forms a circuit that resonates at a higher frequency than that of the oscillator. This circuit is represented as an inductance at the oscillator frequency. This is true because the tank circuit must have at least one inductive element to oscillate and the two reactances must be alike for the feedback to be in the correct phase. In this equivalent circuit the oscillator is the Hartley type. Since the oscillator frequency must be lower than either resonant frequency (plate or grid circuits), the grid or plate circuit, whichever is lower in resonant frequency, controls the oscillator frequency and the other controls the amount of feedback.

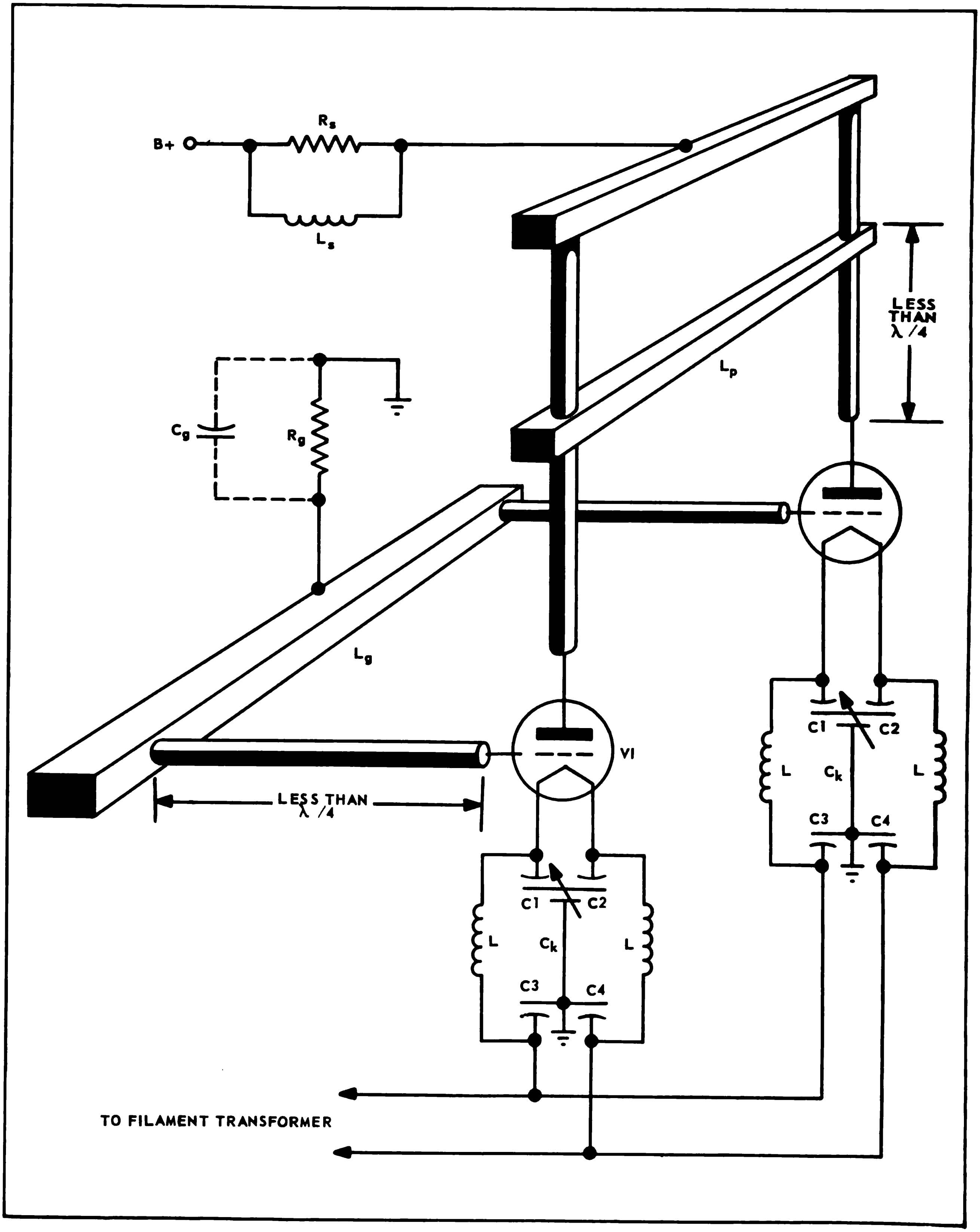


Figure 12–7. Tuned-Plate Tuned-Grid Oscillator using Lecher Lines as Tuned Circuits

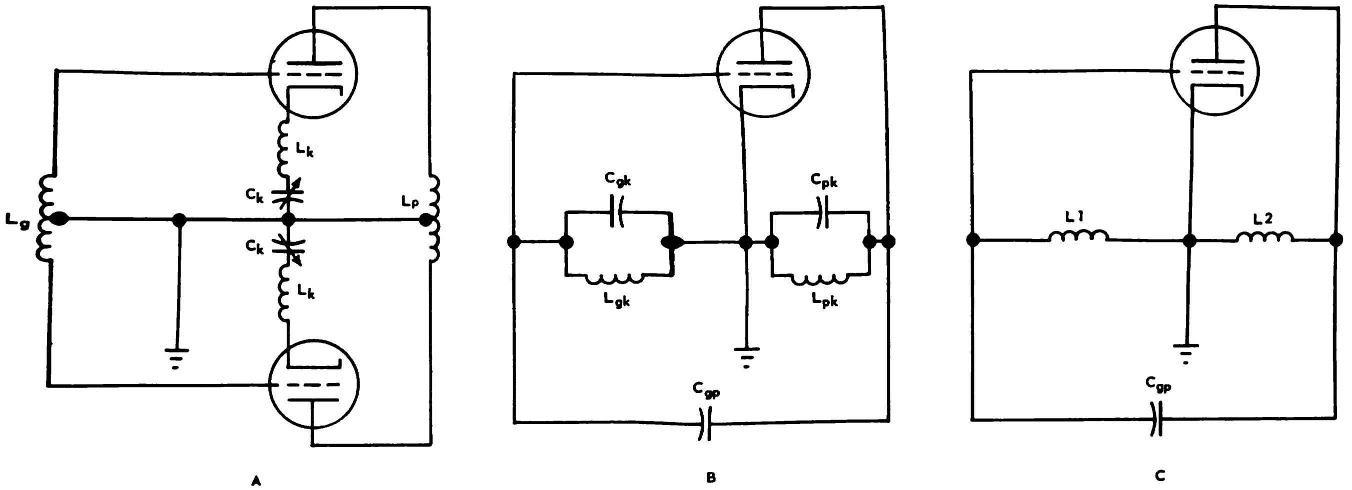


Figure 12–8. Equivalent Circuits of TPTG Oscillator

Tuned-Grid Tuned-Cathode Oscillator. Another push-pull oscillator circuit that is used in several types of UHF radar equipment is the tuned-grid tuned-cathode circuit, Figure 12–9. In this oscillator the plates are effectively short-circuited by tubes that have very little plate lead inductance and that are connected by a section of line. When the lead inductance is large enough to become objectionable, then the plates are connected by a half-wave section of line to insure a short circuit. The DC circuits are the same as in the tuned-plate tuned-grid oscillator.

In the first equivalent circuit, Figure 12–10A, both tubes are shown. The grid line is represented as inductance L_g, the cathode line as reactance X_k, and the interelectrode capacitances as indicated.

The circuit at B shows only one tube. In this circuit L_{gp} represents one half of L_g and X_{pk} represents one half of X_k. By following the line of reasoning similar to that used in the TPTG circuit, you see that the parallel combination of X_{pk} and C_{pk} must be capacitive at the oscillator frequency. Thus, it is shown as capacitor C_k in the equivalent circuit at C.

The oscillator must have an inductive element; hence L_{gp} and C_{gp} in parallel act as an inductance L. For L_{gp} and C_{gp} to be inductive, the grid line must always be less than a quarter-wavelength. Since variations in L have the greatest effect on the frequency of oscillations, *the grid line is the primary frequency tuning device.* The cathode line (X_{pk}) controls only a portion of the total capacitance of the circuit and therefore does not greatly affect the oscillator frequency. However, since X_{pk}, in parallel with C_{pk}, forms a voltage divider with C_{gk}, it has a *considerable effect on feedback.* X_{pk} may be inductive, purely resistive, or capacitive, depending on whether the cathode line length is less than, equal to, or greater than a quarter-wavelength at the frequency of oscillation.

If X_{pk} is a high resistance, the feedback is determined entirely by the voltage divider composed of C_{gk} and C_{pk}. If X_{pk} is capacitive (greater than a quarter-wavelength), then the two capacitances in parallel combine to increase C_k. This results in an increase in feedback. Tuning the cathode line to slightly less than a quarter-wavelength makes it inductive and decreases C_k, which results in a reduction of feedback. Oscillations will cease if X_{pk} and C_{pk} in parallel can no longer be represented as a capacitor. As can be seen by the third equivalent circuit, this oscillator is basically a Colpitts oscillator.

The tuned-grid tuned-cathode type oscillator has three advantages. First, the cathode line offers a convenient means of connecting the load to the oscillator because it is at ground potential with reference to DC. Second, since only DC voltages appear at the plate and only r-f voltages appear in the cathode circuit, the two are not present in the same circuit and the high peak voltages present in a tuned plate circuit are avoided. Third, the tuning of the oscillator is quite simple, the grid line being the frequency

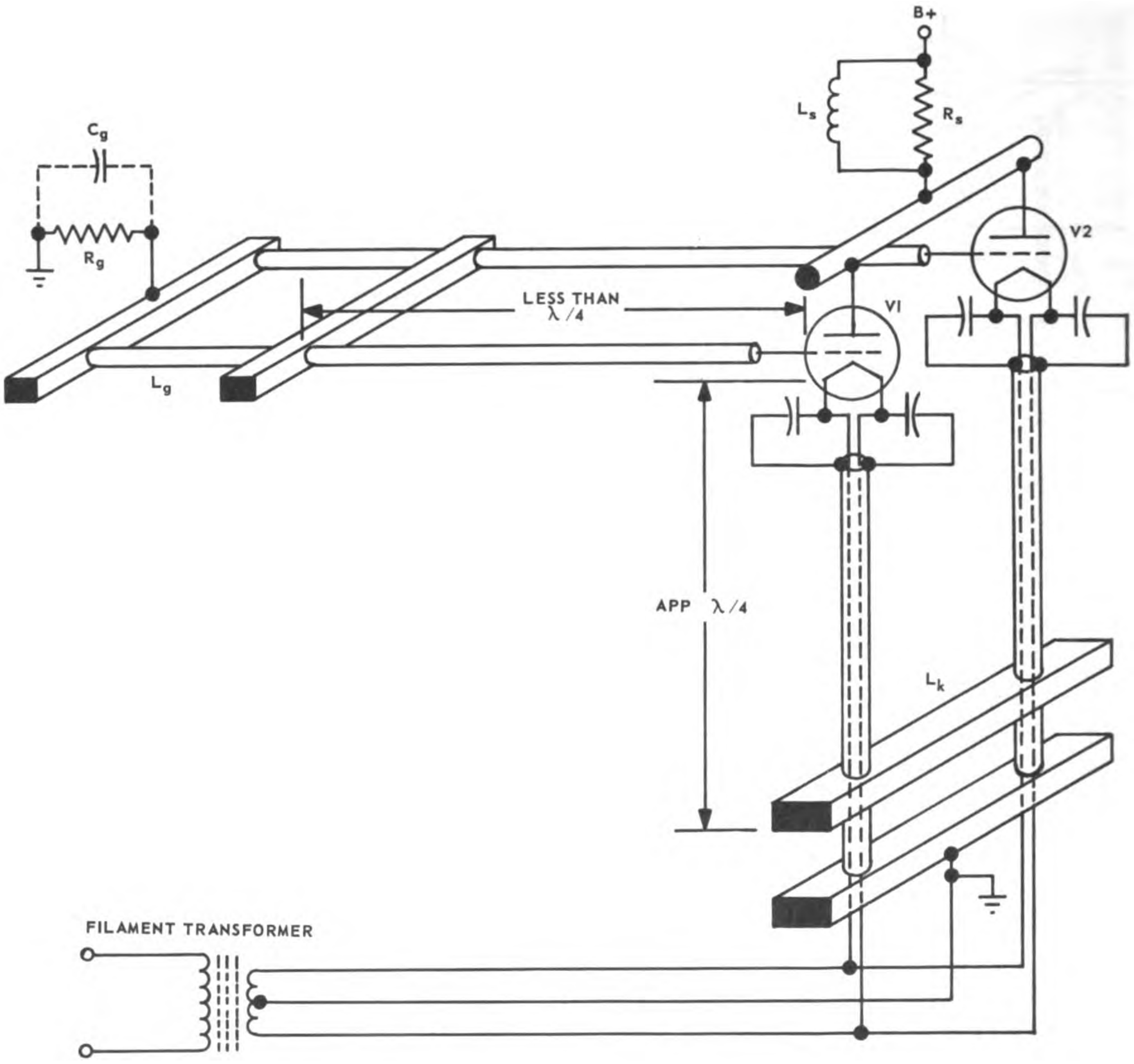


Figure 12–9. Tuned-Grid Tuned-Cathode Oscillator

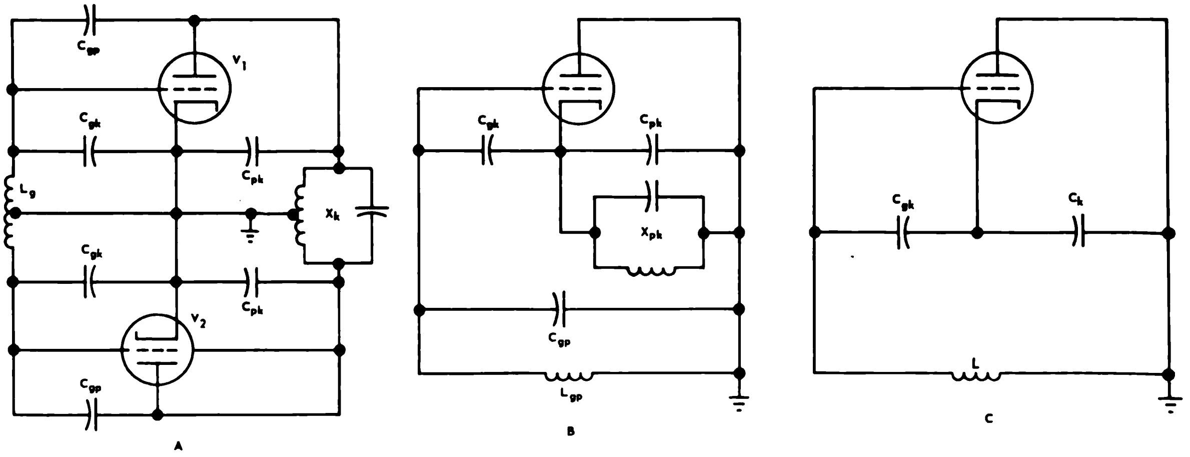


Figure 12–10. Equivalent Circuits of Tuned-Grid Tuned-Cathode Oscillator

control and the cathode line the feedback control. There is some interaction between the tuning controls so that readjustment of each may improve both accuracy and output.

Tuned-Plate, Tuned-Grid, Tuned-Cathode Oscillator Circuit

A number of radar oscillators in the UHF range employ tuned lines in all three leads of the triode, as in the oscillator circuit in Figure 12–11. So far as r-f is concerned, all three tuned lines are grounded at their midpoints. The DC circuits and filament circuits are identical to those described previously. In the circuit shown, the plate and grid lines, which are less than a quarter-wavelength, are labelled L_p and L_g respectively. The cathode line, which may be equal to, less than, or greater than a quarter-wavelength, is labelled X_k.

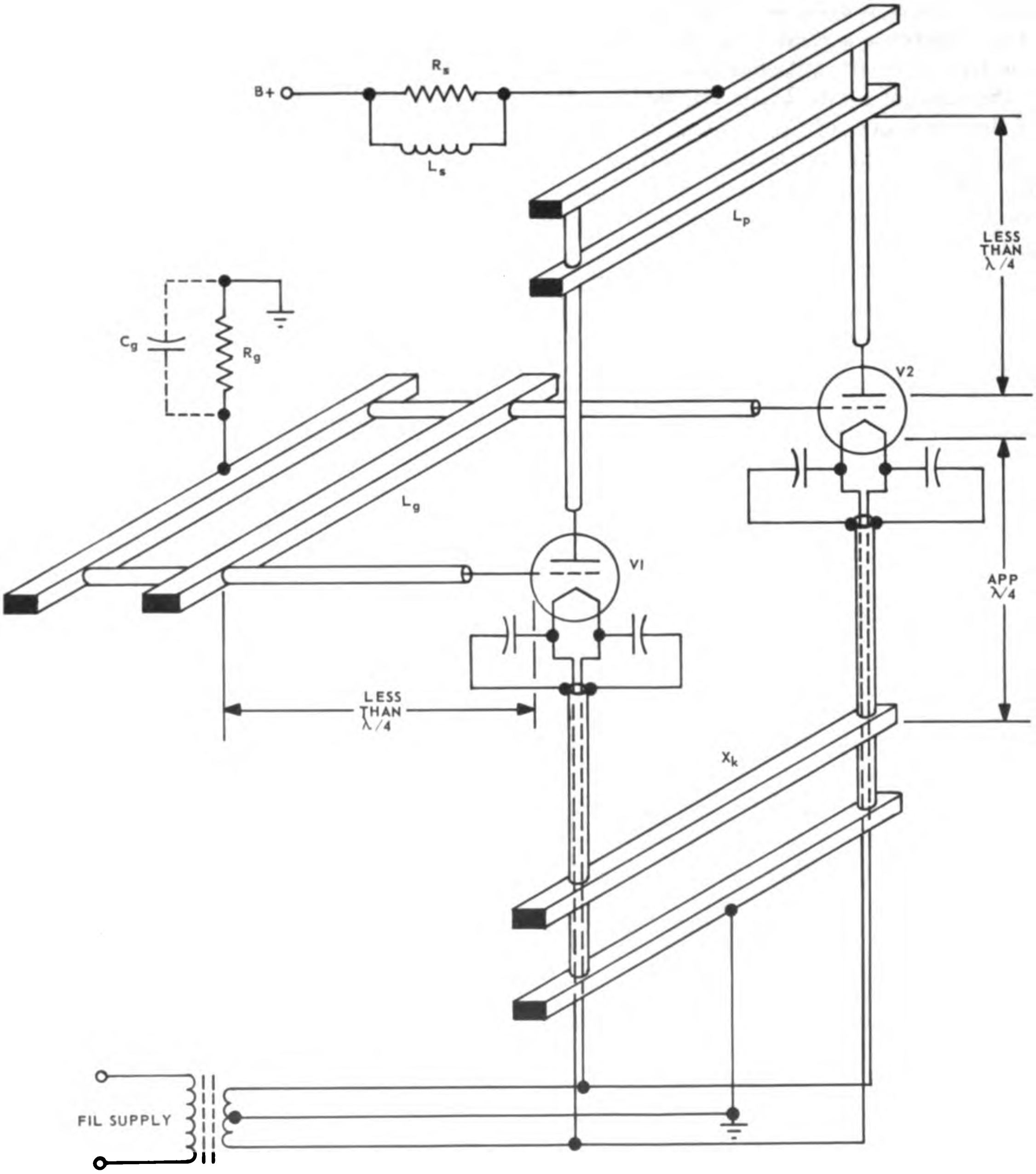


Figure 12–11. Tuned-Plate, Tuned-Grid, Tuned-Cathode Oscillator Circuit

Equivalent circuit, Figure 12–12A, shows both tubes, the inductances L_p and L_g centertapped to ground, and the unknown reactance X_k centertapped to ground. The interelectrode capacitances are not shown in A. Since the circuits of the two tubes are identical, the second equivalent circuit shows only one tube. The inductances L_{pn} and L_{gn} and the reactance X_{kn} represent one-half of L_p, L_g, and X_k. The interelectrode capacitances are added.

Examination of the circuit at B shows that if the cathode line is exactly a quarter-wavelength so that X_{kn} becomes a high resistance, then the circuit is that of a conventional Colpitts oscillator. If the ratio of L_{gn} to L_{pn} is the same as the ratio of the reactances of C_{gk} and C_{pk}, there is no difference in potential between the cathode and ground, and X_{kn} is in effect out of the circuit. This condition also reduces the circuit to that of a Colpitts oscillator. In either case the frequency of oscillation is determined by the sum of the inductances of the plate and grid lines, and the amount of feedback by the ratio of C_{gk} and C_{pk}. Ordinarily this ratio does not give the amount of feedback necessary to cause oscillations for best efficiency; therefore, the cathode line is detuned from the pure resistive quarter-wavelength so that X_{kn} is either capacitive or inductive.

Making X_{kn} capacitive or inductive while keeping the sum of L_{gn} and L_{pn} constant does not change the frequency of oscillation but does increase feedback. The exact amount of feedback depends on the ratio between L_{gn} and L_{pn}. For example, consider the case where L_{pn} is reduced to zero and examine the nature of X_{kn}. Here X_{kn} is in parallel with C_{pk} as shown in the equivalent circuit of Figure 12–10B. Thus, in controlling feedback, the effect of X_{kn} changes is the same as in the tuned-grid tuned-cathode oscillator — that is, the cathode line must be increased in length to increase feedback or be decreased in length to decrease feedback.

On the other hand, if L_{gn} is decreased to zero and L_{pn} is increased to equal the value of their former sum, then X_{kn} is in parallel with C_{gk}. Its effect on feedback then is just the opposite to that described. For example, if the cathode line is greater in length than a quarter-wave, it becomes capacitive, and the parallel combination C_{gk} and X_{kn} becomes a larger capacitance. This reduces the feedback. Reducing the length of the cathode line below a quarter-wavelength makes it inductive, and the parallel combination becomes a smaller capacitance, resulting in an increase in feedback.

This demonstrates that *feedback is a function of both the ratio of* L_{gn} *and* L_{pn} *and of the tuning of the cathode line.* Since the frequency of oscillation is controlled by the sum of L_{pn} and L_{gn} and feedback is controlled both by their ratio

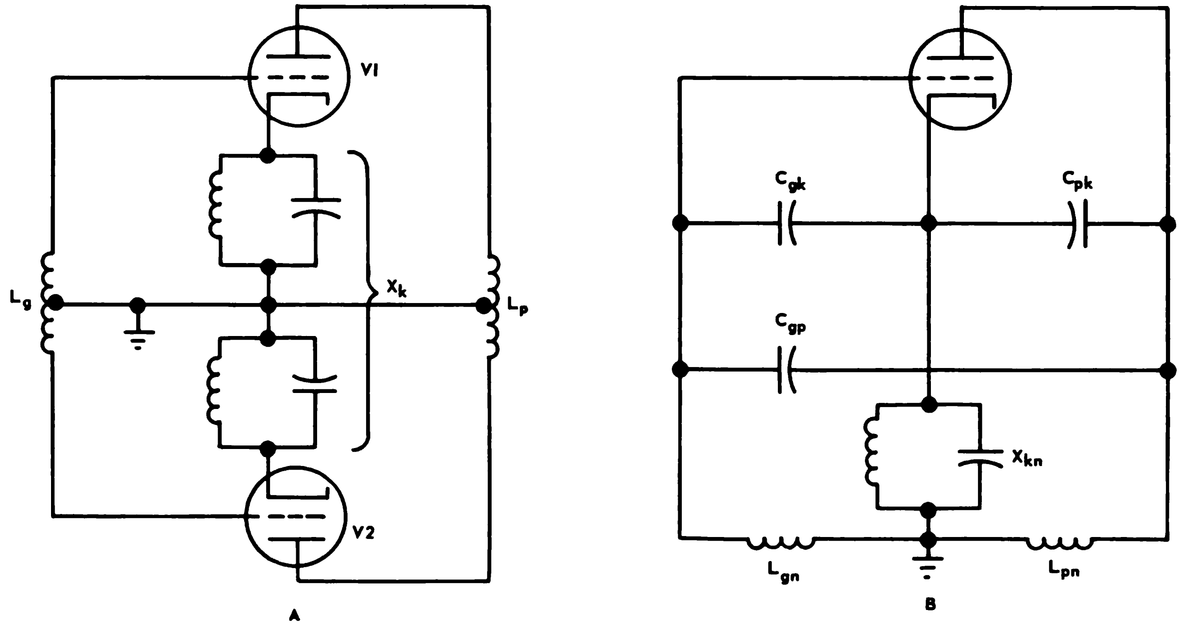


Figure 12–12. Equivalent Circuits of TPTGTK Oscillator

and by the length of the cathode line, there are numerous settings of the three variable line lengths for any given frequency of oscillation and for proper feedback. This is readily understood if you consider that you have three variable elements with which to control two variables, and that there is a certain amount of interaction among the three controls.

Pulsing of Triode Oscillators

There are two methods of pulsing UHF oscillators. One method uses separate modulator tubes; the other employs a self-pulsing oscillator.

THE SELF-PULSING OSCILLATOR. Most oscillators use grid-leak bias or a combination of grid-leak and fixed bias. An advantage of grid-leak bias is that the amplitude of oscillations is limited and stabilized. Although grid-leak bias was taken up in Chapter 4, it is discussed briefly again to introduce the subject of self-pulsing. In this connection notice the graphs of plate current and grid voltage versus time in Figure 12–13. These waveshapes are the same for all oscillators that use grid-leak bias. At zero time plate voltage is applied, but the grids are grounded so that a steady plate current flows. At time t_1 the ground is removed, and any random change in plate current will cause a change in voltage in the plate tank circuit.

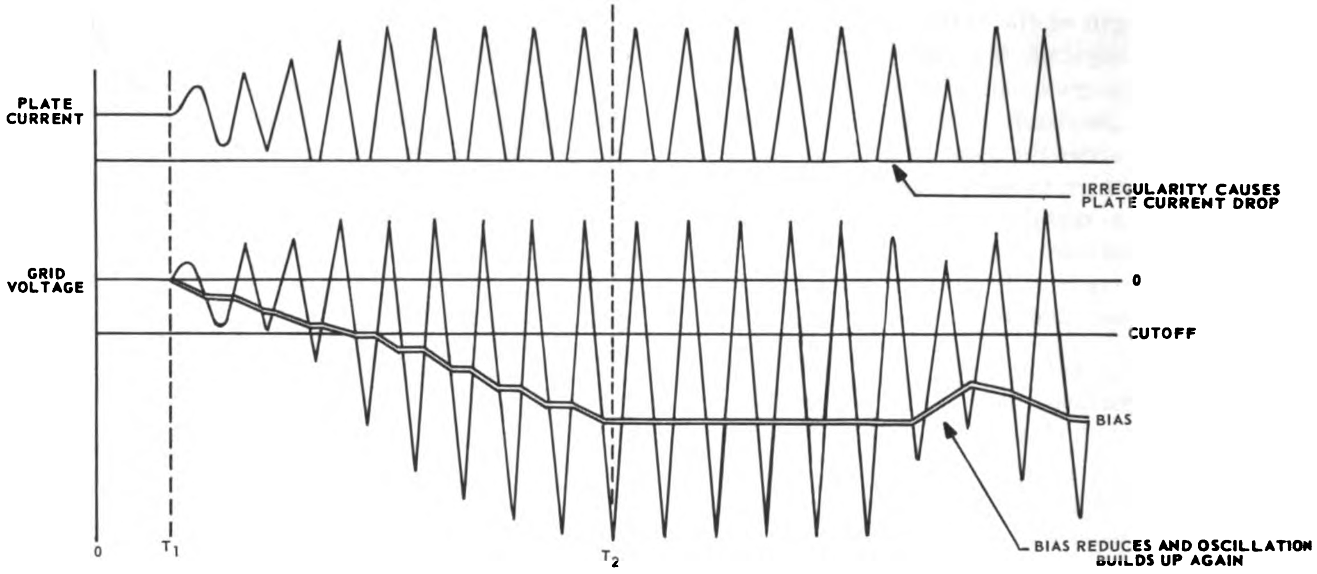
A portion of this change is fed back to the grid circuit. Since this feedback is in the right phase and amplitude, the oscillations will grow in amplitude. Now each time the grid goes positive, grid current flows and charges the grid-leak capacitor more negatively. On each successive oscillation the average grid voltage (the bias) becomes more negative. This continues until a state of balance is reached where just enough grid current is drawn to replace the charge that leaks off through the grid-leak resistor during the remainder of the cycle. In addition, as the average grid voltage becomes more negative, the plate current flows for a shorter portion of each cycle until the flow is just sufficient to replace the energy lost in the tank circuit and to supply feedback to the grid circuit.

With perfect circuit elements and smooth electron emission, the circuit would oscillate indefinitely at this amplitude. But minute irregularities occur in the movement of electrons through resistors and conductors; in addition, emission from a cathode is always in nonuniform bunches. The nonuniformity is slight, but enough to affect the equilibrium in the oscillator circuit. Any slight decrease in current decreases the feedback, and if the time constant of the RC circuit is not too long, the bias is reduced slightly by discharge of the capacitor. The decrease in bias allows a little more plate current, which re-establishes the equilibrium at the old level.

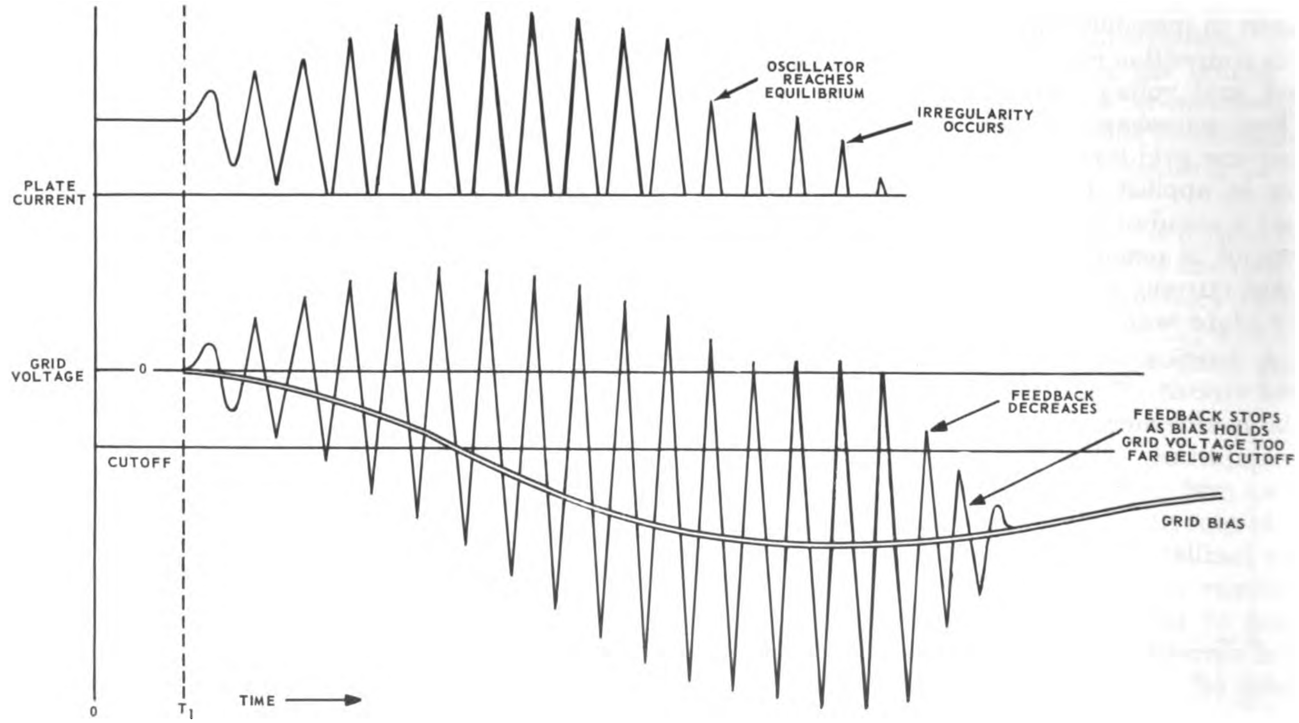
In a circuit with a long RC time constant, however, the circuit starts oscillating, then settles down to a balanced condition as previously described. But any slight decrease in plate current now will not be compensated for by a lowered bias, as shown in Figure 12–13B; the capacitor discharges too slowly. The resulting continued high bias prevents the feedback from being restored to the proper level and after a few cycles, the oscillation dies out completely. The capacitor slowly discharges, and when its voltage reaches cutoff, current again flows and the process repeats.

For continuous operation, then, the RC time constant of the self-biasing circuit must be short enough to allow automatic adjustment of the bias for changing conditions in the circuit. To make the circuit oscillate briefly, then stop, use a long RC time constant. You can see that the time the tube remains cut off depends upon both the grid capacitor and the grid resistance. By properly choosing these values, you can have the oscillator operate for one or two microseconds and then cease operation for 500 to 1000 microseconds, as you desire. You can get an idea of how the variations of grid voltage would look from Figure 12–14A. It is not possible to show the actual time relations since the "off" period is several hundred times as long as the "on" period.

In a radar set that uses a self-pulsing oscillator, synchronizing pulses are taken from the oscillator for starting the sweep on the CRT and for any other circuits that need to be synchronized. In most cases the exact pulse recurrence frequency is unimportant. There are a few sets, however, which use the 400-cps power supply to regulate the PRF. This is done by applying a small synchronizing voltage to the grid as shown in Figure 12–14B and C. Without synchronization the grid voltage rises exponentially and crosses the cutoff line at a small angle. This



A PLATE CURRENT AND GRID VOLTAGE VS TIME SHOWING BUILD UP OF OSCILLATIONS



B EFFECT OF LONG RC

Figure 12–13. Plate Current and Grid Voltage in Oscillators

means that the actual time that oscillations begin may vary. Notice that in C the point at which the grid voltage crosses the cutoff bias line is much more definite. Other ways of controlling the PRF more exactly are to feed in a positive-going pulse when the grid voltage approaches cutoff or to return the grid to a positive voltage rather than to ground.

In order to make the duration of the pulse more definite, some self-pulsed oscillators use an artificial transmission line instead of the grid-leak capacitor. This has the added advantage of

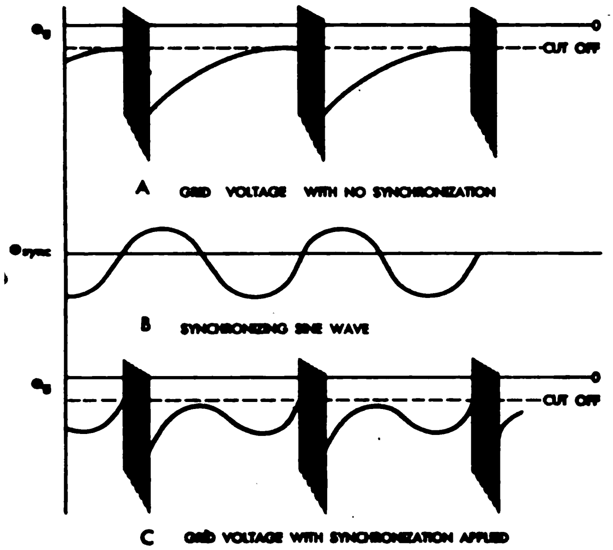


Figure 12–14. Grid Waveshape With and Without Synchronization

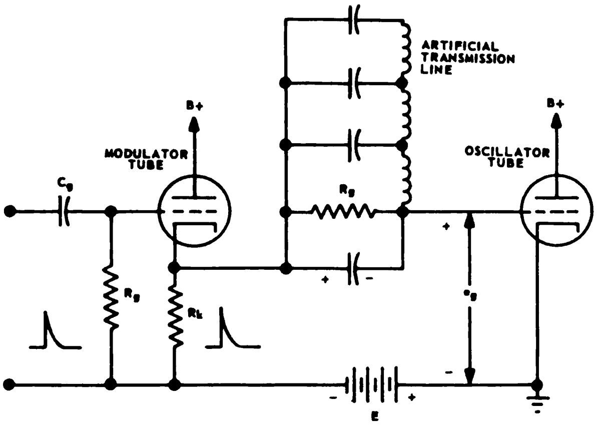


Figure 12–15. Oscillator Circuit with Pulse Line in Grid Circuit

making the operating conditions more nearly constant throughout the pulse. There are two places where the pulse line can be located. One is to put it in the grid circuit as shown in Figure 12–15; the other is in the cathode circuit. (The construction and operation of artificial transmission lines was explained in Chapter 10.)

A pulse line behaves as a long transmission line. Your chief interest in it is that it acts as a resistive circuit until the charging pulse has traveled its length and returned, at which time the line is fully charged. This causes the voltage to change in two steps, the first from the bias voltage to the voltage determined by the product of the grid current times the characteristic impedance of the line. The second step is of equal magnitude. The PRF is controlled by the synchronizing pulses, but the discharge of the pulse line through the grid resistor must be such that the grid voltage approaches the cutoff point by the time the next synchronizing pulse comes in.

The discharge of the line is in steps, as you can see in Figure 12–16. The magnitude of the

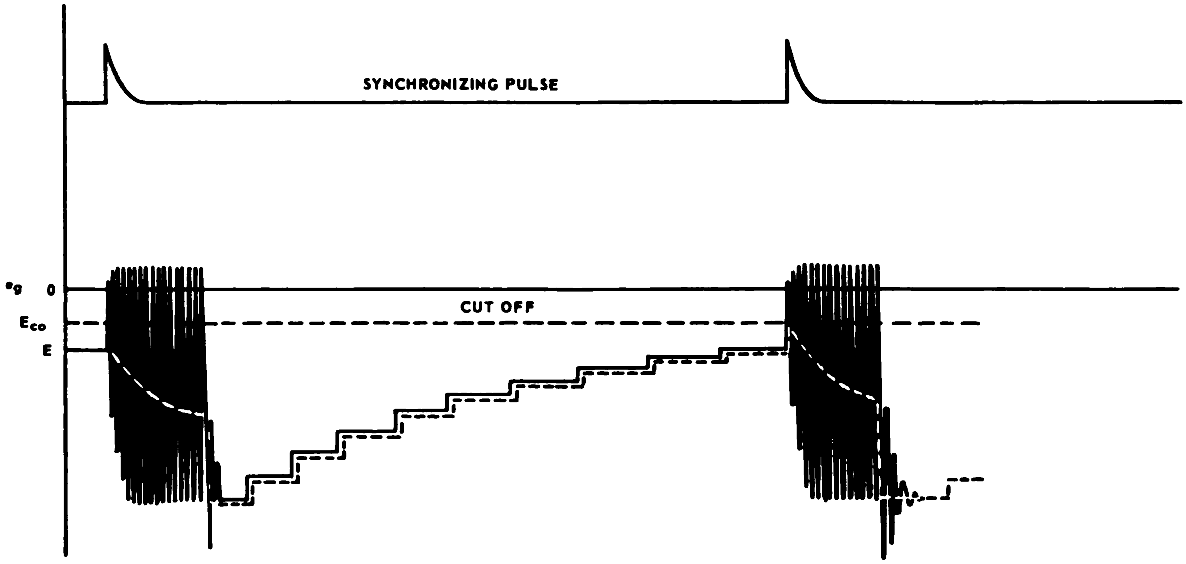


Figure 12–16. Grid Waveshape of a Line-Controlled Self-Pulsed Oscillator

discharge voltage is determined by the relative values of R_g and the characteristic impedance of the line. R_g is usually much larger, hence the steps are small. These steps follow the general path of an exponential discharge curve with a time constant of R_gC_o, where C_o is the capacitance of the pulse line.

Separate Modulators. As mentioned earlier, some triode UHF oscillators are not self-pulsed, but use separate modulators. These modulators are very much like those used with magnetron oscillators which are discussed later in the chapter. Here we merely note that the oscillator may be plate-modulated, grid-modulated, or cathode-modulated. In Figure 12–17, short, positive, rectangular pulses are applied to the grid of the modulator tube resulting in a negative-going pulse at the plate. The transformer is connected so that plate voltage is applied to the oscillator tube, or tubes, in short rectangular pulses. R-f components for the oscillator are not shown in the diagram.

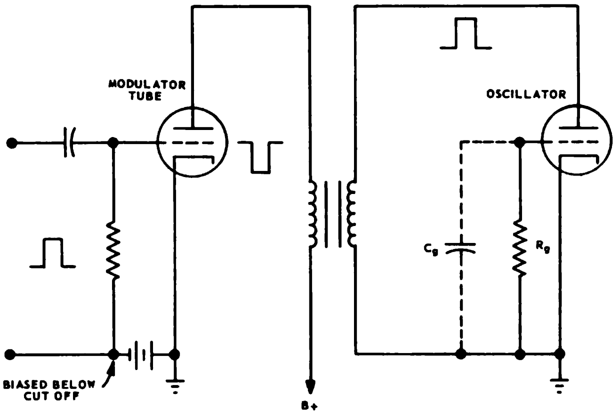


Figure 12–17. Plate-Modulated Oscillator

The transformer is specially constructed so that it passes the rectangular pulse without undue distortion. Since the purpose of the transformer is polarity inversion, why not apply negative-going pulses to the grid of the modulator tube to cut it off and get a positive-going pulse at the plate for use in modulating the oscillator? This would require that the modulator tube, or tubes, conduct heavily during all the cycle except during the pulse and would result in excessive waste of power.

The grid capacitance C_g of the oscillator is quite small, usually consisting of the capacitance-to-ground of the grid-tuned circuit. If made too large, C_g might cause the oscillator to be self-pulsed and cut itself off before the modulator pulse was over. Hence C_g and R_g have values to cause the oscillator to operate continuously if plate voltage were applied all the time. The modulator pulse then is most effective in controlling the output pulse. Oscillations must build up rapidly so that the leading edge of the output pulse will be as steep as possible. The plate voltage of the oscillator must remain constant throughout the pulse.

A cathode-pulsed (modulated) oscillator, Figure 12–18, does away with the necessity of inverting the pulse, hence no transformer is used. The charging diode serves to recharge the coupling capacitor between pulses. The RC time constant of the capacitor discharge path must be quite large so that the capacitor will not discharge appreciably during the pulse and thereby lower the plate-to-cathode voltage.

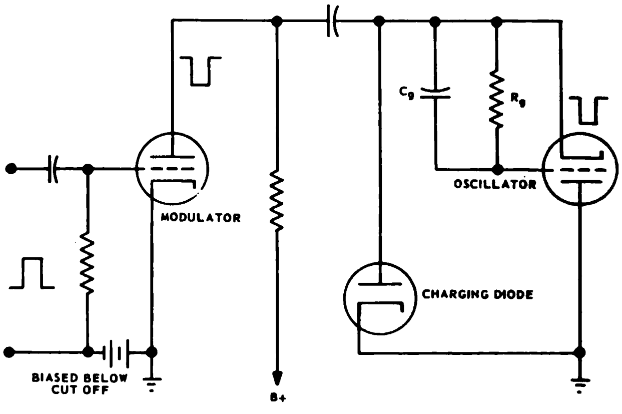


Figure 12–18. Cathode-Modulated Oscillator

Grid modulation requires less power from the modulator for operation than either of the other two types of modulation. It uses a pulse which has not been amplified. In grid modulation, Figure 12–19, a positive-going rectangular pulse is applied to the grid which raises the grid voltage well above the cutoff point, and oscillations start. Oscillations stop when the grid voltage drops to the normal bias level at the end of the pulse. C_g in the grid-pulse oscillator shown is a small capacitance which is in addition to the grid-to-ground

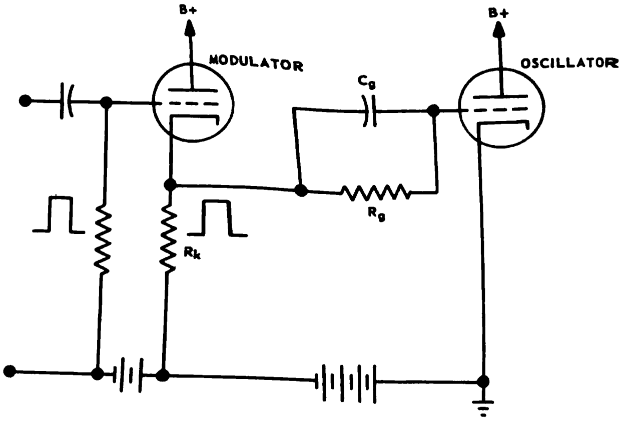


Figure 12–19. *Grid-Modulated Oscillator*

capacitance. Its purpose is to couple the pulse from the cathode of the modulator tube to the grid of the oscillator to insure a rapid rise of voltage at the grid and cause oscillations to build up quickly.

THE MAGNETRON OSCILLATOR AS A TRANSMITTER

Before the days of modern radar, electronic equipment operated at considerably lower frequencies. One reason was because a transmitter could not be constructed to operate in the UHF band and produce a usable output signal. These low frequency transmitters made a large, bulky antenna necessary and it wasn't possible to design an accurate lightweight radar set. Then, with the invention of the magnetron, came a new era in radar. Here was a lightweight, high-frequency oscillator that could produce a usable output with a power level that was unheard of before. The magnetron can produce output signals of up to 30,000 megacycles and power levels up to 6 megawatts.

This discovery was what the military electronics field was looking for. Small, lightweight radar sets with very narrow beams could now be built. Of course, with this discovery came new problems. New circuits had to be designed to operate the magnetrons. New types of modulators came into existence. The line-pulsing modulator seemed to be made for the magnetron. New filament supply circuits were designed.

The main advantage of the magnetron as an oscillator tube is that transit time, which is a determining factor in the upper limit of frequency of operation in conventional tubes, is not a disturbing influence. There are some special types of triodes used in unusual transit time oscillator circuits that will oscillate in the frequency range useful for radar, but unlike the magnetron their power output is too small for them to be useful in most cases. In addition, there are also the velocity-modulated tubes, such as the Klystron and the Shepherd Pierce tube, which operate in the desired frequency range with better power output than the triodes just mentioned.

The magnetron is a diode in which the magnetic field between the cathode and the plate is perpendicular to the electric field. The tuned circuits included in the tube are in the form of cylindrical cavities. As shown in the typical 10-cm magnetron in Figure 12–20, the cathode and filament structure is at the center of the tube. It is supported by filament leads which are large and stiff enough to keep the cathode and filament structure fixed in position under ordinary circumstances.

Exercise care in handling the magnetron tube, since undue jarring of the tube or bending of the filament leads might change the position of the cathode causing it to short to the plate, or change its spacing with the plate. In either case, the operation of the tube would be materially affected. The plate structure, as shown in Figure 12–21, is a solid block of copper in which the resonant cavities are cylindrical holes. A narrow slot opens each cavity into the central portion of the tube and divides the inner plate structure into as many segments as there are cavities. Alternate segments are strapped together to put the cavities in parallel so far as the output is concerned. This makes it possible to take the output from a pickup loop placed inside any one of the cavities.

Since the outer conductor of the output coaxial line is connected to the shell of the magnetron, which is a part of the plate structure, there cannot be a high positive DC voltage applied to the plate. The same result is realized, however, whenever a negative voltage is applied to the cathode. The magnets, which are made of a special alloy called Alnico, are unusually strong. Protect them from sharp blows of metallic substances or any hard material, since such blows will cause them to lose magnetism.

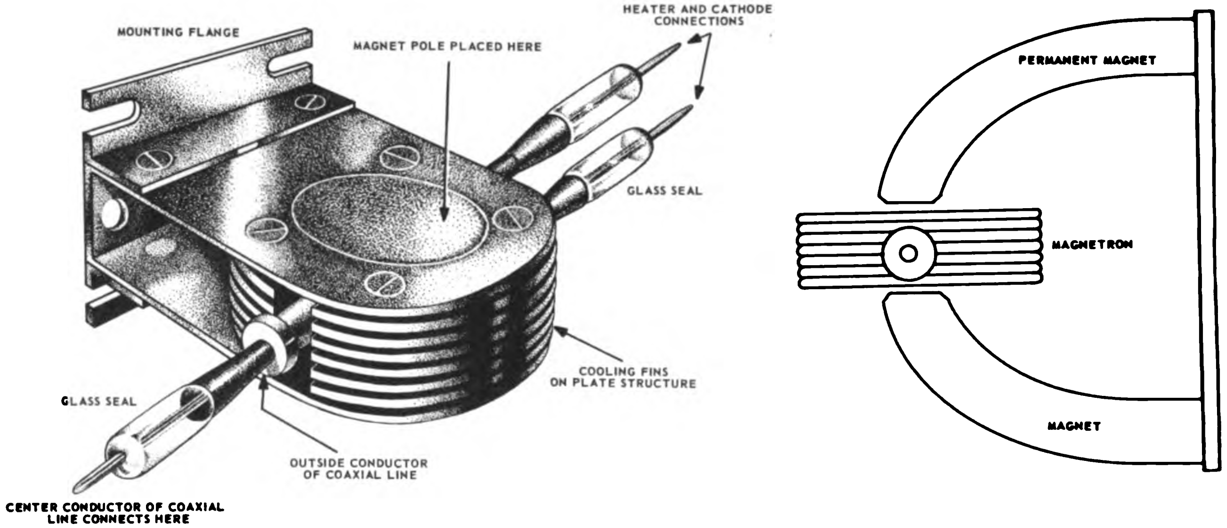

MOUNTING FLANGE
MAGNET POLE PLACED HERE
HEATER AND CATHODE CONNECTIONS
GLASS SEAL
COOLING FINS ON PLATE STRUCTURE
GLASS SEAL
OUTSIDE CONDUCTOR OF COAXIAL LINE
CENTER CONDUCTOR OF COAXIAL LINE CONNECTS HERE
PERMANENT MAGNET
MAGNETRON
MAGNET

Figure 12–20. Typical 10-cm Magnetron

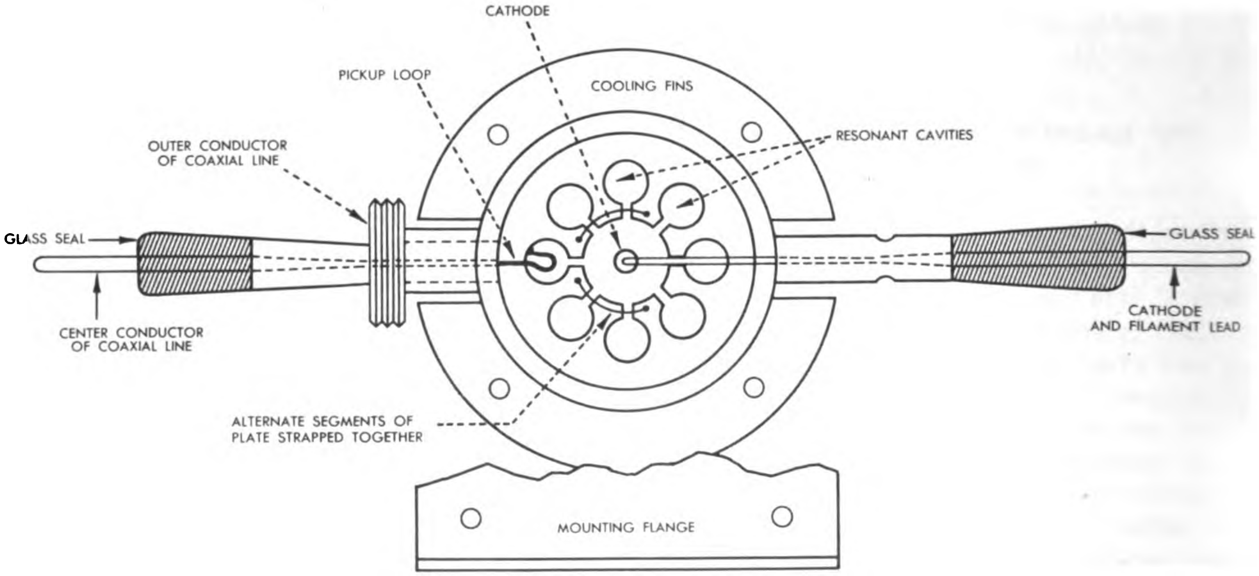

CATHODE
PICKUP LOOP
COOLING FINS
OUTER CONDUCTOR OF COAXIAL LINE
RESONANT CAVITIES
GLASS SEAL
GLASS SEAL
CENTER CONDUCTOR OF COAXIAL LINE
CATHODE AND FILAMENT LEAD
ALTERNATE SEGMENTS OF PLATE STRAPPED TOGETHER
MOUNTING FLANGE

Figure 12–21. Cutaway View of 10-cm Magnetron

Operation of the Magnetron Oscillator

The theory of operation of the magnetron is based on the motion of electrons in combined electric and magnetic fields. The following laws govern this motion.

The direction of an electric field is from the positive to the negative voltage source. The law governing the motion of an electron in an electric field states that the force exerted by an electric field on an electron is proportional to the strength of the field, and the direction of the force is opposite to that of the field. In other words, electrons tend to move from a point of low potential toward one of high potential. Mathematically, this law is stated by the vector equation,

$$F_e = 1.6 \times 10^{-12} E \qquad (1)$$

where E is the electric field intensity in volts per centimeter and F_e the force in dynes.

The law of motion of an electron in a magnetic field states that the force exerted on an electron in a magnetic field is at right angles to both the field and the path of the electron; the

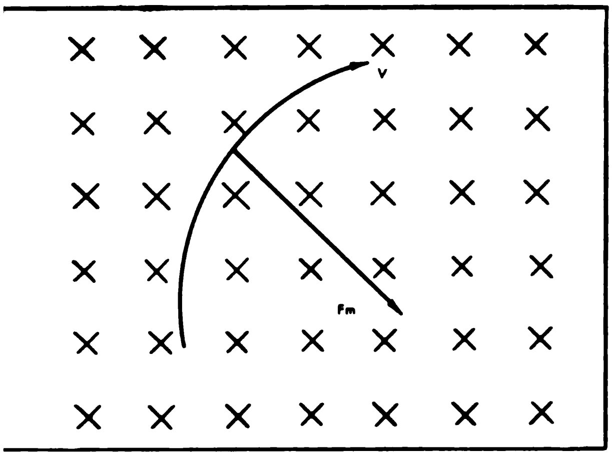


Figure 12–22. Force Acting on an Electron Moving in a Magnetic Field

direction of the force is such that the electron trajectories are clockwise when viewed in the direction of the magnetic field as in Figure 12–22. Mathematically, the law governing the magnitude of a force in dynes is expressed as,

$$F_m = 1.6 \times 10^{-20} V_p B \qquad (2)$$

where V_p is the component of the velocity in centimeters per second of the electron perpendicular to the direction of the magnetic field, and B the magnetic flux density in gausses.

In regions of uniform magnetic field the electrons move in circles of radius given by the equation,

$$R = 5.68 \times 10^{-8} \frac{V_p}{B} \text{ centimeters} \qquad (3)$$

with a period of

$$T = \frac{35.7 \times 10^{-8}}{B} \text{ seconds} \qquad (4)$$

The path that an electron takes when traveling normally with respect to the magnetic field depends upon the initial velocity of the electron. Generally, however, the paths of electrons in a plane normal to the magnetic fields are cycloids (figures generated by a point on the radius of a wheel rolling on a smooth plane). Different paths are obtained by different points along the radius of the wheel. For example, the center point of a wheel generates a straight line. A point on the circumference generates a typical cycloid as shown in Figure 12–23. Points lying beyond the radius generate cycloids also, but these are closed loops.

Paths Generated by Electrons

All paths generated by the rolling wheel may also be generated by electrons traveling at different velocities. This is understandable if you consider that the uniform magnetic field is directed inward perpendicular to this page, and that the electric field is directed downward from the top to the bottom of the page. If an electron is projected horizontally from left to right, there will be two forces acting on it — a magnetic force downward and an electric force upward. If these forces are equal, the path of the electron will be a straight line. Equating equation (1) and (2) gives the velocity required to produce a straight line path. The result is expressed by the equation,

$$V_p = V_s = 10^{8} \frac{E}{B} \qquad (5)$$

where the velocity to produce a straight path is V_s.

For all other velocities, the electron path will be curved as shown in Figure 12–24. All electrons traveling with velocities between zero and V_s take a path similar to that shown in curve b. The qualitative explanation of this path is the following: Because the initial velocity of the electron is less than V_s, there is a smaller magnetic force acting on it. Hence, the electron curves upward at first. But as it moves, it gains kinetic energy from the electric field with a resultant increase in velocity. The magnetic force thus increases and after a time the electron path is bent downward where the electron begins to lose velocity and finally returns to its original state.

Curve c, Figure 12–24, shows the path that an electron takes when it starts with zero velocity. At the start the magnetic field exerts no force on the electron as no magnetic lines of force are being cut. The electric field, however, moves the electron directly upward to a point where the magnetic field acts on it and deflects the electron to the right. As the electron moves upward, it gains velocity. The magnetic force increases, and eventually becomes large enough to bend the trajectory back until the velocity of the electron again becomes zero. The electron now has the same potential in the electric field as when it started.

An electron traveling from right to left (mathematically, a negative velocity) is at first pushed upward by the magnetic and electric fields as

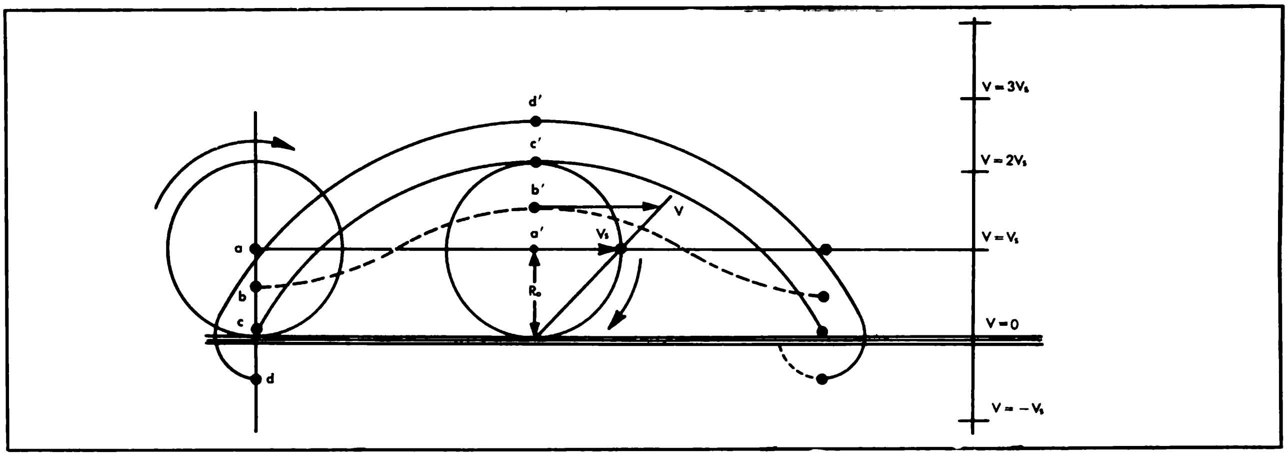


Figure 12–23. Generation of Cycloid

you can see in curve d. Its initial radius of curvature is small. Soon it goes straight upward, gaining velocity rapidly. As the effect of the magnetic force increases steadily, the electron is pushed to the right and finally downward. The path is bent back and the electron finally returns to its initial condition.

The time for each cycle and the horizontal distance covered by each electron in one cycle is *the same* for each of the cases given.

All the paths illustrated in Figure 12–24 are cycloids. It follows that electron *a* corresponds to the center of a wheel rolling on a smooth plane. Electrons *b*, *c*, and *d* correspond to a point on the radius, to a point on the circumference, and to a point outside the wheel, respectively. The period of the wheel is identical to the period of an electron in the magnetic field alone, according to equation (4).

Source of Energy in Magnetron Oscillators

In an electric field alone, a free electron, which always tends to move parallel to the field, gains a large amount of kinetic energy. In combined electric and magnetic fields the electron, which on the average moves at right angles to the field, does not change its average kinetic energy. The magnetic field always exerts a force that tends to change the direction of motion of the electron.

In a magnetron oscillator there are two components in the electric field — a steady component furnished by the DC source and an alternating component furnished by the oscillations of the tank circuit. In a properly designed magnetron, the electrons in transit absorb energy from the DC component of the field and deliver it to the alternating component. The function of the magnetic field, in many respects, is the same

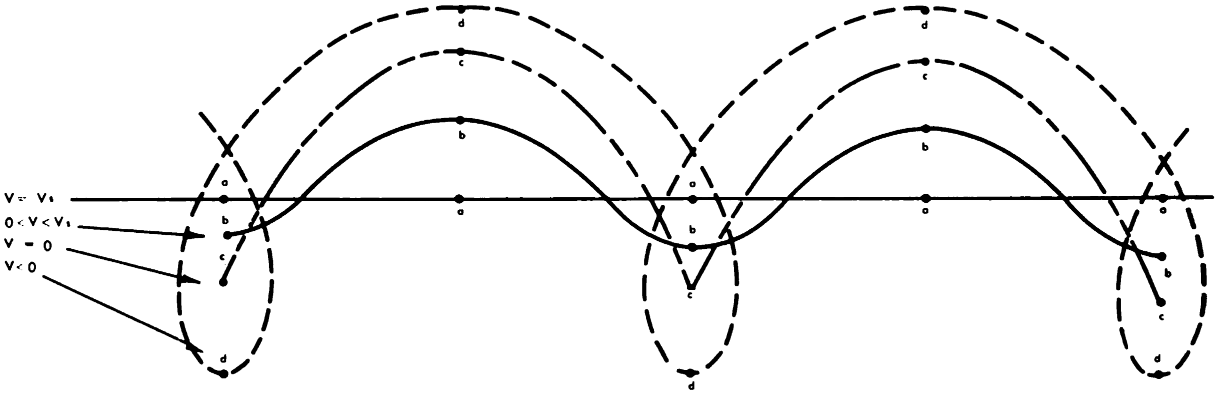


Figure 12–24. Electron Orbits

is that of the positive feedback to the grid of the triode oscillator.

The Split-Anode Transit-Time Magnetron

The two-segmented-plate cylindrical-anode magnetron is a practical oscillator from which the multi-anode magnetron using resonant cavities as tank circuits was developed (Figure 12–25). In it the electric field is the result of the difference of potential between the two plate segments. In addition to this field, there is a steady field (not shown), which is caused by the potential difference between the cathode and the two anodes. Figure 12–26 shows the ideal motion of an electron that delivers energy to the source on alternating cycles between the magnetron plates. Note that the electron trajectory is approximately cycloidal.

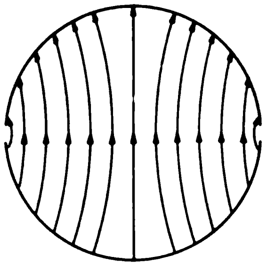


Figure 12–25. Electric Field Between Plate Segments

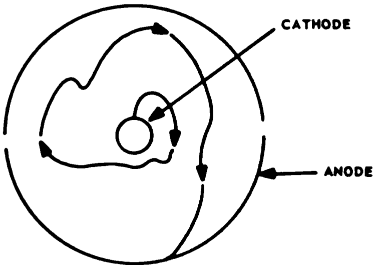


Figure 12–26. Path of an Electron Delivering Energy to the Alternating Field

As you can see, the electric field between plate segments may either increase or decrease the angular velocity of an electron, depending upon the angular position of the electron. If an alternating voltage having a period equal to the time taken for an electron to move once around the filament is impressed between the anode segments, the direction of the field reverses twice in each revolution of the electron about the filament. As a result, the electrons that pass the gaps at the instants in which the field is maximum will experience continuous angular acceleration or deceleration. Electrons that are accelerated gain energy from the source of alternating voltage. The electrons that absorb energy are speeded up and the component of force toward the filament also increases. As a result, these electrons re-enter the cathode after one excursion.

Electrons that deliver energy, on the other hand, are decelerated. This decrease in angular velocity reduces the average force toward the cathode, and causes electrons to drift toward the plate as shown in Figure 12–26. Experiments have proved that electrons which give up energy make between 80 and 100 excursions before reaching the plate. Since the electrons that deliver energy make so many more radial oscillations before striking the plate than the ones that absorb energy, the source of alternating voltage gains energy. Sustained oscillations will take place whenever the oscillatory circuit has sufficiently low dissipation, such as a Lecher line or a resonant cavity.

The Multi-Anode Transit-Time Magnetron

Plane Form. Since the exact theory of the cylindrical multi-anode magnetron is more difficult to understand than that of the plane form, the plane form is discussed first. The ideas presented for the plane form may be carried over in the discussion of the more practical cylindrical multi-anode magnetron.

To understand the operation of the plane form magnetron, consider a magnetron oscillator that is composed of a continuous plane cathode and a segmented anode of which the alternate sections are connected to the opposite sides of its tank circuit. In Figure 12–27, which shows this magnetron, note that the alternating electric field is sketched in the space between the cathode and anode at the instant when alternate anode segments are at their maximum positive and negative values.

Important considerations in the operation of this magnetron are: (1) which electrons tend to sustain oscillations, (2) which electrons tend to

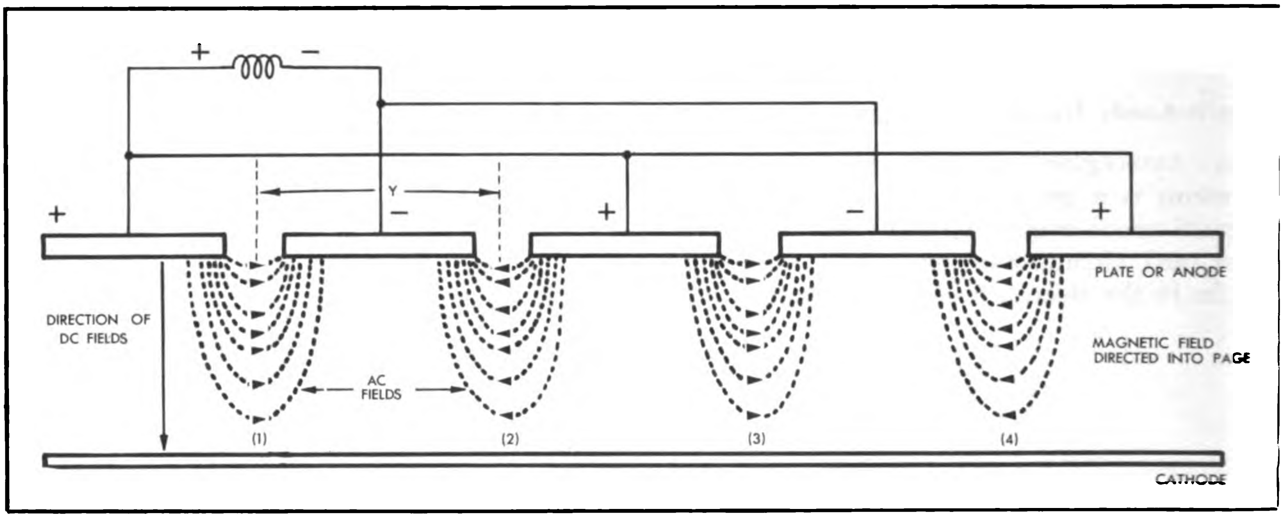


Figure 12–27. AC Electric Field in Plane Magnetron with Multi-Anodes with Maximum Potential Between Adjacent Anode Segments

absorb energy, and (3) the probable electron paths. In this connection consider an electron in a uniform electric field directed as shown in region (1) in Figure 12–27. The direction of the resultant field is obtained by adding the AC and DC fields vectorially at this point. (The DC field, which is not shown, is directed downward.) The approximate direction of the electric field and the path of the electron are shown in Figure 12–28A.

As shown in this figure, the electron makes cycloidal paths. Since the general progression of a cycloid is always at right angles to the resultant electric field, these cycloids progress upward and to the right with the magnetic field directed into the page. On the other hand, an electron in a uniform field whose direction is the same as the resultant shown for region (2) in Figure 12–27 would progress downward and to the right as shown in Figure 12–28B. The direction of the uniform electric field is also indicated.

Since the electron of diagram A moves, on the average, from cathode to anode, it must absorb energy from the source of the steady field. This same electron, however, tends to deliver energy to the alternating field since it moves in a direction that is opposed by the field of the tank circuit. Because the average velocity V of an electron does not change as it progresses toward the anode, it is a very efficient means for converting energy from the DC source into energy of oscillation for the tank circuit. By the same reasoning, the electron in region (2) (Figure 12–27) tends to absorb energy from the alternating field and to transfer it to the steady field.

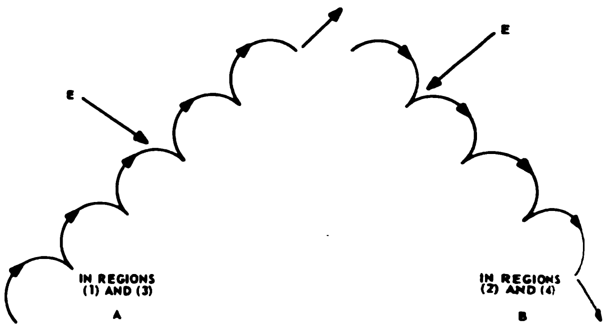


Figure 12–28. Electron Paths

Whenever oscillation is to be maintained, the electron following the path in diagram A must be made to continue its path to the anode or plate. The other electrons — the ones that absorb energy — must be removed from the field quickly. An electron following the path in diagram B, starting from the cathode with a small or zero velocity, will strike the cathode before completing its first loop. Had the fields of regions (1) and (2) been static, the electron of diagram A would tend to turn downward as it approached region (2). Since the fields are alternating, this will not necessarily be the case. In fact, if the left-to-right velocity of the electron of diagram B is such that it arrives in region (2) n exactly one-half cycle, it will encounter there xactly the same kind of field that it encountered n region (1) one-half cycle earlier.

Thus, if this electron, shown in the magnetron n Figure 12–27, continues to progress distance from left to right during each half-cycle, it vill go all the way to the anode, following a path imilar to that of an electron in diagram A, Figure 12–28. This condition is necessary for scillation. Oscillation is sustained since the lectron of diagram A contributes energy to the ank circuit for a much longer time than the lectron of diagram B takes energy away from he tank circuit.

The following analyzes the factors affecting the requency of oscillation in a plane magnetron. The average left-to-right velocity V_s of the elecron depends only on the average downward omponent of the electric field. The DC field is he average electric field. Since an electron should progress horizontally a distance of Y centimeters in half a period (T/2), it follows that

$$(V_s)\frac{T}{2} = Y \qquad (6)$$

and

$$f = \frac{1}{T} = \frac{V_s}{2Y} = \frac{(10^8E)}{B}\,\frac{1}{2Y} \qquad (7)$$

where $V_s = \frac{10^8E}{B}$ from equation (5) and E is the DC component of the electric field.

These conditions clearly indicate that the plane magnetron is a transit-time oscillator, for an electron is allowed a certain definite time to travel from cathode to anode. Thus, in addition to the usual condition of resonance, $f = \frac{1}{2\pi\sqrt{LC}}$, there is equation (7) to be fulfilled also. When this is done you may rewrite equation (7) as

$$\frac{f}{(E/B)} = \text{constant} \qquad (8)$$

The question arises whether you could obtain greater efficiency if, in addition to adjusting the ratio (E/B) to the frequency as required by equation (8) or (7), you also made the time for one cycloidal loop equal to the period of the tank circuit. When both of these conditions are filled, the lengths of the cycloidal loops become equal to 2Y, or the length of two anode segments, as illustrated in Figure 12–29. It develops, however, that the efficiency is not increased by doing this.

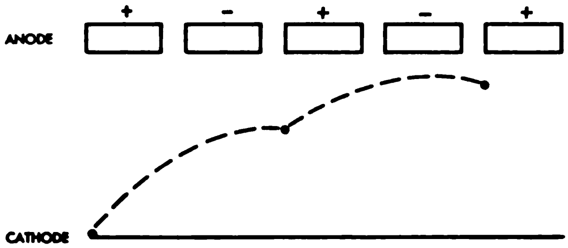


Figure 12–29. Electron Loops

This is why the efficiency is not increased. Since an electron returns to zero kinetic energy at each cusp, the efficiency of conversion from energy to tank-circuit energy is 100% from cusp to cusp. A cusp is a point where two curves or arcs meet. The only DC energy that is not converted into tank-circuit energy by electrons which reach the anode is that which appears as the kinetic energy which arrives at the anode. As this kinetic energy is all acquired after the last cusp, the closer the last cusp is to the anode, the less will be the energy lost relative to the total available DC energy. Therefore, the smaller the cycloids, the smaller will be the fraction of energy lost as kinetic energy at the plate. For these reasons efficiency is not increased by making the looping frequency equal to the tank-circuit frequency.

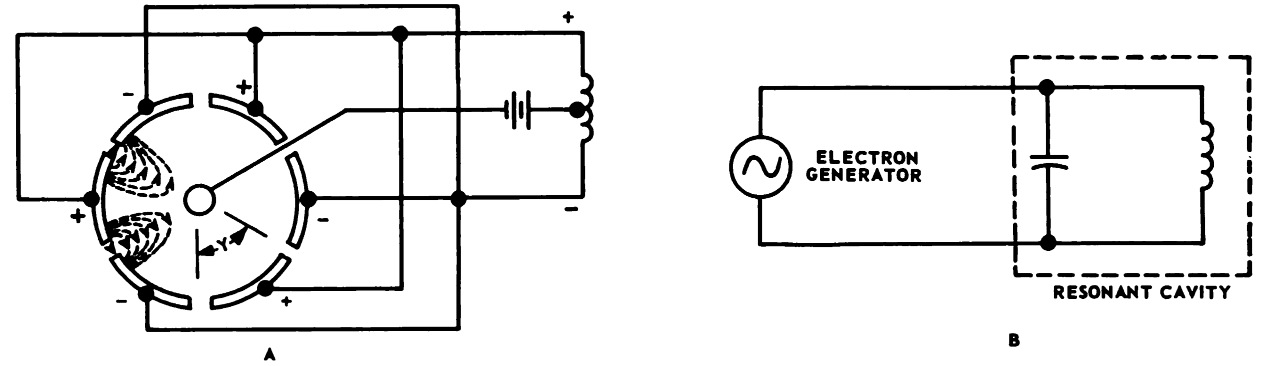
To increase the efficiency in a magnetron, it is necessary to make the cycloids smaller. As can be proved from the properties of a rolling wheel, the size of the orbits depends on the radius of the wheel R, and the equation,

$$R = 5.68\,\frac{E}{B^2} = 5.68\,\frac{(E)}{B}\,\frac{(1)}{B} \qquad (9)$$

Thus, it follows that for smaller cycloids the value of B must be increased for a given value of (E/B). As previously stated the ratio (E/B) is fixed for a given frequency, as this is one of the conditions for oscillation indicated in equation (8).

In practice it is possible for the value of B to be 1.5 to 3 times the value which makes the looping frequency equal to the tank-circuit frequency. Not only does increasing B increase the efficiency, but it also increases the power input since E must be increased in direct proportion to B in order to keep E/B constant.

Cylindrical or Cavity Form. The cylindrical or cavity form magnetron is analogous to



(A) Simplified Tank Circuit with Six-Anode Magnetron, Showing the Alternating Electric Field.

(B) The Equivalent AC Circuit in Which the Generator Corresponds to the Electrons Between Anode Segments.

Figure 12–30. Six-Anode Magnetron

the plane magnetron rolled up into a circle. The number of segments (anodes) which may be used in it varies widely. As many as 32 have been tried on certain occasions. Here, however, for simplicity the discussion considers only a magnetron with six segments. Figure 12–30A shows the circuit of a six-anode magnetron and the approximate AC field between the segments. Figure 12–30B is its equivalent circuit.

To qualitatively understand the performance of the cavity magnetron, assume that the DC field is uniform and that its magnitude is $E_b/(b-a)$, where E_b is the DC supply voltage, b the anode radius in centimeters, and a the cathode radius. Since the AC field is much narrower near the cathode than near the anode in the cylindrical magnetron (shown in Figure 12–30A), the linear distance (the distance which the electron must progress around the cathode per anode segment) is less near the cathode than near the anode. If the theory of the plane magnetron is to be applicable to the cylindrical magnetron, it will be necessary to choose an effective, or average, value of Y. This value is one-sixth of the circumference of the circle halfway between the cathode and the anode. These assumptions about Y and the electric field lead to the same resonance condition as before, equation (8). In addition, as with the plane magnetron, it is inefficient to fulfill the resonance condition for B alone.

In developing the discussion of cylindrical or cavity magnetrons, it is desirable to see how some simpler systems of several resonant cavities work. The buncher and catcher cavities of a nonreflex klystron and a single velocity-modulated electron beam are examples. The theory of buncher and catcher cavities is explained more in detail later in this chapter. Refer to Velocity-Modulated Tubes and the Traveling-Wave Tube for more information on the theory of bunching of electrons.

Figure 12–31 depicts a two-cavity system consisting of two resonant cavities tuned to the same frequency with an electron beam passing through both sets of cavity grids. Assume that the first cavity is excited at its resonant frequency through the coaxial input line, and that the electron beam is velocity modulated. With a properly adjusted system and the correct electron velocity and modulation level, the beam should be bunched at the grids of the second cavity. This being true, oscillations will occur in the second cavity when a large signal is present. The phase of the second cavity oscillations must be such that the bunched electrons are decelerated as they pass through the cavity grids. In the deceleration of the electron beam, energy is given to the cavity.

At any other phase, oscillation will be unaffected by the electron beam if bunched electrons pass through the cavity grids when both are at the same potential. Also, oscillation at any other phase will transfer energy from the cavity to the beam if the bunched electrons pass through the cavity grids at an accelerated rate. Therefore, the electron beam has the capability of transferring energy from one cavity to another.

If the second cavity is moved toward the first, the electron beam then is underbunched at the

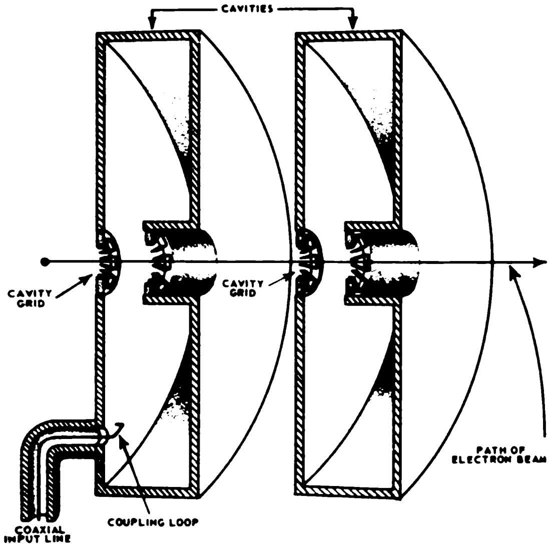


Figure 12–31. Two-Cavity System with Electron Beam Coupling

grids of the second cavity; that is, bunching is not complete. As there is only partial bunching, electron flow is not smooth, but the second cavity is able to absorb energy from the beam. By absorbing energy the phase of oscillation (frequency) in the second cavity automatically adjusts itself in order to slow the greatest possible number of electrons. Those remaining electrons are accelerated as they pass the cavity grids when the second grid is more positive than the first grid. You see, then, that the second cavity slows down over half of the electrons that pass through its grid and accelerates less than half of the electrons. This reduces the average velocity of the electrons and adds to the velocity-modulation of the beam.

In a cavity system that has more than two cavities, it is possible for each cavity, from the second cavity on, to absorb energy from the beam and improve the velocity modulation of the beam for the next cavity. A small portion of the energy comes from the source that feeds the first cavity; most of the energy comes from the electron beam source. Each cavity reduces the average velocity of the electrons in the beam. The system is very sensitive to adjustment, as the energy absorbed by each cavity is dependent upon the relation of the spacing between cavities and the bunching distance.

Let us take a multicavity system, all six cavities tuned to the same frequency, and arrange it as shown in Figure 12–32. Assume that a beam of electrons is influenced so that the beam travels in a closed loop. This loop passes through the associated grids of each cavity. Any irregularity or roughness in the electron beam will cause one or more of the cavities to ring (oscillate). This modulates the beam and energy is delivered to all cavities. The velocity of the electron beam is the source of the microwave energy, and the transfer of energy is a result of the reduction of the average electron velocity. As long as there is a continuous supply of accelerated electrons, while slow electrons are removed from the system, energy will be transferred from the electron beam to all of the cavities. This r-f energy is in the form of electric and magnetic fields. The cavity magnetron operates this same way.

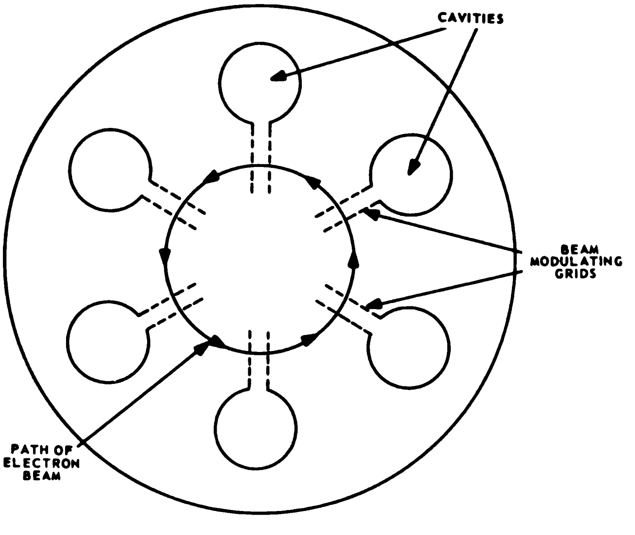
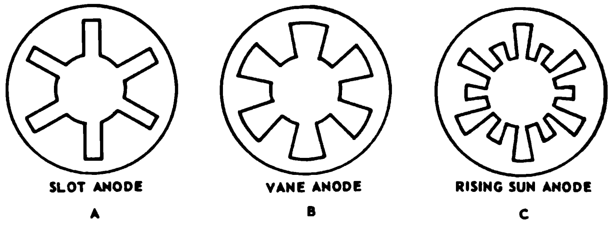


Figure 12–32. Ring System by Resonant Cavities

Figure 12–32 illustrates one of the more common magnetron anodes, a hole-and-slot type. Although six cavities are shown, magnetrons with other numbers and shapes of cavities are not uncommon. Other types of magnetron anodes are shown in Figure 12–33. A shows a slot anode, B a vane anode, and C a rising sun anode. The choice of one anode over another is based largely on design specifications, as all types behave in the same way.



SLOT ANODE
A

VANE ANODE
B

RISING SUN ANODE
C

Figure 12–33. Types of Anodes

MODE OF OSCILLATION OF THE MAGNETRON

The ring system of resonant cavities shown in Figure 12–33, with voltages as applied to the simplified six-anode magnetron shown in Figure 12–30A, appears in Figure 12–34 as six resonant circuits with magnetic coupling between them. While there is usually considerable coupling between inductances that are close together, in actual magnetrons which employ cavities as resonant circuits there is no coupling between adjacent cavities — that is, coupling in a manner characteristic of lumped circuit constants. On the other hand, there can be coupling resulting from radiation of energy from one cavity to another.

As you recall from the theory of coupled circuits, when two circuits tuned to the same frequency are closely coupled, they will resonate not at that frequency but at two different frequencies. The six resonant circuits in Figure 12–33 will resonate at different frequencies separated by a few cycles from each other. A point of interest is whether all these frequencies are associated with the same configuration of the AC field in the space between the cathode and anode. As it is, they are not, but instead, for each frequency there is one configuration or *mode* of oscillation. For example, for three different frequencies, there are three configurations as illustrated in Figure 12–35.

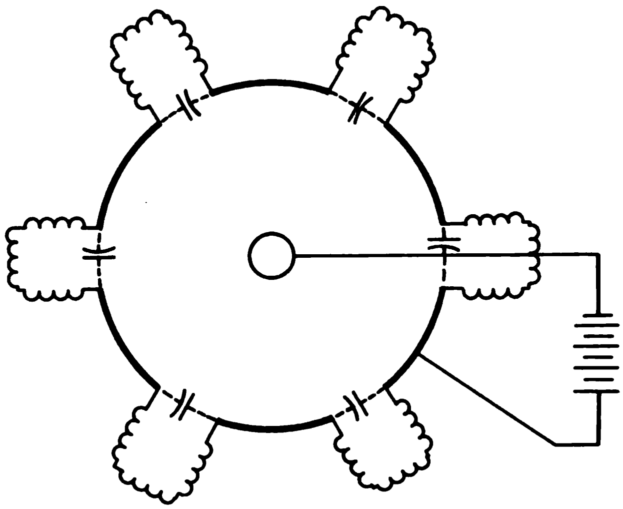
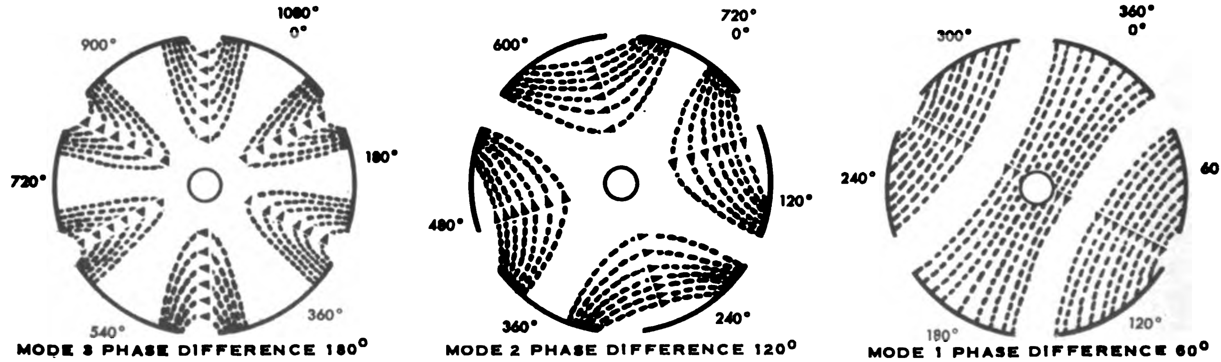


Figure 12–34. Forming Six Magnetically Coupled Resonant Circuits

In connection with the excitation of mode 3, the average velocity, V_a which is equal to 10^8 E/B, must be adjusted so that an electron passes two segments per cycle. In mode 2 the ratio of E/B must be higher since the electron must pass three segments per cycle. It follows that this mode requires very high voltage and is difficult to excite strongly.



MODE 3 PHASE DIFFERENCE 180°
A

MODE 2 PHASE DIFFERENCE 120°
B

MODE 1 PHASE DIFFERENCE 60°
C

Figure 12–35. Configuration of AC Field Between Adjacent Anode Segments in the Various Modes of Oscillation

The radar magnetron should oscillate only in one mode under operating conditions, since the frequencies of these three modes are generally different, and since the radar receiver can be tuned to only one frequency at a time. If the modes are widely separated in frequency, it will be easier to excite only the one desired. Since mode 3 requires the least voltage, it is the mode generally used.

Another method of describing the modes of operation is as follows. In the case of the six-cavity magnetron, the phase-difference between adjacent cavities may be 60°. Then the sum of the six-phase differences is 1 cycle. When the phase difference is 120°, the sum is 2 cycles; if the phase difference is 180°, the sum is 3 cycles. In like manner, in an 8-cavity magnetron the phase difference between adjacent cavities may be 45°, 90°, 135°, or 180°. A phase difference of 180° is the desired mode, and other modes frequently are suppressed by strapping.

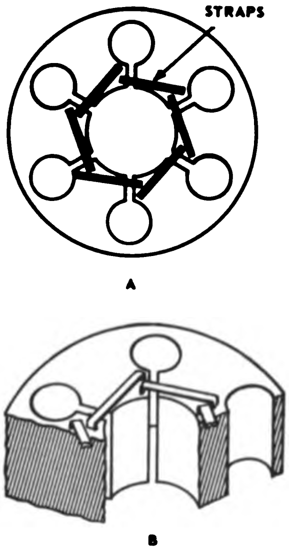


Figure 12–36. Strapped Anode

Strapping is illustrated in Figure 12–36 for a six-cavity hole-and-slot anode. Figure 12–36A shows an end view of the straps and B shows a three-dimensional view. Corresponding points on adjacent cavities are connected by straps. As the straps have an electrical length of a half-wavelength, 180° phase difference is assured. Only one lip of each cavity is shown strapped, but the other lips on the other side of the anode are strapped also. There are other strapping arrangements; all of the straps may be at one end or a complete set of straps may be at each end. A magnetron like this is called a strapped magnetron.

Summary

The following summarizes magnetron theory.

1. By using a segmented anode, you can make the electrons work against an alternating electric field that is crosswise to the steady field. In this way it is possible to get the electrons to go all the way to the anode without much increase in kinetic energy. This results in high power and high efficiency.
2. Oscillations are obtained in mode 3 (π) when the average crosswise velocity V_a is such that the electron passes two plates per cycle. The equation for resonance for these oscillations is,

$$f = 10^8 \frac{E}{2BY}$$

In using this equation replace factor 2 by 3 and 6, respectively, for modes 2 and 1.

3. The frequency of looping, which depends on B alone, is not usually matched to the other frequencies, because doing this causes too low a power input and too low an efficiency.
4. Quantitative calculations show that there are more electrons that give energy to the tank circuit than electrons that remove energy from the tank circuit. Further, it shows that on the average each of the former electrons gives more energy to the tank circuit than each of the latter takes away.
5. Magnetrons in use at present have a wavelength range from about 50 cm down to less than one cm.
6. Efficiencies of magnetrons at moderate power in service at present have values from 30% to 40%, and from 50% to 60% at powers greater than 200 kw.
7. The actual tuned circuit will resonate at several different frequencies, each of which corresponds to a distinct phase difference between anode segments.
8. Magnetrons are readily adaptable to pulsed modulation.

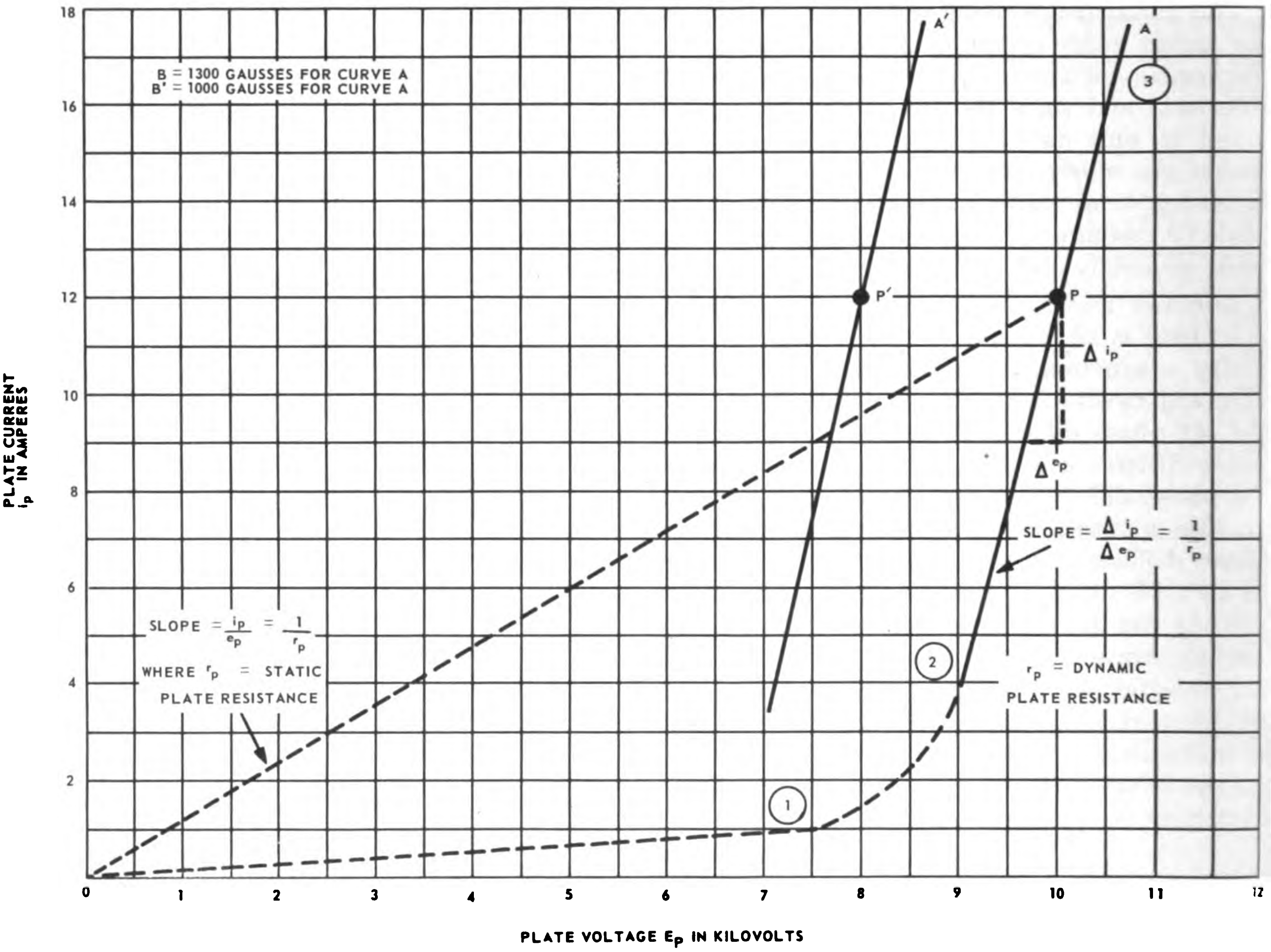


Figure 12–37. Magnetron e_p-i_p Characteristic Curve

Plate Current-Plate Voltage Curves of a Magnetron

A useful curve for studying magnetrons is the e_p-i_p characteristic curve. The curve shown in Figure 12–37 is a typical curve for a 10-cm magnetron operating at about 1300 gauss. Ordinarily, these curves are plotted by holding the magnetron field constant.

These curves are important in that you can get much information by proper interpretation. Keep in mind the following factors about these curves. The resistance between the cathode and anode is nonlinear. The AC and DC resistances are different from each other at any one voltage and vary with the voltage. For example, at point P the static resistance r_p is the ratio of e_p/i_p, while the dynamic resistance is the ratio of $\Delta e_p/\Delta i_p$. Since the magnetron operates on the straight line portion of the curve, the reciprocal of the slope of this straight portion, ② to ③, is the dynamic resistance r_p.

Note that from the origin to point ① on curve A the magnetron current is very small. A negligible coherent r-f energy is produced. Also a considerable amount of random r-f noise is produced. Somewhere between 6 and 8 kilovolts, appreciable oscillation starts. In the region from point ① to point ②, the operation of the magnetron varies greatly, not only from one magnetron to another, but also from one time to another in the same magnetron. Oscillations of more than one wavelength can be detected in the output, indicating that the magnetron is oscillating in more than one mode. The value of E/B which excites oscillations of higher modes is probably a submultiple of that which fulfills the resonance condition for these modes, and at the same time is not very far from the resonance condition for the 180° mode.

In the straight-line portion, between points ② and ③, the magnetron oscillates in the stable mode 3. In this region the frequency varies only a few megacycles over this range. In the strapped 10-cm service magnetrons the straight portion continues to the highest current obtainable. Theoretically, if the range is made great enough, mode 2 might be expected to appear. However, this mode is rarely, if ever, encountered in strapped tubes. Unstrapped magnetrons, however, have an additional multimoding region near or about point ③.

Even though the percentage of variation in frequency is small over the straight line portion, the actual change may be greater than the receiver bandwidth. For this reason it is important to maintain the pulse amplitude constant.

The power input is $e_p i_p$. Since the straight line portion is fairly steep, the input power rises rapidly with increase in voltage.

The magnetron has a very low efficiency in the region 0—① —②. In the single-mode region ② — ③, the efficiency is maximum somewhere in the middle of the straight line portion and decreases slowly toward point ③. In this region you may safely assume that the efficiency is constant and equal to 30–40% over the entire region. It may seem surprising in view of the resonance condition specified earlier, that a single-mode oscillation is excited over such a wide range of voltage (region ② — ③). This is explained by the fact that the point of resonance is not a sharp one. The point of resonance on the characteristic curve is not the only point at which oscillation is excited, but is the point where the *efficiency* of operation is a maximum.

The power output increases approximately as the power input in region ② — ③ since the efficiency is practically constant within this region.

Curve A′ shows the effect of reducing the magnetic flux density to 1000 gauss. To a close approximation, the new operating point P′ will occur at a voltage 25% less than that of P. The current will be the same. As is evident, the dynamic resistance will be essentially unchanged. If B were increased 25%, the curve would be shifted to the right of curve A by about the same amount. The general properties of i_p-e_p curves are not altered except that both the input power and efficiency increases in the linear portion as B is increased. The increase in power input is the result of the higher voltage necessary to keep E/B constant. The efficiency increases because the kinetic energy with which the electrons strike the anode remains roughly constant (from equation (5)), while the total energy available increases with the voltage.

Table 12–1 summarizes the characteristics of the magnetron:

Region	Mode	Power Input	Power Output	Efficiency
0-①	Noise	Very Small	Zero	Zero
①-②	More than one	Small	Very Small	Very Small
②-③	Mode 3	Increases rapidly	Increases rapidly	High and fairly constant (40%)
Beyond ③ Un-strapped	Double-mode	Large	Moderate	Low
Beyond ③ Strapped	Continues same mode	Increasing	Large	Moderate and decreasing

Table 12–1. Summary of Magnetron Characteristics

Modulating the Magnetron Transmitter

The modulator for the magnetron serves a double purpose — it determines the waveform of the output pulse, and it stores energy between pulses and releases it through the magnetron during the pulse.

There are two circuits that make it possible for the modulator to store and release energy. These are the parallel charging circuit and the series charging circuit.

As illustrated in Figure 12–38A, the source of energy in the parallel charging circuit is connected at all times to the storage element through the charging impedance Z_{ch}. During the output

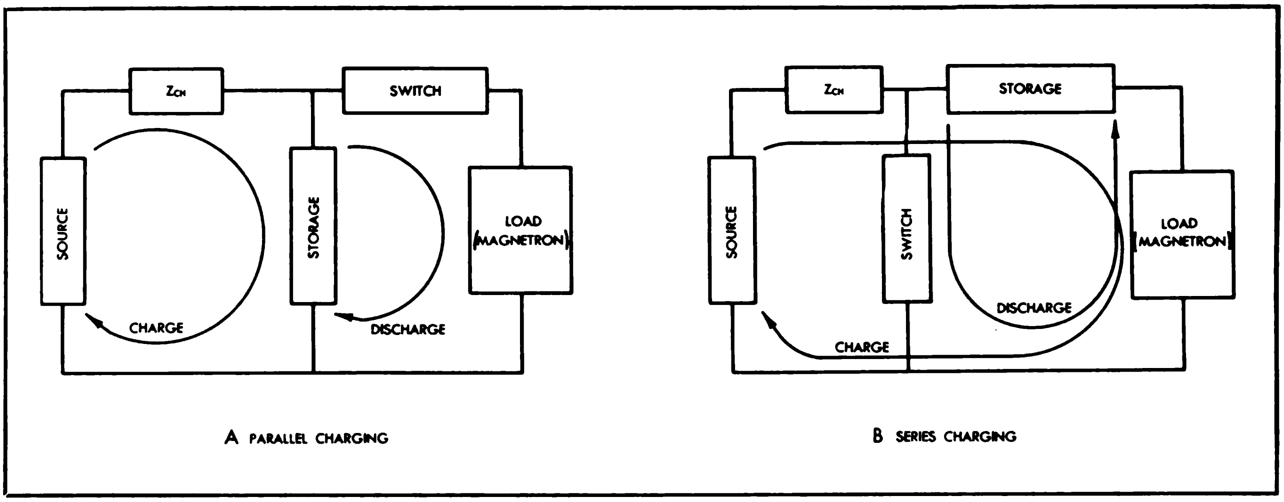


Figure 12–38. Parallel and Series Circuits for Storage and Release of Energy

pulse the closed switch applies the voltage from the storage element to the load.

A disadvantage of this circuit is that it is not possible to prevent grounding of all three of the circuit elements — the source, the switch, and the load.

Because of the disadvantages of the parallel charging circuit, the series charging circuit is generally employed. In this circuit the storage element is charged through the charging impedance and the load. It is discharged through the load when the switch is closed. From an examination of this circuit, you can see that closing the switch also completes a circuit containing the source and Z_{ch}. This has little consequence, however, since the switch is only closed for about a microsecond at a time, and Z_{ch} is large enough to prevent any damage.

It is well to find out the components of some of the modulator circuit elements which are shown as blocks in Figure 12–38. The source from which the storage element is charged may be either a DC voltage or an AC voltage. A magnetron requires a voltage of 10 to 15 kilovolts or higher for proper operation. In some types of equipment a transformer is used between the storage element and the magnetron to step up the voltage, while in many others a direct connection is made between the storage element and the magnetron. Actual charging circuits are discussed in a later section dealing with line-pulsing modulators. In these circuits the charging impedance may be a high resistance or a high inductance, or both. Its purpose is to control the charging time for the storage element and to prevent short-circuiting of the source during the pulse.

Switches for controlling the charging time for storage are of two general types — a soft switch or a hard tube. A soft switch is one in which conduction takes place through a gas. Soft switches include gas thyratron, mercury thyratron, fixed trigger gap, and rotary spark gap switches. A hard-tube switch is a high-vacuum-tube switch. Up to a few years ago, radar equipment used the hard-tube, spark gap, and thyratron switches. The present trend is to use hydrogen thyratron tubes as a switch because of the stability requirements of the Moving Target Indication system.

In the fixed trigger gap switch, Figure 12–39, breakdown occurs at the desired instant when a trigger pulse is applied to electrode C. This, in turn, causes breakdown from C to B resulting

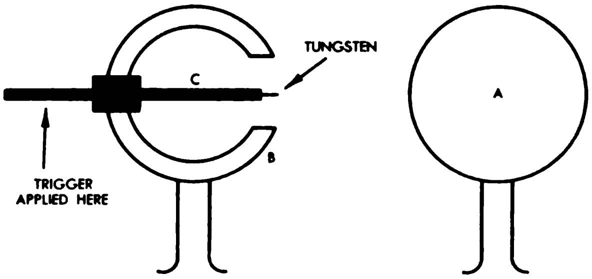


Figure 12–39. Fixed Trigger Gap

in the formation of a sufficient number of ions to cause breakdown between the high-voltage electrodes A and B. One difficulty with this switch is that the trigger wire C corrodes and is consumed in too short a time to make the circuit general in use.

In the rotary spark gap shown in Figure 12–40, there are four electrodes on the rotating wheel, which is grounded. The electrodes pass between two pairs of fixed electrodes when the switch is in the 800-cycle position, and between one pair when it is in the 400-cycle position.

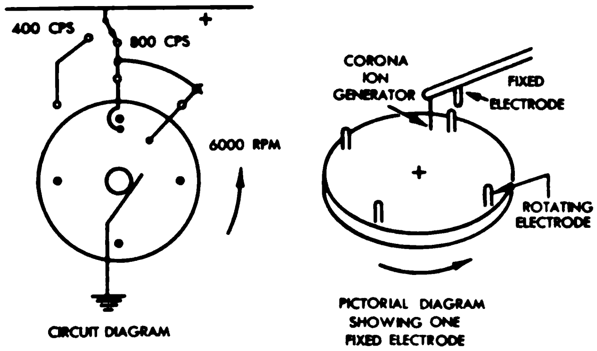


Figure 12–40. Rotary Spark Gap

Each time one of the moving electrodes passes a pair of fixed electrodes, there is a breakdown of voltage and the switch closes for a brief period of time. In operation there is some irregularity in the intervals between breakdown, which may cause the period to vary as much as 50 microseconds. But in a self-synchronous radar set this presents no difficulty. In fact, there is some advantage to the irregularity since it causes pulses from other radar sets to move on the scope, and it also lessens the likelihood of confusion due to echoes occurring on the second trace after the transmitted pulse.

Each type of switch mentioned has certain disadvantages; none is perfect for the job. Therefore, you may expect to find any or all of them in use in various sets.

The storage element can be either a capacitor or a pulse-forming network. A capacitor is most often used with the hard-tube modulator in which the sole purpose is to store energy. In this type of modulator circuit, it is necessary that the time constant be such that the capacitor does not discharge appreciably during the pulse for, as you can see in Figure 12–37, the magnetron current and, in turn, the magnetron output drop very rapidly when the voltage decreases.

The pulse-forming network in the storage element serves the double purpose of storing energy and of shaping the output waveform. In operation it is quite similar to the artificial transmission line; that is, it furnishes a steady output voltage for a certain length of time at which time its voltage drops to zero. It is different, however, in that instead of being made up of a number of identical sections, each section is different. It has the advantage of forming more nearly rectangular pulses than an artificial transmission line with a similar number of sections.

Although the theory of operation of the pulse line shown in Figure 12–41 during discharge is somewhat complex, it may be summed up as follows: C_4 stores most of the energy for the pulse while the components C_1, C_2, L_1, L_2, and L_3, which are in series with it, mainly shape the pulse. During the first part of the pulse (when the charge on C_4 is nearly equal to the supply voltage E_b) the series components offer a high impedance and cause the voltage across Z_c (the load) to jump to $E_b/2$. As the pulse continues, C_4 discharges but the impedance of the series components also drops. This causes the voltage across Z_c to remain at $E_b/2$. Later when the charge on C_4 is less than $E_b/2$, the inductances maintain the level of current flow. This continues until the capacitor is discharged and the magnetic fields are collapsed. At this time the voltage drops rather rapidly to zero.

The components C_3 and R_1 make the leading edge of the pulse steeper. There being no inductance in this branch of the network, the current can rise immediately to its full value. The

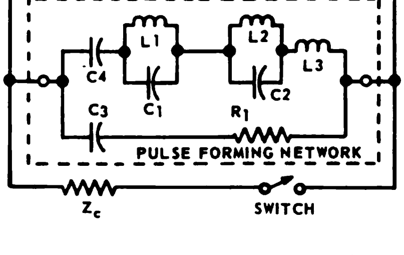


Figure 12–41. Pulse-Forming Network as Storage Element

pulse that appears at the load is essentially a rectangular pulse of the desired duration and equal in amplitude to $E_b/2$. The values of the components determine the duration of the pulse and the characteristic impedance of the line. The characteristic impedance of the line must be matched to that of the load. For this reason, even though the line is charged to E_b, the voltage across Z_c is only $E_b/2$.

Study Figure 12–42 to get a comparison of the output pulse and the discharge curve of C_4. The time of rise of the voltage is about one-tenth of the pulse duration and the time of fall a like amount of time. This type of pulse network is usually sealed in a metal box filled with oil which acts as an insulator. For that reason they are sometimes called potted networks. It is impossible to replace components in the line. Some lines using only a few elements are not potted. Generally, these do not give as nearly rectangular an output pulse as the potted networks, due to the smaller number of sections.

In the circuits discussed thus far the load can be either a magnetron and its associated circuits or the primary of a pulse transformer that has its secondary connected to the magnetron. In addition, a pulse transformer can be used in the amplifier stages preceding the hard-tube modulator. When this is done, all tubes are cut off between pulses. The interstage transformer is a low-voltage and low-turns-ratio device. Its only purpose is polarity inversion. The use of a high-voltage step-up transformer to furnish voltage for the magnetron makes possible the use of lower voltages in the pulse-forming network and thus simplifies the insulating problem.

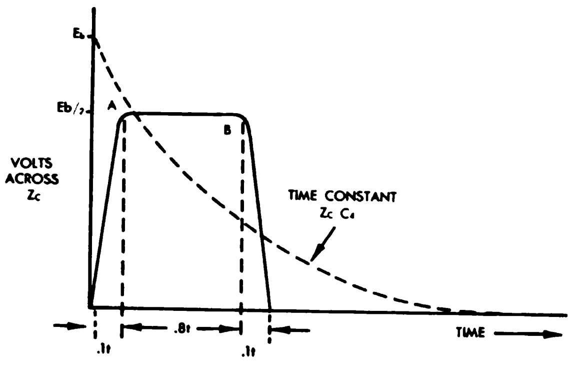


Figure 12–42. Comparison of Pulse from Pulse Line with Exponential Discharge of Capacitor

Another advantage in using a transformer with two secondaries, as shown in Figure 12–43, is that it avoids having high voltage in the magnetron filament transformer. A third need for the high-voltage pulse transformer is matching impedances, particularly where the radar antenna is remote from the set itself. Since the use of long transmission lines between the magnetron and the antenna make the magnetron's operation somewhat erratic, it is desirable to have the magnetron near the antenna. This necessitates the use of long lines to carry the modulator pulse to the magnetron. In order to be able to use a 50-ohm line for this purpose and still get a match of impedance with the approximately 1000 ohms of the magnetron, it is necessary to use a 4½:1 step-up transformer at the magnetron end.

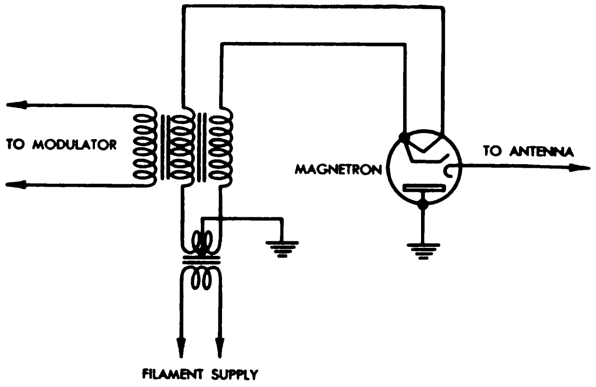


Figure 12–43. A Transformer with Two Secondaries Makes Possible the Grounding of the Filament Transformer Secondary

The pulse transformer must be specially designed because there are very high frequency components present in a rectangular pulse of such short duration. The core is usually silicon steel and has laminations in the order of 0.003 inch in thickness. In general, a good pulse transformer must have low leakage inductance, low interwinding capacitance, and high primary inductance.

Low leakage inductance is necessary in transformers to preserve the steep leading edge of the pulse. This is achieved by very close coupling between primary and secondary and by using fewer turns on each. Low interwinding capacitance causes the oscillations between it and the leakage inductance to be of high frequency, making it easier to damp them out. Since capacitance between windings can be decreased by separating the windings farther, you see that a compromise is necessary. In most cases the spacing is rather close and the number of turns low, the primary and secondary consisting of a single layer each, wound one on top of the other on the same leg of the core.

To damp oscillations, a resistor is shunted across the primary or the secondary, or both. The high primary inductance is desirable to induce a high voltage into the secondary, and to insure that there is little change in the secondary voltage if the primary current remains constant. High primary inductance requires many turns, which is in conflict with the requirement for low leakage inductance. Usually the latter is considered more important so the primary inductance is not as high as would be desirable. For this reason the secondary voltage does drop some during the pulse. The maximum allowable drop is about 5%.

Types of Modulators

There are two types of modulators: the line-pulsing modulator and the driver-hard-tube modulator, Figure 12–44. The line-pulsing modulator stores energy and forms pulses in the same circuit element. This element is usually the pulse-forming network. The driver-hard-tube modulator forms the pulse in the driver and stores the energy in the modulator circuit.

The advantages and limitations of the various types of charging circuits are shown in Figure 12–45.

In circuit 1 of Figure 12–45 the pulse line is represented as C_{st}. It is charged through R_{ch} to the value of the DC voltage. During the pulse one-half of the DC voltage is applied to the magnetron (assuming the line impedance is matched to that of the magnetron) for a period of time determined by the line components. This is only 50% efficient. Therefore, it is necessary that the DC voltage be twice the value needed for the magnetron. The time constant $R_{ch}C_{st}$ must be large in comparison to the pulse length d but small in comparison to the period of the repetition frequency. This insures that the line will fully charge between pulses but will not discharge appreciably through the source and R_{ch} during the pulse.

In circuit 2 of Figure 12–45 resistor R_{ch} is replaced by an inductance L_{ch}. As C_{st} charges, the current through L_{ch} builds up a magnetic field. This field causes current to continue after C_{st} is charged to E_b. The charge will reach a peak of approximately 1.9 E_b after which it starts decreasing. If the modulator is triggered at the instant of maximum charge, the voltage applied to the magnetron is about 95% of E_b. This necessitates that the resonant frequency of L_{ch} and C_{st} be one-half the PRF of the radar. The firing time is very critical if you are to take advantage of the maximum voltage.

The difficulty encountered in the preceding circuit is overcome in circuit 3 by adding a diode

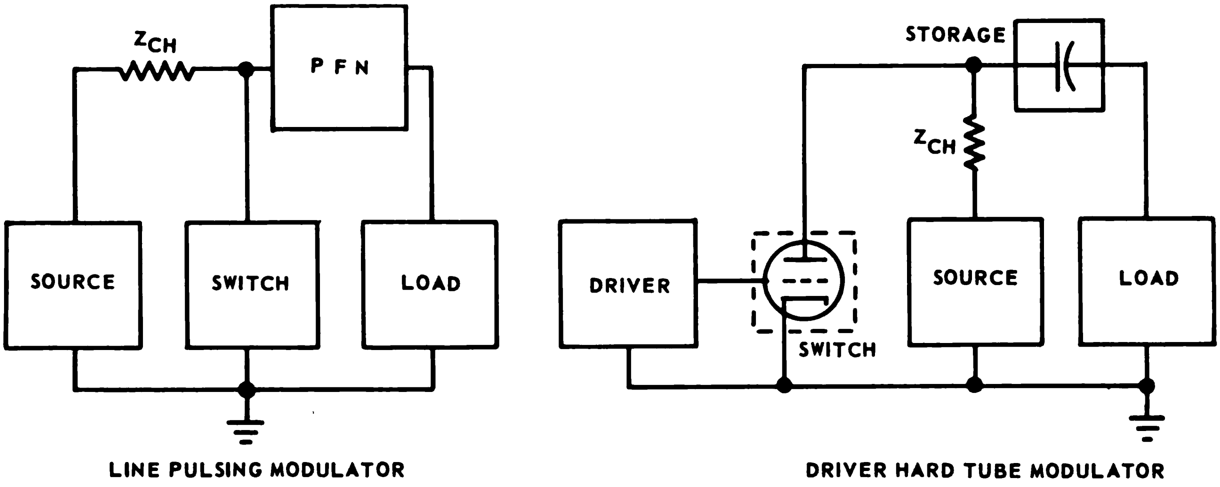


Figure 12–44. Two Types of Modulators

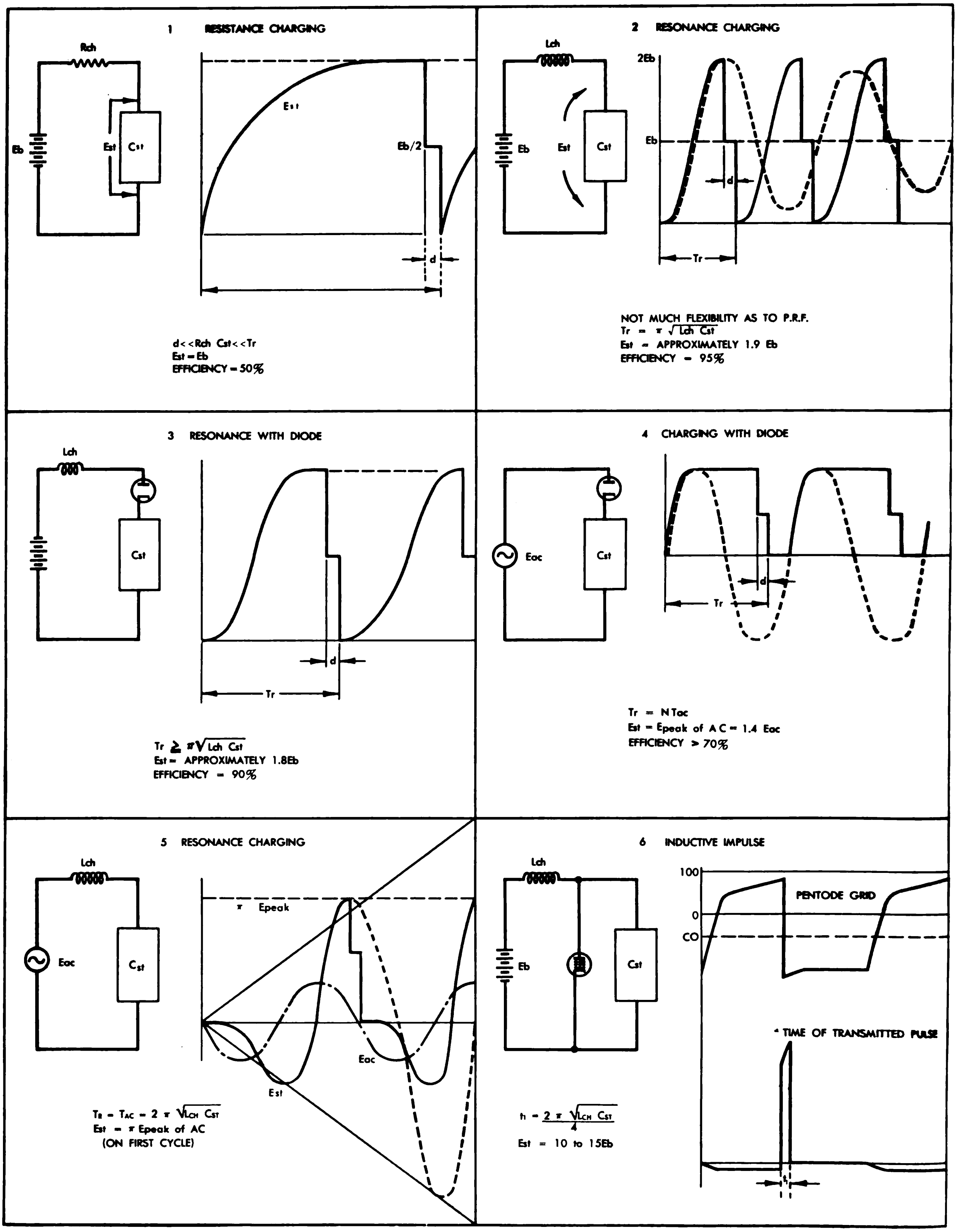


Figure 12–45. Types of Charging Circuits

in series with the charging element. This prevents the line from discharging after reaching its peak. The time of firing can occur any time after the peak is reached. Therefore, the frequency at which L_{ch} and C_{st} resonate is equal to, or greater than, one-half the PRF. The disadvantage of this method of making the firing time less critical is that the voltage to which the pulse-forming line charges is decreased by the drop across the diode to about 1.8 E_b. This makes the efficiency of this circuit about 90%.

Circuit 4 has the pulse line charging from an AC source. In this circuit the charging line is prevented from discharging by a diode in series with it. In this circuit it is necessary that every trigger pulse come at a time when the diode plate is negative; otherwise, the diode and the switch will short-circuit the AC source. Therefore, $T_r = N\ T_{ac}$. The voltage to which the line charges is almost the peak value of the AC wave so that the efficiency is about 70%.

In circuit 5 the diode is replaced by the inductance L_{ch}. This inductance, with C_{st}, forms a resonant circuit that is resonant at the frequency of the AC supply voltage. In this circuit the switch may be closed at any peak of oscillations. Usually, though, it is closed at the first peak. At this time the voltage is approximately π times the peak value of the AC voltage. In addition $T_r = T_{ac} = 2\ \pi\sqrt{L_{ch}C_{st}}$. If the switch is closed at the second peak, the voltage will be nearly twice as high. This type of charging gives voltages higher than the source voltage.

The inductive impulse charging circuit in Figure 12–45 provides voltages 10 to 15 times as great as the supply voltage. It is used where the available supply voltage is of the order of 1 kv and operates as follows: The pentode grid is raised above cutoff and into the positive voltage region so that the current through the inductance builds up a magnetic field around it. At time t_1 (the time before the beginning of the transmitter pulse) the pentode is suddenly cut off. The magnetic field in collapsing sends current through the pulse line and charges it to a very high voltage. To take full advantage of this high voltage, the transmitter must be keyed at the instant of maximum voltage. The correct timing is $t_1 = \frac{1}{4}\ (2\ \pi\ \sqrt{L_{ch}C_{st}})$. The value of t_1 is usually 10 to 15 microseconds. This type of charging circuit is used only in conjunction with a master timer.

Line-Pulsing Modulator. The typical line-pulsing modulator circuit in Figure 12–46 uses an AC resonance-charging circuit. It also has a 115v AC input which is stepped up to 3250v by an input transformer. The resonance-charging system charges the line to approximately 2.8 times the 3250 volts or about 9 kv. The switch is a rotary spark gap which fixes the PRF at 400

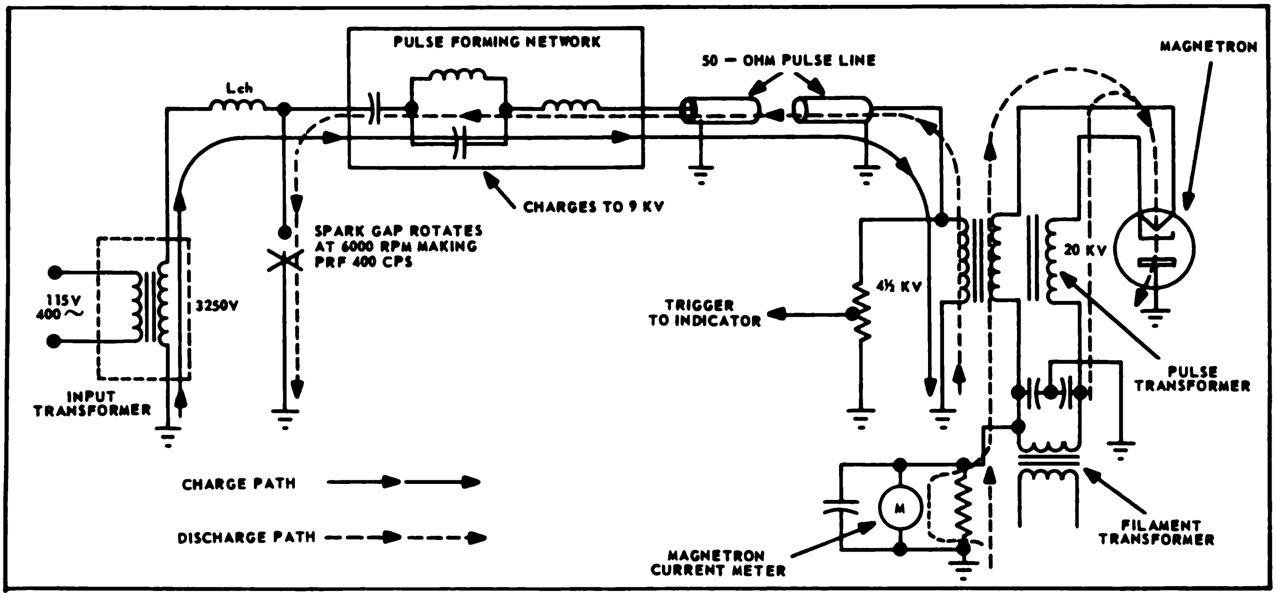


Figure 12–46. Typical Line-Pulsing Modulator

cps. The pulse-forming line is connected through a 50-ohm pulse line to the primary of a pulse transformer. The voltage across the primary is approximately 4.5 kv. This is stepped up to about 20 kv by the transformer.

The Driver-Hard-Tube Modulator. The driver-hard-tube modulator, as the name indicates, consists of two parts — the driver and the hard-tube modulator. The hard-tube modulator is the part that stores energy between pulses to be released when the hard tube conducts during the pulse. The driver is the part that shapes the pulse applied to the grid of the modulator.

In the earlier days of radar the hard-tube modulator was the most commonly used circuit, because at that time there was no good switching arrangement available for the line-pulsing modulator.

A simplified circuit of a hard-tube modulator using DC resonance with diode charging is shown in Figure 12–47. Between pulses from the driver, the modulator tube is nonconducting due to a very high negative voltage on its grid. The storage capacitor charges through the path indicated. This path includes R_1, L, and R_2 and the milliammeter in parallel. As R_2 is about 1000 ohms, most of the current flows through the meter. The purpose of R_2 is to maintain a charge path when the meter is switched to other circuits to measure current. Because the meter reads the charging current, it will indicate the average magnetron current since the charge that is lost during the pulse is replaced between pulses.

When the driver raises the grid potential of the modulator, it starts conducting and the capacitor discharges through the path containing the magnetron and the modulator tube. The charging diode prevents any discharge through that branch. A negligible amount of current will flow through the source, R_1 and L. When the driver lowers the modulator grid voltage below cutoff, the pulse ends.

The amount of voltage that the modulator applies to the magnetron is determined by a voltage divider circuit consisting of the static plate resistances of the magnetron and the modulator. These resistances have values of approximately 1000 ohms and 100 ohms, respectively.

The voltage that is applied to the magnetron is roughly 90% of the capacitor voltage. It is not a true rectangular pulse, however, because of the distributed capacitances in the leads to the two capacitor plates and to the magnetron. The total distributed capacitance, which is 50 to 120 micromicrofarads, must be charged as the voltage rises (in a negative sense) and discharged as the voltage decreases. The time constant for charging the distributed capacitance, which is determined mainly by the static plate resistance of the modulator tube, is on the order of 0.01 microsecond, a time which is small enough to ignore.

Discharge of the distributed capacitance takes place through R_1, L, and the magnetron itself. The calculation of the discharge time constant is rather complicated, but you can estimate it by ignoring the effect of the inductance and figuring the parallel combination of R_1 and plate resistance of the magnetron as the voltage decreases. For example, an average discharge time constant is about 0.5 microsecond as R_1 and the r_p of the magnetron are relatively high. This dis-

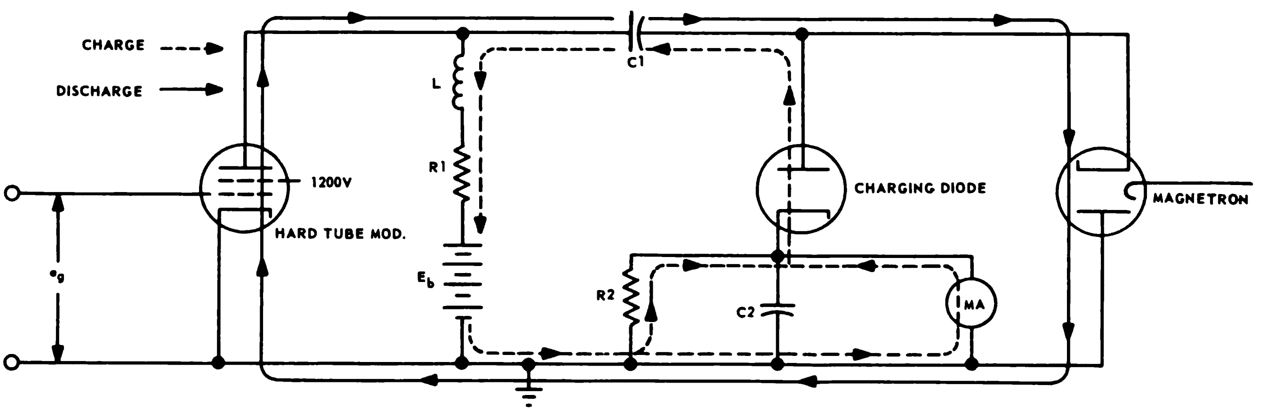


Figure 12–47. Hard-Tube Modulator with Diode Charging

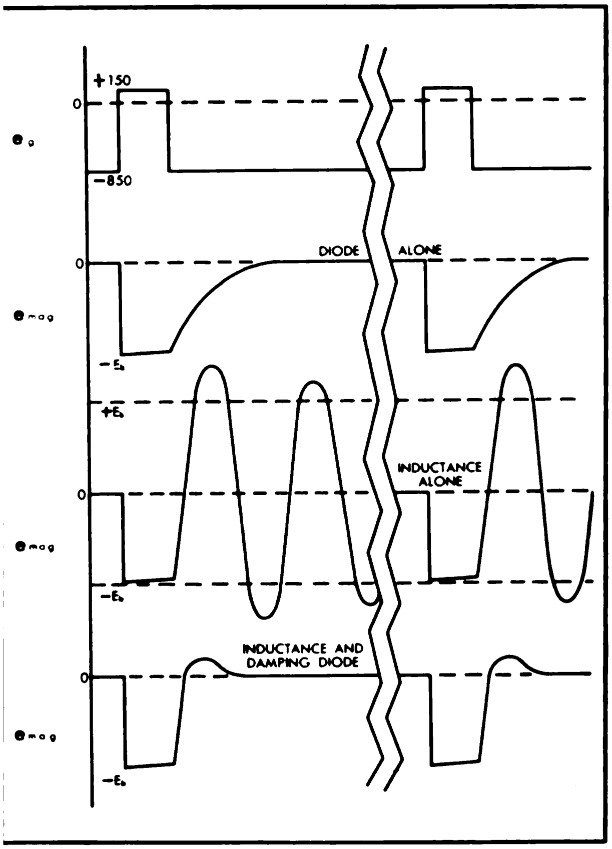


Figure 12–48. Voltage Waveshapes in Hard-Tube Modulator Circuit

charge time causes the voltage to trail off rather slowly as shown in Figure 12–48 and, in turn, produces considerable noise and erratic oscillations for one or two microseconds as the voltage passes through the region below 8 kv. All this makes it impossible to detect a nearby target.

The difficulty is overcome by using a tail-clipping circuit, which consists of an inductance and a diode connected in parallel, to replace the charging diode. In the modulator tail-clipping circuit in Figure 12–49, the inductance (L_2) is small, being 3 to 5 mh. In the charge and discharge paths, most of the charging current flows through L_2. This means that you could leave the diode out of the circuit as far as charging is concerned except for a very important function. The discharge path includes L_2 in parallel with the magnetron. At the beginning of the pulse little current flows through the inductance. Nevertheless, it builds up because the inductance is small. At the end of the pulse the inductance discharges through the distributed capacitance. This causes the voltage to return more rapidly to zero. Another point is that L_2 and the distributed capacitance can be thought of as a resonant circuit. With a small value of L_2 the resonant frequency is such that the period is only two or three microseconds.

The oscillations of this tuned circuit would cause the voltage to swing positive and negative if it were not for the damping diode which conducts when its plate swings positive and ends the oscillations. For the waveshapes with and without the damping diode, see the bottom illustrations in Figure 12–48. Since the current through the inductance has to build up during the pulse, there will be more change in magnetron voltage (faster discharge of C_1) than with diode charging alone. The larger the tail clipping inductance, the less the magnetron voltage will change. Already you have seen that in order to return the magnetron voltage to zero quickly

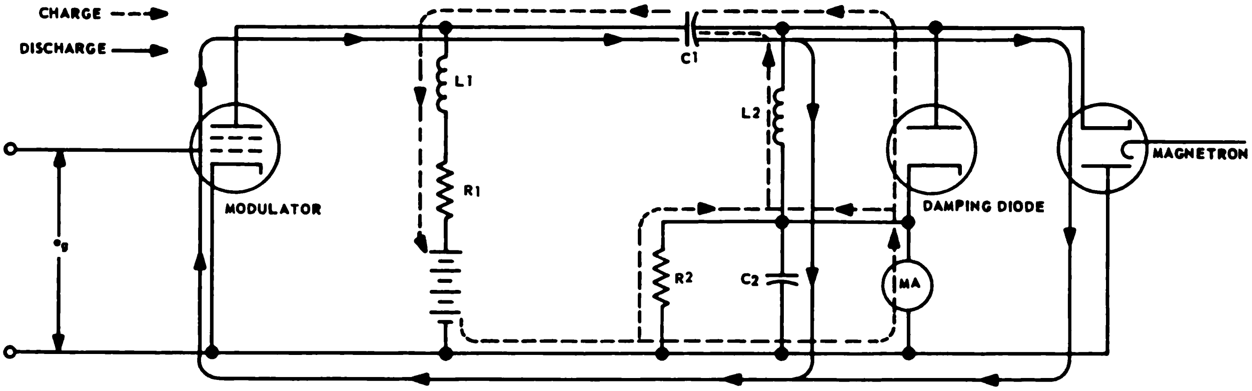


Figure 12–49. Hard-Tube Modulator with Tail Clipping

at the end of the pulse, you need a small inductance. It is evident that the inductance cannot be both large during the pulse and small at the end of the pulse, so a value that is a compromise between the two is used.

TYPES OF DRIVER STAGES. The two principal types of driver stages that are used for shaping the pulse applied to the hard-tube modulator are the boot-strap driver and the blocking oscillator.

The boot-strap circuit was widely used in earlier radar sets because it did not require a pulse transformer, a device that was not then perfected. The boot-strap driver circuit forms the pulse by discharging an artificial transmission line through a gas tube. Since the voltages of the pulse required to trigger the hard-tube modulator exceed the voltages that can be present in a gas tube, an amplifier is added between the gas tube and the modulator. To permit "off" tube operation of all three stages, cathode coupling is used. This coupling is unique in that it accomplishes amplification of the pulse without polarity inversion.

In the basic circuit diagram of the boot-strap driver in Figure 12–50, the circuits in heavy lines are those along which pulse formation and amplification progress. One side of the pulse-forming line is connected to —400v and the other to —1000v. This line charges to 600v when it is allowed sufficient time. In this modulator, the pulses are only 500 microseconds apart and the time constant of the charge circuit is about 300 microseconds. The pulse-forming line has time only to charge to approximately 500 volts.

The discharge circuit which the line uses when the gas tube is made to conduct by the trigger pulse is through the parallel combination of the cathode-to-grid resistance of the 829 tube and the 2.5K cathode resistor in series with the RX233 tube. The resistance of the RX233 is negligible and of the parallel combination is approximately $Z_0/2$, where Z_0 is the characteristic impedance of the line. (In some circuits the parallel combination has a value of Z_0.) With the line discharging through $Z_0/2$, one-third of the voltage to which the line is charged appears across the 2.5K resistor and the grid-to-cathode resistance of the 829 for a period of time determined by the components of the pulse line. This 167-volt pulse applied between grid and cathode of the 829 causes it to conduct heavily since its bias is —100 volts (cathode is at —1000v and grid at —1100v).

When the 829 is conducting heavily, the drop across the tube is about 250 volts so that the cathode rises to about +300 volts. This represents a change of about 1300 volts since in the nonconducting period the cathode is at —1000

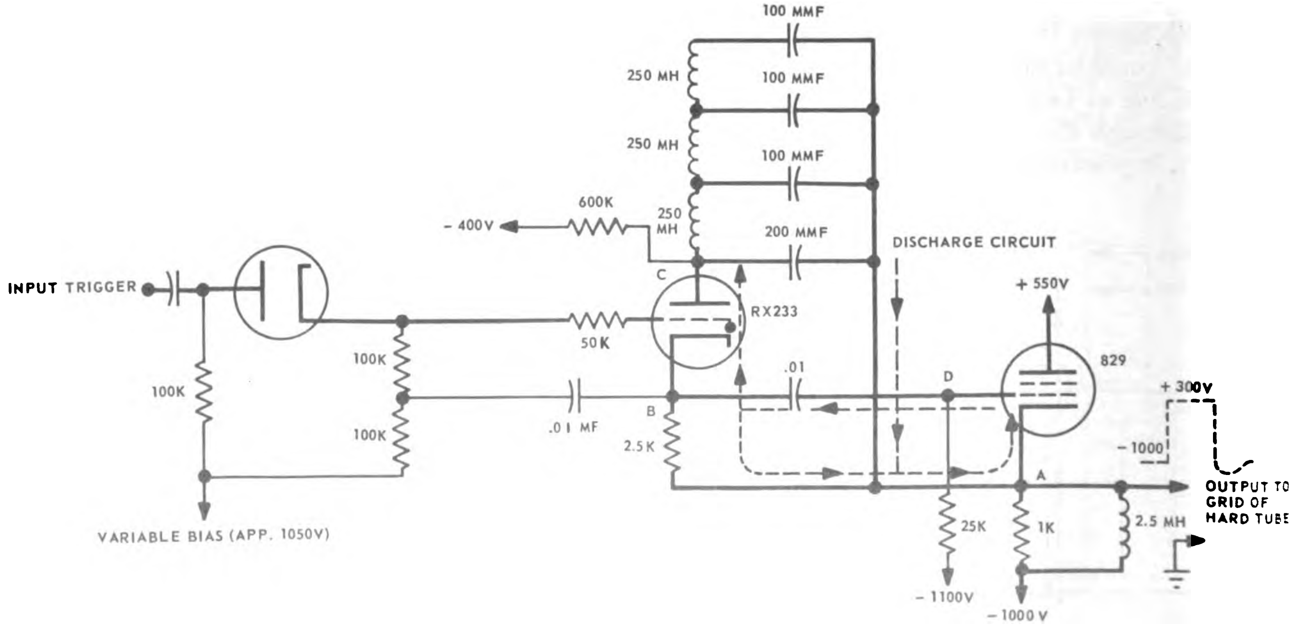


Figure 12–50. The Boot-Strap Driver for Hard-Tube Modulator

volts. The purpose of the 2.5 mh inductance in parallel with the 1K cathode resistor of the 829 tube is to produce the tail at the end of the pulse. The tail results from the collapsing of the magnetic field after the pulse. When the field collapses, current goes through the 1K resistor in the downward direction and causes point A to be more negative than —1000v for a brief time. This action insures a rapid cutoff of the hard tube.

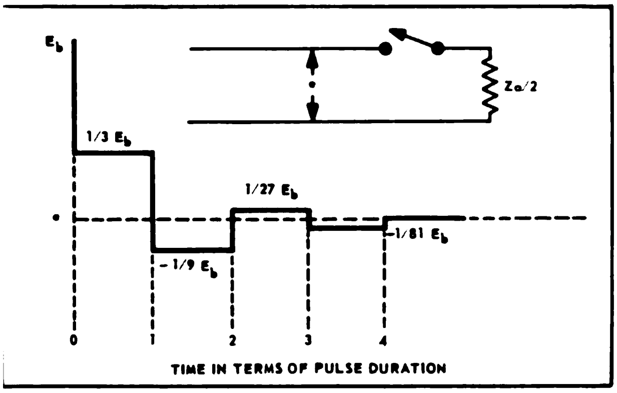


Figure 12–51. Discharge of Pulse Line into $Z_o/2$

The discharge of a transmission line through an impedance of $Z_o/2$ is oscillatory, as shown in Figure 12–51. In this diagram you can see that at the end of the time for a pulse, the voltage across the line swings to $-E_b/9$. (In this circuit this equals —500/9 or —55 volts.) Thus the gas tube and the 829 are cut off very quickly. The purpose of the diode in the input circuit is to isolate the trigger circuit from any transients.

A summation of the action at points A, B, C, and D before and during each pulse is as follows:

	A	B	C	D
Before	—1000v	—1000v	—500v	—1100v
During	+ 300v	+ 467v app.	+467v app.	+ 367v

In a line-controlled blocking-oscillator driver circuit, Figure 12–52, the pulse-forming line controls the duration and the waveshape of each pulse. Normally, the oscillator is cut off by the —140v bias. During the interval between pulses the line is charged to 140 volts since it is connected to the bias supply through the 27K resistor, the transformer secondary, S_1, and R_K (the cathode resistor of the trigger tube). The trigger pulse, which is about 150 volts in amplitude, is sufficient to raise the grid of the blocking oscillator well above cutoff. It is coupled to the grid through the capacitance of the pulse-forming line.

As in an ordinary blocking oscillator, the plate voltage drops when the tube begins to conduct. This drop is coupled back to the grid in opposite phase due to the transformer action and causes the grid to become more positive. This action is

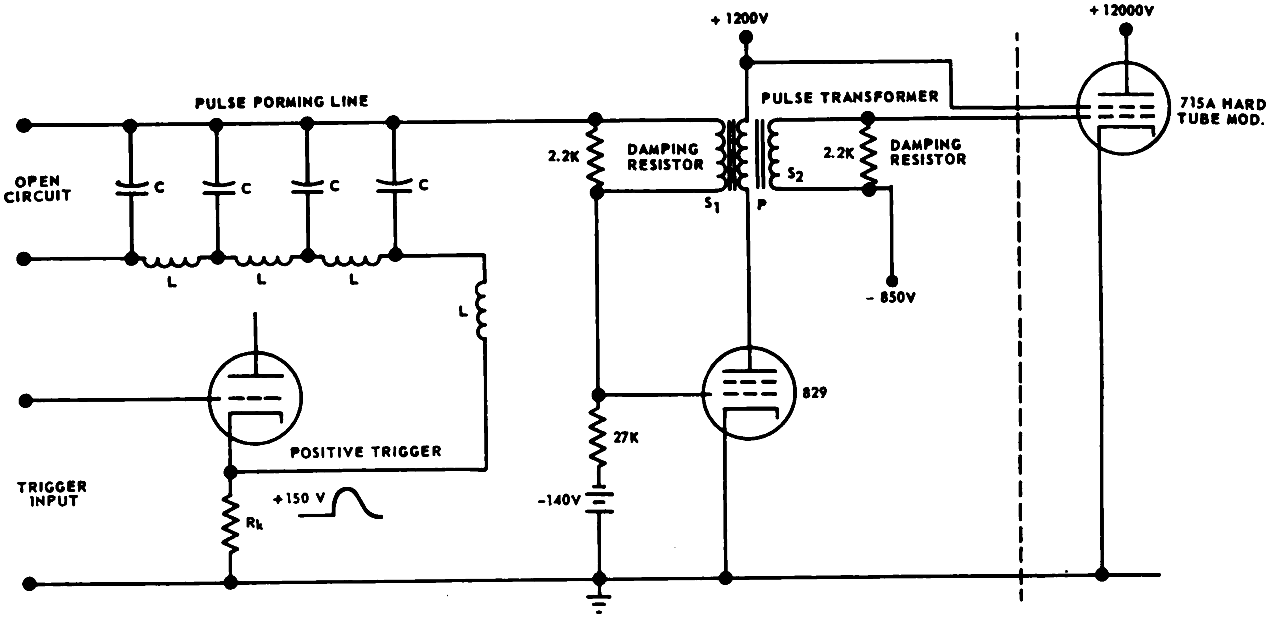


Figure 12–52. Line-Controlled Blocking Oscillator Driver

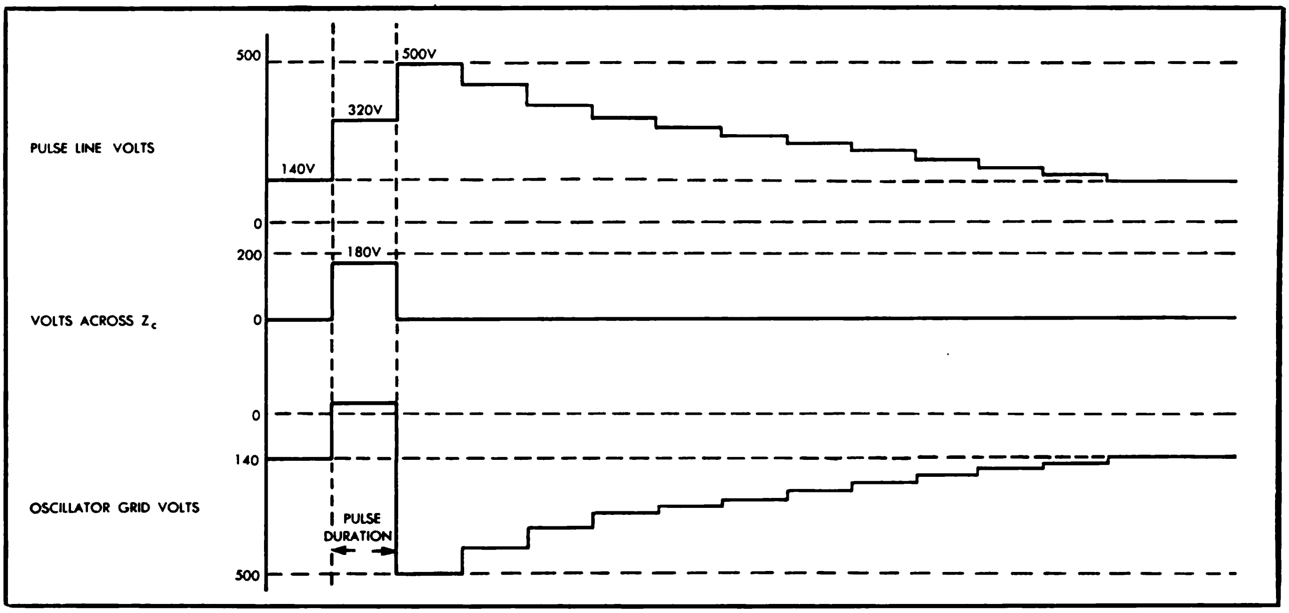


Figure 12–53. Blocking Oscillator Voltages

rapid enough to quickly drive the tube to saturation. In this condition the voltage across the primary P of the transformer is about 1000 volts and that across the secondary S_1 is about 500 volts. Since the pulse line is charged to 140 volts and since it is in series with the secondary voltage of 500 volts, it will charge to the higher voltage through the grid-to-cathode resistance of the oscillator, the secondary S_1, and R_K. Further, the impedance of the circuit is matched to the characteristic impedance of the line. Thus, the voltage across the line goes from 140 to 500 volts in two steps as shown in Figure 12–53.

The length of the step at 320 volts is the time it takes the voltage impulse to travel the length of the line, be reflected and return to the sending end. At that time the line is fully charged and current ceases to flow. The current flow through the grid-to-cathode resistance is practically constant during this time and keeps the grid-to-cathode potential steady, insuring little change in the primary voltage. At the time the line becomes charged, current flow ceases through the charging circuit and causes a drop in the oscillator grid voltage. This decreases the plate current and causes a rise in plate voltage. Through the transformer this drop appears as a drop in voltage at the grid and still further reduces conduction.

The action is cumulative, eventually resulting in the tube being cut off quickly. In this condition the line is charged to 500 volts. As it has only 140 volts applied, it starts discharging through the 27K resistor, S_1 and R_K. As this path has a much higher impedance than the Z_o of the line, the discharge appears as a series of steps that follows the general exponential curve. The output voltage applied to the grid of the hard-tube modulator is taken from another secondary winding S_2. This is a 1000-volt pulse. Damping resistors shunt both secondaries.

TYPICAL MAGNETRON TRANSMITTER

Block Diagram Analysis

The line-pulsing network in the modulator supplies a negative pulse to the input primary of the pulse transformer, Figure 12–54. The output of the high-voltage power supply determines the amplitude of the pulse, and the characteristics of the pulse-forming network determine its duration. The amplitude will vary generally from a negative 5kv to a negative 15kv. The duration will vary from $0.5\mu s$ to $2\mu s$. The step-up ratio of the pulse transformer increases the amplitude of the signal to a value that is optimum for operation of the magnetron. The impedance of the pulse transformer should match the characteristic impedance of the pulse-forming network to have maximum transfer of power.

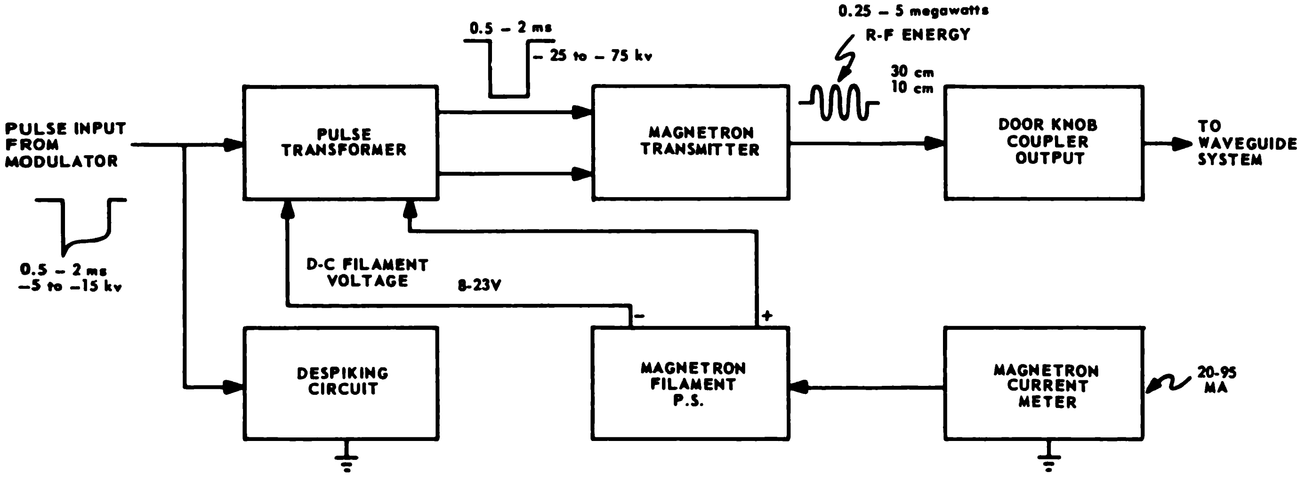


Figure 12–54. Typical Magnetron Transmitter Block Diagram

A high, negative voltage pulse from the secondary of the pulse transformer is applied to the cathode of the magnetron by the bifilar secondary windings. This negative pulse is considerably higher in amplitude but of the same duration as the input pulse. The usual value of the pulse is between a negative 25kv and a negative 75kv, depending upon the type of magnetron used. The conducting impedance of the magnetron is actually reflected back to the pulse transformer and becomes the impedance of the secondary windings. This impedance is determined by the type of magnetron used, voltage applied to the cathode, and strength of the transverse magnetic field.

This secondary impedance directly affects the primary winding impedance as shown by the formula:

$$A = \frac{B}{X^2}$$

where

A = primary winding impedance

B = secondary winding impedance

X = turns ratio, primary to secondary

You can see by using this formula that the impedance match and the power transfer from modulator to transmitter may be affected by several things. Among these are high-voltage power supply output, magnetic field strength, and pulse transformer step-up ratio. Magnetron characteristics and arcing in the pulse-forming network discharging circuit also will affect the impedance match and power transfer. Although the magnetron transmitter will operate over a relatively large variation of input voltages, it will be most efficient at the voltage for which the circuit was designed. Normally, the magnetrons are, at best, about 50% efficient.

A despiking RC network is generally used in parallel with the primary of the pulse transformer. The despiking network eliminates the undesirable effects of a spike on the leading edge of the pulse applied to the magnetron. The spike is caused by a momentary impedance mismatch before the magnetron starts oscillating. This network is discussed more in detail later in this chapter.

Filament voltage for the magnetron is usually a low DC voltage, although in some sets a 60-cycle voltage is used. The voltage for the filament ranges from 8 to 30 volts and the current from 4 to 10 amperes. The filament power supply is usually a low-voltage, high-current type of power supply. The filament voltage is applied to the magnetron through the bifilar wound secondary of the pulse transformer as illustrated in Figure 12–55.

The impedance of the pulse transformer to the DC or AC filament current is very low. During the time the transmitter is firing, high current is flowing in the secondary of the transformer. The peak current ranges in value from 20 to 50 amperes and lasts only for the duration of the pulse. In order to isolate the filament power supply from the voltage that would be developed across the power supply if the peak current were

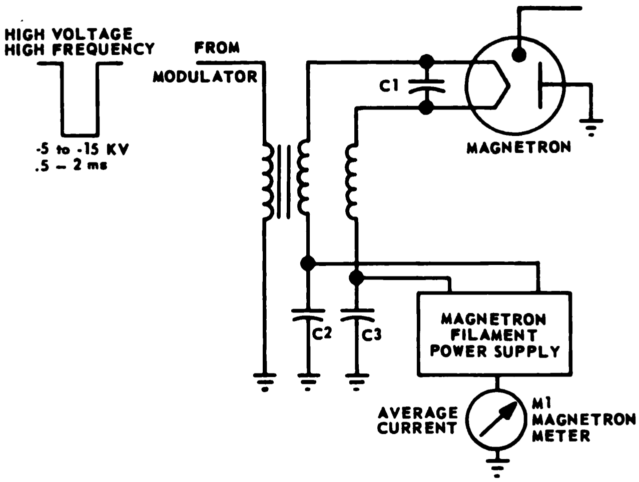


Figure 12–55. Basic Magnetron Filament Circuit

allowed to flow through it, capacitors C_2 and C_3 are placed as shown in Figure 12–55.

These capacitors filter out the current peaks, leaving an average current. The average current is normally 30 to 95 milliamperes. The average current flow from cathode to plate of the magnetron will also flow from ground through the filament power supply and pulse transformer secondary to the magnetron cathode. Now, an ammeter can be placed in series from ground to the filament power supply to measure average magnetron current.

Consider the high-voltage pulse from the modulator as a half-cycle of voltage. Because of the short duration, it is a high-frequency signal. You can see this by the formula:

$$f = \frac{1}{T}$$

where T is the time for one cycle. Since the negative pulse represents one-half of a cycle, the time for one cycle will be two times the pulsewidth. It is possible to isolate the high-voltage pulse from the filament voltage by using high- and low-pass filters. Capacitor C_1, Figure 12–55, protects the filament of the magnetron. By placing this capacitor in parallel with the filament, both sides of the filament are placed at the same potential for the high-frequency pulse.

The output from the magnetron is coupled from one of the cavities by an inductive loop. A doorknob coupler couples the energy into the waveguide. This type of coupling gives maximum power transfer over a relatively wide range of frequencies.

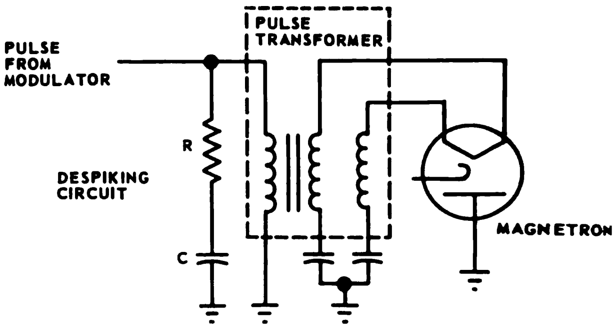


Figure 12–56. Despiking Circuit

Despiking Circuit

The despiking circuit compensates for the failure of the magnetron to oscillate instantaneously when a negative high-voltage pulse is applied. This circuit may consist of either an RCL or an RC network, as shown in Figure 12–56.

Just before the magnetron oscillates, the impedance of the pulse transformer and magnetron reflected into the ATL discharge circuit is high. The voltage will build up to high values that may cause the magnetron to arc without oscillating. Some precaution must be taken to prevent the magnetron from arcing. The despiking network acts as a low-pass filter in the pulse line, thereby eliminating the high-frequency components of the sharp leading edge of the pulse. Figure 12–57A shows the spike on the leading edge of the pulse. The impedance of the ATL and the impedance of the pulse transformer primary are of approximately the same value when the magnetron is oscillating. Regardless of what the reflected impedance is, the total impedance cannot exceed the value of R because the total impedance of a parallel circuit cannot exceed that of the smallest resistance.

By selecting a value of R that is slightly greater than the impedance of the ATL, a fairly good impedance match is obtained for the leading edge of the pulse. C (Figure 12–56) charges up very rapidly and then the RC circuit has a high impedance. However, by this time the reflected impedance has decreased greatly because the magnetron has started to oscillate. In some ground radar sets a coil is used in series with the ATL to slope the leading edge of the pulse (Figure 12–57B) and therefore ease the magnetron into oscillation.

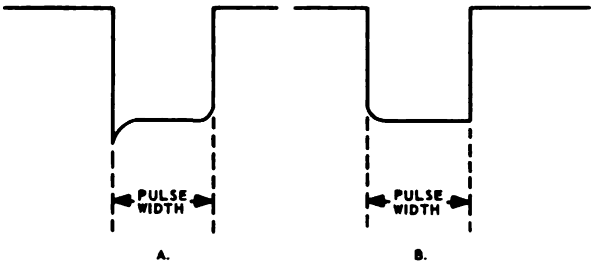


Figure 12–57. Effect of Despiking

CARE OF MAGNETRONS

Magnetrons are handled with extreme care at the factories where they are produced. Special shipping cartons are provided to insure a maximum amount of protection for the magnetron during shipment. Magnetrons are very sensitive to vibration and shock because the cathode may become off-centered slightly and thus cause the tube to be destroyed when used in a transmitter. Packaged magnetrons (those magnetrons with integral magnets) are damaged when subjected to an external magnetic field or when placed in contact with ferromagnetic materials. Many magnetrons are destroyed by careless handling before they are placed in a transmitter. The following set of rules for handling magnetrons is stressed highly.

1. Use the correct tools for opening shipping crates containing any type of electronic equipment.
2. When it is necessary to store a magnetron for a period of time or to ship it to another destination, be certain that the dust covers are placed over the cathode bushing and r-f output windows. The danger of destroying an expensive magnetron because of accumulated dirt can only be attributed to neglect.
3. Do not strike any surface with the magnetron nor strike the magnetron with any other object during installation, since it may be very harmful to both the magnetron and the magnet. BE VERY CAREFUL.
4. Do not try to force the magnetron into position when installing it. This results in shattered ceramic parts and broken seals.
5. Use nonmagnetic wrenches when installing magnetrons. This protects the magnetron from damage, and also lessens the chance of skinned knuckles when working in a magnetic field.
6. Use only the grade of oil specified by the equipment manufacturer for the pulse transformer and magnetron. Failure to do this may result in pitted bushings and eventual failure of the magnetron.
7. Never use steel wool or any other metallic abrasive to clean a magnetron. If cleaning is necessary, use a nonmetallic cleansing agent. Steel particles from steel wool, because of the strong magnetic field, can adhere to insulating parts and thus cause high-voltage breakdown.

By following the preceding simple set of rules, you will prevent many magnetron failures resulting from mishandling.

Some other faults which can cause magnetron failure are:

1. High-voltage standing-wave ratio in waveguide. This causes burned and cracked r-f output windows.
2. Low oil level in pulse transformer. Result of this is heavy internal arcing of magnetron and subsequent damage to cathode and ceramic insulators.
3. Defective ATR and TR tubes, insufficient waveguide pressure, and dirt or moisture in waveguide. These troubles may cause high energy to be reflected back to the transmitter. This would result in a high-voltage standing-wave ratio and a damaged magnetron.

Remember, microwave transmitter tubes are very fragile and subject to damage. Handle these tubes with as much care as you would a CRT.

VELOCITY-MODULATED TUBES AS LOCAL OSCILLATORS

In radar most receivers use 30- or 60-mc intermediate frequencies. A highly important factor in receiver operation is the stability of the local oscillator which generates the frequency that beats with the incoming signal to produce the i-f. For example, if the local oscillator frequency is 3000 mc, a frequency shift of as much as 0.1% would be a 3-mc frequency shift. This is equal to the bandwidth in most receivers and would cause a considerable loss in gain.

Still another consideration in radar is a receiver that uses a crystal mixer. In such a receiver the power required of the local oscillator is small, being only 20 to 50 mw in the 3000-mc region. Due to the very loose coupling, only about one milliwatt actually reaches the crystal.

A third requirement of a local oscillator is that it must be tunable over a range of several megacycles. This is to compensate for changes in the transmitted frequencies and in its own frequency. In cases where the r-f units are located in a remote position, it is desirable to mount the converter section of the receiver remotely. This insures that the intermediate frequency, rather than the transmitted frequency, is cabled back to the receiver, and makes it necessary that the local oscillator be tuned from a position some distance away, preferably by varying the voltage applied to it.

Because the reflex-velocity-modulated tube meets these three requirements, it is used for local oscillators in microwave radar receivers. The following deals with its operation. In it the cavity resonators are treated as parallel resonant circuits, as was explained in the theory of the resonant cavity in the preceding chapter.

Theory of Operation

In the circuit of the reflex-velocity-modulated tube, Figure 12–58, note the arrangement of electrodes and the voltages involved for operation. Electrons are emitted by an indirectly heated cathode. These electrons are attracted by the cavity grids which are more positive than the cathode by the amount of voltage E_a. The control grid, which is located between the cathode and the cavity grids, has as its purpose the control of electron flow. It also has a *positive* potential which is usually about 200 or 300 volts. The electrons emitted from the cathode travel toward the cavity grids at a velocity determined mainly by E_a.

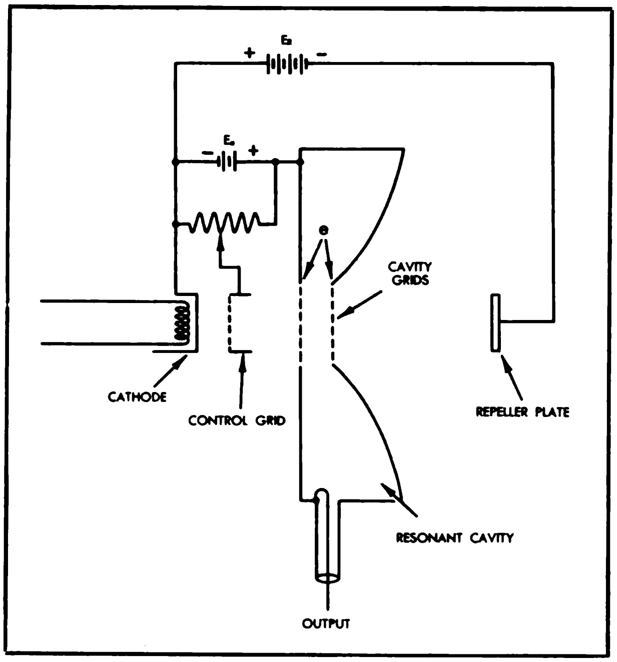


Figure 12–58. Circuit of Reflex-Velocity-Modulated Tube Oscillator

Most of the electrons pass through the control grid, the cavity grids, and continue on toward the repeller plate. After passing the cavity grids they come to a region where the electrical field opposes their motion, since the repeller plate is negative with respect to the cathode by the voltage E_R. This voltage is variable with a maximum of about −100 volts. This, in turn, makes the voltage from repeller to cavity grids 300 or 400 volts and slows down the electrons, causing them to come to a stop. After stopping they reverse direction and pass back through the grids. They are collected either by the control grid, the shell, or the cathode of the tube.

An important consideration is the cause of oscillations in resonant cavities. In most oscillators, oscillations start from some irregularity in current flow, such as a transient that results from voltage being suddenly applied to the tube or from shot effect. With this in mind, assume that the oscillations in the resonant cavities are already taking place. From this assumption examine the source of the energy needed to sustain these oscillations.

With the tank circuit oscillating, a high-frequency voltage, *e*, appears between the two cavity grids. This makes the electric field between these grids reverse twice each complete cycle of operation. In this condition as the electrons approach these grids, the electron stream is uniform. The time that is required for the electrons to pass through the short distance between the grids is small compared to the period of oscillations. Electrons that enter the space between the grids when *e* is zero will encounter no AC electrical field and will pass on through at the same velocity. The electrons which enter the space when *e* makes the left grid negative with respect to the right will encounter a field that tends to accelerate them. The amount they are accelerated is a function of *e*. Electrons entering the space when *e* is reversed in polarity are decelerated.

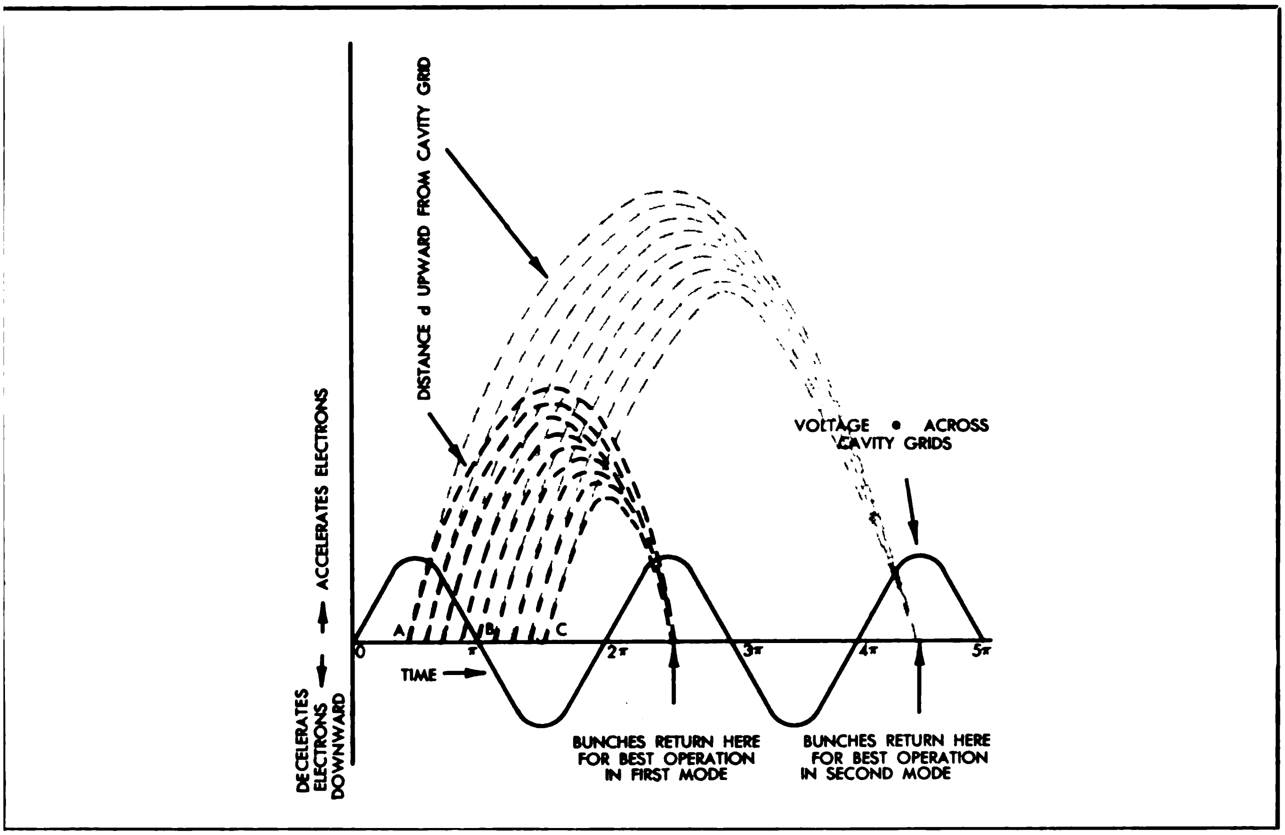

DISTANCE d UPWARD FROM CAVITY GRID
ACCELERATES ELECTRONS
DECELERATES ELECTRONS DOWNWARD
VOLTAGE e ACROSS CAVITY GRIDS
A
B
C
0
π
2π
3π
4π
5π
TIME
BUNCHES RETURN HERE FOR BEST OPERATION IN FIRST MODE
BUNCHES RETURN HERE FOR BEST OPERATION IN SECOND MODE

Figure 12–59. Bunching Action of Reflex-Velocity-Modulated Tube

The change in velocity caused by acceleration and deceleration is small in comparison with the original velocity. Those electrons which are accelerated most will travel farther toward the repeller plate before being turned back, while those that are decelerated most will be turned back before approaching very close to the repeller. Thus it is conceivable that with the proper magnitudes of e, E_a and E_R, electrons returning to the cavity grids will arrive in bunches.

Figure 12–59 shows the position of electrons in the tube at various times during their transit. The zero distance position is midway between the cavity grids. Electron A, which arrives when e is positive, is accelerated and travels farther before being turned back; electron B is unaffected; electron C is decelerated and turns back after a shorter excursion. Hence, in the diagram these electrons and the ones passing through at intermediate times are shown as arriving back at the grids at the same instant of time. This is the ideal situation, but it is not difficult to see that electrons will return to these grids in a stream that varies in intensity at the frequency of the oscillations. Therefore, this tube is called a velocity-modulated tube. It is called a *reflex*-velocity-modulated tube because the electrons reverse direction and travel through the interelectrode space twice.

On the return trip the electric fields set up by the voltage e again act upon the electrons. Since they are now traveling in the opposite direction, they will be decelerated if they return when e is positive and accelerated if they arrive when e is negative. You know that an electron which is accelerated by an electric field has its kinetic energy increased and that this additional energy is taken from the electric field. Also, an electron that is decelerated gives up energy to the electric field. If the bunches of electrons can be made to arrive back at the grid when e is positive, they will give up energy to the alternating field. For maximum transfer of energy the bunches must arrive when e is maximum positive.

The question arises as to where this energy originated. You have seen that if the electron stream from the cathode is uniform, some electrons are accelerated and some are decelerated, on the outbound trip, by the electric field of e. On the average, as many electrons absorb energy from the field as give up energy to it. Hence, very little net energy is taken from the oscillating circuit during the bunching process. The average kinetic energy of the electron is that imparted to it by the DC voltage E_a. Thus, you see how energy is taken from the DC electric field and transferred to the AC field to sustain the oscillations.

Modes of Operation

It is not necessary that the bunches of electrons return to the grids on the first positive swing of e after they leave them. Figure 12–59 indicates that if the bunches arrive on the second positive swing, the net result is still the same. You can see that the time in transit for the average electron B is three-fourths of a cycle, one and three-fourths cycles, two and three-fourths cycles, etc. In actual practice there are three or four "modes" in which it is possible for the reflex-velocity-modulated tube to oscillate. The black lines in the figure indicate paths of electrons operating in the first mode while the grey lines indicate paths of electrons operating in the second mode. A third mode is possible when the average transit time is two and three-fourths cycles, etc.

The following shows you how the transit time is controlled to produce oscillations in the different modes. If you reason a little, you can understand that since the original velocity of an electron depends on the DC voltage E_a, and since the distance that the electron travels before turning back and the speed with which it returns depend upon the difference between E_a and E_R, it is possible to adjust the two voltages E_a and E_R for any of the modes. The voltage E_a is usually fixed in magnitude since varying it produces greater initial velocity, which in turn causes a farther excursion and a greater return velocity. Since it is not feasible to make E_a variable, E_R is variable. For operation in the first mode, the round trip must be completed in the shortest time. This is accomplished by making the repeller plate most negative. For greater time in the interelectrode space, the repeller is made less negative.

Figure 12–60 shows power output and frequency of oscillations as functions of the repeller voltage for three modes of operation. Notice that the frequency at the point of maximum output is the same for all three modes and is the resonant frequency of the cavity. In addition, note that the power outputs for the various modes at the resonant frequency are not the same and that the output is least in the highest mode. This can be explained by examining the factors which limit the amplitude of oscillations and which, in turn, limit the power output.

Power and amplitude limitations are due to overbunching as well as the usual losses in the oscillatory circuit. Overbunching occurs in the following way. As oscillations build up and e becomes greater, the amount of acceleration and deceleration increases. This causes bunching to occur in a shorter period of time, that is, in a time before the electrons reach the grids on the return trip. This tends to reduce the magnitude of oscillations. In the higher modes of oscillations where the bunches are formed more slowly, the electrons are more susceptible to overbunching. The magnitude of e that results in overbunching is therefore lower, and oscillations are limited by this action to a lower amplitude than in the lower modes of operation.

As shown in Figure 12–60, the frequency of oscillations in a reflex-velocity-modulated tube is variable to a limited degree in any of the modes of operation by varying the repeller voltage. When the repeller voltage is varied, it causes a bunch to return either a little sooner or a little later than normal. Off resonance, the amplitude of oscillations decreases by an amount depending on the Q of the cavity. In this tube the tuning range is small in comparison with the frequency of oscillations and varies somewhat from one mode to another. It is greatest in the highest mode, because bunching and debunching take place at a slower rate and because greater variation from the ideal time of return is possible without debunching, which would cause the amplitude of oscillations to drop below the usable output level.

Another way to look at it is that in the highest mode the interval between leaving the grids and returning is greater, and the change in period represented by a given change in frequency is a smaller portion of the interval. To illustrate, in the third mode the interval before return must

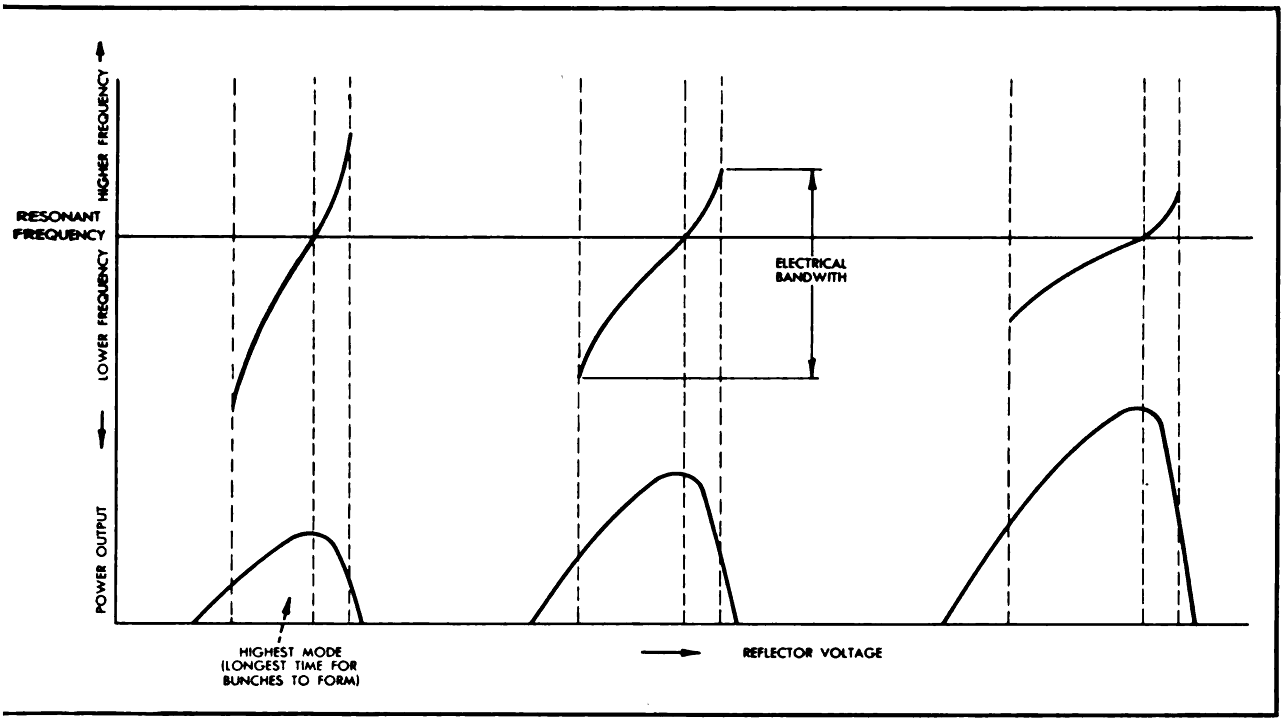


Figure 12–60. Power Output and Frequency Versus Repeller Voltage in Different Voltage Modes for a Reflex-Velocity-Modulated Tube

be about two and three-fourths cycles. A small change in the period of *e* would therefore be only 3/11 as great a portion of the interval as t would if operation were in the first mode where the ideal time interval is three-fourths of a cycle.

The band of frequencies which can be obtained by varying the repeller voltage lies between the half power points shown in Figure 12–60. This range of frequencies is known as the *electrical bandwidth.* The output curves of the bandwidth are unsymmetrical about the maximum output points. This results from the fact that if E_R is increased, not only does the bunching voltage *e* decrease and cause bunches to form at a later time, but the repeller voltage causes a quicker return. Also, the effects of the two actions add to cause poor bunching at the time the electrons return, resulting in a rapid drop in output on the high side of the hump. At lower voltages, however, even though the bunching voltage *e* decreases and causes slower bunching, the decreased repeller voltage causes a later return to the grids. In this way the two effects are counteracting and a greater change in repeller voltage is possible before the output drops below the usable level.

The choice of the point and mode of operation is a compromise among several factors. To begin with, there are three or four modes that have the necessary power output. On the whole, then, it would seem that the correct choice would be the highest mode, for it gives the largest tuning range. The highest mode, however, is too sensitive to a change in voltage to be very well regulated. A change of one volt may cause a change of 0.5 mc in the 3000-mc oscillator. Since the humps are unsymmetrical, the point of operation is usually chosen a little below the point of maximum output. This makes possible the tuning above the operating frequency by a greater amount than if the maximum point were used.

In practice, the reflex-velocity-modulated tube is usually used in conjunction with an automatic frequency control circuit. This circuit controls the repeller voltage in such a way as to maintain the correct intermediate frequency. The details of the discriminator circuit and its operation were discussed in Chapter 9. Keep in mind that the frequency of oscillations is primarily determined by the dimensions of the cavity and that the repeller voltage is effective in making small

No.	Name	Mfr.	Freq. (Mcps)	Acc. Voltage	Beam Cur. (Ma)	Repeller Voltage	Control Grid Voltage	Power Output (mw)	Elec. Tuning (Mcps)
K417	Klystron	Sperry	3000	300-600	5-30	+50 to −500	+5 to +50	150	5
707A	McNally	W. E. and Raytheon	3000	250-325	25-35	0 to −250	same as acc.	75	30
726A	Shepherd-Pierce	W. E.	3000	300	22	−20 to −300	same as acc.	100	20
723A	Shepherd-Pierce	W. E.	9400	300	18-25	−20 to −300	same as acc. (internal connection)	20	45

Table 12–2. Typical Reflex-Velocity-Modulated Tubes

changes in the frequency. Hence, in most reflex-velocity-modulated tubes there is a coarse frequency adjustment that varies the cavity size in some way. The repeller voltage is the fine frequency adjustment.

Tubes

Table 12–2 gives some of the operating characteristics of reflex-velocity-modulated tubes. The data in this table will give you an idea of the order of magnitude of tube quantities. There is wide variation between different tubes and different conditions of operation.

The K417 reflex klystron is one of the earlier types that was used for 10-cm operation. One feature of this tube was that in its early application it did not have provision for controlling the frequency through a change in the repeller voltage since both coarse and fine frequency controls changed the cavity grid spacing.

Another 10-cm tube is the 707A (McNally) tube shown in Figure 12–61. In it the cavities are external to the tube and are not evacuated. This makes them susceptible to changes in temperature which results in changes in frequency. To get good frequency stability in it, it is necessary to control the cavity temperature. The coarse frequency control consists of plugs which, when screwed into or out of the cavity, change its size. Fine frequency is controlled by the variable repeller plate voltage control.

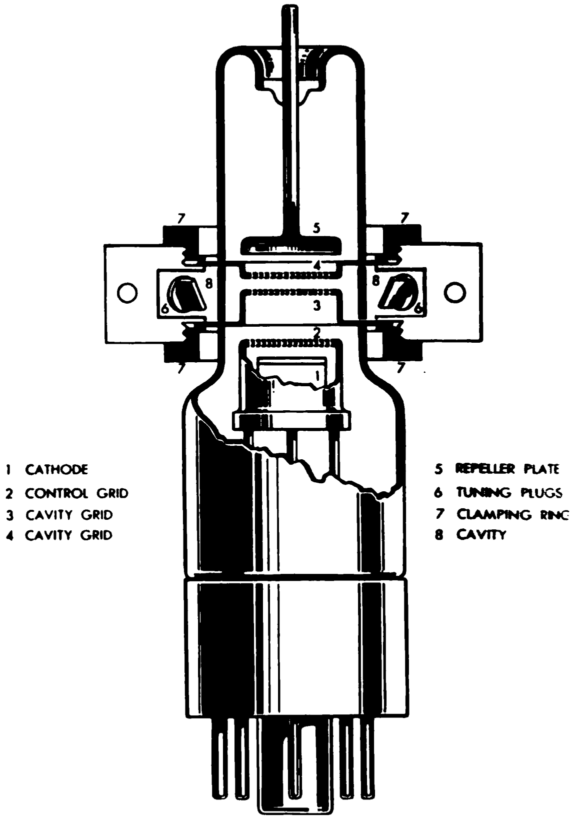


Figure 12–61. 707A (McNally) Tube

The Shepherd-Pierce tube (Figure 12–62) is an all metal tube which is available for both 10-cm and 3-cm operation. In it the cavities are located inside the tube. Mechanical coarse tuning is accomplished with struts on the side of the tube. The struts are adjusted by a screw which, in turn, varies the size of the cavity. The repeller voltage control also serves as the fine frequency control. The 10-cm and 3-cm type Shepherd-Pierce tubes differ in the shape of the cavity and in the method of coupling the output.

VELOCITY-MODULATED TUBES AS AMPLIFIERS OF POWER

In the following discussion a brief explanation of the two-cavity klystron, and a detailed explanation of the three-cavity klystron will be given. In addition, this type of power amplifier is compared with the heretofore conventional magnetron. It will be seen that the three-cavity klystron, like the two-cavity type, possesses a cathode for emission of electrons, an input cavity, a drift tube, an output cavity, and a collector that is positive with respect to the cathode. The three-cavity klystron differs in the addition of a third cavity in the drift tube, in size, and in gain.

Two-Cavity Klystron

Figure 12–63 shows the construction and essential components of a double-cavity klystron. In this klystron the indirectly heated cathode emits electrons that are focused into a sharp beam by the control grid, which is at a low positive potential. The beam is then accelerated by a very high, positive DC potential on grids G_2 and G_3, which are very close together and connected to a cavity resonator. Superimposed on this DC potential is

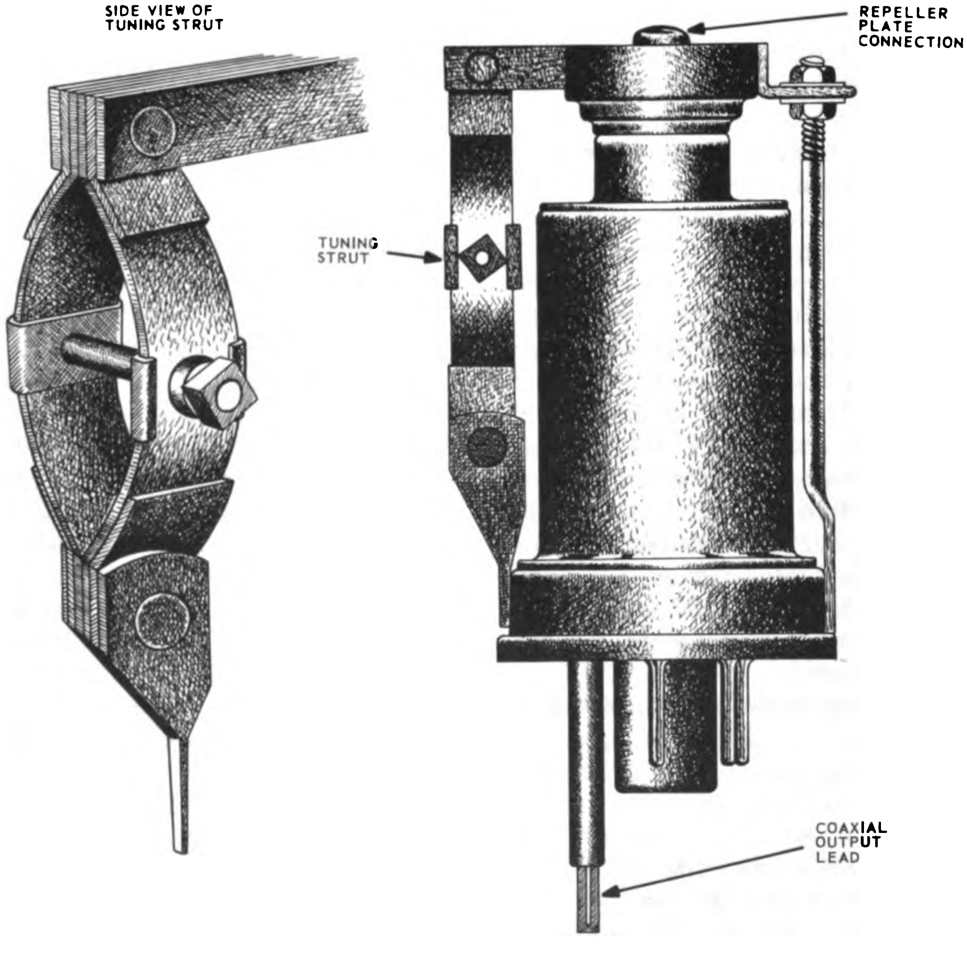


Figure 12–62. External View of Shepherd-Pierce Reflex Klystron

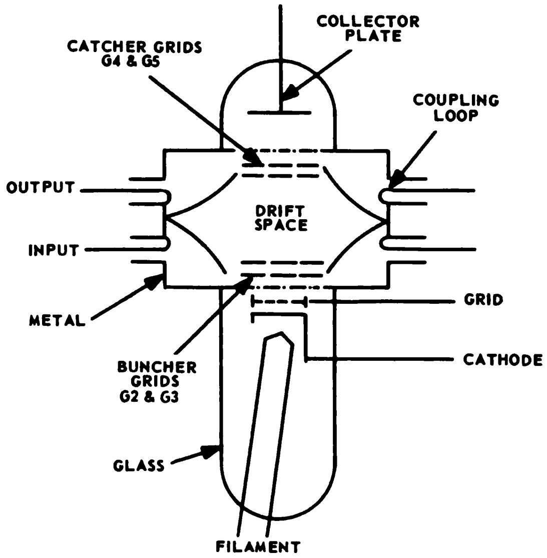


Figure 12–63. Two-Cavity Klystron

an AC voltage that changes the acceleration of the various electrons differently. Those electrons that reach grids G_2 and G_3 at a time when the AC voltage is zero will be accelerated either more or less than normal, depending upon whether the AC voltage is going positive or going negative at the time they reach the grids.

Grids G_2 and G_3 are called *buncher grids* because the electrons, which have been accelerated differently, travel at different velocities through the drift space and will bunch at some point. If these bunches of electrons arrive at grid 4 and grid 5 and are there decelerated, they deliver energy to the cavity resonator connected to these grids. Since the function of grids 4 and 5 is to absorb, or catch, the energy contained in these bunches, they are called *catcher grids*. Electrons that continue through the catcher grid hit the collector plate and are returned to the cathode through an external circuit.

The bunching action that occurs in the klystron is illustrated by the Applegate diagram in Figure 12–64. In it the lines all start at the top of the diagram at the buncher grids; they represent the paths of the various electrons in their travel through the drift space. The AC voltage waveshape at the top represents the AC component of the voltage on the buncher grids; the horizontal axis represents time. The lines are therefore tilted to the right in their passage from buncher to catcher grids. The difference in the slope of the various lines as they leave the buncher grids represents the different velocities of the electrons, which are due to the different accelerations imparted to these electrons by the AC component of the voltage on the buncher grids. Note that the electrons which arrive at the buncher grids between times t_0 and t_1 form a bunch at the catcher grids. There is only one bunch of electrons in each cycle; hence, the frequency of bunches arriving at the catcher grids is the same as the input frequency.

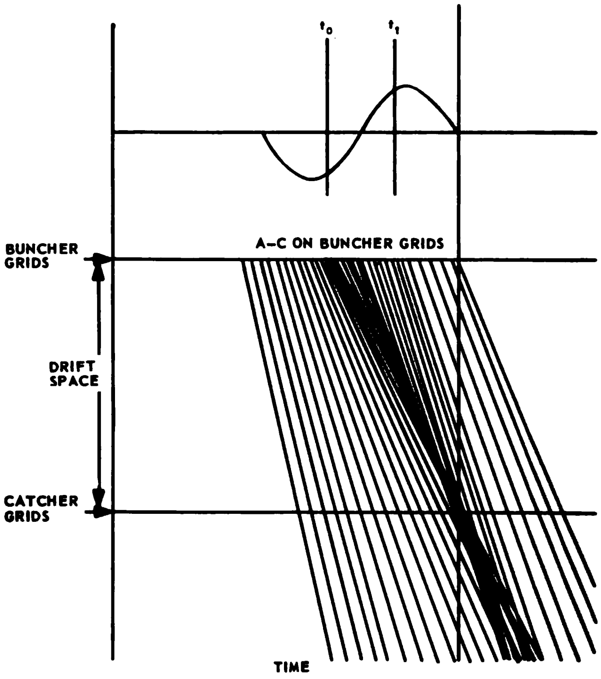


Figure 12–64. Two-Cavity Bunching Action, Applegate Diagram

For the klystron to function most efficiently, the drift space should not be too long. If the distance between catcher and buncher grids is made too great, the bunches disappear or tend to merge, thus making the beam no longer usable. There is a certain spacing that gives best results with given DC and AC voltages. Since the spacing (drift space) is fixed in a tube, the voltages must be accurately adjusted for the tube to function properly. Also, the resonant circuit of the catchers must be tuned to the correct frequency. The strength and phase of the oscillation must be such as to absorb maximum energy from the electron bunches.

The AC, which is superimposed upon the DC pplied to the buncher grids, may be applied rough the input terminal and then by a oupling loop to the buncher cavity. The output ay be taken from the catcher with a coupling op through the output terminal. The klystron ower amplifier may be used in pulse operation y applying in pulses the DC voltage, the r-f oltage, or both.

hree-Cavity Klystron

In this type the cathode is heated by an AC perated heater. The heater and cathode asemblies, with leads brought out at the base of he tube, are insulated from the entrance to the rift tube by a glass insulating sleeve. The input avity is located close to the drift tube entrance. The middle cavity is located farther along the rift tube, and the output cavity is at the other nd of the drift tube. The collector is beyond he output cavity. The entire drift tube assemly, the three cavities, and the collector are at round potential. This eliminates hazards and omplications involved in tuning the cavities, nd in liquid cooling of the body and the colector. With the drift tube assembly at ground otential, the cathode is pulsed with a negative oltage to accelerate electrons from the cathode oward the drift tube entrance. The negative ulse is supplied from a pulse-forming network nd is stepped up to the desired value by a pulse ransformer.

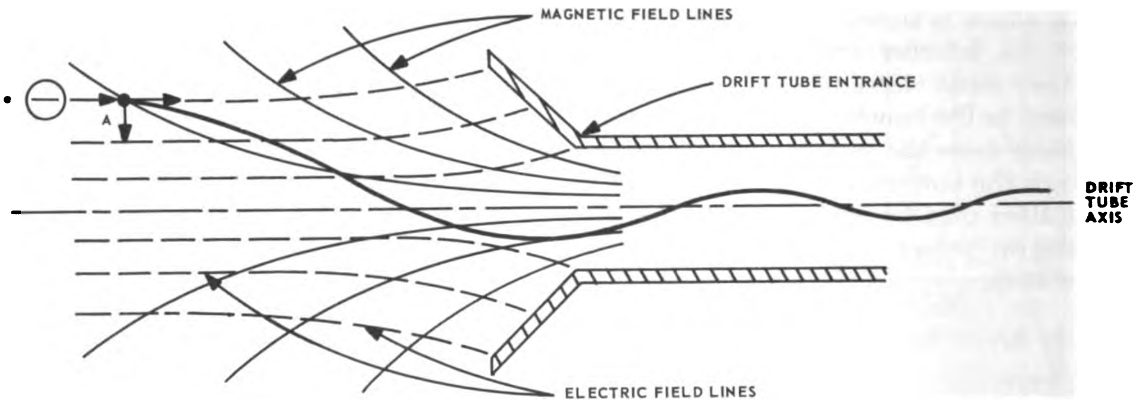
In high-power klystrons the amplitude of the athode pulse is measured in hundreds of kiloolts, and the peak current flowing for the duraion of the pulse is measured in hundreds of mperes. The high values of accelerating voltages roduce dangerous amounts of X-ray radiation, articularly around the collector, where most of the electrons strike. To protect maintenance personnel from harmful radiation effects, the entire klystron is housed in a heavy lead shield. Because of the great current density and the distance of electron travel, electromagnetic focusing is used to control the electron stream.

The choice of cathode requires much consideration. For a desired r-f power output, a certain amount of DC power must be available. If a peak r-f output of one megawatt is desired, with an expected efficiency of 33%, a peak DC power input of three megawatts is required. The DC input power is the product of beam voltage and current. The same amount of power can be supplied at high voltage and low current or at low voltage and high current. There is a practical limit to the maximum pulse voltages that can be generated consistently at the PRF's common in modern radar systems. On the other hand, an electron beam of very great density is difficult to control and produces severe focusing problems. The beam voltage and current values used are therefore a compromise between high beam voltage and current density.

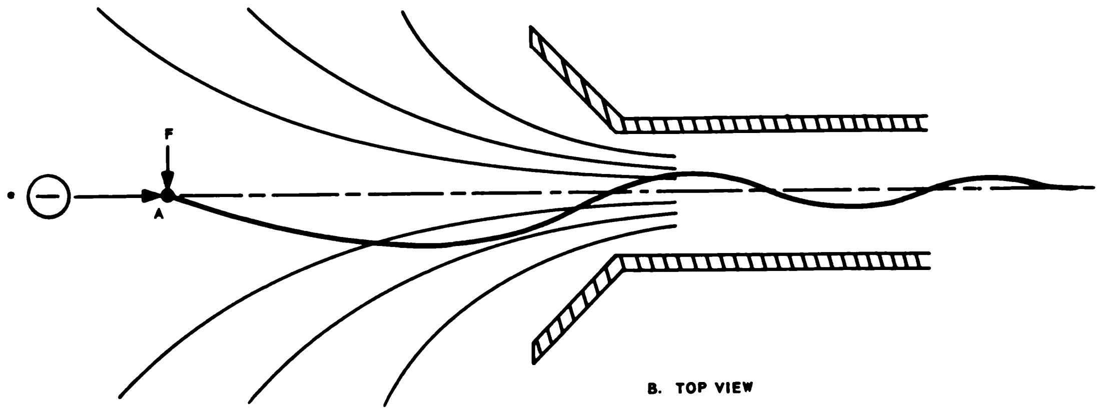
The relation between beam voltage and beam current determines the characteristics of the cathode. The cathode characteristic, which is of chief importance, is *perveance*. This is the ratio of cathode current to the beam voltage raised to the three-halves power. It is a measure of the ability of the cathode to emit electrons at various beam voltages, and is determined by the emitting material of the cathode, the cathode area, and the operating temperature. The impedance presented to the pulse transformer secondary by the klystron during conduction is equal to the ratio of beam voltage to beam current. Since this ratio is determined by the perveance of the cathode, the perveance determines the impedance reflected through the pulse transformer to the pulse-forming network supplying the cathode pulses.

Current flows in the pulsed klystron for the duration of the cathode pulse. Between pulses no potential difference exists between the cathode and the drift tube entrance and, therefore, no current flows. The remaining discussion will apply only to the period of conduction, or during the cathode pulse time. When the cathode is pulsed negatively, electrons are accelerated toward the drift tube entrance. The total acceleration given to the electrons by the cathode-to-drift-tube potential, resulting from the negative cathode pulse, occurs while the electrons are in transit between the cathode and the drift tube. Figure 12–65A presents an approximate picture of the electric field lines (shown dotted) existing between the cathode and the drift tube entrance.

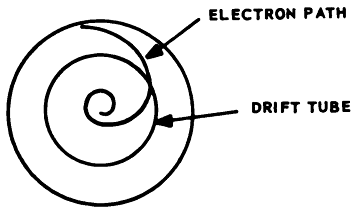
The electric field lines do not extend into the drift tube for any appreciable distance. With only the electric field present, the electrons would tend to follow the field lines. Under this condition, the electrons would travel from the cathode to the drift tube entrance, and the energy imparted to the electrons by the cathode pulse



A. SIDE VIEW



B. TOP VIEW



C. END VIEW OF DRIFT TUBE ENTRANCE SHOWING ELECTRON PATH

Figure 12–65. Convergence of Electron Stream at Entrance of Drift Tube in Klystron

would be dissipated at the drift tube entrance as heat generated by electron bombardment of the metal surface. To avoid this, a magnetic focusing system is employed to divert the electron stream into the drift tube. Such a field must be symmetrical about the drift tube axis. A few lines of such a field (solid lines) are shown superimposed on the electric field lines of Figure 12–65A. The focusing action at the drift tube entrance can be seen by referring to Figure 12–65B. This figure shows a cross section of a magnetic field, which is symmetrical about the drift tube axis. The path of a single electron under the influence of the electric field alone is shown as a heavy solid line.

At point A in Figure 12–65B, where the electron path intersects a magnetic field line, the line is resolved into its components. One component (axial component) is seen to be parallel to the direction of the electron path and the other (radial component) is at right angles to the electron path. The radial component force causes the electron to rotate out of the plane of the paper, around the axis of the drift tube. This ngential force causes the electron path to beme helical as it approaches the drift tube ennce. At the drift tube entrance the magnetic ld intensity is much greater than that at the thode, causing the spiral of the electron path become tighter about the drift tube axis. so, at the drift tube entrance, the magnetic field es are almost entirely axial.

The tangential component of the electron locity in the presence of the axial field causes e electron to be accelerated radially inward. he electron path as viewed from the cathode d of the drift tube is shown in Figure 12–65C. he greater the initial distance (perpendicular) the individual electrons from the drift tube is, the greater the forces tending to direct the ectrons toward the drift tube axis. Electrons leased from the cathode at or near the drift ube axis experience relatively slight effects of he magnetic focusing field and travel an almost raight line path from the cathode to the drift ube entrance. The over-all effect of the magnetic eld is to force electrons emitted from the cathode to the drift tube, thus keeping to a minimum he number striking the metal surrounding the rift tube entrance.

Once the electrons enter the drift tube, they re no longer under the accelerating effects of the athode-to-drift-tube potential, and they coast, r drift, at constant speed until they encounter he fields across the gaps of the three resonant avities arranged along the length of the drift ube. The axial field produced by the entire ocus coil assembly extends the length of the rift tube. The electrons within the drift tube ravel parallel to the axial magnetic field. Whenver, for any reason, an electron is deflected from he axial path, a radial component of velocity s introduced that causes the electron to spiral bout the axial magnetic lines. It is impossible o provide the degree of focusing required to revent all electrons from striking the walls of he drift tube. By proper adjustment of the nagnetic field, however, it is possible to hold he number to a practical minimum.

Current produced by electrons striking the drift tube walls is referred to as body current. A high body current is undesirable since it indicates that a large number of electrons are giving up energy as heat in collisions along the drift tube walls rather than contributing energy to the r-f output cavity. Several factors influence the amount of body current produced. The most important is the focusing adjustments. Another is the interaction of electrons in a dense electron stream. Since all electrons are negatively charged, they tend to repel each other; this produces beam spreading and subsequent collisions along the drift tube walls. Another effect is also produced by the concentration of negative charge within an electron stream.

Electrons along the outer edges of the stream see a concentration of negative charge at the center of the stream (corresponding to the drift tube axis). The drift tube walls, being at ground potential, appear positive with respect to the concentrated charge along the axis of the tube. The fringe electrons are, therefore, accelerated toward the drift tube walls because of this apparent potential difference between the center of the drift tube and the walls.

As previously stated, electron behavior is influenced by the cavity gap voltages once the electrons have entered the drift tube. Cavities of the gridless type are used because of the unusually great electron velocities produced by the high beam voltage. The edges of the cavity gaps are flush with the drift tube wall to avoid intercepting electrons in the beam.

Cavity resonator construction for klystron tubes is of two primary types. One type is built into the tube and is evacuated as part of the tube structure. Tuning of this type of cavity is performed by some type of bellows-driving mechanism that alters the physical size of the cavity. Another type of cavity is mounted externally to the evacuated tube section. The portion of the drift tube comprising the resonant cavity gap is a cylinder of ceramic or other low loss insulating material. The tuned cavity can be removed from the drift tube in such an assembly.

The chief factor in cavity gap spacing is electron transit time. The ideal condition is zero transit time but, of course, this is not possible at the desired operating frequencies. There is a finite time interval involved in the passage of electrons across the gap of the resonator. The time required for passage is determined by the initial electron velocity upon entering the gap and upon the width of the gap. The operating frequency determines the passage time required in terms of a complete r-f cycle. If the width of the gap is made very small to reduce transit time, the cavity impedance is lowered, since this is equivalent to increasing the capacitance of a tuned parallel-resonant circuit. Also, if the gap is made very small, possibility of a breakdown across the gap exists, because of the large r-f voltages developed.

In addition, with a small gap there is heavy "loading" of the cavity. This is a result of secondary electrons, emitted around the cavity gap, being driven across the gap by the r-f potentials across the gap. Movement of these electrons under the influence of the r-f field across the gap is an energy drain on the cavity and performs no useful function. This waste of energy is reflected as a reduced Q of the cavity with associated losses in power. A cavity gap that produces a transit time of a quarter-cycle or slightly more at the operating frequency provides satisfactory beam coupling and, at the same time, avoids the undesirable effects of too small a gap spacing.

The drift tube diameter is also a compromise. While a small diameter gives a high degree of coupling between the cavities and the electron beam, it complicates the focusing problem. A large tube diameter simplifies the focusing problem but reduces beam coupling. However, the diameter of the drift tube must be less than the cutoff dimension at the operating frequency. This is necessary to avoid standing waves along the drift tube. The drift tube diameter is, therefore, a compromise value that gives satisfactory beam coupling to the cavity gaps and at the same time permits practical focusing of the beam.

The output of any klystron (regardless of the number of cavities employed) is developed by velocity modulation of an electron beam. Velocity modulation, in turn, produces electron bunching. The bunching process is a result of the velocity modulation, and is a function of the time required for electrons to travel the distance from the input cavity to the output cavity. The electrons that are initially accelerated by the beam voltage (considered to be DC for the duration of the cathode pulse) are acted upon by r-f fields developed across the input and middle klystron cavities.

Some electrons are accelerated, some are decelerated, while others are unaffected, depending on the amplitude and polarity of the cavity r-f voltages when the electrons cross the cavity gaps. For the interval during which the electrons travel from one cavity to the next, the accelerated electrons tend to overtake the decelerated ones, thus causing the formation of groups, or bunches, of electrons. As a result, bunches of electrons arrive at the output cavity at the proper instant of each r-f cycle to deliver energy to the output cavity. Transit time is thus seen to be a necessity rather than a hindrance to the klystron operation.

The bunching process is best explained by reference to Figure 12–66, an Applegate diagram of the bunching process in a three-cavity klystron. The vertical axis represents the distance measured along the drift tube. The horizontal axis represents time measured either in millimicroseconds or in degrees of an r-f cycle. The diagonal lines are electron paths plotted as a function of time and of distance along the tube.

The slope of a line is proportional to the velocity of the electron whose path is described by that particular line. The steeper the slope, the greater the electron velocity. A change in slope, therefore, indicates a change in velocity. An increased slope indicates an increase in velocity, and a decreased slope indicates a decrease in velocity. Between the cathode and the first cavity the electrons emitted from the cathode receive equal accelerations and therefore travel at equal velocities. All electrons entering the input cavity gap are, therefore, seen to have equal slopes.

A sinusoidally varying r-f field exists across the first (input cavity) gap as a result of the signal input supplied from an external r-f source. The input cavity is resonated at the input frequency. The effects of the r-f voltage across the input cavity gap on individual electrons depend upon the instantaneous magnitude and polarity of the r-f gap voltage. The word "instantaneous" implies that the transit time is zero. As previously explained, this is not true, since an appreciable portion of an r-f cycle (over one quarter) elapses during the electron transit across the gap. More correctly, the average magnitude and polarity of the r-f cycle during the electron transit must be considered.

An electron we will designate as electron 1 in Figure 12–66 crosses the input cavity gap when the potential is zero. The velocity of electron 1 is unaffected by the input gap voltage and there is no change in its velocity. This is shown by no change in the slope of the line describing the path of electron 1. Electron 3 crosses the input

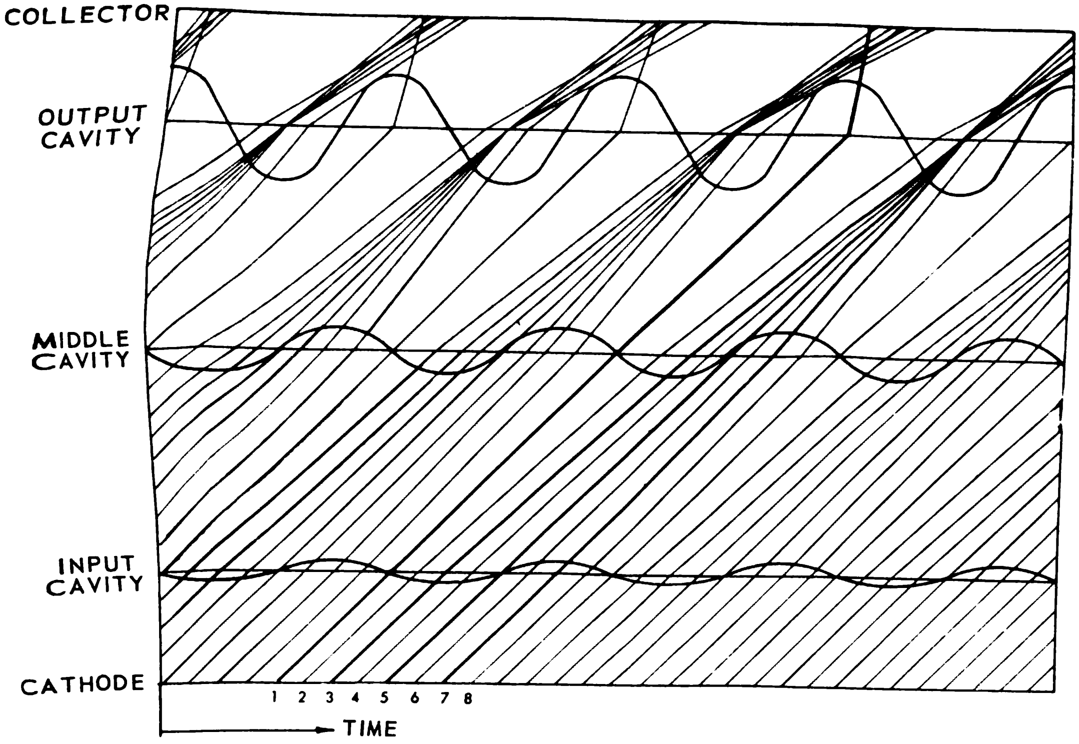


Figure 12–66. Klystron Bunching Graph

cavity gap when the r-f voltage is maximum in a negative direction. This electron is decelerated during its passage through the gap and emerges from the gap traveling at a reduced velocity. This is indicated by a decrease in the slope of line 3 beyond the input cavity.

Electron 5, like electron 1, passes through the gap when the r-f signal is zero and therefore emerges with no change in velocity. Electron 7 crosses the input gap when the r-f voltage is maximum in a positive direction. This electron is accelerated during its transit through the input gap and emerges from the gap at increased velocity. This is indicated by an increase in the slope of line 7 beyond the input cavity gap. Electrons 2 and 4 cross the input gap when the r-f voltage is negative and are decelerated by an amount proportional to the amplitude of the r-f voltage. Electrons 6 and 8 cross the input gap when the voltage is positive and are accelerated by an amount proportional to the amplitude of the r-f voltage.

When the electrons pass the input cavity gap, they enter the drift tube section between the first and second cavities. In this region there are no DC or r-f fields and the electrons travel at whatever velocities they acquired as a result of the combined effects of the beam voltage (cathode pulse) and the r-f voltage across the input cavity gap. In passing through the first gap, electrons emitted from the cathode at successive intervals are accelerated or decelerated as explained above. This process is called *velocity modulation.* In the drift tube section, between cavities one and two, the velocity modulation produces *density modulation* of the beam.

Electron 3 leaves the cathode earlier than electron 5. Initially, both have the same velocity. Since electron 3 is decelerated in passing through the input cavity gap and electron 5 is unaffected, electron 5 will overtake electron 3 if the two drift for a time in a space where no fields are present. Electron 7 is emitted from the cathode at a later time interval than electron 5, but both have the same initial velocity. Electron 5 is unaffected by the r-f voltage across the input cavity gap, but electron 7 is accelerated. Therefore, electron 7 will eventually overtake electron 5 if they travel undisturbed for a sufficient time. The over-all effect is that electrons crossing the input cavity gap before and after electron 5 tend to converge on, or group around, electron 5. This is slightly evident at the middle cavity.

Velocity modulation of the DC beam by the input cavity does not produce serious loading of the input cavity. The fact that a DC beam is being velocity-modulated means that the number of electrons crossing the input cavity gap when the r-f voltage across the gap is negative is equal to the number crossing the gap when the r-f voltage across the gap is positive. If electrons cross the cavity gap when the r-f voltage across it is negative, energy is supplied to the cavity by the electrons, because in slowing down the electrons give up kinetic energy to the gap. If the electrons cross the cavity gap when the r-f voltage across it is positive, energy is supplied to the electrons from the cavity, since the electrons are accelerated. The r-f input to the first cavity, therefore, is of sufficient magnitude to overcome the inherent losses in the cavity in maintaining the desired r-f voltage across the cavity gap.

Only a small degree of density modulation (bunching) occurs within the electron beam in the interval of travel from the input cavity to the middle cavity. The amount is very small compared to the degree of bunching required at the output cavity. The amount of density modulation is sufficient, however, to excite the middle cavity and, because of the high Q of this cavity, to maintain a large oscillatory voltage across it. The voltage thus produced across the middle cavity gap is much larger than that across the input gap. This voltage is responsible for most of the velocity modulation, and subsequent density modulation, produced within the klystron.

There is a negligible net transfer of energy from the beam to the middle cavity, or from the cavity to the beam. The amount of energy coupled to the cavity from the beam is the small amount necessary to overcome the cavity losses in maintaining the oscillatory r-f voltage across the gap. The degree of density modulation present in the electron stream at the middle cavity, as a result of the velocity modulation at the input cavity, is so slight compared to that required at the output cavity that it can be considered as DC for practical purposes. Therefore, the net energy delivered to, or taken from, the electron beam is quite small, as explained for the input cavity.

The large voltage across the middle cavity gap produces a much larger degree of velocity modulation than that of the input cavity. This is evident in the Applegate diagram of Figure 12–66 by sharper changes in the slopes of the electron paths crossing the middle cavity. Electrons 1 and 5 again cross the gap when the r-f voltage is zero, and continue with no change in velocity. Electrons 2, 3, and 4 cross the gap when the r-f voltage is negative and are slowed down, as shown in a decrease in slope of lines 2, 3, and 4. Electrons 6, 7, and 8 cross the gap when the r-f voltage is positive and are accelerated, as shown by increases in the slopes.

After the electrons cross the middle cavity gap, they once more drift in the field region between the middle cavity and the output cavity. The bunching process continues in this region, but at a more rapid rate than that between the input cavity and the middle cavity because of the greater degree of velocity modulation imparted to the beam by the middle cavity. The effect of velocity modulation is shown at the output cavity. Electrons 3, 4, 5, 6, and 7 are bunched and cross the output cavity gap when the gap voltage is maximum negative. Under this condition, maximum energy transfer from the electrons to the output cavity occurs.

The energy given up by the electrons is the kinetic energy of high velocity, imparted to the electrons by the beam voltage (cathode pulse). Giving up this energy produces a decrease in velocity as shown by the sharp changes in slopes at the output cavity. Electrons 2 and 8 did not receive sufficient velocity modulation to cause them to bunch with the others. Electron 1, having traveled unaffected through the input and middle cavities, reaches the output cavity when the r-f voltage across it is maximum positive. The voltage imparts to the electron a sharp acceleration. This is shown by the sharp increase in slope. This acceleration of electron 1 represents a load on the cavity, since energy from the cavity is required to accelerate the electron.

Optimum bunching reduces the number of such electrons to a minimum. The bunching process ctured in Figure 12–66 is ideal, particularly the output cavity as the bunched electrons are own arriving at the output cavity at the same stant. This is not true in practice. The passage an electron bunch at the output cavity occurs ver a period covering a considerable portion of half-cycle. Also, the history of only eight ectrons is discussed. Actually, billions of elec- ons are emitted from the cathode during the me interval between the emission of electrons and 8. The effects of mutual repulsion pro- uced by such a dense electron stream make it possible to approach, in practice, the degree bunching shown in Figure 12–66.

Since it has been stated that the bunching rocess is a function of time, it would appear hat if an electron beam is velocity-modulated y a small r-f voltage and the electrons are per- itted to drift for a sufficiently long time, the esired degree of bunching could be obtained ith a minimum of input power. The defocusing ffects previously discussed prevent this, how- ver. The debunching effects of a dense electron tream would spread the beam in the drift space fter a short time interval. The drift time is herefore limited to a time interval corresponding o only a few r-f cycles, and input power is ad- usted to produce the desired bunching in this ime interval.

The three-cavity klystron is the equivalent of pair of two-cavity klystrons in cascade, and is eferred to as a cascaded amplifier klystron. The hree-cavity klystron is more efficient than a pair f two-cavity klystrons in cascade, since the iddle cavity replaces the output cavity of one air and the input cavity of the other. In this vay cavity losses are cut in half and the further ivision of power produced is avoided. (A maxi- num of 50% efficiency is realized when a matched ource and load are coupled.) Another advan- age is the simplified circuitry involved. There re only three tuned circuits instead of four; nly one electron beam is used, and power sup- ly provisions are also simplified.

SUMMARY. The mechanics of bunching within klystron cause the output power to be critically ependent on input power. For a given fre- uency, drift tube length, and beam voltage, there is only one value of input power that gives maximum output if all three cavities are tuned for maximum gain. The degree of bunching achieved in a given drift tube space depends on the amount of velocity modulation imparted to the beam electrons while crossing the gaps of the input and middle cavities. The velocity modulation depends on the amount of r-f voltage appearing across the input and middle cavity gaps.

The greater the voltage developed across the input cavity gap, the greater the velocity modulation produced at this gap and the greater the degree of density modulation produced in the interval of electron travel from the input cavity to the middle cavity. The excitation of the middle cavity depends on the amount of density modulation in the beam arriving from the first cavity. The greater the density modulation seen at the middle cavity gap, therefore, the greater the r-f voltage developed across the middle cavity gap, and the greater the velocity modulation imparted to the beam. If the input power is at the optimum value, the maximum bunching point will occur at the output cavity gap.

If the input power is too low, there is insufficient velocity modulation of the beam, and maximum bunching will occur slightly later, or at a point beyond the output cavity gap. If there is too much input power, too large a degree of velocity modulation is produced, and maximum bunching occurs before the velocity-modulated electrons reach the output cavity. The amount of bunching at the output cavity determines the power delivered to the cavity from the electron beam. Either of these conditions, overdriving or underdriving the input cavity, produces maximum bunching in the klystron at some point along the drift tube other than the output cavity gap with a resulting decrease in output power. A curve of klystron power output versus power input is shown in Figure 12–67.

PERFORMANCE. Another consideration is expressed in the curves of Figure 12–67. The power output of a three-cavity klystron can be increased beyond that corresponding to maximum gain (all three cavities tuned to the same frequency). The bunches of electrons arriving at the output cavity contain components contributed by velocity modulation produced both at the input cavity and at the middle cavity. With all three cavities tuned to the same frequency, the bunched current contributed by velocity modulation at the input gap is not ideally phased with respect to the velocity modulation contributed by the middle cavity; therefore, the bunching components

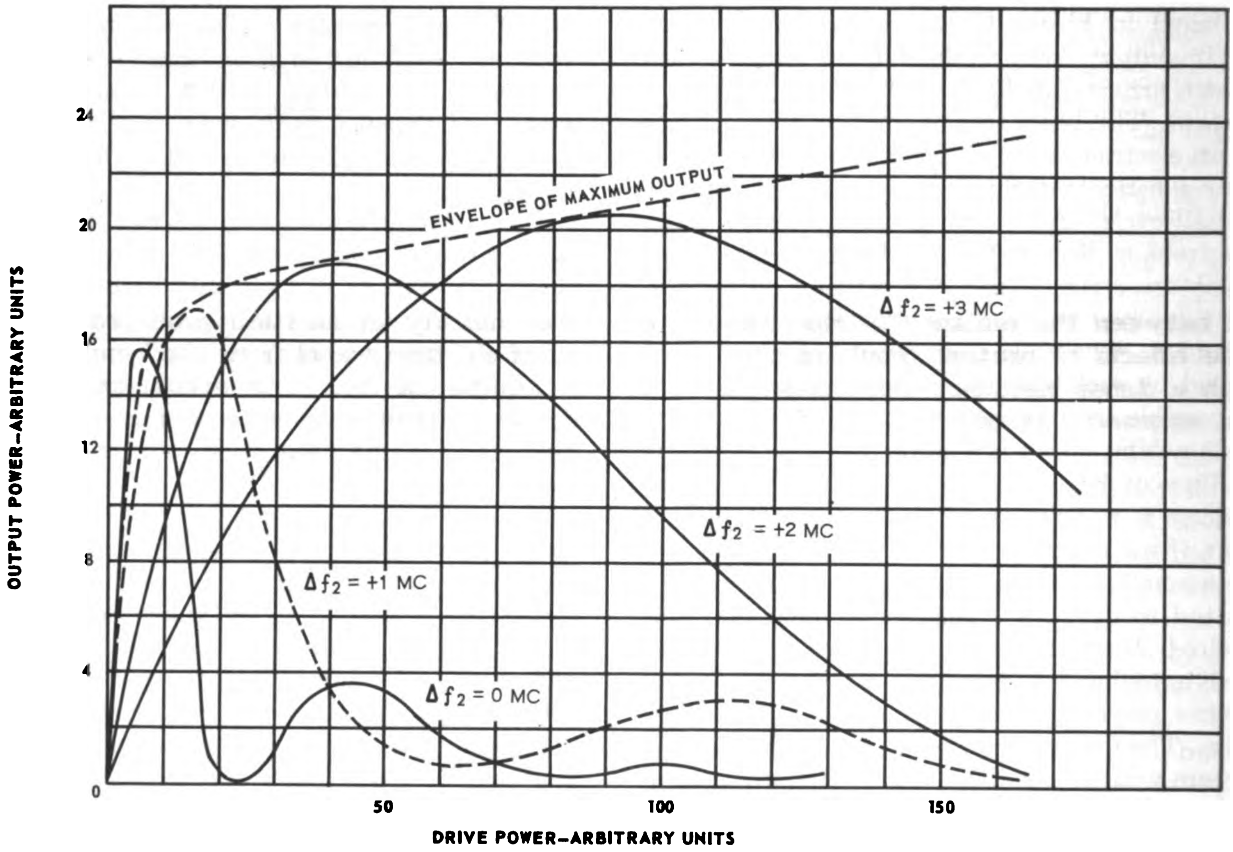


Figure 12–67. Graph of Klystron Input Versus Output Power

of the input cavity do not add with those of the middle cavity to produce optimum bunching at the output cavity.

By detuning the middle cavity the components contributed by the input cavity are phased more advantageously with respect to those contributed by the middle cavity, and the bunching at the output cavity is increased. When the second cavity is detuned to change the phase of the r-f voltage across it with respect to that across the input cavity, the amount of density modulation required to maintain a given r-f voltage across the middle cavity gap is increased. Since the density modulation appearing at the middle cavity is a result of the velocity modulation contributed by the input gap, the power supplied to the input gap must be increased to supply the required increase in velocity modulation. Further detuning of the second cavity results in a further increase in output power, provided the input power is increased to the new required value.

The curves of Figure 12–67 show the results obtained with various amounts of detuning of the middle cavity. Notice that the increase in output power is not proportional to the increase in input power required to maximize the output for a particular degree of middle cavity detuning. Increases in output power obtained by detuning the middle cavity and increasing the input power are obtained at a sacrifice of gain. At the higher power outputs obtained by detuning the middle cavity, the output curve is much broader, and the klystron output is less critically dependent on variations of input power.

The final major consideration in power klystron performance is the shape of the cathode voltage pulse. The amplitude of the pulse determines the transit time of the electrons from the cathode to the collector. Any change in pulse amplitude affects the electron transit time. This, in turn, affects the points of occurrence of maximum electron bunching in the region of the output cavity. If the r-f input to the klystron is constant and the cathode pulse amplitude varies over the duration of the pulse, the points of occurrence of maximum bunching will vary with pulse amplitude. This results in phase shift of the r-f signal output with respect to the input during the cathode pulse.

Since the transit time from the input cavity to the output cavity is usually equal to the time of several r-f cycles, a small percentage phase shift may still be undesirably large. Any phase shift is undesirable in a moving target indicator system, which yields information based on pulse-to-pulse phase comparisons. Phase shift of the r-f output must therefore be held to a value that will not limit the moving target resolution possible in the cancellation system employed in the equipment. Thus the cathode voltage pulse must be flat-topped.

Klystron Versus Magnetron

The advantages of the klystron as compared to the magnetron are many. First of all, physical considerations are simplified. In the magnetron the cathode and the anodes are included in the frequency determining system and are a compromise between desired power output and required size. In the klystron, the cathode and collector are external to the frequency determining fields and can be designed independently of the r-f section to handle the desired power. Cooling methods are simplified, since most of the heat generation occurs in the collector and can be dissipated by a cooling system without consideration of the r-f portion of the tube. Figure 12–68 illustrates a three-cavity klystron.

From the standpoint of electrical performance, the klystron again excels. The magnetron is a power oscillator and is the frequency determining element of the radar system. Its frequency stability is dependent on the combination of a magnetic and an electric field, either of which is subject to variations over short periods of time. In a moving target indicator (MTI) system, discussed in Chapter 15, the stable local oscillator is continuously varied by an AFC system to track with the magnetron. A lock (transmitter) pulse is supplied to the coherent oscillator at the beginning of each output pulse to provide a reference phase for the echo returns.

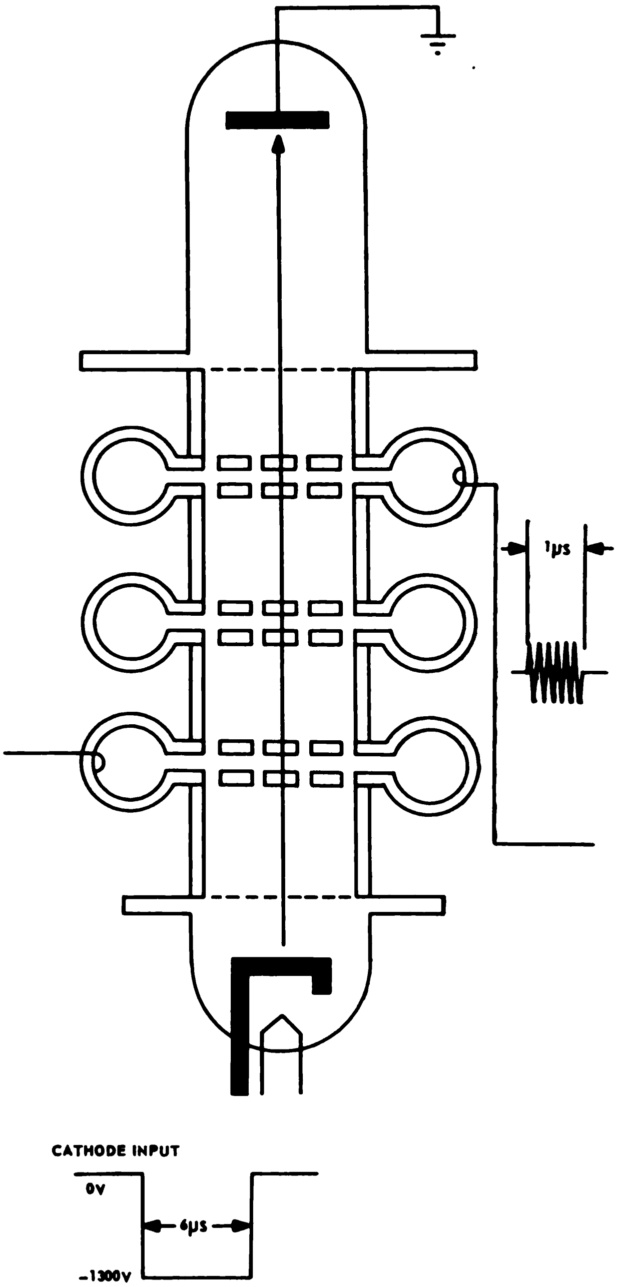


Figure 12–68. Three-Cavity Klystron

The pulse-to-pulse frequency of the magnetron is subject to considerable variation for which the AFC system cannot compensate instantaneously. As a result, pulse-to-pulse phase differences occur because of magnetron frequency changes; these changes limit fixed target cancellation. The klystron, on the other hand, is simply a power amplifier and has no influence on the frequency determining system. A reference oscillator (stable local oscillator) is maintained at an almost constant frequency and is effectively isolated from the output stage by intermediate amplifiers.

The reference oscillator output which is the input to the klystron amplifier is mixed with the coherent oscillator output to produce the desired output frequency.

Because the frequency determining oscillators operate at a low level, both can be made extremely stable and the pulse-to-pulse frequency variations are almost negligible. No AFC is needed in such a system and the pulse-to-pulse phase variations of the transmitter output stages are limited by providing a modulator pulse having a suitably flat-topped portion. The fixed target cancellation of an associated MTI system is considerably improved by the low pulse-to-pulse phase variations, and the radar transmitter is no longer considered the limiting factor in the MTI performance of a radar system.

The disadvantages of the klystron are chiefly the circuit complexities of the transmitter. A series of amplifiers are inserted between the frequency determining oscillators and the power amplifier klystron. These stages must be individually tuned when the operating frequency is changed. Most of these stages are pulsed, and synchronizing problems arise. Power supply requirements are also increased. The bulk of the equipment required in a high-power klystron radar system limits the mobility of such a system, and the radar installation employing klystrons is either a fixed station or one of limited mobility. Regardless of the disadvantages, the klystron system complexity and bulk are justified in that they insure consistent MTI performance which excels that possible with magnetrons.

Klystron Mixer Tube

Another utilization of the multicavity klystron, other than a power amplifier, is that of a mixer. The klystron is used as a mixer tube primarily when the power amplifier klystron is used in the transmitting system. The mixer is used to mix the stable local oscillator (stalo) signal and the coherent oscillator (coho) signal to produce the transmitter frequency. A typical klystron mixer circuit is shown in Figure 12–69 and will be referred to as the synchrodyne tube.

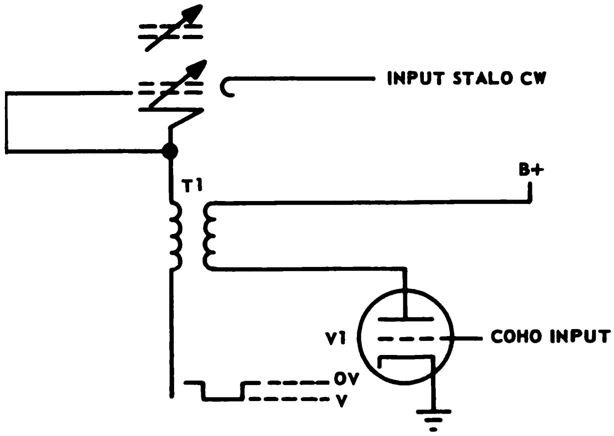


Figure 12–69. Simplified Klystron Mixer Circuit

The synchrodyne tube mixes the coho output with the stalo output to produce pulsed energy of the proper frequency for application to the first cavity of the power output klystron in the transmitter. The synchrodyne tube is a three-cavity, low-power klystron. The mixing procedure is effected in the flow of electrons in the drift tube from the cathode to the collector. Tuning of the cavities is accomplished by three tuning knobs (one for each cavity) which determine the resonant frequency of each cavity. These tuning knobs are accessible from the top of the tube.

The drive pulses applied to the cathode are negative; they are remodulated by the pulses of coho energy fed into modulation transformer T1 by driver tube V1. The drive pulse applied to the cathode causes beam current to flow for the entire period. But the r-f energy is applied for only a period during the drive pulse, and the mixing process is in operation during this time. The output of the stalo is applied to the first cavity of the synchrodyne tube continuously. The second and third (output) cavities of the synchrodyne tube klystron are so tuned that they are resonant only to a frequency that is the difference between the stalo and coho frequencies. Accordingly, the synchrodyne has an output only during the intervals when the coho pulses are applied to the synchrodyne tube's klystron cathode.

The mixing process is accomplished in the density modulated electron beam in the synchrodyne. The beam is modulated by the coho energy

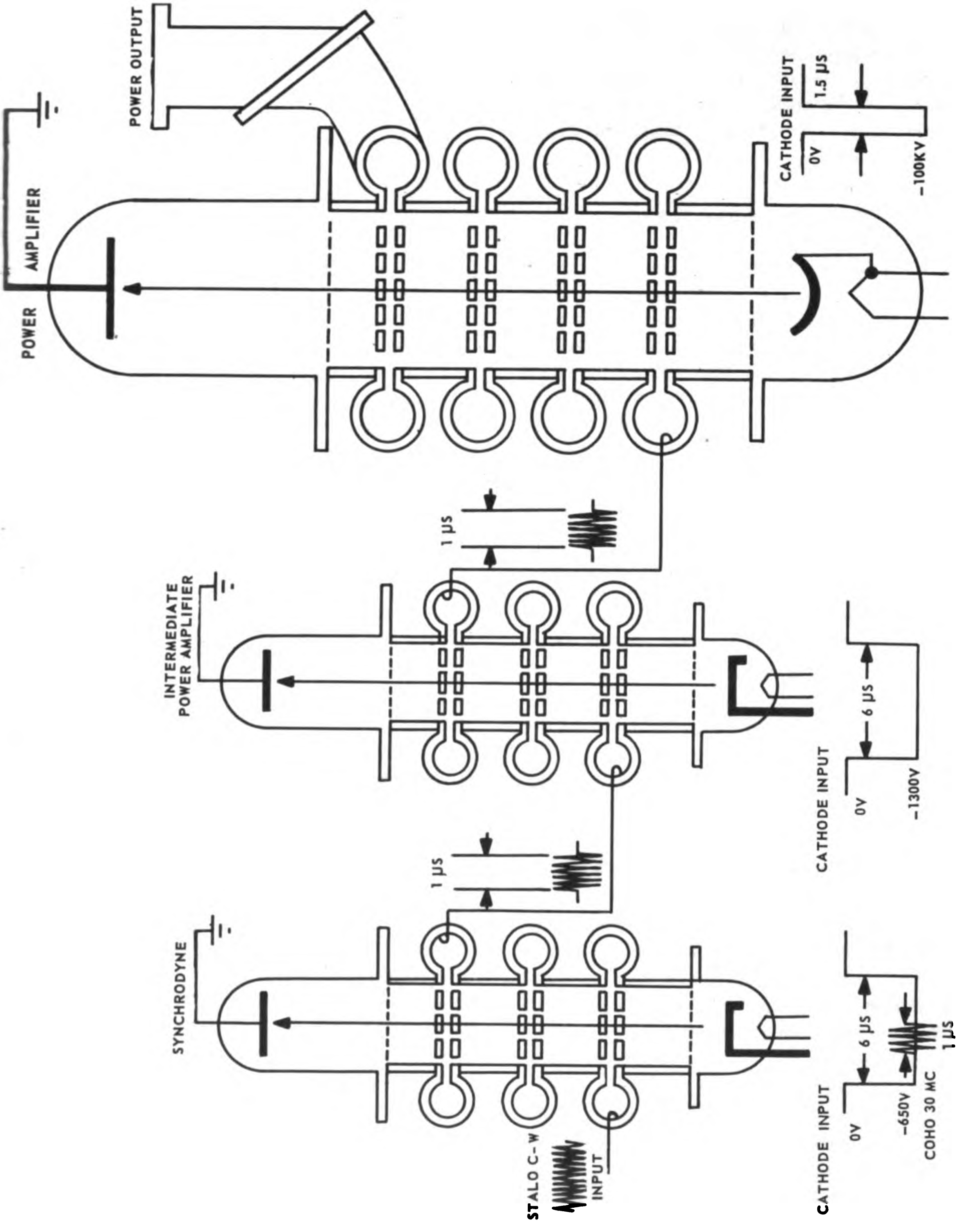


Figure 12–70. Multicavity Klystron Stages

and the first cavity is tuned to the frequency of the stalo. The second and third cavities are tuned to the difference between the coho and stalo frequencies. The collector of the synchrodyne tube is grounded since the beam voltage is applied as a negative pulse on the cathode. Figure 12–70 illustrates how the synchrodyne is used as a mixer tube in conjunction with an intermediate stage.

TRAVELING-WAVE TUBE

The traveling-wave tube has, since the 1950's, become important to the circuit designer and the systems engineer in the field of microwave amplification. This does not mean that the traveling-wave tube is better than other microwave tubes in all respects. As yet it is somewhat inefficient when compared to magnetrons, although efficiencies of over 10% have been obtained. A related device, the "M" type *carcinotron* or magnetron amplifier, shows quite high efficiencies and essentially operates on the traveling-wave principle.

It is true that gains of 40 db have been achieved by the traveling-wave tube which can be rivaled only by the newer multiresonator klystrons. However, gain is not an all important consideration of the traveling-wave tube although gain is one of its major virtues. This factor, together with its low noise figure (some specially designed tubes have a noise figure of 6 db), make it an extremely useful device at microwave frequencies.

The simple structure of the traveling-wave tube has enabled manufacturers to build tubes that operate over a vast portion of the spectrum. By 1961 successful tubes had been built that operated as low as 300 mc and as high as 50,000 mc.

The major advantage of the traveling-wave tube is its tremendous bandwidth. It is this characteristic that makes it superior to other microwave devices. Bandwidths of one octave are not uncommon, and even greater bandwidths have been achieved. (A bandwidth of one octave is one in which the upper frequency is twice the lower frequency.) This feature allows the traveling-wave tube to be used in exploiting vast portions of the microwave spectrum.

Characteristics

Most microwave amplifiers have a limited bandwidth. The bandwidth of a klystron, for example, is no greater than that of low-frequency pentodes. High-frequency triodes do exhibit a wider bandwidth than klystrons, but this is not significant when compared to the traveling-wave tube. Microwave triodes and klystrons also suffer the same limitation as do conventional tubes. As the bandwidth is widened, the gain is decreased to the point where no gain is available. Tube capacities and electron transit time also limit these tubes to relatively low microwave frequencies. (In the case of the triode, successful operation cannot be obtained much above 4000 mc.)

In the traveling-wave tube these limitations are overcome completely. There is no lumped input capacitance nor any resonant circuit. The tube is a transmission line with a negative attenuation in the forward direction and a positive attenuation in the backward direction. Negative attenuation is the exponential *increase* of voltage in the direction of propagation. Positive attenuation is the exponential *decrease* of reflected voltage. The bandwidth is limited to some extent by the input and output transducers connected to the tube, but these have been successfully made to cover a bandwidth greater than one octave. In some cases the bandwidth is so great that we have by no means fully exploited what is available.

In general, the traveling-wave tube compares very favorably with other microwave devices as to gain, noise figure, and simplicity of construction. Admittedly, it is not as efficient as the magnetron, although reasonable efficiencies have been obtained. Finally, it definitely provides amplification over a bandwidth commensurate with microwave frequency space available, and it is this feature that has caused it to be incorporated in radars, countermeasures equipment, repeaters and communications equipment, and radio astronomy devices.

Construction

The traveling-wave tube was first announced in 1946. Since that time the large research organizations of the world have devoted much time

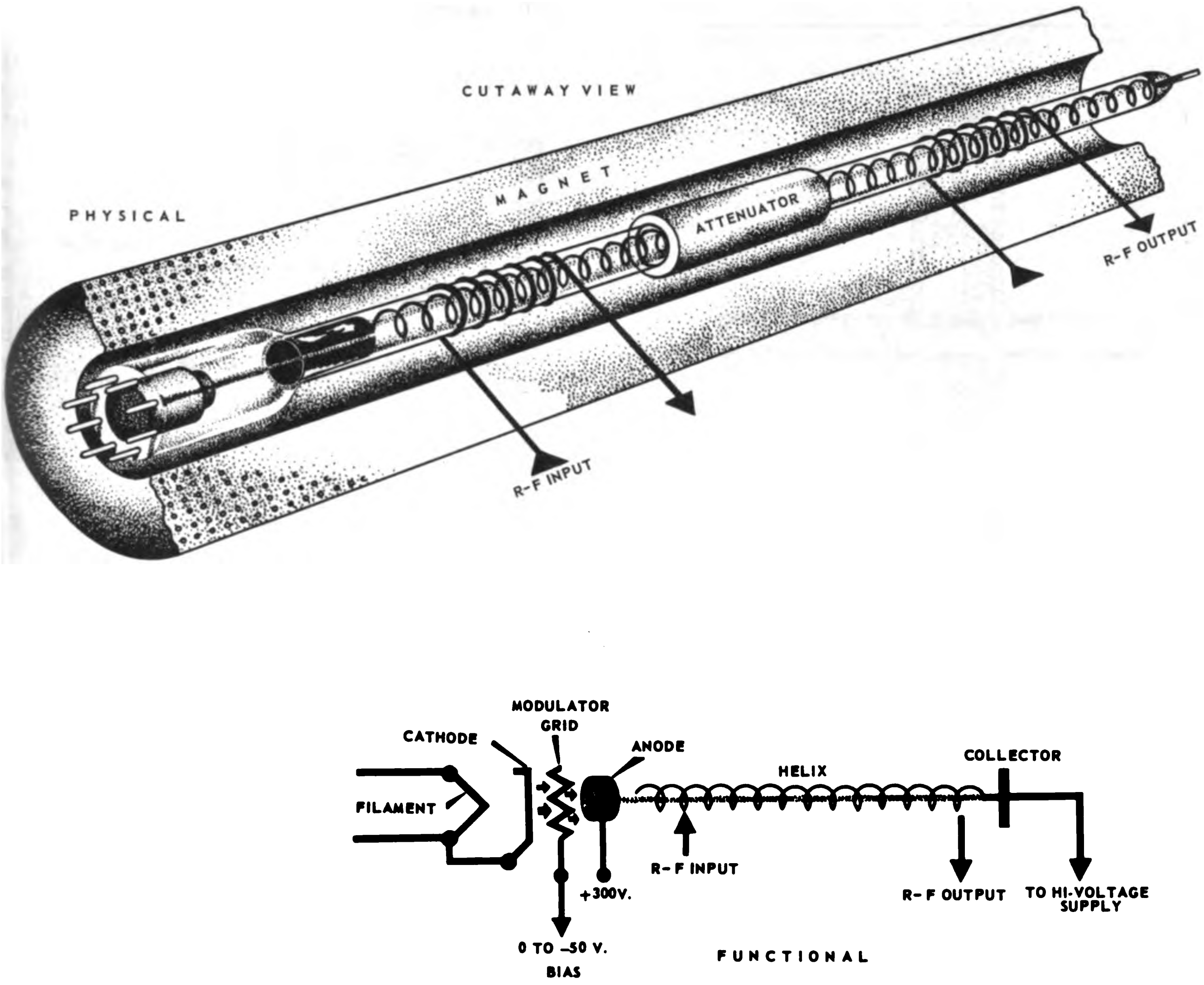
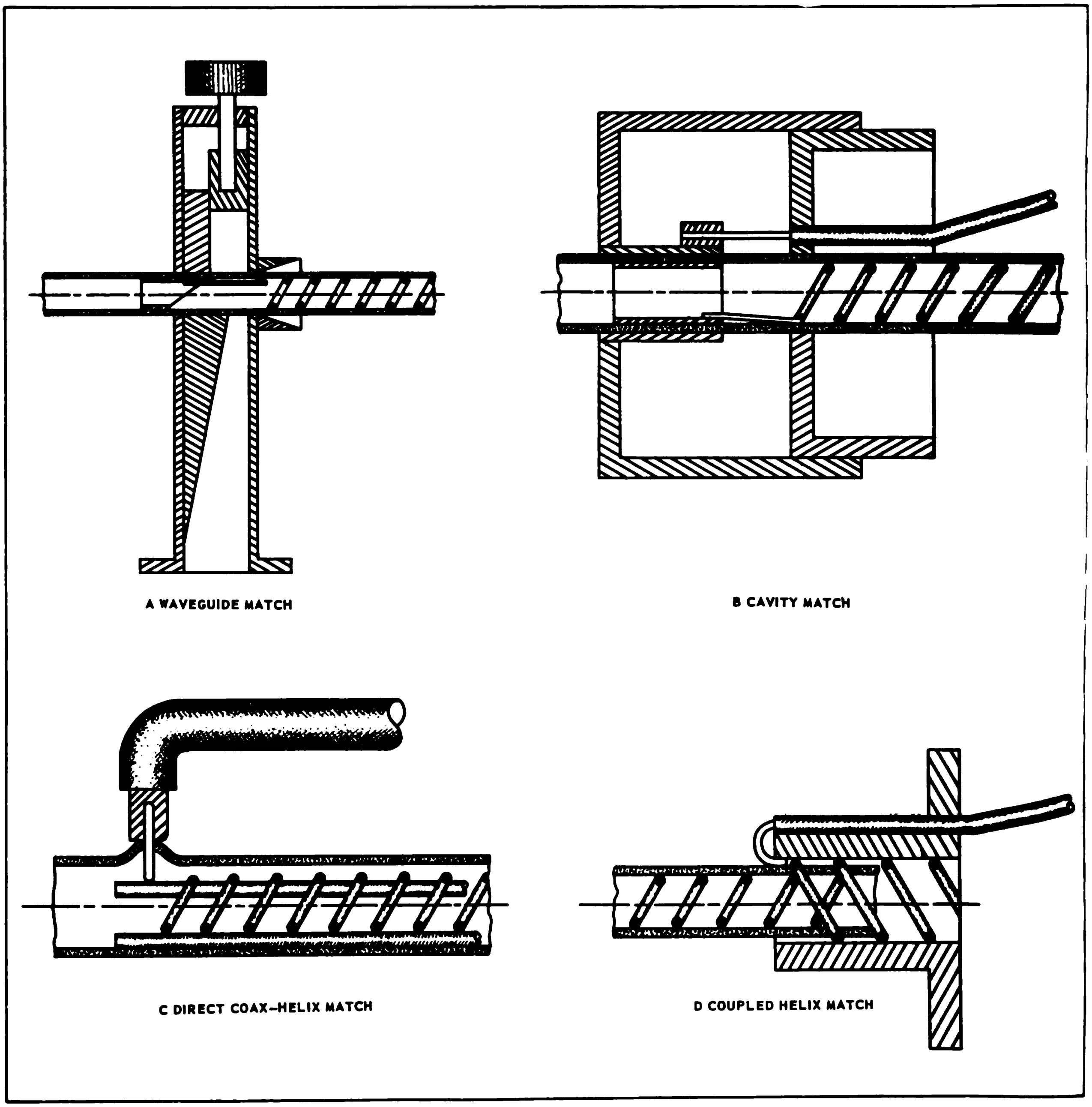


Figure 12–71. Traveling-Wave Tube

and money to perfect it. The traveling-wave tube consists of an electron gun that produces a stream of electrons which are kept in a tight beam with a magnetic focusing field supplied by a solenoid magnet. A cutaway view of the traveling-wave tube is shown in Figure 12–71. The electrons are accelerated by two different means. Initially they are accelerated by the specially designed electron gun that originally produced them. In the electron gun the heater boils off electrons from a parabolic-shaped cathode. This tends to focus the electron stream; small accelerator anodes increase the velocity. The result is a narrow beam that has the relative velocity of an electron accelerated by 1500 volts.

A positive potential on the helix and a collector anode at the end of the tube continue to speed the electron flow. The electron beam is made to flow down the central axis of a long, loosely wound helix on its way to the collector. The beam is kept from spreading by the solenoid magnet surrounding the tube. The focusing system that is generally used is a long, wirewound, magnetic solenoid that completely surrounds the loosely wound helix portion of the tube. The axis of the solenoid magnetic field coincides with the central axis of the helix.

The ability of this magnetic lens to keep the electron beam focused into a tight area is a direct function of the current through the magnet. The higher the solenoid current, the tighter the beam. If the magnetic field of the solenoid were



A WAVEGUIDE MATCH

B CAVITY MATCH

C DIRECT COAX–HELIX MATCH

D COUPLED HELIX MATCH

Figure 12–72. Traveling-Wave Tube Couplers

lost for an instant, the electron beam would spread, intersect the helix, ionize, and destroy the traveling-wave tube. The weight of the electromagnet and the associated power supply add to the bulk and complexity of traveling-wave tubes. In fact, this has prevented their use in some instances. An improvement has been solenoids wound with aluminum foil instead of wire.

Coupling

Some means must be provided to apply r-f energy to one end of the helix, and remove it from the other end. A transformer is one way of doing this. Another method is by waveguide matching (shown in Figure 12–72A). The waveguide is terminated in a nonreflecting impedance.

The helix is inserted into the waveguide. The coupling action is similar to that of a quarter-wave section. The efficiency of the system is rather good, but the waveguide has a far higher Q than the traveling-wave tube. This means that the broadband characteristics of the traveling-wave tube suffer in that the entire bandwidth is not available for amplification. The waveguide will not respond over such a wide spectrum.

The cavity match, illustrated in Figure 12–2B, is very similar to the waveguide match. Cavities can be made to resonate over wider ranges than waveguides, but they still have a igh Q compared to the traveling-wave tube. The elix is placed at the mouth of the cavity ready to absorb any E-fields produced. The r-f is led to the cavity by coaxial cable and is introduced into the cavity by a quarter-wave section. A imilar system removes the r-f from the other nd of the helix.

The direct coax-helix match, shown in Figure 2–72C, is sometimes referred to as direct pin match. It is the simplest system of all. The center conductor of the input coax is connected directly to the helix of the traveling-wave tube. Although this is the method usually used in the high power traveling-wave tubes of today, it has ome disadvantages. The high-voltage standing-wave ratio causes heating around the pin. Since his pin passes through the glass envelope on he way to the helix, the envelope is subject to heating and breaking at this point.

Figure 12–72D shows the coupled helix match, a widely used type of coupling. In this system the center of the coax input conductor is attached to a small helix. The major helix of the traveling-wave tube is inserted within this input helix where it acts as the secondary of a transformer. The input r-f energy is magnetically coupled to the main helix; the output r-f is removed by magnetic coupling also. Although the structure of a helix has good VSWR characteristics and is broader in bandwidth than cavities and waveguides, it is as yet mechanically unable to handle large amounts of power.

Operation

To understand the operation of the traveling-wave tube, it will be helpful to compare it to that of the two-cavity klystron amplifier. In the klystron an electron gun which is capable of great emission densities provides the electron beam that travels through input and output resonant cavities and is finally captured by a collector electrode.

The input cavity is tuned to the signal frequency to be amplified. Thus an r-f voltage is concentrated across the first gap. This alternating voltage has an E-field whose direction is parallel to the electron stream. Thus it will either add to or subtract from the velocity given to the electrons by the beam voltage. Electrons passing through the gap are either accelerated or decelerated. Therefore the electron beam, which has a uniform velocity given by the beam accelerating voltage, is also velocity-modulated at an r-f rate by the input r-f signal. Fast electrons catch up with slow electrons at specific distances beyond the input cavity to form bunches. The frequency at which these bunches of electrons arrive at points beyond the buncher cavity corresponds to the frequency of the input signal. One bunch will arrive during each cycle.

If the catcher cavity is tuned to the same frequency and placed at a point along the electron stream where bunches form, oscillations that follow the input signal will be induced in the cavity. There is high-power gain because the power required to bunch the beam is much less than the power that can be derived from it. The additional energy comes from the kinetic energy imparted to the beam by the DC accelerating voltage. The interaction between the electron stream and the r-f fields takes place only in the rather narrow gap. The Q of the cavities must be high to build up the r-f voltage across the gaps to high values for efficient interaction. This high Q cavity means that the klystron is a narrow band device.

The tremendous bandwidth of the traveling-wave tube compared to that of the klystron is the traveling-wave tube's major advantage. The traveling-wave tube differs from the klystron principally in that the electrons interact with a *traveling* wave, rather than with a *standing* wave. This interaction is distributed, rather than localized, as in a klystron. There are no high Q resonant circuits in a traveling-wave tube; therefore, amplification can occur over appreciably broad frequency ranges.

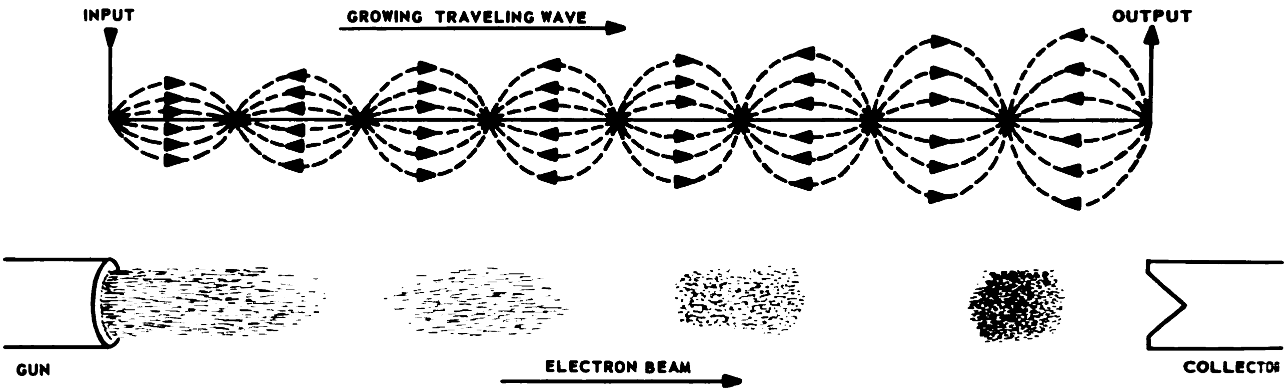


Figure 12–73. Simplified Traveling-Wave Concept

The simplest concept of a traveling wave is illustrated in Figure 12–73. An electron beam is passed along, or around, a transmission line consisting of a straight wire. This wire has an input and an output terminal. The electrons are gathered by a collector electrode, just as in the klystron. If the output is terminated in the characteristic impedance of the line, a wide range of radio frequencies will travel along the line; the line is nonresonant. Consider these frequencies. Part of their electric fields are parallel to the direction of travel of the wave and to the direction of the electron beam. This means that there will be interaction between the electrons in the beam and the electromagnetic wave on the transmission line wire. This interaction takes place the length of this hypothetical tube and for a wide range of frequencies.

If the electrons could be accelerated so that they traveled faster than the electromagnetic wave on the wire, they would become bunched due to the effect of the field parallel to the electron beam. Some of the electrons would be slowed down, while others would be accelerated. These bunches of electrons give part of their energy to the wave on the transmission line. As a result, the traveling wave increases in amplitude, as shown in Figure 12–73. Since the same action takes place for a wide range of frequencies, the traveling-wave tube has a very wide bandwidth.

However, this simple form of a traveling-wave tube will not work because waves travel along a straight wire at the speed of light. At present there is no known way of accelerating a beam of electrons to such speeds. For this reason traveling-wave tubes are designed to use a transmission line that will delay the r-f wave so that the electron stream can be accelerated to the same or higher speeds.

A common type of delay line is a wire that is wound in the form of a long coil, or helix, as shown in Figure 12–74. The diameter and the number of turns per unit length are selected so that while the wave travels along the wire at the usual speed, its velocity along the axis of the helix is about one-tenth the velocity of light. Now it is possible to accelerate the electrons to the point where their speed matches that of the wave as it proceeds down the tube. The helical type of delay line works very well because there will be a large proportion of the electric field parallel to the electron beam. There will be longitudinal fields both inside and outside the transmission line (helix). Usually the beam of electrons is passed down the center of the coil because the field is more concentrated there.

If any of the signal reflected at the output end reaches the input, it will be greatly amplified. This would produce spurious oscillation. Figure 12–74 shows a lossy wire attenuator placed near the midpoint of the traveling-wave tube envelope. Another type of attenuator consists of a split cylinder of graphite which is placed around the coil near the center. These attenuators serve to isolate the input from the output, thus preventing oscillation. In the lower portion of Figure 12–74 is shown the traveling wave for the tube. Notice the loss in gain due to the attenuator. The bunches of electrons emerging from the attenuator build up a new c-w signal on the helix.

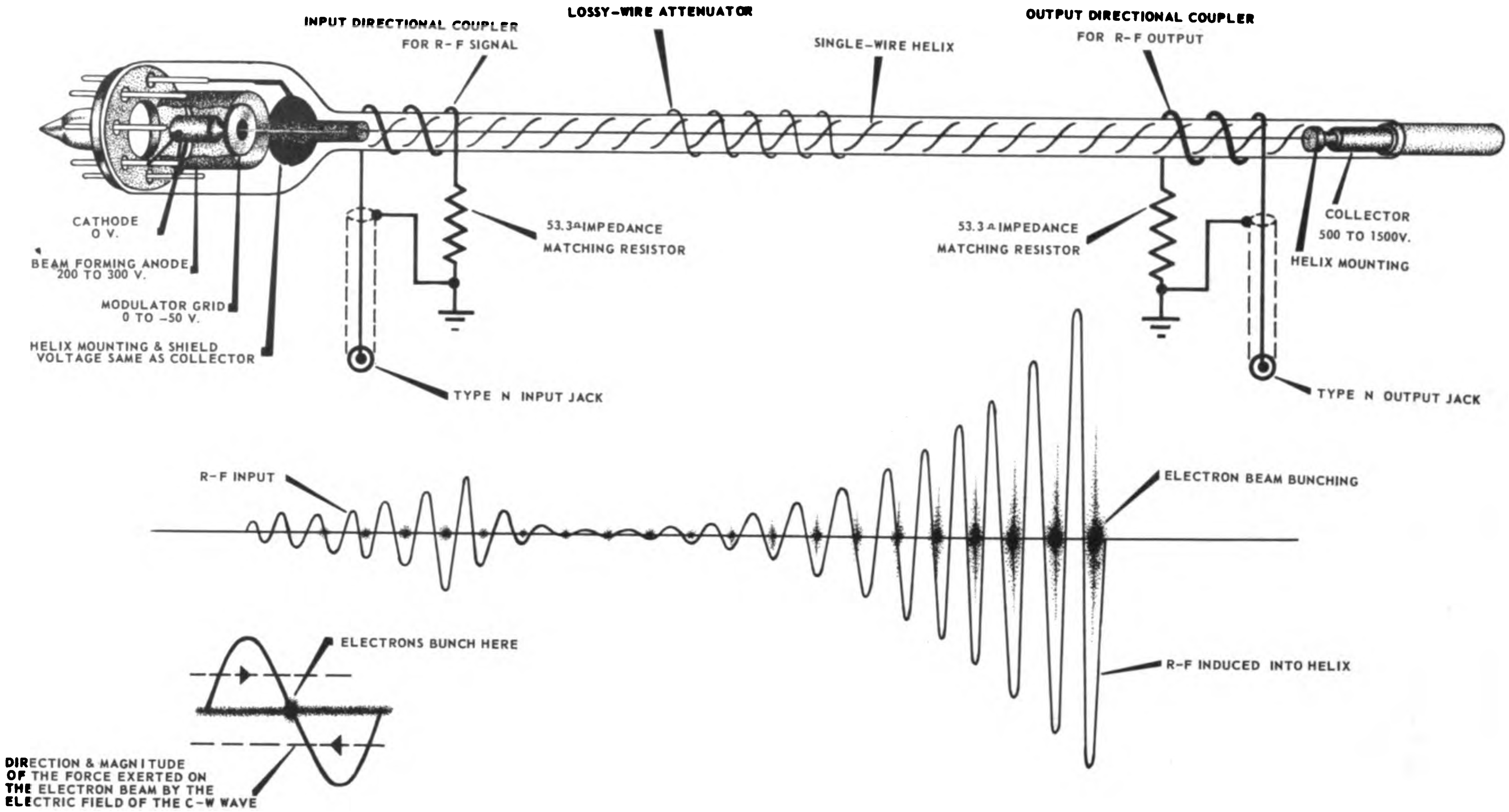


Figure 12–74. Functional Diagram of a Traveling-Wave Tube

This signal has the same frequency as the original input signal.

Summary of Action in a Traveling-Wave Tube

In the typical low- and medium-power traveling-wave tubes the electron beam is directed through the center of a transmission line in the form of a coil, or helix. A continuous-wave signal of a predetermined frequency is coupled into the helix. Electrons are bunched due to the action of the fields. Amplification of the signal on the helix begins as the field formed by the bunches interacts with the electric field of the c-w signal. Each newly formed electron bunch adds a small amount of energy to the c-w signal on the helix. The slightly amplied c-w signal then causes a denser electron bunch which, in turn, adds a still greater voltage to the c-w signal. This process is continuous as the wave and the bunches progress down the helix.

Amplification increases as the greater velocity of the electron beam and the retarding effect of the negative points in the wave pull the electron bunches more nearly in phase with the electric field of the c-w signal. The additive effect of the two fields, exactly in phase, produces the greatest resultant of amplification. Attenuators placed near the center of the helix reduce all the waves traveling along the helix to approximately zero. This prevents reflected waves from returning to the tube input and causing spurious oscillations. The electron bunches pass through the attenuator unaffected. The electron bunches, after emerging from the attenuator, induce a new c-w signal on the helix. This new c-w signal has the same frequency as the original input signal. The electric field of the newly induced c-w signal interacts with bunched electrons to begin afresh the amplification process.

For a short distance, the velocity of the electron bunches is reduced appreciably due to the large amount of energy absorbed by the formation of the new c-w signal. Amplification increases as the greater velocity of the electron beam and the E-fields of the wave pull the electron bunches more nearly in phase with the electric field of the newly induced c-w signal. At a point of predetermined power, the amplified c-w signal is coupled out of the helix. Note that the amplified c-w signal is a "new" signal whose energy is completely supplied by the bunched electron beam.

Types of Traveling-Wave Tubes

There are three basic types of traveling-wave tubes: the low level, the driver, and the power amplifier. The helix tied to the collector and separated from the accelerating anode is a distinguishing characteristic of the low level type. The driver traveling-wave tube has the helix separated from the collector and connected to the accelerating anode. The power amplifier traveling-wave tube has the collector, helix, and accelerating anode all separate and at the same potential.

Advantages and Disadvantages

The traveling-wave tube is used because of its very wide bandwidth characteristics. This unique advantage is almost offset by several disadvantages. The traveling-wave tubes in use today must rely on external magnetic focusing of the electron beam. For the S-band this amounts to a solenoid current of 11 amperes with a voltage drop across the solenoid of approximately 15 to 85 volts DC. None of this power expenditure can be recovered in the output.

The traveling-wave tube is also a low efficiency device. The lost power is in the form of heat, which calls for a high volume of air conditioning or liquid cooling. Should a power amplifier traveling-wave tube be turned on without forced cooling, the tube would fail almost immediately. Even slight heating can deform the helix, and a nonuniform helix causes "holes," or areas of no transmission, in the output. Holes that exist in normal traveling-wave tubes are due to errors in the manufacturing process.

Because these holes are known to exist, their position (frequency) must be accurately determined. This leads to another disadvantage in that a traveling-wave tube must be thoroughly checked out before it can be placed in its intended equipment. Perhaps one of the most elaborate of tube testers has been constructed for traveling-wave tube checkout. Such a piece of equipment must always be available when equipment using these tubes is to be serviced. Each traveling-wave tube must be tested and evaluated independently. At this stage of development, it is impossible to substitute one traveling-wave tube for another. So much difference exists among them that they must be calibrated separately. Then, only after a comparison has been made, can one be substituted for another of the same type.

BACKWARD-WAVE OSCILLATOR

The backward-wave oscillator provides a flexible source of microwave energy that can be voltage-tuned to produce a frequency range as large as 2:1. The output frequency of the backward-wave oscillator is determined by a frequency-selective feedback and amplification process rather than by resonant circuits as used in conventional microwave oscillators.

Structure of Helix Type

The backward-wave oscillator tube consists of an electron gun, a helix structure, and a collector at the far end of the helix. Physically, the backward-wave oscillator resembles the traveling-wave amplifier tube, although for comparable frequencies it is larger in diameter and somewhat shorter in length. Another difference, not apparent from a visual inspection of the tube, is that the helical backward-wave oscillator uses a hollow electron beam with a strong concentration of the electrons near the helix. This hollow electron beam is focused along the length of the helix by a strong magnetic field supplied by an axial solenoid surrounding the tube.

The r-f output of the backward-wave oscillator is a result of the interaction between the electron beam and the electric fields accompanying a microwave signal present on the helix. The term backward-wave oscillator is quite appropriate for this tube since the r-f energy moves and builds up in a direction opposite to that of the electron beam and is coupled out at the gun end of the tube via the helix terminal.

Theory of Operation

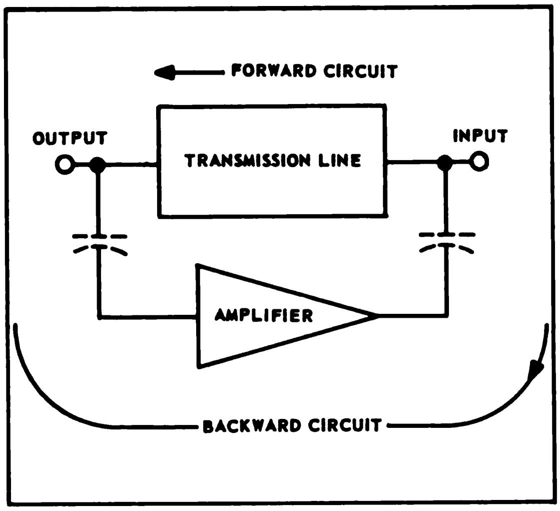


Figure 12–75. Typical Feedback Loop

The operation of the backward-wave oscillator tube may be explained in terms of a series of feedback loops similar to those common to low-frequency electronic circuits. Each of these regenerative loops can function as an amplifier or an oscillator and is designed so that the phase shift around the loop is one cycle. One of these feedback loops is shown in Figure 12–75 where, using conventional circuit terminology, the forward circuit consists of a section of transmission line and the backward circuit is a unilateral amplifier connecting the output of the transmission line to the input.

In this circuit positive feedback will occur when the amplifier gain becomes sufficiently high to overcome the loss in the transmission line. The loop will oscillate at a frequency for which the total phase delay is one or more cycles. If the amplifier is designed for limited high-frequency response, oscillations will occur only when the phase delay is one cycle. The frequency of oscillation can be shifted by changing the phase delay in the amplifier. The essential feature of the voltage-tuned backward-wave oscillator is that the frequency of oscillation can be changed electrically by changing the phase delay in the amplifier.

FEEDBACK LOOPS. Figure 12–76 shows a chain of identical regenerative feedback loops. Along the top of the chain is a series of transmission line sections that will support a wave moving either to the right or to the left. Along the bottom of the chain is a series of unilateral amplifiers in which signals can pass only in the left-to-right direction. Each loop consists of a transmission line, coupling capacitance and an amplifier transmitting from left to right. In operation, positive feedback, which leads to regenerative amplification and oscillation, makes use of a

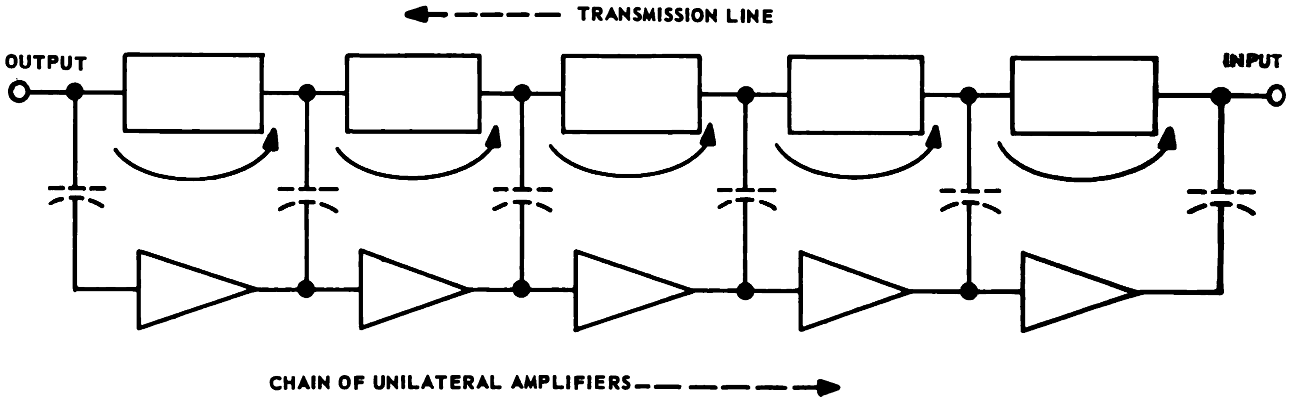


Figure 12–76. Typical Chain of Feedback Loops

wave going from right to left on the transmission line when the phase delay in a single loop is just one cycle. The total phase delay around a group of n loops then will be n cycles.

For low values of amplification, the chain of loops will act as a regenerative amplifier operating at the frequency which provides positive feedback. But if the transmission line is terminated in its characteristic impedance at the input and the amount of amplification is increased, oscillations will start. The frequency of oscillation is controlled by the phase delay in the amplifier chain.

With this background, we can now examine the actual functioning of the backward-wave oscillator tube. Figure 12–77 shows a cross section of the helix, and a portion of the electron beam. The helix structure consists of a cylindrically-wound flat-wire tape; the electron beam is hollow and passes very close to the helix turns. The strong axial magnetic field focuses the electrons in the beam and allows movement only in the direction of the axis of the tube. The lines of force of the electric fields associated with an r-f wave traveling along the helix are also shown in Figure 12–77. Although these fields rotate around the helix at the velocity of light, the axial velocity is much slower, depending on the ratio of the turn-to-turn spacing of the helix to its circumference. The axial velocity of the r-f wave is approximately equal to the velocity of the beam wave.

The lengthwise electric fields are strong between helix turns and very weak directly under the wire itself (see Figure 12–77), since the electric fields cannot exist parallel to a conductor. The field between the helix turns has a great effect on the velocity of the electrons in

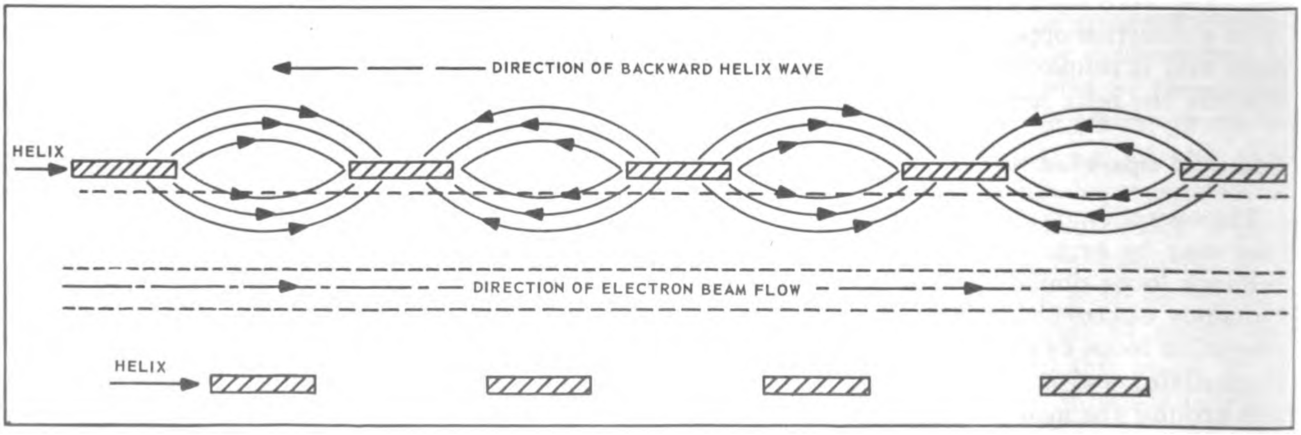

DIRECTION OF BACKWARD HELIX WAVE
HELIX
DIRECTION OF ELECTRON BEAM FLOW
HELIX

Figure 12–77. Cross Section of Helix, Electron Beam and Helix Wave

the beam, producing an interaction process. This process is represented by the capacitive coupling between the transmission line and the amplifier chain shown in Figure 12–76. In this way feedback loops are formed between the midpoints of adjacent helix gaps.

Although the concept of individual feedback loops is a useful device for explanation, the backward wave interaction is actually a continuous process. The maximum coupling between the helix wave and the beam occurs midway between gaps and gradually tapers off to a minimum directly under the helix turns. One of these regenerative loop chains exists at each angular position around the helix. Each of these regenerative loop chains is independently coupled to the helix transmission line so that the net effect is a continuous amplification and feedback process occurring down the entire length of the tube.

Amplification. The basic mechanism of amplication is a velocity modulation process that causes the electrons to bunch in the beam. Figure 12–78 shows the sinusoidal variations in amplitude of the electric field at the midpoint between helix turns. The phase relationship between the backward wave on the helix and the velocity of the electron beam is such that each specific portion of the electron beam will be affected by an electric field of the same phase as it passes successive gaps down the helix.

An electron at point A in Figure 12–78 experiences a continuous decelerating effect caused by fields of the same phase and direction of force as it proceeds down the tube. In a like manner an electron at point B will be continuously accelerated in its journey down the tube. In this way some parts of the electron beam are slowed down while others are advanced. The net effect is a bunch formed at the midpoint between the accelerating and the decelerating fields (Figure 12–78). Since a bunch will be formed in the beam for each wave on the helix, 20 to 40 bunches will be traveling along the beam (each surrounded by a wave on the helix) at any given instant.

At this point it should be mentioned that the average electron velocity of the beam is slightly faster than the effective phase velocity of the amplifier chain. This means that the electron bunches will advance a quarter of a cycle as they approach the collector end of the tube. Thus they encounter the full decelerating effects of the electric field and give up a maximum amount of kinetic energy to the wave on the helix.

Figure 12–79A shows that the density of the electron bunches increases according to an exponential relationship. Figure 12–79A shows the envelope of the bunching, rather than instantaneous amplitudes, since many r-f cycles exist

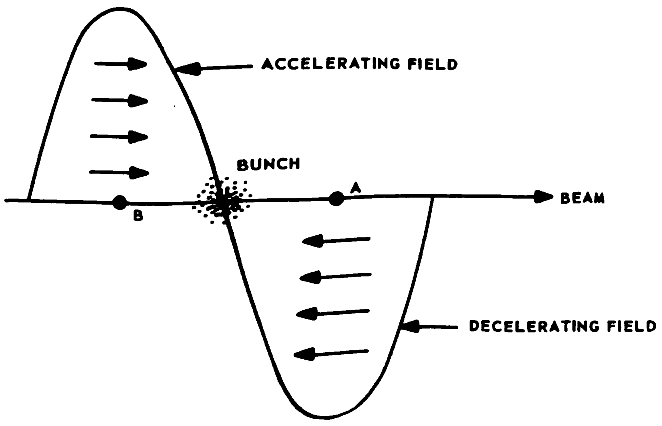


Figure 12–78. Axial Electric Field of Helix Wave Provides Velocity Modulation and Bunching of Electron Beam

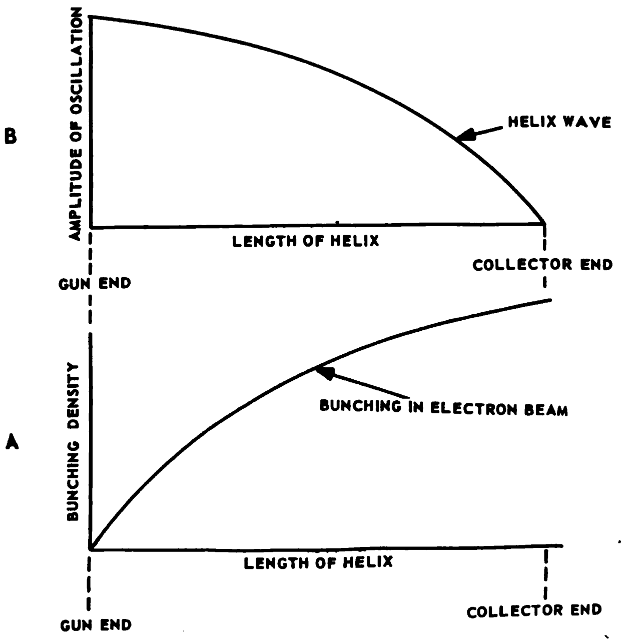


Figure 12–79. Relative Amplitude of the Helix Wave and Electron Bunching Depicted in Envelope Form

along the length of the backward-wave oscillator tube. The wave on the helix moves from right to left toward the gun end of the tube and gains amplitude according to the degree of electron bunching in the beam. In this way the envelope of the wave on the helix shown in Figure 12–79B is the result of the bunching envelope so the maximum energy transfer from the beam to the wave on the helix occurs at the collector end of the tube.

BEAM VELOCITY CONTROLS FREQUENCY. Now that a correspondence between the chain of lumped regenerative loops and the helix backward-wave oscillator tube has been established, it can be seen that if the velocity of the electron beam is varied, the phase delay around each of the regenerative loops will be changed. If the electron beam is at the correct velocity, the chain of regenerative loops will oscillate at a frequency where the phase delay of each loop is equal to one cycle.

Oscillations begin in the backward-wave oscillator in much the same way as they begin in other oscillators. Noise waves are established on the helix from the shot noise coupled from the electron beam and from the thermal energy developed in the termination at the input end of the tube. The waves traveling backward on the tube (to the left) velocity modulate the beam. Velocity modulation causes the electrons to bunch and, in turn, reinforce the wave that exists on the helix at the frequency where the single-loop delay is equal to one cycle. In this way oscillations are built up at a single frequency determined solely by the electron beam velocity, which is a function of the cathode-to-helix voltage of the tube.

Crossed-Field Oscillator

There has been extensive research and development in a newer oscillator called the crossed-field oscillator. This type of oscillator provides many advantages over other types of existing backward- and forward-wave oscillators. In comparison with the helix backward-wave oscillator, the crossed-field oscillator offers higher average c-w power, higher efficiency, broader bandwidth, and smaller size and weight per unit of power output. The crossed-field oscillator uses a permanent magnet with the advantage of elimination of solenoid power requirements and heat generation.

The principle of operation of the crossed-field oscillator is the same as that of the traveling-wave tube with the exception that the basic electron interaction is of magnetron interaction rather than that of klystron interaction.

CHAPTER 13

Antennas

This chapter deals with the basic principles of antennas. It discusses antennas in general, the principles of electromagnetic radiation and its application to radar antennas, various antenna arrays, and typical airborne radar antenna systems.

FUNCTION OF THE ANTENNA

An antenna can be defined as a conductor or system of conductors used either for radiating electromagnetic energy into space or for collecting electromagnetic energy from space. To have adequate signal strength at the receiver, either the power transmitted must be extremely high, or the efficiency of the transmitting and receiving antennas must be high because of the high losses in wave travel between the transmitter and the receiver, and the lower sensitivity of the ultra-high frequency range receivers. Since it is difficult to generate large amounts of power efficiently in the higher frequency ranges, it is especially important that as much of the available signal at the transmitter be converted into radiated energy as possible, and that as much of the radiated energy as possible be picked up at the receiver.

ANTENNA RECIPROCITY

Fortunately, separate antennas seldom are required for transmitting and receiving radio energy. Any antenna transfers energy from space to its input terminals with the same efficiency with which it transfers energy from the output terminals into space, assuming, of course, that the frequency is the same. This property of interchangeability of the same antenna for transmitting and receiving operations is known as *antenna reciprocity*. Antenna reciprocity is possible chiefly because antenna characteristics are essentially the same regardless of whether an antenna is sending or receiving electromagnetic energy. Because of antenna reciprocity, most radar sets use the same antenna both for receiving and for transmitting.

An automatic switch in the radio frequency line first connects the antenna to the transmitter, then to the receiver, depending upon the sequence of operation. Because of reciprocity of radar antennas, this chapter treats antennas from the viewpoint of the transmitting antenna with the understanding that the same principles apply equally well when the antennas are used for receiving electromagnetic energy.

DIRECTIONAL PROPERTIES

Usually, the most important characteristic of a radar antenna is its directional property, or its directivity. Directivity means that an antenna radiates more energy in one direction than in another. In radar operations some antennas are required to send all energy in one direction in order that as much as possible of the electromagnetic energy generated by the transmitter strike an object in a given direction. In other systems it is desirable for the energy to be radiated equally well in all directions from the source. An example of an antenna system that radiates energy in a given direction is that of an airborne navigation and bombing set. In this set there is a limited amount of power available at the transmitter. To achieve maximum benefit from this minimum power, all of it is sent in the same direction.

Since the antenna in this set is also used for reception, it likewise receives electromagnetic energy from only one direction. Because of design features, it is possible to tell the direction of an object at which this directional antenna is sending energy or the direction of the object from which the antenna is receiving energy. Furthermore, the physical position of the antenna is indicative of the direction of the object. An example of a nondirectional radar antenna is the antenna installed in a radar beacon. This antenna must receive energy equally well from all directions in order that radar-equipped aircraft can ascertain position regardless of direction from the beacon antenna.

ANTENNA EFFICIENCY AND GAIN

An efficient antenna is one which wastes very little energy. The higher the antenna efficiency, the less the energy loss. The greatest cause of decreased antenna efficiency during the conversion of guided waves to free-space waves, is the loss of energy in the form of heat. Heat losses will be discussed later. Actually, antenna efficiency may be high; in a single antenna element the efficiency may be as great as 90%.

An entire antenna system, properly designed, can concentrate all its radiated energy in a given direction with virtually none being lost in other directions. The gain of the antenna is the ratio of the maximum radiation intensity of the antenna in question to the maximum radiation intensity from a reference antenna with the same power input. The gain of a directional antenna compared to a nondirectional, or isotropic, antenna can be measured by measuring the intensity of the radiated field on the principal axis at a known distance from the antenna.

The field intensity that would exist with the isotropic radiator with that same power input is calculated. The ratio of the intensity of the directional antenna to the intensity of the nondirectional antenna is the gain. The effectiveness of a directional antenna in a particular direction compared to a standard dipole is also known as antenna gain. It is usually expressed as the ratio of the standard antenna power to the directional antenna power that will produce the same field strength in the desired direction.

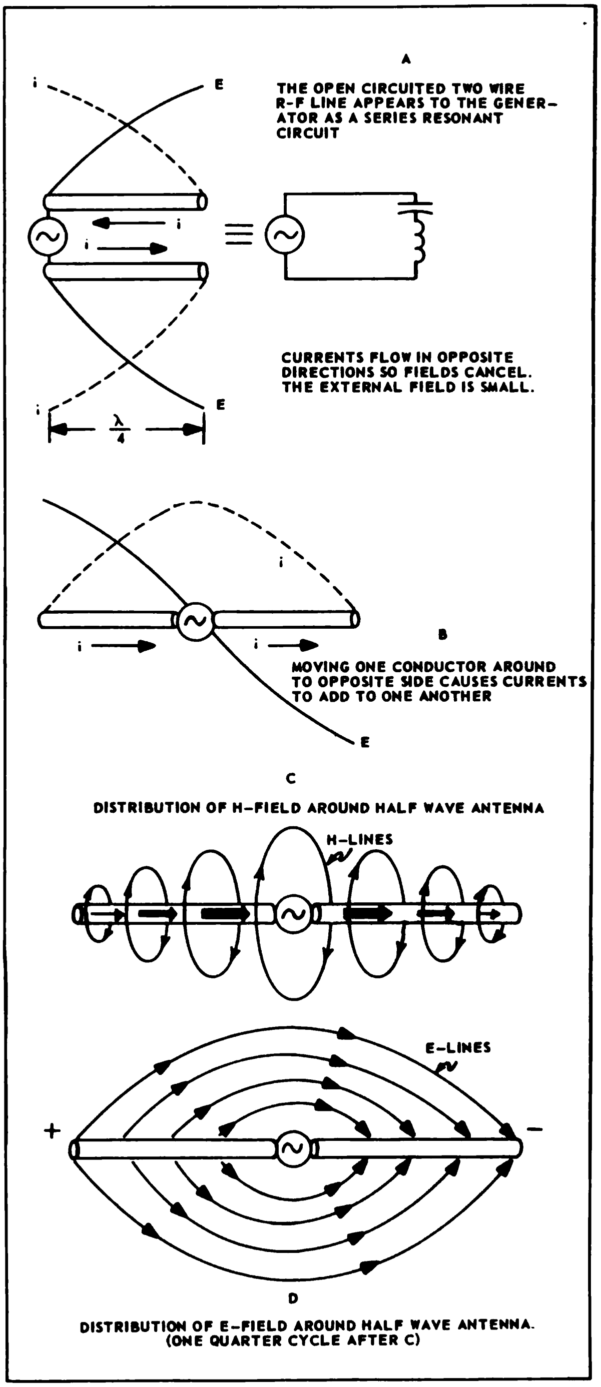

A
THE OPEN CIRCUITED TWO WIRE R-F LINE APPEARS TO THE GENERATOR AS A SERIES RESONANT CIRCUIT
i
E
i
i
≡
CURRENTS FLOW IN OPPOSITE DIRECTIONS SO FIELDS CANCEL. THE EXTERNAL FIELD IS SMALL.
i
E
λ/4
i
i
i
B
MOVING ONE CONDUCTOR AROUND TO OPPOSITE SIDE CAUSES CURRENTS TO ADD TO ONE ANOTHER
E
C
DISTRIBUTION OF H-FIELD AROUND HALF WAVE ANTENNA
H-LINES
E-LINES
+
−
D
DISTRIBUTION OF E-FIELD AROUND HALF WAVE ANTENNA. (ONE QUARTER CYCLE AFTER C)

Figure 13-1. Development of Half-Wave Antenna from Open-End Quarter-Wave R-F Line

ELECTROMAGNETIC ENERGY IN SPACE

Because the half-wave dipole antenna is the ndamental element in an antenna system, it ill be used as a starting point for discussing he radiation of electromagnetic energy into pace. A dipole antenna can be developed from n open-end transmission line one-quarter waveength long. Figure 13–1A shows this quarter-ave section and the standing waves of current nd voltage developed on the line. The current low, as indicated, is flowing in opposite direcions in the two conductors. The magnetic fields ue to these currents cancel. The electrostatic ields, too, are in opposite directions.

Efficient radiation requires a very heavy field ensity; therefore, the fields must reinforce, or id, each other. Suppose the ends of the two onductors are spread apart. The direction of urrent flow doesn't change, but the magnetic ield is being "opened" into space, and less canellation occurs. The electrostatic field is opened ut instead of being concentrated in the small space between the two conductors. If the two conductors are moved until they are in line with each other as shown in Figure 13–1B, the current flow in the two conductors is now in the same direction. The standing waves are exactly as they were before.

The magnetic (H) field about the wire is shown in Figure 13–1C. The direction of current flow is indicated by the arrows, from left to right. The magnetic field is in the direction indicated. Since maximum change of current takes place at the center of the dipole, the magnetic field is greatest at the center. Figure 13–1D shows the electrostatic (E) field about the conductor when the difference in potential between the ends of the conductor is the greatest. This does not occur at the same time as the maximum magnetic field. Since the voltage and current on the dipole are 90° out of phase, the electrostatic and magnetic fields surrounding the antenna are also 90° out of phase.

Figure 13–2 shows the relationship between the variations in field intensities. When maximum current is flowing, there is no difference in potential between the ends of the conductors. The magnetic field about the conductor is maximum (Figure 13–2A). As the current continues to flow, a charge is built up on the conductor and the current decreases. The charge built up increases and the electrostatic field which accompanies the charge also builds up. The current flow

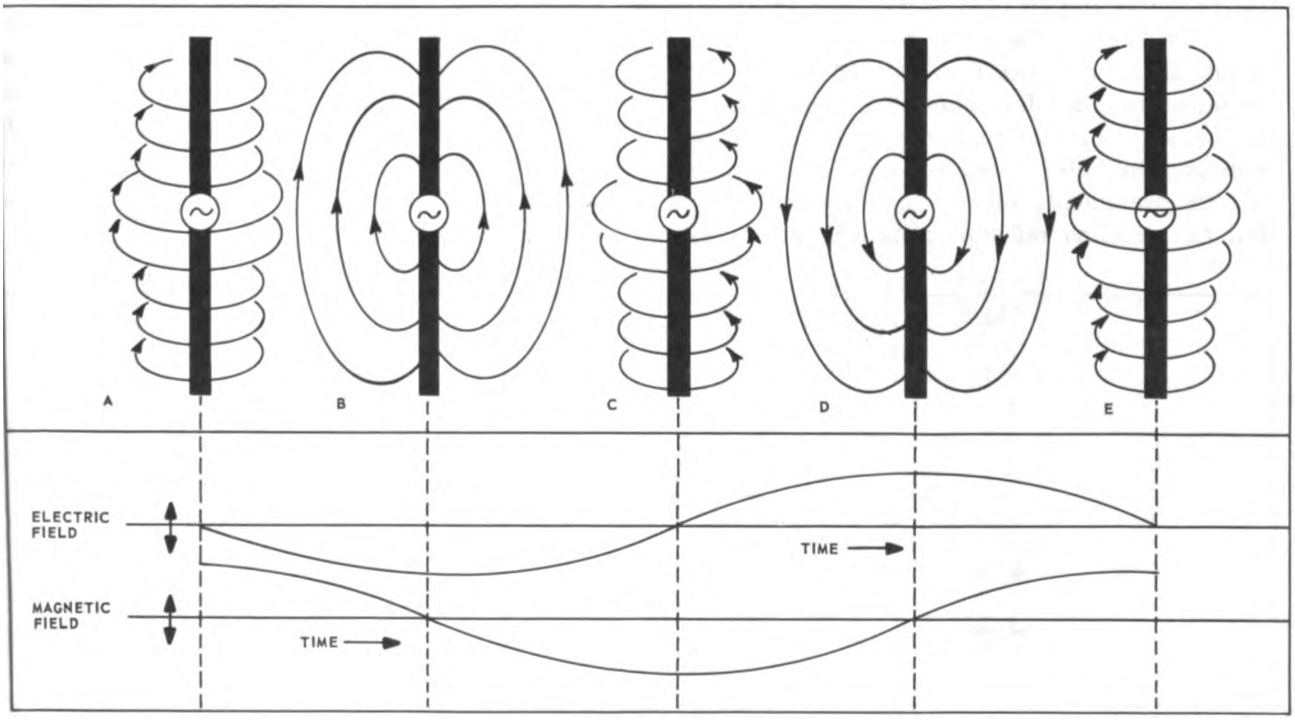


Figure 13–2. Variations in Field Intensities

and its accompanying magnetic field decrease. When the charge has built up to maximum, the current flow has decreased to zero. The electrostatic (or electric) field is at its maximum intensity (Figure 13–2B). When the current starts to flow in the opposite direction through the conductor, the magnetic field builds up as the electric field decreases. Figure 13–2C shows the fields when current is maximum in the direction opposite to A.

Figure 13–2D shows the electric field at a maximum when the preceding current flow has stopped momentarily and the magnetic field is zero. In E there is no charge and the flow of current is again maximum. Notice that the magnetic and electrostatic lines of force are at right angles to each other. In addition, these fields reach their maximum intensity a quarter of a cycle, or 90 electrical degrees, apart. It is said that these two fields are in *space and time quadrature*. Quadrature is the term used to express the phase relationship between two periodic quantities of the same period when the phase difference between them is one-fourth of a period. The E and H fields are 90° out of phase (time) and the lines of force are at right angles to each other (space).

The fields associated with the energy stored in the resonant circuit (antenna) are called the *induction* fields. These fields decrease with the square of the distance from the antenna. They are only local in effect and play no part in the transmission of electromagnetic energy. They represent only the stored energy in the antenna and are responsible only for the resonant effects that the antenna reflects to the generator. The fields set up in the transfer of energy through space are known as the *radiation* fields. Although these fields decrease with distance from the antenna, this decrease is much less rapid than the decrease of the induction fields. This is because the decrease is linear. Therefore, the radiation fields reach great distances from the antenna. It is the radiation fields that are responsible for electromagnetic radiation.

Mechanics of Radiation

There are many theories as to the why and how of radiation. While these theories are responsible for the advance in antenna design, they do not provide a simple and clear physical picture of the mechanics of radiation. The simplified physical picture does not tell the entire story. But the important fact to remember is that radiation does take place.

Figure 13–3 gives a simple picture of an E-field detaching itself from an antenna. The H-field will not be considered, although it is present. In A the voltage is maximum and the electric field has maximum intensity. The lines of force begin at the end of the antenna that is positively charged and extend to the end of the antenna that is negatively charged. Note that the outer E-lines are stretched away from the inner ones. This is due to the repelling force between lines of force in the same direction. As the voltage drops, the separated charges come together, and the ends of the lines move toward the center of the dipole (Figure 13–3B). But, since lines of force in the same direction repel each other, the centers of the lines are still being held out.

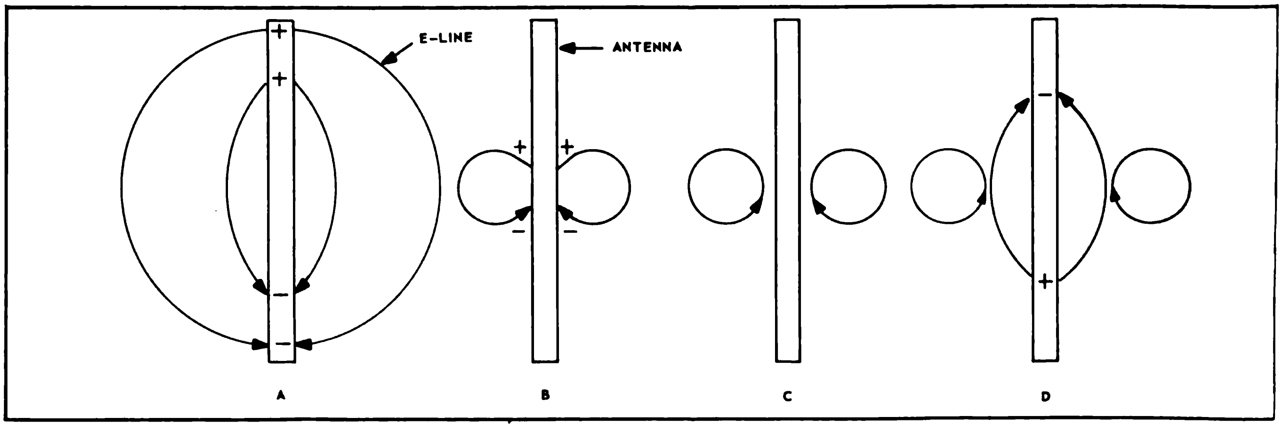


Figure 13–3. E-Field Detaching Itself from an Antenna

As the voltage approaches zero, some of the nes collapse back into the dipole, but in others ne ends of the line come together and form a omplete loop. Notice the direction of these lines f force next to the dipole in Figure 13–3C. At his point the voltage on the antenna is zero. As he charge starts to build up in the opposite diection, electric lines of force again begin at the ositive end of the antenna and stretch to the egative end of the antenna. These lines of force, eing in the same direction as the sides of the losed loops next to the antenna, repel the closed oops and force them out into space at the speed f light. As they travel, they generate a magnetic ield in phase with them.

Another factor to remember about radiation is hat the ease with which it may occur varies with requency. At lower frequencies, such as 60-cycle ower frequency for example, voltage on the anenna changes so slowly that the component of nergy radiated is extremely small and is of no practical value. At higher frequencies, such as 50,000 cps and up, the radiated energy is great enough to meet communications requirements.

Fields in Space

Figure 13–4 shows the manner in which radiation fields are propagated from the antenna. The E-field and the H-field are shown as separate sets of flux lines about the antenna. The electric flux lines are the closed loops on each side of the antenna. The magnetic flux lines are closed circular loops which have their axis around the antenna, or in other words, the antenna is the center of each loop. They are represented as dots and crosses. The sine waves that are labeled the curves of radial variation of flux density indicate the relative field strength at various distances and angles from the antenna. Since the field configuration is not a standing wave, but a traveling wave phenomenon, the magnetic and electric

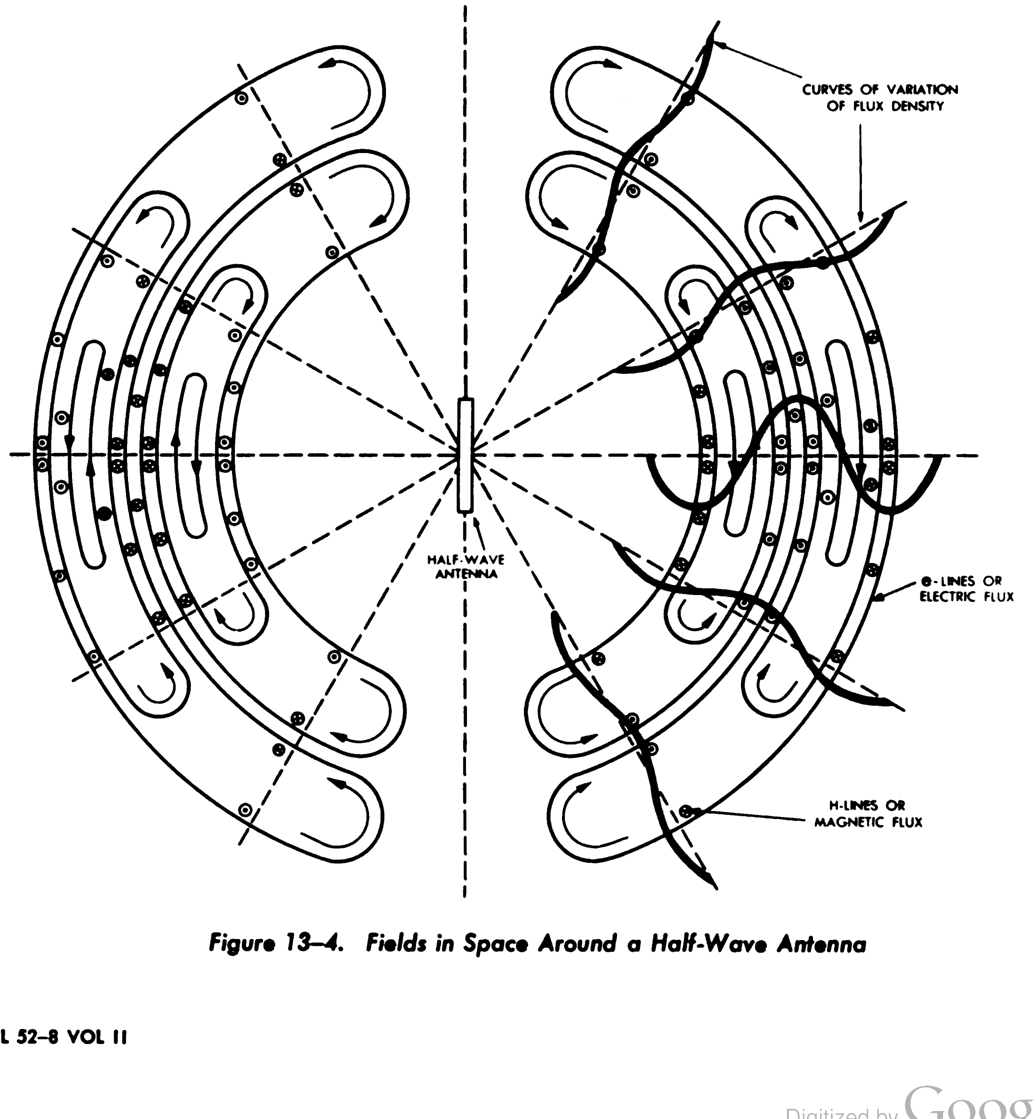


Figure 13–4. Fields in Space Around a Half-Wave Antenna

fields are in phase and thus the sine waves apply to the flux density of either field.

In the direction perpendicular to the antenna, both the H-field and the E-field are strong, for this is the direction where both fields originally formed. In the direction parallel to the antenna, or off the ends, no H-field forms at all and only a very small E-field. The flux density, therefore, is small in these two directions. Due to the directional properties of the half-wave antenna, most of the radiated energy travels in the direction perpendicular to the antenna, while very little energy travels in the direction along its axis.

As previously stated in the discussion of waveguides, no moving E-field can exist without an H-field, and no H-field can be propagated without an associated E-field. Similarly, with the propagation of electromagnetic energy into space, no moving electrostatic forces can exist without magnetic stresses existing in space, and no magnetic force in motion can exist without an electrostatic stress. Each creates the other, and one cannot exist without the other.

The direction of motion is the travel of the fields away from the antenna. The current associated with the magnetic field does not flow because in space there is no conductor to carry current, but the field exists, nevertheless. To visualize a field existing without current flow, think of a moving magnetic field cutting a glass rod. A voltage (electrostatic stress) is induced in the rod, but there is little actual electron movement because the rod is a good insulator. Magnetic lines move in space and set up electric stresses in space in a similar manner.

Propagation into Space

Any part of the electromagnetic field set up by an antenna travels away from the antenna in a straight line. There are many parts to this field and many directions in which energy travels. Shown in Figure 13–5 are four of the large number of paths that the energy can take. These and other paths that energy takes affect reception of radiated energy. If there is nothing between the emitting antenna and the receiving station (R_1), some energy will travel directly to R_1 via path 3. Receiving station R_2 cannot receive energy because the earth is between the two points and because the energy cannot go through the earth for any distance. Some of the energy will follow path 2 out into space.

At some height above the earth there is a heavily-ionized layer of atmosphere, called the *ionosphere* (refer to Chapter 15). This constitutes a change of media through which the energy must travel, and is sufficient to refract a wave. In the case of path 2 the refraction is sufficient to bend the energy wave out of the ionosphere back to the earth. Therefore, receiving station R_3, which is beyond the horizon and not located far

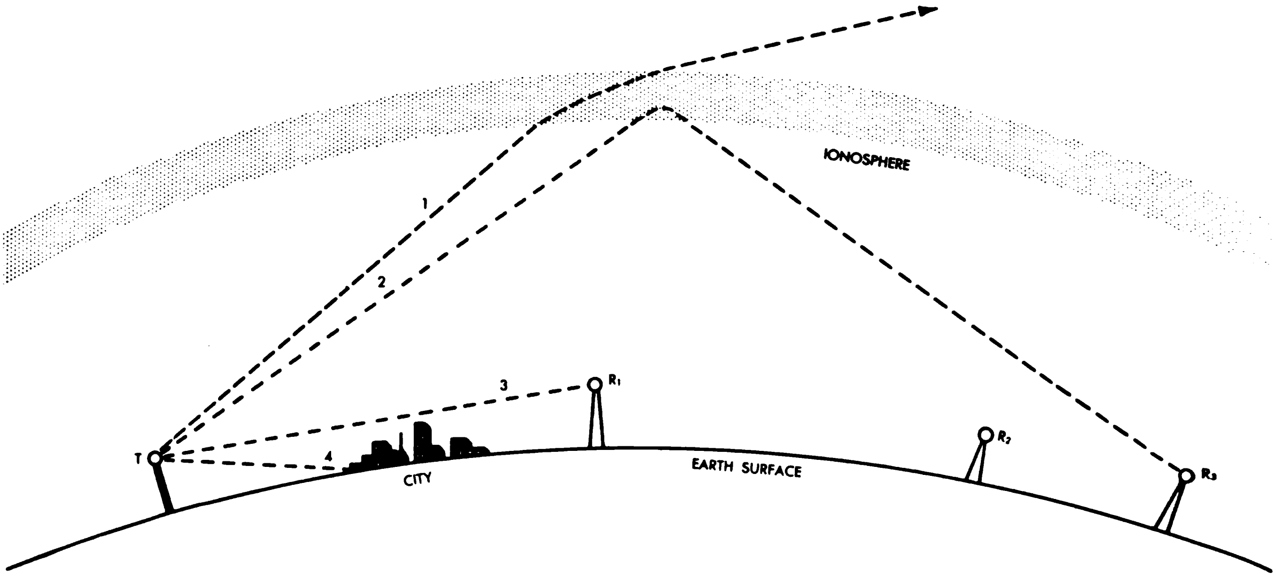


Figure 13–5. Effectiveness of Energy Traveling in Various Directions from an Antenna

receiving a direct ray, receives the reflected ray from the ionosphere.

It is the ionosphere that makes possible round-the-world communications. If the angle of incidence of an energy wave with the ionosphere is too great, the angle of refraction may not be sufficient to bend the energy back toward the earth and it may go on through the ionosphere and be lost, as shown by path 1. Paths 3 and 4 follow along the surface of the earth and are called *ground waves.* The waves bent back to the earth by the ionosphere are called *sky waves.*

The angle of refraction depends greatly upon the frequency of the wave. The higher the frequency, the less bending of the wave.

THE HALF-WAVE ANTENNA

As stated previously in the discussion of electromagnetic radiation, a half-wavelength conductor is the simplest of the radiating elements. Considerable radiation occurs in this element because of its resonant characteristics and its ability to store large amounts of energy in induction fields. Resonance causes high voltages and high circulating currents and they, in turn, produce strong fields around the antenna.

In Figure 13–6 you can see that the voltage standing wave is high at the ends of the antenna and low at the center. As previously explained, this is also the case with the quarter-wave, open-circuit, two-wire line, from which the half-wave antenna is developed.

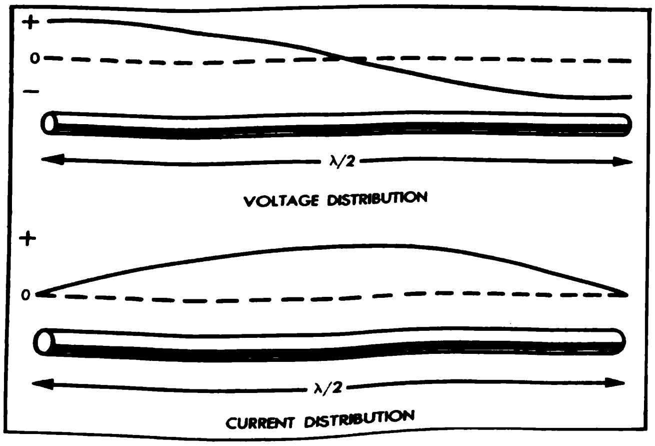


Figure 13–6. Current and Voltage Distribution in Half-Wave Antenna

An examination of the current distribution curve shows that the current standing wave reaches maximum a quarter-cycle after the voltage reaches maximum at which time the current is maximum at the center and zero at the ends. In practical applications the ends of a half-wave antenna must be insulated due to the high voltages there, and the center of the antenna must have low resistance in order to minimize the I^2R losses because of the high current there.

RESONANCE AND DIMENSIONS

Electromagnetic waves travel through space at a speed of 300 million meters per second. The length of one cycle in space depends upon frequency and is called the *wavelength.* Mathematically, the length of an electromagnetic wave is expressed by the formula,

$$\lambda = \frac{3 \times 10^8}{f}$$

where λ is the wavelength in meters, and f is the frequency in cycles per second.

Due to the resistance of the wire, the movement of waves along a wire (antenna) is somewhat slower than wave movement in space. Wavelength on a wire, therefore, is slightly less than that of a wave traveling in space and is expressed by the equation,

$$l = \frac{3 \times 10^8 \times 0.94}{f} \text{ meters.}$$

Physically, an antenna is about 6% shorter than a half-wave traveling in space. (This accounts for the fact that it is necessary to multiply the wavelength in the antenna formula by 0.94.) In this manual assume that the correction in antenna length has been made whenever the length of the antenna is given in wavelengths.

Effect of Length on Antenna Impedance

An antenna of the correct length acts as a resonant circuit and presents pure resistance to the excitation circuit. (See Figure 13–7A.) An antenna having other than the correct length displays both resistance and reactance to the excitation circuit. An antenna slightly longer than a half-wave, for example, acts as an inductive circuit. This is understandable if you think of the antenna in terms of a quarter-wave r-f line. When the antenna is excited at the center, it is

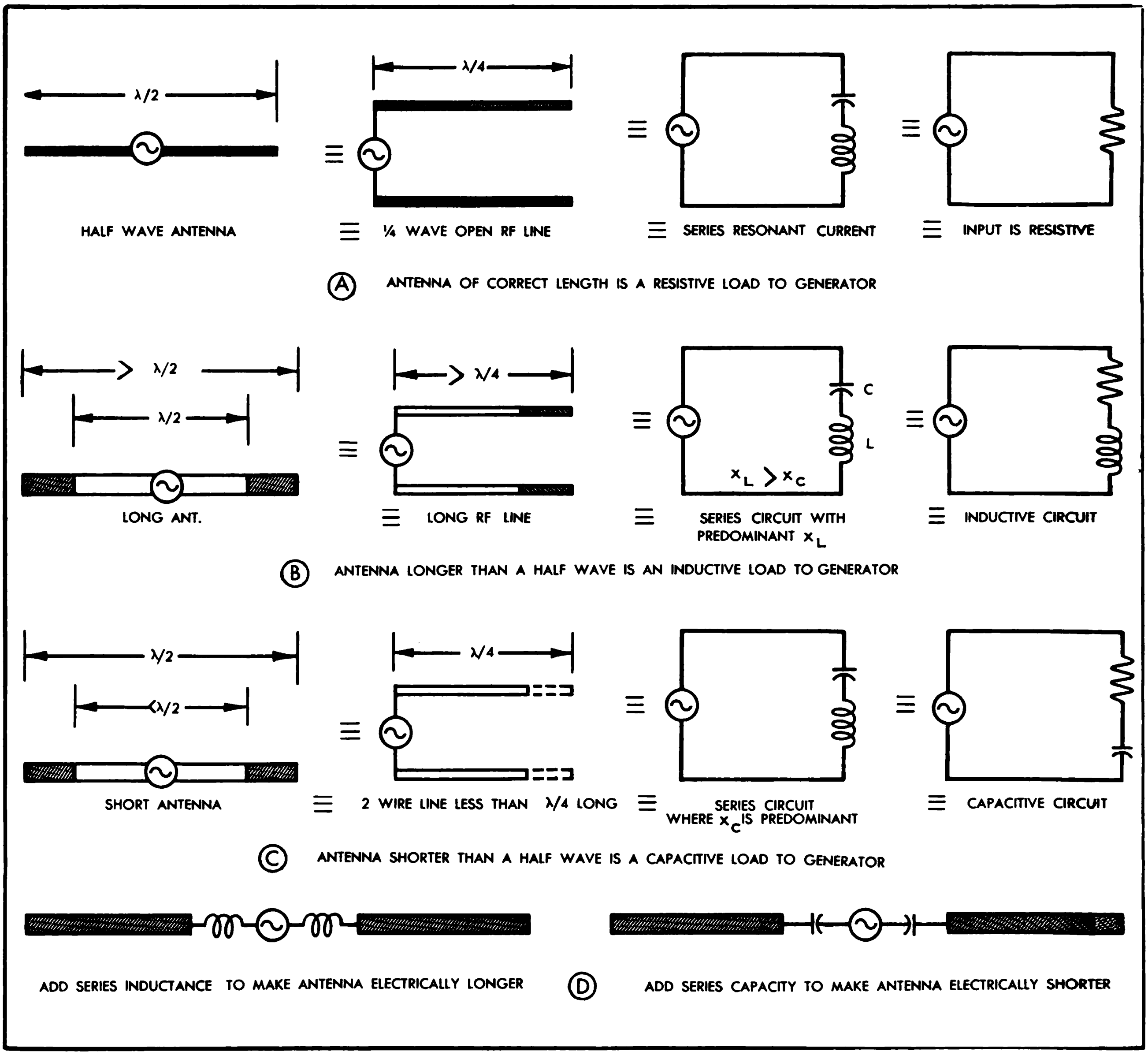

λ/2
λ/4
HALF WAVE ANTENNA
≡ ¼ WAVE OPEN RF LINE
≡ SERIES RESONANT CURRENT
≡ INPUT IS RESISTIVE
Ⓐ ANTENNA OF CORRECT LENGTH IS A RESISTIVE LOAD TO GENERATOR
> λ/2
λ/2
> λ/4
C
L
X_L > X_C
LONG ANT.
≡ LONG RF LINE
≡ SERIES CIRCUIT WITH PREDOMINANT X_L
≡ INDUCTIVE CIRCUIT
Ⓑ ANTENNA LONGER THAN A HALF WAVE IS AN INDUCTIVE LOAD TO GENERATOR
λ/2
<λ/2
λ/4
SHORT ANTENNA
≡ 2 WIRE LINE LESS THAN λ/4 LONG
≡ SERIES CIRCUIT WHERE X_C IS PREDOMINANT
≡ CAPACITIVE CIRCUIT
Ⓒ ANTENNA SHORTER THAN A HALF WAVE IS A CAPACITIVE LOAD TO GENERATOR
ADD SERIES INDUCTANCE TO MAKE ANTENNA ELECTRICALLY LONGER
Ⓓ
ADD SERIES CAPACITY TO MAKE ANTENNA ELECTRICALLY SHORTER

Figure 13–7. Effect of Length on Antenna Impedance

equivalent to a quarter-wave r-f line, looking at it from the generator end. Any two-wire line, longer than a quarter-wave (but less than a half-wave), is like a quarter-wave section with an open-circuited section (less than a quarter-wave long) added to it. The open section, which is capacitive, is inverted to inductance at the input terminals by the quarter-wave. In the same manner a slightly long antenna looks inductive.

According to the equivalent lumped circuit at B, the inductance is not entirely balanced by the capacitance. The remedy for correcting the length is either cutting the conducter shorter, or tuning out the inductive reactance by adding capacitance in series.

If the antenna is physically shorter than its resonant length, the input impedance becomes capacitively reactive. The two-wire line shorter than a quarter-wavelength displays capacitance at its input terminals. (See Figure 13–7C.) The correction in this case is either to add inductance in series with the antenna to bring it back to resonance or to add physical length to the antenna.

Increasing Antenna Length

With radar equipment a wide band of frequencies — rather than a single frequency — must be handled by an antenna. This means that an antenna which is designed for the center frequency is short for frequencies in the lower (low frequencies) sidebands, and long for frequencies in the upper (high frequencies) sidebands. The antenna must be corrected for both ong and short conditions at the same time.

Note in Figure 13–8 that each side of an antenna must be less than a quarter-wave (in space) in order to present zero reactance. This agrees with the previous statement that the physical length of the half-wave element must be 5% shorter than the half-wave in space. Two curves are shown. One is for a large diameter antenna, and the other for a small diameter antenna. Note that the larger diameter displays ess reactance at lengths off resonance.

Antenna diameters as great as 1/20 wavelength are found in radar equipment. A large diameter ncreases the capacitance of the antenna. The nductance is decreased for the same resonant frequency. Lower inductance with the same resistance lowers the X_L/R ratio, or Q, of the antenna. A lower Q causes the resonance curve o be broader and gives the antenna a more uniform response to a greater band of frequencies.

IMPEDANCE AT INPUT TERMINALS

In practice an antenna acts as a resonant or near resonant circuit. As a series resonant circuit, its input terminals display zero impedance; as a parallel resonant circuit, it displays infinite impedance. Further, if it is not of the correct length, it has reactance as well as resistance.

Practical antennas, like practical resonant circuits, have losses which must be replaced by the source. These losses make the input impedance of the antenna somewhat resistive. This resistance is a combination of two resistances — the resistance of the conductors themselves, which is increased by the skin effect at high frequencies, and the radiation resistance.

Radiation resistance is a fictitious resistance that dissipates the same amount of power in the form of heat as is actually dissipated as radiated energy. Because of radiation resistance, an antenna allows part of its fields to escape into space. This makes it necessary for additional current to flow into the antenna from the source of power to replace the part of the field that escaped. Therefore, the source must provide the energy lost by radiation. The energy lost is power (P) and its ratio to the square of the current (I) is the radiation resistance, R_r. Mathematically, it is expressed,

$$R_r = \frac{P}{I^2}$$

Radiation resistance and conductor resistance constitute the total input resistance to an antenna. It is possible to make the conductor resistance small in comparison to the radiation resistance by using large diameter, low resistance conductors. It is desirable that the radiation resistance be a major portion of the input resistance. In practice radiation ratios as high as 9:1 are obtainable. This means that an antenna can be 90% efficient as a radiator.

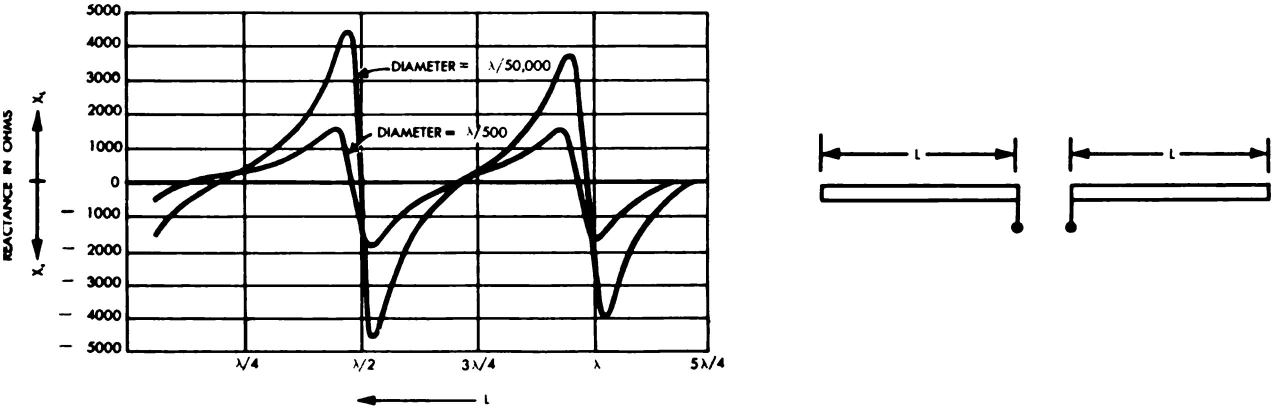


Figure 13–8. Reactance at Input of a Center-Fed Antenna of Arbitrary Length

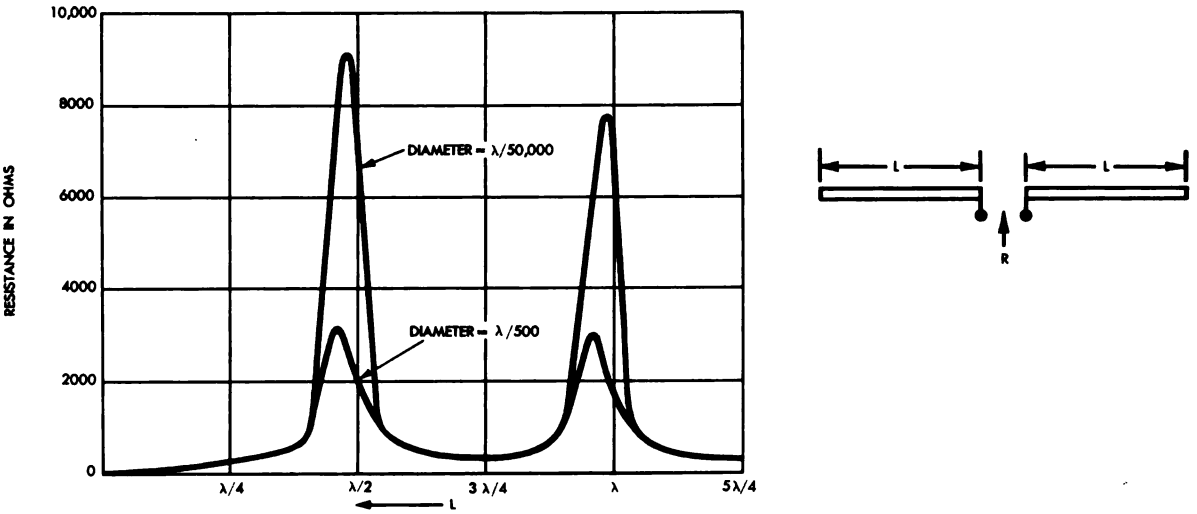


Figure 13–9. Resistance at Input of a Center-Fed Antenna of Arbitrary Length

Figure 13–9 is a graph showing the input resistance at the center of antennas of various lengths. For the half-wave dipole ($L = \lambda/4$), the input is 73 ohms. The input resistance of a full-wave antenna ($L = \lambda/2$) can be as high as 9000 ohms, depending on the diameter of the conductor.

For the lengths of antennas in which the reactance is zero, the input resistance is the input impedance of the antenna. However, for lengths in which the reactance is not zero, the input impedance is the vector sum of the resistance (Figure 13–9) and the reactance at the input of the antenna (Figure 13–8).

Other factors which affect the input impedance of an antenna are nearby conducting objects, such as the earth, and other antennas or the skin of the aircraft. The graphs in Figure 13–8 and Figure 13–9 apply only to a center-fed antenna which is located in free space and not close to any conductor.

POLARIZATION OF AN ELECTROMAGNETIC WAVE

The radiated energy from an antenna is in the form of an expanding sphere. A small section of this sphere is called a *wave front;* it is perpendicular to the direction of travel of the energy. All energy on this surface is in phase. Usually all points on the wave front are at equal distances from the antenna. The farther from the antenna, the less spherical the wave appears. At a considerable distance the wave front can be considered as a plane surface at right angles to the direction of propagation.

Linear Polarization

The radiation field is made up of magnetic and electric lines of force which are always at right angles to each other. Most electromagnetic fields in space are said to be *linearly polarized.* The *direction* of polarization is the direction of the electric vector. That is, if the electric lines of force are horizontal, the wave is said to be *horizontally polarized* (Figure 13–10A), and if the E lines are vertical, the wave is said to be *vertically polarized.* As the electric field is parallel to the axis of the dipole, the antenna is in the plane of polarization. The horizontally-placed antenna in Figure 13–10 produces a horizontally-polarized wave, and the vertically-placed antenna produces a vertically-polarized wave.

For maximum absorption of energy from the electromagnetic fields, it is necessary that a dipole be located in the plane of polarization. This places the conductor at right angles to the magnetic lines of force that are moving through the antenna and parallel to the electric lines.

In general, the polarization of a wave does not change over short distances. Therefore, transmitting and receiving antennas are oriented alike, especially if short distances separate them.

Over long distances, the polarization changes. The change is usually small at low frequencies, ut at high frequencies the change is quite rapid.

With radar transmissions, a received signal is reflected wave from some object. As the polari-ation of the reflected signal varies with the type f object, no set position of the receiving antenna s correct for all returning signals. Where sepa-ate antennas are used, the receiving antenna is enerally polarized in the same direction as the ransmitting antenna.

When the transmitting antenna is close to the round insofar as propagation is concerned, verti-ally-polarized waves cause a greater signal trength along the earth's surface. On the other and, antennas high above the ground should be orizontally polarized to get the greatest signal trength possible to the earth's surface. For this eason most airborne radar systems emit hori-ontally-polarized waves.

ircular Polarization

A factor that determines the effective range of radar set is the manner in which the r-f energy s radiated into space. The radiation of electro-nagnetic energy during normal weather condi-ions gives good target returns. However, during a heavy rainstorm the individual raindrops act as targets and reflect the polarized electromagnetic energy, which may completely illuminate the indicator. This condition is undesirable and is overcome by radiating energy in a type of polarization that is designed to cancel returns from symmetrical targets. This type of polarization is *circular* polarization.

Imagine the electric lines of force in vertical polarization being changed from their fixed position in space and rotated through 360° with every cycle of r-f energy fed to the antenna. Constant amplitude electric lines of force are continuously changing their angular position as they are radiated into space. The resulting effect of this circular motion of the electric field is called circular polarization.

When a reflecting surface is hit by a cycle of r-f energy, currents are induced in the reflector. These induced currents, in turn, cause the reflecting surface to act as a transmitting antenna, radiating a cycle of energy at the same frequency as the original energy, polarized in the same plane but reversed in phase. The intensity of this transmitted (reflected) energy will be dependent

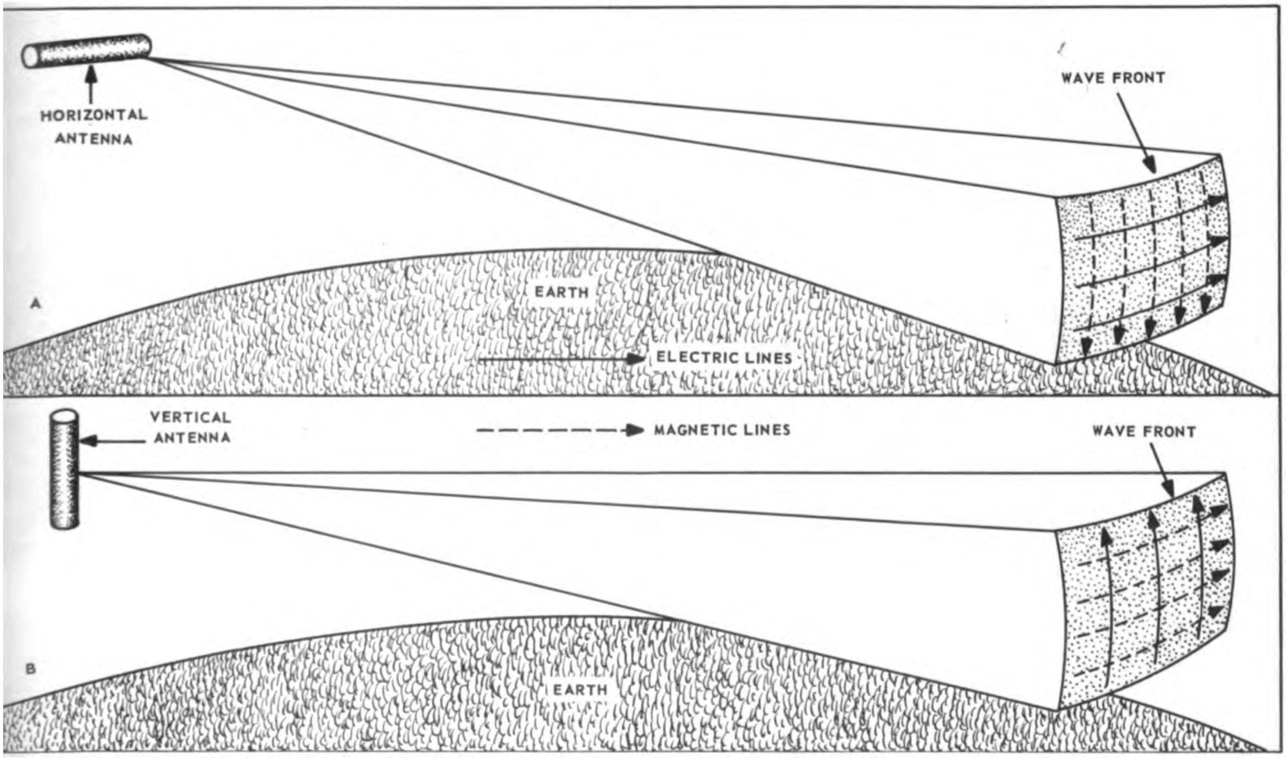


Figure 13–10. Horizontal and Vertical Polarization

on (1) the intensity of the energy that induced the current, (2) the efficiency of the reflecting surface as an antenna at that frequency, and (3) the response of the target to energy polarized in that plane.

A given reflector might be a very good radar target for vertically polarized energy and yet be a very poor radar target for horizontally polarized energy at the same frequency. Symmetrical reflectors, such as spheres, however, would respond to vertically and horizontally polarized energy equally well. In fact, they give equal response to energy polarized in any plane. When the plane of polarization changes, constantly rotating 360° with each cycle radiated, symmetrical targets give constant returns. On the other hand, nonsymmetrical targets, such as aircraft, do not respond equally well to the energy polarized in any plane. When the energy is polarized in one plane, the reflected energy may be very weak. This results in returns of different amplitudes.

Polarizer. The polarizer is designed to reject the circularly polarized waves reflected by raindrops and accept the energy reflected from a nonsymmetrical target (elliptically polarized returns, which are stronger in one plane than the other). Although there are several methods that will give the electromagnetic energy circular polarization, we will explain only a typical system. The piece of equipment that changes either vertical or horizontal polarization to circular polarization (Figure 13–11) is called the polarizer. The polarizer is a circular section of waveguide containing a fixed dielectric slab. The material used for the slab, *rexolite*, is known for its dielectric strength and ability to withstand the high voltages that exist across the waveguide. The polarizing feed is physically an extension of the waveguide.

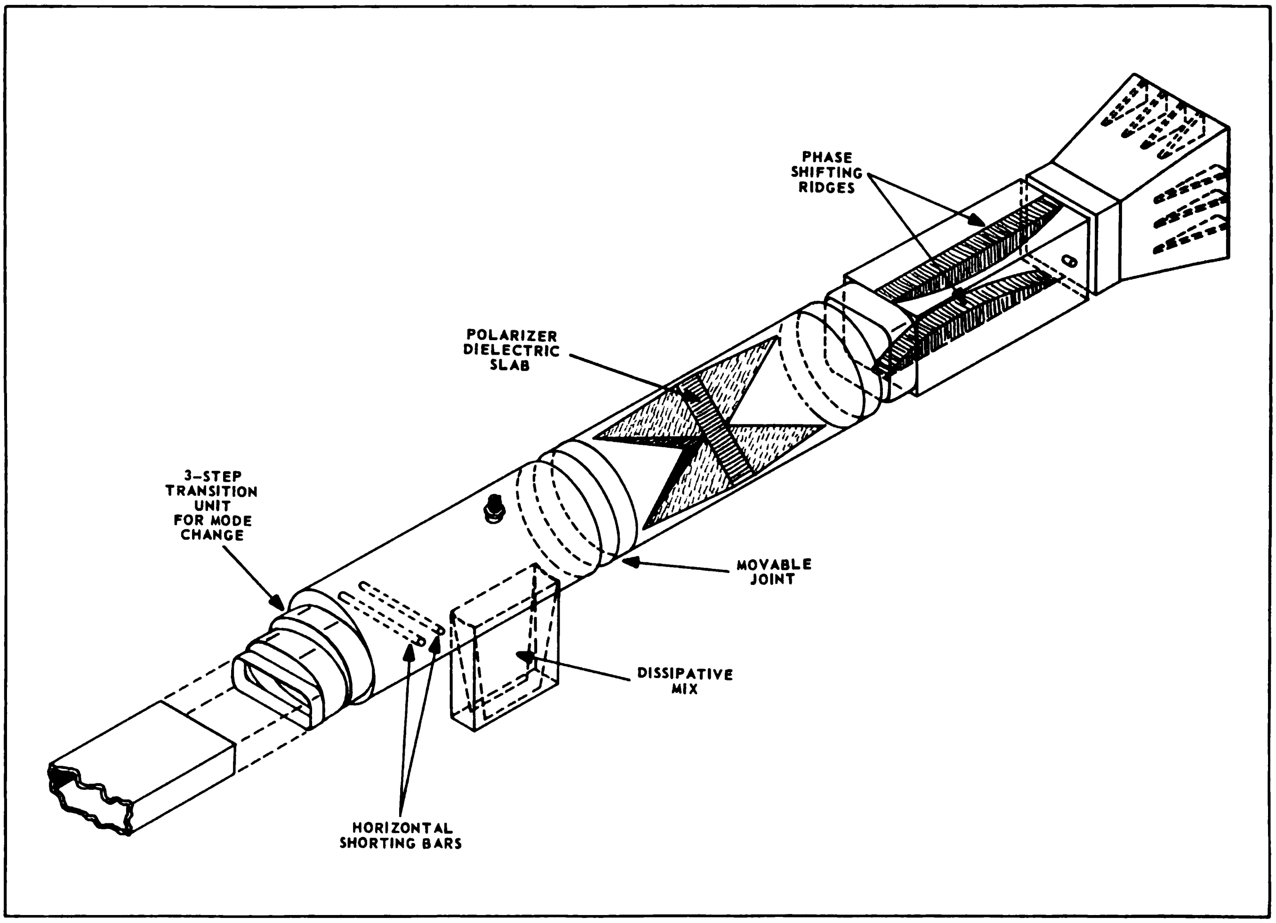


Figure 13–11. Simplified Diagram of Polarizer

A three-step change is made in connecting the ctangular waveguide to the circular waveguide, us converting from the $TE_{0,1}$ mode of operaon to the $TE_{1,1}$ mode. (See Chapter 11, Analogy Waveguide Action by Electric Waves.) To see hy this is true, represent the energy being transitted through the waveguide by a vertical vecr, V_A, as shown in Figure 13–12A. Now break e vector into two perpendicular components, $_B$ and V_C, Figure 13–12B. These two components ay be assumed to represent the two sine waves own. As long as these two sine waves are in phase, they will combine to form the original sine wave represented by vector V_A. Furthermore, they will remain in phase as long as neither of them is intercepted by the dielectric slab. This is the condition for vertical polarization.

The three-step move makes a smooth transition and thus minimizes the reflections that come with rapid changes in the waveguide. The movable section of the guide is the polarizer, and can be rotated 45° to produce either vertically- or circularly-polarized r-f energy. To make the change in the dielectric material as gradual as

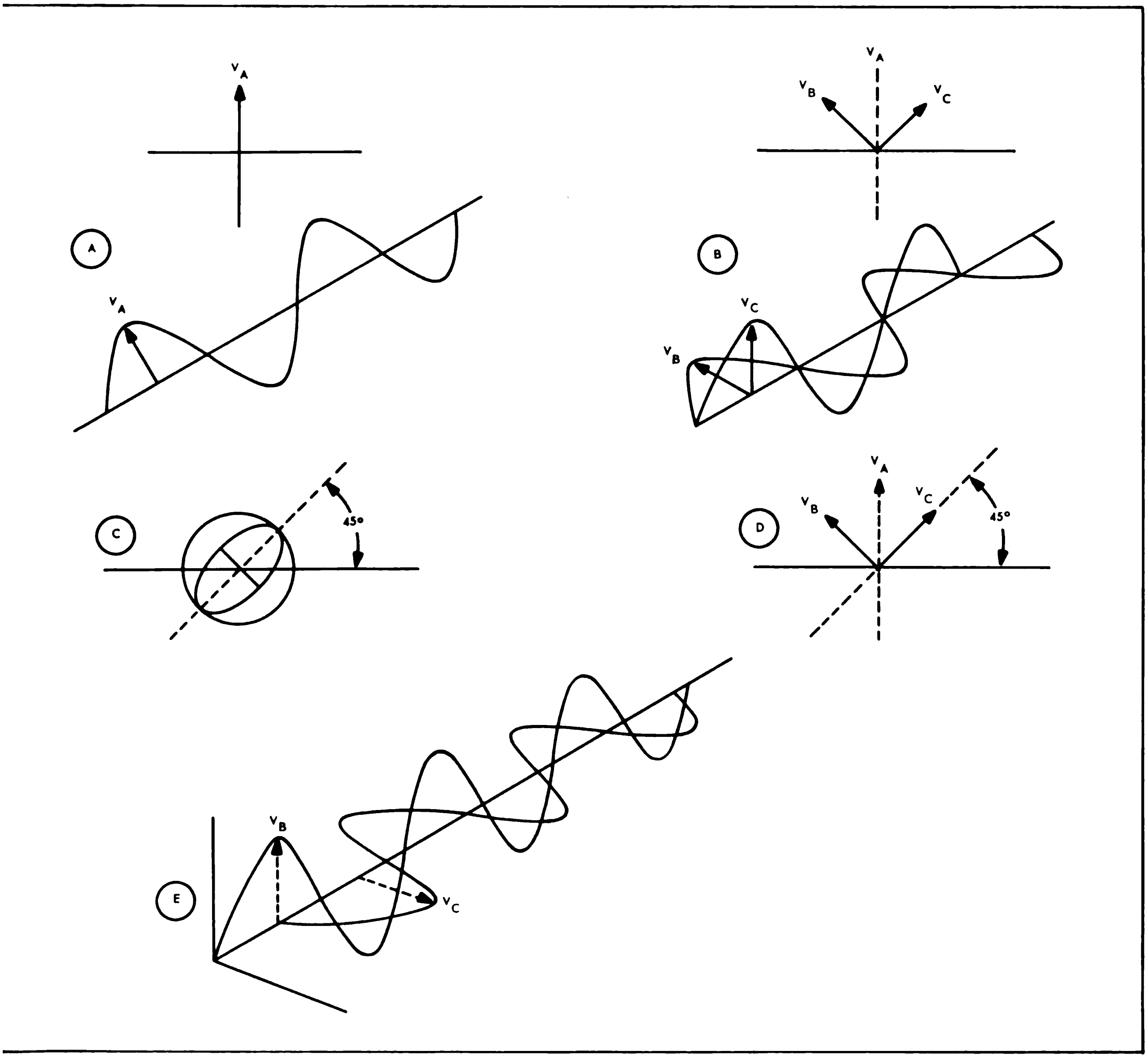


Figure 13–12. Development of Vectors and Phase Shift

possible and obtain minimum reflections, the slab is sloped at each end (Figure 13–11). The position of the slab in the waveguide determines the degree to which the energy is intercepted which, in turn, determines the degree of phase shift.

The dielectric slab is manually controlled for maximum operating efficiency. A reversible motor, through a gear reduction system, can also change the slab from its normally horizontal position. Adjustable mechanical stops ensure proper alignment of the waveguide in either the vertical or circular polarizing position. The polarized energy then passes on through a one-step circular-to-rectangular (square) waveguide transition. As the diameter of the round guide and the distance between the walls of the square waveguide are nearly equal, the transition can easily be made in one step. The square waveguide is constructed with phase-shifting ridges that prevent the circularly-polarized energy from becoming elliptical. The polarized energy passes into a feed horn that feeds the microwave energy to the reflector.

Dielectric Slab Effects. The dielectric slab is normally horizontal, effectively perpendicular to the vertically-polarized electric field. The slab is shaped to have minimum effect on the electric field when in this position, and any small effect that the element does have will act equally on the entire cross section of the electric field. When the dielectric slab is in the horizontal position, it is perpendicular to all lines in the electric field, intercepting every line equally, so that the sine waves represented by vectors V_B and V_C, Figure 13–12, remain in phase.

On the other hand, if circular polarization is desired, one of the vectors, either V_B or V_C, must be intercepted by the dielectric slab. If the slab is rotated to an angle of 45° with the horizontal, as shown in Figure 13–12C, then the component vector V_C will be intercepted by the slab (Figure 13–12D). The operation of the slab is very similar to that of a phase shifter, whose action is based on the fact that the wavelength of energy in a waveguide depends upon the dielectric used within the waveguide.

In the waveguide air is the dielectric, and the wavelength of the propagated energy is shorter than it would be in free space. The slab appears to the energy vector V_C as a more dense dielectric, resulting in a shorter wavelength for the sine wave represented by vector V_C. A shorter wavelength for V_C has the same effect in the waveguide as a phase shift for V_C. The number of degrees of shift in the sine wave represented by V_C is dependent upon the length of the dielectric slab. As a result there are two sinusoidal components which are 90° out of phase with each other. The two quadrature components of the electric field are indicated, showing instantaneous directions and amplitude, in Figure 13–12E.

The resultant field (vector sum) at any instant is an electric field of constant amplitude changing its direction of polarization at a uniform rate through 360° with every cycle of radiated r-f energy. Figure 13–13A illustrates the resultant waveform with the arrows depicting the electric field. Figure 13–13B shows the direction of the E field at a point in space at various instants of time over a complete cycle.

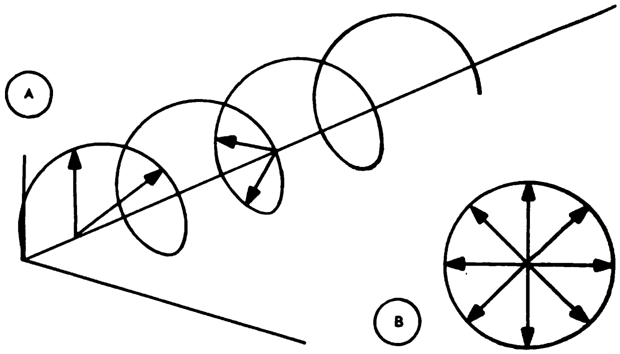


Figure 13–13. Wave Depicting Circular Polarization and Vectors

When the circularly-polarized energy hits a symmetrical target (a raindrop) and returns, the two quadrature components remain equal in amplitude, both having been changed the same amount. This is because the raindrop is essentially a sphere and therefore has a symmetrical surface. As the reflection from the raindrop returns to the polarizer, part of the energy (vector V_C) is again intercepted by the dielectric slab. Once again the sine wave represented by V_C is shifted 90°. Since it had already been shifted 90° during the transmit cycle, this wave is now 180° out of phase with the sine wave vector as represented by V_C in Figure 13–12D. Figure 13–14A represents the return vectors and Figure 13–14B the sine waves after returning

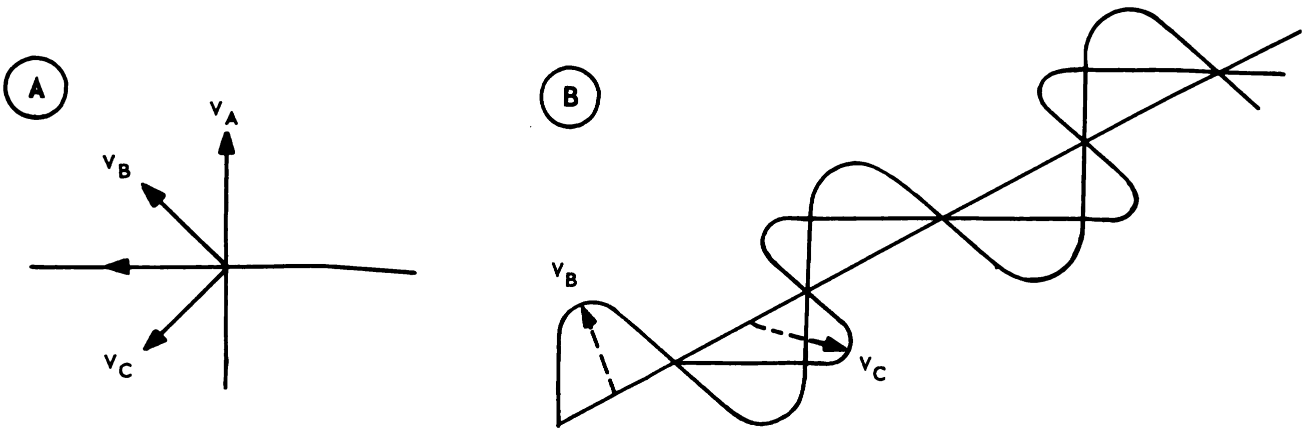


Figure 13–14. Return Phase Shift for V_C

through the polarizer. Note that the resultant energy is now horizontally polarized. Horizontal shorting bars (Figure 13–11) cancel these returns, and the power is dissipated in the filter load (Figure 13–11).

When the circularly-polarized energy strikes a target that is not spherical, the results are different. Perhaps it will fail to reflect all of the energy represented by the vector V_C. In this case the sine wave resulting from vector V_C will have a different amplitude from that of the sine wave resulting from vector V_A. The resultant will have a vertical component which, as the energy passes through the polarizer, will *not* be shorted by the horizontal shorting bars but instead will pass on to the receiver. This is illustrated in Figure 13–15A with V_X being the vertical component. This is called elliptical polarization. Where the magnitude of the electric vector changes over the cycle, the wave is said to be *elliptically polarized.* Figure 13–15B illustrates the electric field at various instants of time over a complete cycle. Figure 13–15C illustrates the condition where the vertical field is maximum at 45°.

Tests have proven that returns from dense rain clouds can be reduced 30 db by using circular polarization. While returns from the nonsymmetrical targets are weaker, which is a radar range factor when circular polarization is used, the elimination of most interference from heavy rain storms has proved the positive value of the circular polarization feature in search radars.

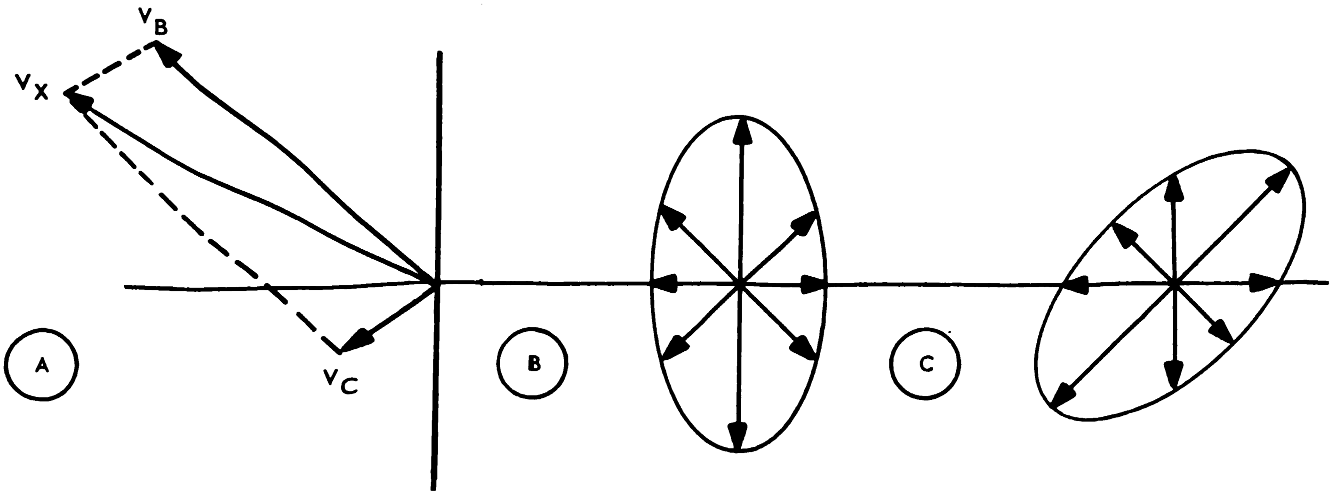


Figure 13–15. Vertical Component of Return Wave Elliptically Polarized

DIRECTIONAL CHARACTERISTICS

If you measured the radiation from a single point in space, you would find that this point radiates equally well in all directions. Since the strength of the radiated energy varies inversely as the distance, if all the points where the energy was of the same strength were plotted, the points would form a sphere, with the radiating point at the center.

If radiation occurs from more than one point in space, the radiated signal from each point will add vectorially to produce the total radiation strength.

Note the two points in space in Figure 13–16. Each point is radiating a similar field in all directions. Concentric circles are shown around each point. The circles represent positive and negative peaks of the radiation field. The solid lines are positive peaks. For simplicity the points are placed a half-wave apart. The picture is stopped just as each point is ready to generate a negative peak.

Note the peaks as they cross the vertical line YY′. Adding the two fields there doubles the field strength. Next observe the peaks along the line XX′. Wherever a positive peak from one point appears, a negative peak from the other also appears. This means that cancellation occurs along the line XX′.

The radiation from a pair of points is directional. Most of the energy is detected along YY′, while little or no energy is detected along XX′. The strength at angles between X and Y varies as the cosine of the angle.

When you apply these facts to a half-wave antenna, you can look at the conductor as a series of points arranged in a straight line. The radiation from points equally spaced from the center will add in directions perpendicular to a line through them, but will cancel along the line through them. A plot of all points of equal strength would produce a figure which is doughnut-shaped. This three-dimensional figure is shown in Figure 13–17. Note that a cross section resembles two ellipses which are adjacent to one another.

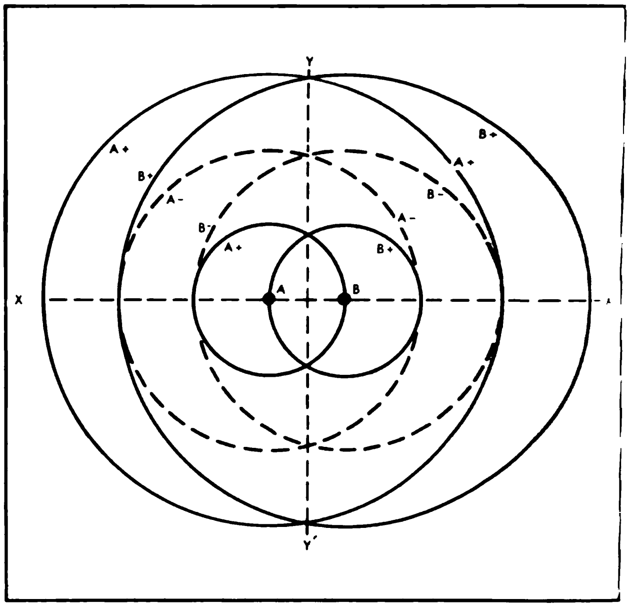


Figure 13–16. Vector Addition of Electromagnetic Energy from Points A & B

The cross-sectional view is called a *polar diagram*. This sort of diagram is standard for delineating antenna performance on paper.

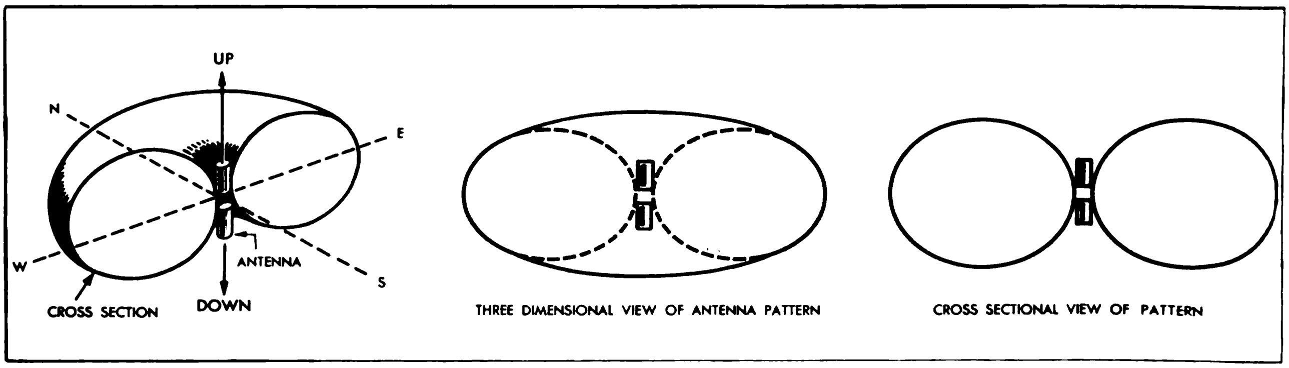
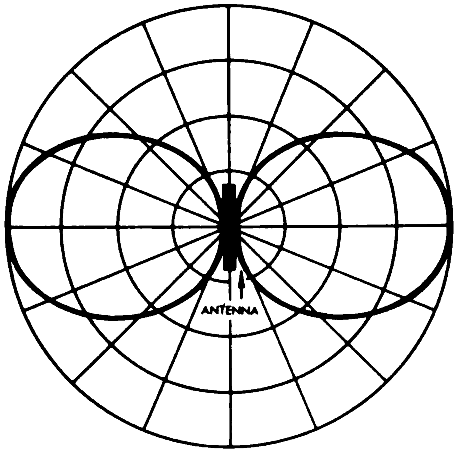

UP
N
E
W
S
ANTENNA
CROSS SECTION
DOWN
THREE DIMENSIONAL VIEW OF ANTENNA PATTERN
CROSS SECTIONAL VIEW OF PATTERN

Figure 13–17. Cross-Sectional View of Vector Addition Figure

Constructing Polar Diagram

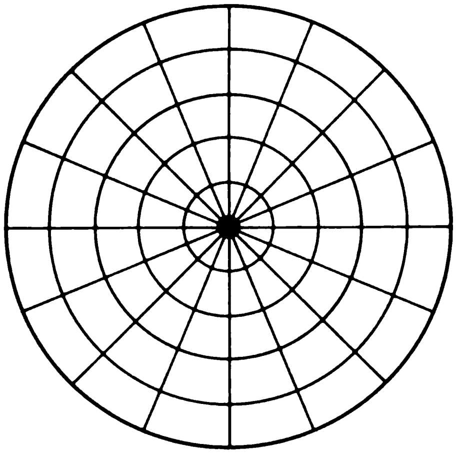
There are two methods of constructing a polar diagram. One method consists of measuring the field strength at the same distance in all directions from the antenna, and plotting the angle against field strength on polar coordinate paper. The other method consists of finding the points where the field strength is the same, and plotting the angle against distance on polar coordinate paper. Both types of measurement produce identical polar diagrams.

Note the polar diagrams for the dipole in Figure 13–18. Diagram A holds good for any plane containing the antenna. For the plane perpendicular to the half-wave antenna, the antenna forms the single point which was derived previously. The polar diagram for this point becomes a circle with the point at the center, as in diagram B. Thus, the simple dipole is bidirectional in any plane containing the antenna, but nondirectional in the plane perpendicular to the antenna.



POLAR DIAGRAM FOR HALF-WAVE ANTENNA IN ANY PLANE CONTAINING THE ANTENNA

A



POLAR DIAGRAM FOR HALF-WAVE ANTENNA IN PLANE PERPENDICULAR TO ANTENNA

B

Figure 13–18. Polar Diagram of Dipole

Field Strength

The strength of a radiation field is called *field strength* and is measured in units called *volts-per-meter*. One volt-per-meter is defined as the field strength that will induce one volt in a conductor one meter long. As field strengths of a volt-per-meter are seldom encountered, weaker fields are measured in *microvolts-per-meter* or *millivolts-per-meter*.

IMAGE ANTENNAS

The preceding discussion dealt with antennas that are isolated from any conductor. However, in all practical cases, there are usually conductors somewhere near the antenna. The effect of the conductors is often undesirable but can sometimes provide advantages.

Theory

The effect of a nearby conductor is illustrated diagrammatically in Figure 13–19A. A real antenna is shown perpendicular to a horizontal per-

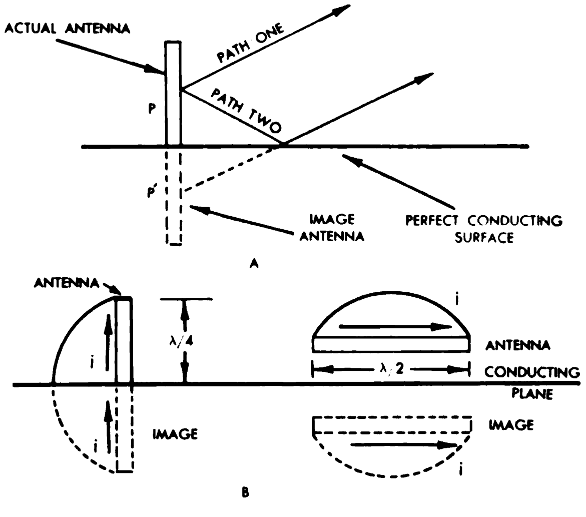


Figure 13–19. Image Antennas

fectly conducting plane. When a field is radiated, energy will be sent out in all directions. There are two possible paths for the field to take. Path one starts out directly toward a distant point. The energy in path two starts out toward the conducting plane. Upon striking the perfect conductor, it is reflected in reverse phase. (This is analogous to an incident wave being reflected in reverse phase from the shorted end of an r-f line.)

As with light waves, the angle of reflection is the same as the angle of incidence. The reflected wave also proceeds toward some point in space. The sum of the two waves makes up the total wave at any point in space. So far as the action in the conducting plane is concerned, it can be replaced by another antenna which is a mirror image of the actual antenna. The reflected wave can be assumed to have originated at point P′ on the image antenna.

In general, current in vertical image antennas flows in the same direction as in actual antennas, while in horizontal antennas the current flows in opposite directions. Note in Figure 13–19B that the combination of the real and image antennas for a vertical quarter-wave makes a dipole.

Marconi Antenna

A quarter-wave grounded antenna is a common type of grounded antenna. This type is called a *Marconi* antenna, as contrasted with the half-wave (ungrounded dipole) antenna, which is called a *Hertz* antenna. Note the amplitude of the standing waves of current and voltage on the Marconi antenna shown in Figure 13–20A. Note also the similarity to the half-wave element when the image is included.

In the Marconi antenna the vertical field strength pattern (polar diagram) shown in Figure 13–20B is the same as that of the half-wave element, except that the conducting plane cuts it off at the center. The image is only effective above the plane because no energy penetrates the conducting plane.

In the horizontal polar diagram in Figure 13–20C the vertical field strength pattern can be rotated with the antenna as the axis to form the horizontal polar diagram of a Marconi. It is nondirectional in a plane perpendicular to the length of the antenna.

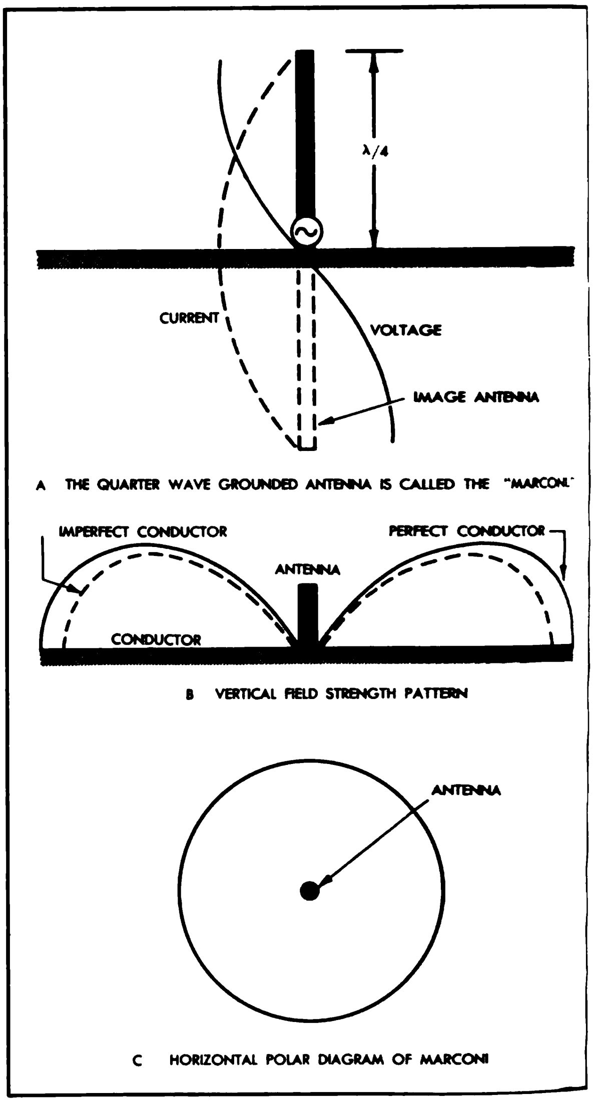

λ/4
CURRENT
VOLTAGE
IMAGE ANTENNA
A THE QUARTER WAVE GROUNDED ANTENNA IS CALLED THE "MARCONI"
IMPERFECT CONDUCTOR
PERFECT CONDUCTOR
ANTENNA
CONDUCTOR
B VERTICAL FIELD STRENGTH PATTERN
ANTENNA
C HORIZONTAL POLAR DIAGRAM OF MARCONI

Figure 13–20. Marconi Antenna

The input impedance to the Marconi is approximately 37 ohms when the antenna is fed at its base as illustrated in Figure 13–20A. In addition a quarter-wave Marconi is resonant and displays zero reactance just like a half-wave antenna.

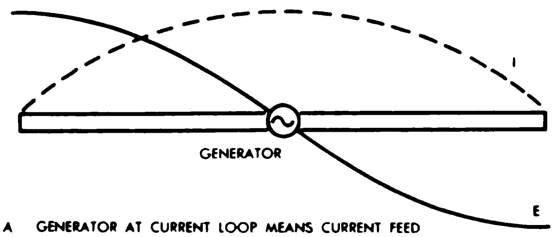
This discussion has assumed that the conducting plane is a perfect conductor. If it is not a perfect conductor, as is the usual case, some of the conditions just discussed must be altered. The principal alteration is the reduction in size of the polar diagram. This results in decreased signal strength from the antenna (Figure 13–20B). The conducting plane is usually the skin of the aircraft where airborne equipment is concerned; with ground equipment it is the earth's surface, or an artificial ground called a *counterpoise.*

Other types of conductors that might be near an antenna are the aircraft tail pieces, the aircraft wing, or steel antenna towers. As the radiation field passes any conductor, currents are induced in the conductors. These currents vary at the same radio frequency and make the conductor itself a radiator. In other words, when the conductor is excited it reradiates, creating a radiation field of its own. This is the way in which radar "echoes" are produced.

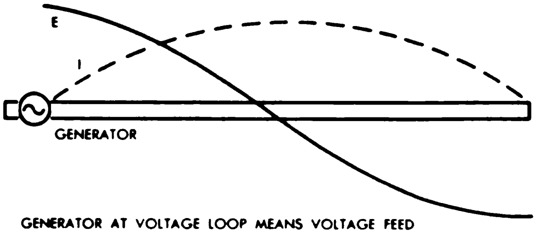
When the radiation field of the conductor is strong, it will change the radiation pattern of the original antenna in two ways. First, I^2R losses in the conductor absorb power from the radiation field. This reduces the field strength in areas beyond the conductor. Second, the reradiated field combines with the antenna radiation field producing random phase relationships. If they occur in phase, a strong resultant field will be present in the area. If they are out of phase, the field strength will be low. The presence of conductors is undesirable unless they are deliberately placed near the antenna to distort the field strength pattern. This is discussed later under *arrays*.

INTRODUCING ENERGY TO THE ANTENNA

Energy may be fed to an antenna in a variety of ways; the most common are shown in Figure 13–21. When the excitation energy is introduced to the antenna at the point of high circulating currents, the antenna must be *current-fed*. When the energy is introduced at the point of maximum voltage, the antenna must be *voltage-fed*.



A GENERATOR AT CURRENT LOOP MEANS CURRENT FEED



B GENERATOR AT VOLTAGE LOOP MEANS VOLTAGE FEED

Figure 13–21. Current and Voltage Feed

It is seldom possible to connect a generator directly to an antenna. Instead, it is necessary to use r-f lines to carry the energy from the generator to the antenna. The r-f lines which carry the excitation energy may be resonant lines, nonresonant lines, or a combination of both.

Feeding Antennas with Resonant Lines

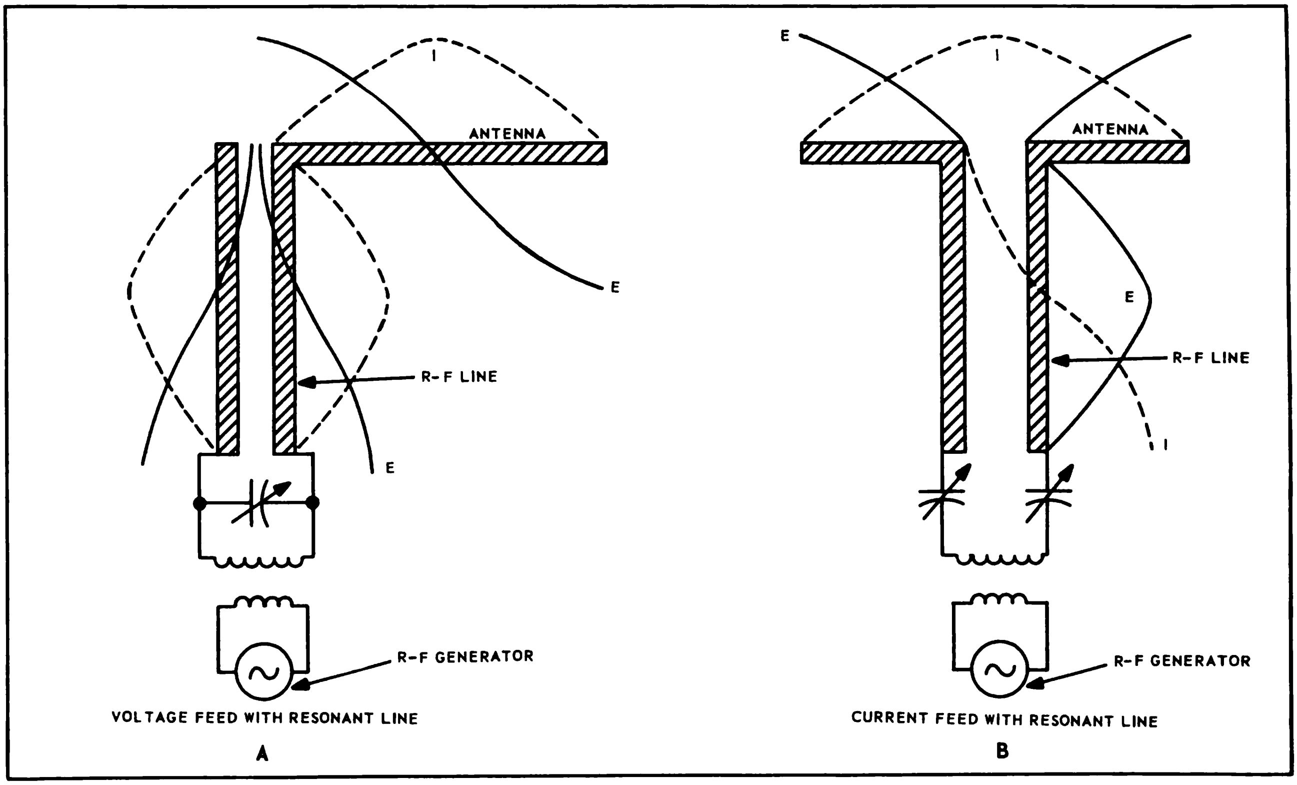
In a voltage-fed half-wave antenna that is excited by a resonant r-f line, as shown in Figure 13–22A, the end of the antenna is connected to one side of the line. Voltage changes at that point excite the antenna into oscillation. The impedance at the end of the antenna is high, since there the voltage is high and the current is low. The r-f line shown is a half-wavelength two-wire line. The impedance of such a line is usually low so that the high impedance at which it is terminated will set up standing waves on it. In addition, it is always a multiple of half-wavelengths electrically in order that it will act as a resonant circuit. This makes the input to the line also a high impedance. A parallel resonant circuit is used to develop a high voltage across this high input impedance. Small irregularities in line lengths can be compensated for by tuning the parallel resonant circuit at the input.

In the current-fed antenna with a resonant line shown in Figure 13–22B, the r-f line is connected to the center of the antenna. This antenna has a very low impedance at the center and, like the voltage-fed antenna, has standing waves on it. It also is a multiple of half-wavelengths and is a resonant circuit. Making it exactly a half-wavelength causes the impedance at the sending (generator) end to be low. A series resonant circuit is used to develop the high current necessary to excite the line. Adjusting the capacitors at the input compensates for irregularities in line and in antenna length.

Although these examples of feeding antennas are simple ones, the principles described apply to antennas and to lines of any length — providing both are of resonant length.

The line which is connected to the antenna can be a two-wire or a coaxial line. In radar equipment the coaxial line is preferred.

One advantage of the resonant line is that it makes matching devices unnecessary. In addition, it makes it possible to compensate for incorrect antenna lengths by transmitter tuning. Its disadvantages are increased I^2R losses from



VOLTAGE FEED WITH RESONANT LINE
A

CURRENT FEED WITH RESONANT LINE
B

Figure 13–22. Feeding Antennas with Resonant Lines

high standing waves of current on the resonant feed lines, increased possibility of arc-over between lines due to standing waves of voltage, very critical length, and production of radiation fields by the two-wire line because of the standing waves on it. Radiation fields are undesirable because they distort the normal radiation pattern of the antenna.

Feeding Antennas with Nonresonant Lines

Figure 13–23 shows the excitation of a half-wave antenna by nonresonant lines. As the input to the center of the antenna at A is 73 ohms and as a coaxial cable with the highest Q has a characteristic impedance of 73 ohms, the most common method of feeding this antenna is through a coaxial cable connected directly to the center of the antenna. This method of connection produces no standing waves on the line (coaxial cable) when the generator is matched to the line. Coupling to the generator is usually made, as you see at A, through a simple untuned transformer secondary. Another means of efficient transfer of energy to the antenna can be accomplished by a twisted pair line as shown at B. In a twisted pair the impedance is about 70 ohms. It, like the coaxial cable, provides a good impedance match.

When a line does not match the impedance of the antenna, it is necessary to use special impedance matching devices. Any of the impedance matching r-f lines discussed in Chapter 10 are adequate for this purpose. An example of a type of impedance matching device is the delta match shown at C. It gets its name because the wires form the Greek letter Delta (Δ) at the antenna.

The delta match is used to match a line having more than 73 ohms impedance to the center of an antenna. The line in the example shown is a 600-ohm line. The connections from the line are spread apart at the antenna until the impedance between connections is equal to 600 ohms.

As the connections are moved away from the center in the delta match, the voltage becomes

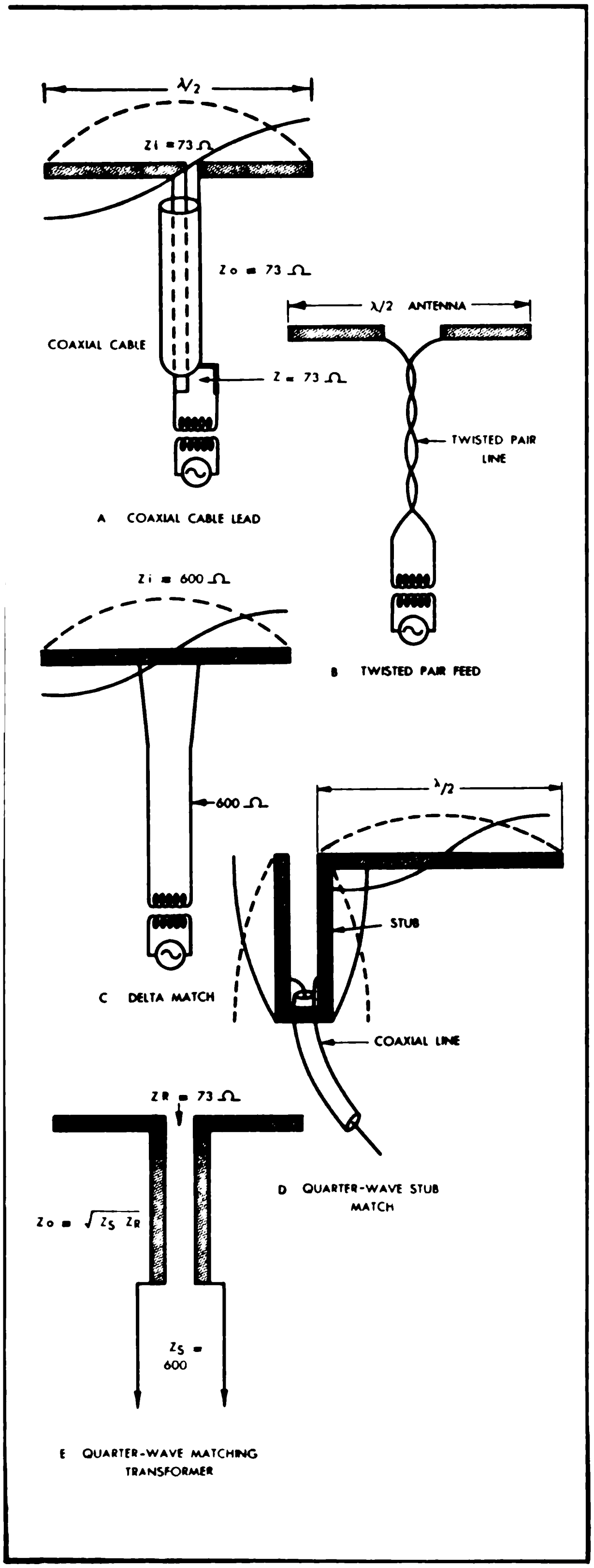


Figure 13–23. Feeding Antennas with Nonresonant Lines

higher and the current becomes lower. The ratio between these, known as the E/I ratio, increases from a very low value at the center to several thousand ohms at the ends. The two connections are set at the point where this ratio is equal to 600 ohms. This is the same as matching various impedances along a shorted quarter-wave section of line.

Another method of matching impedance is the quarter-wave stub match shown in Figure 13–23D. In the quarter-wave stub match the high impedance at the end of the antenna matches the open end of the stub. The impedance on the stub varies from zero at the short circuit to several thousand ohms at the open end. This makes it possible to connect a 70-ohm coaxial line a short distance from the shorted end at the 70-ohm point. It is possible to match almost any impedance along the length of the stub.

Still another impedance matching device is the quarter-wave transformer (Figure 13–23E). This device is used for matching the low impedance at the antenna to a line of higher impedance. In the example shown, the characteristic impedance, Z_0, of the quarter-wave section is 210 ohms. With this matching device standing waves will exist on the antenna and the quarter-wave section but not on the 600-ohm line itself.

Nonresonant lines are characterized by small radiation, low voltage at all points for any given power, and noncritical length. In addition they are preferred for the transfer of power over distances greater than one or two wavelengths.

Exciting Antennas by Radiation Fields

The antenna may be excited either by a direct connection or by radiation (induction fields) from nearby antennas. A half-wave antenna driven by a directly connected generator is shown in Figure 13–24. The flux from the driven antenna expands and cuts the other element and induces in it a current that varies at the exciting frequency of the driven antennas and which, in turn, causes additional radiation fields. When the antenna elements are properly designed, the radiation field resulting from the induced current will be nearly as strong as the original fields.

In addition to being excited by a nearby antenna, an antenna may also be excited by radiation fields from a distant transmitting antenna. This takes place in any receiving antenna. In a

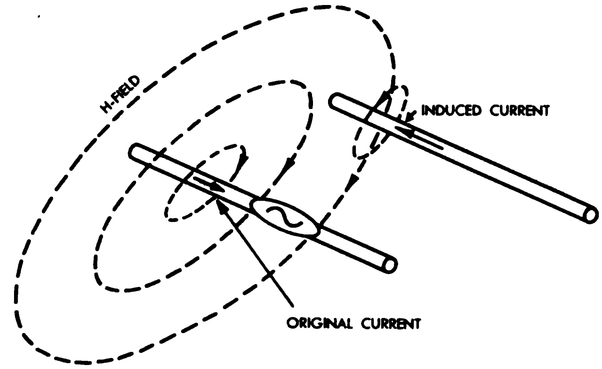


Figure 13–24. Exciting Antenna by Radiation Fields from Nearby Antenna

receiving antenna the current that is conducted to the receiver is the result of current induced by radiation fields generated by the transmitting antenna.

ANTENNA ARRAYS

There are uses for antennas where it is desirable that the radiation be absolutely nondirectional, and other uses where the antenna must be very sharply directional. In the latter case the antenna system usually consists of two or more simple half-wave elements so spaced that the fields from the elements add in some directions and cancel in others. The set of antenna elements is called an *antenna array*. There are two types of antenna arrays — ***driven*** and *parasitic* arrays.

Driven Arrays

The driven array consists of two or more elements with all elements connected to the generator. They may be divided into three basic types — the *broadside* array, the ***end-fire*** array, and the *colinear* array.

Broadside Array. When two or more half-wave elements placed one half-wave apart and parallel to each other are excited in phase, Figure 13–25A, most of the radiation is broadside to the plane of the elements. The array is termed a *broadside* array.

While the radiation around each element is nondirectional (looking at the antennas end on), these fields combine to form a directional field. When the wavefront from antenna 1 reaches an-

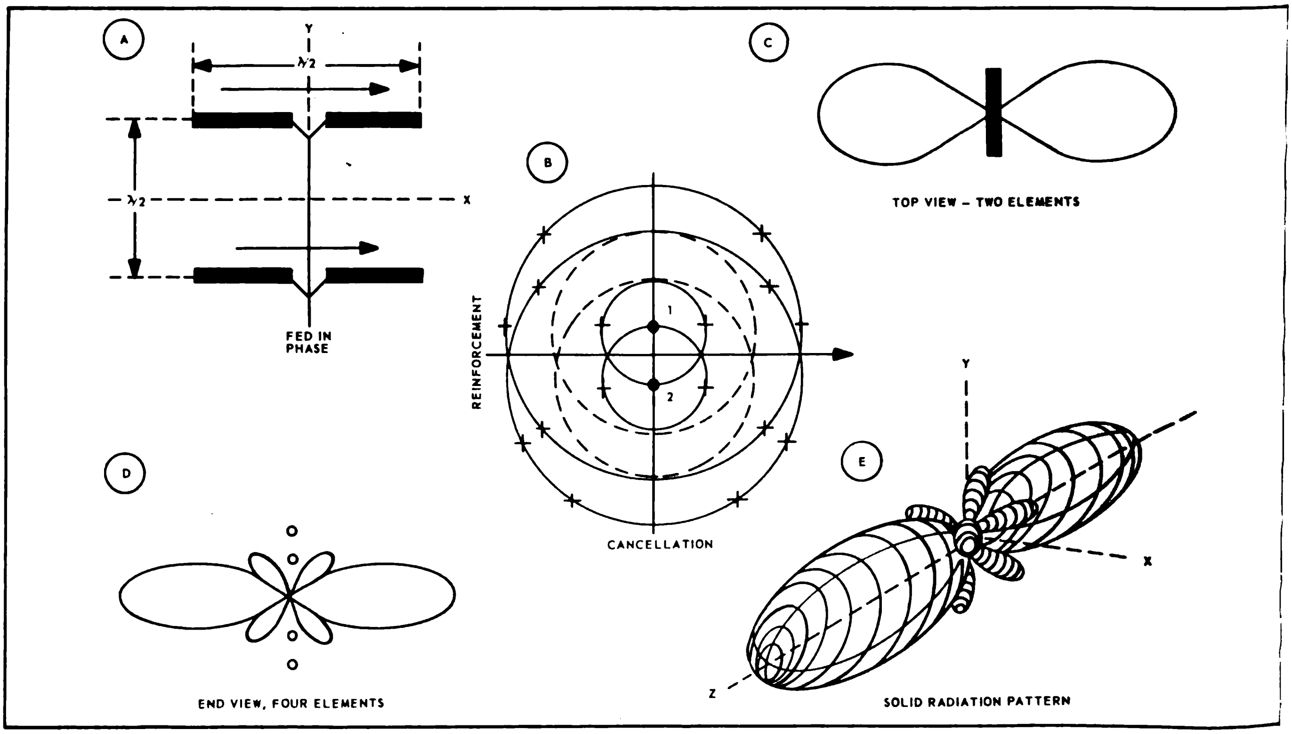


Figure 13–25. Broadside Array

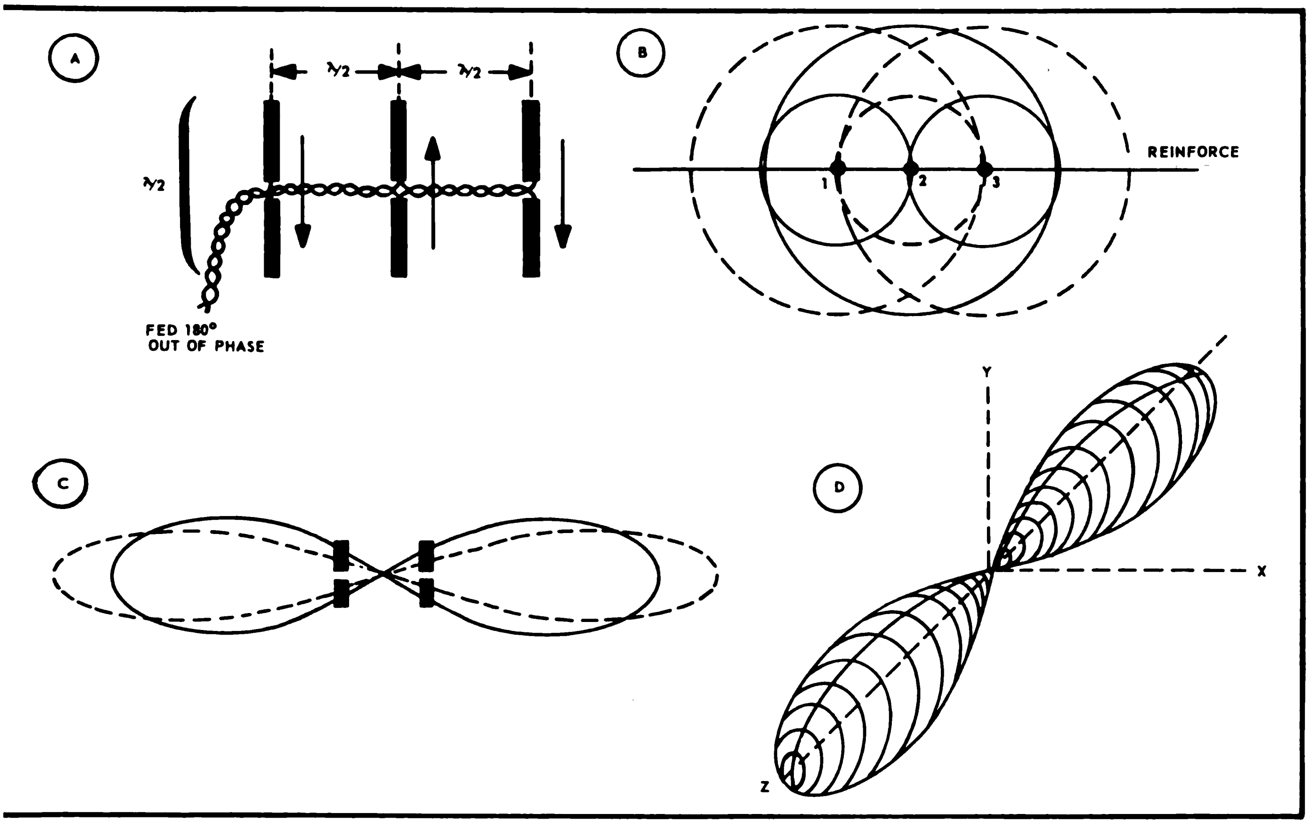


Figure 13–26. End-Fire Array

tenna 2, it is 180° out of phase with the energy now leaving antenna 2. As shown in Figure 13–25B, these fields cancel in the plane containing the antennas. However, in the plane at right angles to the plane of the antennas and midway between them the fields reinforce. The gain in the broadside direction, or the direction perpendicular to the plane of the antennas (each antenna is also parallel to this plane), is twice the gain of the individual antenna. Figure 13–25C is a view of the radiation pattern looking down on the antennas, arranged one above the other. As more elements are added, the pattern becomes more directional and side lobes appear (Figure 13–25D and Figure 13–25E).

END-FIRE ARRAY. If half-wave antenna elements spaced a half-wavelength apart and parallel are fed 180° out of phase (Figure 13–26A), the radiation field is strongest at the ends of the row of dipoles. This array is known as an *end-fire* array.

Figure 13–26B shows the wavefronts traveling out from each antenna, looking at the ends of the antennas. Since the antennas are a half-wavelength apart and each is excited 180° out of phase with the adjacent antennas, when the wavefront from one antenna reaches the next antenna, it is in phase with the wave just leaving that antenna. Thus the radiation fields reinforce each other along the axis of the antennas.

From above the radiation pattern appears as shown in Figure 13–26C. The effect of additional elements is shown in Figure 13–26D.

CARDIOID ARRAY. If two dipoles are spaced a quarter-wavelength apart and are excited one-fourth cycle (90°) out of phase, a unidirectional pattern results. In Figure 13–27A the excitation of antenna *a* leads that of antenna *b* by 90°. When the wave from *a* reaches antenna *b*, the wave just leaving *b* will be in phase with it, due to the 90° lag and $\lambda/4$ the first wave has traveled. These will reinforce each other. However, when the wave from antenna *b* reaches antenna *a*, it is 180° out of phase with the wave now leaving antenna *a*, due to the 90° lag in excitation and the quarter-wavelength it has traveled. Waves traveling toward antenna *a*, then, are 180° out

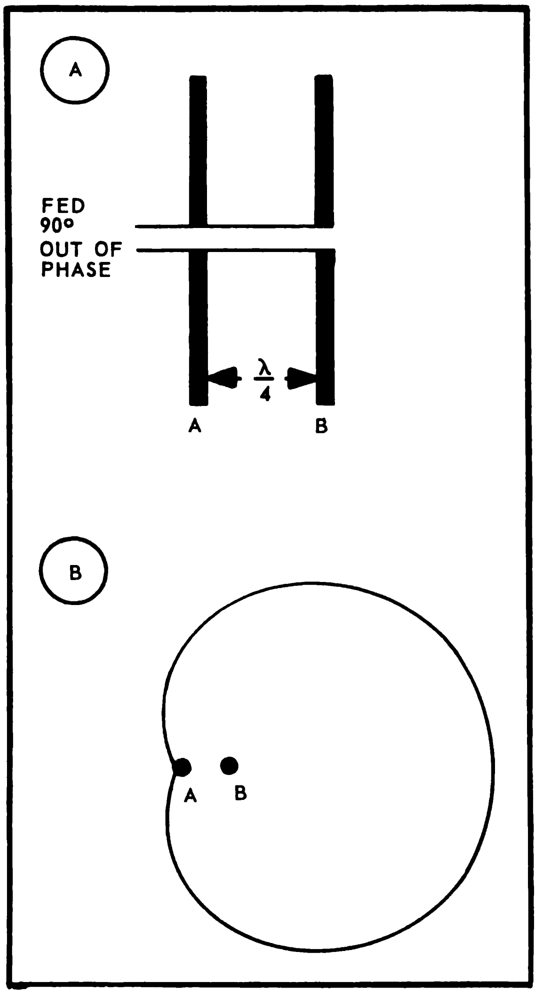


Figure 13–27. Cardioid Array

of phase and cancel out radiation in that direction. The radiation pattern appears to be heart-shaped (Figure 13–27B) so this array is often termed a *cardioid* array. The pattern is unidirectional in the direction of the antenna fed by the *lagging* excitation.

Colinear Array. If two half-wave elements are placed end to end as shown in Figure 13–28A and excited in phase, they form a *colinear* array. Looking at the end of the array the pattern is circular, just as is that of a single dipole. The effect of the colinear array is to squeeze the doughnut pattern into a flat disc (Figure 13–28B). As more elements are added, the disc of radiation becomes flatter and small secondary lobes appear. The radiation pattern of a four-element colinear array is shown in Figure 13–28C.

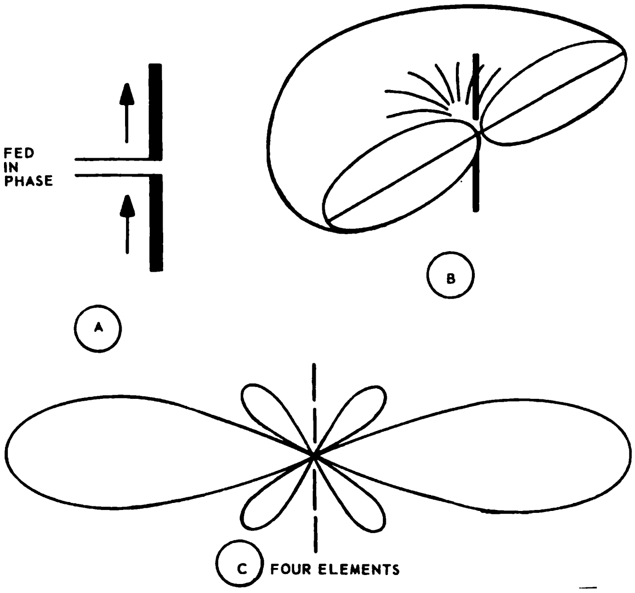


Figure 13–28. Colinear Array

Interconnection of Elements in Arrays

There are many ways to connect the elements in an antenna array to obtain the required phase of excitation. The most convenient method of changing the phase from one element to another element a half-wave distant is with an r-f line. In the broadside array shown in Figure 13–29A one element is excited directly from the transmitter. This element is connected to the other element by a half-wavelength of r-f line. The phase of the voltage along the r-f line is shifted 180° for each half-wavelength. The leads to the second driven element are reversed. This causes another 180° phase shift. The second element is then excited in the same phase as the first.

It is possible to connect any number of elements in this manner. In addition, increasing the number of elements makes the array more directive.

In the end-fire array in Figure 13–29B the elements are also interconnected with r-f lines. But instead of reversing the leads to the elements, they are connected directly to take advantage of the phase shift. With half-wave spacing the interconnecting line will be a half-wave long and will provide a phase shift of 180°. If quarter-wave spacing is used, the interconnecting line will be a quarter-wave long. This arrangement produces a 90° phase shift.

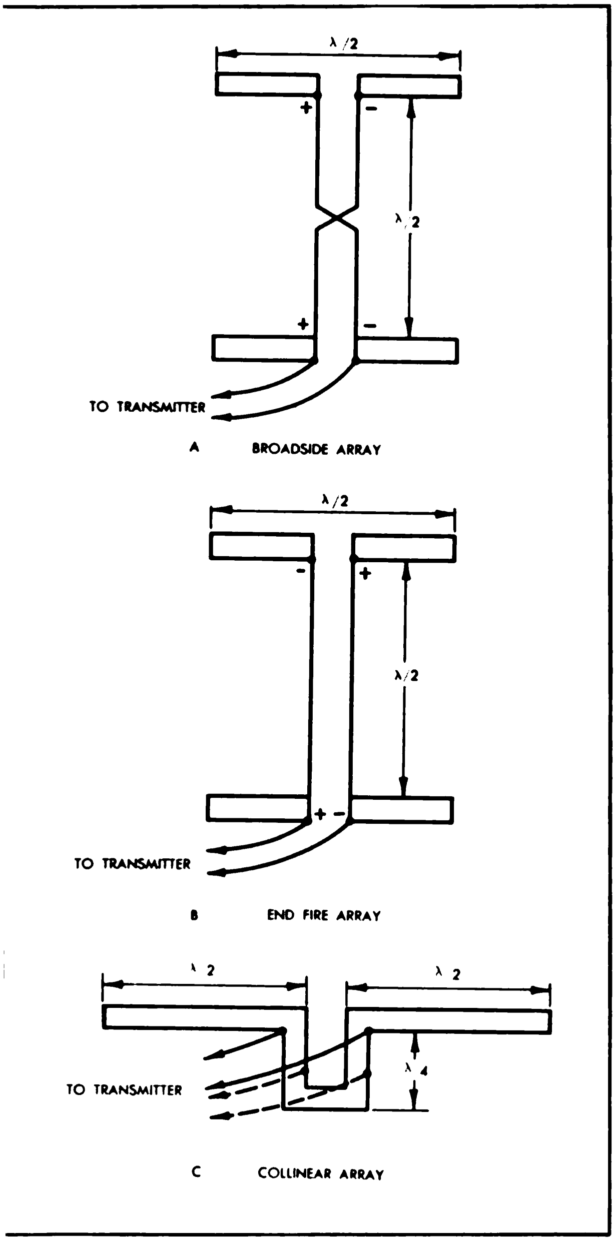


Figure 13–29. Connecting Half-Wave Elements to Obtain Proper Phase Relationship

The usual method for providing the correct phase in a colinear array is shown in Figure 13–29C. Since current direction changes for each half-wavelength of an antenna, it is not possible to connect the half-wave sections together directly. Instead, the half-wave section that carries the current in the wrong direction is folded to form a quarter-wave section of r-f line. This brings together the ends of the sections in which current flows in the same direction.

In terms of voltage, it is necessary that the two antenna sections have voltages that are of opposite polarity at the open end. Further, note that the line to the transmitter is connected at a point of very high impedance. This requires the use of a resonant line. If a nonresonant line is to be used, it must be connected as shown by the dotted lines. In Figure 13–29C the connection near the closed end of the quarter-wave section is at a point of low impedance. Another way of correctly exciting a colinear array is by a waveguide. This method is described later.

Parasitic Array

A parasitic array is an antenna system that consists of two or more elements, one or more of which are excited by the induction and radiation fields produced by the driven element or elements. With such arrays it is possible to obtain highly directional patterns without changing the length of the antenna.

The parasitic elements are placed parallel to the driven half-wave antenna, the spacing depending upon the effect desired. Because of currents set up in the parasitic elements by the induced emf, they, too, radiate energy. At some points the energy reradiated by the parasitic element will be in phase with that radiated directly by the driven element; at other points the fields will be 180° out of phase, and there will be cancellation. The unidirectional pattern can be found by plotting the signal strength measured in all directions about the array. When the radiation pattern is strongest in the direction of the driven element, the parasitic element is called a *reflector.* When the maximum radiation is in the direction of the parasitic element, this element is called a *director.*

The phase difference between the current in the driven element and that in the parasitic element depends upon the distance between the driven element and the parasitic element and the relative length of the parasitic element. If the parasitic element is a half-wavelength, it will act as a pure resistance. If the two elements, both a half-wavelength long, are very close together, the current in the parasitic element will be 180° out of phase with the current in the driven element, as is the case in the secondary and the primary of untuned transformers. This is normal transformer action. It has the same effect as though both elements were excited 180° out of phase.

If the two elements are a quarter-wavelength apart, then the radiated energy reaching the parasitic element will be 90° behind that at the driven element, due to the distance traveled. While the induced current is 180° out of phase with the radiated energy inducing it, it is, in effect, leading the current in the driven element by 90°. It has the same effect as feeding the parasitic element with an excitation leading that of the driven element by 90°. The radiation pattern, then, will effectively be that of Figure 13–27B. Since the radiation is strongest in the direction of the driven element, the parasitic element is called a reflector.

If the two elements are placed closer together, the voltage induced in the parasitic element will be greater. For a reflector, maximum gain over a half-wave antenna alone is obtained with a spacing of 0.15 wavelength; for a director; 0.1 wavelength.

When a driven element and a parasitic element are placed 0.15 wavelength apart, as shown in Figure 13–30A, the induced voltage in R (parasitic element) lags that in the driven element by 0.15 wavelength, or 54°. This induced voltage is that acting at the center of R causing the parasite to be excited as though by a transmission line. If the radiation field due to the induced current is to reach the driven element in phase with the field leaving that element, there must be an additional phase shift. If the reflector (R) is slightly longer than a half-wavelength, it will present an inductance to the transmission line (in this case the induced voltage), and the current will lag the voltage. The amount of lag is varied by adjusting the length of the element. If the lag is properly adjusted, the reradiated voltage from R (Figure 13–30A) reaches A in phase and the radiation pattern will be as shown in the polar diagram.

The field passing from the reflector cuts the driven element and induces a voltage in it. This voltage changes the input current. The input impedance, which is a function of this current, is less than the 73 ohms for the antenna alone. The actual input impedance depends upon the diameter of the elements and the spacing, shape, and length of the reflector. It is usually about 50 ohms.

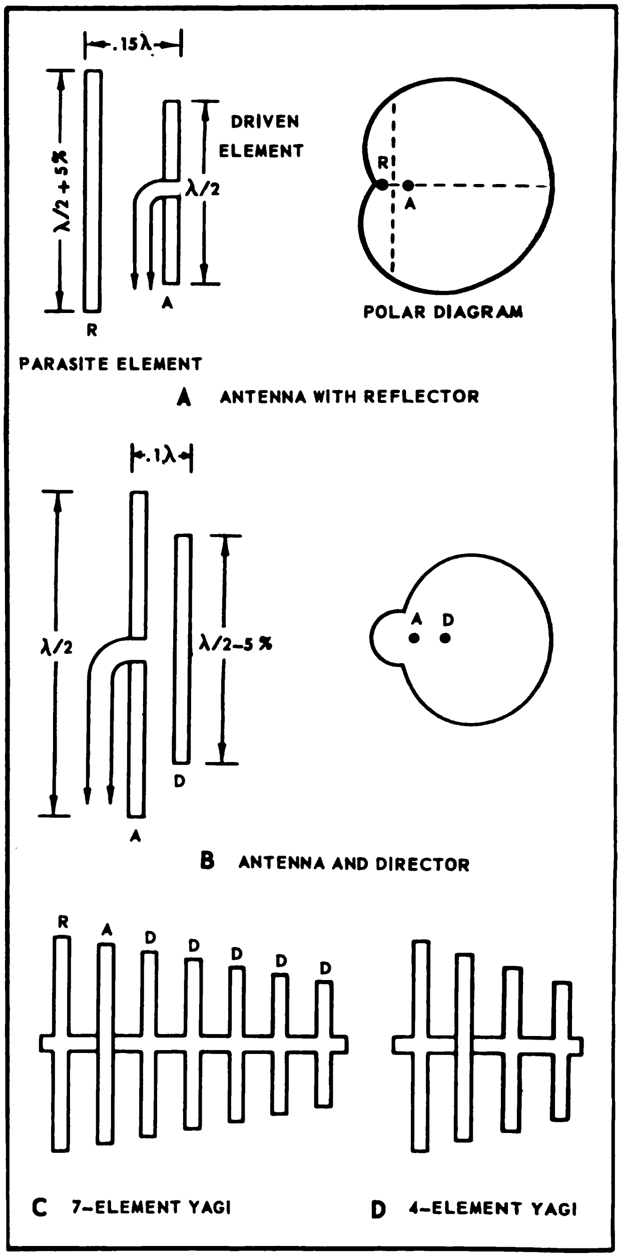


Figure 13–30. Parasitic Array

If the parasitic element is made shorter than the driven element and placed parallel to it with a spacing of 0.1 wavelength or slightly more, it will act as a director (Figure 13–30B). Most of the radiation will be in the direction of the parasitic element.

When this shorter parasitic element is placed 0.1 wavelength from the driven element, the leading effect caused by the capacitive action of the short element makes the reradiated energy from the parasitic element reach the driven element approximately 180° out of phase with the field now leaving the antenna. This will produce cancellation in the direction of the driven element and maximum radiation in the direction of the director. Like the reflector, a director decreases the radiation resistance of the antenna.

If the spacing between the driven element and the director is increased, the director must be cut shorter to provide the proper phase shift; the farther away, the shorter the element.

Directivity

The conventional method for describing the directivity of an antenna array is in terms of either the ratio of the power in the best lobe to the power radiated by a simple half-wave antenna, or the ratio of the power in the direction of the best lobe to the power in the opposite direction. The direction of the best lobe is usually called *forward* direction, while that in the opposite direction is called the *backward* direction. In other words, this second ratio is the power in the forward field to the power in the backward field. For example, the front-to-back ratio of the antenna array illustrated in Figure 13–30B is about 5:1. In decibels, this represents approximately a 7-db gain. The ratio between forward power and power from a single half-wave antenna is about 4 db.

The length and spacing of antenna elements which give maximum forward gain do not provide maximum attenuation in the opposite direction. In order to have the highest front-to-back ratio the tuning or length of the parasitic element must be adjusted. A reflector, for example, is tuned slightly lower or made slightly longer than for maximum gain. Adjusting the director for maximum attenuation of the rear lobe can increase the front-to-back ratio from about 5.5 to 17, with a decrease in the gain (compared to a simple half-wave antenna) of about 1 db.

Comparisons between the more directive types of arrays are made in terms of the beam angle. This is the angle between half-power points in the main lobe. In arrays the half-power points are the points where the electric field strength is 0.707 times as great as that along the axis of the beam. The narrower the beam, the more directive it is considered.

Yagi Array

A comparison of two-element parasitic arrays will show little difference in gain between that using a reflector and the one using a director. However, by combining a driven element with both reflector and director, a more directional array can be built. Such an array is called a *Yagi* array. Yagi arrays are also designed with more than one director. Additional reflectors are not used since there is little radiation field behind the single reflector. Spacings are the same as for single elements — the reflector is usually 0.15 wavelength behind the driven element, and the director 0.1 wavelength ahead of the driven element for maximum gain in the forward direction.

Figures 13–30C and D show two Yagi arrays, one of seven elements, the other of four. The driven element (A) is insulated from the support, but the reflector and directors are all welded to this piece of tubing running parallel to the direction of propagation. Chapter 15 contains more on Yagi arrays.

The gain of parasitic arrays is difficult to calculate or to measure. However, based on ground measurements and actual experience, the average gain for a 3-element Yagi is about 6 to 7 db.

One of the disadvantages of the Yagi array is that tuning of the elements is critical. If the antenna is used for frequencies other than the one for which it is designed, gain, front-to-back ratio, and radiation pattern change completely.

When used with the frequency for which it was designed, the antenna shown in Figure 13–30D has a beam width of about 50°. The 7-element Yagi in Figure 13–30C has a beam width of about 19°.

Folded Dipole

In multielement arrays the input impedance drops as low as 15 ohms. In these arrays special matching devices are needed for matching this low impedance to the higher impedance of most r-f lines. One method for making the match is to use the Delta matching system previously described. Another method, the one most often used in radar equipment, uses the folded dipole as shown in Figure 13–31.

A folded dipole is a full wavelength conductor that is folded to form a half-wave element. A better description is that it consists of a pair of

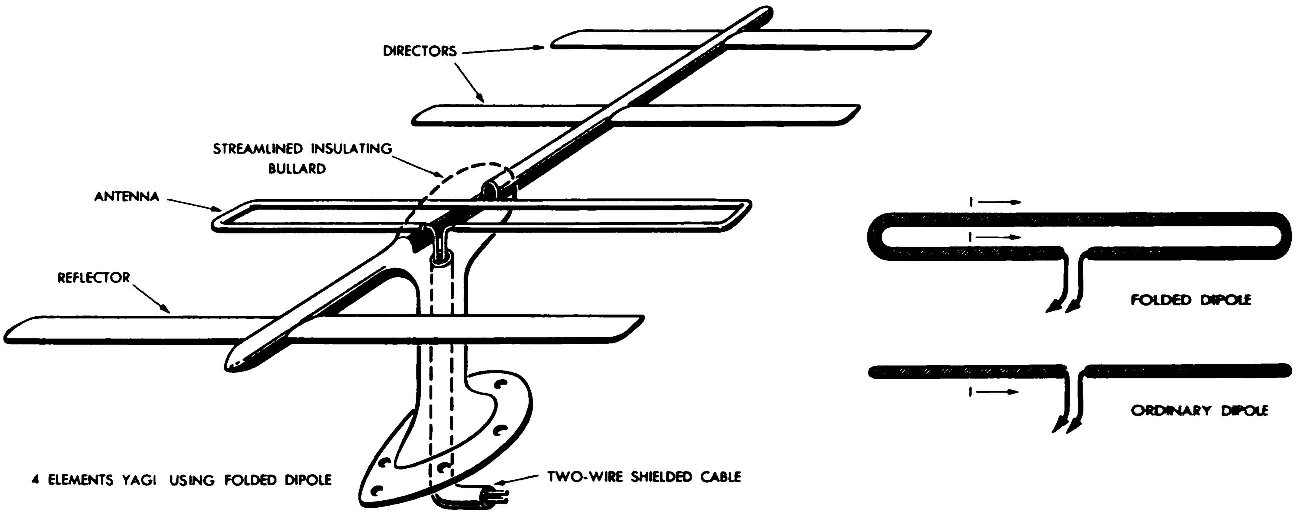


Figure 13–31. Folded Dipole

half-wave elements connected together at the ends. In it the voltage at the ends of each element must be the same. In operation the field from the driven element induces a current in the second element. This current is the *same* as the current in the driven element.

An ordinary dipole with a given current I produces a certain field intensity in space. Due to this field, there is also a certain power density per square meter in space. This power density is produced by the input power P. The relationship between the input resistance, the current, and the input power is expressed by the equation,

$$R = \frac{P}{I^2}$$

Another consideration with the folded dipole is that when the same current I exists in each of the two sections, the field strength in space is doubled. This causes the power density per square meter to increase four times. In turn, the input power must be four times as great. In this case to balance the equation, it is necessary to multiply R by 4 as follows:

$$\frac{4P}{I^2} = 4R$$

As long as each section of a dipole has the same diameter, the input resistance is four times that of the simple (half-wave) dipole. Increasing the diameter of one section makes the increase in impedance still greater. While the input impedance to the driven element of a parasitic array drops to about a fourth of the value of the coaxial cable impedance, the use of a folded dipole increases the impedance by about four times. In this way a good impedance match is effected.

REFLECTORS

Parabolic Reflectors

A spherical waveform spreads out as it travels because the energy travels in a direction perpendicular to the wave front. This produces a pattern that is not too sharp or directive. On the other hand, a plane wave front does not spread out because all the wave front moves forward in the same direction. For a sharply defined radar "beam" we need to change the spherical wave front from the antenna into a plane wave. A parabolic reflector is one way of doing this.

As the transmitting frequencies become higher, the wavelength shorter, radio waves become more like light waves. Microwaves (with wavelengths measured in centimeters) travel in straight lines as do light rays. They can be focused and reflected just as light rays can.

A metallic parabolic reflector is used to focus light rays in an automobile headlight. The light source is placed at the focal point. Light rays striking the parabolic reflector are reflected at angles that result in parallel paths for all the light rays. The same thing can be done with microwaves. While light rays require a highly olished surface, microwaves can be reflected by large metal surface.

In Figure 13–32A a point radiation source is laced at the focal point F. The field leaves this ntenna with a spherical wave front. As each art of the wave front reaches the reflecting surace, it is shifted 180° in phase and sent outward t angles that cause all parts of the field to travel n parallel paths. Because of the shape of a arabolic surface (paraboloid), all paths from to the reflector and back to line XY are the ame length. Therefore, all parts of the field arive at line XY the same time after reflection.

If a dipole is used as the source of radiation, here will be radiation from the antenna out into pace as well as toward the reflector. This energy hat is not directed toward the paraboloid has a ide beam width which would destroy the narrow pattern from the parabolic reflector. To prevent this, a parasitic reflector may be used with the half-wave driven element. In this way all of the radiated energy is directed into the parabolic reflector, and a narrow beam is produced.

If part of the antenna is outside the focal point, energy from that part will not be reflected parallel to the rest (Figure 13–32B). For this reason, the antenna is often physically shorter than a half-wave. For the radiating element to approximate a point source, the parabolic reflector should be at least ten times the size of the dipole being used.

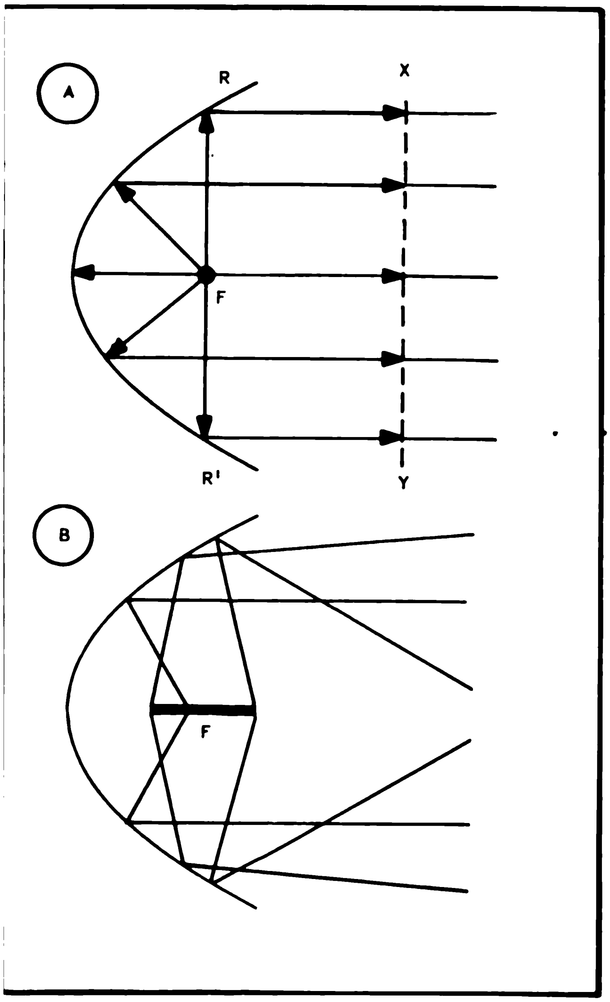


Figure 13–32. Parabolic Reflector

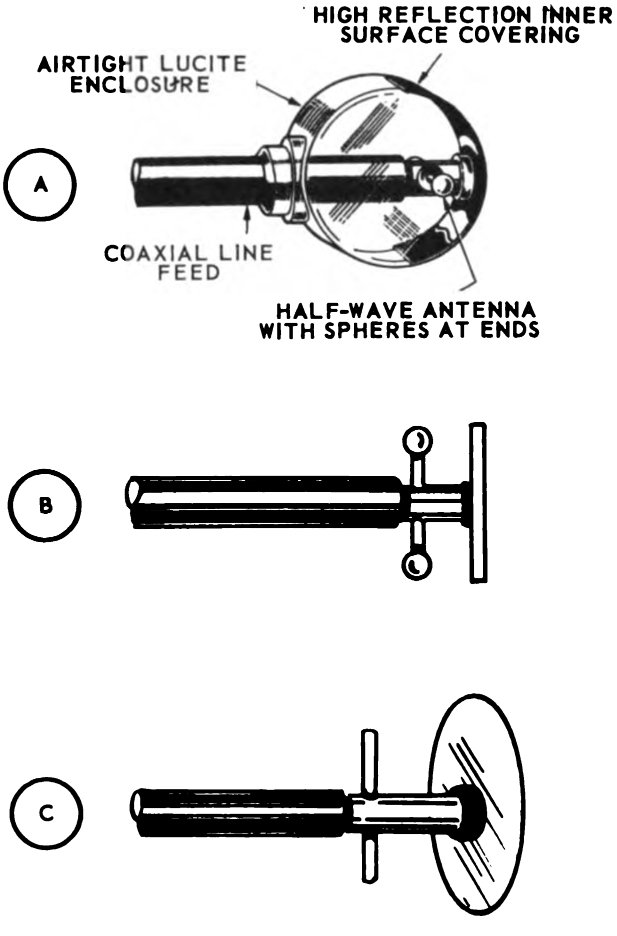

HIGH REFLECTION INNER SURFACE COVERING
AIRTIGHT LUCITE ENCLOSURE
A
COAXIAL LINE FEED
HALF-WAVE ANTENNA WITH SPHERES AT ENDS
B
C

Figure 13–33. Dipole for Exciting Parabolic Reflector

Figure 13–33A shows the exciting antenna for a parabolic reflector for a 10-cm radar system. The dipole is mounted a quarter-wave back from the short on the coaxial line. To sharpen the focal point the antenna is physically less than a half-wave long. However, the balls at the end make it electrically a half-wave long. This broadens the band of frequencies it will handle. The airtight cylinder in which the antenna is enclosed permits the coaxial line to be pressurized. The inner surface of half of the cylinder, that is, the side away from the parabolic reflector, is coated with a reflecting foil. This reflecting surface directs energy from that side of the antenna into the parabolic reflector. Without this reflector half of the antenna radiation would be nondirectional.

The natural directivity of a dipole causes the pattern from a parabolic reflector to be somewhat sharper in the plane containing the dipole than in the other plane. For this reason the dipole is mounted horizontally for maximum azimuth accuracy in radar systems. If vertical accuracy is of primary importance, the dipole is mounted vertically.

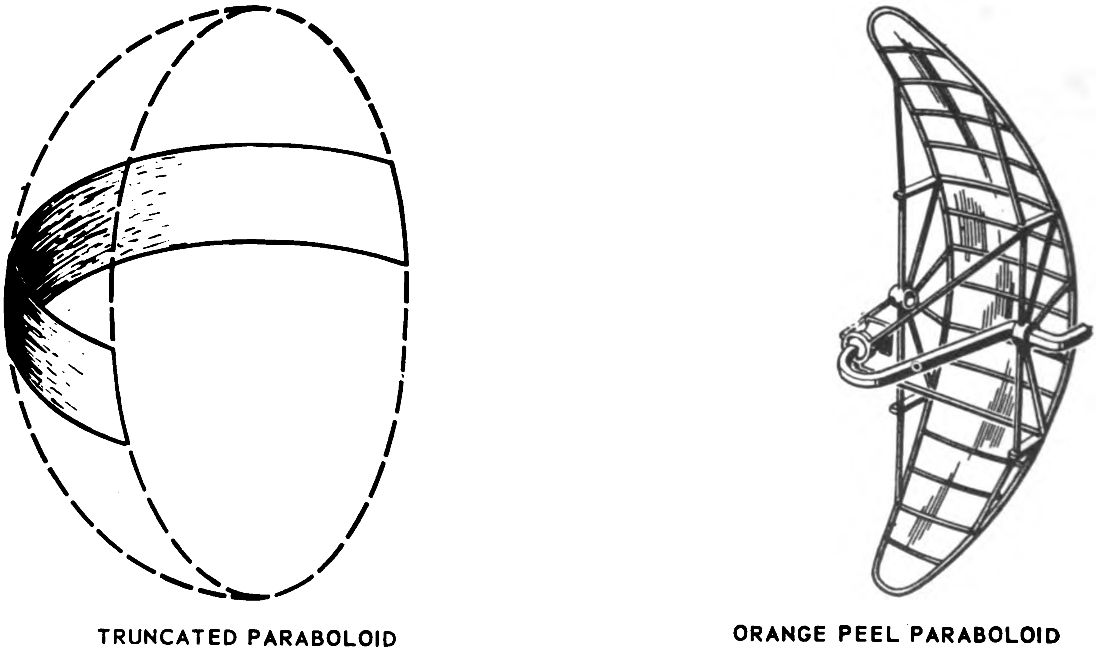
Figure 13–33B shows the use of a parasitic reflector with the dipole; Figure 13–33C shows a disc that is placed in front of the dipole.

There are many forms the parabolic reflector may take. The larger the reflector with respect to the wavelength, the narrower the beam width. This fact makes it possible to shape a beam by changing the dimensions of the reflector.

Truncated Paraboloid

Figure 13–34A shows a truncated paraboloid. Since the reflector is parabolic in the horizontal plane, the energy is focused into a narrow beam. Since the reflector is truncated, or cut, so that it is shortened vertically, the beam spreads out vertically instead of being focused. Such a fan-shaped beam is used to determine azimuth accurately. Since the beam is wide vertically, it will detect aircraft at different altitudes without changing the tilt angle of the antenna.

The truncated paraboloid reflector can also be used in height-finding equipment. Since the reflector is parabolic in the vertical plane, the energy is focused into a narrow beam; since the reflector is truncated, or cut, so that it is shortened horizontally, the beam spreads out horizontally instead of being focused. Such a fan-shaped beam is used to determine elevation very accurately. Since the beam is wide, a greater area can be scanned in a shorter time than is possible with a narrow, pencil-like beam.


TRUNCATED PARABOLOID

A

ORANGE PEEL PARABOLOID

B

Figure 13–34. Paraboloid Reflectors

Orange-Peel Reflector

A section of a complete circular paraboloid, often called an *orange-peel* reflector because of its shape, is shown in Figure 13–34B. Since the reflector is narrow in the horizontal plane and wide in the vertical, it produces a beam that is wide in the horizontal plane and narrow in the vertical. In shape it resembles a huge beaver tail. The r-f energy is sent into the parabolic reflector by a horn radiator fed by a waveguide. The horn nearly covers the shape of the reflector, so almost all of the r-f energy is directed into the reflector, very little escaping at the sides. This type of antenna system is generally used in height-finding equipment.

Cylindrical Paraboloid

When a beam of radiated energy that is noticeably wider in one cross-sectional dimension than in the other is desired, a cylindrical paraboloidal section approximating a rectangle can be used. Figures 13–35A and 13–35B illustrate this antenna. A parabolic cylinder has a parabolic cross section in one dimension only; therefore, the reflector is directive in one plane only. The cylindrical paraboloid reflector can be fed by a linear array of dipoles, by a slit in the side of a waveguide, or by a thin waveguide radiator. Rather than a focal point, this type of reflector has a series of focal points forming a straight line. Placing the radiator, or radiators, along this focal line produces a directed beam of energy. As the width of the parabolic section is changed, different beam shapes are obtained. This type of antenna system is used in search and in Ground Control Approach (GCA) equipments.

Cosecant-Squared Reflector

Some antennas are designed to produce special pattern shapes or to produce a nonuniform distribution of radiated energy. Airborne equipment has a special need for such equipment. In

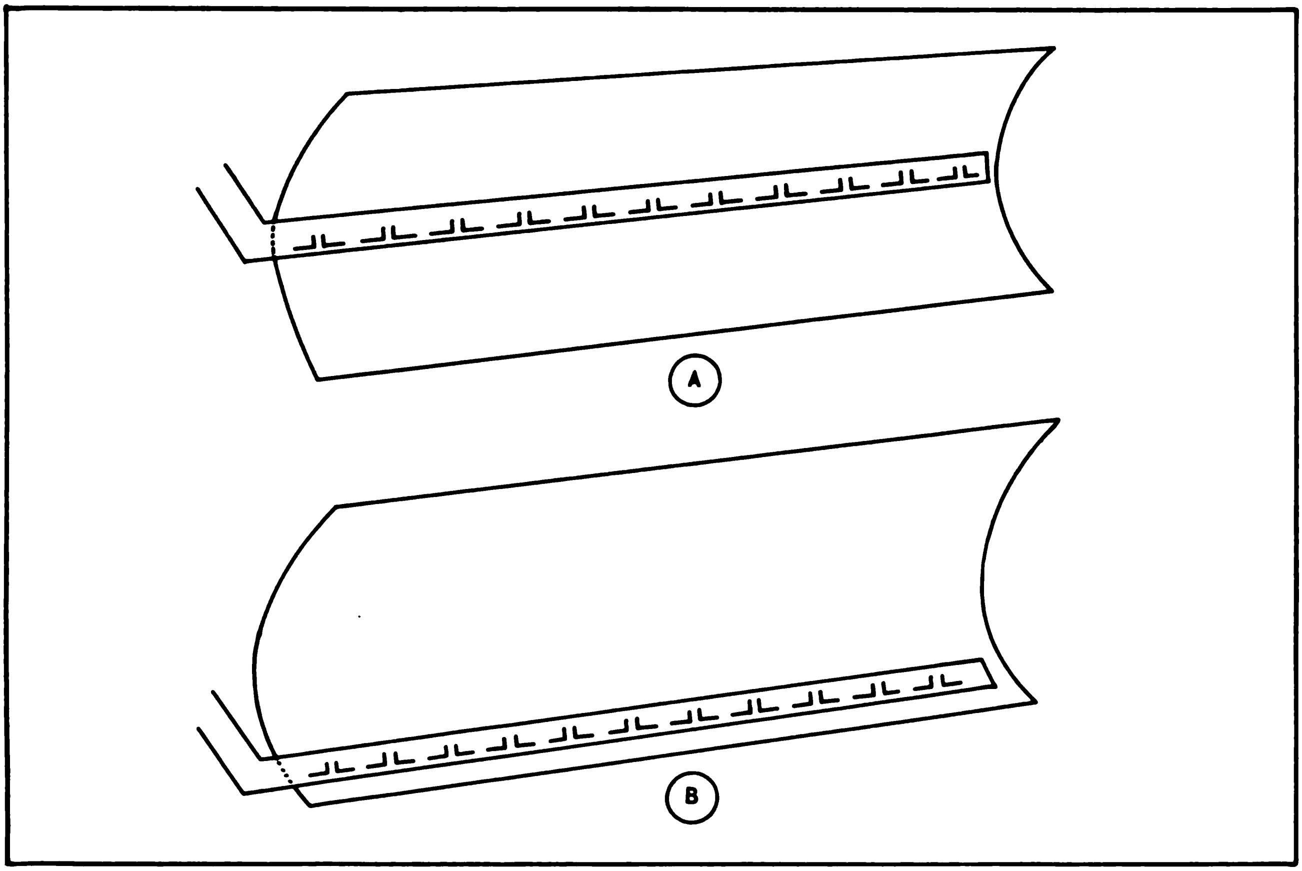


Figure 13–35. Cylindrical Paraboloid

scanning the surface of the earth in a forward direction, the surface some distance ahead is the area of importance. Normally this area would receive and return the weaker signal. The antenna, then, must be designed to reduce the energy that would be radiated downward and concentrate it in a forward direction.

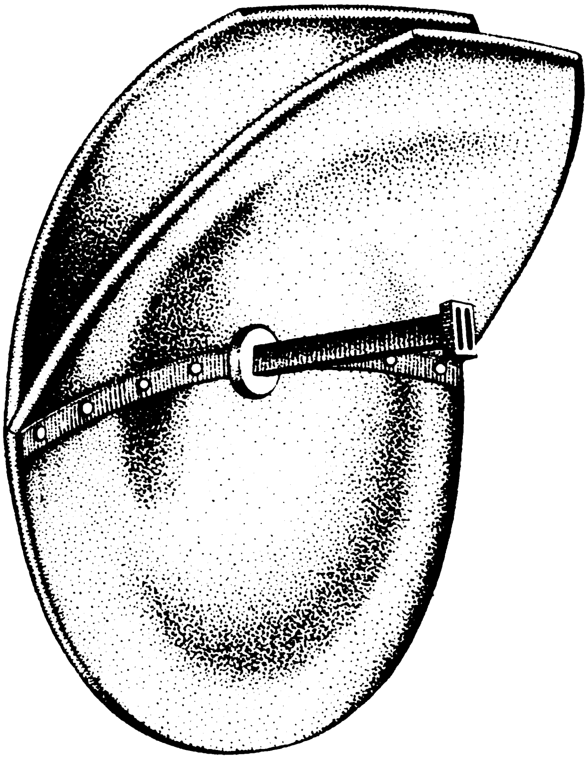


Figure 13–36. Cosecant-Squared Reflector

If the top of the parabolic reflector is bent forward, the desired pattern will be produced. Rather than change the actual shape of the parabolic reflector, a shaped piece of metal, known as the cosecant-squared ***skirt,*** is placed inside the upper part of the reflector. The signal strength now varies directly as the cosecant of the angle between the horizontal and the direction of the antenna. Since the power varies directly as the square of the field intensity, this antenna reflector is called a cosecant-squared reflector. The cosecant-squared principle can be used on equipment other than airborne; however, Figure 13–36 shows a type of cosecant-squared reflector used on a piece of airborne equipment.

Conical Scanning with Parabolic Reflector

Conical scanning is used in radar equipment to direct antiaircraft fire and in automatic tracking systems. It is possible to obtain angular precision of only a few hundredths of a degree. In this type of scanning a point on the radiated beam forms a circle at the base of the cone, as illustrated in Figure 13–37. The axis of the beam forms the cone. When a target is within the cone but off the axis, it will receive a different amount of energy as the beam makes its conical scan. As the target approaches the axis of the cone, variations of reflected energy become less and less. The reflected power does not vary when the target is on the axis of the cone. By detecting the amplitude variations of the received pulses, an

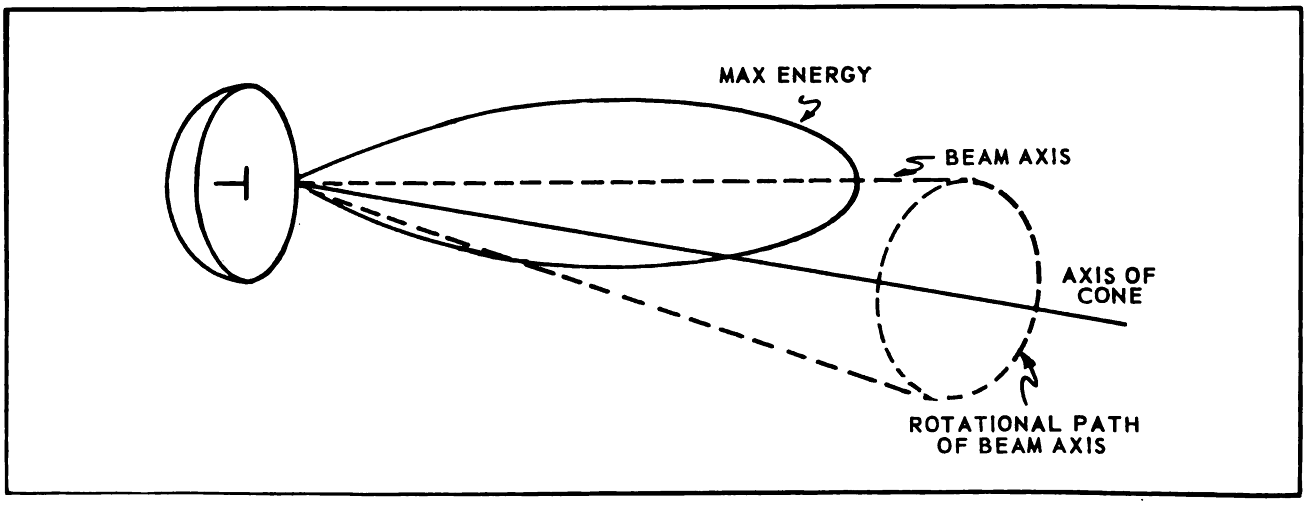


Figure 13–37. Conical Scanning

automatic tracking system is made possible. Conical scanning may be produced by rotating the dipole antenna around the axis of the parabolic reflector.

TYPICAL AIRBORNE ANTENNA SYSTEMS

General

The form and size of aircraft antennas are dependent upon their use. One of the prime considerations is that the antenna must not interfere with the flight of the aircraft. Normally the antenna is located away from all dielectric obstructions, so that nearby objects will not absorb any radiated energy. The ground for the antenna system is practically perfect as it is the metallic skin of the aircraft. Short transmission lines with their inherent lower loss can be used as the antenna is usually near its associated equipment.

The prop-driven aircraft that are still being used have fixed wire or stub-mast antennas for communications. Some are flush-mounted into the skin of the aircraft and some are mounted in the tail section. They are located so that the antennas offer minimum drag to the aircraft during flight.

Because of the speed of jet aircraft, the communications antennas used are different from those used in prop-driven aircraft. UHF, VHF, and HF antennas are a part of the rudder section, located at the top of the rudder. The physical size of the antennas is determined by the wavelength of the frequencies used. Each antenna is insulated from the others. Installed in this manner the antenna offers no additional drag to the aircraft. Where other communications antennas are needed, they are flush-mounted, generally in the nose of the aircraft.

Navigational Antenna

The AN/APN-69 is an Airborne Radar Beacon set useful in navigation. This set has found wide use in mid-air refueling operations. It was designed and constructed to be used with 3-cm Airborne Search Radar equipment. For successful operation of the AN/APN-69 there must be a search radar set operating in the vicinity as a beacon-interrogator.

The antenna, as shown in Figure 13-38, serves to receive radio-frequency energy from the interrogating aircraft or radiate r-f energy to the interrogating aircraft. The antenna is designed to provide omnidirectional coverage. The radiating element, which protrudes outside of the aircraft structure, is covered by a radome made of a nylon cloth shell filled with a foam material. The antenna is installed in a fixed position and does not require operating adjustments.

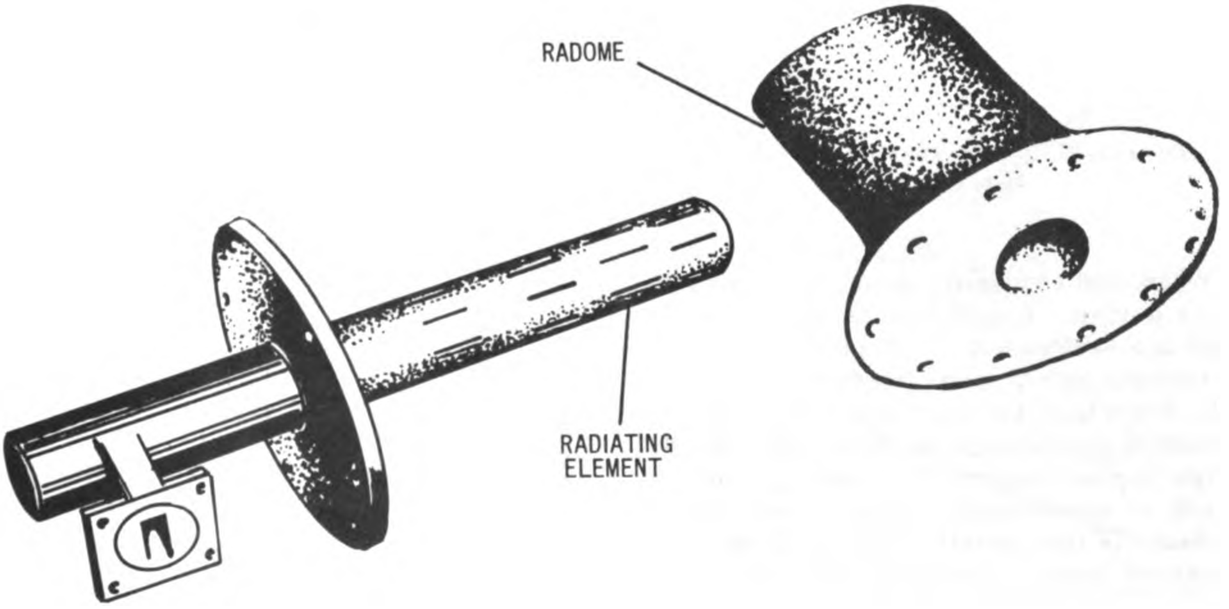


Figure 13-38. Antenna and Radome AN/APN-69

Airborne Search Radar

Radar Set AN/APN–59 is a small, lightweight radar system which operates as a navigational and search radar, a weather observation radar, or radar beacon interrogator-receiver.

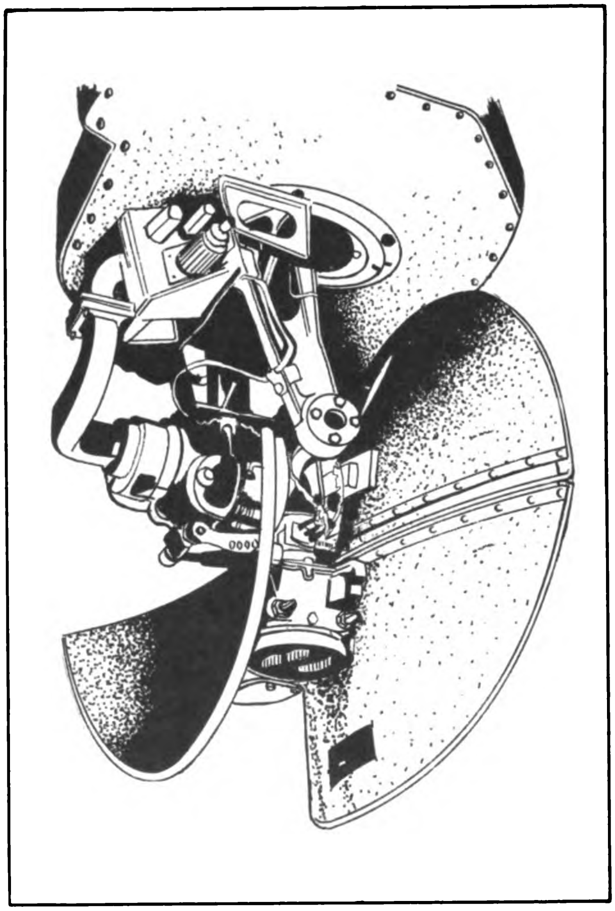


Figure 13–39. Antenna Reflector AN/APN–59, Right Side Detail

When used as a search radar, the system displays a visual, map-like scope picture. When used as a weather radar, it displays less substantial objects such as storm fronts and heavy rainfall. When used for radar beacon navigation, it transmits an interrogating signal and then displays, in plan position, the space-coded identification of the automatic radar beacon reply or replies. As the aircraft pitches and rolls, any displayed signal is prevented from shifting position or disappearing by the use of a gyro-stabilized r-f beam pattern.

The transmitted signal may be radiated as either a fan or a pencil beam. The fan beam is shaped to sweep out at a vertical angle of nearly 60° downward from the level of flight with a power pattern that returns echoes of nearly equal strength from remote or nearby targets. The pencil beam is similar to a searchlight beam. Tilt is provided so that the beam may be directed downward or slightly upward, as required.

The antenna may be set to rotate about an axis perpendicular to the deck plane of the aircraft, or gyro-stabilized to rotate about an axis perpendicular to the earth. The antenna may be driven in full rotation either clockwise or counterclockwise, or stopped; and it may be set to scan a sector across the aircraft heading. The antenna has two speeds of scan that are chosen automatically by the function and range in use. Both an 18-inch and a 30-inch reflector and feed assembly are available. The 30-inch reflector gives better definition due to the narrow horizontal beam width of 3°. This larger reflector will be used where space and weight are not at a premium.

The waveguide in the antenna is all of the rigid type. This requires three rotating joints: (1) azimuth, (2) level (pitch), and (3) cross level (roll). Between the level and cross-level joints of the 18-inch reflector on the top side of the waveguide is a machined surface for placing level test instruments. Figure 13–39 shows the 18-inch and 30-inch reflector units of the AN/APN–59.

The antenna shown in Figure 13–40 is used with the AN/APS–42A, an airborne search set. The illustration shows a portion of the r-f waveguide ① which conducts the transmitter pulses to the antenna and the received pulses from the antenna, the 18-inch paraboloid reflector ② and spoiler grid ③ which shape the transmitted beam into the desired radiation pattern, the azimuth drive motor ④ that moves the antenna on full or sector scan at one of two speeds. A gyroscope ④ functions with associated circuits to stabilize the antenna when the aircraft rolls or pitches. The antenna tilt system ⑤ enables the operator to tilt the antenna from 10° above the horizontal plane to 15° below the horizontal plane for additional coverage. The pattern change system makes possible the use of two radiated patterns: pencil or obstacle beam, and mapping beam.

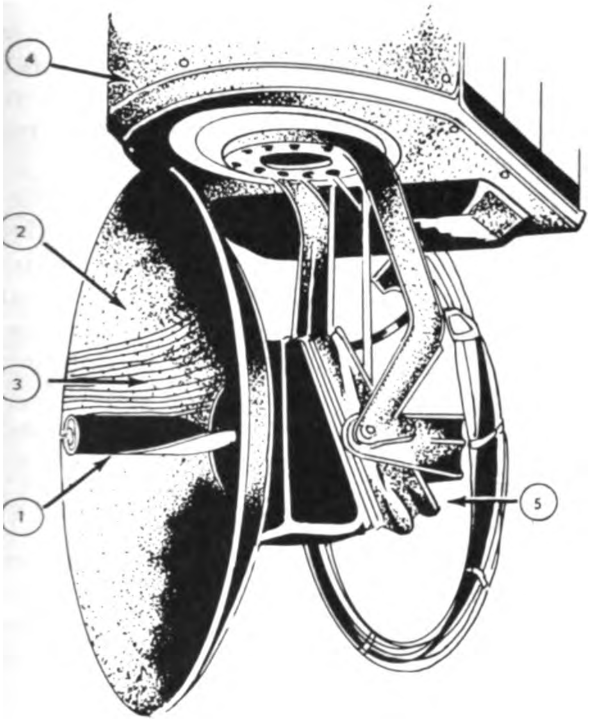


Figure 13–40. Antenna Unit AN/APS–42A

AEW Equipment

The largest antennas used in airborne equipment are those used in the Aircraft Early Warning Radar System. The aircraft used for this system is the RC–121D, shown in Figure 13–41. Two unclassified pieces of equipment used in an airborne early warning system are the AN/APS–45, height finder, and the AN/APS–95, search set. Figure 13–41 shows the location of the AN/APS–45 antenna on the aircraft.

The height-finder antenna for the AN/APS–45 is shown in Figure 13–42. The widest part of the orange-peel reflector is about 4 feet; the height is about 9½ feet.

This antenna provides for focusing, radiating, and receiving the radar pulses, and for conveying target azimuth and elevation information to the control-indicator. The beam position is stabilized in elevation with respect to line-of-sight, and in azimuth, with respect to north. The antenna can be rotated continuously clockwise or set to any desired azimuth position. In addition it can be set to any desired elevation angle within the 16° limits of the elevation sector scan.

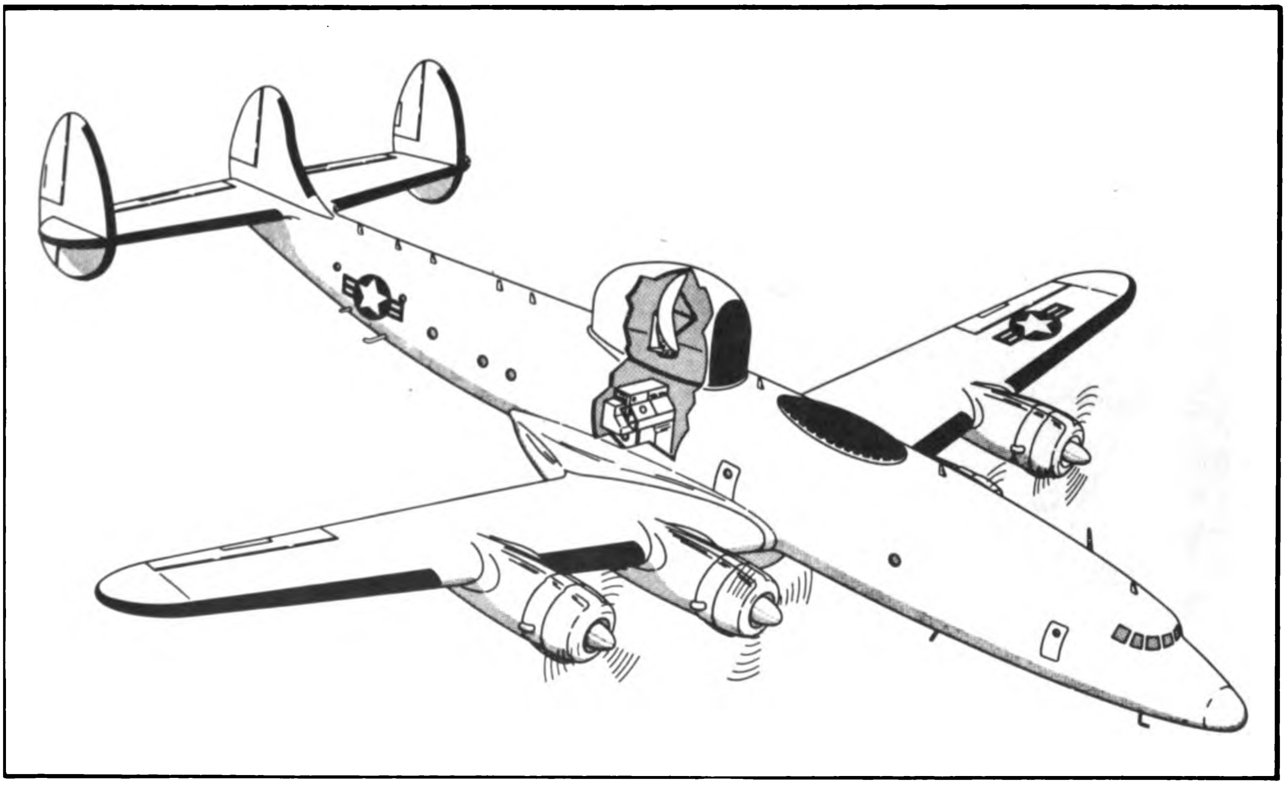


Figure 13–41. RC-121D AEW Aircraft

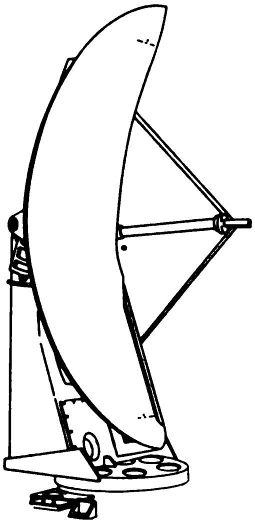


Figure 13–42. Antenna for AN/APS–45

The search set antenna, Figure 13–43, functions to radiate radar and IFF transmitted power, and receive radar and IFF return signals. The radar portion of the antenna consists of a 12-element linear array of bow-tie-slot resonant cavities. Each element consists of a rectangular cavity containing one or two bow-tie slots in the front panel. Each bow-tie is fed by a rigid coaxial feed; this, in turn, is fed by a rigid coaxial corporate feed. The corporate feed is fed by a hybrid power divider which contains a sum and difference channel. The sum channel is used during both transmission and reception, while the difference channel is used during reception only.

The antenna is mounted to the rotating portion of the pedestal casting which protrudes through the aircraft fuselage, so that the antenna is mounted below the aircraft. The rotary joint provides continuity between the fixed and rotating sections of the antenna pedestal. Rotation in either direction is possible at a speed of 2, 4, or 6 rpm. The antenna assembly is protected by a weatherproofed radome housing. The physical length of the antenna is approximately 17 feet and the height is approximately 4 feet.

A special feature of the APS–95 is that it uses the principle of a Displaced-Phase-Center Antenna. The function is to initiate phase relationships in order to cancel off-axis clutter in the MTI unit.

The IFF antenna is a 2-dimensional linear array of two rows of 24 identical slot-fed dipoles. They are vertically positioned and each pair is covered with a radome for protection. This is illustrated in Figure 13–43. The IFF antenna uses the front panel of the radar antenna as a reflecting surface.

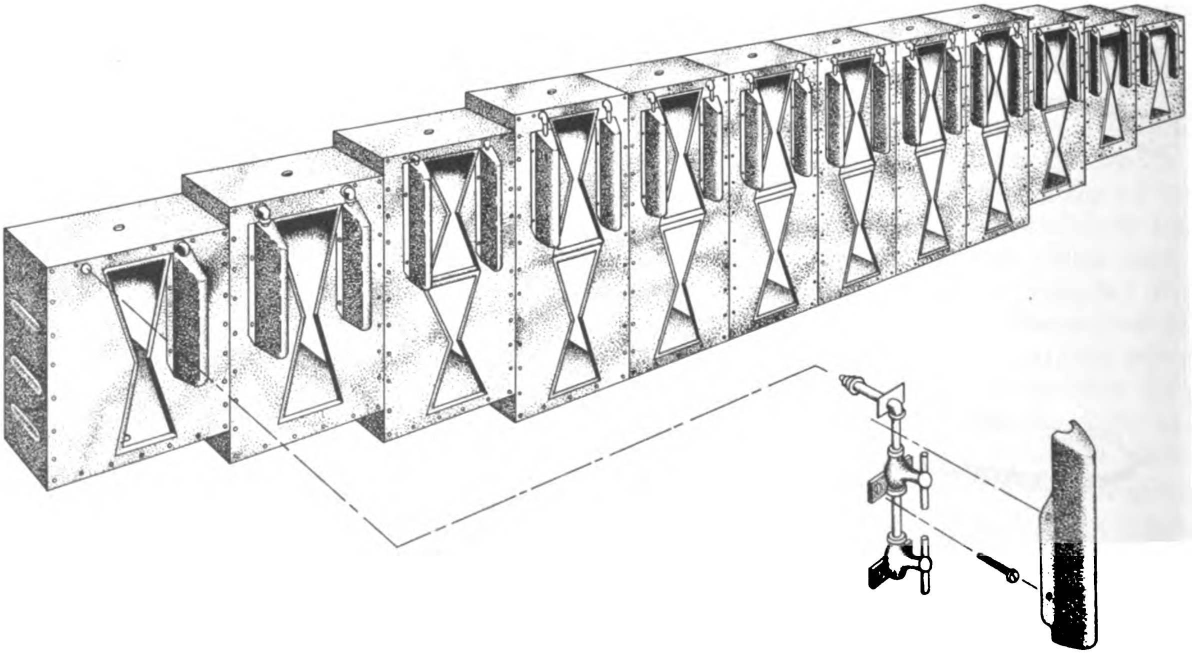


Figure 13–43. Antenna Assembly AN/APS–95

GROUND RADAR ANTENNAS

General

The effectiveness of any radar set is influenced to a great degree by the nature and quality of the radar's antenna. The range, accuracy, and resolution of the set is affected by the electrical properties of the antenna. The directional features of radar antennas makes it possible to illuminate certain selected targets and to receive the returns from these same targets for any given time. Directivity allows all of the energy to be concentrated in one general direction. Radar antennas are as numerous in kind as is radar equipment. They must be designed according to the character and particular function of the radar.

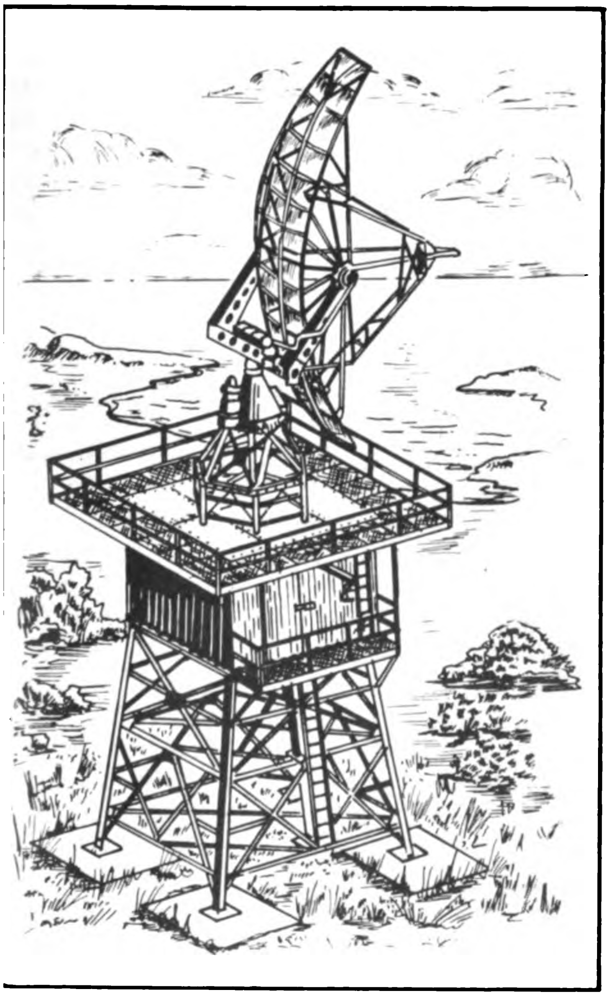


Figure 13–44. AN/FPS–6 Antenna

Height Finder Radar

The AN/FPS–6 is an air-transportable, high-power, long-range, fixed-station height-finder radar used in aircraft control and warning. AC&W systems consist of ground search radar equipment combined with height-finding equipment. The AN/FPS–20 and the AN/FPS–6 form such a combination. The FPS–6 radar set in conjunction with the FPS–20 enables the aircraft controller to place the enemy aircraft accurately in azimuth, range, and height. This three-dimensional picture allows the controller to easily vector the intercepting aircraft to the point of interception.

The reflector of the FPS–6 antenna is a truncated orange-peel type. The reflector is horn fed, has a height of 30 feet and is 7½ feet in width. The reflector nodding scan is from −2° to +32° with a nodding frequency of 20 or 30 npm. The antenna has a 360° azimuth bearing capability; it can operate while rotating continuously or at a stationary position at any azimuth. Figure 13–44 shows an AN/FPS–6 antenna on a fixed tower.

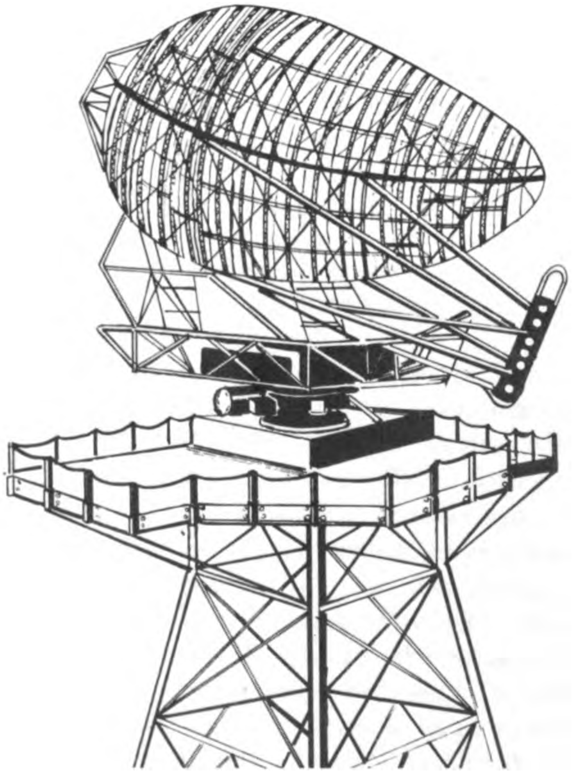


Figure 13–45. AN/FPS–20 Antenna

Ground Search Radar

The AN/FPS–20 is a long-range, high-powered ground search set. It furnishes a very important addition to the Air Defense system by providing a range-azimuth coverage. In conjunction with height and identification information, this completes the AC&W system.

In a rotating antenna system, such as used in the AN/FPS–20 shown in Figure 13–45, a transition must be made between the fixed waveguide system below the antenna pedestal and the rotating waveguide system above it. The FPS–20 uses a rotary joint in the antenna pedestal as the transition from fixed to rotating waveguide.

A high-powered, three-cavity klystron is used as a power amplifier in the transmission system. The desired antenna beam pattern coverage of only one beam is accomplished with this type of transmission and a line feed reflector. The energy pattern is narrow in the horizontal plane and very broad in the vertical plane. Such a beam gives good azimuth definition and also detects high flying targets.

The azimuth rotation system rotates the antenna through 360° of azimuth search. The pedestal, drive motor, and the azimuth data generator unit which is mounted on the pedestal is included in this system. The antenna reflector and the rotary coupler are mounted on the antenna bull gear, which is driven by a 20-hp three-phase motor through a gear reducer. The antenna rotates at speeds of 3.3, 5.0, 6.6, and 10.0 rpm.

The reflector shape is a section of a paraboloid rotated around its axis. The outline of the reflector is an ellipse; the long dimension of the reflector is approximately 40 feet long, and the short dimension is approximately 16 feet long. Figure 13–45 shows the AN/FPS–20 antenna mounted on a fixed tower.

CHAPTER 14

Synchros and Servomechanisms

Up to now we have analyzed a great variety of circuits used in electronics in general, and in a number of radar sets in particular. This chapter is to acquaint you with devices which transmit position between these circuits and remotely located gages, dials, and controls. The chapter discusses the basic theory of two of these devices — the synchro, a machine which converts mechanical position into electrical position, or electrical position into mechanical position; and the servomechanism, a system that amplifies and transmits mechanical position from one place to another by electrical means.

USES

There are many operations in radar where it is desirable for two shafts to rotate in synchronization. But because the distance between the two shafts is too great, or because one shaft cannot develop enough torque to rotate the other one, it is not always feasible for the shafts to be connected together mechanically. Therefore, shafts which are designed to rotate in synchronization are generally connected electrically.

Electronic systems for rotating two shafts in synchronization are called *remote indication* or *remote control* systems. A system in which the torque of the shaft which controls the operation (the *control* shaft) is amplified before it is applied to the shaft that is connected to the load (the *load* shaft), is called a *servomechanism* or *servo* system.

As you can see by the name, a remote indication system is designed either to produce at one position an indication of an operation at another position, or to transfer information from one position to another. Sometimes this system is called a *data transmission system.* Many radar antennas, for example, are designed to rotate 360° but are located in places where it is not possible for the operator to see them. Therefore, a data transmission system is employed to give the operator the direction in which the antenna is pointed.

A pointer attached to a shaft rotates in synchronization with a shaft connected to the antenna, and registers the position of the antenna on a dial at the operator's position. The devices through which the position of the antenna is transmitted to the dial are called *synchros.* They resemble small electric motors. The one at the antenna is called a *synchro generator* or *transmitter;* the one at the operator's position is called the *synchro motor* or *receiver.*

The word synchro, as you can see, is a contraction of the word *synchronous.* Trade names which are frequently used for these devices are *Selsyn, Autosyn, Magnesyn,* and *Synchrotie.*

In the remote indication system described, it is possible for the operator to rotate the antenna itself from his position. However, this requires more torque than can be transmitted efficiently through the system. To provide the necessary torque a servo system must be used. In the servo system, the operator turns a hand wheel (or some other control). This moves the shaft connected to the hand wheel out of step with the shaft connected to the antenna.

The out-of-step condition produces a voltage called the error voltage. This voltage is amplified by the torque amplifier and applied to a motor that rotates the antenna. The motor rotates in a direction that counterbalances the error produced by the out of step relationship between the hand shaft and the antenna shaft, and thus causes the antenna to take the same position as established by the hand control.

POSITIONING A SHAFT WITH DC VOLTAGES

Although the synchro is an AC device, DC voltages are sometimes used for positioning shafts. Since an understanding of shaft positioning with DC will help you to analyze the operation of the synchro, it is well to take it up first.

There are a number of ways for positioning a shaft with DC voltages. One is to mount a bar magnet on the shaft and to locate it near a solenoid (coil). When a voltage is applied to the coil, the shaft will rotate to either of the two positions shown in Figure 14–1. When you reverse the polarity of the DC voltage, the magnet will likewise reverse its position. The position of the magnet shown is arbitrarily designated the 0° position. Furthermore, the positive angles represent rotation in the counterclockwise direction. In examining this arrangement closely you can see that only two positions of rotation are possible. Therefore, it would not be very satisfactory as a remote indicating system.

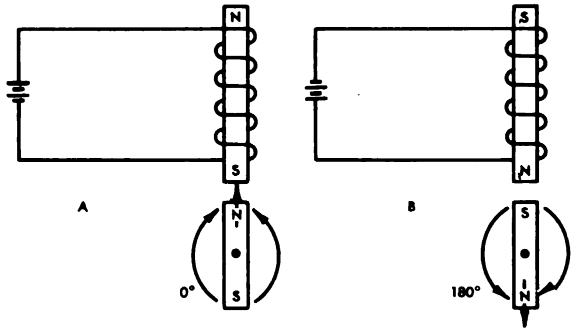


Figure 14–1. Using One Coil to Position a Magnet

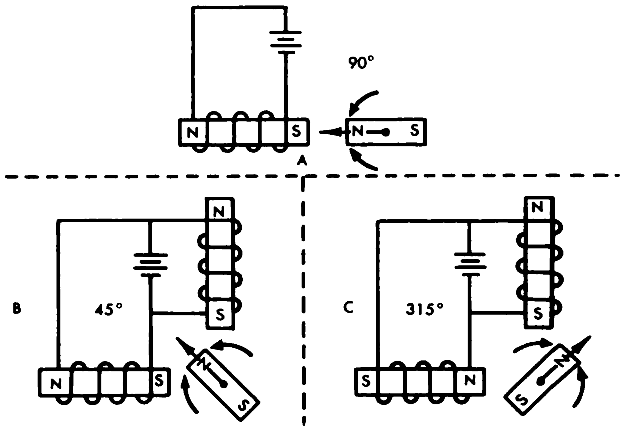


Figure 14–2. Using Two Coils Placed at Right Angles

Another system capable of rotating a shaft to more positions than a system using one coil, uses two coils that are placed at right angles about the magnet. When a single coil is placed as shown in Figure 14–2A, the magnet will rotate to the 90° position; when the polarity of the coil is reversed, the magnet will rotate to the opposite, or 270° position. If, however, you connect two identical coils at right angles into the circuit (Figure 14–2B), the magnet will rotate to the 45° position. If you reverse the polarity of both coils, the magnet will reverse its position and come to rest at the 225° position.

Reversing the polarity of the coil in the horizontal position causes the magnet to assume the 315° position shown in Figure 14–2C. If the polarity of both coils in Figure 14–2C is reversed, the magnet will take a 135° position. Thus, you can see that by applying the same voltage to one or both of two coils that are placed

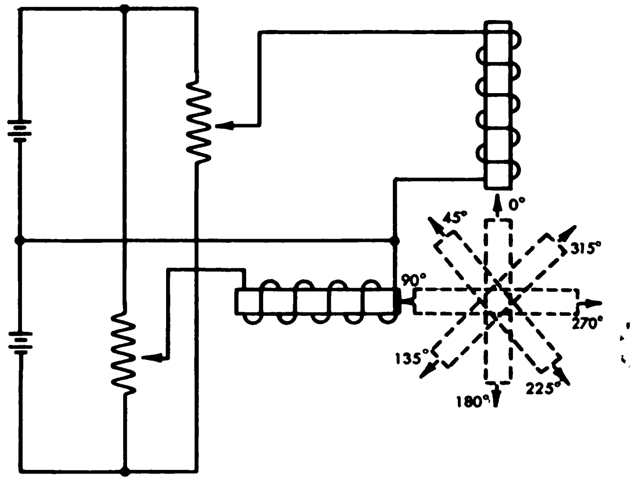


Figure 14–3. Using Two Coils and Potentiometers

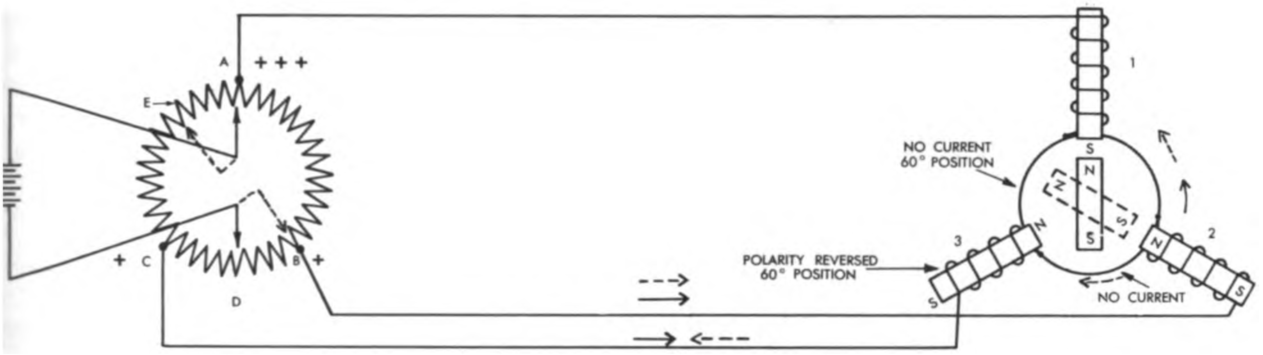


Figure 14-4. Potentiometer and Magnet Remote Indicator

at right angles, and by reversing the polarity of the coils, the magnet will take any one of eight positions.

A further method of positioning a shaft with DC, as shown in Figure 14-3, involves using a potentiometer for controlling the polarity and magnitude of the voltage applied to each coil. Since the potentiometer makes it possible to vary the magnitude of the voltage, the magnet will take any position around the circle. While this arrangement produces accurate results, it is not satisfactory as a practical remote indicating device since it requires independently controlled potentiometers for each coil.

POTENTIOMETER AND MAGNET REMOTE INDICATING SYSTEM

A practical DC remote indicating system that uses the principles described in positioning shafts with DC is the Potentiometer and Magnet Indicating System as shown in the illustration in Figure 14-4. Sometimes this system is called a DC synchro.

The potentiometer in this remote indicating system is circular. DC voltage is applied to it through contacts that slide along the inner side of the potentiometer. These contacts are insulated from each other to prevent the DC voltage from being shorted out. In the three-coil permanent magnet arrangement at the right of the figure, the coils may be connected either in delta or in wye. In either connection there is little difference in the operation of the system.

The potentiometer is step-wound to compensate for the shunting effect between the potentiometer coils. This type of winding, however, causes the voltages at points A, B, and C to vary as the potentiometer rotates in a counterclockwise direction, as shown in Figure 14-5 in the graph of the voltages at different positions of the potentiometer contacts. All are with respect to the negative (ground) side of the DC voltage supply.

As previously mentioned, there is little difference between the operation of a wye- and a delta-connected system. Thus, if you follow through the theory of operation of the wye-connected type of three-coil-permanent magnet assembly, you will be able to understand the delta-connected type also.

During operation of the wye-connected system when the contacts are at the 0° position (+ to A and − to D), the voltages at A, B, and C respectively are 27v, 9v, and 9v. One end of coil 1 is connected to point A; one end of coil 2 is connected to B; and one end of coil 3, to C. The other ends of each coil are connected together. Current flows from B to A through coils 2 and 1, and from C to A through coils 3 and

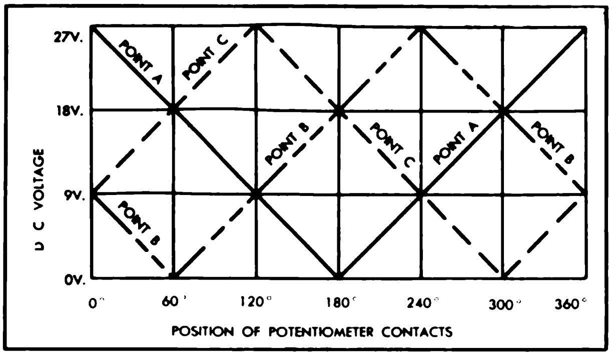


Figure 14-5. Graph of Voltage vs. Position of Contacts

1. Since B and C are at the same potential, there is no current flow between them. Coils 1, 2, and 3 become magnetized, with their poles as shown in Figure 14–4. Coil 1 carries more current and therefore exerts a strong attraction for the north pole of the magnet. Coils 2 and 3 carry equal currents and attract the south pole of the magnet with equal force. The magnet will therefore assume the position shown (the 0° position).

When the potentiometer contacts move to the 60° position, then E is at 27v, and B at 0v. As you can see in the graph in Figure 14–5, A is at 18v, and C at 18v. This causes current to flow from B to A through coils 2 and 1 and from B to C through coils 2 and 3. No current flows from A to C. In this condition coil 2 now carries more current than the other coils and strongly attracts the south pole. The current in coil 3 is reversed. Its polarity is also reversed. Thus, coils 1 and 3 attract the north pole with equal force, and the magnet assumes the 60° position corresponding to the potentiometer contacts.

As the potentiometer contacts move from the 0° position to the 60° position, the voltages vary uniformly as follows: A from 27v to 18v; B from 9v to 0v; and C from 9v to 18v. Hence the current through coil 1 decreases; the current through coil 2 increases; that through coil 3 decreases to zero at 30° and then increases in the opposite direction. The rotation of the magnet is uniform and follows the rotation of the contacts.

Since the smooth variation of voltages is upset if the coils draw much current, this system is limited to applications where the current in the coils is not appreciable when compared with the current in the potentiometer. Since the potentiometer current is not useful except for maintaining the voltages, the system wastes considerable power. Its principal use is as an indicating device that tells the operator the angle of elevation of the antenna. It may be used to indicate the azimuth to the operator, but he usually gets this information from the CRT.

PHYSICAL CHARACTERISTICS OF SYNCHROS

Before going into the operation of the synchro generator and motor, it is well to first consider the construction of each. Both resemble small electric motors and are identical except for several details. Each unit contains a fixed element called the *stator*, Figure 14–6, and a movable element called the *rotor*, Figure 14–7. The stator consists of a number of coils that are placed in slots around the inside of a laminated iron field structure, very much like an ordinary AC motor. These coils are divided into three groups, spaced 120° apart around the inside of the field. Actually the groups overlap somewhat so that the attractive force tending to pull the rotor into position is the same for all positions of the rotor. The 120° spacing of windings sometimes leads a person to believe that three-phase voltages and currents are used, but such is not the case. *Only single phase voltage and current are present.*

In a typical synchro generator, the rotor consists of a single coil of wire wound on a soft iron core and mounted on a shaft in a way that the axis of the coil is perpendicular to the shaft. The ends of the core are curved so that the air gap between them and the stator is small and uniform. To make friction low, the shaft is mounted on ball bearing mounts. The rotor turns inside the stator and is electrically connected through two slip rings on the shaft.

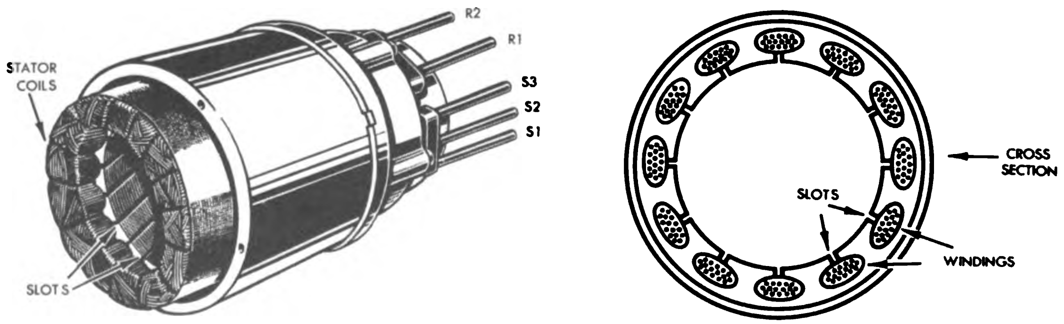


Figure 14–6. Synchro Stator

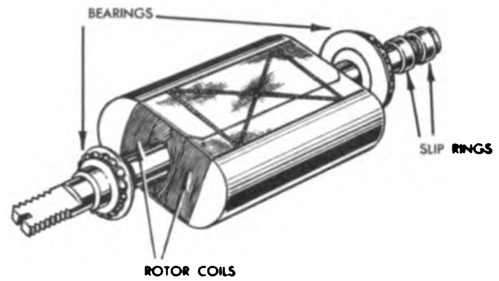


Figure 14-7. Synchro Rotor

Because a synchro motor is similar to an ordinary AC motor, it tends either to oscillate violently or to spin continuously under some conditions. This is particularly likely to happen when the shaft is turned suddenly as, for example, when power is first applied to the system. To prevent this undesirable oscillation a heavy metal flywheel, called an *inertia damper*, is mounted on one end of the shaft, Figure 14-8. This flywheel is mounted so that it turns freely on the shaft for 45° or so, and then runs into a keyed bushing. This bushing, which is fastened to the shaft through a friction disc, also turns on the shaft but with a great deal of friction.

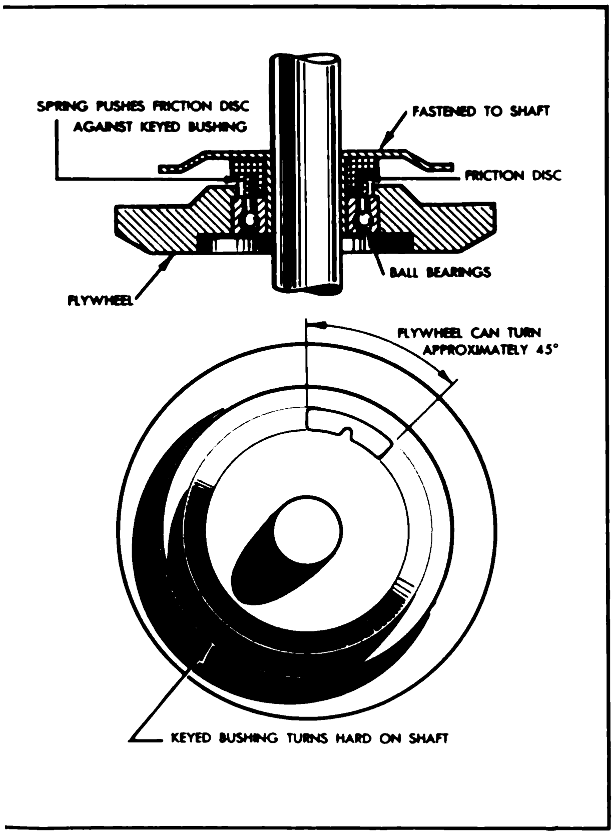

SPRING PUSHES FRICTION DISC AGAINST KEYED BUSHING
FASTENED TO SHAFT
FRICTION DISC
BALL BEARINGS
FLYWHEEL
FLYWHEEL CAN TURN APPROXIMATELY 45°
KEYED BUSHING TURNS HARD ON SHAFT

Figure 14-8. Typical Inertia Damper

For slow changes in position of the shaft, the flywheel follows along without much oscillatory effect. But if the shaft tries to turn suddenly, the flywheel tends to stand still, and the friction disc acts as a brake to slow down the motion of the shaft. This keeps the shaft from reaching a speed fast enough to start oscillating or spinning. If oscillation or spinning does occur, it is usually a certain indication that something is wrong with the damper.

In the synchro the leads to the rotor and stator are brought out through two insulating strips at the back of the motor or generator. The stator connections are marked S1, S2, and S3; the rotor connections R1 and R2, as shown in Figure 14-6. This system of marking connections is standard among the various manufacturers.

OPERATION OF SYNCHRO SYSTEM

Positioning a Shaft with AC Voltages

Figure 14-9 shows a solenoid located close to a permanent magnet that is mounted so that it can rotate on an axis. When a 60 cps voltage

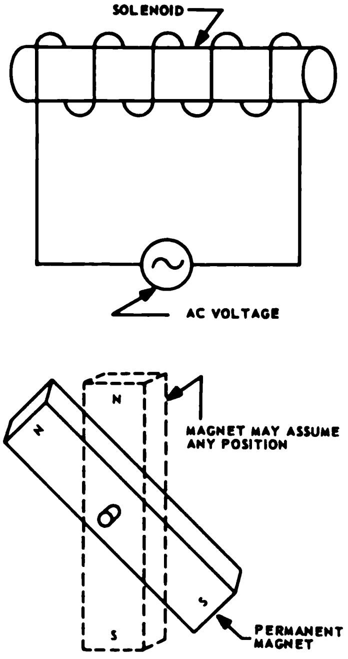


Figure 14-9. Effect of AC Solenoid on Magnet

is applied to the solenoid, the torque exerted on the magnet will reverse direction 120 times per second, first in one direction, then in the opposite direction. Since the magnet cannot reverse direction this rapidly, it will respond only to the average torque and since the average torque is zero, the magnet will assume some position independent of action by the coil. Thus, since the magnet is not affected by the coil, it is obvious that it cannot be used with an AC coil as it can be with a DC coil.

But when you substitute another AC coil for the permanent magnet, connected as in Figure 14–10, the current in both solenoids reverses at the same instant. This causes the same end of the rotating solenoid to be attracted to the nearest pole of the fixed solenoid throughout the entire cycle. As shown, the C-end of the rotating solenoid is attracted to the B-end of the fixed solenoid at all times. Reversing the connections on either coil (but not on both coils), reverses the rotating coil and causes B to attract D. This reversal of connections is equivalent to a reversal of phase in that coil.

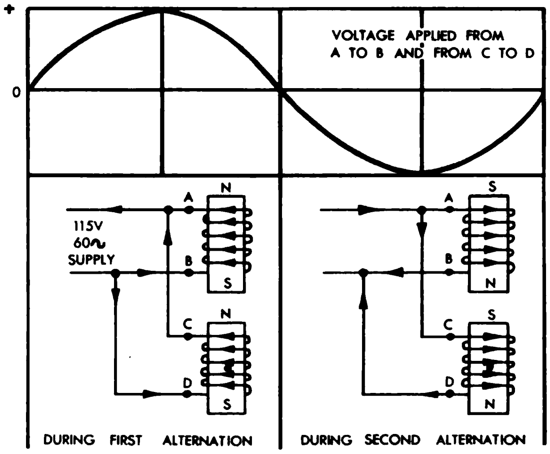

+
0
VOLTAGE APPLIED FROM A TO B AND FROM C TO D
115V
60~
SUPPLY
A
B
C
D
N
S
N
S
A
B
C
D
S
N
S
N
DURING FIRST ALTERNATION
DURING SECOND ALTERNATION

Figure 14–10. Effect of One AC Coil on Another

Symbols for Synchro Diagrams

It is not very practical to show synchro circuits in the form of photographs or drawings of the actual apparatus, since they merely show the outer appearance and leave the inside obscure. Therefore, to make it easier to describe the operation of a synchro motor (or generator), standard diagrams which are referred to as *synchro diagrams* (Figure 14–11) are used. In forming these diagrams, there is a standard way for describing the shaft position. This is necessary to insure that if the rotor shaft is properly connected mechanically in the system, it will turn to the correct position when it is connected electrically to a synchro generator.

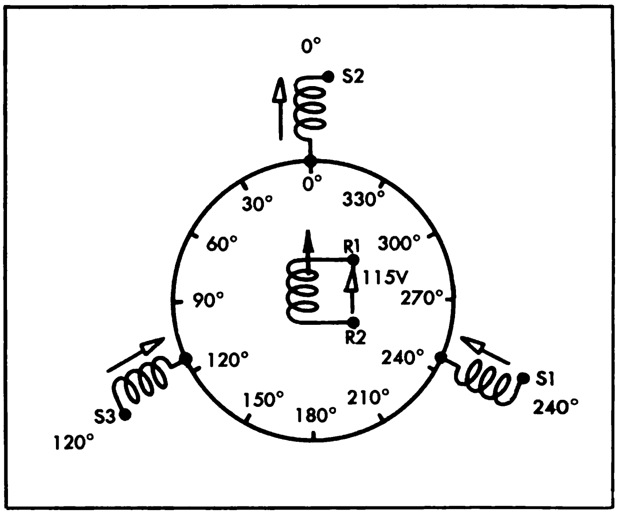


Figure 14–11. Synchro Diagram

In Figure 14–12 the position the rotor takes when it lines up with the stator coil that is connected to S2 is called the *electrical zero* position. Other positions are measured in degrees in a counterclockwise direction (looking at the shaft end of the unit).

In studying the synchro diagram think of the rotor as turning around an axis in the center of the diagram, and that the arrow on the R1 end of the rotor points at the number indicating the electrical position of the rotor.

Another point to consider is that the rotor coil in the diagram is assumed to be wound in the same direction, going from R1 to R2, and that each of the stator coils is wound from the outside to the inside.

The open-headed arrows are important in that they indicate the following *relative phase* polarity. If two arrows point in the same direction, the voltages are in phase; if the arrows point in opposite directions, they are 180° out of phase.

All voltages are either in phase or 180° out of phase with each other. The diagram indicates that the voltages from R2 to R1, from

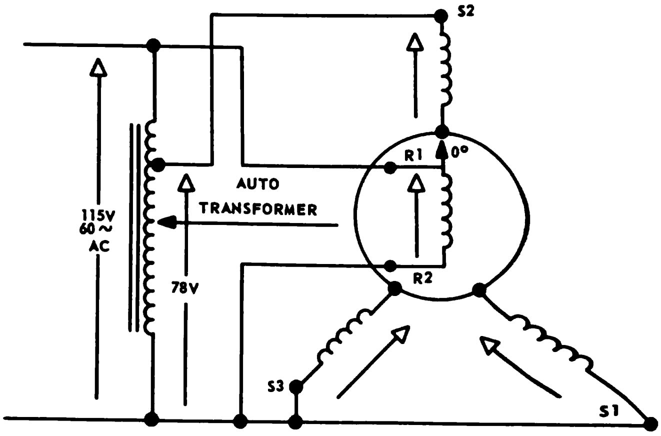


Figure 14–12. 0° Position

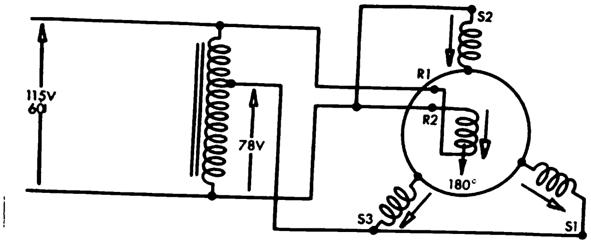


Figure 14–13. 180° Position

common to S2, from S1 to common, and from S3 to common are in phase, while from S2 to common they are 180° out of phase with either S1 to common or S3 to common.

Voltages Required to Position a Synchro Motor

For a standard synchro motor to turn to the electrical zero position, 115v AC (rms value) must be applied to the rotor leads, zero volts between S1 and S3, 78v between S2 and S1–S3 in such a way that the voltage from R1 to R2 is as shown in Figure 14–12. In this condition the coils connected to S1 and S3 exert equal attractive forces on the R2 end of the rotor. The coil connected to S2 attracts the R1 end of the rotor with greater force, for it carries the current of both the other coils. Therefore this causes the rotor to assume the 0° position.

Reversing the leads to the 78v supply causes the voltages across the stator coils to reverse in phase and the rotor coil to reverse in position, causing it to point to the 180° position, Figure 14–13.

Connecting together the other pairs of stator leads and applying the 78v to the third lead either in phase or out of phase with the R1 or the R2 voltage will cause the rotor to turn to the four positions — 60°, 120°, 240°, 300° — as shown in Figure 14–14.

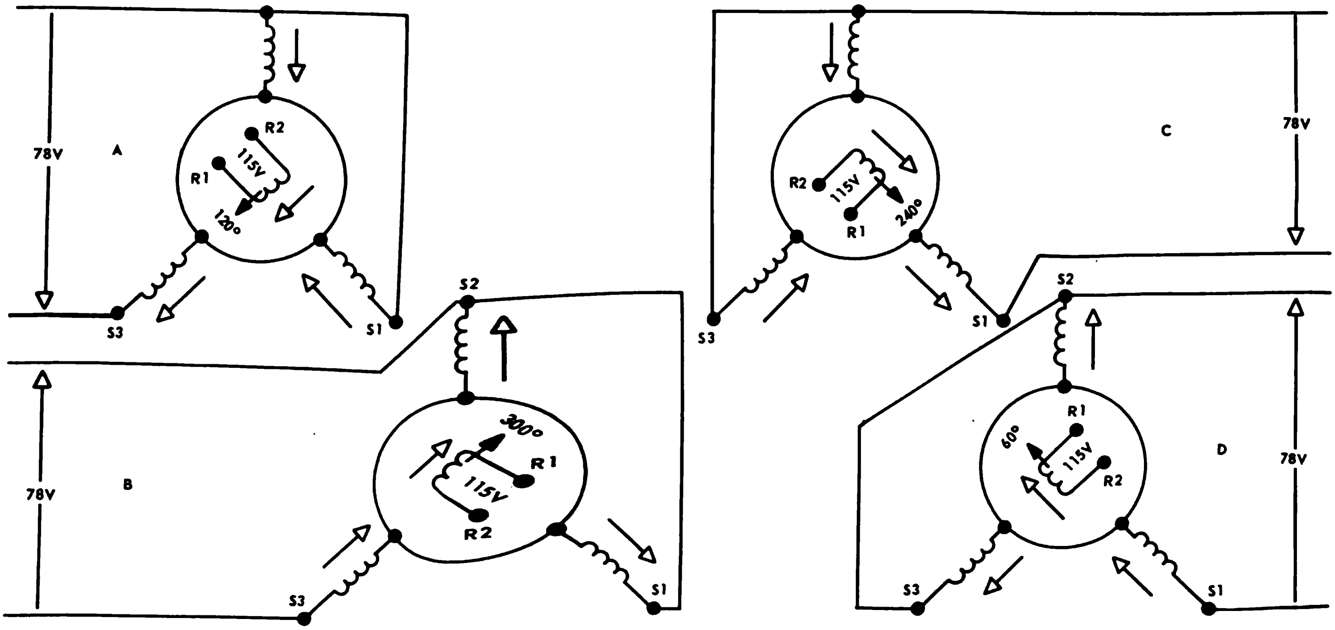


Figure 14–14. Four Positions Possible With Two Leads Connected

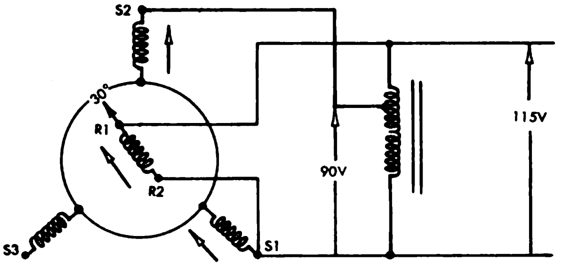


Figure 14–15. 30° Position

When S3 is left open and 90v is applied between S1 and S2 (90v is required to produce the same attraction as before), the rotor will come to rest in the 30° position. This results because the R1 end of the rotor is attracted to the coil connected to S2 with the same force as the R2 end is attracted to the S1 coil, as you can see in Figure 14–15. Reversing the phase polarity of the 90v causes the rotor to turn to the 210° position (top of Figure 14–16). By leaving S1 or S2 open and applying 90v to the remaining two terminals, either in phase or 180° out of phase, you can produce the four other positions shown in Figure 14–16.

So far, only the means of obtaining 12 positions, spaced 30° apart, has been described. Next, consider methods for obtaining positions at intermediate points between these positions.

Figure 14–17 shows a setup that gives positioning at any point from 0° to 60°. In it, fixed AC voltages are applied to S1 and S2. The voltage at S3 is variable from the value of the voltage at S1 to that at S2 with a variable autotransformer. If the voltage at S3 is the same as that at S2, the rotor will turn to the 60° position as shown in Figure 14–14D. If the voltage at S3 is equal to the voltage at S1, the rotor turns to the 0° position as indicated in Figure 14–12. These two conditions occur when the variable tap is at N and P respectively. When the tap is at M, midway between N and P, S3 is at zero potential with respect to the common connection of the coils and exerts no attraction or repulsion. Hence, the rotor turns to the 30° position just as it did when S3 was disconnected.

At intermediate points between M and N, the phase of the voltage in coil S3 will be as indicated by the dotted phase arrow and the rotor will assume a position between 30° and 60°, depending upon the magnitude of the voltage at

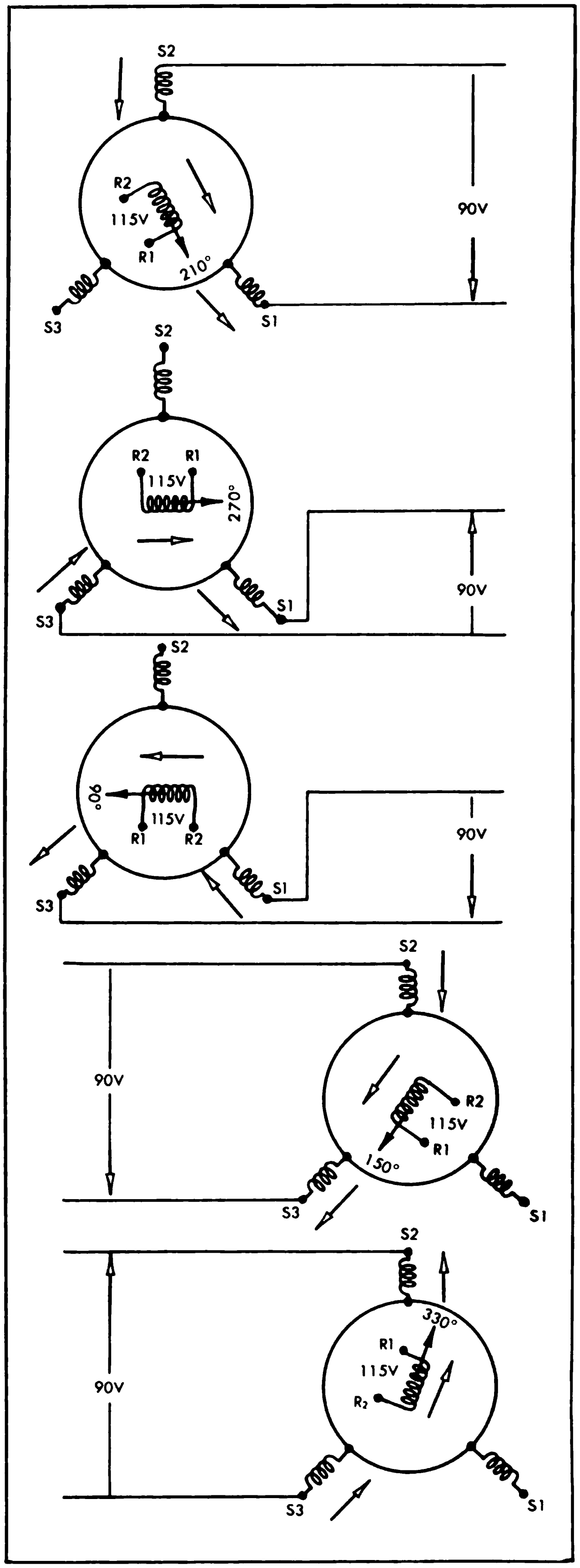


Figure 14–16. Other Positions with One Lead Open

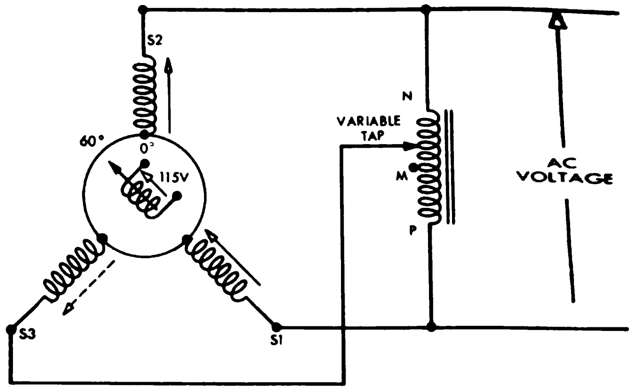


Figure 14–17. Using Variable Tap to Obtain Positions from 0° to 60°

S3. Between N and P the rotor turns to some point between 30° and 0° which depends on the magnitude of the voltage at S3, the phase polarity of which is opposite to that indicated by the arrow.

By using stator voltages intermediate to the voltages for the 12 basic positions, it is possible for the rotor to assume any position around the circle. The voltages necessary to produce any angular position may be plotted as in the graph in Figure 14–18. The three curves show the voltages between pairs of stator leads. The voltages above the zero axis are in phase with the voltage from R1 to R2, and those below the axis are opposite in phase. Keep in mind that the curve of the S2 to S1 voltages will be 180° out of phase with the curve shown for the S1 to S2 voltage.

Due to the method of arranging the rotor and stator windings on a standard synchro, the curves shown in the graph in Figure 14–18 are in the shape of sine curves. In other words, the curves look like time graphs of sinusoidal voltages. They show something entirely different, however, for time is not involved in any way in the preceding graph. Another thing to remember is that while they have the appearance of the voltages in three phase, they are really single phase. What is actually shown in the curves is the effective values of the three AC voltages plotted against position of the rotor, and whether the voltages are in phase or 180° out of phase with the rotor voltage.

Next, suppose you check the voltages in Figure 14–18 against the voltages in some of the cases previously discussed. Notice that when the rotor turns to the 0° position, the voltage from S3 to S1 is 0v, and the other two are 78v. This is the same situation shown in the 0° position illustrated in Figure 14–12. When the voltage between S1 and S2 is increased to 90v, each of the others is 45v; the shaft will turn to the 30° position as in Figure 14–15. In Figure 14–15, S3 is open and the 90 volts from S2 to S1 divide equally between the coils, placing 45 volts across each coil. Again using voltages from Figure 14–18, when S3 to S1 and S1 to S2 is 78 volts, you have the same situation as in Figure 14–14D; the shaft turns to 60°. At each position of the rotor there is a certain definite value and phase condition for each of the three voltages.

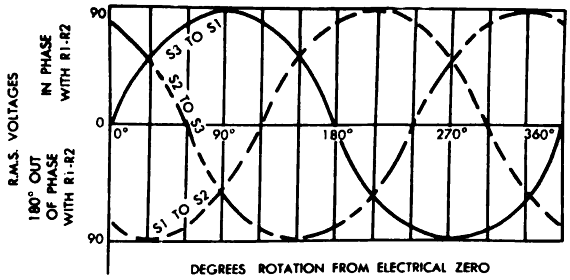


Figure 14–18. Voltage vs. Rotor Positions

Transformer Action of Synchro

Another important factor in synchro operation is transformer action. This deals with the voltages induced in the stator windings when an AC voltage is applied at various positions of the rotor. In studying this action, assume that at first the stator leads are open. Figure 14–19 shows one stator coil with the rotor in three positions. At A the axis of the rotor and stator are aligned, and the flux produced in the rotor due to the 115v applied to it induces a voltage in the stator coil. Due to the turns ratio this induced voltage in a standard synchro is 52 volts. This voltage is in phase with the rotor voltage as indicated by the open-headed arrows.

In B the rotor is in the 90° position. In this position the flux set up in the rotor is not effective in inducing a voltage in the stator because the flux that passes through the coil is at right angles to its axis and induces equal and opposite voltages on the opposite sides of each turn. Thus, at this position the net voltage is zero.

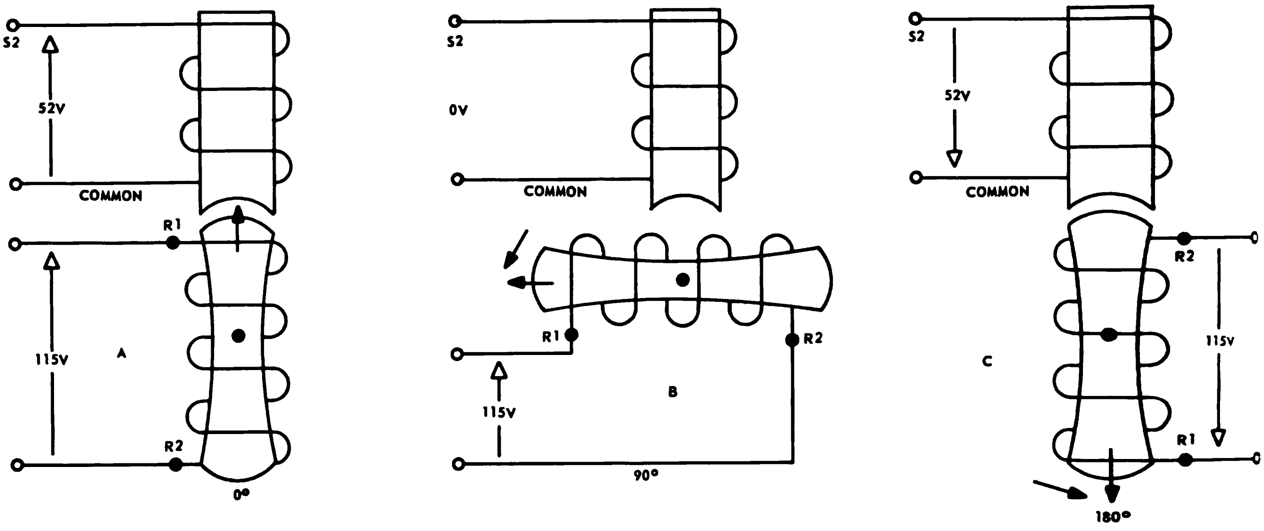


Figure 14–19. Transformer Action

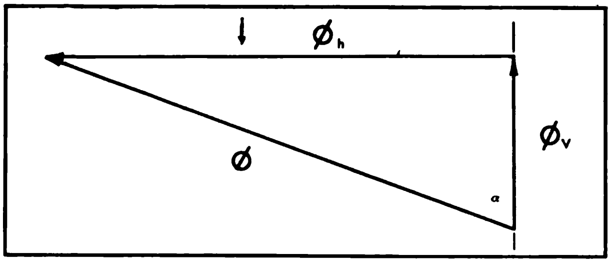


Figure 14–20. Components of Flux ϕ

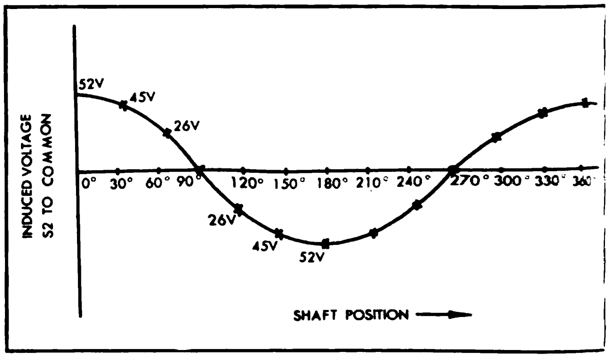


Figure 14–21. Induced Voltage vs. Shaft Position

In C the rotor is in the 180° position. Here the flux at any instant is 180° out of phase with what it would be in the 0° position. Thus, the voltage induced in the stator coil is 180° out of phase with the rotor voltage (R1 to R2). The magnitude is the same as for the 0° position, 52 volts.

At intermediate points the magnitude and relative phase polarity depend upon the flux threading the coil. If you resolve the flux (ϕ) into the components ϕ_h and ϕ_v, as shown in Figure 14–20, you can determine the relation between the flux and the induced voltage. The horizontal component is ineffective, while the vertical component of flux induces a voltage in the coil. But ϕ_v/ϕ is the cosine of angle α between the axis of the stator and rotor. Thus, you can see that for any angle of rotation the voltage induced in the stator coil S2 is 52 cos α volts. (Figure 14–21 is a graph of these voltages vs. shaft position.) Due to the 120° spacing of the stator coils, maximum voltage is induced in S3 at the 120° position and in S1 at the 240° position.

Since the connections are made to points S1, S2 and S3, the voltages appearing between those points are of special interest. These voltages are the sums of, or differences between, the voltages induced in the individual stator coils, depending on whether the voltages are in phase or 180° out of phase. For example, the voltage from S1 to S2 is the sum of the voltages from S1 to the common point and from the common point to S2. Figure 14–22 shows the magnitude of the voltages between stator connections for any position of the rotor. In comparing the voltages shown in this figure and those shown in Figure 14–18, note that the voltages induced in the stator coils

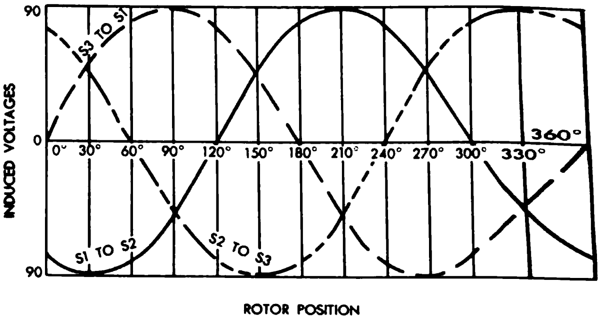


Figure 14–22. Induced Voltage vs. Rotor Position

are the same as the voltages required to position the rotor to the same angle.

How a Synchro Motor Follows a Generator

The synchro system in its simplest form consists of one motor connected to one generator as shown in Figure 14–23. With this connection, whenever the shaft of the generator turns, the shaft of the motor turns. The *electrical position* of the motor shaft is the same as that of the generator shaft. Thus, for example, when the generator shaft turns to electrical zero, the motor shaft turns to 0°, or when the generator shaft turns to 30°, the motor shaft turns to 30°, and so on. In order to see how the motor shaft follows the generator shaft, first examine the internal conditions of each synchro. To do this, consider both shafts at the same position and then both shafts at different positions.

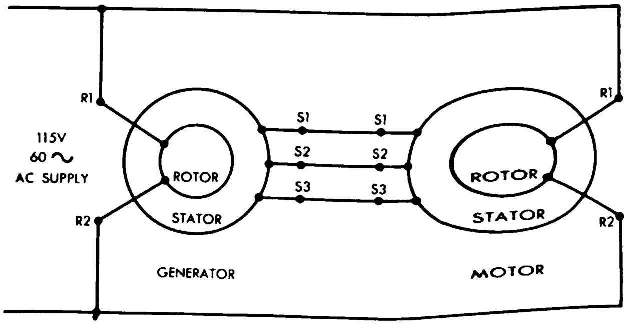


Figure 14–23. Synchro System Connections

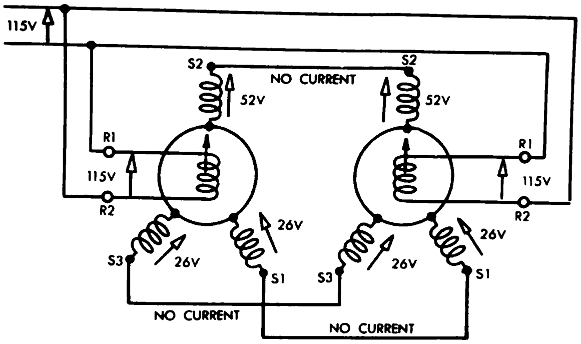


Figure 14–24. Generator and Motor Both at 0° Position

Figure 14–24 shows both shafts turned to the 0° position. In this position there will be voltages induced in the stator windings of both generator and motor, with the phase polarity and magnitude indicated. Note that the voltages of the motor are equal to those of the generator, but because of the method of connection, the voltages in one oppose the voltages in the other. This causes no current to flow in the stator coils. Hence, there is no magnetic field set up by them and thus there is no attraction or repulsion to cause either rotor to turn. The rotors will remain in the 0° position indefinitely if no change in outside conditions occurs. The same situation exists if the two rotors are at any other position, except that the magnitudes of the induced voltages vary in accordance with the curves shown in the induced voltage versus rotor position graph in Figure 14–22.

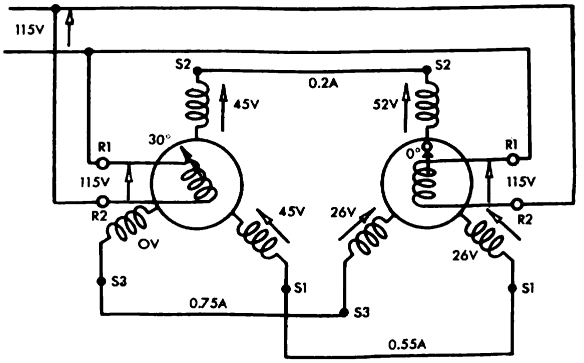


Figure 14–25. Shafts in Different Positions

Figure 14–25 shows generator and motor shafts at different positions. When the two shafts occupy different positions, the voltages induced in the stator windings are not balanced as they are when the shafts occupy the same position. Suppose, for example, that the generator rotor is at the 30° position and that the motor rotor is at the 0° position. By referring to Figure 14–22 showing rotor position, you can determine the value of the induced voltages. Note that there is no balance among the voltages (Figure 14–25) and that currents flow in all three stator leads and all stator coils of both generator and motor.

The currents are greatest in the circuits where the voltage unbalance is greatest, that is, in the circuit composed of S3 coils and common. The effect of these currents is to produce magnetic fields that tend to turn the motor shaft to the same position as the generator shaft. Thus, if the motor shaft is free to turn it will take the 30° position. As the motor shaft turns, the misalignment decreases and, in turn, the unbalance of voltages decreases and the currents produced by them decrease. When the shaft reaches the 30° position, the same conditions that prevailed when both shafts occupied the same position will prevail.

A torque also is set up in the generator which causes it to turn in the clockwise direction, resulting in both rotors tending to align themselves. In the usual system the generator shaft is attached to some device so that the shaft is not free to rotate.

To see more clearly the magnetic effects producing the torques influencing motor operation, note in Figure 14–26 the conditions in the circuit *at a given instant.* The same conditions do not hold except for that particular instant. Examination of the figure shows the following action: (1) the lower end of the S2 coil of the generator attracts the R1 end of the rotor, (2) the S end of the S1 coil repels the R2 end of the rotor, and (3) the N end of the S3 coil attracts the R2 end of the rotor. All these forces tend to cause rotation in the clockwise direction.

In the motor the magnetic poles of the stator coils are the reverse of those of the generator. Consequently, the forces of attraction and repulsion are reversed and the torque is counterclockwise. At one-half cycle (of the input voltage) later, the polarities of the magnets and the directions of current flow will reverse. The forces of attraction and repulsion do not change, however, since the polarities of the stator and rotor magnets reverse at the same instant.

How Currents Flowing in a Synchro System Depend upon Shaft Positions

As was shown, current flows in the stator circuits whenever the two shafts occupy different positions. In any stator lead the amount of this current flow depends on the difference between the voltage induced in the two coils to which that lead connects. Suppose that the current in any one stator lead (S2 for example) is measured and that the actual shaft positions are adjusted so that this current is maximum for each difference of the shaft positions. From Figure 14–27, which shows this maximum possible current versus the difference in shaft positions, you see that currents as high as 3 amperes are possible if the shafts are held in positions differing by 180°.

But in practical operation the shafts are seldom more than a degree apart, since the motor shaft starts turning as soon as the generator shaft starts to rotate. Hence, the current in any lead seldom becomes more than one-tenth of an ampere. The current in one stator lead depends not only on the difference of shaft positions, but

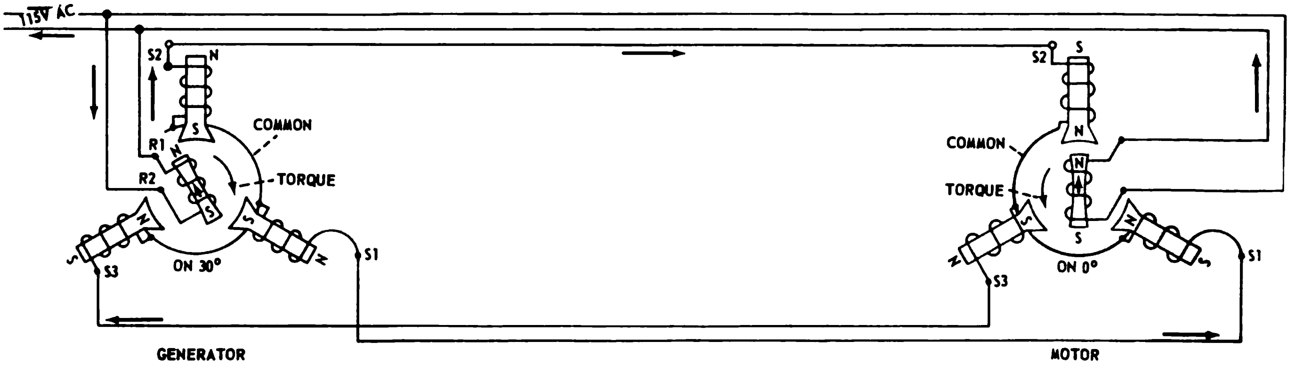


Figure 14–26. Instantaneous Forces, Shafts in Different Positions

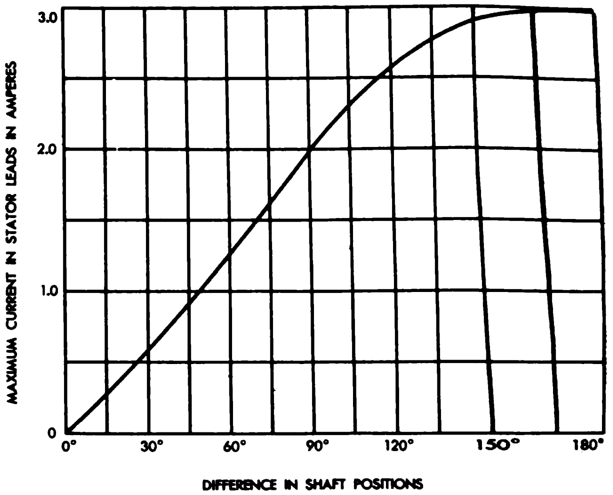


Figure 14–27. Stator Current vs. Difference in Shaft Positions

also on the positions themselves. Figures 14–28 and 14–29 show the percent of maximum current in each lead for the various mean shaft positions (average positions between the two shaft positions).

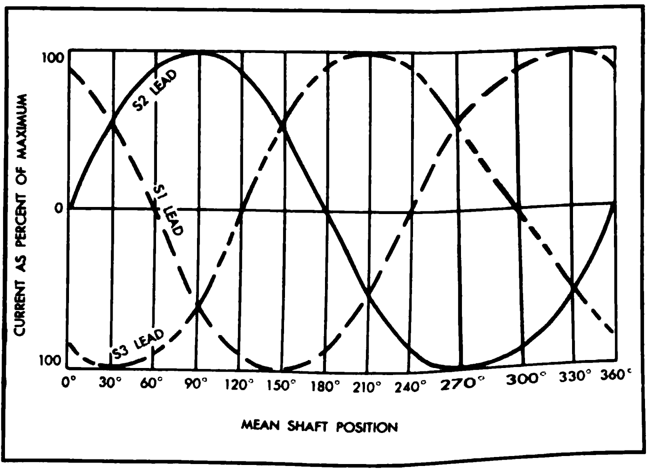


Figure 14–28. Stator Current vs. Mean Shaft Position

To illustrate how these two figures may be used in conjunction, take the case where the generator shaft is at 60° and the motor shaft is at 120°, a difference of 60°. By referring to the difference in shaft position diagram, you can see that the maximum possible current in any one lead is 0.95 ampere. By referring to the mean shaft position chart you see what percentage of this maximum current flows in each lead with a mean shaft position of 90°. The S2 lead carries 100% or 0.95 ampere, and the S1 and S3 leads carry 50% or 0.475 ampere. The fact that the current S1 and S3 carry is shown below the zero axis means that the phase is opposite that of the current flowing in the S2 lead. Remember that currents are usually not high since small angular differences are the usual or normal condition.

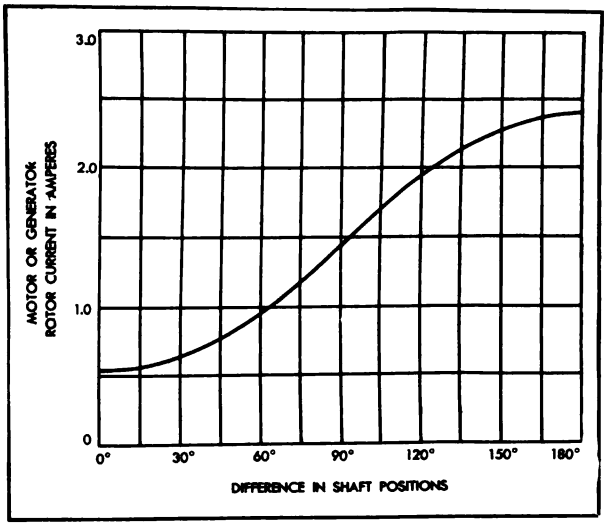


Figure 14–29. Rotor Current vs. Difference in Shaft Positions

Since the synchro generator or motor acts like a transformer, an increase in stator current causes a corresponding increase in rotor current. (See Figure 14–29). Although all three stator currents are zero when the shafts are aligned, the rotor current is still approximately 0.5 ampere. As in any transformer the primary draws some current with no load on the secondary. This current produces magnetization of the rotor and supplies its losses.

The following statements summarize the factors concerning the relative shaft positions:

- Reversing the rotor leads on either motor or generator causes a 180° difference in shaft positions.
- Interchanging the S1 and S3 leads on either the generator or the motor causes rotation of the motor shaft in the opposite direction to that of the generator.
- Shifting the stator leads so that S1, S2, and S3 of the generator connect to S2, S3, and S1 of

the motor produces a fixed difference of 120° in shaft positions.

Since both the synchro motor and the load attached to it have some friction and inertia, some torque is required both during motion and during acceleration. Since no torque is developed when the two rotor positions coincide, perfect accuracy is impossible. Some angular displacement is necessary to overcome friction and inertia. This lag is not appreciable; in most cases it is only a fraction of one degree. To minimize the inaccuracy, the friction of the motor must be as low as possible and the torque gradient (torque per degree displacement) must be as high as practicable. The torque gradient depends upon the internal impedance of its stator coils. Hence, both the motor and generator must be large enough, for if either is too small, the higher impedance of the windings will reduce the current for small unbalances of voltages and the torque will accordingly be less.

Note in the graph of the torque versus angular displacement relations for a typical synchro (Figure 14–30), that for small angles the graph is practically a straight line. The term *torque gradient* is simply the slope of the curve in that region. Mamimum torque is developed at approximately 100°. Figure 14–30 shows zero torque at 180° displacement. This is true because the powerful forces that are exerted are in a state of balance. This is, therefore, an unstable condition and any small change in either rotor position results in a return to the zero displacement condition.

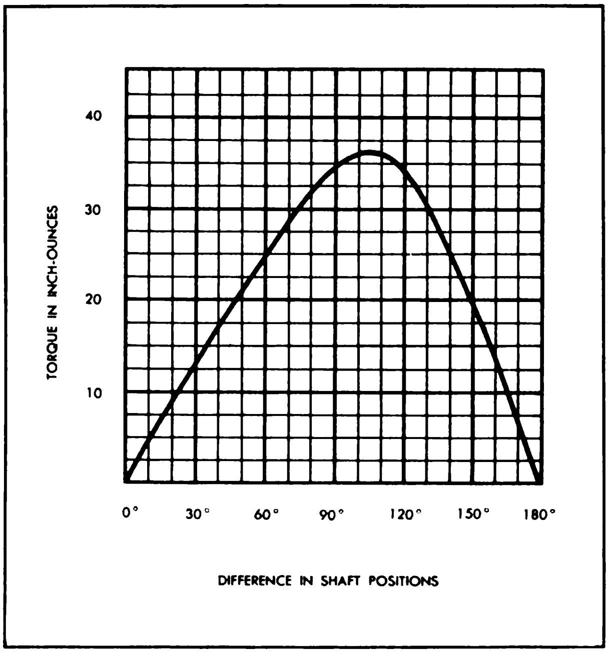


Figure 14–30. Torque vs. Displacement

One synchro generator is sometimes used to transmit data to several motors at various locations. For example, the synchro generator connected to the gyrocompass aboard a plane is used for driving synchro motors at several of the compass repeater stations. If there are more than two motors, a larger generator must be used to maintain accuracy. If any synchro motor becomes jammed, it affects the accuracy of the entire system since it acts as a generator competing with the real synchro generator. As shown in Figure 14–31 all the corresponding stator and rotor coils are connected in parallel.

THE DIFFERENTIAL SYNCHRO

In a mechanical system a differential connects three shafts together in such a way that the amount that one shaft turns is equal to the difference between the amounts that the other two turn. The differential synchro is named according to the two functions it performs. If the shaft of the differential synchro serves to indicate the difference in shaft positions of two other synchros, the other two synchros are generators and the differential synchro is a *motor*. If the differential synchro shaft position is to be subtracted from that of a synchro generator (or vice versa) and the difference to be indicated by the rotor position of a motor, the differential synchro is then a *generator*.

The stator of the differential generator or motor is very similar to the stator of the ordinary synchro. It consists of three sets of coils wound in slots and spaced 120° apart around the inside of the field structure. The rotor, however, differs considerably from that of an ordinary synchro. It is cylindrical in shape and has *three* sets of coils wound in slots and equally spaced around the circumference, Figure 14–32. Connections to the external circuits are made through three brushes riding on three slip rings on the rotor shaft. As in the ordinary synchro, the motor has an oscillation damper and the generator does not. There is no connection to the 110v AC as in the case of

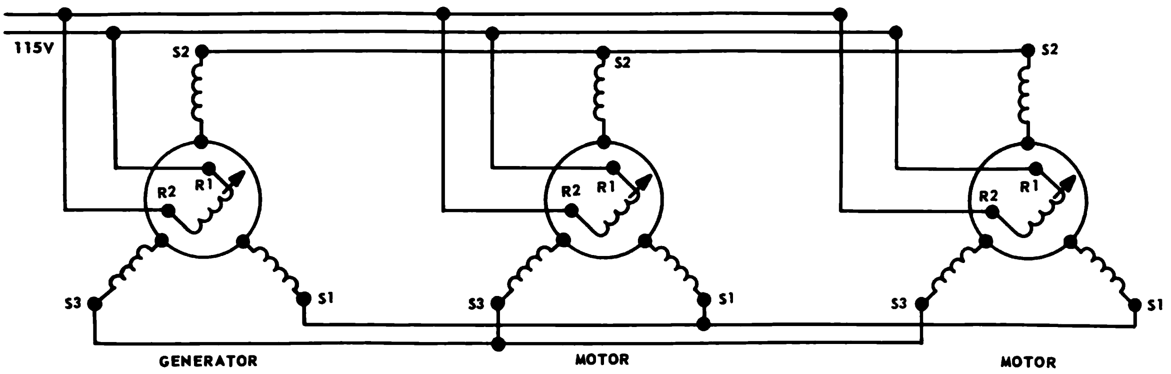


Figure 14–31. Two Motors Connected to One Generator

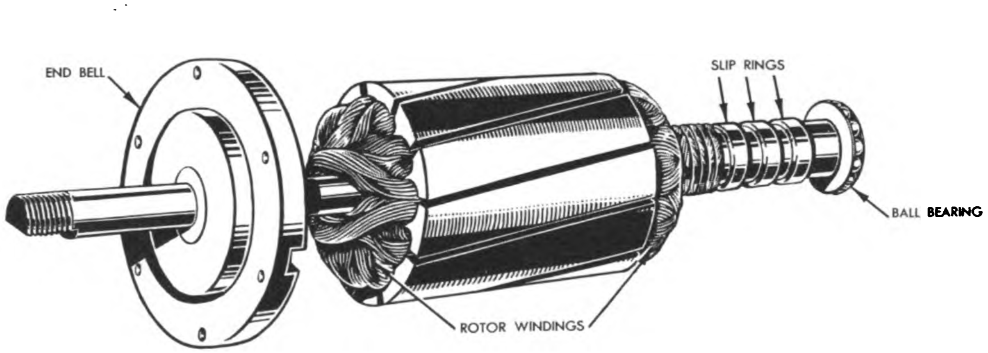


Figure 14–32. Rotor of Differential Synchro

the ordinary synchro. The three leads to the rotor are designated R1, R2, and R3. Electrical zero is the position in which rotor coil R2 is aligned with stator coil S2, R1 with S1, and R3 with S3.

Electrically, the differential synchro acts as a 1:1 transformer with the stator coils as the primary and the rotor coils as the secondary. Due to the air gap, there must be a few more turns on the stator coils than on the rotor; hence, the transformation is not a 1:1 ratio when the rotor coils are connected as the primary.

The synchro generator in Figure 14–33 is connected to a differential synchro in which the rotor leads are open. Both shafts are in the 0° position. Since the stator coils of the differential synchro connect to the stator coils of the synchro generator, the voltages are equal in magnitude and are in phase. Since the differential is a 1:1 transformer, the induced voltages are equal in magnitude and are in phase with the stator voltages. When the generator shaft is turned to some other position, the voltages in the stator coils of the differential and, consequently, those induced in the rotor coils, will be the same as the voltages in the stator coils of the generator.

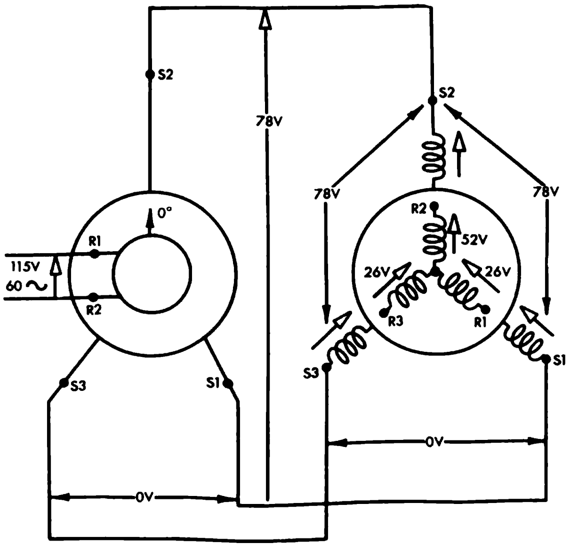


Figure 14–33. Differential Transformer Action at 0° Position

As the generator shaft is rotated with the differential shaft remaining in the 0° position, the rotor voltages of the differential will be as shown in the rotor position diagram in Figure 14–24. A motor with its stator windings connected to the rotor windings of the differential will have the correct voltages applied causing it to assume the same position as the generator rotor. This means that with the differential rotor in the 0° position, the system functions as it would if the differential were out of the circuit and connections were directly from generator to motor.

When the generator rotor is in the 0° position, and the rotor of the differential is turned to 15° as shown in Figure 14–34, the voltages induced in the rotor coils will be as indicated. The voltages between terminals R1–R2, R2–R3 and R3–R1 are respectively 63v, 87v, and 24v, and the R2–R3 voltage is in phase with the R1–R2 voltage across the rotor of the generator. The other voltages are 180° out of phase. These voltages correspond to the voltages induced in the stator windings of a synchro generator in which the rotor is at the 345° position. To understand this action, consider the relative angle of the inducing magnetic field axis and the axis of the secondary coils.

When a synchro generator rotor is in the 345° position, the field axis is 15° *clockwise* from the axis of the S2 secondary coil; the voltage induced in it equals 52 cos (−15°). Likewise the voltages induced in S1 and S3 equal 52 cos (−255°) and 52 cos (−135°). These voltages set up a field in the motor that equals the vector sum of the three fields of the stator coils, resulting in a field with an axis at the 345° position. The rotor of a synchro motor accordingly moves to the 345° position. Now in the differential synchro, the composite primary field has reached the 0° position while the secondary (rotor) coils have rotated 15°. This makes the field axis 15° *clockwise* from the axis of the R2 coil.

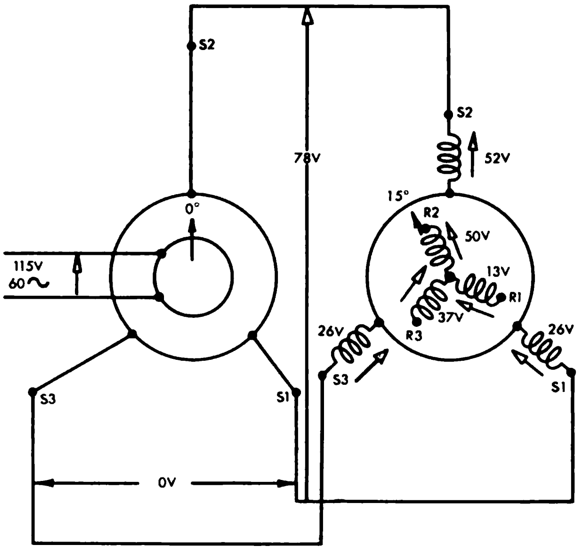


Figure 14–34. Transformer Action at 15° Position

The induced voltages then have the values previously stated and shown in the transformer action illustrated in Figure 14–34. If the motor had its stator coils connected to the terminals R1, R2, R3 of the differential, the voltages applied to it would cause the rotor of the motor to move to the 345° position. Conversely, if the rotor of the differential were turned to the 345° position, the motor would turn to the 15° position. If the generator shaft position is at 0°, the voltages induced in the rotor of the differential synchro are the same as if the rotor of the differential were at 0° and the rotor of the generator were rotated *in the opposite direction.*

Thus, whenever the shafts of both generator and differential rotate, the resultant field and induced voltages vary with the difference of the angular position of the shafts. A motor connected to the rotor coils of the differential will assume a shaft position equal to the difference of the angular positions of the other two shafts. The mathematical proof, quite a long one, is beyond the scope of this manual. Briefly, it involves setting up equations for the voltages induced in each of the rotor coils of the differential by each of the stator coils and adding the induced voltages in pairs.

A qualitative explanation of what causes the motor shaft to turn to the indicated position is fairly simple to understand. For example, take the case illustrated in Figure 14–35. The generator rotor is at the 180° position. Hence, the voltages induced in S1, S2, and S3 are 26v, 52v, and 26v, respectively, in the phases indicated. All the voltages in S1, S2, and S3 of the differential are identical. These voltages set up a magnetic field in the differential that has the same position, 180°, for its axis as the field in the generator. If the axis of the rotor of the differential generator

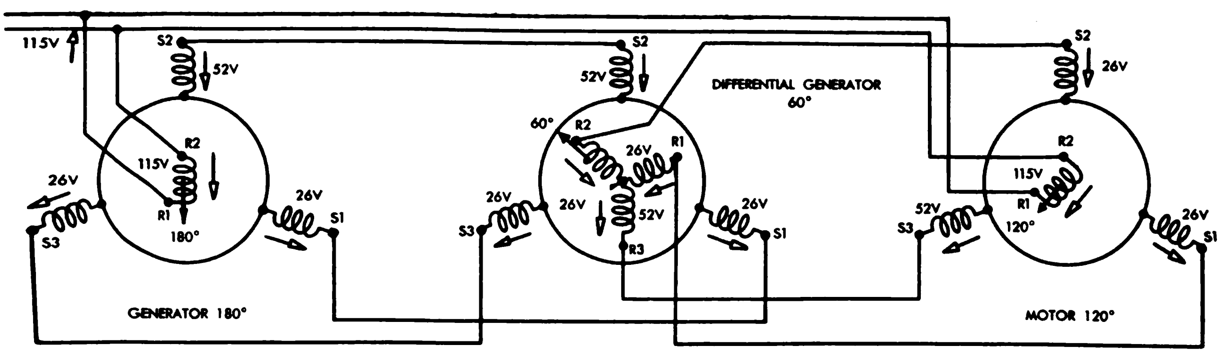


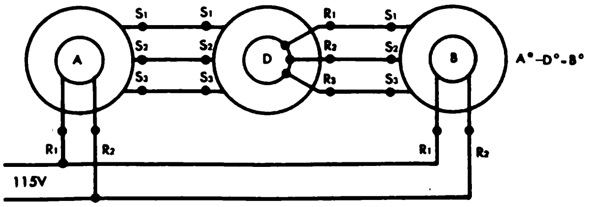
Figure 14–35. Synchro System with Differential Generator

were aligned with the 0° position, the induced voltages would be identical to the stator voltages. But since the rotor coil axis is shifted to the 60° position, the axis is only 120° from that of the magnetic field.

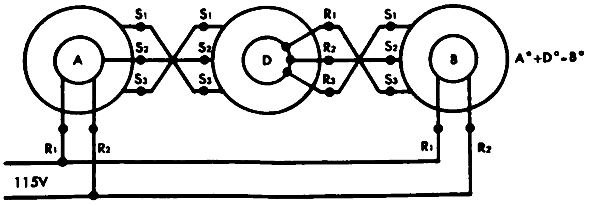
Therefore, the voltages induced in R1, R2, and R3 are 52 cos 120°, 52 cos 240°, and 52 cos 0° volts, respectively. The voltages from R1 to R2, from R2 to R3 and from R3 to R1 are therefore 0v, 78v, and 78v with R2–R3 180° out of phase with R1–R2 of the generator. When these voltages are applied to the motor, the rotor will turn to such a position that the voltages induced in its stator coils by the rotor just balance these voltages. This happens when the rotor is at the 120° position, the difference between 180° and 60°.

By reversing pairs of leads either between the generator and the differential or between the differential and the motor, you can make any one of the shafts assume a position equal to the sum or difference of the angular positions of the other shafts. The proper connections for various types of operation are shown in Figure 14–36.

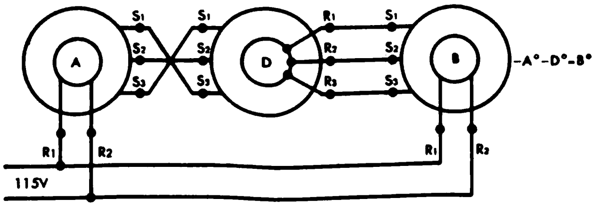
A differential synchro cannot be a 1:1 transformer in both directions. Therefore, the perfect balance of voltages indicated is not strictly accurate. The differential synchro is so designed that the voltages induced in the rotor coils are equal to the voltages applied to the stator windings provided the shaft is at 0°. When a differential is connected between a generator and a motor, some current flows in the generator's stator circuit, reducing the output voltages due to the IR drop. These lower than normal voltages are induced in the rotor windings of the differential. They are too small to buck out the motor voltages, and some current flows in the motor stator circuits. The generator currents, usually higher, reduce the accuracy of the system. Therefore, when a very high degree of accuracy is required, the use of a differential is avoided if at all possible.



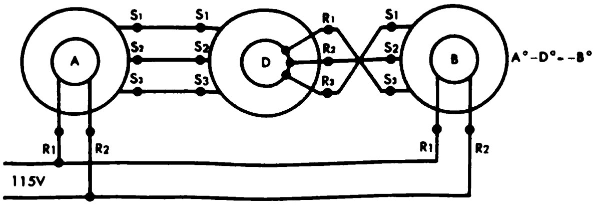
DIRECT CONNECTIONS



DIFFERENTIAL REVERSED



"A" REVERSED



"B" REVERSED

Figure 14–36. Various Connections for Synchro System

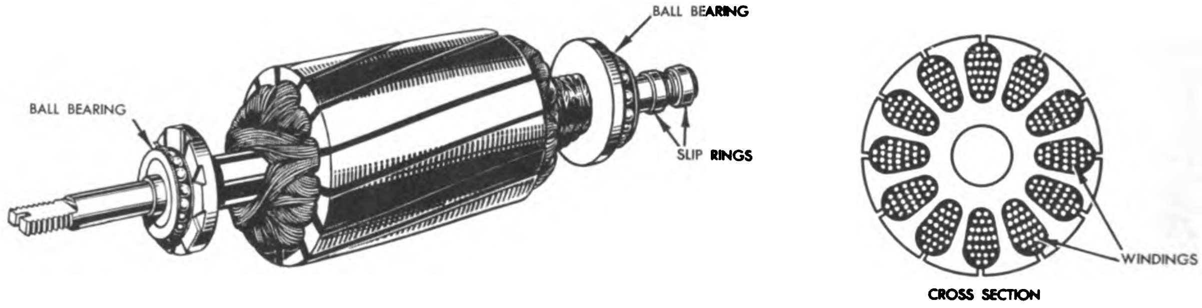


Figure 14–37. Typical Control Transformer Rotor

The Control Transformer

Synchros are used to cause two shafts to rotate in synchronization and to produce an error voltage that indicates the difference in position of two shafts. The systems discussed up to this point are for synchronization purposes. For the purpose of producing error voltages, there is a type of synchro known as the control transformer that is used in conjunction with a synchro generator. Both the generator and the transformer have their shafts connected to loads. The voltages induced in the stator coils of the generator are applied to the stator of the control transformer. These voltages induce in the rotor of the control transformer (not connected to the 115v AC) a voltage whose magnitude and polarity depend upon the relative positions of the two shafts.

The stator of a control transformer is very similar to that of a synchro generator or motor except that the coils are wound with more turns and finer wire. The purpose of this is to make the impedance high enough to prevent high currents from flowing. Remember that no voltage is induced in the stator coils of the control transformer to buck the applied voltage and keep the current within safe limits. The stator current is determined by the impedance of the stator windings and is practically independent of the rotor's position.

The rotor of a control transformer (Figure 14–37) looks somewhat like the rotor of a differential synchro. Yet there are some important differences. Instead of three groups of windings spaced 120° apart, all the windings are in series and have their external connections through two slip rings and brushes. The shaft always connects to a load making an oscillation damper unnecessary The coils contain many turns of fine wire. The turns ratio is such that the maximum output voltage, with normal stator voltages, is 55 volts. There is no appreciable current in the rotor windings. The rotor is not free to turn to any particular position when voltages are applied to the stator windings. The electrical zero position of the rotor is such that no voltage is induced in the rotor windings. This means that the coil axis is at the right angle position as shown in Figure 14–38. Note that the terminals are labeled exactly as are those of a synchro motor or generator.

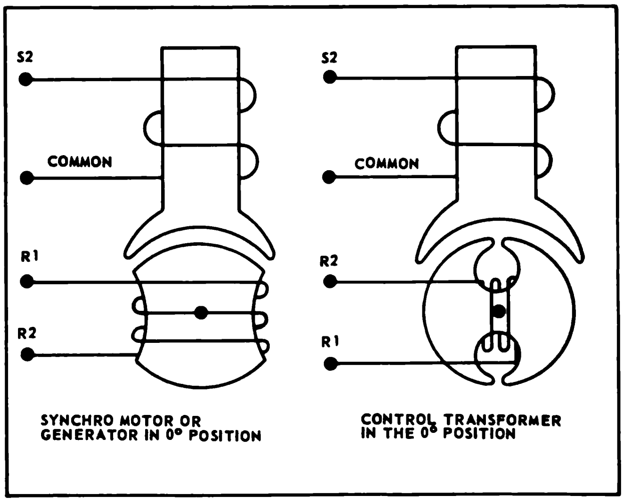

S2
COMMON
R1
R2
SYNCHRO MOTOR OR GENERATOR IN 0° POSITION
S2
COMMON
R2
R1
CONTROL TRANSFORMER IN THE 0° POSITION

Figure 14–38. Comparison of Electrical Zero Position

The control transformer operates in this manner. In Figure 14–39, the generator shaft is at 0° and the control transformer shaft is at 0° position. The axis of the magnetic field set up by the stator voltages is in the 0° direction — at right angles to the rotor windings. It induces no voltage in the rotor windings. Now assume that

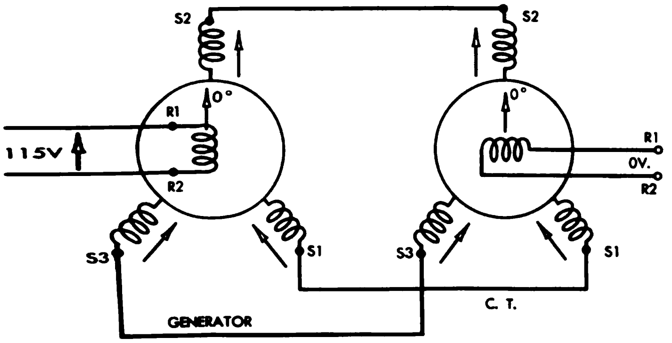


Figure 14–39. Rotor at 0° Position

the control transformer rotor is turned to the 90° position. The axis of the coil will then be aligned with the composite magnetic field and there will be a maximum (55v) voltage induced in it. The phase polarity of the R1–R2 voltage will be identical to the R1–R2 voltage of the generator rotor.

At other positions you can determine the magnitude of the induced voltage by resolving the magnetic field into two components, one of which is perpendicular to the axis of the coil and induces no voltage, and the other which is aligned with the coil and induces a voltage. The component of the field that is parallel to the coil axis is proportional to the sine of the angle to which the control transformer rotor is turned. Note in the induced voltage vs. shaft position graph, Figure 14–40, that voltages above the zero axis are in phase with the generator rotor voltage, R1–R2, and that voltages below the axis are 180° out of phase. Obviously, then, the voltages from the 180° position to the 360° position are 180° out of phase.

When the rotor of the generator is turned to some position other than 0°, the magnetic field of the control transformer will follow and assume the same position. When the rotor of the control transformer is turned to the same position, the output will be zero volts, since the axis of the coil is perpendicular to the field. The rotor, starting from this position, on rotating in a counterclockwise direction will cause the voltages induced in the control transformer to assume the values indicated in Figure 14–40, where the abscissa is the difference in angular position of the two rotors. This action indicates that if the control transformer (CT) shaft *leads* the generator shaft by any number of degrees up to 180, the output is *in phase* with the generator rotor voltage, but that if the CT shaft *lags* the generator shaft by any number of degrees up to 180, the output is opposite in phase.

SERVOMECHANISMS

You have already seen how synchros transmit data from one place to another. When the shaft to be driven at the remote position is connected to an indicating device or some light load, the synchro motor is capable of developing the necessary torque. If the load is greater and more torque is necessary, then torque amplification is

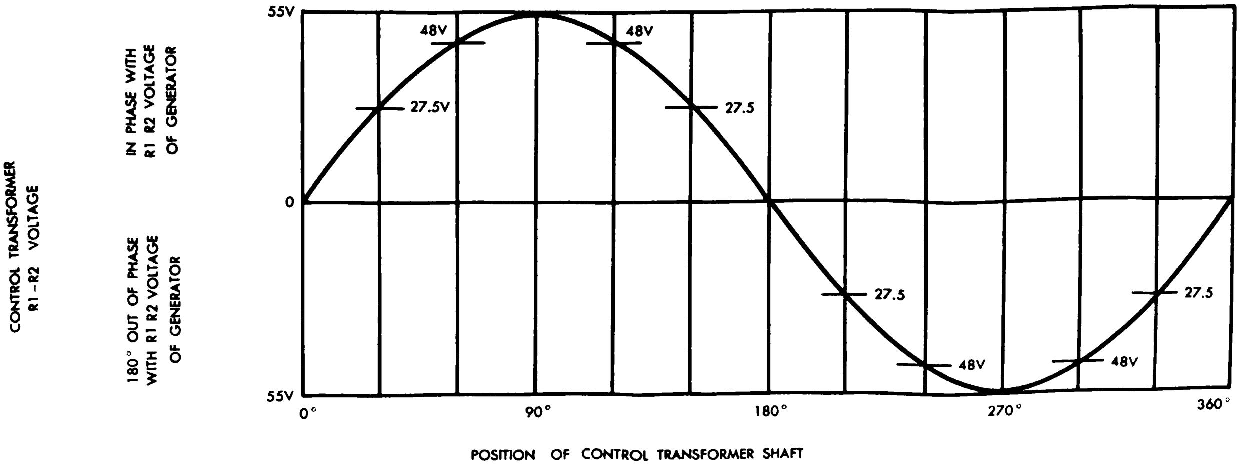


Figure 14–40. Induced Voltage vs. Shaft Position

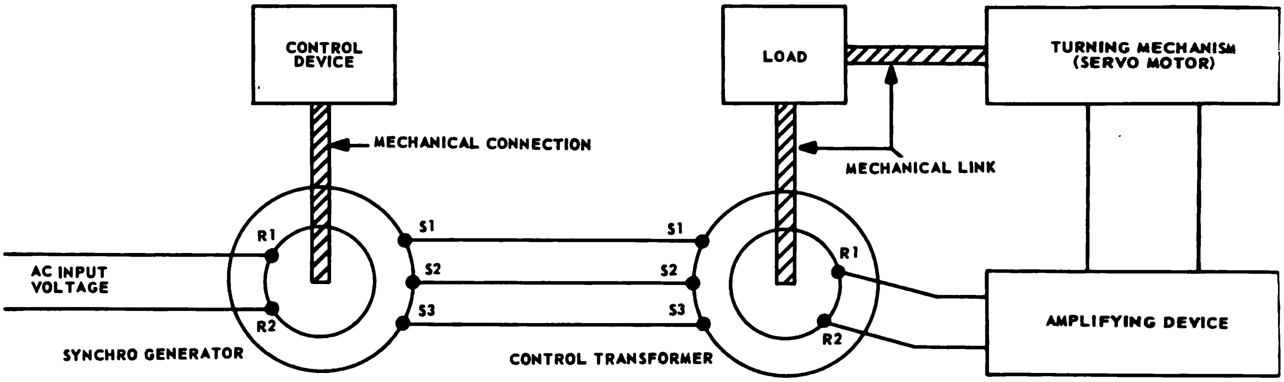


Figure 14–41. Simple Servomechanism

required. In this case the load is driven by a reversible adjustable-speed motor which is called a servo motor. The complete system, including the motor and its control devices, is called a *servomechanism.* The speed and direction of rotation of the motor are controlled by the magnitude and phase of an "error voltage" that is produced by any misalignment of the two shafts (the load shaft and the control shaft). Obviously, the direction of rotation must be such as to reduce the misalignment causing the error voltage.

The principles of the servo system in its simplest form are shown in Figure 14–41. Assume that the control device is a crank or a handwheel and the load is the antenna. The handwheel is connected to a synchro generator, and the antenna to a control transformer. The stator windings of the generator and control transformer are connected electrically as shown. The output of the control transformer is an AC voltage that depends upon the relative shaft positions for magnitude and phase. There is no output when the shafts are in the same position, and accordingly no output from the amplifying device. Hence, the servo motor does not turn and the antenna remains stationary. To change the position of the antenna, you merely turn the handwheel to the desired position. This produces an error voltage in the control transformer. This voltage is rectified and increased in magnitude by the amplifying device and then applied to the servo motor, causing it to turn the antenna to the desired position.

The synchro generator and control transformer may be interchanged in position. However, such an interchange necessitates five electrical leads from one position to the other and gives no advantages. If the control device rotates continuously in one direction, the servo motor will rotate in the same direction at the same rate but with a small angle of lag. Remember that an error voltage is necessary before torque is developed. The greater the torque required, the greater will be the angle of lag.

The Servo Motor

An AC motor, inherently a constant speed device, is not suitable for the requirements of a servomechanism and therefore a DC motor is used. There are two methods of controlling the speed and direction of rotation — controlling the voltage applied to the field coils, and controlling the armature voltage. Controlling the field voltage permits control of speed only from a certain minimum speed upward, since speed varies inversely as the flux set up by the field. However, reduction of speed to zero at times is necessary; hence, the control in the servo system is always accomplished by varying the armature voltage.

Placing a variable resistor in series with the armature to control the armature voltage is wasteful since the resistor itself dissipates considerable power. Usually armature voltage is controlled by a DC generator driven by an AC motor. The Ward Leonard drive, discussed later, is an example of this system.

Servo motors operate with constant fields. The field is supplied either by permanent magnets or by DC applied to the field coils. The method of applying DC voltage to the field coils is wasteful of power, since power is consumed even when the motor is not turning. In addition, the design of

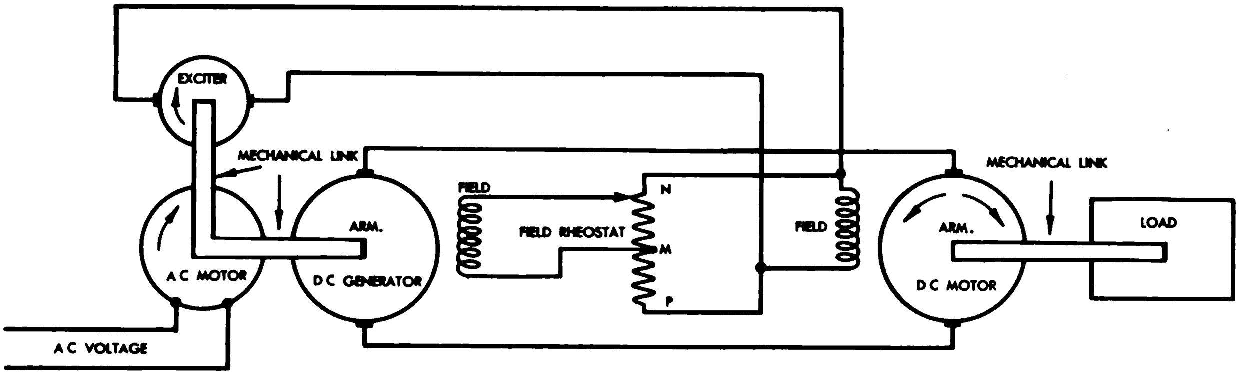


Figure 14–42. Ward Leonard Drive

the equipment is much more complicated than the permanent magnet arrangement. Mainly for this reason the permanent magnet is the more widely used.

One problem in the use of permanent magnets is the danger of demagnetization of the magnet by the field set up by current flow in the armature. This is overcome by the inclusion of a special winding that is connected in series with the armature and wound in the opposite direction. In this winding, called the compensating winding, a compensating field is set up that results from current flow through it. Since the current in the compensating winding varies with the armature current, the compensating field balances out the armature current and protects the permanent magnets.

The Ward Leonard Drive

In the Ward Leonard drive shown in Figure 14–42, an AC motor turns at a constant speed always in the same direction. It drives the armatures of the large and small DC generators. The small generator provides excitation voltages for both the large DC generator and the motor. The DC motor field is constant, being supplied by field coils. The generator field is not constant. Both its polarity and magnitude are controllable by a field rheostat.

When the variable arm is at the point M, no voltage is applied to the generator field coils and, consequently, there is no output voltage, no voltage applied to the armature of the DC motor, and no motor operation. When the variable arm is between M and N, a voltage of a certain polarity is applied to the field coils of the generator; the generator output has a certain polarity and causes the motor to turn in a certain direction. When the variable tap is located between M and P, the generator field voltage is reversed in polarity, the generator output is reversed in polarity, and the motor turns in the opposite direction.

In either case the motor speed depends upon the voltage applied to the field coils and is controllable by the rheostat. Thus, you can control both the speed and direction of rotation of the motor. It will therefore function very well as a servomechanism if the generator field is controlled with an error voltage instead of the field rheostat.

Servo System Using the Ward Leonard Drive

When the Ward Leonard drive is used for a turning mechanism, the servo system operates as shown in Figure 14–43. The error voltage produced by the control transformer (CT) is fed into the control amplifier, whose output is a DC voltage having a polarity and magnitude that is determined by the phase and magnitude of the CT voltage. The output of the control amplifier is applied to the field of the DC generator of the Ward Leonard drive, starting the action just described. The circuits of the control amplifier are explained later. The purpose of synchro generator G2 and synchro motor M1 is to furnish information to the operator as to the position of the antenna. In some cases this could be dispensed with since the scope pattern furnishes the same information when the scan is of the B or the PPI type.

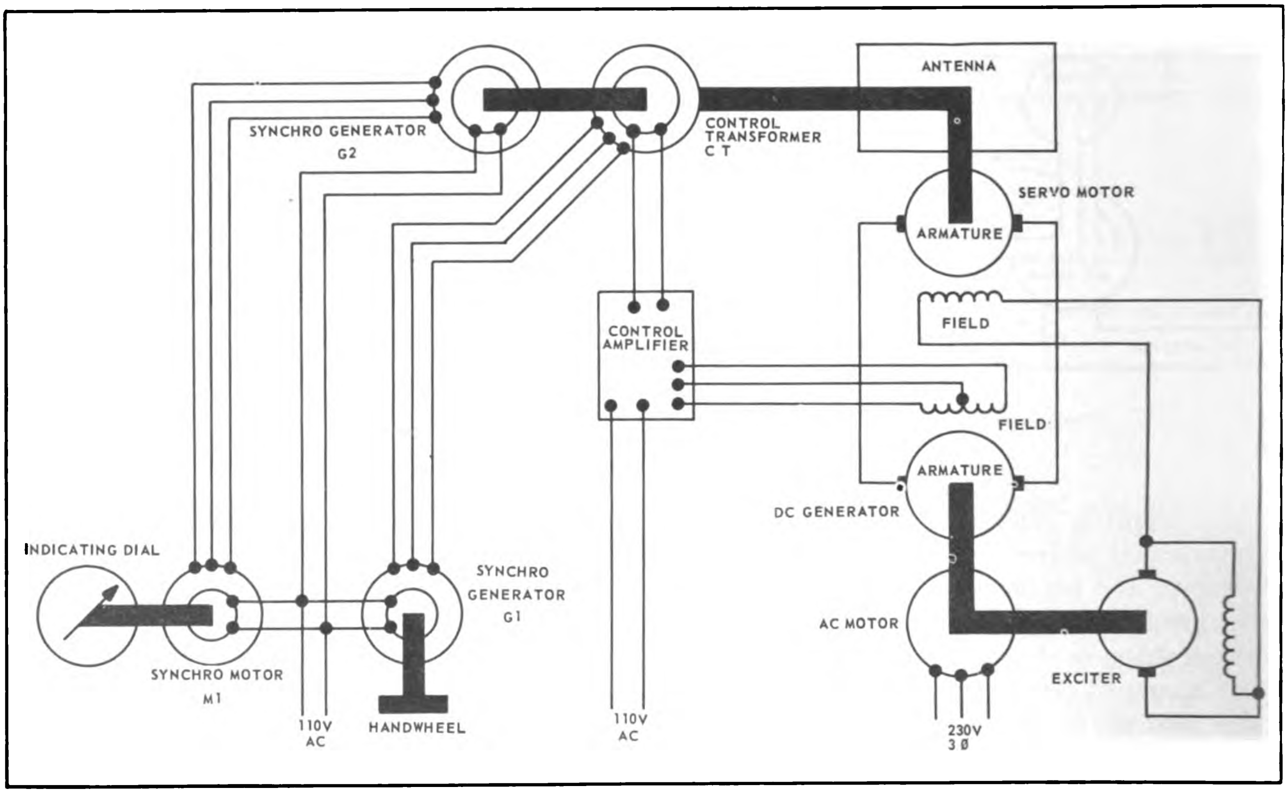


Figure 14–43. ***Servo System Using Ward Leonard Drive***

The Amplidyne

The amplidyne is a special type of DC generator commonly used for the DC generator in a system similar to the Ward Leonard. Such a system is known as an amplidyne servomechanism. The word amplidyne means dynamo amplifier.

The DC generator may be considered an amplifier since a small amount of power applied to the field coils controls 10 to 100 times as much power in the output of the generator. Of course, the additional power comes from the mechanical power that turns the armature. This mechanical power is comparable to the B+ voltage of a vacuum tube amplifier.

If greater amplification is desired than possible with a single DC generator, then two are used in cascade; that is, the output of the first (the exciter) is the input to the second. While an exciter was used in the Ward Leonard drive system, it may not be considered two stages of amplification, since the input to the exciter was uncontrolled. When two generators are used in cascade as shown in Figure 14–44, the over-all amplification is the product of the two amplifications. For example, if each amplifies the power 30 times, the total amplification is 900.

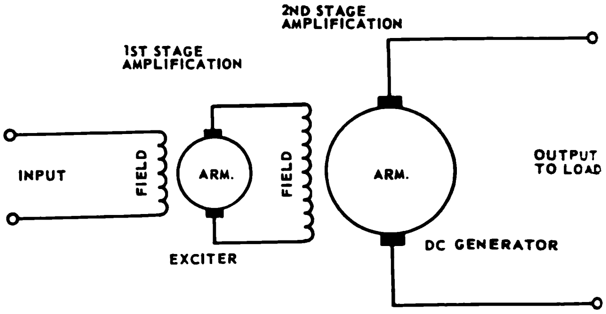


Figure 14–44. ***DC Generators as Cascade Amplifiers***

The amplidyne performs as a two-stage DC generator-amplifier with the two stages combined into one armature, Figure 14–45. The input goes to the control field which is on the direct axis of the machine. Rotation of the armature in the control field sets up a voltage at right angles to the field as shown in Figure 14–45.

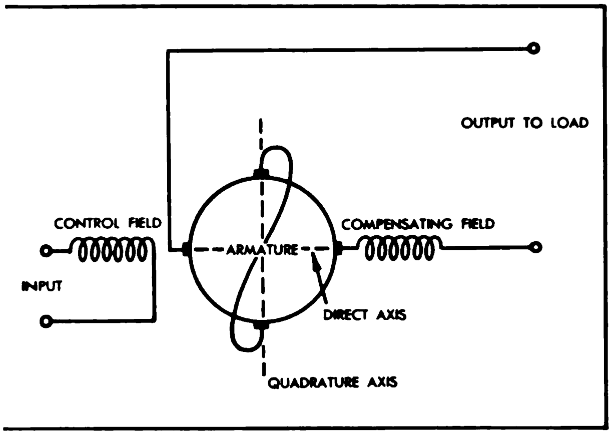


Figure 14–45. The Amplidyne

Brushes are placed on this axis. If a load were connected to these brushes, the machine would become an ordinary DC generator. However, in the amplidyne these brushes are short circuited and quite high currents flow in the armature producing a magnetic field along the quadrature (vertical) axis. This field is much stronger than the control field. This is known as armature reaction, the same as it is in an ordinary generator. The rotation of the armature through this quadrature field sets up a voltage at right angles to it, that is, along the direct (horizontal) axis.

Brushes located on the direct axis have the load connected to them. When the load draws current, a magnetic field is set up along the direct axis because of the armature reaction of this second stage. Since the field due to armature reaction in each stage is 90° from the field producing it, the reaction field of the second stage is 180° from the control field and tends to cancel it out, thereby reducing the amplification of the amplidyne. This is comparable to degeneration in a vacuum tube amplifier. In order to prevent this cancellation of the control field, a compensating winding on the direct axis and in series with the brushes on the direct axis sets up a field that opposes and cancels out the armature reaction of the second stage.

As shown in Figure 14–46A the various magnetic fields of an amplidyne occupy certain positions and have certain directions. The symbol $\phi 1$ is the control field, $\phi 2$ the field due to armature reaction of the first stage or the quadrature field, ϕa is the armature reaction of the second stage and ϕb is the field due to the compensating windings. Since, by design, ϕb equals ϕa but is opposite in direction, the net result is as shown in Figure 14–46B.

In order to guard against residual magnetism in the direct axis, a demagnetizing winding called a "killer" winding is sometimes found on the stator. It is excited with alternating current from a magneto generator mounted on the same frame. The magneto consists of a bar magnet revolving within a separate field winding to generate the AC voltage. The alternating current flowing in the stator windings forms a demagnetizing system that neutralizes any residual magnetism when the control field current is zero.

Actually there are two control field windings in an amplidyne when it is used with a control amplifier. The reason is that the vacuum tubes of the control amplifier conduct only in one direction and in order to control the polarity of the output voltage on the amplidyne, it is necessary to reverse the direction of the control field.

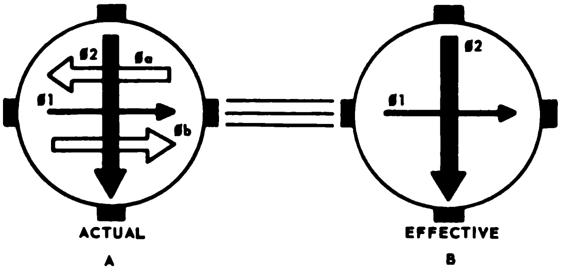


Figure 14–46. Magnetic Fields in Amplidyne

A practically constant input ratio makes it necessary that there be no saturation of the frame and the armature core. The use of a *killer* winding assists in making the gain constant. The power gain of the amplidyne is of the order of several hundreds or thousands and may even be tens of thousands. The full load input is about 0.25 to 0.75 watt, which can easily be supplied by the control amplifier. The output varies from hundreds of watts to several thousand watts. The speed of response to changes in input depends upon the time constants of the control field, the quadrature windings of the armature, and the load circuit or direct axis armature windings. Under the usual operating conditions, the time lag is about 0.1 second.

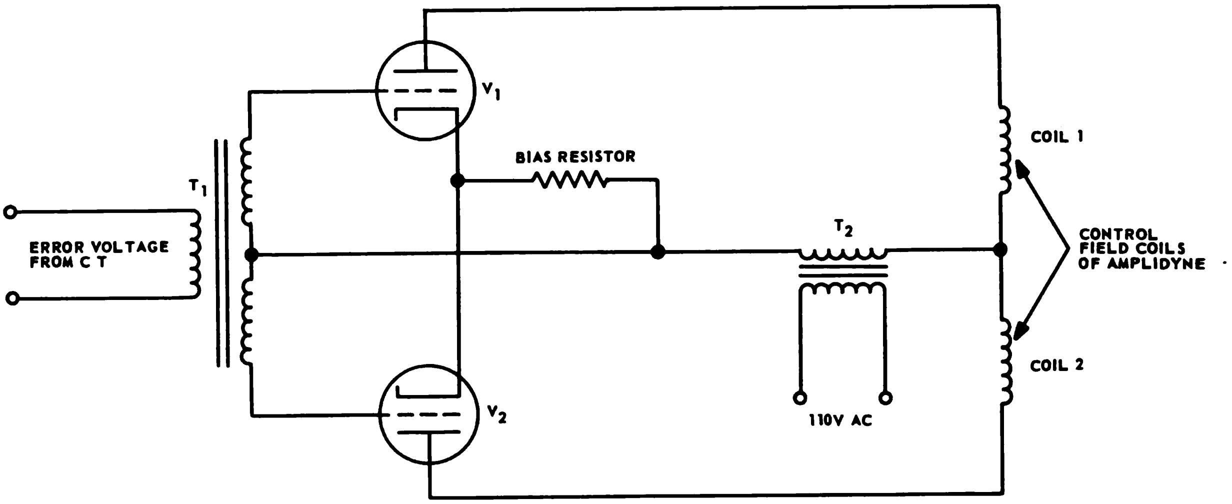


Figure 14–47. Basic Control Amplifier Circuit

The Control Amplifier

As mentioned previously, the function of the control amplifier is to take the error voltage produced by the control transformer and to convert it into a DC voltage. The magnitude and polarity of this voltage is determined by the magnitude and relative phase of the error voltage. In the control amplifier circuit in Figure 14–47 the two triodes are operated in push-pull, and an AC voltage from a 110-volt line is applied to the plates through transformer T_2. Bias for the tubes is provided by a common cathode resistor. The error voltage is applied to the grids through transformer T_1 in such a way that the two grids are 180° out of phase. It is either in phase with or 180° out of phase with the 110-volt line voltage depending upon the relative position of the two shafts (synchro generator and control transformer).

To follow through the operation of the control amplifier, assume that the error voltage is in phase with the 110v and that the grid of V_1 goes in a positive direction at the same time that the plate goes positive. This causes an increased current flow through V_1 and this, in turn, an increase through coil 1. At the same time the grid of V_2 has gone in a negative direction, and there is a decrease in current through V_2 and coil 2. Then in the amplidyne the field set up by coil 2 will decrease.

There will then be an output voltage of a certain polarity from the amplidyne causing the servo motor to turn in a certain direction. During the alternation of the 110v when the plates are negative, no current flows through either tube. Hence the current through coils 1 and 2 is pulsating direct current. (Figure 14–48 shows current and voltage relationships.) If a greater voltage is applied to input transformer T_1, it will result in more current passing through coil 1 and less through coil 2, which causes the servo motor to turn faster.

When the error voltage is 180° out of phase with the 110 volts, the grid of V_2 will go positive and the grid of V_1 negative at the time the plates go positive. As a result, the current flow through coil 2 will increase and that through coil 1 will decrease. This causes the flux in the amplidyne to increase but in the opposite direction so that the output voltage is opposite in polarity. The servo motor will turn in the opposite direction.

In actual practice some sort of filter circuit is employed to smooth out the pulsations of the control amplifier output. This makes the output steadier. One way of accomplishing this is to connect resistors in parallel with the amplidyne control field windings. During the negative alternation of the plate voltage, the self-induced voltage of the inductance of the control field maintains the field current, which finds a complete circuit through the shunting resistor. Another way of making the output more nearly pure DC is to put an RC circuit in to act as the load impedance and use this practically constant voltage to feed into a DC amplifier.

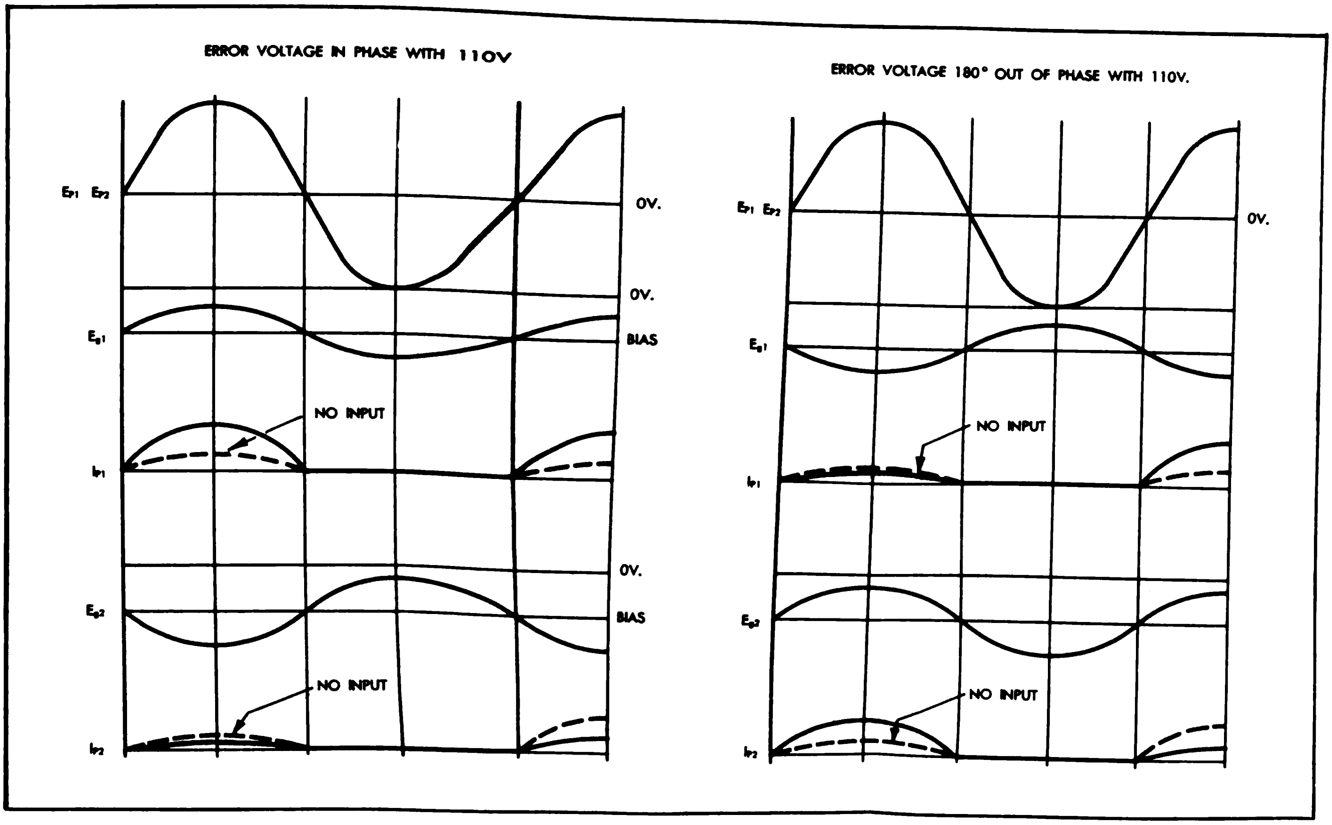


Figure 14–48. Control Amplifier Currents and Voltages

Hunting is quite likely to take place in a servomechanism unless some method is devised to prevent it. Hunting means that the servo motor drives the load past the desired position causing an error voltage that reverses the motor's direction; the motor will swing back and forth past the final position several times before stopping. Under some conditions continued oscillation may take place.

The usual anti-hunt circuit consists of feeding back a portion of the amplidyne output into the input of the control amplifier. This voltage is proportional to the speed of the motor and is applied so that it makes the grid of the conducting tube more negative (in other words, it decreases the gain of the amplifier) and decreases the amplifier output and the motor speed. In this way the motor, in effect, acts as a brake on its own speed. The magnitude of the feedback decreases as the speed decreases and allows the motor to rotate the load to the desired position without unnecessary hunting. The amount of feedback is adjustable by a potentiometer.

USE OF SERVOMECHANISMS FOR AZIMUTH STABILIZATION

The heading of an aircraft is quite changeable. For this reason, it is sometimes desirable for the presentation of data on the indicator scope to remain in the same position, regardless of changes in heading of the aircraft. For example, if the 0° position of the PPI scan is always true north it is much easier to compare the scope picture with the map of the region. In order to accomplish this, it is necessary to change the relative positions of the antenna and sweep on the scope by an amount equal to the change in heading. This can be effected by applying azimuth stabilization to the antenna drive assembly.

Synchro System Employing Azimuth Stabilization

The circuits shown in Figure 14–49 are those of a typical set employing stabilization. Stabilization is obtained by shifting the sweep coils of the CRT whenever the aircraft changes its heading. The antenna is driven by the azimuth motor

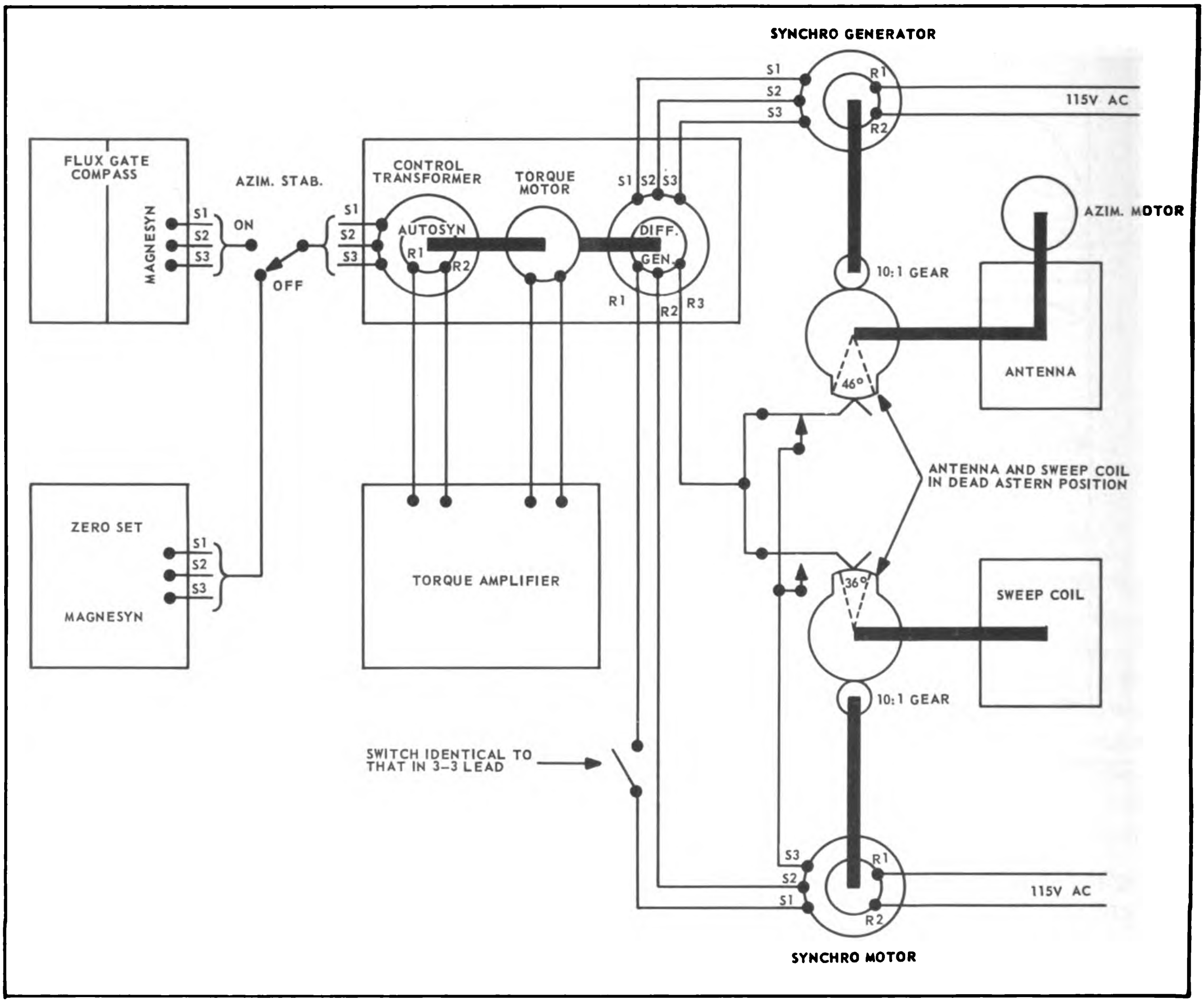


Figure 14–49. Typical Synchro System Using Azimuth Stabilization

at a rate of approximately 24 rpm. A synchro generator is geared to the antenna through a 10:1 gear. The generator is connected to a differential synchro which has its output connected to a synchro motor. This motor is geared, through another 10:1 gear, to the sweep coils and causes them to rotate about the neck of the CRT. If the differential generator is in the electrical zero position, the rotation of the sweep coils is in synchronization with the rotation of the antenna.

The purpose of the 10:1 gears is to reduce to $\frac{1}{10}$ of its value the lag inherent in a synchro system. In this system, if the synchro motor lags the generator by 10°, it produces only 1° lag between the antenna and sweep coils. The actual error is a small fraction of 1°. The increased accuracy is gained by the possibility of the antenna and sweep coils being 36°, or some multiple thereof, out of synchronization. Of course the synchros are always in step but since 36° rotation of the antenna (or sweep coils) will mean one complete rotation of the synchro motor, there is the possibility that this error could occur.

To guard against such an occurrence, cams are attached to the shafts of the antenna and the sweep coils. These are called phasing cams and operate switches in the R1–S1 and R3–S3 leads from the differential generator to the synchro motor. The cam attached to the antenna shaft *closes* the two switches during 46° of rotation centered about the dead astern position. The cam attached to the sweep coil shaft *opens* switches in the same leads during 36° of rotation, also centered about the astern position. If the two shafts are in synchronization, the antenna cam closes the switches 5° ahead of the position where the sweep coil cam opens the switches and holds them closed for 5° past the position where the sweep coil cam again closes its switches. Thus, the circuits are complete at all times and the synchro motor drives the sweep coils in step with the antenna.

Operation

To illustrate the action, assume that when the system is turned on and the antenna is pointing somewhere in the aft sector the sweep coils are at a position corresponding to dead ahead. Both sets of phasing switches are closed, and the antenna and sweep coils start rotating. There is large error between the antenna and sweep coils. Figure 14–50 shows the cams and switches at successive steps in the rotation. This out-of-step rotation will continue until both switches are open at the same time (see C), then the sweep coils will stop and remain in that position (180°) until the antenna reaches the position where it will close the switch. This occurs at 23° before the dead astern position. The sweep coil rotor will then electrically interlock and cause the antenna and sweep coils to be in step once more.

The description of the operation thus far assumes that the differential generator was in the electrical zero position. This is true if the AZIM. STAB switch is in the OFF position. In this type operation, the control transformer in the phasing unit has its stator windings connected to a zero set magnesyn. The magnesyn is a device similar in performance to a synchro generator. The control transformer in this particular setup is known as an autosyn. The output of the control transformer, after being amplified by the torque amplifier, causes the torque motor to turn the CT rotor and differential synchro rotor to the desired position which is the electrical zero position of the differential generator.

When using azimuth stabilization, turn the switch AZIM. STAB to the ON position. This disconnects the control transformer from the zero set magnesyn and connects it to the flux gate compass magnesyn. The switch also shorts out the phasing switches and connects the differential generator directly to the synchro motor. The magnesyn in the flux gate compass produces voltages to set up a field in the control transformer, which is aligned with the position of the rotor in the magnesyn. This position is dependent upon the position of the compass. The error voltage produced by the control transformer, after

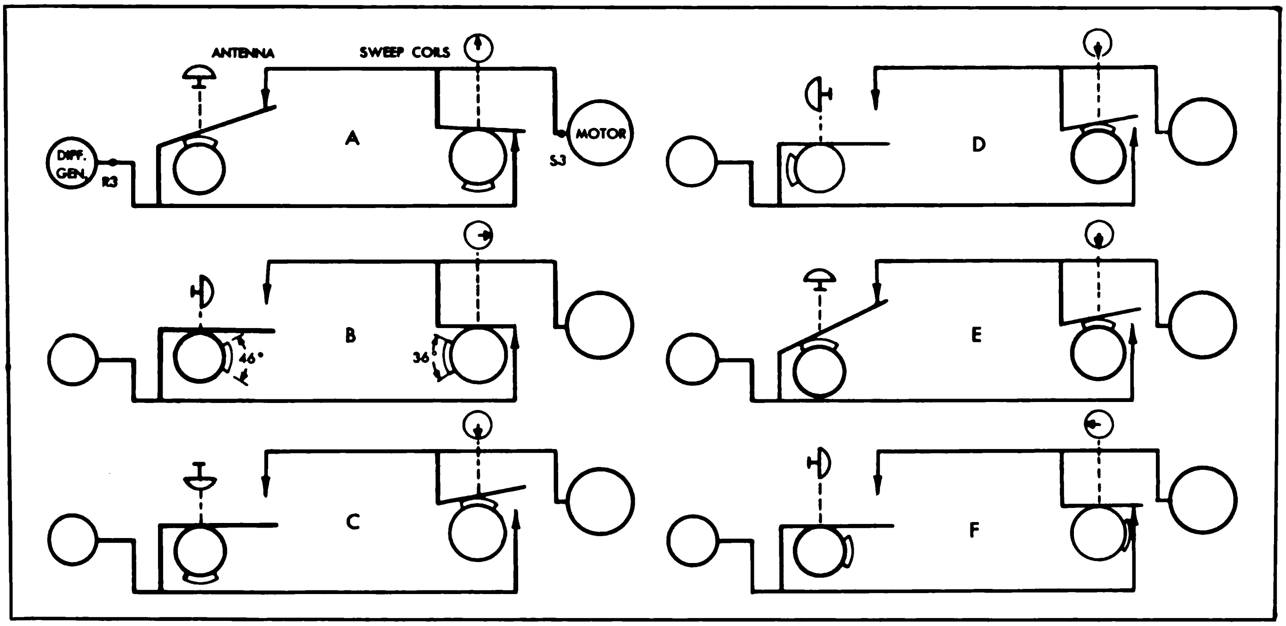


Figure 14–50. Phasing Switch Action

amplification, turns the control transformer rotor and that of the differential synchro to the position corresponding to the compass position.

Since the rotor of the differential generator is shifted by an angle equal to the difference between the aircraft's heading and true north, its output voltages will be such as to cause the sweep coil motor to lead or lag the antenna generator by the same angle. By proper connection and adjustment the sweep coils move in such a way that the 0° position of the scope is true north. In order to determine the aircraft's heading from the scope pattern a switch connected to the antenna causes the bias of the CRT to decrease when the antenna points dead ahead. This results in a brighter trace which indicates the aircraft's heading. The purpose in shorting out the phasing switches is to permit the antenna and sweep coils to be out of step, as is the case when the aircraft's heading is other than north.

MAGNETIC AMPLIFIERS

Magnetic amplifiers, for the most part, are made up of controlled variable inductances *(saturable reactors)*, dry-disk rectifiers, fixed and adjustable resistors, capacitors, and conventional-type transformers. The magnetic amplifier uses the saturation of a magnetic core, which is regulated by the control circuit, to determine the AC circuit resistance. In this way electrical devices can be controlled and electrical signals can be amplified.

Magnetic amplifiers have been in use since the early 1900's. In 1901 a saturable reactor was used in a "wireless signaling" system to amplify the output of a carbon microphone and to modulate a transmitter. During this same period, a saturable transformer was being used to measure currents from 100 to 1000 amperes with an accuracy of about ±0.5%. In 1915 the development of electronic amplifiers did away with the magnetic amplifier.

Later, experimentation with magnetic amplifiers introduced feedback, push-pull circuits, and self-saturating circuits. From 1930 on, much of the work was for the military. Military application required unusual durability, simplicity, and reliability. New core materials and efficient and reliable dry-disk rectifiers brought about still further development in magnetic amplifiers.

Reliability is the main advantage of the magnetic amplifier. There are no moving parts, no contacts, no filaments or other features that are prone to fail in electron-tube amplifiers. Magnetic amplifiers are sturdier than electron-tube amplifiers; they absorb vibrations and shocks easily. Since they have no filaments, they require no warm-up time and operate the minute the switch is turned on. Magnetic amplifiers are sturdy; since they have no moving parts, they do not require servicing or replacement. They can be hermetically sealed. Magnetic amplifiers are suitable for measuring or amplifying extremely small voltages or currents.

The size of the core of the saturable-reactor elements in a magnetic amplifier is inversely proportional to the power-supply frequency. When a 400-cycle supply is used, the weight and size of the magnetic amplifier can be less than that of a vacuum-tube amplifier for the same circuit. Since the output of the magnetic amplifier can be completely isolated from the input, multiple input windings can be used. This allows the simultaneous application of many control signals without electric interference among them. Power gains of 10,000 to 200,000 in a single-stage circuit can be obtained with magnetic amplifiers. The range of input impedances that can be matched by the control winding is from 0.1 ohm up to about 100,000 ohms. The output impedances that can be matched by the load circuit range from about 5 to 5000 ohms with a power supply frequency of 60 cycles per second.

Disadvantages of the magnetic amplifier are poor frequency response and the long time of response.

Magnetic amplifiers, vacuum-tube amplifiers, and transistor amplifiers should not be considered as rival devices. They may be combined to take the greatest advantage of the properties of each. Certain types of amplifiers are better than others in each application; the one best suited for the job should be used.

SATURABLE REACTORS

In order to understand the operation of a magnetic amplifier, it is necessary to understand each of the components in the circuit. All except the saturable reactor have been discussed in previous chapters.

A saturable reactor, or *transductor*, consists of a closed magnetic core on which are mounted one or more load windings and one or more control windings. Since there are no moving parts, it is one of the simplest of electrical devices. Properly designed and constructed, the saturable reactor requires practically no maintenance and has an extremely long life expectancy.

The core material for saturable reactors requires certain magnetic properties. Losses due to hysteresis and eddy currents must be minimum. Resistivity must be high, and coercive force low. Saturation flux density should be high so that a given weight of core material will have large power-handling capabilities. The hysteresis loop should be nearly rectangular. The core material must also have stable magnetic characteristics under conditions of shock, of changing temperature, and of changing magnetic strain. It must also be possible to form the material into thin tapes or laminations. A lamination is a thin sheet of the metal from which the core is made.

Construction

There are many types of core constructions used in saturable reactors. A ring core (toroid) may be made of a stack of flat circular-shaped laminations with a circular opening in the center. The material is preferably nondirectional in magnetic characteristic. The most common core used in the better saturable reactors is made by winding a tight spiral from a thin strip of grain-oriented core material. For best results the direction of orientation should be parallel to the long side of the strip. (Grain-oriented materials magnetize more easily in one direction than in others.)

Insulation can either be wound on the core or be in the form of cylinders to fit inside and outside the core, with a ring-shaped collar at each end. The latter type of insulation is used where the voltage between winding and ground is below 1300 volts, because of the danger of arc-over at the joint between the cylinders and the collars.

Rectangular cores are often made of rectangular laminations that can be stacked into the prewound coil. Two-legged cores are made from U- and I-shaped laminations, the I-shaped pieces fitting across the end of the U-shaped piece. Three-legged cores made from E- and I-shaped pieces of core material are put together with the I-shaped piece at opposite ends of the core in adjacent laminations. For best performance, the air gap must be as small as possible and the reluctance kept to a minimum. The usual interleaving technique used in ordinary transformers is unsatisfactory, as the many small air gaps provided by the butt joints of the standard E-core distort the magnetization curve.

An improvement is the four-legged core with the openings between the legs in the laminations punched out and only the two inner legs loose at one end. The AC windings are slipped on the two inner legs with the control windings around both. When the windings are on the legs, the legs are bent back into position. The very small joint in this type of core gives it an advantage over the other rectangular cores. However, the joints are not normally perfect because of the oxide coating or other insulation on the edges of the pieces at the joint. This adds to the reluctance of the core.

Three-phase saturable reactors can be made from three toroidal (ring) cores, each with one winding, by stacking them and wrapping the control winding around all three. Another type of three-phase saturable reactor consists of a three-legged core with a load, or gate, winding on each leg and a control winding around all three legs. Various other types of construction may also be used.

The physical arrangement of the various toroidal windings on the cores will greatly affect the performance of the magnetic amplifier. They must be wound in a way that will reduce leakage effects to a minimum and minimize the circulation of alternating current in the DC control, bias, and feedback windings.

Regardless of the size of the saturable reactor, it controls some device or process that has dependent on it huge sums of money, or even human life. For this reason, protection of the elements of the saturable reactor is very important. The insulation, protection, and enclosure must be evaluated, not on the value of the reactor itself, but on the value of the process it controls.

Classification

The saturable reactor permits control of large amounts of electric power with much smaller control power. This means that smaller control equipment can be used than when the load is controlled directly. The saturable reactor and the load are placed where they are needed; the control equipment is placed at a convenient location without excessive loss in transmission between the control and the load.

Magnetically controlled saturable reactors can be divided into two main groups — those controlled by a constant magnetomotive force, and those controlled by a variable magnetomotive force. Under the first classification come frequency transformers and constant current devices, which use either a constant magnetic field or a constant electric current. The second group contains devices using a variable magnetic field, such as various current and voltage measuring devices and magnetic amplifiers. (The main difference between magnetic amplifiers and flux-gate magnetometers is that in the amplifier the saturable reactor is controlled by the electric current or voltage to be amplified, whereas in the flux-gate magnetometer, the saturable reactor is controlled by the magnetic field to be measured.)

Principles of Operation

During steady-state operation the saturable reactor may be looked at as an impedance to the AC circuit that is very high through one part of the half-cycle of the supply voltage and very low during the rest of the half-cycle.

As you know, the inductance value of a particular coil depends upon its physical characteristics. An air-core coil can have its inductance increased by the insertion of a magnetic core, since inductance is directly proportional to the permeability of the core. This means that if an iron-alloy core with a permeability of 1250 is put in an air-core coil, the inductance becomes 1250 times greater than the value with the air core.

As the iron core is moved in and out of the winding, it alternately increases and decreases the effective permeability of the coil's core. Changing the permeability changes the inductance. This will change the inductive reactance of the coil. Figure 14–51 shows an air-core inductance in series with a load and a source of power. The inductive reactance is minimum and the current is maximum. The greater part of the applied voltage is dropped across the load. When the iron core is moved into the coil, the inductance, and thus the inductive reactance, increases. As a result, the current decreases, and the voltage drop across the load decreases proportionately. Moving the iron core in and out of the coil will alternately increase and decrease the inductance and, as a result, decrease and increase the voltage across the load.

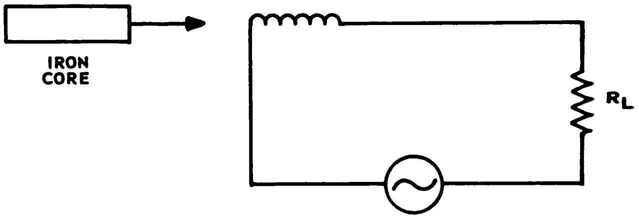


Figure 14–51. Inductance

A comparatively small amount of energy is enough to control the output of the source, which may have a rating of several horsepower. It is for this reason that the circuit acts as a magnetic amplifier.

A more practical way of varying the permeability of the core is shown in Figure 14–52. The winding in series with the load has a fixed magnetic core. The core's permeability is varied by saturating or unsaturating the material with a relatively small current applied to the control winding.

To better understand how the circuit works, let us first review some of the basic magnetic concepts. When current flows in a coil, it produces a magnetomotive force, which can be compared to the electromotive force in an electric circuit. The strength of the magnetomotive force depends on the ampere turns (current flow times the number of turns in the coil).

The mmf produces magnetic flux, which consists of closed lines of force, that is comparable to current in an electric circuit. Like current, the flux varies directly with the force but indirectly with the opposition (reluctance) of the path making up the magnetic circuit. Magnetic reluctance is a property of the material of the

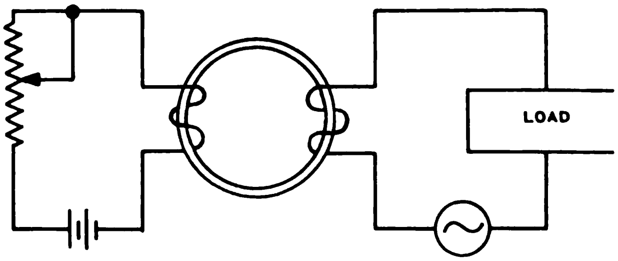


Figure 14–52. Load Control

magnetic path. In an electromagnet the value of the reluctance depends primarily on the core.

The reluctance of an air core is constant, but the reluctance of a ferromagnetic material can change with the change in flux. In an iron-core coil, when the magnetizing current first begins to flow, the reluctance is low and the flux is very high compared with that in an air-core coil under similar conditions. The reluctance will increase if the current is increased, but the rate-of-flux increase falls off.

As the current increases further, the reluctance increases still more rapidly, its value approaching the reluctance of air. When further increases in the magnetizing current produce very little increase in the total flux, the core is said to be saturated.

Figure 14–53 is the permeability curve for the ferromagnetic material used as core in Figure 14–52. From this graph you can see that permeability (B/H) is not constant. It has different values for different magnetizing forces. Between points 1 and 2 the curve is nearly a straight line. The permeability is high and the coil has a large value of inductance. Between points 2 and 4, a change in the magnetizing force (H) produces a much smaller change in the flux density (B) than the same change in H for the linear portion of the curve. The permeability between points 2 and 4 is much lower; therefore, the inductance is also smaller. Beyond point 4 the curve begins to flatten out. As H is increased, B increases very little, if any. The core is saturated and the inductance of the coil is minimum.

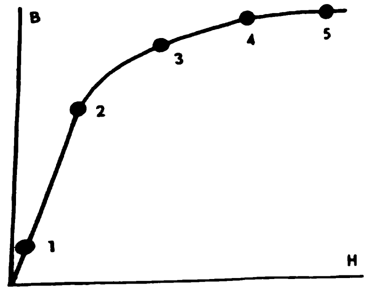


Figure 14–53. Permeability Curve

The control action of a magnetic amplifier depends on change in inductance. If the control signals vary the magnetization of the core above and below point 3 (from 2 to 4), small changes in control voltage (input) result in comparatively large variations in the output circuit impedance and the output voltage. The control current in Figure 14–52 is adjusted by a potentiometer. This current sets up a flux in one direction through the coil with an intensity determined by the ampere-turns of the control circuit. The load winding is connected in series with the load resistance and an AC power source.

An increase in control current lowers the inductance of the load winding because the magnetization of the core is shifted toward point 4 on the curve in Figure 14–53 where the permeability is less. This lowers the inductive reactance of the load winding. The total impedance of the load circuit decreases; the load current increases, and the power in the load is increased. If the increase in control current is enough to saturate the core, the reactance of the load winding drops to an extremely low value, and maximum supply voltage is applied to the load.

Decreasing the control current increases the reactance of the load winding. In this case the magnetization of the core shifts in the opposite direction, toward point 2 on the curve of Figure 14–53, and the permeability increases. When the load coil has maximum inductance, minimum load current flows and the load power is minimum.

This arrangement is very inefficient because of the transformer action between the load winding and the control winding. The induced AC voltage in the control winding alternately adds to and subtracts from the control voltage.

To eliminate the transformer action between the load and the control windings, a three-legged magnetic core may be used. Figure 14–54 shows a device used for controlling large alternating currents. There is an AC winding on each outer leg. These are arranged so that their alternating flux components cancel (dotted line) in the center leg on which the DC control winding is placed. There will be complete cancellation if the AC windings have the same number of ampere turns and are wound in the right direction.

The alternating flux effectively does not pass through the center leg. However, the two flux components are in the same direction in the outer legs. This composite flux at one instant is indicated by the broken line in Figure 14–54. The control winding produces a magnetic flux that magnetizes the entire core. The direction of the control flux is indicated by the solid lines. The

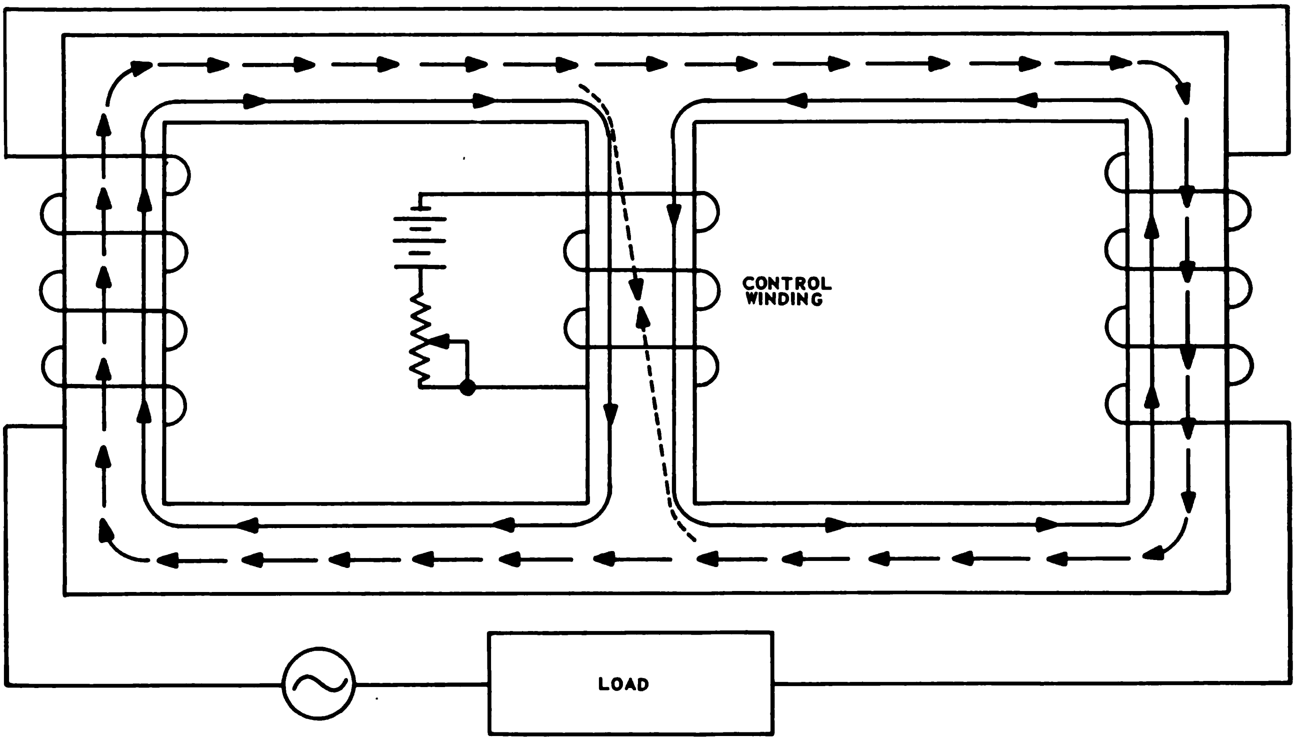


Figure 14–54. Three-Legged Magnetic Core Circuit

DC flux affects the operation of the load circuit, but there is no transformer action from the load circuit to the control winding.

The preferred method of saturable reactor construction uses two separate circular cores that are stacked (Figure 14–55B). Each core has an AC winding. The cores are stacked so that these windings are wound in opposite directions. The control winding is wound around both cores after stacking. Figure 14–55A shows the schematic diagram of this arrangement. The control winding is shown as two windings connected series aiding.

At any instant the flux in the two cores, due to the alternating current flowing in the load windings, is in such a direction that the induced currents in the control windings cancel each other.

The direction of the flux in the cores at the instant the AC source has the polarity indicated

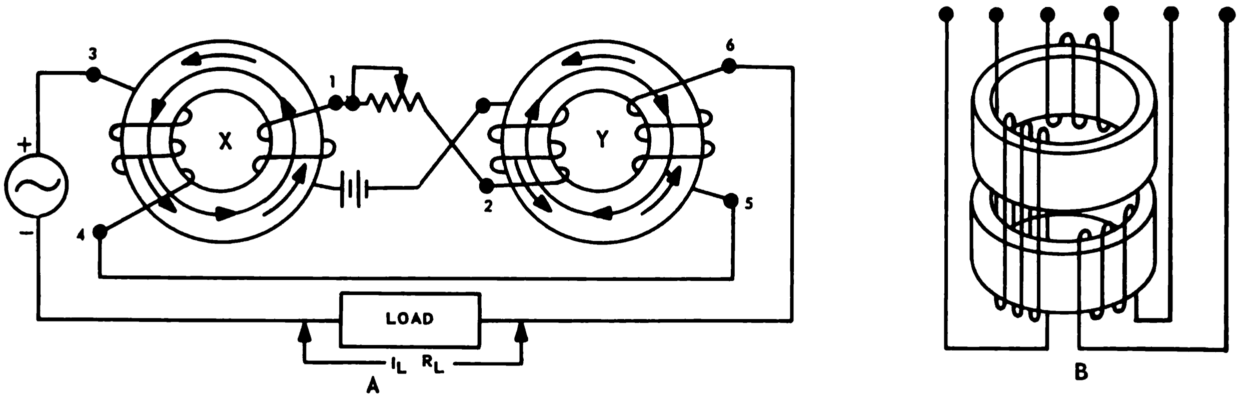


Figure 14–55. Saturable Reactor Control

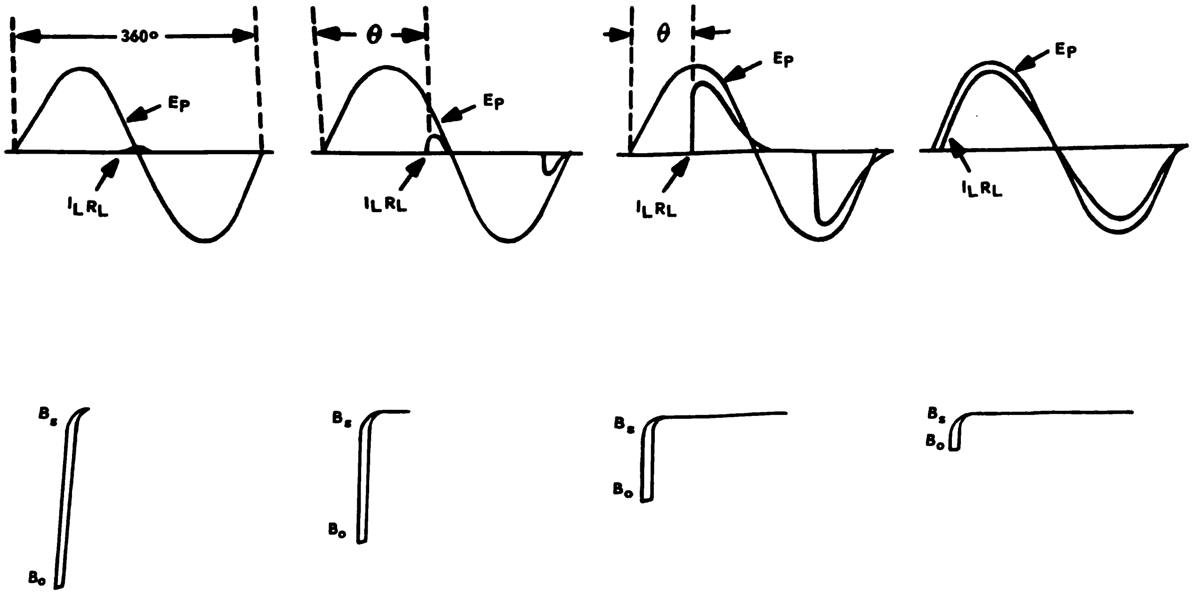
in Figure 14–55 is indicated by the broken lines. The direction of the flux due to the current in the control winding is shown by solid lines. When the polarity of the AC source reverses, the direction of that flux reverses, but the flux due to the control current continues to be in the original direction, since the control source is DC.

When the control current is minimum, the reactance of the load windings is very high, and minimum load current flows. If the control voltage source is adjusted so that current flows in the control windings, the flux produced aids the AC flux in core X and opposes that in core Y on the half-cycle shown in Figure 14–55A. On the other alternation, the DC flux opposes the AC flux in core X and aids that in core Y. Alternately, core X and core Y are saturated, letting pulses of current flow in the load circuit. When core X is saturated, core Y will act as a transformer. The load winding will induce a pulse of current into the control winding. When core Y is saturated, core X acts as a transformer, and its load winding induces a pulse into the control winding. This pulse of current will be in the same direction as that induced by the winding on core Y.

While building to saturation in each core, the reactance in the circuit is high, and there is little current flow in the load circuit. At saturation there will be no voltage across the windings on the saturated core since the reactance will be zero for all practical purposes. Effectively, this part of the control winding shorts that part on the unsaturated core. When the secondary of a transformer is shorted, the primary appears shorted. Thus, there is no voltage appearing across either core, and almost the entire supply voltage appears across the load resistance.

Figure 14–56 shows the waveshapes of the circuit in Figure 14–55. The upper part of the illustration shows the applied voltage and the voltage drop across the load, due to the load current. The lower part of the illustration shows the hysteresis loop for one core. The type of material in the core produces a very narrow, rectangular, hysteresis loop.

Figure 14–56A shows the waveshape from Figure 14–55 with minimum control current. The voltage drop across the load (I_LR_L) is extremely small. In Figure 14–56B the control current is larger, and the core becomes saturated for a part of each cycle. The load current begins to



A
MIN I_c

B
LOW I_c

C
MED I_c

D
HI I_c

Figure 14–56. Output Voltage and Hysteresis Loops at Various Control Currents

flow earlier and reaches a higher value. The distance along the time axis from the starting point of the applied voltage to the time when the load current begins to flow is the *phase angle*, or *firing angle*. The size of the phase angle depends upon the initial magnetization of the core.

With a steady current flowing in the control winding, the hysteresis loop is much shorter than with no DC at all. B_o, the minimum flux density (the flux density at the start of each cycle in this arrangement), depends upon the magnetization due to the control current; the higher the control current, the closer to saturation the starting flux density.

In Figure 14–56C the control current is increased to a still higher value than in Figure 14–56B. Notice how abruptly the load current begins. It rises quickly to a maximum, and the voltage across the load at that time is almost equal to the applied voltage. Before the load current starts, all of the applied voltage is across the AC windings; after load current begins to flow, only a very small portion of the applied voltage (because of the resistance of the windings themselves) appears across the AC windings. Figure 14–56D shows the load voltage when the control current is high enough to keep the cores saturated all of the time. The load voltage waveshape follows that of the applied voltage.

The output voltage waveshapes of Figure 14–56B and Figure 14–56C resemble those of a thyratron with a resistive load. The operation of a saturable reactor can also be compared to that of a synchronous switch. The saturable reactor acts as an open switch between the load and the supply voltage during the first part of each half-cycle when no load current is flowing. During the other half-cycles, the saturable reactor is like a closed switch, allowing current to flow through the load. The point at which the saturable reactor allows current to flow in the load circuit determines the amplitude of the load, or output voltage. By increasing the control current, you can decrease the phase angle, and thus control the point at which the saturable reactor saturates and also thereby control the amplitude of the output.

One of the disadvantages of the simple magnetic amplifier circuit shown in Figure 14–55 is the requirement for relatively large control currents. The large control current is needed to overcome the effects of the varying magnetic flux caused by the large AC load current. Without any flux due to a direct current flowing in the control winding, the magnetic flux density of the core varies above and below zero in accordance with the alternating current applied by the power source. Thus, during one half-cycle the flux in one core due to the load current will aid that due to the control current, and during the other half-cycle the load current will oppose that due to the control current. This can be considered as feedback. Since the feedback takes place because of the circuit itself, it is called internal, or *intrinsic*, feedback. When the feedback is aiding the input, it is called positive; when opposing the input, it is called negative.

Enough flux must be provided to balance out the oscillating flux of the load current and to set the operating point at the desired place on the magnetization curve. This can be accomplished in two ways. Either the control circuit ampere turns must exceed those of the load circuit, which results in an inefficient amplifier, or else some feedback from an external source must be used to cancel the effect of the changes induced by the load current flux.

CHAPTER 15

Special Circuits

DOPPLER SYSTEM

Doppler Effect

The Doppler effect, by definition, is the apparent change in the frequency of light, sound, or electromagnetic radiation that is observed when the source and the observer are in relative motion. This frequency shift is external to the transmitter. You have all heard a train's whistle appear to change its pitch from a high tone as the train approaches to a lower tone as it moves away from you. As the train approaches, there is an apparent increase in frequency, a raising of the pitch; as the train moves away from you, there is an apparent decrease in frequency, a lowering of the pitch. This is the Doppler effect.

Analysis of the Doppler Principle

What causes this apparent change in pitch? Let us assume that a transmitter emitting an audio signal at a frequency of 60 cycles per second is traveling at a velocity of 360 feet per second. Therefore, at the end of one second the transmitter will have moved from T to T1 in Figure 15–1A. The total distance from T to O, the position of the observer, is 1080 feet. We know that the velocity of sound is 1080 feet per second. Thus, a sound emitted at T will reach O in one second. To find the wavelength (λ) of the transmitted signal, divide the velocity of the signal (1080 feet per second) by the frequency of the signal (60 cps). The result is 18 feet.

We know that in one second the transmitter moves 360 feet toward the observer and transmits 60 cycles. At the end of one second the first cycle of the transmitted signal reaches the observer just as the sixtieth cycle is leaving the transmitter at T1. Under these conditions the 60 cycles emitted are between the observer and T1. Notice that this distance is only 720 feet (1080 − 360). The 60 cycles spread over the distance from T1 to O have a new wavelength, which is only 12 feet (760/60).

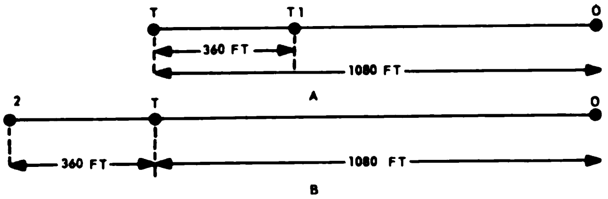


Figure 15–1. Transmitter Moving Relative to an Observer

To find the new frequency, use the formula for frequency,

$$f = \frac{V}{\lambda}$$

$$= \frac{1080}{12}$$

$$= 90 \text{ cps}$$

The original frequency of 60 cps has changed to an apparent frequency of 90 cps. This new frequency only applies to the observer at O. Notice that the Doppler frequency variation is directly proportional to the velocity of the approaching transmitter. The faster the transmitter moves toward the observer, the greater the number of waves that will be crowded into the space between the transmitter and the observer.

If the transmitter were stationary and the observer moving, the observer would encounter more waves per unit of time as he approached the transmitter. As a result, he would hear a higher pitch than the transmitter was actually emitting.

When the transmitter is traveling away from the observer, as in Figure 15–1B, the first cycle leaves the transmitter at T and the sixtieth at T2. The first cycle reaches the observer at O when the transmitter reaches T2. There are 60 cycles stretched out over 1080 plus 360 feet, a total of 1440 feet. The wavelength of these 60 cycles is 1440/60, or 24 feet. Their apparent frequency is 1080/24, or 45 cps.

Continuous-Wave System

Another name which is given to the Doppler system is continuous-wave system. This system, as the name suggests, transmits continuously. The transmitting antenna directs energy to the target, which reflects a small fraction of the energy to the receiving antenna. It is not practical to use a single antenna in the continuous-wave system because the transmitter and the receiver are operating simultaneously and continuously.

These antennas are so oriented that only a small fraction of the transmitted power leaks directly to the receiver. This is done because at microwave frequencies the best practical detector is a crystal, and a crystal would burn out if directly connected even to a low-power transmitter.

Applications of a Continuous-Wave System

There are several applications of the continuous-wave, or Doppler, system. In one radar application the radar set differentiates between the transmitted and reflected waves, and thus determines the speed of the moving object. The Doppler method is the best means of detecting fast-moving objects that do not require range resolution.

The Doppler effect here can be explained as follows. As a moving object approaches the transmitter it encounters, and therefore reflects, more waves per unit time, just as an observer approaching a transmitter receives more waves per unit time. The amount of the frequency shift is very small in relation to the carrier frequency, since the velocity of propagation of the signal is very high compared to the speed of the target. However, as the carrier frequencies used in radar are high, appreciable frequency shifts are produced in the audio-frequency range by proportionate speeds of hundreds of miles per hour. One-quarter cycle shift at 10,000 mc will give speed measurements accurate to a fraction of a percent.

The separation of the background and the target is based on this shift in frequency (Doppler effect) that is caused by the reflection of the signal from a moving target. A disadvantage of the system is that it does not determine the range of the target, nor is it able to differentiate between targets when they lie in the same azimuth plane. Moreover, it does not "see" stationary or slowly moving objects, which a pulse system radar can detect.

A second use is to provide information through severe clutter. A pulse system, even with moving target indicator equipment, may have difficulty reading through clutter. Certain Doppler systems can, and do, provide useful and adequate intelligence in such instances.

A third use is involved in air navigation equipment where the continuous longitude and latitude (position) of an aircraft is maintained.

The cost of continuous-wave equipment is less than that of a complex pulse system and, too, this system can work down to zero range, which is something a pulse system cannot do.

Detection of Doppler Frequency

The detection of the Doppler frequency may be accomplished by various means. The transmitted and received frequencies may be mixed in a detector. The output of this circuit is an audio frequency. In continuous-wave system radars, the output is an audio-frequency sine wave that is heard over a headphone or loudspeaker. In pulsed radars, the output is observed on an oscilloscope.

In addition to a shift in frequency, there will be a change in phase in pulse equipment. Here phase detection is used. The phase of successively received pulses is compared to a reference pulse in order to find any change in phase caused by the Doppler shift. There is no phase shift in echoes from stationary targets. By insuring cancellation of signals showing no phase shift, all evidence of stationary targets is removed; at the same time, echoes from moving targets are preserved.

FORWARD TROPOSPHERIC AND IONOSPHERIC SCATTER COMMUNICATIONS SYSTEMS

Ionosphere

The ionosphere is that region of the earth's atmosphere in which the gases are ionized by meteor activity and by ultraviolet light from the sun. Ions and electrons are distributed throughout the entire region, which is between 30 to 250 miles above the surface of the earth. Several layers have been formed by these ions and electrons in which the ionization density appears to have reached a maximum. These layers have been given the alphabetical symbols D, E, and F in their respective order of height, D being the lowest region. What is the quality of the ionosphere that gives it the properties of a mirror a great distance above the earth? What is it about the ionosphere that makes it return radio waves to earth, that permits long-distance radio communication? Really, the characteristics of the radio waves have as much to do with their being reflected as do the characteristics of the ionosphere.

The following definitions and facts concerning radio waves and the ionosphere will be helpful in understanding ionospheric scatter. The speed of a radio wave through space is about 186,000 miles or 300 million meters per second, the speed of light. The radio wave is made up of electric and magnetic fields, perpendicular to each other and to the direction of the wave. The ionosphere consists of ions and free electrons and is formed by meteors and by the bombardment of ultraviolet rays from the sun. The ionospheric gases have the characteristics of an electrical conductor, whereas ordinary air acts as an insulator.

Reflection is that phenomenon which causes a wave that strikes a medium of different characteristics to be turned back; the angles of incidence and of reflection are equal and lie in the same plane. *Refraction* is that phenomenon which causes a wave entering another medium obliquely to undergo an abrupt change in direction when the velocity of the wave in the second medium is different from that in the first. *Diffraction* is the effect produced when waves (light, sound, or radio) encounter a barrier and bend around it. The mechanics of tropospheric and ionospheric scatter include reflection, refraction, and diffraction.

Ionospheric Scatter

Until about 1950 it was considered impractical to use VHF communication for any distance greater than 200 miles. In early 1951, however, distances greater than 600 miles were found to be practical in the lower portion of the VHF band. These distances were obtained by a scattering process in the ionized E region of the atmosphere. Where operational requirements call for circuit paths from 600 to 1200 miles in length and for low susceptibility to long range jamming, ionospheric scatter type of communication has proven particularly useful. Since 1952 VHF ionospheric scatter communications systems have been used by the military to provide exceptionally reliable service in areas where high frequency and medium frequency systems are subject to severe propagational disruptions.

A simple analogy can be used to explain how ionospheric scatter is accomplished. Using a searchlight to illuminate a cloud layer at night, an observer can see a diffused reflection at distances well beyond line-of-sight. Most of the light passes through the cloud, but a small portion is scattered earthward. Substitute for the searchlight a VHF transmitter and directional antenna, for the cloud the ionized E region of the atmosphere, for the observer a receiver and another directional antenna (as shown in Figure 15–2), and you have the basic scatter system. Since the ionized E region is always present and dependable, it permits exceptionally reliable communication when the proper equipment is used.

There are two concepts used to help explain certain principles and techniques used in ionospheric scatter: scattering volume and common

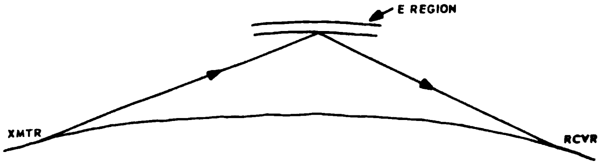


Figure 15–2. Geometry of Ionospheric Scatter

scattering volume. Scattering volume is the volume in the ionospheric scattering region included by both the transmitting and the receiving beams, as illustrated in Figure 15–3.

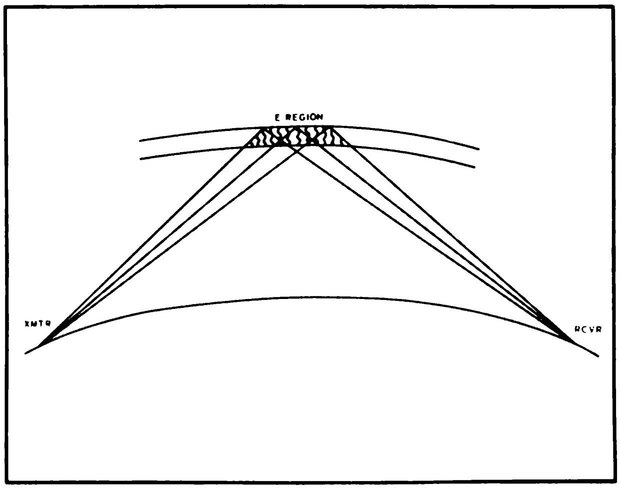


Figure 15–3. Scattering Volume

Common scattering volume is that region which is visible from both terminals, as illustrated in Figure 15–4. The surface path between the transmitter and the receiver must not be too long.

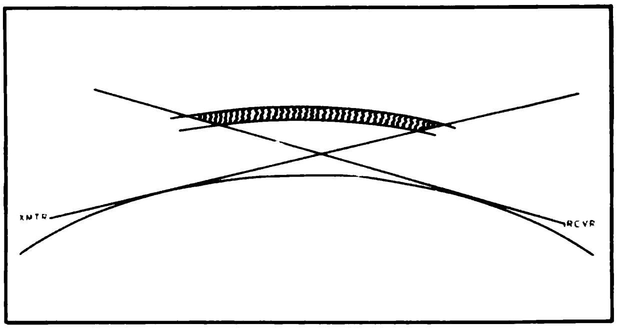


Figure 15–4. Common Scattering Volume

In summary, the scattering volume is that part of the common volume which is simultaneously within the beams of both antennas. The common scattering volume includes all of the ionospheric scattering region that can be simultaneously *seen* from both the transmitting and the receiving antennas.

General Theory of Scattering

A number of theories have been proposed to explain the nature of ionospheric scatter. To the present time, however, no single theory has been universally accepted as a complete explanation. Each theory, while acceptable in most respects, has at least one disadvantage that prevents it from being completely acceptable.

One group of theories takes into consideration variations in the texture of the ionosphere. These may be due to acute variations in density with height (partial-reflection theory), or to discontinuities caused by turbulent fluctuations of ionization in the lower atmosphere. These theories state that energy in small amounts is reflected or scattered from the ionosphere to the receiving antenna. Although small, the amount of energy is greater than would be expected to be bent, or diffracted, around the earth's surface. The rapid fading of the received signal with time normally observed in ionospheric scatter can be explained well enough by the theory involving turbulence, but not so easily by the partial-reflection theory. In the latter theory, changes in density with time must be theorized to take place to account for the fading.

A different theory states that ionospheric scatter is largely the result of scattering from meteor trails. Meteors, traveling at high speeds, upon entering the earth's upper atmosphere generate heat that ionizes the atmospheric gases. The ionized gases will reflect radio energy. Rapid fading can be explained with this theory, depending as it does on the combined efforts of a large number of meteors. This theory can also be used to explain the variations in the direction of arrival of received energy, an observed characteristic of ionospheric scatter transmission.

It is most likely that the true method of ionospheric scatter propagation is actually a combination of the theories just described. All actions take place at the same time but with relative importance, changing with time of day and season in a more or less systematic manner. Whatever the final conclusion may be, there are certain recognizable characteristics that have to be considered. One is the effect of solar radiation. Signal intensities change with the time of day and seasons. This indicates that the effect the sun has on the ionosphere is an important factor.

Another fact that has to be considered is the effect of the meteors. The correlation of increased signal intensity and increased meteoric activity proves that ionospheric-scatter signals are affected by the passage of meteors. Other factors that must be included in the general theory are variable directions of signal arrival at the receiver, rapid fading, and corpuscular radiation, presumably of solar origin.

It is very interesting to note the effects of distance and received signal intensities. The turbulence theory indicates that the signal does not begin to reach a usable level until the path length exceeds 600 miles. From that point the received signal intensities increase until a broad maximum is reached at about 800 miles. After this distance the signal intensity gradually decreases. This discussion, of course, assumes the proper antenna height.

As the frequency is increased, there is a reduction in signal strength. Frequencies between 30 and 40 mc are the most efficient ones for ionospheric scatter. The polarization has little effect on received signal strength, although horizontal polarization is slightly better than vertical polarization. The signal at the receiving antenna has both horizontal and vertical components regardless of transmitted polarization.

System Capabilities

Voice, teleprinter, and facsimile can be transmitted by ionospheric scatter propagation. The vulnerability of such circuits to long range jamming is very low. Continuity of service is exceptionally high because of the ability to operate continuously at a single frequency. Ionospheric scatter systems may be designed to have very high equipment reliabilities and hence low maintenance outage. These qualities are provided through dual operating equipment. This feature is important, as experience has shown that overall system reliability tends to be governed by equipment reliability rather than propagational difficulties.

The capability of the system to handle various transmission rates and types of information is normally provided by the use of conventional multiplexing and modulation techniques. Both time-division and frequency-division multiplexing methods may be used. Time-division multiplexing is the orderly transmission of signals of the same frequency by automatic electronic switching. Frequency-division multiplexing is the orderly transmission of signals of different frequencies by automatic electronic switching.

TRANSMITTING EQUIPMENT. Transmitter power amplifiers used in ionospheric scatter installations have the following major characteristics: (1) output frequency of 30 to 60 mc, (2) output power of 20 to 60 KW, and (3) high power-conversion efficiency, on the order of 50%.

Careful attention should be given to linearity, to internally generated noise, and to shielding of strong r-f fields. It is also desirable to have an easy changeover from linear operation to Class C operation.

RECEIVING EQUIPMENT. Important characteristics of teleprinter channel receivers are: (1) low noise figure, 2 db for the high band (45 to 60 mc) and 3 db for the low band (30 to 45 mc); (2) provision for diversity reception; (3) narrow i-f bandwidths; and (4) frequency stability.

For voice receivers, a narrow-band FM or single-sideband receiver may be used. If voice signals are frequency-division multiplexed with teleprinter carriers, the receiver selectivity must be adequate to reject teleprinter channels that may be only a few kilocycles away.

ANTENNA EQUIPMENT. The performance of an ionospheric scatter system is directly related to the qualities of the antennas used. The most common are the rhombic array, the Yagi array, and the corner-reflector.

Forward Tropospheric Scatter

A tropospheric scatter facility is capable of two-terminal communication over distances up to 300 miles. Since ionospheric scatter communication is usually not effective on paths under 400 miles, and since microwave links, because of terrain, are sometimes not practicable on long paths, forward tropospheric scatter communication fills a gap in the USAF communication system. To provide reliable communications on a path of 300 miles, high engineering standards must be met in both propagation and equipment. The propagation design may be controlled to fit the existing conditions; however, the equipment, once manufactured, is fixed.

The logistical and engineering requirements of a forward tropospheric scatter facility are little different from those of most radio systems. The scatter equipment requires 130 to 400 KVA of power, the amount dependent upon the type of system that is chosen. As with similar facilities, high, level land areas are best suited for the installation and operation of this type of communications.

Each forward tropospheric scatter communications system must be individually designed, because the operating command has specific requirements. The most important consideration, usually decided by the operating command, is the reliability of the system. Two factors control the design of the system reliability. A breakdown of the equipment will lower the reliability of the service, but this condition may be minimized by providing dual equipment. The second factor, which is completely a design function, is propagation reliability.

The system is generally designed with enough flexibility to obtain any reasonable degree of reliability. Over a 300-mile path, reliability of 99.99% may be attained, although certain factors may limit the number of voice channels or require high power and large "dish" antennas. The important fact, however, is that this type of communications may be achieved. When the reliability requirement is lower, the number of voice channels may be increased, and the use of smaller antennas and lower power may prove sufficient. Reliability is a function of path distance and path conditions, then, to which all computations and design requirements are made to conform.

For nonstrategic voice transmission, 95% reliability is considered adequate; this lower reliability usually results in a saving of thousands of dollars. For a missile guidance system, 95% reliability could prove disastrous; therefore, such a system requires a higher reliability. The reliability is determined from the type of service required. Various bandwidths and numerous channels can be used. The forward tropospheric scatter terminal equipment is compatible with voice systems that terminate in telephone line pairs and use 600-ohm and 135-ohm line circuits for ringdown.

GENERAL THEORY OF SCATTERING. As with ionospheric scatter, many theories have been presented on the mechanics of tropospheric scatter. Some theories have been withdrawn because of new data and experimentation with this phenomenon. The theory presented here is probably the most widely used and also the most conservative presented to date. It has been compiled from the latest and most reliable reports and articles.

The troposphere is the region of the earth's atmosphere that extends from the ground to a height of slightly over six miles. It lies several miles below the stratosphere and the ionosphere. The difference between the ionosphere and the troposphere is that there is practically no ionization of the air molecules in the troposphere. Also, there is a decrease in air pressure as you ascend from the earth's surface toward the stratosphere.

The ability of the troposphere to act as a smooth refractive medium is based upon the variation of the dielectric constant. The greater the decrease in dielectric constant, the greater the bending of the beam toward the earth. Water vapor content is the main cause of the variation of the dielectric constant. The greatest variations in heating and cooling take place near the surface of the earth; therefore, the greatest variations in the dielectric constant also occur near the surface of the earth. It can be said that the vapor variations play no great part above four miles, and have no serious effect on predicting the performance of tropospheric propagation.

As the earth's atmosphere is in a constant state of motion with respect to the earth, small irregularities, or eddies, occur. These irregularities occur in "blobs" that are large compared to the wavelength used in scatter communications; thus, they present a different index of refraction from that of the surrounding medium. The index of refraction is the ratio of the velocity of the wave in free space to that in the medium. The changes in the refractive index are a result of the variations in the dielectric constant. These variations, as we have said, are a result of the turbulent air motion and the water content.

This abrupt change in the index of refraction produces a "scattering" of an electromagnetic wave. However, most of the propagated energy continues in the forward direction, although enough energy is scattered toward the earth to be usable. Effectively, the blob has reradiated the signal toward the earth. The signals received will be affected by the actual conditions of the atmosphere, since scatter depends upon turbulences in the atmosphere. The number of blobs in the volume common to both the transmitting and the receiving antenna beams (Figure 15–5) determines the amplitude of the signal received. The scatter model in Figure 15–5 is idealized to

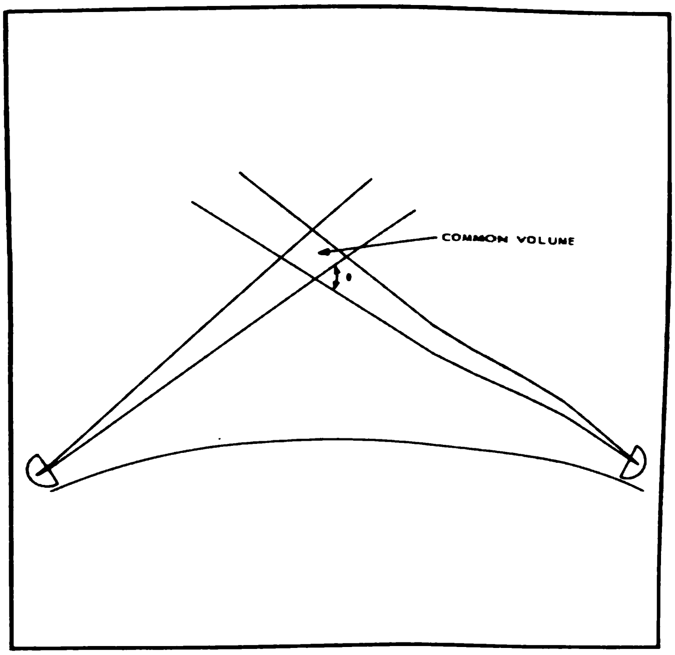


Figure 15–5. Tropospheric Scatter Geometry

indicate the path geometry. Notice that both the receiving and the transmitting antennas are directed at the horizon. A certain volume of the atmosphere is common to both antennas. It is within this volume that the scatter phenomenon takes place.

It can be proven mathematically that the scatter angle (θ in Figure 15–5) is the most important factor in determining the received power. θ is the angle between a ray from the transmitter antenna and one to the receiving antenna. If the scatter angle is increased, the received power falls off rapidly. With wide-beam antennas, the effective scatter volume is increased, but the effective received power is decreased. With narrow-beam antennas, the entire common volume contributes significantly to the received signal. For each ray of the entire beamwidth considered, a slightly different angle exists. It is necessary to sum up the infinite number of rays over the entire volume to obtain the sum of the individual contributions. It is correct to assume, therefore, that the scatter radiation comes from an infinite number of small scatterers within the common volume and that each scatterer effectively reradiates according to the amount of energy impinging upon it.

SCATTER CAPABILITIES. There is a great similarity between forward tropospheric scatter and ionospheric scatter in that the upper atmosphere must have different refractive indexes for communication to be possible. Ionospheric scatter communications are most effective on paths exceeding 400 to 500 miles, and their propagation techniques are slightly different from those used in tropospheric scatter communications. While tropospheric scatter is efficient and reliable up to approximately 300 miles, ionospheric scatter is commonly successful on links exceeding 1000 miles. Reliability requirements usually dictate the total path length of either type. Tropospheric scatter is used when a large number of channels is required, because its shorter path lengths make it more efficient than ionospheric scatter. Ionospheric scatter has a limited total information bandwidth.

Tropospheric scatter may be called upon to transmit any type of signal, although voice transmission is usually used. Normally, some teletype will be transmitted, using equipment that feeds four teletype channels into one voice channel. With modifications to certain equipment, facsimile, telemetry, video, and other types of communications may be transmitted and received. Total path attenuation due to the scatter phenomena is dependent upon frequency, distance and horizon angle.

ATMOSPHERIC CONSIDERATIONS. The atmosphere that surrounds the earth is stratified, and all the strata have varying characteristics and relationships. Scattering blobs are actually relatively small volumes in space at which the index of refraction varies from the surrounding atmosphere. Usually they are caused by the meeting of moisture-laden air masses and warm air masses, which effects a change in refractive index. Rain, snow, and similar weather conditions do not have a persistently noticeable direct effect on VHF and UHF waves.

Cosmic noise has an effect on radio propagation. Cosmic noise encompasses the radio interference caused by eruptions from sun and stars. Eruptions cause an abnormal ionization of the strata affecting the reflection and refraction conditions necessary for propagation. It is impossible to predict accurately the nature of these disturbances or even the effect of the phenomena. But, at the frequencies used in tropospheric scatter, cosmic noise and cosmic interference are usually so small that they do not interrupt communications. Man-made noise at the tropospheric scatter frequencies are usually of some consequence, however, and must be considered in the design and the location of scatter systems. Usually the least costly and simplest solution is to locate the system away from vehicular traffic, aircraft, machinery, electrified fences, power lines and other causes of man-made interference.

ANTENNAS. Two basic antenna systems are associated with Air Force tropospheric scatter systems. A wide variety of communication facilities is provided by the combination of these antenna systems with other equipment. It is possible to use either a 28-foot or a 60-foot parabolic reflector illuminated by either a vertically- or a horizontally-polarized feedhorn. If two reflectors are used, a vertically-polarized feedhorn to one reflector and a horizontally-polarized feedhorn to the other will provide a higher order of diversity. However, this necessitates cross-polarized feedhorns for reception at both terminals.

Due to the narrow beamwidth of the antennas, it is necessary that the equipment be made capable of changing both elevation and azimuth.

MOVING TARGET INDICATOR SYSTEM

General

The Moving Target Indicator (MTI) system effectively cancels clutter caused by fixed target signals, and thus reveals moving target signals. The unwanted echoes can consist of ground clutter (echoes from surrounding objects on the ground), sea clutter (echoes from the irregular surface of the sea), or even of echoes from storm clouds. The problem is to find the desired echo in the midst of the clutter. To do this, the MTI system must be able to distinguish between fixed and moving targets and then eliminate only the fixed targets. This is accomplished by phase detection and pulse-to-pulse comparison.

Target echo signals from fixed objects have a definite fixed phase relationship from one receiving period to the next, whereas moving objects produce echo signals that have a different phase relationship from one receiving period to the next. It is on this principle that the MTI system operates to discriminate between fixed and moving targets.

Elimination of fixed targets is obtained by delaying the signals received from each transmitted pulse for a period of time exactly equal to the pulse recurrence time, and then combining the delayed signals with the signals received from the next transmitted pulse in such a manner that the amplitudes subtract from each other. Since the fixed targets give the same amplitude on each successive pulse, they will be eliminated. But the moving target signals are, in general, of different amplitudes on each successive pulse so that the signals do not completely cancel but leave a small signal that can be amplified and presented on the indicators.

Block Diagram Analysis

The over-all operation of the MTI system may be seen by an examination of the block diagram of the Signal Comparator Unit of a typical radar set shown in Figure 15–6. This unit contains those stages that are the difference, essentially, between a normal receiver and an MTI receiver.

As shown in Figure 15–6, 30-megacycle signals from the signal mixer are applied to the 30-megacycle amplifier, where they are amplified, limited, and fed to the phase detector. Another 30-megacycle signal, obtained from the coho (coherent oscillator) mixer, is applied as a lock pulse to the coherent oscillator. The coho lock pulse is originated by the transmitted pulse and is used to synchronize the coherent oscillator to a fixed phase relationship with the transmitted frequency at each transmitted pulse. The 30-megacycle c-w signal output of the coherent oscillator is applied, together with the 30-megacycle echo signal, to the phase detector.

The phase detector produces a video signal whose amplitude is determined by the phase difference between the coho reference signal and the i-f echo signals. This phase difference is the same as that between the actual transmitted pulse and its echo. The resultant video signal may be either positive or negative. This video output, called coherent video, is applied to the 14-megacycle carrier oscillator.

The 14-megacycle c-w carrier frequency is amplitude-modulated by the phase-detected coherent video. The modulated signal is amplified and applied to two channels. One channel delays the 14-megacycle signal for a period equal to the time between transmitted pulses. The signal

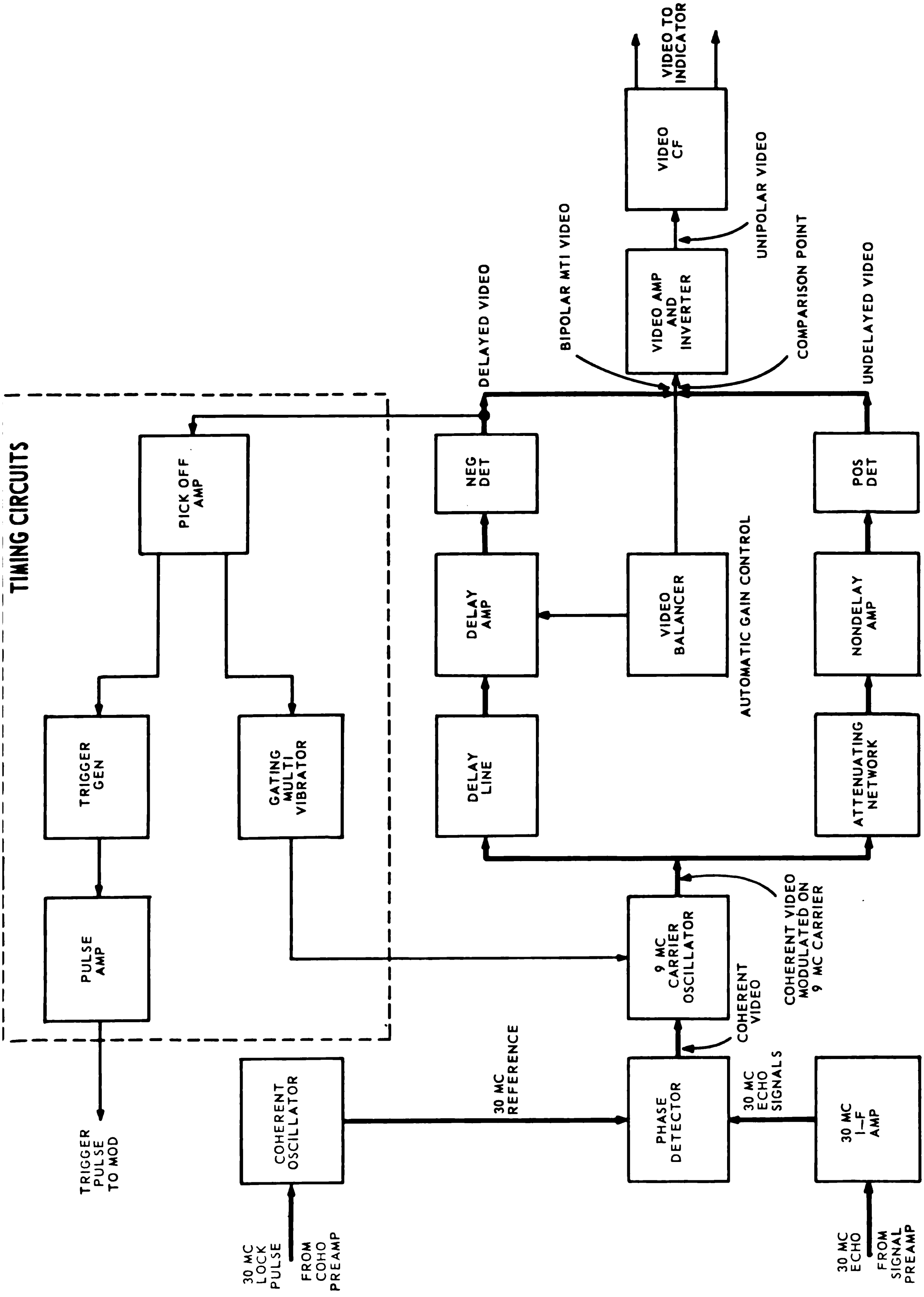


Figure 15–6. Block Diagram, MTI System

is then amplified and detected. The 2500-microsecond delay required (the period between transmitted pulses) is obtained by using a mercury delay line or a fused quartz delay line, which operates ultrasonically at 14 megacycles.

The signal to the other channel is amplified and detected with no delay introduced. This channel includes an attenuating network that introduces the same amount of attenuation as does the delay line in the delayed video channel. The resulting nondelayed video signal is combined in opposite polarity with the video signal resulting from the *preceding* echo signal, that is, the delayed signal.

The amplitude difference, if any, between the two video signals is amplified, and since the signal may be of either polarity, it is made unipolar. The resultant video signal, which represents only moving targets, is sent to the indicator system for display.

An analysis of the MTI system operation just described shows that signals from fixed targets, which have an unchanging phase relationship to their respective transmitted pulses, produce in the phase detector recurring video signals of the same amplitude and polarity. Thus, when one video pulse is combined with the preceding pulse of opposite polarity, the video signals cancel, and no information is passed on to the indicator system.

Signals from moving targets, however, will have a varying phase relationship to the transmitted pulse. As a result, the signals from adjacent receiving periods produce signals of different amplitudes in the phase detector. When such signals are combined, the difference in signal amplitude provides a video signal that is sent to the indicator system for display.

The timing circuits shown in Figure 15–6 are used to accurately control the transmitter pulse recurrence frequency to insure that the pulse repetition period remains constant from pulse to pulse. This is necessary, of course, for the pulses arriving at the comparison point to coincide in time and thus achieve cancellation of fixed targets.

As shown in Figure 15–6, a feedback loop is used from the output of the delay channel to the trigger generator and gating multivibrator circuits. The leading edge of the square wave produced by the detected carrier wave in the delayed video channel is differentiated at the pick-off amplifier and used to activate the trigger generator and gating multivibrator. The trigger generator sends a trigger to the modulator, causing the set to transmit.

The gating multivibrator is also triggered by the negative spike from the differentiated square wave. This stage applies a 2400-microsecond negative gate to the carrier oscillator. Thus, the 14-megacycle oscillator operates for 2400 microseconds and then is cut off. Since the delay line time is 2500 microseconds, the 14-megacycle oscillations stop before the initial waves reach the end of the delay line. This wave train, when detected and differentiated, turns the gating multivibrator on, producing another 2400-microsecond wave train. The 100 microseconds of the delay period when no signals are applied to the delay line is necessary in order that the mechanical waves within the line have time to damp out before the next pulse repetition period.

In this manner the pulse repetition period of the set is controlled by the delay of the mercury or quartz delay line. Since this delay line is also common to the video pulses going to the comparison point, the delayed and the undelayed video pulses will arrive at exactly the same time.

BASIC TELEVISION

Introduction

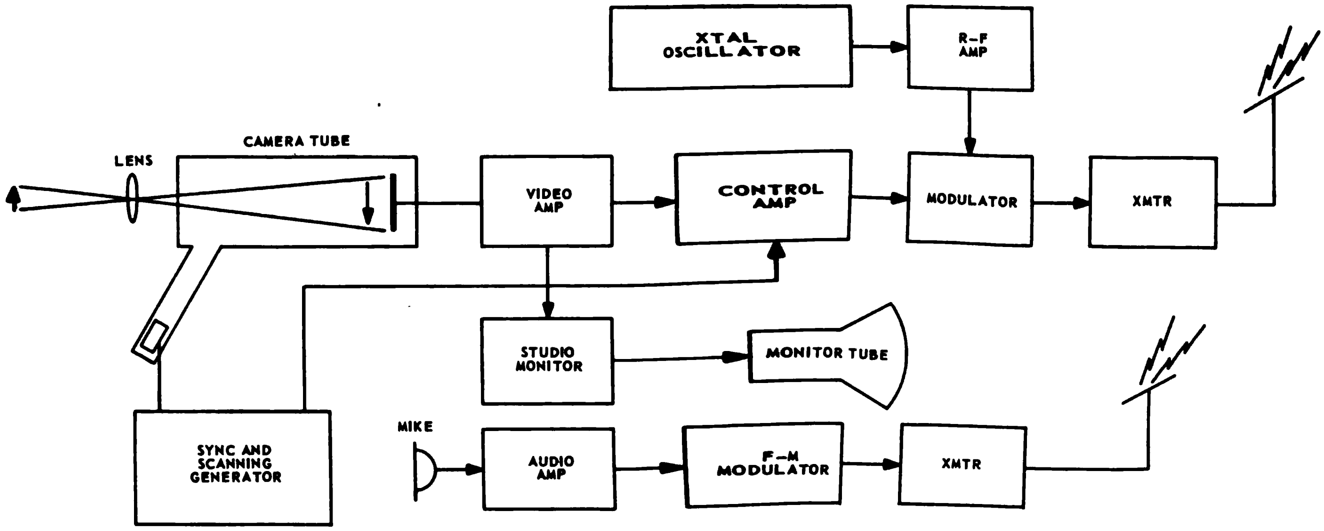
The scope of this discussion is intended only to familiarize you with the underlying principles involved in the transmission and reception of television signals. By definition, television is the system of telecommunication for the transmission of transient images of fixed or moving objects. Television is a comprehensive subject. Almost every type of electronic circuit is involved in its transmission and reception. A few years ago the radio was the principal means of communication via air waves. Today, television is a vital factor in the home, and closed circuit TV is rapidly becoming a new approach to instruction in the school.

Television Signal

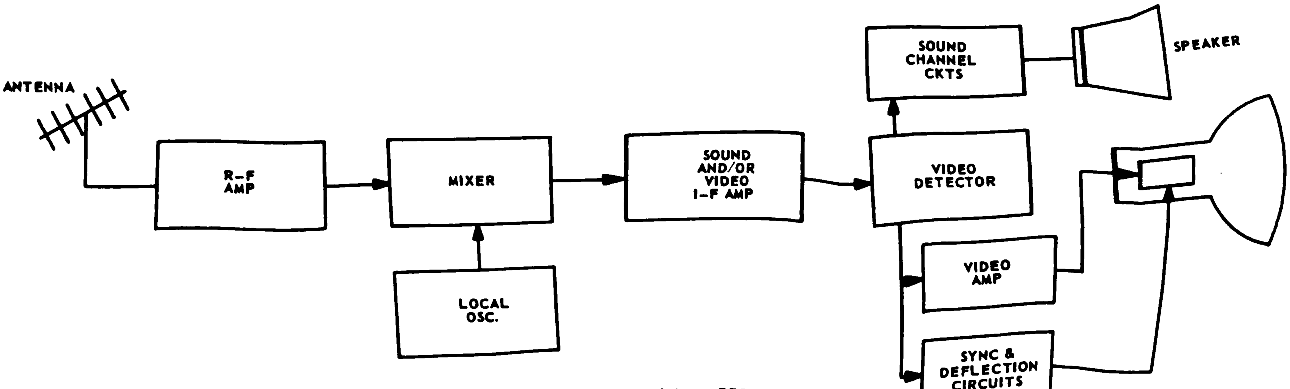
There are certain requirements that must be met for the television signal to be saitsfactory from an observer's point of view. The image should have the same smooth appearance as that of a good photograph. There must be absolutely no flicker. The size of the picture should be such that it can be viewed by several people who are eight feet or more from the screen. There must be an adequate amount of light from the receiver screen to meet the changing requirements of day or night viewing. The *aspect ratio* must be such that the picture "looks right" to the viewer. The aspect ratio is the ratio of the width to the height of the picture. This has been standardized as 4 to 3, making the picture height three-fourths of its width.

The essentials of a basic, modern television system are shown in Figure 15–7. In A, two transmitters are shown — one for picture and one for sound. Both transmitted signals, however, are picked up by a single receiver (B).

In the picture transmitter, a lens focuses the scene to be televised onto the photosensitive plate of the camera tube. (This plate is called a mosaic and will be explained later.) The mosaic is scanned by a thin stream of electrons from the electron gun of the camera tube. This scanning of the mosaic releases a series of electrical impulses, which become the video signal. The picture does not consist of one electrical impulse but of a series of signal impulses, each of which represents a very small area of the scene. These impulses are amplified and used to modulate the carrier, so that it can be transmitted into space. For the electron beam in the receiver tube to be synchronized with the beam in the camera tube, synchronizing pulses are inserted.



(A) TRANSMITTER



(B) RECEIVER

Figure 15–7. Simplified Basic Television System

The sound is transmitted by a conventional frequency modulation system, which consists of audio amplifiers, frequency modulators and a high frequency transmitter. Television is merely an extension of radio that includes both sound and picture, because in both cases, electromagnetic waves are used to transmit the signal.

As illustrated in Figure 15–7B, the video and sound signals are received by the same antenna and fed to the r-f amplifier. The diagram illustrates the sound being separated from the video after the mixer. The amplification of the sound signal at this point is less than that of the video signal. This is to prevent the sound signal from producing interference on the picture screen. After separation, the sound signal goes through one or more i-f amplifiers, an f-m detector, audio amplifiers, and the speaker.

After the mixer, the video signal is conveyed to the video i-f amplifiers, which produce two signal outputs. One is the video signal and blanking pulse. The other is the sync pulses that are fed to the sync and sweep circuits. There must always be a correlation between the sweeping electrons in the picture tube and the camera tube. The point at which the electrons strike the mosaic image must be the same point in the picture tube. This is the responsibility of the sync pulses. For black and white images, only vertical and horizontal sync pulses are required.

Television Camera Tubes

The quality of image at the receiver is no better than that received and transmitted by the camera tube. There are several types in use. The one best suited to the purpose is used.

The *iconoscope* camera tube converts variations of light intensity in the picture elements into an orderly sequence of electrical impulses that correspond to the picture information. To do this, the photoelectric properties of the mosaic image plate are used.

It is known that certain metals have the characteristic of emitting electrons when light rays strike their surface. A light sensitive surface is usually prepared by evaporating a thin layer of caesium oxide on a silver base. A positive collecting plate is placed close to the emitting surface. The emitted electrons can be taken from the positive plate to produce an electric current that corresponds to the specific light ray.

The rectangular mosaic plate in the iconoscope is made up of many microscopic globules of sensitized caesium-silver compound that have been deposited on a thin sheet of insulating mica. The globules do not come into contact with each other, since they are insulated by the mica. On the opposite side, a continuous layer of a conducting substance is deposited; from this layer there is an electrical connection to an external circuit.

This construction actually forms a capacitor. The dielectric is the mica; the globule forms one plate, and the conducting substance on the backside of the mica the second plate.

The object to be televised is focused on the front face of the mosaic plate. Each globule emits electrons according to the light ray intensity reaching it; thus, each globule will assume a different positive charge, and will retain this charge, since it is insulated from the other globules. The mica also prevents any leakage of the charge to the conducting layer. The conversion of light variations into electrical impulses has now been accomplished. Figure 15–8 illustrates the construction of the iconoscope. The electron gun and deflection coils are similar in purpose to those of other CRT's. The electrical impulses are amplified and fed to the next stage.

The iconoscope will produce a good picture if the scene is sufficiently illuminated, but it has a tendency to produce background shading due to the secondary emission of electrons from the mosaic. A camera tube that is more sensitive than the iconoscope is the *image orthicon* (see Figure 15–9). The greater sensitivity of the image orthicon tube gives it two important advantages. One is the ability to televise scenes that have insufficient illumination to produce an acceptable image with other camera tubes. The other is the greater depth of field, permitting the inclusion of background that would otherwise appear blurred or obscured on the receiver screen.

In operation, the light rays from the scene to be televised are focused on a transparent photosensitive plate by a lens. From each point on the inner surface of this plate, electrons are emitted in proportion to the intensity of the incident light. The emitted electron image (the electron density at each point corresponds to the light at that point) is drawn to the target by a positive

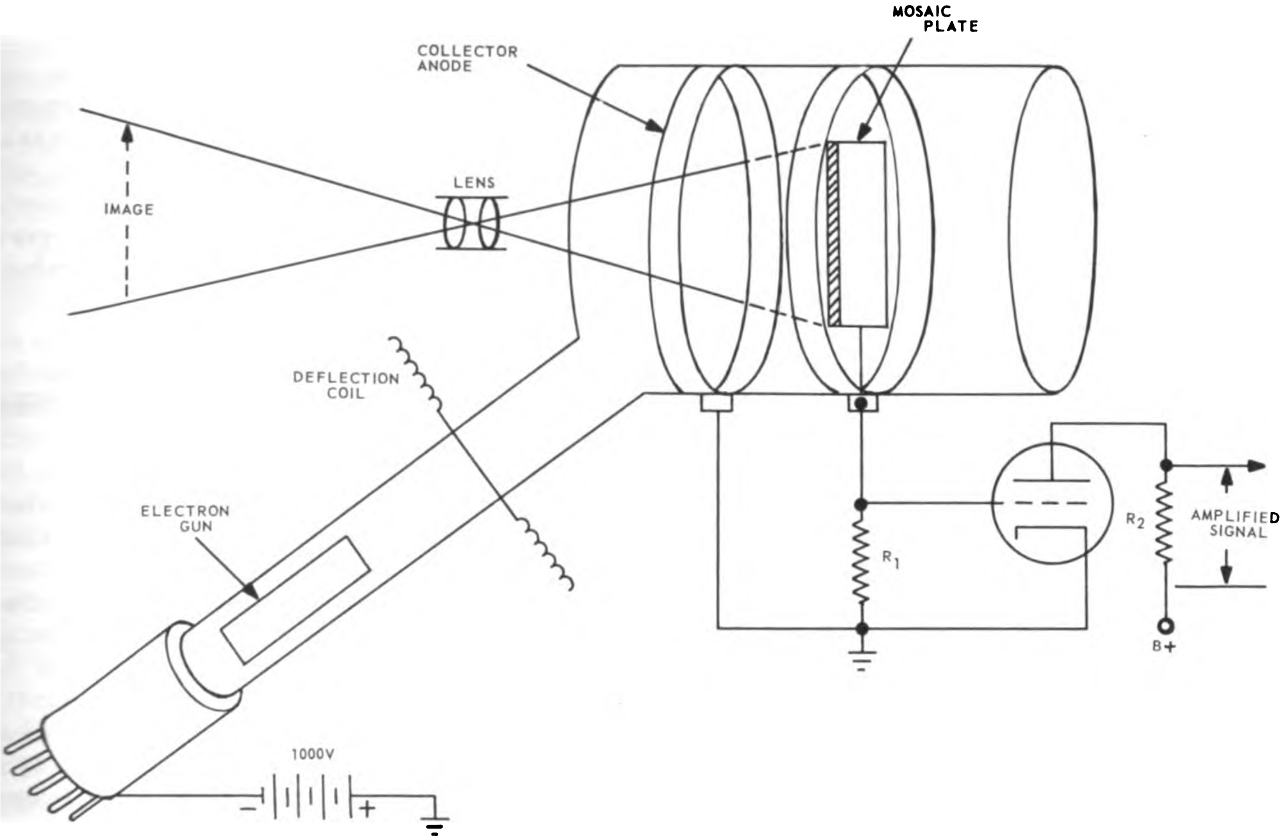


Figure 15–8. Internal Construction of Iconoscope

wall coating. At the target the accelerated electrons produce secondary emission, leaving a pattern of positive charges on the target due to the deficiency of electrons. This pattern corresponds to the pattern of light from the scene, with the most positive parts of the target representing the lightest parts of the scene.

At the same time the pattern is being formed on the target plate, the plate is being scanned by the beam from the electron gun. The velocity of the electron beam, as it approaches the target, is such that it is deflected and returns to the source. However, when the scanning beam moves over a section of the target containing a positive

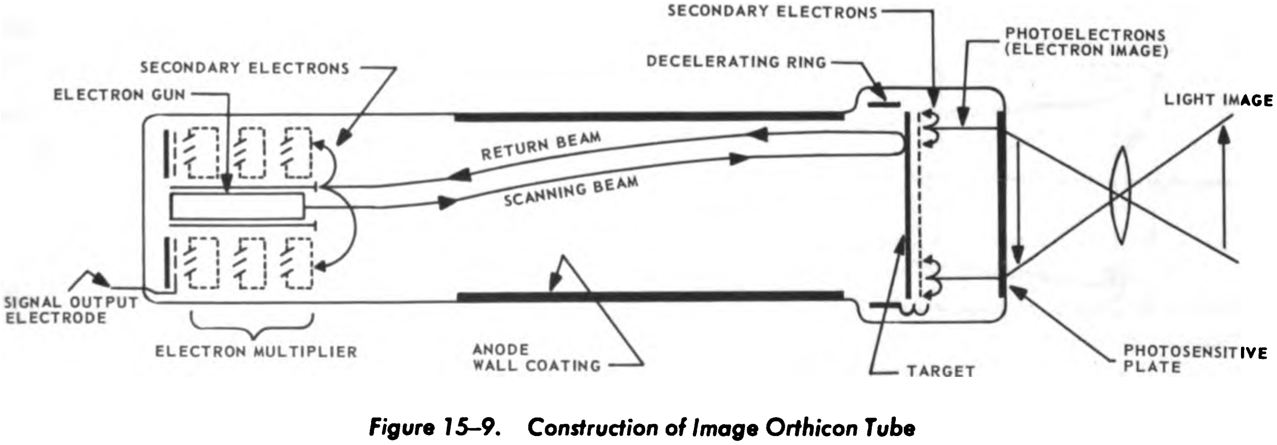


Figure 15–9. Construction of Image Orthicon Tube

charge, electrons are deposited to neutralize the positive charge. The returning electron beam, then, contains variations in magnitude that correspond to the charge distribution on the target plate and the picture elements in the optical image. Hence, the most positive points on the plate return the least number of electrons from the original scanning beam, and the voltage across the output resistor is inversely proportional to the positive charge intensity on the target.

The added sensitivity of the image orthicon is a result of several factors. The increased efficiency of the storage action on the target allows storage over a greater portion of the frame period. The combination of the higher photosensitivity obtainable with a conducting photocathode surface and the secondary-emission gain of the electron image at the target provides an increase of camera sensitivity. By the use of a two-sided target, the charge pattern can be formed on one side while the opposite side is scanned by the low-velocity electron beam. This reduces shading difficulties and provides stable operation over a wide range of scene brightness. An additional gain is provided by the electron-multiplier arrangement where a number of electrons are produced for each electron impinging thereon.

The image orthicon is used extensively in field television operations outside the studio, such as on-the-spot reporting of news, sports events, and conventions. Any scene with sufficient illumination for direct viewing by an audience can be televised satisfactorily with the image orthicon camera because of its sensitivity and its ability to operate over a wide range of light values. In addition, the optical system used with the image orthicon is sufficiently compact to make it suitable for portable field equipment. While the image orthicon is also used for studio cameras, the iconoscope and image dissector tubes are preferred in many instances — particularly when televising motion-picture film.

The *image dissector* tube is considered to be an instantaneous scanner because the principle of storage is not involved. The tube, shown in Figure 15–10, contains a flat photosensitive cathode located at one end of the tube. The scene to be televised is focused on this cathode by a suitable lens system located outside the opposite end of the tube. As the light reaches the cathode, electrons are emitted in direct proportion to the amount of light striking any one point. Thus, the electron distribution leaving the cathode is the same as the distribution of light in the optical image. This electron image extends from the cathode surface and is attracted toward the aperture of the electron multiplier by the voltage on the nickel anode coating. As the electrons pass through the aperture, they are multiplied by secondary emission in the multiplier.

Since the aperture that receives the electrons (to provide the signal current) is stationary, the entire electron image must be moved past the opening in such a way that electrons are taken in the required succession of picture elements and

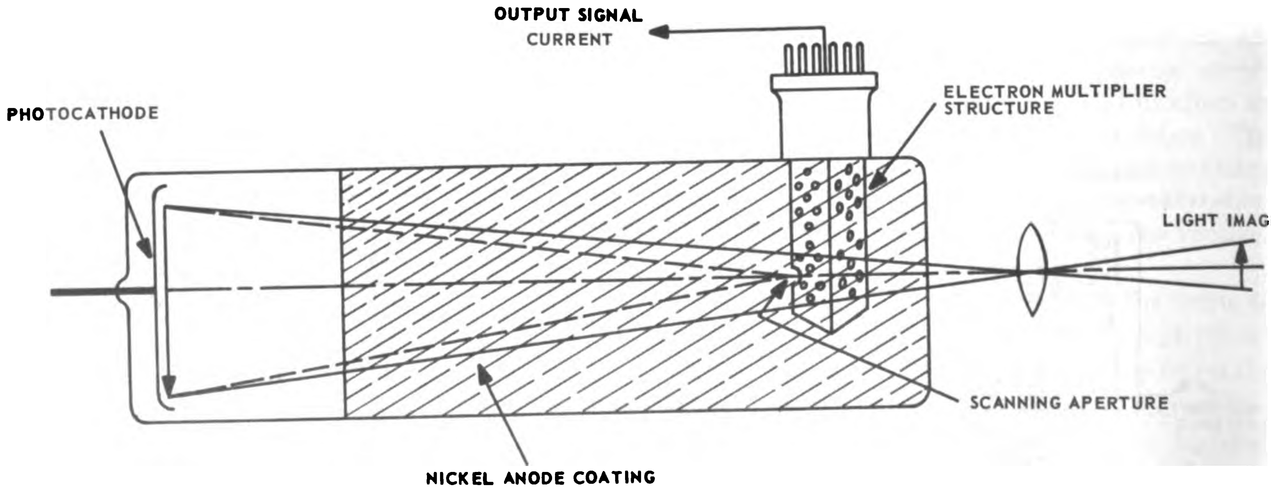


Figure 15–10. Construction of Image Dissector Tube

scanning lines. The electron image is moved past the aperture in the desired scanning pattern by vertical and horizontal deflecting coils mounted externally around the tube. The electrons that are not being scanned are attracted to the anode coating, but since the cathode provides a steady stream of electrons, the image is not destroyed.

The vidicon camera tube is similar to the image orthicon. The image is focused optically on a photoconductive target. When light strikes the target area, the scanning beam neutralizes the charged elements and current flows through the load resistor. The vidicon is physically smaller than the orthicon and is portable. Therefore, it finds wide application in industrial and tactical television systems. In fact, the portability of this tube has permitted the development of a man-carried television transmitter to be used for observation in forward combat areas.

Television Scanning

In transmitting a picture, it is possible to transmit at one time all elements that compose the picture, or to transmit each element separately in a specific order. The complexity of the first system has been a determining factor in adopting the system of transmitting each element separately. Other methods of scanning that might be employed are narrow horizontal strips, narrow vertical strips, and spiral scanning.

To explain television scanning, let us review the scanning process of the conventional A-scan CRT. A horizontal sawtooth is applied to the horizontal deflection plates or coils and this causes the electron beam to travel from left to right across the face of the tube. On the negative-going portion of the sawtooth, the electron beam is retraced sharply back to the left side. Therefore, with no signal applied to the vertical deflection plates, a straight line appears on the screen.

Now apply a sawtooth voltage of one frequency to the vertical plates and a sawtooth voltage of a much higher frequency to the horizontal plates. The resultant is an electron beam traveling from left to right on the screen and also moving from top to bottom, as shown in Figure 15–11. A vertical sawtooth frequency of 60 cycles per second and a horizontal sawtooth of 15,750 cycles per second produced the lines shown. It is evident that the speed of the horizontal sawtooth is much greater than that of the vertical sawtooth. In fact, each line appears as an almost straight horizontal line. There are approximately 525 lines present from top to bottom of the television screen.

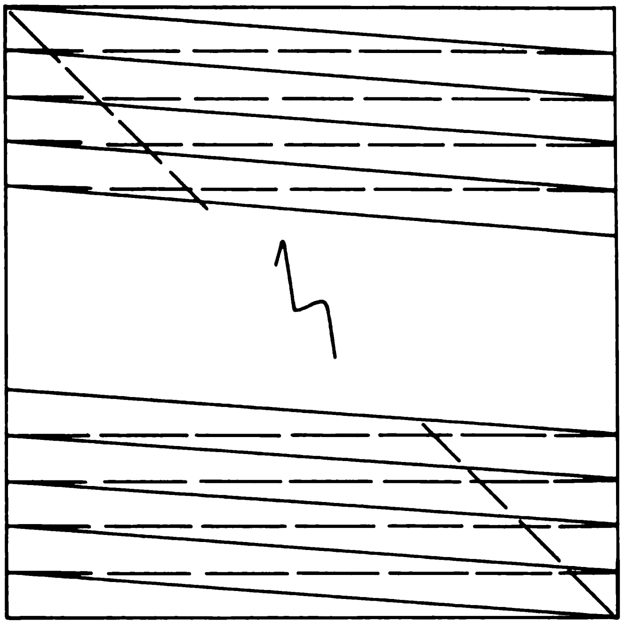


Figure 15–11. Basic Television Scanning

Let us now apply the scanning process to the iconoscope camera tube (Figure 15–8), and visualize the resulting action. Assume that the camera is focused on the scene to be televised and that each globule on the mosaic signal plate has a positive charge proportional to the light sensitivity it received.

At the first instant the electron beam, focused in the neck of the tube, is accelerated toward the upper left-hand corner of the mosaic signal plate. There, under the influence of the vertical and horizontal sawtooth voltages, it moves to the right and downward. As the electron beam moves to the right, it passes over the charged globules that are across the top of the mosaic plate. As each globule is scanned, electrons from the electron beam restore those electrons that have been emitted. This brings each globule back to its neutral state.

During the time that each globule is being neutralized, the charge that is held on the conducting surface of the opposite side of the mosaic is released. The releasing of this charge causes a small current to flow in R1 of Figure 15–8. The strength of the current is proportional to the charge on the neutralized positive globule which, in turn, was proportional to the intensity of light focused on the mosaic at that particular point. In this manner, then, light rays have been transformed into equivalent electrical currents.

Interlaced Scanning

In the modern television system, 525 lines constitute the scanning *raster*. By definition, raster is the predetermined pattern of scanning lines which provides substantially uniform coverage of an area. Since 15,750 lines are scanned per second, 30 frames are scanned per second. By definition, a frame constitutes the entire picture or image. The fundamental rate of 30 frames (images) was chosen because of its relation to the commercial power frequency. This frequency rate requires less filtering of the AC ripple, which causes the hum in the audio system.

In order to eliminate all possibility of flicker, the raster is scanned at an effective rate of 60 frames per second. To do this, the downward travel rate of the scan is increased so that every other line is scanned. This pattern is shown in Figure 15–12. The odd-numbered lines are scanned first, and then the beam returns and scans the even-numbered lines. As the odd and even lines are transmitted in $\frac{1}{60}$ of a second each, both add up to $\frac{1}{30}$ of a second. The scanning of both lines takes $\frac{1}{30}$ of a second, so 30 frames per second is still the fundamental rate. But the *effective* rate is 60 frames per second and there is no flicker. This method of transmitting television images is called interlaced scanning.

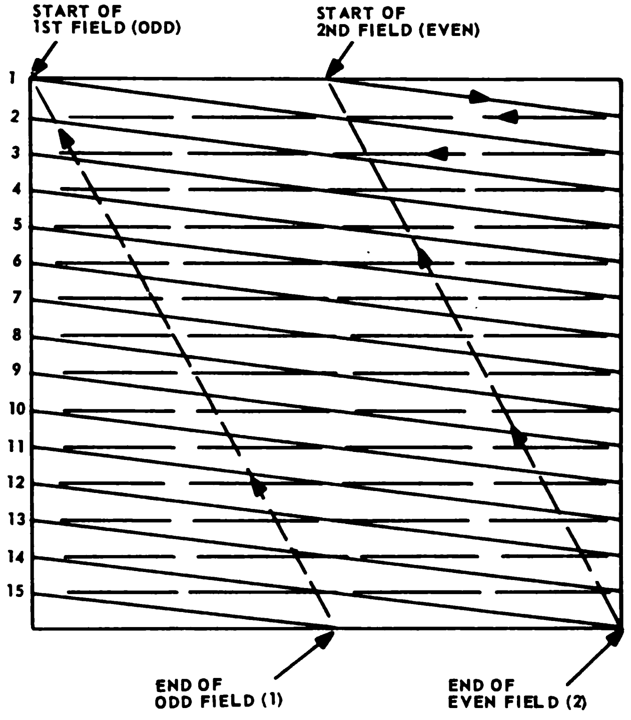


Figure 15–12. Interlaced Scanning

Compound Television Signal

The television signal is composed of three types of information. *Video signals* are the orderly sequence of pulses that represent the light intensity of the televised scene. The *sync pulses*, both horizontal and vertical, are rectangular pulses for locking-in both camera tube and picture tube. The *blanking pulses*, horizontal and vertical — are rectangular pulses for blanking out the screen during retrace time.

Another group of signals is transmitted to insure proper spacing of the interlaced scanning lines and to prevent loss of synchronization of the horizontal circuits during interlaced scanning. These are called *equalizing pulses*.

Picture Signal

Figure 15–13 illustrates the entire transmitted signal. It depicts, from left to right, the last four lines being scanned, the vertical blanking time, and the first three lines of the next frame. During the vertical blanking time, the vertical pulse intervals hold the picture stationary vertically. These pulses of very short duration insure equidistant spacing between scanning lines. The equalizing pulses also permit transmission of identical vertical intervals between frames and between fields, and maintain horizontal synchronism during vertical trace intervals.

Figure 15–14 expands that portion of the video signal which depicts scanned lines. Notice that of the total amplitude available, approximately 80% is used for video signal variations. Where the signal voltage stops, the blanking voltage begins. Notice also that the blanking and sync voltages always have the same amplitudes, regardless of signal voltage, because of requirements in the television receiver.

Figure 15–14 also illustrates the polarity of signal as used in receivers in the United States. The reverse is used in England. The type used in the United States is designated as negative picture transmission, and the picture is said to

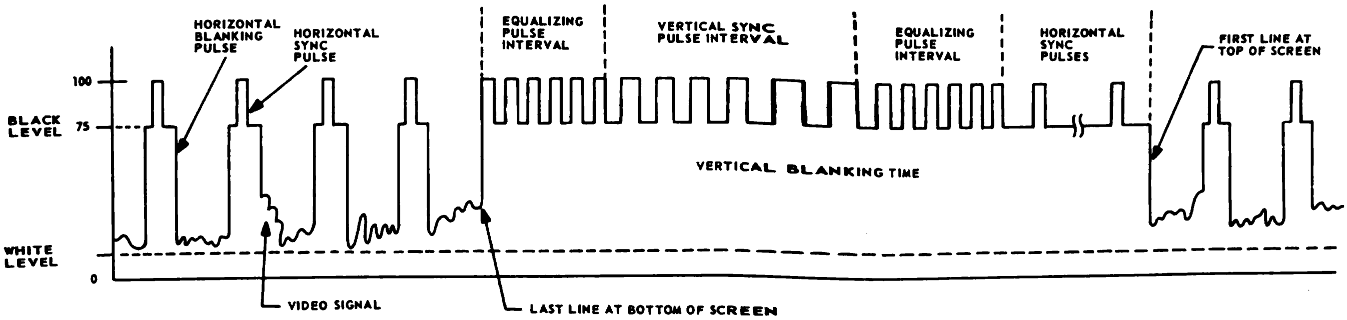

HORIZONTAL BLANKING PULSE
HORIZONTAL SYNC PULSE
EQUALIZING PULSE INTERVAL
VERTICAL SYNC PULSE INTERVAL
EQUALIZING PULSE INTERVAL
HORIZONTAL SYNC PULSES
FIRST LINE AT TOP OF SCREEN
100
BLACK LEVEL
75
WHITE LEVEL
0
VERTICAL BLANKING TIME
VIDEO SIGNAL
LAST LINE AT BOTTOM OF SCREEN

Figure 15–13. Complete Video Signal

be in the negative picture phase. The signal voltages from the iconoscope have been completely reversed. In the receiver the voltages must be changed again before being applied to the cathode of the picture tube. It is said that less interference and better over-all reception is obtained under this negative picture transmission.

Signal Standards and Frequencies

Commercial television channels are 6 megacycles wide. In contrast, the entire radio broadcast band is slightly more than 1 megacycle wide; each broadcast channel is only 10 kilocycles wide. It is necessary to have a wide television channel in order to transmit pictures that have clarity and sharpness. Within the 6-megacycle band are found both picture and sound carriers and corresponding sidebands. Picture carrier and sidebands occupy approximately 5.75 megacycles, a very large percentage of the total band. The picture carrier is amplitude modulated, and the sound carrier frequency modulated.

Vestigial Sideband Transmission

An unsymmetrical distribution of the frequency band of the picture carrier allows transmission with much better definition. This is called *vestigial sideband transmission,* with the high-frequency sideband being 4.5 mc wide and the low-frequency sideband being only 1.25 mc wide. Since the modern television station broadcasts signal components up to 4 megacycles wide, a 9-megacycle channel would be required to transmit a symmetrical sideband. In a 6-megacycle channel, leaving a frequency for the sound carrier and transmitting with symmetrical sidebands, the highest frequency picture component is a little over 2.75 megacycles. Picture components would be lost in transmitting just these frequencies. However, by partially suppressing one sideband, it is possible to transmit a 4-megacycle component in a 6-megacycle band. Figure 15—15 illustrates the unsymmetrical distribution in vestigial sideband transmission.

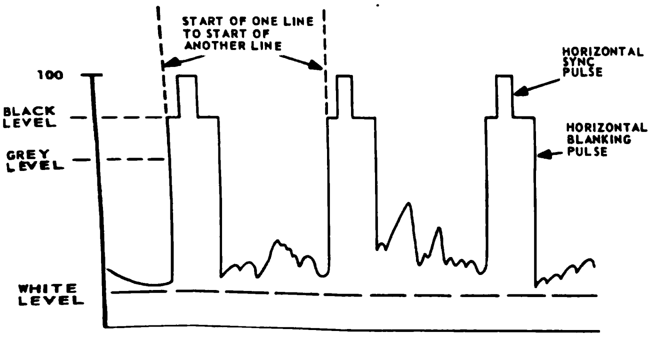

START OF ONE LINE TO START OF ANOTHER LINE
100
BLACK LEVEL
GREY LEVEL
WHITE LEVEL
HORIZONTAL SYNC PULSE
HORIZONTAL BLANKING PULSE

Figure 15–14. Expanded Video Signal

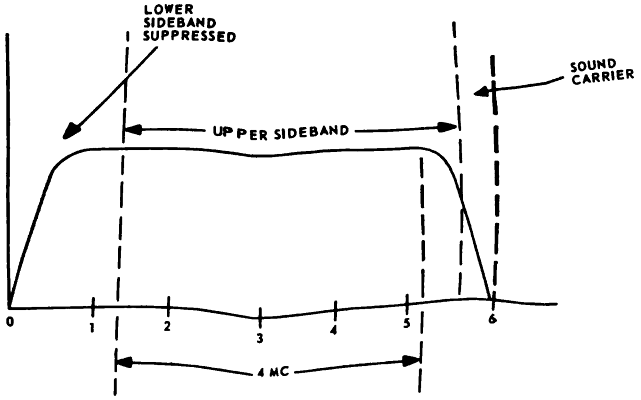


Figure 15–15. Frequency Distribution in Vestigial Sideband Transmission

The image to be telecast is focused on the image plate of the camera tube, which is comparable to the mike of the audio transmission. The iconoscope and the image orthicon are the tubes most generally used: the iconoscope for film projection and studio, the image orthicon for remote telecasts. Each tube has characteristics that make it desirable for its particular use.

The current resulting from the scanning of the image plate is amplified by a stage that is normally a part of the camera unit. The signal then goes to the distribution and control amplifiers, where the signal is mixed with signals from other cameras. Here the signal can be switched in and out or superimposed on another. Blanking and sync pulses are added at this time, being inserted with the proper polarity in respect to the black and white polarity of the picture signal. Let it be emphasized at this point that the sync generator is the timing unit of the entire system. In addition to acting as the basic timing unit, the sync generator initiates, shapes, and arranges in the correct sequence all the pulses required for control of the 525-line interlaced scanning.

After leaving the control amplifiers, the composite television signal goes to the modulator. The modulator has a unique assignment. It must amplify a single-polarity signal which is varying with the average brightness of the televised scene. The modulator must maintain a constant level of the blanking and sync pulses irrespective of the variations of the picture content and average brightness. The modulator is composed of DC amplifiers and/or DC restorers which maintain the level of the composite television signal.

The signal, after leaving the modulator, is fed to the transmitter unit, which is comprised of the frequency determining device, multipliers, amplifiers, and transmitter tube. The transmitter unit is comparable to any high-frequency transmitter.

TELEVISION RECEIVERS

We have stated previously that the television receiver uses the same antenna to pick up the picture and sound signals. Therefore, it is necessary for both the antenna and the r-f section to have a response over the 6-megacycle band of frequencies that is uniform, yet selective enough to discriminate against undesired frequencies. In a fringe area the antenna should be directional toward the transmitter site and should have a broad, flat bandpass.

In one early type of receiver, the video and sound signals are separated at the mixer and fed to their respective i-f amplifiers. The signals are then treated separately, the sound going to an FM detector and the video going to an AM detector.

In the *intercarrier-type* receiver, the video and sound signals are not separated until after they pass through a common i-f amplifier designed to yield a sound carrier with an amplitude of only about 5% of the maximum picture carrier amplitude. In the following video detector, heterodyne action produces a 4.5 mc sound i-f and all of the video frequencies. Even after passing through the video amplifier, the 4.5 mc sound i-f is low enough not to affect the picture, and must be amplified before going to the FM detector and audio amplifier.

Controls

Figure 15–7B showed us the block diagram of a conventional-type television receiver, simplified. Before analyzing the circuitry of the receiver, let us briefly discuss the panel controls.

CONTRAST. This is the most important control on the front panel. This control adjustment is as necessary to the picture as the volume control is to the sound. The higher the video voltage applied to the control grid of the CRT, the more intense the image on the screen.

BRILLIANCY CONTROL. The brilliancy (brightness) control, usually tied to the cathode of the CRT, adjusts the DC bias on the picture tube, and is a working partner with the contrast control. The brilliancy control should not be adjusted until the contrast has been adjusted to the desired intensity of image. A proper setting of this control just biases the CRT to the point where the blanking voltage level cuts off the electron beam.

FINE TUNING. This control is a variable capacitor across the oscillator-mixer circuit. This control is important in that at high frequencies, small variations mean large frequency changes. The frequency desired is selected by either push button or switch, and the fine tuning compensates for small variations in the oscillator tuning circuit to center the signal on the i-f bandpass.

Hold Control. There are two hold controls, ne for the vertical circuit and one for the horiontal circuit. If the picture should get out of ync, these controls enable the viewer to bring ne oscillators within the range of control of the ync pulses.

Other variable controls, generally placed in he rear of the set, include horizontal linearity, ertical linearity, width control, height control, ertical centering, horizontal centering, and ocus. There may be additional controls, dependng on the complexity of the receiver.

eneral Theory of Operation

Using Figure 15–7 for reference, let us now xplain in a general sense the operation of the arious blocks. The r-f amplifier has a three-old function. First, it provides amplification of ignals where the voltage value of the signal is he lowest. The r-f amplifier must operate inearly over a 6-megacycle band of frequencies or each channel. Gain is sacrificed to obtain mplification that is linear over a wide bandpass. igure 15–16 illustrates a typical r-f amplifier or a TV circuit. Note the loading resistors across he tuned circuits.

Second, the r-f amplifier helps to discriminate gainst close frequencies in adjacent channels. Especially is this true for image frequencies. Tube interference, snow on the screen, is also essened with proper circuit design. Third, local oscillator radiation can interfere with other receivers that are nearby.

Detection in television is no different from that in radio; the intelligence (picture modulation) is removed from the i-f carrier. The modulation contains picture, blanking, and sync information. This is the composite television signal. As with any diode operation, there is no current flow when the cathode is more positive than the plate. The effect of this action, then, is a unipolar signal with the circuit designed to develop a signal of the proper polarity for use on the grid of the picture tube. At the same time the circuit must be designed to detect the highest and lowest frequencies of the i-f spectrum.

Since the American receivers use negative picture transmission, a positive output from the video detector is designated as negative picture phase. The signal is inverted before being applied to the grid of the picture tube. The simplified circuit in Figure 15–17 is used to show the inversion of signal for proper application to the grid of the picture tube.

The i-f system of a television receiver is much more complex than that of a radio receiver. Picture and sound must be amplified and separated. Figure 15–18 is a simplified drawing illustrating the separation of picture and sound. Gain over a band of frequencies 4 to 5 megacycles wide must be linearly amplified. Since the selectivity of the i-f stages is a factor that must be carefully

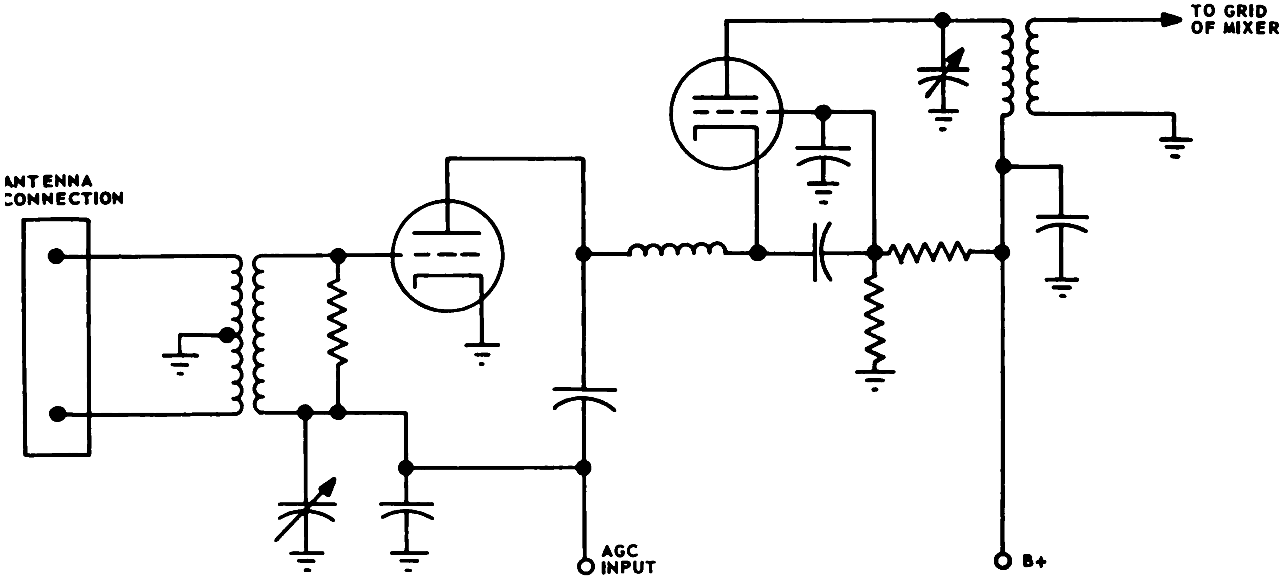


Figure 15–16. Typical R-F Amplifier

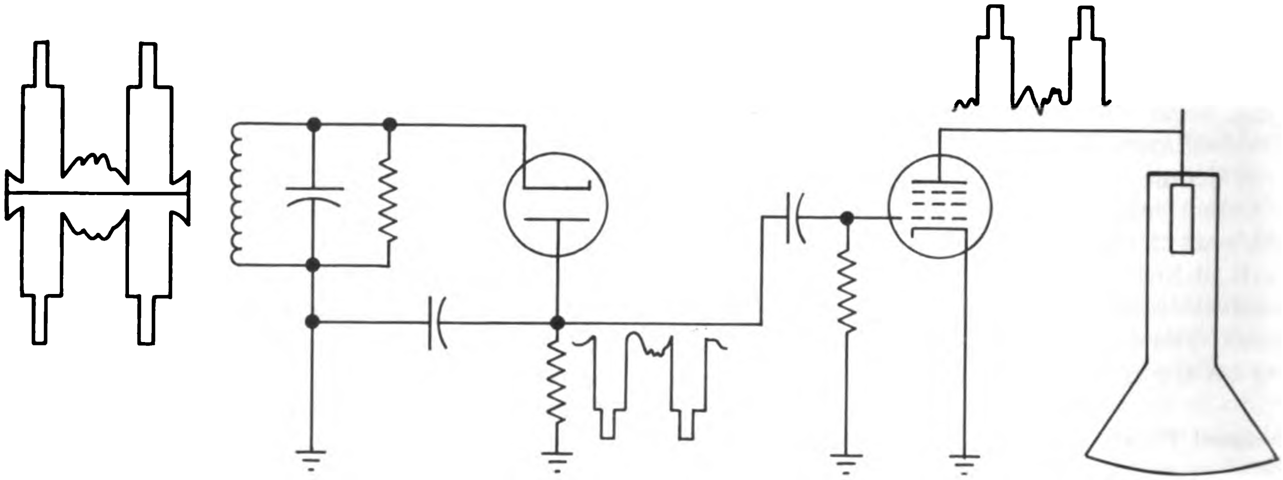


Figure 15–17. Detector, Inverter Amplifier Stages

considered, stagger tuning is used extensively. In fact, the majority of television sets of today use single-tuned coils between the stages of video i-f amplifiers. Each coil is tuned to a different frequency that is within the 4-megacycle bandpass of the system. The desired over-all response of all stages is a broad, flat bandpass. Unwanted frequencies that are present are rejected with wavetraps. The wavetraps are specifically designed to reject sound i-f frequencies that cause undesirable bar patterns on the picture tube screen.

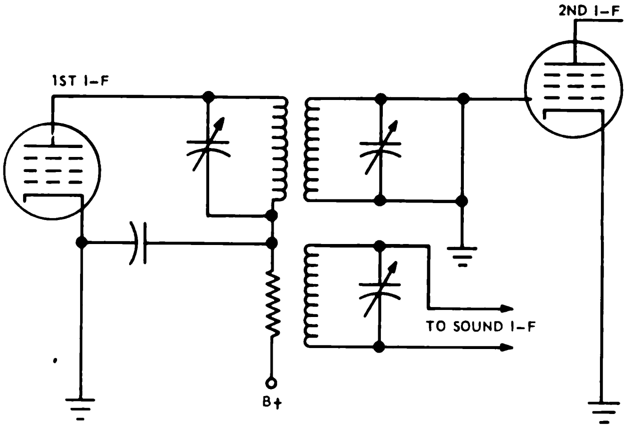


Figure 15–18. Picture-Sound Separation, Simplified

Many receivers use an Automatic Gain Control (AGC) system to prevent the undesirable effects of variations in signal strength. AGC provides a voltage that varies the gain of the i-f amplifiers and keeps constant the peak amplitude of the signal applied to the video detector.

So far the discussion has taken the received TV signal through an r-f amplifier, converted it to an i-f frequency, further amplified it with i-f amplifiers, and detected it. More amplification is now necessary because the amplitude of the signal from the detector is not sufficient to provide the required contrast on the CRT.

While each i-f amplifier stage is required to pass a 4-mc band, the video amplifier must be designed to have a flat response over a band that extends from 30 cycles to 4 megacycles. The rectified video signal contains the picture, and blanking and synchronizing information, which must be passed without attenuation or phase distortion. Associated with the video amplifier is a DC restorer. The purpose of the DC restorer is to return all blanking voltages to the same level as they were in the incoming signal. The shift in the blanking level is caused by the charge and discharge characteristics of any RC coupling between stages. If the shift of the blanking level is not corrected, we will have either a picture that is too dark or too bright.

The composite video signal shown in Figure 15–13 includes the synchronizing pulses that are required to synchronize transmitter and receiver scanning. The sync pulse is applied during the blanking period to prevent it from interfering with the picture information contained in the camera signal. The horizontal sync pulse at the end of each line causes the beam to return to the left side of the screen in position for the next line. The vertical sync pulse at the end of each field causes the beam to return to the top of the image in position to begin the next field.

Note that the amplitude of the signal is divided into two sections; the lower 75% is devoted to the active camera signal, and the upper 25% is used for the sync pulses. The smallest amplitudes of the signal represent the whitest parts of the picture, while the higher amplitudes represent the darker picture information until the black level is reached. The black level is maintained at a constant percentage of the peak signal amplitude in order to maintain a brightness level in the television system.

The combination of blanking pulses, sync pulses, and image signal is shown in Figure 15–13 in which four complete horizontal lines of the picture are represented. At the end of each line, the blanking pulse is applied to the beam. This prevents the electrons from reaching the mosaic of the camera tube or the screen of the receiver. While the blanking signal is being applied, the sync pulse causes the horizontal deflection coils to move the position of the electron beam from the right side of the picture to the left. When the blanking voltage is removed, the electron beam starts scanning again. This process continues until all the lines in one field have been scanned. The time between the beginning of the blanking pulse and the beginning of the sync pulse is called the *front porch*, while the time between the end of the sync pulse and the end of the blanking pulse is called the *back porch.*

The vertical motion stops at the bottom of the field. Therefore, it is necessary to bring the beam quickly to the top of the picture so that the next field can be scanned. As the vertical triggering and retrace require a longer period of time than the horizontal triggering and retrace, a longer blanking signal must be used.

The left side of Figure 15–13 shows the last four lines at the bottom of a field with the required horizontal blanking and synchronizing pulses. After the last active line, the signal is brought up to the black level by the vertical blanking pulse in preparation for the vertical retrace. To provide exact timing of the vertical retrace motion in successive fields, the vertical blanking period begins with six equalizing pulses spaced at half-line intervals and ends with six equalizing pulses.

The vertical sync pulse is broken up to permit the horizontal sync voltages to continue without interruption. Otherwise, the horizontal oscillator that controls the retrace in the receiver would slip out of control with a resulting loss of synchronization. The serrated pulse permits both retraces to occur at the proper time. One vertical pulse occurs at the end of every 262½ lines, while a horizontal pulse is applied at the end of each line.

TELEVISION ANTENNAS

Introduction

The antenna system is the link between individual transmitting stations and the multitude of widely scattered receivers. Regardless of the form of the television antenna, its main purpose is to radiate or receive signal-bearing r-f energy. The considerations involved in the design of a television receiving antenna are somewhat different from those of the broadcast receiving antenna. First, the television antenna must obtain a good signal over an extremely wide range of frequencies. Second, the amount of signal voltage that can be induced is limited by the short lengths of television antennas. Third, the problem of obtaining adequate signal is increased by the low signal strengths encountered in VHF. The problems that are particularly important in the television antenna system include selecting and positioning of a suitable antenna, selecting and installing a suitable transmission line, and proper matching of antenna, transmission line, and receiver impedances. Each of these items is critically important in receiving good pictures.

Propagation of Direct, Sky, and Ground Waves

In considering propagation in the 30- to 1000-megacycle band, which contains the television channels, only the direct-wave mode of propagation is effective at all times. The direct wave, also called the space wave, is defined as *that part of the radiated energy which travels directly from the transmitting antenna to the receiving antenna. No portion of the direct wave is returned from the ionosphere above or reflected from the surface of the earth or objects on or above it.*

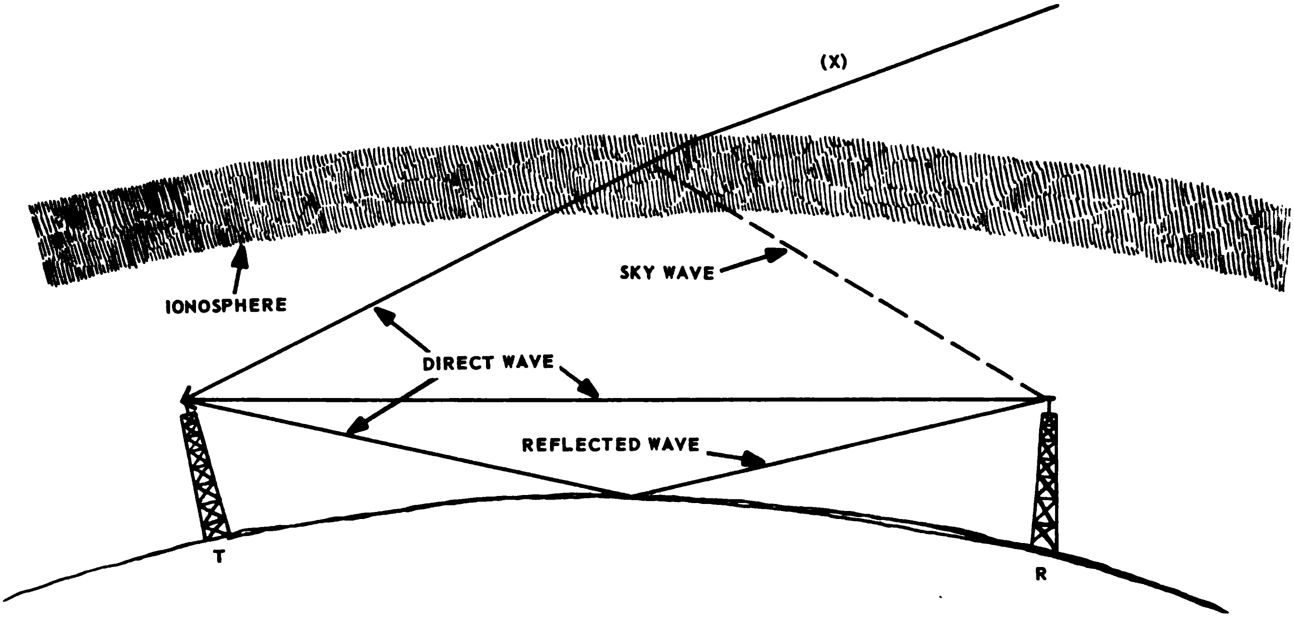


Figure 15–19. Direct, Sky, and Reflected Waves

Sky-wave propagation and ground-wave propagation, which account for the long distances covered by radio waves at lower frequencies, become less effective as the frequency is raised above 30 mc. At frequencies of 100 mc and above, they are of little practical use except under unusual circumstances.

Figure 15–19 shows the portions of the radiated wave in the sky wave, the reflected wave, and the direct wave. The sky wave is considered to be that portion of the radiated wave (shown by the dotted line) that reaches the receiver antenna after being deflected downward from its original direction. This wave becomes very weak when the frequency is increased. At 30 mc and above, most of the radiated energy passes through the ionosphere with only slight downward bending, excepting under unusual ionospheric conditions.

Ground wave propagation refers to the part of the radiated energy that moves along the surface of the earth at and closely above the boundary between the surface and the air above it. At frequencies below 3 mc, the ground wave (sometimes called the surface wave) is strong enough to permit communication at distances far beyond the optical horizon. At the higher frequencies, however, the attenuation of the ground wave is great, and the true ground or surface wave decreases in strength so rapidly that it is useless for practical purposes. Because of these conditions, sky and ground waves are relatively unimportant in television transmission most of the time.

The radio energy intercepted by a receiving antenna located relatively near but above the surface of the earth is actually the resultant of the direct and reflected waves which reach that point. As a result, the actual strength of the signal generated in the receiving antenna may be either stronger or weaker than that which would be produced by the direct wave alone; the strength of the signal depends on the phase relationship between the direct and reflected waves at the antenna. The reflected wave is usually weaker because of some absorption and scatterings, and the two waves seldom will cancel each other completely, even when they are exactly out of phase at the receiving antenna.

Because of the relative ineffectiveness of sky-wave and ground-wave propagation in the VHF and UHF bands, only those points on or above the surface of the earth that can be reached by the direct wave are normally within effective communication range. The general effect on communication can be seen in Figure 15–20. Antennas located at B and C receive good signals, but an antenna at D receives only a very weak ground-wave signal.

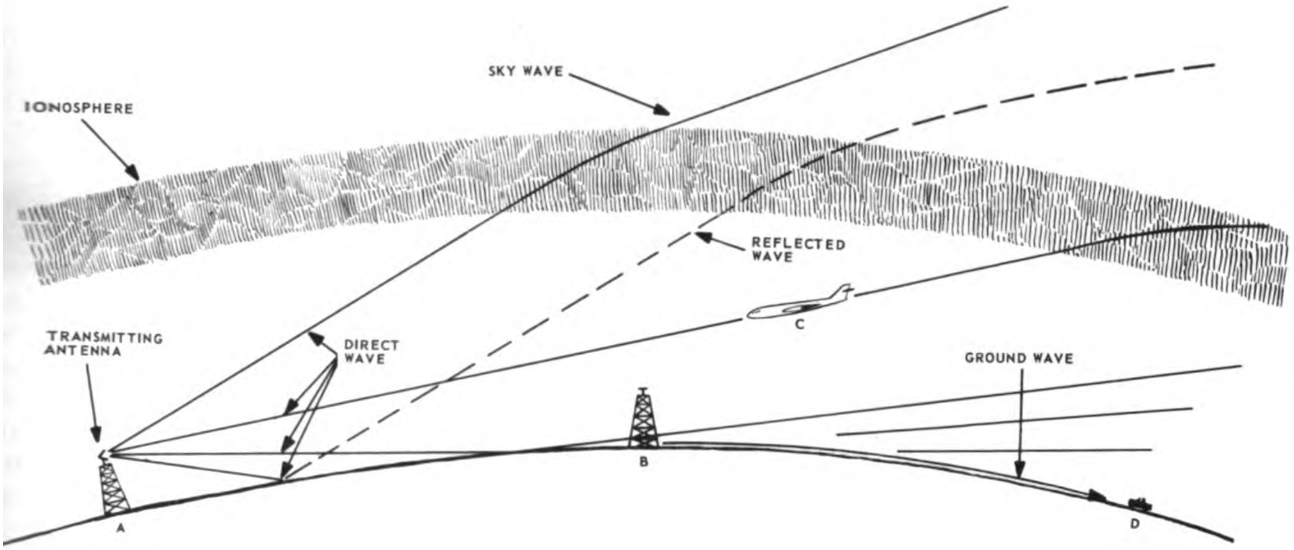


Figure 15–20. Practical Propagation at VHF

Radio waves above 30 mc are often referred to as quasioptical to indicate that they behave similarly to light waves. Since radio waves in this frequency range behave similarly to light waves, their propagation characteristics are often called line-of-sight transmission and reception. Because electromagnetic waves move in straight lines in free space, it might seem at first that communication beyond the range at which the receiving antenna actually can be seen from the transmitting antenna would be impossible. Experiment has shown that this is not true.

The radio horizon is somewhat beyond the optical horizon, even under normal conditions of the troposphere (lower part of the atmosphere). The radius of the troposphere considered with the earth is 1.33 times as great as the actual radius of the earth alone. A straight line drawn from the transmitting antenna to the visible horizon of this larger earth will indicate the true radio line-of-sight distance. The difference in the behavior of light and radio waves in this frequency range is due almost entirely to the difference in wavelength.

Effects of Reflections of Radio Waves

The degree of reflection will depend on the conductivity of the obstacle and its physical size as compared with the wavelength. Reflections from objects of good conductivity are particularly strong, and since many of the works of men are metallic, reflections from them often are troublesome in television transmission. Whenever the waves of the desired signal reach the receiving antenna over paths of different lengths, there is the possibility of multipath interference.

Obviously, when the difference in path length is great enough, the receiver receives two signal impulses carrying the same information, but separated by a short time interval. If the two signals are of approximately the same amplitude, the interference thus produced can be serious. When television signals from multipath reception are received, a ghost image, or images, is produced. This image is displaced to the right of the main image on the face of the picture tube.

Common Types of Television Antennas

DIPOLE WITH REFLECTOR. Although the simple half-wave antenna will provide satisfactory reception within reasonable distances from the transmitter, in outlying areas the signals become weaker and are affected more by noise and interference. In order to increase the signal gain of the antenna, a reflector is added to the antenna. Not only is the gain of the antenna increased, but the width of the response area is decreased (Figure 15–21). Therefore, a small displacement of the antenna one way or the other alters the strength of the received signal appreciably. This property is useful in preventing reflected signals from reaching the receiver.

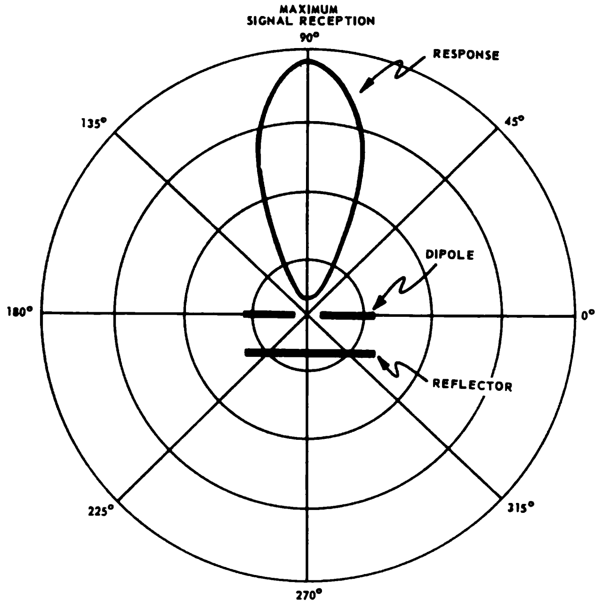


Figure 15–21. Directional Response Curve of a Dipole Antenna with Reflector

Folded Dipole. Another type of antenna in wide use is the folded-dipole antenna. The folded dipole has the same bidirectional pattern as the simple dipole and approximately the same gain, but the impedance is close to 300 ohms as compared with the 73 ohms of the simple dipole. The response of the folded dipole, however, is more uniform over a band of frequencies than the simple dipole. The addition of a reflector will increase the directivity of the folded dipole.

Conical Antenna. The conical antenna shown in Figure 15–22 is one of a large number of special antennas that have been developed to operate satisfactorily over a wide frequency band. The front elements are veered forward while the rear elements (the reflectors) generally extend straight out. Due to the position of the elements, the response pattern of the antenna contains a major lobe on each channel. One array is satisfactory for all VHF channels.

Stacked Antennas. In some areas where the signal strength is low, the gain of the receiving antenna can be increased by *stacking* the basic dipoles, as shown in Figure 15–23. This antenna is sometimes known as the "Lazy H" because it looks like the letter H lying on its side. The two dipoles are mounted one above the other; the

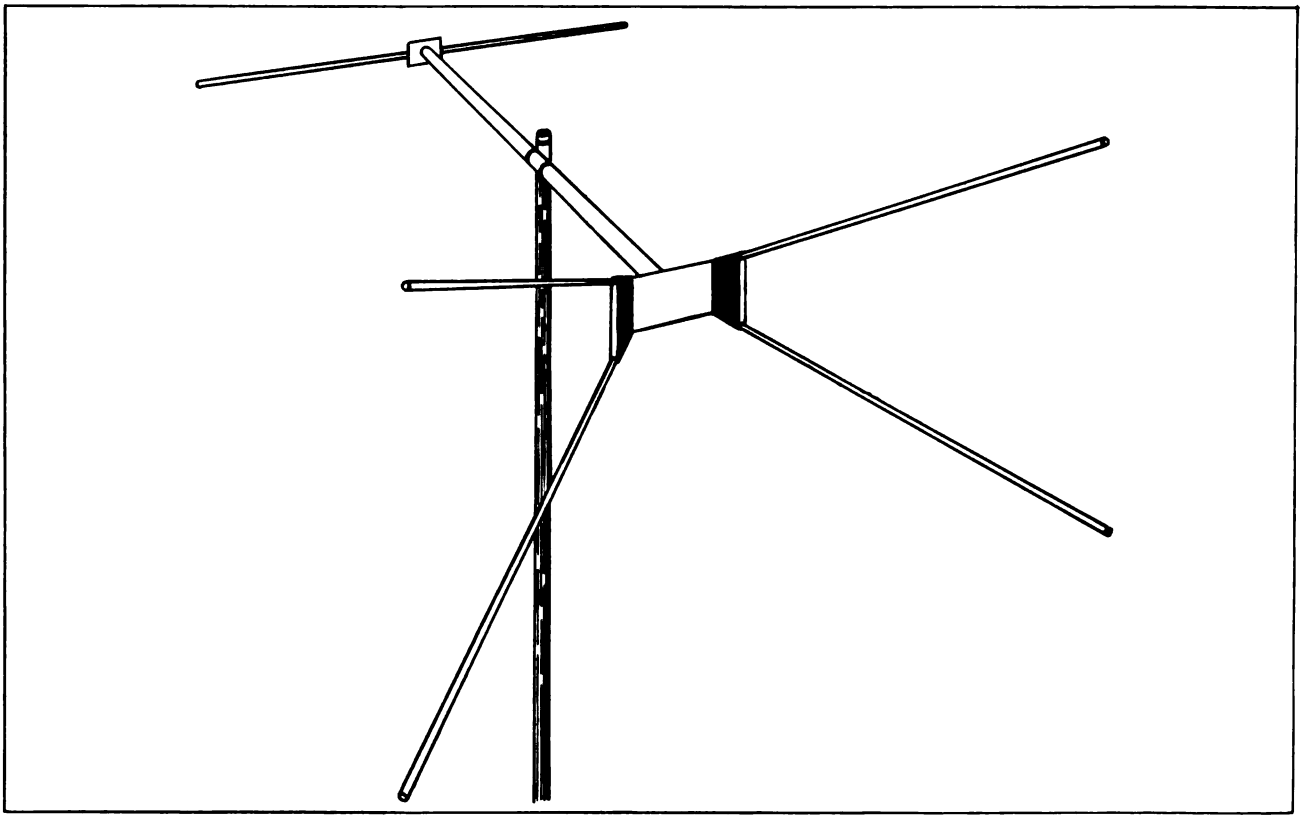


Figure 15–22. Conical Antenna

enter terminals of each dipole are connected with parallel wire. Each conductor of the transnission line is attached to this parallel wire at a oint midway between the dipoles. A reflector an be placed behind each dipole to make the rray more directive. Also, any one of the basic ntennas can be stacked to provide greater gain.

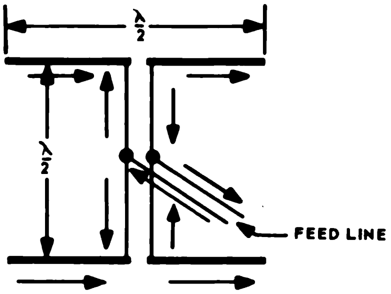


Figure 15–23. Stacked Dipole Antenna

MULTIELEMENT ANTENNAS. The directional response of an antenna can be further improved by adding directors to the array (Figure 15–24). Whereas the reflector is slightly longer than the dipole and placed behind it, the directors are slightly shorter than the dipole and placed in front of it. This arrangement is known as the Yagi antenna; it has a sharp unidirectional response pattern. The radiator may be either a simple or a folded dipole, and the number of directors may vary from one to twelve or more.

Transmission Lines

A transmission line is used to couple energy from the transmitter to the transmitting antenna, and from the receiving antenna to the receiver. Its purpose is to transfer power from one to the other with least possible loss. How well this purpose is accomplished depends on the characteristics of the transmission line used. For maximum transfer of power the characteristic impedance of the transmission line should match the characteristic impedance of the antenna.

Normally, it is most convenient and economical to use an antenna that is capable of receiving all the VHF television stations. This requires a fairly uniform response from 54 to 216 mc. Since a resonant dipole has an impedance of 73 ohms at its center, the connecting transmission line should match this value to effect a maximum transfer of power. However, a dipole cut to resonate at 50 mc will have an impedance of 2000 ohms at 100 mc, hence a transmission line of higher impedance is required.

Since line loss is inversely proportional to the characteristic impedance, it is desirable to use as high an impedance value as possible. However, the size of the line must be considered; a compromise has been effected so that in current

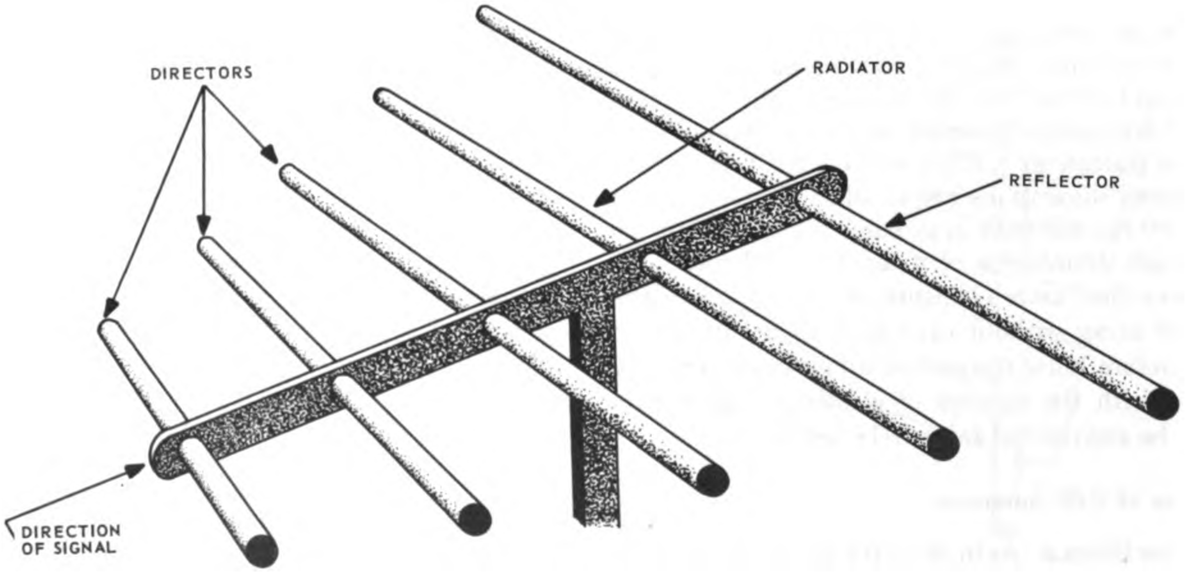


Figure 15–24. Yagi Antenna

practice a standard 300-ohm transmission line is used. It has been found that a 300-ohm line used with a dipole produces a broad frequency response without excessive loss due to mismatching. Transmission lines that are used in television were included in Figures 10–21, 10–22, and 10–24, Chapter 10.

After the correct antenna and transmission line have been chosen, the installation must be made with certain principles in mind. The higher the antenna, the stronger the signal received. Before fixing the supports permanently, the antenna should be tested by connecting it to the receiver. When more than one station is to be received, the final placement of the antenna must be a compromise. In extreme cases, it may be necessary to install more than one antenna.

UHF Antennas

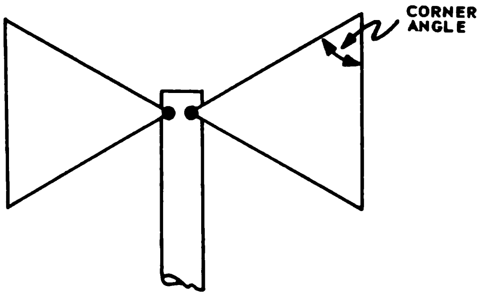
The addition of the UHF band to the original VHF band created new problems in television communications. At the higher frequencies, transmission line losses are larger and noise becomes a greater problem in the receiver. The first UHF receivers fed the signals directly to a crystal mixer, where they were converted to a lower frequency and then amplified. Since the early UHF receivers did not use r-f preamplifiers before the mixer, they were normally less sensitive than VHF receivers. However, new tube design has made it possible to use preamplifiers in UHF receivers. This increases the sensitivity.

Because of the poorer receiver sensitivity, UHF antennas must be erected more carefully than VHF antennas. Not only is it necessary to select a good location, but the proper height must also be determined. However, high-gain antennas are more feasible at UHF than at VHF because the antenna dimensions are smaller. Thus, a dipole at 800 mc will have approximately one-tenth the over-all dimensions of a dipole at 80 mc. This means that more elements can be added to the UHF array without causing it to be unwieldly. Therefore, since the gain of an antenna generally rises with the number of elements, higher gain can be anticipated from UHF arrays.

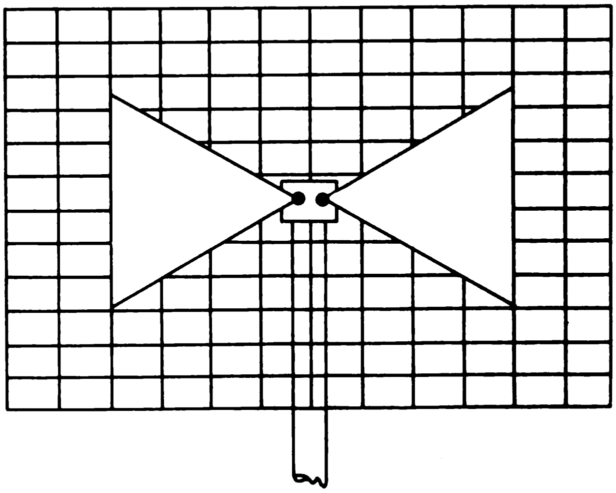
Types of UHF Antennas

FAN DIPOLE. As in the VHF band, the simplest type of antenna in the UHF band is the dipole. In order to provide the broad band characteristics necessary to receive all signals within the television UHF band, triangular sheets of metal are used instead of rods, as shown in Figure 15–25A. The array is normally designed with a corner angle of 70° in order to have an input impedance of approximately 300 ohms. The radiation pattern of a fan dipole is a figure-eight, but if a screen is placed behind the dipole as shown in Figure 15–25B, the field becomes unidirectional. For greatest gain, the over-all length of the fan dipole should be about 16 inches.

Since the gain is only slightly greater than that of a rod dipole, these units provide satisfactory reception in strong signal areas where there are relatively few ghost signals. However, fan dipoles can be stacked two and four high to provide increased gain and better discrimination against ground-reflected signals. Notice that the reflector



BOW–TIE
A



WITH SCREEN REFLECTOR
B

Figure 15–25. Fan Dipole Antenna

in Figure 15–25B is a wire screen instead of the rods usually used at VHF. Screens are much more efficient reflectors than rods, but are seldom used at VHF because they are too bulky at the lower frequencies. Screens are as effective as solid metallic reflectors provided the openings are no larger than two-tenths of a wavelength at the highest operating frequency. Reflector dimensions are not critical, but the edges should extend for a short distance beyond the dipole elements.

RHOMBIC ANTENNAS. The rhombic antenna shown in Figure 15–26 has been used successfully for VHF and UHF reception in weak signal areas. It has good gain, broad bandpass characteristics, and a sharply defined unidirectional response pattern. Since an increase in the length of the sides, or legs, results in higher gain and a sharper directivity pattern, the length of each side should be at least several wavelengths.

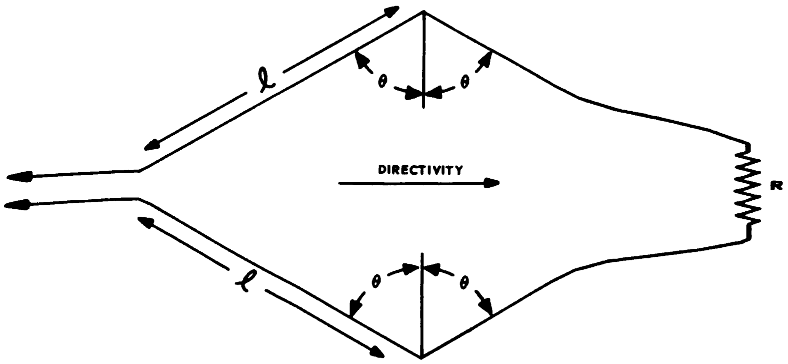


Figure 15–26. Rhombic Antenna

In order to obtain the unidirectional effect, a resistor must be inserted at one end of the array. For use with UHF television receivers, the resistor will have a value of 470 to 500 ohms. The impedance reflected into the transmission line is slightly less than the terminating resistance and will match a 300-ohm line without a serious loss of gain. In weak signal areas, rhombic antennas can be stacked for greater gain. The array is then oriented with the resistor end pointing in the direction of the station.

CYLINDRICAL PARABOLIC REFLECTOR. A parabolic reflector can be used to obtain a high concentration of electromagnetic energy. This was explained in Chapter 13. The use of the reflector increases the gain and directivity. It is
to use only a section of a parabolic
Adding a cylindrical parabolic reflector
onant dipole results in a gain of about
array will provide good results where
is desired and the ghost problem is n

CORNER REFLECTORS. Two flat su
also be used to form a reflector as
Figure 15–27. This type of antenna i
corner reflector antenna. The receivin
usually a dipole placed at the center o
angle but at some distance from the
response pattern of this array not o

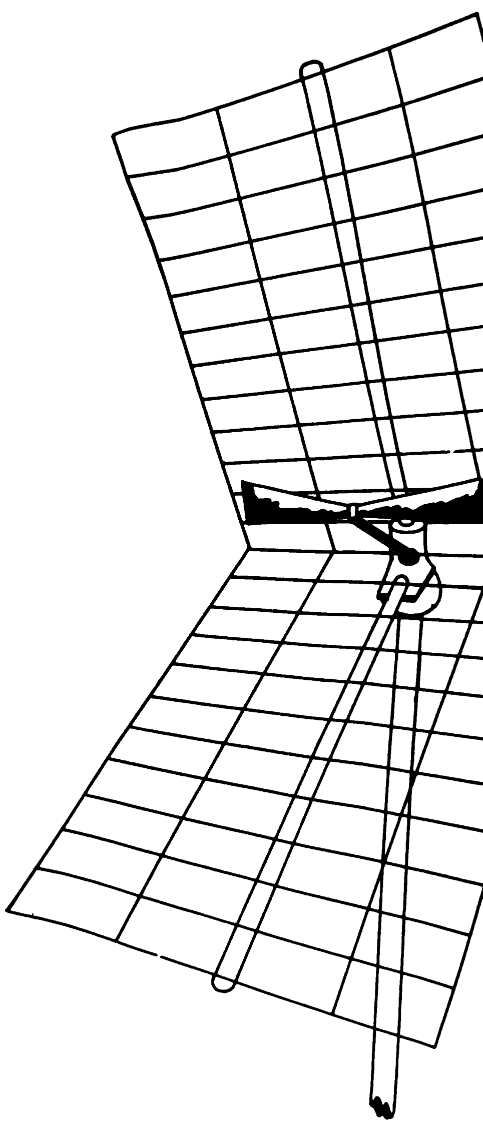


Figure 15–27. Fan Dipole with Corner R

on the corner angle but also on the distance between the dipole and the reflector corner. If the dipole is located too far from the vertex, the response pattern will have several lobes. If it is brought in too close, the vertical response will be broadened, and the array will become more susceptible to ground-reflected signals.

OTHER ANTENNAS. Actually, any of the VHF antennas can be used for UHF television if the size is reduced to the proper dimensions. The design formulas remain the same and no other changes are required. However, because of the smaller dimensions, stacked arrays are more commonly found among UHF antennas. This is understandable since greater gain is obtainable from the increased number of elements.

COLOR TELEVISION

Engineers have been experimenting with color television since 1946. Several systems were submitted, but were rejected because they were unsuitable for commercial use. The line and field frequencies were different from those of the black and white receivers. The National Television Systems Committee (NTSC) system was finally adopted in 1953.

Color Elements

If one is to see the true color of an object, it must be seen under a light which has all of the wavelengths of light. The color of an object is the function of the wavelengths of light that are not absorbed by the object. Anyone who has worked in a theater with projector lamps has found that when different colored lights are combined, the resulting color, as seen by the audience, is different in hue from any of the projected lights. Experimentation has found that the colors green, red, and blue, when combined in various proportions, will produce a wide range of colors. These three colors work better, in this respect, than any other combination of three colors, and have been chosen as the primary colors of television.

The National Television Systems Committee (NTSC) System

This system was designed so that the signal would not occupy more than 6 mc, carrying both the entire black and white signal and the color information. It was found that in the 4 megacycles assigned to the video signal, video information was not contained in every *cycle*. Instead, the monochrome signal appeared in clusters of energy that were located close to the harmonics of the 15,750-cycle line-scanning frequency. This meant that empty spaces were available for additional information. This is illustrated in Figure 15–28.

To make the second signal fall between the clusters of energy of the first signal and thus cause no interference, odd multiples of 7,875 (half of 15,750) are used. This placing of clusters of energy of one signal between clusters of energy of another signal is called interleaving (Figure 15–29). From this we see that the entire color signal is actually composed of two components — the monochrome signal and the color information signal.

The monochrome signal (black and white portion of the entire signal), for all practical purposes, is the same as the black and white signal in non-color sets. This signal is produced by the combination of the green, blue, and red signals in certain proportions.

$$Y = 0.60G + 0.30R + 0.10B$$

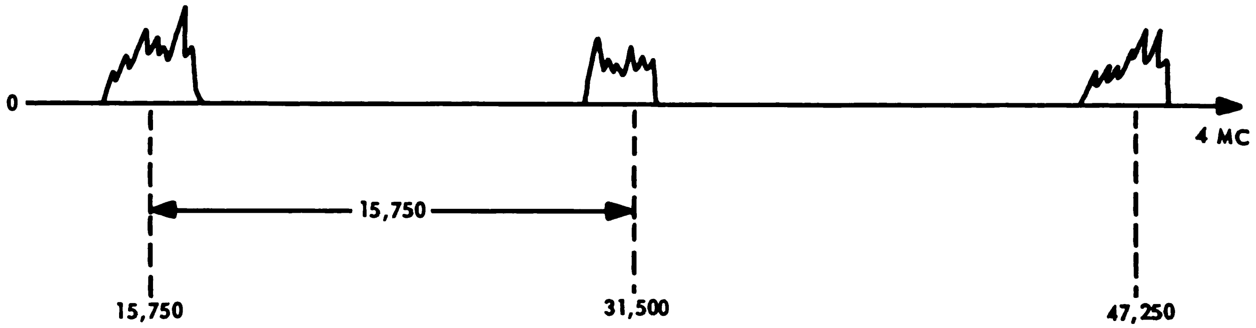


Figure 15–28. Spectrum Analysis of Monochrome Signal

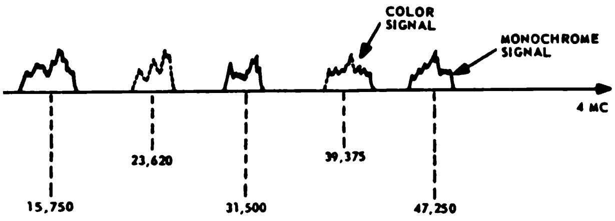


Figure 15–29. Interleaving of Signals

The letter *Y* is used to denote the monochrome signal; the letters *G*, *R* and *B* stand for green, red, and blue. To the human eye, green is brighter than red and much brighter than blue. That is why these proportions were chosen. Only the variations in amplitude of the picture signal are contained in the monochrome video signal, and it is these variations that give us the light intensity changes at the picture tube screen.

The color information signal is interleaved with the monochrome signal. We determine the amount of information this component must carry to form the color image from the reaction of the eye to color. The average human eye, when viewing a relatively large area, sees a full color range; but when the area decreases, distinguishing between colors becomes difficult. When the area is very small, the eye can only detect changes in brightness. The NTSC system utilizes these color seeing characteristics of the eye by sending the large and medium size areas in color and the very small areas in black and white. We find, too, that more color is given to the large objects and areas than to the medium size areas.

The first step in transmitting color is to produce a type of subcarrier, with a frequency of approximately 3.58 mc, and its associated sidebands. The next step is to modulate this subcarrier with the proper voltages representing the three colors — red, green, and blue.

In the color signal, we find that the Y signal (monochrome) has components from 0 to 4 mc and that the subcarrier (color) frequency is 3.58 mc. Two color signals modulate the subcarrier; these are designated as the I and Q signals. The I color signal (red) frequencies extend from 0 to 1.5 mc; the Q color signal (blue) frequencies extend from 0 to 0.5 mc.

It is sufficient to transmit two color signals, I and Q, as the Y signal contains voltages from all three colors.

The color subcarrier is suppressed in the NTSC system to reduce the interference that would be formed in connection with the sound carrier, a 920-kc beat frequency. When the image is televised in black and white, the entire color signal is removed.

A disadvantage of a suppressed subcarrier is that it must be reinserted in the receiver so that detection can be accomplished. It is important that the subcarrier have the correct phase. The information concerning the frequency and phase of the subcarrier is provided in a color burst that is transmitted with the signal. This burst is located on the "back porch" of each blanking pulse. The information in this burst is used to synchronize the frequency and phase of the 3.58-mc oscillator in the receiver.

Receivers

Circuits in the color receiver which are relatively the same as those in the black and white receiver are r-f tuner, mixer, video i-f, sound i-f, audio detector, audio amplifiers, video detector, video amplifiers, sync circuits, and high-voltage power supply. Those circuits which are not common are chrominance circuits, color sync circuits, and the tri-gun color picture tube and associated circuits.

There are a relatively large number of stages in the chrominance channel. These stages are used to demodulate the color signal and reobtain the original I and Q voltages. The I and Q signals, with the Y signal, are mixed in a matrix; the resulting voltages represent the red, green, and blue components in the I, Q, and Y signals. After amplification, the three voltages are fed to the respective control grids of the tri-gun color tube.

The color sync section has the distinct function of generating a 3.58-mc signal that possesses the proper phase relationship to the transmitted subcarrier. The necessary information is obtained from the color burst that is located on the "back porch" of each horizontal sync pulse. If the phase relationship is wrong, there is a very noticeable form of distortion.

The tri-gun color picture tube uses three electron guns, while the black and white picture tube uses only one electron gun. The screen of the black and white tube has only one type of phosphor, while the tri-gun color tube has three different color-emitting phosphors. While globules were used on the mosaic image plate of the camera tube, phosphor dots are used on the tri-gun color tube. They are placed in an orderly arrangement of triangular groups to form approximately 340,000 trios of dots. Each of the electron guns activates only one type of phosphor. We see the light that is given off by the phosphor when it is activated by the electron beam; the color of light we see is determined by the phosphors that are being struck by the electron beam, and by the number of electrons in each beam.

Since any electron beam could activate any phosphor dot on the screen, there must be some way of allowing each beam to strike only one type of phosphor dot. This is accomplished by using a shadow mask. The mask is in front of, and parallel to, the screen; it contains circular holes equal in number to the dot trios. The holes and electron guns are so aligned, with respect to each group, that an electron gun beam can strike only one phosphor dot of a trio. The other two dots are concealed by the shadow mask. Each beam strikes one phosphor dot of the trio. The associated convergence circuits insure that each beam is properly converged at every point of the screen.

The components that are mounted externally on the tri-gun color picture tube are a little different from those mounted on the black-and-white tube. The deflection yoke is more complex because three beams are being deflected instead of one. Another component is the purity coil or magnet. This component adjusts the axis of each electron beam so that the beam will strike the correct phosphor dot. Proper alignment of each beam with respect to phosphor dot plate and shadow mask is the function of the purity coil. The convergence coils, as stated previously, insure that each beam is properly converged at every point on the screen. Ion traps are not used in this tube because the color screen is aluminized. The layer of aluminum is the barrier to the onrushing ions.

CLOSED CIRCUIT TELEVISION

Components of Video Signal Generating Equipment

Basically, the closed circuit system consists of the following equipments: pickup cameras, camera control equipment, monitors, synchronizing generator, projectors and multiplexers, and a parallel audio system.

Since motion picture film is frequently used in closed circuit TV, television film projectors are an integral part of a television system. They must be able to operate continuously for long periods, start and stop almost instantly without serious speed variations, and run at constant speed for long periods of time.

Television projectors produce a sharp, stable, optical image from the film and project it with sufficient intensity of light on the photosensitive surface of the camera pickup tube. Either 16-mm or 35-mm motion picture film or slides can be used. Black and white or color, positive or negative, film may be used.

Motion picture film has a standard projection rate of 24 frames per second, while television systems operate at 30 frames per second. Therefore, film projectors for television must be designed to synchronize the 24-frame film with the 30-frame television system. This will permit any standard 16-mm or 35-mm film to be shown. In addition, the television film projector must be synchronized with the 60-cycle power line if it is used with pickup cameras such as the iconoscope.

Multiplexers make possible the use of several film and slide projectors with only one pickup camera. The multiplexer is the optical link that selects one of several film, slide, or opaque projector outputs for projection into a film pickup camera. Movable mirrors, driven by motors or relays, change the optical paths of the several optical picture sources so that the selected picture may be directed to the camera pickup tube. The mirrors, motors, and control panel are all in one unit, or housing, which constitutes the multiplexer.

Video Signal Distribution Equipment

After a television signal has been produced and processed, it must be distributed or transmitted to the video signal display equipment. This is accomplished by the video signal distribution equipment. It includes distribution amplifiers, equalizers, r-f generators, r-f distribution, microwave relay, and switching equipments.

Distribution amplifiers are divided into two groups: (1) video distribution and (2) pulse distribution amplifiers. Their function is to drive the television signals through the low-impedance coaxial cables. They must pass the signal with the least distortion, provide gain (or amplification), insure isolation between input and output, and add the sync signal to the video signal to form the composite television signal.

Video equalizers are inserted into the coaxial lines to correct for distortion introduced by the cables. Coaxial cables have relatively poor amplitude-versus-frequency response. Long cables or improperly terminated cables may cause high-frequency attenuation and accompanying phase distortion. Such distortion will cause reduced resolution, ringing, smearing, or streaks. Equalizers are composed of variable combinations of inductors, capacitors, and resistors. They are designed to complement the cable characteristics and to produce an amplitude-versus-frequency response that is uniform. Although correction of poor amplitude response is the primary use, video equalizers will correct most of the phase distortion also.

Coaxial cable distributes the television signal to the immediate area; for remote distribution, however, r-f generators and r-f distribution systems are required. R-f generators are low-power transmitters usually tuned to one of the VHF channels. Instead of feeding an antenna, they feed a coaxial line that carries the VHF signal to one or more television receivers. The video signals modulate the output of the r-f generator. The first type of r-f generator accepts only video signals and uses a single television channel. The second type will accept both video and associated audio signals, and can usually operate on any of the standard VHF television channels. These r-f generators have the capability of accepting both video and audio signals simultaneously and distributing them on a single coaxial cable to one or more television receivers.

The r-f distribution system is a highly versatile and effective means of distributing a number of video signals to several standard receivers. The number of receivers used is limited to the length of coaxial line used and the degree of isolation desired between receivers.

Microwave equipment is used for the distribution of television signals over distances of 5 miles or more, where the use of coaxial cable is difficult and costly. A television microwave relay system actually is a complete television system, containing transmitter, transmitting antenna, receiving antenna, and receiver. The system is designed to operate at UHF frequencies ranging from 1000 mc to 15,000 mc. With a minimum of transmitter power and highly directional antennas, it is possible to relay television signals over distances of 30 miles or more without serious attenuation or distortion.

Typical applications of a microwave relay system are from studio to transmitter, from studio to a remote building, between two widely separated buildings, on a large Air Force installation, and from mobile studio to main studio.

Switching equipment is a necessity in any but the simplest, one-camera television system. The equipment selects the output from one of several signal sources and connects it to one outgoing path. In a complex system the switching equipment provides for program monitoring and previewing; video switching between studio cameras, film cameras and remote pickups; and special effects such as fading and lap dissolving.

Video switchers use pushbutton switching and may operate either directly or remotely by relay.

A zero time interval during transfer is ideal, but as this is not practical, the switching equipment is designed to make the transfer by using either gap switching or overlap. Both of these methods permit picture transfer with a minimum of disturbance. Also, there are several other functions that the switching equipment performs. By the use of synchronization, the transfer is smoothly accomplished with fade-ins and fade-outs, lap dissolves, and diagonal wipes. The equipment also provides for two signals to be superimposed, or for two signals to be mixed in split screen montages. Provisions are also provided for previewing the video and for monitoring the output line.

The following components are included in the switching equipment: basic switching unit (relay chassis), fader assembly, gain and isolation stages, special effects and lap dissolve amplifiers (may be fed back to input for previewing), and mixing or output amplifiers (synchronization is added).

Television monitors take a video frequency signal and reproduce a picture image. The signal may be taken directly from a video line or from a demodulated signal from a television receiver. By the addition of r-f and i-f stages, the monitor may function as a receiver. The size of the monitor picture tube is determined by its particular function. A 21-inch tube may be used in a classroom or office; a 17-inch tube may be used in a studio control room; an 8-inch tube is sufficient for cueing work or continuity checks.

Kinescope Recorders

Kinescope recording is the name given to the recording of television programs on motion picture film. This is often called video tape. Special equipment was designed in order to have recordings of high quality for closed-circuit and other applications. A kinescope recording system normally includes a high quality television monitor, a specially designed motion picture camera, and the necessary equipment for recording the sound simultaneously with the picture.

A requirement of the kinescope recorder is that it must synchronize with the television system and resolve the timing difference between it and the TV system. Other requirements are adequate light from the monitor kinescope, and camera exposure time accurately controlled and synchronized with the television system. The motion picture cameras, either 16-mm or 35-mm, should be capable of producing either negative or positive recorded film prints. The complete equipment should have a high degree of reliability and should be ruggedly built for continuous operation.

Typical Closed-Circuit Video System

A closed-circuit system may consist of simply one camera and one monitor combination or it may be quite extensive, including several studios feeding program material to many different locations for viewing. A one-camera, one-monitor system is limited in its usage for different programming, but it might be adequate for a function such as simple surveillance or telemetering.

A moderate closed-circuit system, which is comparable to a small television broadcasting station, is shown in Figure 15–30. Different camera angles and subjects are available for alternate showing without any breaks in program continuity. This system is suitable to Air Force application in classrooms and briefing rooms, and where there are no requirements for film repro-

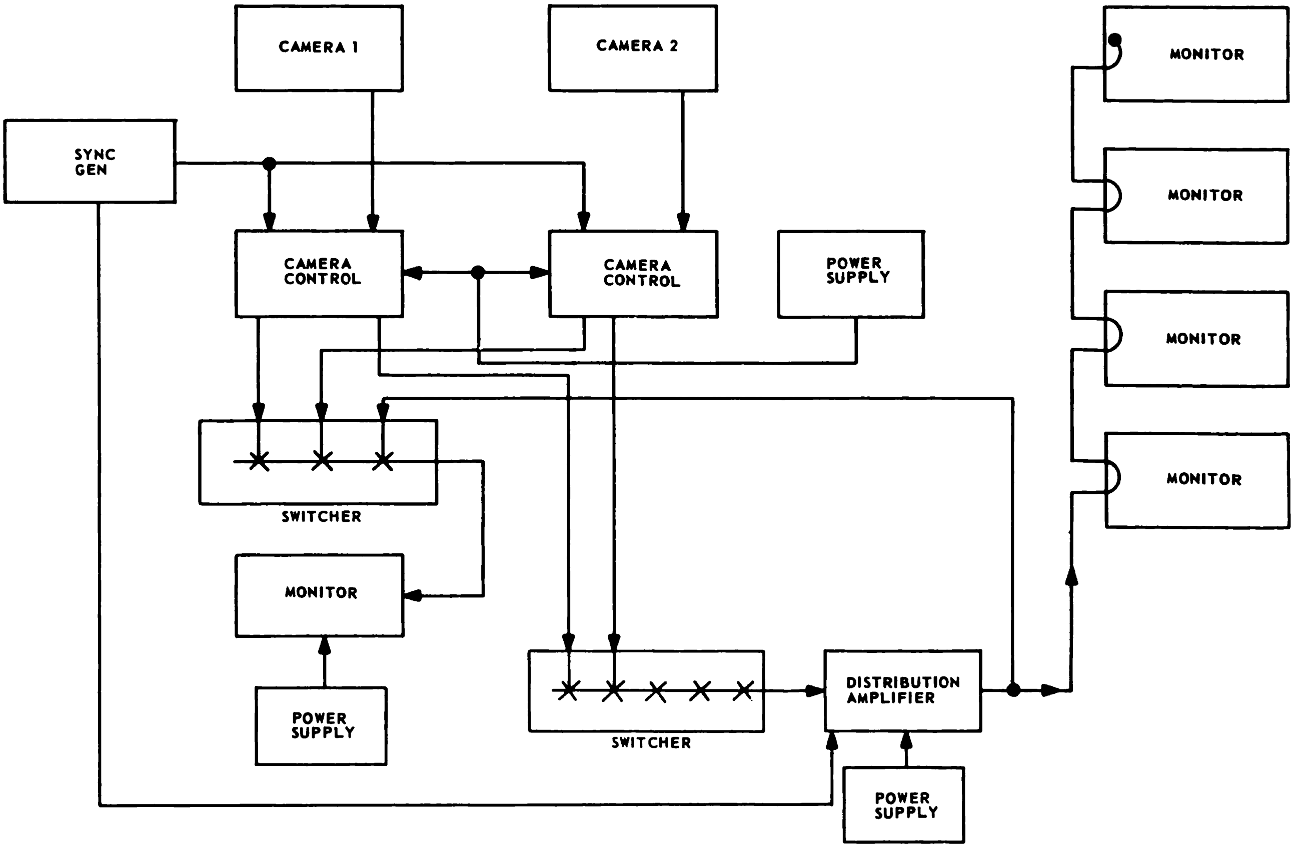


Figure 15–30. Moderate Closed-Circuit Television System

duction. A moderate variety of program material can be presented through this system.

A complete closed-circuit system contains the necessary equipment to provide a versatile, top-quality program. The complete system is comparable to a medium-sized commercial broadcasting station, or to an elaborate closed-circuit system used in some colleges and universities. The complete system is acquired by adding a camera, film reproduction equipment, a video tape recorder, and the necessary distribution equipment to the moderate system. Like the moderate system, the complete system is excellent for classroom application and has an increased capability for a very efficient presentation of a technical lecture with illustrations.

A multiple studio system is the same as the two previously described with the exception of added equipment. The main addition is that of one or more studios, with the distribution, control, and switching equipment that is a part of a studio.

Audio System

As previously stated, the audio system is completely separate from the video equipment in a television system. Film projectors, video tape recorders, and television receivers are the only pieces of equipment in the video system that have any audio stages or functions. In the planning of a television system, this separation of audio and video often leads to a neglect of the audio. The video installation will be limited in scope if the sound facilities are not adequate. In the planning of a closed-circuit system, future expansion should be anticipated as well as the satisfying of present needs for audio and video facilities.

The audio requirements parallel the different video systems. Just as the simple television system consists of one pickup camera and one monitor, a simple audio system consists of one microphone and one speaker. For the presentation of varied program material, as technical instruction, a more complex audio system is demanded. The system should include facilities for intercommunication, program audio, and talkback.

Applications of Closed-Circuit Television

There are two major classes of application of closed circuit TV: program television and telemetering, or remote monitoring. The dividing line between the two is not always completely distinct.

Under program television, there are broadcasting, educational, kinescope recording, and communications. Under telemetering and remote monitoring, there are hazardous and inaccessible locations, data remoting and collection, magnification, ultraviolet, X-ray, and infrared sources. We will discuss here the types the Air Force is concerned with.

Educational TV has an enormous potential. The consensus of leading educators is that a carefully prepared course taught by television has many advantages over conventional classroom techniques. This assumes that all possible equipment and potential of television programming is used.

Kinescope recording has many advantages over direct photography. Through the use of kinescope recording and television techniques, it is possible to produce certain types of films more cheaply than by direct photography.

Community system is the use of a community antenna system where receivers in a town are connected by cable to a central distribution box. This system could well be used in closed-circuit for programs produced locally.

Closed-circuit communications may include several types of applications. These include the use of television at large meetings and intercity hookups.

Examples of hazardous or inaccessible locations where TV might be used are inside of an aircraft wing, the observation of missile firing, in atomic radiation areas, and in high temperatures and high noise level areas.

In data remoting and collection, information is transmitted from a point where it would be expensive or inconvenient to obtain it directly. Information can be brought in from several points to a single viewing location and, conversely, it is possible to bring information from a single control location to a number of branch points. Examples are — use in military operations control centers, and weather briefings.

Image magnification can be used in classroom explanations on small parts or adjustments in electronic equipment. Long focal length lenses make it possible for small objects to be brought closer for more detailed observation. Thus, small objects can be viewed by a large number of students.

TV makes possible the observation of objects illuminated by X-ray, ultraviolet, and infrared sources.

Specific Applications

Only a small cross section of the tasks that closed-circuit television can perform have been given. Some areas in which tasks can be performed by closed-circuit TV are: (1) training and instruction, (2) security, (3) general surveillance, (4) traffic control, (5) weather service, and (6) hazardous monitoring. Each area will be discussed briefly with specific applications in each area.

TRAINING AND INSTRUCTION. The Air Force has found closed-circuit TV to be of much value in its military schools. A greater number of students can be served by the best instructor available by using closed-circuit TV. Television is an excellent visual aid as it combines the use of charts, slides, and films. The value of television as a teaching aid is unquestioned. For the present, television is accepted as a supplement to, rather than a replacement for, conventional classroom instruction.

SECURITY. Continued surveillance of a restricted area is normally done by uniformed guards. Sometimes it is desirable to conceal the surveillance. A hidden closed-circuit system can do this task much better and more consistently than the best continuous surveillance of guards. Other areas where security can be checked by a hidden camera are offices, file rooms, mail rooms, entrances and exits.

GENERAL SURVEILLANCE. While closed-circuit TV is used in this area principally by industry at the present time, military applications may be expected to include those closely paralleling the civilian applications. Examples of general surveillance are data remoting, radar remoting, administrative use, medical applications, hospital wards, Air Police applications, aerial reconnaissance, ground reconnaissance, warehouse functions, fire warning surveillance, and personnel supervision.

TRAFFIC CONTROL. A very good usage of closed-circuit television for the Air Force is that of observing, recording, and directing of aircraft ground movement. Railroad switching and vehicular movement are also expedited by television. Air traffic control towers rely upon visual observation. Television equipment would extend the vision of the personnel in several ways. All parts of the field would be visible.

WEATHER SERVICE. Flight crew briefings on weather station data is greatly facilitated by the use of closed-circuit television. The forecaster has all his data always at hand and only the briefest preparation is necessary. The forecasting map and the winds aloft chart are the principal information sources that demand picture transmitting equipment.

Color Closed-Circuit Television

As in monochrome closed-circuit television, color closed-circuit television has unlimited capabilities. Since the addition of color closely approximates reality, the introduction of color into television as an educational medium presents to its users the most versatile type of closed-circuit educational program possible. In technical training programs, the use of color is a necessity rather than a luxury. Especially is this true in the fields of electronics, communications, weather, aircraft engine repair, and hydraulics.

Color plays a major role in providing identification for small parts, subassemblies, test points, and control functions in each of these technical areas. Without color, oral instructions, via television, would require an extensive amount of explanation. As the need for newer and faster methods of technical training is ever increasing, the introduction of color closed-circuit television could change this condition and could create a new standard in military technical instruction.

BY ORDER OF THE SECRETARY OF THE AIR FORCE

CURTIS E. LEMAY
Chief of Staff

OFFICIAL

R. J. PUGH
Colonel, USAF
Director of Administrative Services

Symbols and Formulas

ELECTROSTATICS

$= \frac{Q_1 Q_2}{K d^2}$	Force between charges
$= \frac{Q}{Kd^2}$	Electric field intensity
$= \frac{V}{d}$	Field between plates
$= QE$	Force on Q in field E
$= QV = eV$	Work
$= \frac{Q}{V}$	Capacitance
$= \frac{Q}{d}$	Potential

DIRECT CURRENT

$= IR$	Ohm's Law
$= EI = I^2R = \frac{E^2}{R}$	Power

eries Circuits

$_t = R_1 + R_2 + R_3 + \ldots$	Total resistance in a series circuit
$_t = E_1 + E_2 + E_3 + \ldots$	Total voltage in a series circuit
$_t = I_1 = I_2 = I_3 = \ldots$	Total current in a series circuit
$_t = P_1 + P_2 + P_3 + \ldots$	Total power in a series circuit

arallel Circuits

$_t = \frac{R_1 R_2}{R_1 + R_2}$	Total resistance — Two resistors in parallel
$R_2 = \frac{R_t R_1}{R_1 - R_t}$	Unknown resistance — Two resistors in parallel with total resistance known
$G_t = \frac{1}{R_t}$	Total conductance
$G_t = G_1 + G_2$	Total conductance
$R_t = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots}$	Total resistance with any number of resistors in parallel
$E = E_1 = E_2 = E_3 = \ldots$	Total voltage in a parallel circuit
$I_t = I_1 + I_2 + I_3 + \ldots$	Total current in a parallel circuit
$P_t = P_1 + P_2 + P_3 + \ldots$	Total power in a parallel circuit

Resistance

$R = P \frac{l}{d^2}$	Value of resistance as a function of the material and its physical dimensions

where R = resistance in ohms
P = resistance in ohms per *circular-mil-foot* of the material (specific resistance)
l = length of conductor *in feet*
d = diameter of wire *in mils*

Meters

$R_{shunt} = \frac{I_m R_m}{I_t - I_m}$	Shunt required to extend range of ammeter

where I_m = current for full scale deflection
R_m = meter resistance
I_t = extended range of current

$\text{Ohms per volt} = \frac{\text{Resistance of meter}}{\text{Full scale reading in volts}}$ Sensitivity of a meter

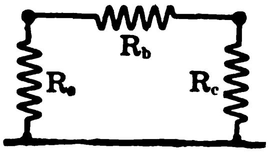
$\text{or} = \frac{1}{\text{full scale current}}$

$R_m = (E \times (E \, x\Omega/V) - R_V$ Multiplier required to extend range of voltmeter

where E = extended range of voltage
Ω/V = sensitivity of meter
R_V = meter resistance

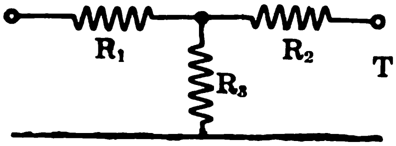
Networks

$$R_1 = \frac{R_a R_b}{R_a + R_b + R_c}$$



π to T transformation

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$



T to π transformation

MAGNETISM AND ELECTROMAGNETS

$F = \frac{m_1 \, m_2}{\mu d^2}$ Force between two magnetic poles

where μ = permeability of the medium

$H = \frac{F}{m}$ oersteds Magnetic field intensity

$H = \frac{2I}{10r}$ oersteds Magnetic intensity about a conductor

where I = current in amperes
r = radius in centimeters

$H = \frac{4\pi NI}{10\,l}$ oersteds Magnetic intensity for long coils

$H = \frac{2\pi NI}{10r}$ oersteds Magnetic intensity for short coils

$\text{mmf} = \frac{4\pi NI}{10}$ gilberts Magnetomotive force

$\phi = \frac{\text{mmf}}{R}$ maxwells Flux

$R = \frac{L}{\mu A}$ rels Reluctance

$B = \frac{\phi}{A}$ gauss Flux Density

$F = \frac{B I l}{10} \sin\theta$ dynes Force exerted on conductor

$\mu = \frac{\beta}{H}$ Permeability

Induction

$e = -N\frac{\Delta\phi}{\Delta t} \times 10^{-8}$ Average Induced voltage

$e = -M\frac{\Delta i}{\Delta t}$ Induced voltage

where M is mutual inductance

$M = K\sqrt{L_1L_2}$ Mutual Inductance

where k is the coefficient of coupling

$L = \frac{N\phi}{I} \times 10^{-8}$ Inductance

where N is number of turns
I is current in amperes

$e_L = -L\frac{\Delta i}{\Delta t}$ Voltage across inductance

$e_L = -N\frac{\Delta\phi}{\Delta t} \times 10^{-8}$ Voltage across inductance

ALTERNATING CURRENT

General

$e = E_{max} \sin\omega t$ Instantaneous voltage

where $\omega = 2\pi f$

$= I_{max} \sin \omega t$ Instantaneous current

where $\omega = 2\pi f$

$= \frac{E^2}{R} = I^2R = EI \cos \theta$ Power in AC circuits

where θ is the phase angle

$= \frac{R}{Z} = \cos \theta$ Power factor

$= \text{arc tan } \frac{X}{R} =$ angle of lead or lag (degrees) Phase angle

$= K\frac{A}{d}$ Capacitance in micromicrofarads

where A is area of plates
d is distance between plates
K is dielectric constant

$_L = 2\pi fL$ Inductive reactance

$_c = \frac{1}{2\pi fC}$ Capacitive reactance

$= \sqrt{R^2 + (X_L - X_c)^2}$ Impedance — triangular form

$= R \pm jX$ Impedance — rectangular form

$_r = \frac{1}{2\pi \sqrt{LC}}$ Frequency at resonance

$= \frac{X_L}{R}$ Merit of a coil or tank

where R is total resistance in series with coil

$= \frac{R}{X_L}$ Merit of a coil or tank

where R is total resistance in parallel with coil

$_T = L_1 + L_2 \pm 2M$ Total inductance of two coils

where M is the mutual inductance

$_T = L_1 + L_2 + L_2 \text{ - - - -}$ Inductance in series

$1/L_t = 1/L_1 + 1/L_2 + 1/L_3 \text{ - - -}$ Inductance in parallel

$C_t = C_1 + C_2 + C_3 \text{ - - -}$ Capacitances in parallel

$1/C_t = 1/C_1 + 1/C_2 + 1/C_3 \text{ - - -}$ Capacitances in series

$Q_t = Q_1 = Q_2 = Q_3 \text{ - - -}$ Total charge on condensers in series

$Q_t = Q_1 + Q_2 + Q_3 \text{ - - -}$ Total charge on condensers in parallel

+ is used when the fields add
— is used when the fields oppose

Transformers

$\frac{E_p}{E_s} = \frac{N_p}{N_s} = \frac{I_s}{I_p}$ Relationships among voltages, turns, and currents in transformers

$\frac{Z_p}{Z_s} = \left(\frac{N_p}{N_s}\right)^2$ Impedances and turns

$Z = Z_1 + Z_p - \frac{{Z_m}^2}{Z_2 + Z_s}$ Impedance when looking into primary of a transformer

where Z_1 is in series with primary
Z_p is impedance of primary windings
Z_m is mutual impedance
Z_s is impedance of secondary windings
Z_2 is impedance in series with secondary

$Z_r = \frac{-{Z_m}^2}{Z_2 + Z_s}$ Reflected impedance

Transients

$t = R \times C$ Time constant for RC circuits

where t is in seconds
R is in ohms
C is in farads

$t = \frac{L}{R}$ Time constant for L/R circuits

where t is in seconds
R is in ohms
L is in henries

$$e_c = E_a\left(1 - \epsilon^{-\frac{t}{RC}}\right)$$ Voltage change across a capacitor in an RC circuit

where E_a is the applied voltage

$$e_R = E_a\epsilon^{-\frac{t}{RC}}$$ Voltage across a resistor in an RC circuit

$$i = \frac{E_a}{R}\epsilon^{-\frac{t}{RC}}$$ Current in an RC circuit

$$e_L - E_a\epsilon^{-\frac{Rt}{L}}$$ Voltage across coil in an L/R circuit

$$e_R = E_a\left(1 - \epsilon^{-\frac{Rt}{L}}\right)$$ Voltage change across a resistor in an L/R circuit

$$i = \frac{E_a}{R}\left(1 - \epsilon^{-\frac{Rt}{L}}\right)$$ Current in an L/R circuit

$$E_R = \left(E_a - E_{co}\right)\epsilon^{-\frac{t}{RC}}$$ Voltages in a direct current restorer

where E_{co} is the voltage on the capacitor at the start

VACUUM TUBES AND AMPLIFIERS

$$\mu = \frac{\Delta e_p}{\Delta e_g}\ (i_p \text{ constant})$$ Amplification factor

$$r_p = \frac{\Delta e_p}{\Delta i_p}\ (e_g \text{ constant})$$ Plate resistance

where r_p is in ohms
e_p is in volts
i_p is in amperes

$$g_m = \frac{\Delta i_p}{\Delta e_g}\ (e_p \text{ constant})$$ Transconductance

where g_m is in mhos
i_p is in amperes
e_g is in volts

$$\mu = g_m r_p$$ Relationship among g_m, r_p and μ

where g_m is in mhos
r_p is in ohms

$$A = \frac{E_o}{E_s} = \frac{E_o}{E_g} = \frac{\mu Z_L}{r_p + Z_L}$$ Voltage gain of triode operated Class A

where E_o is the output voltage
E_s is the input voltage
E_g is the grid-to-cathode voltage
Z_L is the equivalent load impedance
r_p is the plate resistance

$$db = 20 \log \frac{E_o}{E_{in}} = 20 \log \frac{I_o}{I_{in}}$$ Voltage or current gain in decibels, if R_{in} equals R[illegible]

$$db = 10 \log \frac{P_o}{P_{in}}$$ Power gain of stage operated Class A

where P_o = output power
P_{in} = input power

$$A_t = A_1 \times A_2 \times A_3 \ldots\ldots A_n$$ Total voltage gain of stages in cascade

$$db_t = db_1 + db_2 + db_3 \ldots db_n$$ Total power gain of stages in cascade

$$F_h = \frac{1}{2\pi R_L C_s}$$ Highest frequency in band passed by a video amplifier

where R_L is the total load resistance
C_s is total shunt capacitance

$$F_1 = \frac{1}{2\pi R_g C_c}$$ Lowest frequency in band passed by a video amplifier

where R_g is the grid resistor
C_c is the coupling capacitor

$$A' = \frac{E_o}{E_g} = \frac{A}{1 - BA}$$ Gain in a feedback amplifier

where A is the gain without feedback
B is the fraction of the output voltage that is fed back

Cathode Followers

$$Z_o = \frac{R_k r_p}{r_p + R_k(\mu + 1)}$$ Output impedance of a cathode follower

$$A = \frac{E_o}{E_s} = \frac{\mu R_k}{r_p + R_k(\mu + 1)}$$ Gain of a cathode follower

OSCILLATORS

$$= \frac{1}{2\pi\sqrt{LC}}$$

Output frequency of LC oscillator

$$= \frac{1}{2\pi\sqrt{R_1 C_1 R_2 C_2}}$$

Output frequency of a Wien bridge oscillator

$$= \frac{1}{2\pi R_1 C_1}$$

Output frequency of a Wien bridge oscillator when $R_1C_1 = R_2C_2$

$$= \frac{1}{2\pi RC\sqrt{6}}$$

Output frequency of a phase shift oscillator

ock Excited Oscillators

$$= \frac{R_s}{X_L} = \frac{R_s}{L/C}$$

Merit of oscillator tank

where R_s is the shunt resistance

$$X_L = X_c = \sqrt{L/C} \text{ at resonance}$$

$$_o = \frac{1}{2\pi\sqrt{LC}}$$

Frequency of oscillator when Q is 5 or greater

$$_o = \frac{1}{2\pi\sqrt{LC}}\sqrt{1-\left(\frac{1}{2Q}\right)^2}$$

Frequency of oscillator when Q is less than 5

$$_{max} = I_p\sqrt{L/C}$$

Maximum output voltage

TRANSMISSION LINES

$$_o = \sqrt{L/C} = \frac{E_I}{I_I}$$

Characteristic impedance

where E_I is the incident voltage
I_I is the incident current

$$WR = \frac{E_{max}}{E_{min}} = \frac{I_{max}}{I_{min}} = \sqrt{\frac{Z_{max}}{Z_{min}}} = \frac{1+|r|}{1-|r|}$$

Standing wave ratio

where $|r| = \frac{p-1}{p+1}$

$$WR = \frac{Z_L}{Z_o}$$

Standing wave ratio (Resistive load only)

where Z_L is larger than Z_o

$$SWR = \frac{Z_o}{Z_L}$$

Standing wave ratio (Resistive load only)

where Z_o is larger than Z_L

$$Z_o = \sqrt{Z_{in} Z_L}$$

Impedance of matching transformer

where Z_{in} is input impedance
Z_o = characteristic impedance
Z_L is output impedance

$$t = n\sqrt{LC}$$

Time for a wave to travel one length of a delay line

where n is the number of sections in the line

$$PW = 2n\sqrt{LC}$$

Pulse width of a pulse forming line

$$f_{co} = \frac{1}{\pi\sqrt{LC}}$$

Cutoff frequency of an artificial line

WAVE GUIDES AND CAVITY RESONATORS

$$\lambda_a = k\frac{300}{f_{mc}}$$

Wavelength in free space (air)

where k is the velocity correction factor
f_{mc} is frequency in megacycles

$$V_p = f\lambda_g$$

Phase velocity

where λ_g is wavelength in guide

$$V_a = f\lambda_a = \sqrt{V_p V_g}$$

Velocity in free space

where λ_a is wavelength in free space (air)

$$\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

Wavelength at cutoff

where m is the first subscript in TE or TM mode designations

n is the second subscript in TE or TM mode designations

a is the short dimension of the wave guide

b is the long dimension of the wave guide

MICROWAVE OSCILLATORS

$F_e = 1.6 \times 10^{-12} E$ Force in dynes acting on an electron in an electric field

where E is the electric field strength in volts/cm

$F_m = 1.6 \times 10^{-20} V_p B$ Force in dynes acting on an electron in a magnetic field

where V_p is the component of electron velocity perpendicular to the direction of the magnetic field

B is magnetic flux density in gausses

$K.E. = 1/2\, mv^2 = eV$ Kinetic energy in ergs

where m is mass of electron in grams

v is velocity of electron in cm/sec

e is charge on electron in coulomb

V is potential difference through which the electron travels

ATTACHMENT 2

Tables

Powers of Ten

10^{10} = 10,000,000,000
10^{9} = 1,000,000,000
10^{8} = 100,000,000
10^{7} = 10,000,000
10^{6} = 1,000,000
10^{5} = 100,000
10^{4} = 10,000
10^{3} = 1,000
10^{2} = 100
10^{1} = 10
10^{0} = 1
10^{-1} = .1
10^{-2} = .01
10^{-3} = .001
10^{-4} = .000 1
10^{-5} = .000 01
10^{-6} = .000 001
10^{-7} = .000 000 1
10^{-8} = .000 000 01
10^{-9} = .000 000 001
10^{-10} = .000 000 000 1

COMMON LOGARITHMS

N	0	1	2	3	4	5	6	7	8	9
0	....	0000	3010	4771	6021	6990	7782	8451	9031	9542
1	0000	0414	0792	1139	1461	1761	2041	2304	2553	2788
2	3010	3222	3424	3617	3802	3979	4150	4314	4472	4624
3	4771	4914	5051	5185	5315	5441	5563	5682	5798	5911
4	6021	6128	6232	6335	6435	6532	6628	6721	6812	6902
5	6990	7076	7160	7243	7324	7404	7482	7559	7634	7709
6	7782	7853	7924	7993	8062	8129	8195	8261	8325	8388
7	8451	8513	8573	8633	8692	8751	8808	8865	8921	8976
8	9031	9085	9138	9191	9243	9294	9345	9395	9445	9494
9	9542	9590	9638	9685	9731	9777	9823	9868	9912	9956
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
N	0	1	2	3	4	5	6	7	8	9

COMMON LOGARITHMS

N	0	1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996
100	0000	0004	0009	0013	0017	0022	0026	0030	0035	0039
N	0	1	2	3	4	5	6	7	8	9

NATURAL LOGARITHMS

N	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
	....	0.0000	0.6931	1.0986	1.3863	1.6094	1.7918	1.9459	2.0794	2.1972
10	2.3026	2.3979	2.4849	2.5649	2.6391	2.7081	2.7726	2.8332	2.8904	2.9444
20	9957	3.0445	3.0910	3.1355	3.1781	3.2189	3.2581	3.2958	3.3322	3.3673
30	3.4012	4340	4657	4965	5264	5553	5835	6109	6376	6636
40	6889	7136	7377	7612	7842	8067	8286	8501	8712	8918
50	9120	9318	9512	9703	9890	4.0073	4.0254	4.0431	4.0604	4.0775
60	4.0943	4.1109	4.1271	4.1431	4.1589	1744	1897	2047	2195	2341
70	2485	2627	2767	2905	3041	3175	3307	3408	3567	3694
80	3820	3944	4067	4188	4308	4427	4543	4659	4773	4886
90	4998	5109	5218	5326	5433	5539	5643	5747	5850	5951
100	6052	6151	6250	6347	6444	6540	6634	6728	6821	6913
110	7005	7095	7185	7274	7362	7449	7536	7622	7707	7791
120	7875	7958	8040	8122	8203	8283	8363	8442	8520	8598
130	8675	8752	8828	8903	8978	9053	9127	9200	9273	9345
140	9416	9488	9558	9628	9698	9767	9836	9904	9972	5.0039
150	5.0106	5.0173	5.0239	5.0304	5.0370	5.0434	5.0499	5.0562	5.0626	0689
160	0752	0814	0876	0938	0999	1059	1120	1180	1240	1299
170	1358	1417	1475	1533	1591	1648	1705	1761	1818	1874
180	1930	1985	2040	2095	2149	2204	2257	2311	2364	2417
190	2470	2523	2575	2627	2679	2730	2781	2832	2883	2933
200	2983	3033	3083	3132	3181	3230	3279	3327	3375	3423
210	3471	3519	3566	3613	3660	3706	3753	3799	3845	3891
220	3936	3982	4027	4072	4116	4161	4205	4250	4293	4337
230	4381	4424	4467	4510	4553	4596	4638	4681	4723	4765
240	4806	4848	4889	4931	4972	5013	5053	5094	5134	5175
250	5215	5255	5294	5334	5373	5413	5452	5491	5530	5568
260	5607	5645	5683	5722	5759	5797	5835	5872	5910	5947
270	5984	6021	6058	6095	6131	6168	6204	6240	6276	6312
280	6348	6384	6419	6454	6490	6525	6560	6595	6630	6664
290	6699	6733	6768	6802	6836	6870	6904	6937	6971	7004
300	7038	7071	7104	7137	7170	7203	7236	7268	7301	7333
310	7366	7398	7430	7462	7494	7526	7557	7589	7621	7652
320	7683	7714	7746	7777	7807	7838	7869	7900	7930	7961
330	7991	8021	8051	8081	8111	8141	8171	8201	8230	8260
340	8289	8319	8348	8377	8406	8435	8464	8493	8522	8551
350	8579	8608	8636	8665	8693	8721	8749	8777	8805	8833
360	8861	8889	8916	8944	8972	8999	9026	9054	9081	9108
370	9135	9162	9189	9216	9243	9269	9296	9322	9349	9375
380	9402	9428	9454	9480	9506	9532	9558	9584	9610	9636
390	9661	9687	9713	9738	9764	9789	9814	9839	9865	9890
400	9915	9940	9965	9989	6.0014	6.0039	6.0064	6.0088	6.0113	6.0137
410	6.0162	6.0186	6.0210	6.0234	0259	0283	0307	0331	0355	0379
420	0403	0426	0450	0474	0497	0521	0544	0568	0591	0615
430	0638	0661	0684	0707	0730	0753	0776	0799	0822	0845
440	0868	0890	0913	0936	0958	0981	1003	1026	1048	1070
450	1092	1115	1137	1159	1181	1203	1225	1247	1269	1291
460	1312	1334	1356	1377	1399	1420	1442	1463	1485	1506
470	1527	1549	1570	1591	1612	1633	1654	1675	1696	1717
480	1738	1759	1779	1800	1821	1841	1862	1883	1903	1924
490	1944	1964	1985	2005	2025	2046	2066	2086	2106	2126
500	2146	2166	2186	2206	2226	2246	2265	2285	2305	2324
N	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0

NATURAL LOGARITHMS

N	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
500	6.2146	6.2166	6.2186	6.2206	6.2226	6.2246	6.2265	6.2285	6.2305	6.2324
510	2344	2364	2383	2403	2422	2442	2461	2480	2500	2519
520	2538	2558	2577	2596	2615	2634	2653	2672	2691	2710
530	2729	2748	2766	2785	2804	2823	2841	2860	2879	2897
540	2916	2934	2953	2971	2989	3008	3026	3044	3063	3081
550	3099	3117	3135	3154	3172	3190	3208	3226	3244	3261
560	3279	3297	3315	3333	3351	3368	3386	3404	3421	3439
570	3456	3474	3491	3509	3526	3544	3561	3578	3596	3613
580	3630	3648	3665	3682	3699	3716	3733	3750	3767	3784
590	3801	3818	3835	3852	3869	3886	3902	3919	3936	3953
600	3969	3986	4003	4019	4036	4052	4069	4085	4102	4118
610	4135	4151	4167	4184	4200	4216	4232	4249	4265	4281
620	4297	4313	4329	4345	4362	4378	4394	4409	4425	4441
630	4457	4473	4489	4505	4520	4536	4552	4568	4583	4599
640	4615	4630	4646	4661	4677	4693	4708	4723	4739	4754
650	4770	4785	4800	4816	4831	4846	4862	4877	4892	4907
660	4922	4938	4953	4968	4983	4998	5013	5028	5043	5058
670	5073	5088	5103	5117	5132	5147	5162	5177	5191	5206
680	5221	5236	5250	5265	5280	5294	5309	5323	5338	5352
690	5367	5381	5396	5410	5425	5439	5453	5468	5482	5497
700	5511	5525	5539	5554	5568	5582	5596	5610	5624	5639
710	5653	5667	5681	5695	5709	5723	5737	5751	5765	5779
720	5793	5806	5820	5834	5848	5862	5876	5889	5903	5917
730	5930	5944	5958	5971	5985	5999	6012	6026	6039	6053
740	6067	6080	6093	6107	6120	6134	6147	6161	6174	6187
750	6201	6214	6227	6241	6254	6267	6280	6294	6307	6320
760	6333	6346	6359	6373	6386	6399	6412	6425	6438	6451
770	6464	6477	6490	6503	6516	6529	6542	6554	6567	6580
780	6593	6606	6619	6631	6644	6657	6670	6682	6695	6708
790	6720	6733	6746	6758	6771	6783	6796	6809	6821	6834
800	6846	6859	6871	6884	6896	6908	6921	6933	6946	6958
810	6970	6983	6995	7007	7020	7032	7044	7056	7069	7081
820	7093	7105	7117	7130	7142	7154	7166	7178	7190	7202
830	7214	7226	7238	7250	7262	7274	7286	7298	7310	7322
840	7334	7346	7358	7370	7382	7393	7405	7417	7429	7441
850	7452	7464	7476	7488	7499	7511	7523	7534	7546	7558
860	7569	7581	7593	7604	7616	7627	7639	7650	7662	7673
870	7685	7696	7708	7719	7731	7742	7754	7765	7776	7788
880	7799	7811	7822	7833	7845	7856	7867	7878	7890	7901
890	7912	7923	7935	7946	7957	7968	7979	7991	8002	8013
900	8024	8035	8046	8057	8068	8079	8090	8101	8112	8123
910	8134	8145	8156	8167	8178	8189	8200	8211	8222	8233
920	8244	8255	8265	8276	8287	8298	8309	8320	8330	8341
930	8352	8363	8373	8384	8395	8405	8416	8427	8437	8448
940	8459	8469	8480	8491	8501	8512	8522	8533	8544	8554
950	8565	8575	8586	8596	8607	8617	8628	8638	8648	8659
960	8669	8680	8690	8701	8711	8721	8732	8742	8752	8763
970	8773	8783	8794	8804	8814	8824	8835	8845	8855	8865
980	8876	8886	8896	8906	8916	8926	8937	8947	8957	8967
990	8977	8987	8997	9007	9017	9027	9037	9048	9058	9068
1000	9078	9088	9098	9108	9117	9127	9137	9147	9157	9167
N	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0

Values of ϵ^x and ϵ^{-x}

0.00—1.99

x	Function	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	ϵ^x	1.0000	1.0101	1.0202	1.0305	1.0408	1.0513	1.0618	1.0725	1.0833	1.0942
	ϵ^{-x}	1.0000	0.9900	0.9802	0.9704	0.9608	0.9512	0.9418	0.9324	0.9231	0.9139
0.1	ϵ^x	1.1052	1.1163	1.1275	1.1388	1.1503	1.1618	1.1735	1.1853	1.1972	1.2093
	ϵ^{-x}	0.9048	0.8958	0.8869	0.8781	0.8694	0.8607	0.8521	0.8437	0.8353	0.8270
0.2	ϵ^x	1.2214	1.2337	1.2461	1.2586	1.2712	1.2840	1.2969	1.3100	1.3231	1.3364
	ϵ^{-x}	0.8187	0.8106	0.8025	0.7945	0.7866	0.7788	0.7711	0.7634	0.7558	0.7483
0.3	ϵ^x	1.3499	1.3634	1.3771	1.3910	1.4049	1.4191	1.4333	1.4477	1.4623	1.4770
	ϵ^{-x}	0.7408	0.7334	0.7261	0.7189	0.7118	0.7047	0.6977	0.6907	0.6839	0.6771
0.4	ϵ^x	1.4918	1.5068	1.5220	1.5373	1.5527	1.5683	1.5841	1.6000	1.6161	1.6323
	ϵ^{-x}	0.6703	0.6637	0.6570	0.6505	0.6440	0.6376	0.6313	0.6250	0.6188	0.6126
0.5	ϵ^x	1.6487	1.6653	1.6820	1.6989	1.7160	1.7333	1.7507	1.7683	1.7860	1.8040
	ϵ^{-x}	0.6065	0.6005	0.5945	0.5886	0.5827	0.5769	0.5712	0.5655	0.5599	0.5543
0.6	ϵ^x	1.8221	1.8404	1.8589	1.8776	1.8965	1.9155	1.9348	1.9542	1.9739	1.9939
	ϵ^{-x}	0.5488	0.5434	0.5379	0.5326	0.5273	0.5220	0.5169	0.5117	0.5066	0.5017
0.7	ϵ^x	2.0138	2.0340	2.0544	2.0751	2.0959	2.1170	2.1383	2.1598	2.1815	2.2034
	ϵ^{-x}	0.4966	0.4916	0.4868	0.4819	0.4771	0.4724	0.4677	0.4630	0.4584	0.4538
0.8	ϵ^x	2.2255	2.2479	2.2705	2.2933	2.3164	2.3396	2.3632	2.3869	2.4109	2.4351
	ϵ^{-x}	0.4493	0.4449	0.4404	0.4360	0.4317	0.4274	0.4232	0.4190	0.4148	0.4107
0.9	ϵ^x	2.4596	2.4843	2.5093	2.5345	2.5600	2.5857	2.6117	2.6379	2.6645	2.6912
	ϵ^{-x}	0.4066	0.4025	0.3985	0.3946	0.3906	0.3867	0.3829	0.3791	0.3753	0.3716
1.0	ϵ^x	2.7183	2.7456	2.7732	2.8011	2.8292	2.8577	2.8864	2.9154	2.9447	2.9743
	ϵ^{-x}	0.3679	0.3642	0.3606	0.3570	0.3535	0.3499	0.3465	0.3430	0.3396	0.3362
1.1	ϵ^x	3.0042	3.0344	3.0649	3.0957	3.1268	3.1582	3.1899	3.2220	3.2544	3.2871
	ϵ^{-x}	0.3329	0.3296	0.3263	0.3230	0.3198	0.3166	0.3135	0.3104	0.3073	0.3042
1.2	ϵ^x	3.3201	3.3535	3.3872	3.4212	3.4556	3.4903	3.5254	3.5609	3.5966	3.6328
	ϵ^{-x}	0.3012	0.2982	0.2952	0.2923	0.2894	0.2865	0.2837	0.2808	0.2780	0.2753
1.3	ϵ^x	3.6693	3.7062	3.7434	3.7810	3.8190	3.8574	3.8962	3.9354	3.9749	4.0149
	ϵ^{-x}	0.2725	0.2698	0.2671	0.2645	0.2618	0.2592	0.2567	0.2541	0.2516	0.2491
1.4	ϵ^x	4.0552	4.0960	4.1371	4.1787	4.2207	4.2631	4.3060	4.3492	4.3929	4.4371
	ϵ^{-x}	0.2466	0.2441	0.2417	0.2393	0.2369	0.2346	0.2322	0.2299	0.2276	0.2254
1.5	ϵ^x	4.4817	4.5267	4.5722	4.6182	4.6646	4.7115	4.7588	4.8066	4.8550	4.9037
	ϵ^{-x}	0.2231	0.2209	0.2187	0.2165	0.2144	0.2122	0.2101	0.2080	0.2060	0.2039
1.6	ϵ^x	4.9530	5.0028	5.0531	5.1039	5.1552	5.2070	5.2593	5.3122	5.3656	5.4195
	ϵ^{-x}	0.2019	0.1999	0.1979	0.1959	0.1940	0.1920	0.1901	0.1882	0.1864	0.1845
1.7	ϵ^x	5.4739	5.5290	5.5845	5.6407	5.6973	5.7546	5.8124	5.8709	5.9299	5.9895
	ϵ^{-x}	0.1827	0.1809	0.1791	0.1773	0.1755	0.1738	0.1720	0.1703	0.1686	0.1670
1.8	ϵ^x	6.0496	6.1104	6.1719	6.2339	6.2965	6.3598	6.4237	6.4883	6.5535	6.6194
	ϵ^{-x}	0.1653	0.1637	0.1620	0.1604	0.1588	0.1572	0.1557	0.1541	0.1526	0.1511
1.9	ϵ^x	6.6859	6.7531	6.8210	6.8895	6.9588	7.0287	7.0993	7.1707	7.2427	7.3155
	ϵ^{-x}	0.1496	0.1481	0.1466	0.1451	0.1437	0.1423	0.1409	0.1395	0.1381	0.1367

Values of ϵ^x and ϵ^{-x}

x	Function	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.0	ϵ^x ϵ^{-x}	7.3891 0.1353	7.4633 0.1340	7.5383 0.1327	7.6141 0.1313	7.6906 0.1300	7.7679 0.1287	7.8460 0.1275	7.9248 0.1262	8.0045 0.1249	8.0849 0.1237
2.1	ϵ^x ϵ^{-x}	8.1662 0.1225	8.2482 0.1212	8.3311 0.1200	8.4149 0.1188	8.4994 0.1177	8.5849 0.1165	8.6711 0.1153	8.7583 0.1142	8.8463 0.1130	8.9352 0.1119
2.2	ϵ^x ϵ^{-x}	9.0250 0.1108	9.1157 0.1097	9.2073 0.1086	9.2999 0.1075	9.3933 0.1065	9.4877 0.1054	9.5831 0.1044	9.6794 0.1033	9.7767 0.1023	9.8749 0.1013
2.3	ϵ^x ϵ^{-x}	9.9742 0.1003	10.074 0.0993	10.176 0.0983	10.278 0.0973	10.381 0.0963	10.486 0.0954	10.591 0.0944	10.697 0.0935	10.805 0.0926	10.913 0.0916
2.4	ϵ^x ϵ^{-x}	11.023 0.0907	11.134 0.0898	11.246 0.0889	11.359 0.0880	11.473 0.0872	11.588 0.0863	11.705 0.0854	11.822 0.0846	11.941 0.0837	12.061 0.0829
2.5	ϵ^x ϵ^{-x}	12.182 0.0821	12.305 0.0813	12.429 0.0805	12.554 0.0797	12.680 0.0789	12.807 0.0781	12.936 0.0773	13.066 0.0765	13.197 0.0758	13.330 0.0750
2.6	ϵ^x ϵ^{-x}	13.464 0.0743	13.599 0.0735	13.736 0.0728	13.874 0.0721	14.013 0.0714	14.154 0.0707	14.296 0.0699	14.440 0.0693	14.585 0.0686	14.732 0.0679
2.7	ϵ^x ϵ^{-x}	14.880 0.0672	15.029 0.0665	15.180 0.0659	15.333 0.0652	15.487 0.0646	15.643 0.0639	15.800 0.0633	15.959 0.0627	16.119 0.0620	16.281 0.0614
2.8	ϵ^x ϵ^{-x}	16.445 0.0608	16.610 0.0602	16.777 0.0596	16.945 0.0590	17.116 0.0584	17.288 0.0578	17.462 0.0573	17.637 0.0567	17.814 0.0561	17.993 0.0556
2.9	ϵ^x ϵ^{-x}	18.174 0.0550	18.357 0.0545	18.541 0.0539	18.728 0.0534	18.916 0.0529	19.106 0.0523	19.298 0.0518	19.492 0.0513	19.688 0.0508	19.886 0.0503
3.0	ϵ^x ϵ^{-x}	20.086 0.0498	20.287 0.0493	20.491 0.0488	20.697 0.0483	20.905 0.0478	21.115 0.0474	21.328 0.0469	21.542 0.0464	21.758 0.0460	21.977 0.0455
3.1	ϵ^x ϵ^{-x}	22.198 0.0450	22.421 0.0446	22.646 0.0442	22.874 0.0437	23.104 0.0433	23.336 0.0429	23.571 0.0424	23.807 0.0420	24.047 0.0416	24.288 0.0412
3.2	ϵ^x ϵ^{-x}	24.533 0.0408	24.779 0.0404	25.028 0.0400	25.280 0.0396	25.534 0.0392	25.790 0.0388	26.050 0.0384	26.311 0.0380	26.576 0.0376	26.843 0.0373
3.3	ϵ^x ϵ^{-x}	27.113 0.0369	27.385 0.0365	27.660 0.0362	27.938 0.0358	28.219 0.0354	28.503 0.0351	28.789 0.0347	29.079 0.0344	29.371 0.0340	29.666 0.0337
3.4	ϵ^x ϵ^{-x}	29.964 0.0334	30.265 0.0330	30.569 0.0327	30.877 0.0324	31.187 0.0321	31.500 0.0317	31.817 0.0314	32.137 0.0311	32.460 0.0308	32.786 0.0305
3.5	ϵ^x ϵ^{-x}	33.115 0.0302	33.448 0.0299	33.784 0.0296	34.124 0.0293	34.467 0.0290	34.813 0.0287	35.163 0.0284	35.517 0.0282	35.874 0.0279	36.234 0.0276
3.6	ϵ^x ϵ^{-x}	36.598 0.0273	36.966 0.0271	37.338 0.0268	37.713 0.0265	38.092 0.0263	38.475 0.0260	38.861 0.0257	39.252 0.0255	39.646 0.0252	40.045 0.0250
3.7	ϵ^x ϵ^{-x}	40.447 0.0247	40.854 0.0245	41.264 0.0242	41.679 0.0240	42.098 0.0238	42.521 0.0235	42.948 0.0233	43.380 0.0231	43.816 0.0228	44.256 0.0226
3.8	ϵ^x ϵ^{-x}	44.701 0.0224	45.150 0.0221	45.604 0.0219	46.063 0.0217	46.525 0.0215	46.993 0.0213	47.465 0.0211	47.942 0.0209	48.424 0.0207	48.911 0.0204
3.9	ϵ^x ϵ^{-x}	49.402 0.0202	49.899 0.0200	50.400 0.0198	50.907 0.0196	51.419 0.0195	51.935 0.0193	52.457 0.0191	52.985 0.0189	53.517 0.0187	54.055 0.0185

Values of ϵ^x and ϵ^{-x}

4.00–5.99

x	Function	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
4.0	ϵ^x ϵ^{-x}	54.598 0.0183	55.147 0.0181	55.701 0.0180	56.261 0.0178	56.826 0.0176	57.397 0.0174	57.974 0.0172	58.557 0.0171	59.145 0.0169	59.740 0.0167
4.1	ϵ^x ϵ^{-x}	60.340 0.0166	60.947 0.0164	61.559 0.0162	62.178 0.0161	62.803 0.0159	63.434 0.0158	64.072 0.0156	64.715 0.0155	65.366 0.0153	66.023 0.0151
4.2	ϵ^x ϵ^{-x}	66.686 0.0150	67.357 0.0148	68.033 0.0147	68.717 0.0146	69.408 0.0144	70.105 0.0143	70.810 0.0141	71.522 0.0140	72.240 0.0138	72.966 0.0137
4.3	ϵ^x ϵ^{-x}	73.700 0.0136	74.440 0.0134	75.189 0.0133	75.944 0.0132	76.708 0.0130	77.478 0.0129	78.257 0.0128	79.044 0.0127	79.838 0.0125	80.640 0.0124
4.4	ϵ^x ϵ^{-x}	81.451 0.0123	82.269 0.0122	83.096 0.0120	83.931 0.0119	84.775 0.0118	85.627 0.0117	86.488 0.0116	87.357 0.0114	88.235 0.0113	89.121 0.0112
4.5	ϵ^x ϵ^{-x}	90.017 0.0111	90.922 0.0110	91.836 0.0109	92.759 0.0108	93.691 0.0107	94.632 0.0106	95.583 0.0105	96.544 0.0104	97.514 0.0103	98.494 0.0102
4.6	ϵ^x ϵ^{-x}	99.484 0.0101	100.48 0.0100	101.49 0.0099	102.51 0.0098	103.54 0.0097	104.58 0.0096	105.64 0.0095	106.70 0.0094	107.77 0.0093	108.85 0.0092
4.7	ϵ^x ϵ^{-x}	109.95 0.0091	111.05 0.0090	112.17 0.0089	113.30 0.0088	114.43 0.0087	115.58 0.0087	116.75 0.0086	117.92 0.0085	119.10 0.0084	120.30 0.0083
4.8	ϵ^x ϵ^{-x}	121.51 0.0082	122.73 0.0081	123.97 0.0081	125.21 0.0080	126.47 0.0079	127.74 0.0078	129.02 0.0078	130.32 0.0077	131.63 0.0076	132.95 0.0075
4.9	ϵ^x ϵ^{-x}	134.29 0.0074	135.64 0.0074	137.00 0.0073	138.38 0.0072	139.77 0.0072	141.17 0.0071	142.59 0.0070	144.03 0.0069	145.47 0.0069	146.94 0.0068
5.0	ϵ^x ϵ^{-x}	148.41 0.0067	149.90 0.0067	151.41 0.0066	152.93 0.0065	154.47 0.0065	156.02 0.0064	157.59 0.0063	159.17 0.0063	160.77 0.0062	162.39 0.0062
5.1	ϵ ϵ^{-x}	164.02 0.0061	165.67 0.0060	167.34 0.0060	169.02 0.0059	170.72 0.0059	172.43 0.0058	174.16 0.0057	175.91 0.0057	177.68 0.0056	179.47 0.0056
5.2	ϵ^x ϵ^{-x}	181.27 0.0055	183.09 0.0055	184.93 0.0054	186.79 0.0054	188.67 0.0053	190.57 0.0052	192.48 0.0052	194.42 0.0051	196.37 0.0051	198.34 0.0050
5.3	ϵ^x ϵ^{-x}	200.34 0.0050	202.36 0.0049	204.38 0.0049	206.44 0.0048	208.51 0.0048	210.61 0.0047	212.72 0.0047	214.86 0.0047	217.02 0.0046	219.20 0.0046
5.4	ϵ^x ϵ^{-x}	221.41 0.0045	223.63 0.0045	225.88 0.0044	228.15 0.0044	230.44 0.0043	232.76 0.0043	235.10 0.0043	237.46 0.0042	239.85 0.0042	242.26 0.0041
5.5	ϵ^x ϵ^{-x}	244.69 0.0041	247.15 0.0040	249.64 0.0040	252.14 0.0040	254.68 0.0039	257.24 0.0039	259.82 0.0038	262.43 0.0038	265.07 0.0038	267.74 0.0037
5.6	ϵ^x ϵ^{-x}	270.43 0.0037	273.14 0.0037	275.89 0.0036	278.66 0.0036	281.46 0.0036	284.29 0.0035	287.15 0.0035	290.03 0.0034	292.95 0.0034	295.89 0.0034
5.7	ϵ^x ϵ^{-x}	298.87 0.0033	301.87 0.0033	304.90 0.0033	307.97 0.0032	311.06 0.0032	314.19 0.0032	317.35 0.0032	320.54 0.0031	323.76 0.0031	327.01 0.0031
5.8	ϵ^x ϵ^{-x}	330.30 0.0030	333.62 0.0030	336.97 0.0030	340.36 0.0029	343.78 0.0029	347.23 0.0029	350.72 0.0029	354.25 0.0028	357.81 0.0028	361.41 0.0028
5.9	ϵ^x ϵ^{-x}	365.04 0.0027	368.71 0.0027	372.41 0.0027	376.15 0.0027	379.93 0.0026	383.75 0.0026	387.61 0.0026	391.51 0.0026	395.44 0.0025	399.41 0.0025

Natural Sines, Cosines and Tangents

0°-14.9°

Degs.	Function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
0	sin	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157
	cos	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999
	tan	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157
1	sin	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332
	cos	0.9998	0.9998	0.9998	0.9997	0.9997	0.9997	0.9996	0.9996	0.9995	0.9995
	tan	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332
2	sin	0.0349	0.0366	0.0384	0.0401	0.0419	0.0436	0.0454	0.0471	0.0488	0.0506
	cos	0.9994	0.9993	0.9993	0.9992	0.9991	0.9990	0.9990	0.9989	0.9988	0.9987
	tan	0.0349	0.0367	0.0384	0.0402	0.0419	0.0437	0.0454	0.0472	0.0489	0.0507
3	sin	0.0523	0.0541	0.0558	0.0576	0.0593	0.0610	0.0628	0.0645	0.0663	0.0680
	cos	0.9986	0.9985	0.9984	0.9983	0.9982	0.9981	0.9980	0.9979	0.9978	0.9977
	tan	0.0524	0.0542	0.0559	0.0577	0.0594	0.0612	0.0629	0.0647	0.0664	0.0682
4	sin	0.0698	0.0715	0.0732	0.0750	0.0767	0.0785	0.0802	0.0819	0.0837	0.0854
	cos	0.9976	0.9974	0.9973	0.9972	0.9971	0.9969	0.9968	0.9966	0.9965	0.9963
	tan	0.0699	0.0717	0.0734	0.0752	0.0769	0.0787	0.0805	0.0822	0.0840	0.0857
5	sin	0.0872	0.0889	0.0906	0.0924	0.0941	0.0958	0.0976	0.0993	0.1011	0.1028
	cos	0.9962	0.9960	0.9959	0.9957	0.9956	0.9954	0.9952	0.9951	0.9949	0.9947
	tan	0.0875	0.0892	0.0910	0.0928	0.0945	0.0963	0.0981	0.0998	0.1016	0.1033
6	sin	0.1045	0.1063	0.1080	0.1097	0.1115	0.1132	0.1149	0.1167	0.1184	0.1201
	cos	0.9945	0.9943	0.9942	0.9940	0.9938	0.9936	0.9934	0.9932	0.9930	0.9928
	tan	0.1051	0.1069	0.1086	0.1104	0.1122	0.1139	0.1157	0.1175	0.1192	0.1210
7	sin	0.1219	0.1236	0.1253	0.1271	0.1288	0.1305	0.1323	0.1340	0.1357	0.1374
	cos	0.9925	0.9923	0.9921	0.9919	0.9917	0.9914	0.9912	0.9910	0.9907	0.9905
	tan	0.1228	0.1246	0.1263	0.1281	0.1299	0.1317	0.1334	0.1352	0.1370	0.1388
8	sin	0.1392	0.1409	0.1426	0.1444	0.1461	0.1478	0.1495	0.1513	0.1530	0.1547
	cos	0.9903	0.9900	0.9898	0.9895	0.9893	0.9890	0.9888	0.9885	0.9882	0.9880
	tan	0.1405	0.1423	0.1441	0.1459	0.1477	0.1495	0.1512	0.1530	0.1548	0.1566
9	sin	0.1564	0.1582	0.1599	0.1616	0.1633	0.1650	0.1668	0.1685	0.1702	0.1719
	cos	0.9877	0.9874	0.9871	0.9869	0.9866	0.9863	0.9860	0.9857	0.9854	0.9851
	tan	0.1584	0.1602	0.1620	0.1638	0.1655	0.1673	0.1691	0.1709	0.1727	0.1745
10	sin	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822	0.1840	0.1857	0.1874	0.1891
	cos	0.9848	0.9845	0.9842	0.9839	0.9836	0.9833	0.9829	0.9826	0.9823	0.9820
	tan	0.1763	0.1781	0.1799	0.1817	0.1835	0.1853	0.1871	0.1890	0.1908	0.1926
11	sin	0.1908	0.1925	0.1942	0.1959	0.1977	0.1994	0.2011	0.2028	0.2045	0.2062
	cos	0.9816	0.9813	0.9810	0.9806	0.9803	0.9799	0.9796	0.9792	0.9789	0.9785
	tan	0.1944	0.1962	0.1980	0.1998	0.2016	0.2035	0.2053	0.2071	0.2089	0.2107
12	sin	0.2079	0.2096	0.2113	0.2130	0.2147	0.2164	0.2181	0.2198	0.2215	0.2232
	cos	0.9781	0.9778	0.9774	0.9770	0.9767	0.9763	0.9759	0.9755	0.9751	0.9748
	tan	0.2126	0.2144	0.2162	0.2180	0.2199	0.2217	0.2235	0.2254	0.2272	0.2290
13	sin	0.2250	0.2267	0.2284	0.2300	0.2318	0.2334	0.2351	0.2368	0.2385	0.2402
	cos	0.9744	0.9740	0.9736	0.9732	0.9728	0.9724	0.9720	0.9715	0.9711	0.9707
	tan	0.2309	0.2327	0.2345	0.2364	0.2382	0.2401	0.2419	0.2438	0.2456	0.2475
14	sin	0.2419	0.2436	0.2453	0.2470	0.2487	0.2504	0.2521	0.2538	0.2554	0.2571
	cos	0.9703	0.9699	0.9694	0.9690	0.9686	0.9681	0.9677	0.9673	0.9668	0.9664
	tan	0.2493	0.2512	0.2530	0.2549	0.2568	0.2586	0.2605	0.2623	0.2642	0.2661
Degs.	Function	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′

Natural Sines, Cosines and Tangents

15°-29.9°

Degs.	Function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
15	sin	0.2588	0.2605	0.2622	0.2639	0.2656	0.2672	0.2689	0.2706	0.2723	0.2740
	cos	0.9659	0.9655	0.9650	0.9646	0.9641	0.9636	0.9632	0.9627	0.9622	0.9617
	tan	0.2679	0.2698	0.2717	0.2736	0.2754	0.2773	0.2792	0.2811	0.2830	0.2849
16	sin	0.2756	0.2773	0.2790	0.2807	0.2823	0.2840	0.2857	0.2874	0.2890	0.2907
	cos	0.9613	0.9608	0.9603	0.9598	0.9593	0.9588	0.9583	0.9578	0.9573	0.9568
	tan	0.2867	0.2886	0.2905	0.2924	0.2943	0.2962	0.2981	0.3000	0.3019	0.3038
17	sin	0.2924	0.2940	0.2957	0.2974	0.2990	0.3007	0.3024	0.3040	0.3057	0.3074
	cos	0.9563	0.9558	0.9553	0.9548	0.9542	0.9537	0.9532	0.9527	0.9521	0.9516
	tan	0.3057	0.3076	0.3096	0.3115	0.3134	0.3153	0.3172	0.3191	0.3211	0.3230
18	sin	0.3090	0.3107	0.3123	0.3140	0.3156	0.3173	0.3190	0.3206	0.3223	0.3239
	cos	0.9511	0.9505	0.9500	0.9494	0.9489	0.9483	0.9478	0.9472	0.9466	0.9461
	tan	0.3249	0.3269	0.3288	0.3307	0.3327	0.3346	0.3365	0.3385	0.3404	0.3424
19	sin	0.3256	0.3272	0.3289	0.3305	0.3322	0.3338	0.3355	0.3371	0.3387	0.3404
	cos	0.9455	0.9449	0.9444	0.9438	0.9432	0.9426	0.9421	0.9415	0.9409	0.9403
	tan	0.3443	0.3463	0.3482	0.3502	0.3522	0.3541	0.3561	0.3581	0.3600	0.3620
20	sin	0.3420	0.3437	0.3453	0.3469	0.3486	0.3502	0.3518	0.3535	0.3551	0.3567
	cos	0.9397	0.9391	0.9385	0.9379	0.9373	0.9367	0.9361	0.9354	0.9348	0.9342
	tan	0.3640	0.3659	0.3679	0.3699	0.3719	0.3739	0.3759	0.3779	0.3799	0.3819
21	sin	0.3584	0.3600	0.3616	0.3633	0.3649	0.3665	0.3681	0.3697	0.3714	0.3730
	cos	0.9336	0.9330	0.9323	0.9317	0.9311	0.9304	0.9298	0.9291	0.9285	0.9278
	tan	0.3839	0.3859	0.3879	0.3899	0.3919	0.3939	0.3959	0.3979	0.4000	0.4020
22	sin	0.3746	0.3762	0.3778	0.3795	0.3811	0.3827	0.3843	0.3859	0.3875	0.3891
	cos	0.9272	0.9265	0.9259	0.9252	0.9245	0.9239	0.9232	0.9225	0.9219	0.9212
	tan	0.4040	0.4061	0.4081	0.4101	0.4122	0.4142	0.4163	0.4183	0.4204	0.4224
23	sin	0.3907	0.3923	0.3939	0.3955	0.3971	0.3987	0.4003	0.4019	0.4035	0.4051
	cos	0.9205	0.9198	0.9191	0.9184	0.9178	0.9171	0.9164	0.9157	0.9150	0.9143
	tan	0.4245	0.4265	0.4286	0.4307	0.4327	0.4348	0.4369	0.4390	0.4411	0.4431
24	sin	0.4067	0.4083	0.4099	0.4115	0.4131	0.4147	0.4163	0.4179	0.4195	0.4210
	cos	0.9135	0.9128	0.9121	0.9114	0.9107	0.9100	0.9092	0.9085	0.9078	0.9070
	tan	0.4452	0.4473	0.4494	0.4515	0.4536	0.4557	0.4578	0.4599	0.4621	0.4642
25	sin	0.4226	0.4242	0.4258	0.4274	0.4289	0.4305	0.4321	0.4337	0.4352	0.4368
	cos	0.9063	0.9056	0.9048	0.9041	0.9033	0.9026	0.9018	0.9011	0.9003	0.8996
	tan	0.4663	0.4684	0.4706	0.4727	0.4748	0.4770	0.4791	0.4813	0.4834	0.4856
26	sin	0.4384	0.4399	0.4415	0.4431	0.4446	0.4462	0.4478	0.4493	0.4509	0.4524
	cos	0.8988	0.8980	0.8973	0.8965	0.8957	0.8949	0.8942	0.8934	0.8926	0.8918
	tan	0.4877	0.4899	0.4921	0.4942	0.4964	0.4986	0.5008	0.5029	0.5051	0.5073
27	sin	0.4540	0.4555	0.4571	0.4586	0.4602	0.4617	0.4633	0.4648	0.4664	0.4679
	cos	0.8910	0.8902	0.8894	0.8886	0.8878	0.8870	0.8862	0.8854	0.8846	0.8838
	tan	0.5095	0.5117	0.5139	0.5161	0.5184	0.5206	0.5228	0.5250	0.5272	0.5295
28	sin	0.4695	0.4710	0.4726	0.4741	0.4756	0.4772	0.4787	0.4802	0.4818	0.4833
	cos	0.8829	0.8821	0.8813	0.8805	0.8796	0.8788	0.8780	0.8771	0.8763	0.8755
	tan	0.5317	0.5340	0.5362	0.5384	0.5407	0.5430	0.5452	0.5475	0.5498	0.5520
29	sin	0.4848	0.4863	0.4879	0.4894	0.4909	0.4924	0.4939	0.4955	0.4970	0.4985
	cos	0.8746	0.8738	0.8729	0.8721	0.8712	0.8704	0.8695	0.8686	0.8678	0.8669
	tan	0.5543	0.5566	0.5589	0.5612	0.5635	0.5658	0.5681	0.5704	0.5727	0.5750
Degs.	**Function**	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′

Natural Sines, Cosines and Tangents

30°–44.9°

Degs.	Function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
30	sin	0.5000	0.5015	0.5030	0.5045	0.5060	0.5075	0.5090	0.5105	0.5120	0.5135
	cos	0.8660	0.8652	0.8643	0.8634	0.8625	0.8616	0.8607	0.8599	0.8590	0.8581
	tan	0.5774	0.5797	0.5820	0.5844	0.5867	0.5890	0.5914	0.5938	0.5961	0.5985
31	sin	0.5150	0.5165	0.5180	0.5195	0.5210	0.5225	0.5240	0.5255	0.5270	0.5284
	cos	0.8572	0.8563	0.8554	0.8545	0.8536	0.8526	0.8517	0.8508	0.8499	0.8490
	tan	0.6009	0.6032	0.6056	0.6080	0.6104	0.6128	0.6152	0.6176	0.6200	0.6224
32	sin	0.5299	0.5314	0.5329	0.5344	0.5358	0.5373	0.5388	0.5402	0.5417	0.5432
	cos	0.8480	0.8471	0.8462	0.8453	0.8443	0.8434	0.8425	0.8415	0.8406	0.8396
	tan	0.6249	0.6273	0.6297	0.6322	0.6346	0.6371	0.6395	0.6420	0.6445	0.6469
33	sin	0.5446	0.5461	0.5476	0.5490	0.5505	0.5519	0.5534	0.5548	0.5563	0.5577
	cos	0.8387	0.8377	0.8368	0.8358	0.8348	0.8339	0.8329	0.8320	0.8310	0.8300
	tan	0.6494	0.6519	0.6544	0.6569	0.6594	0.6619	0.6644	0.6669	0.6694	0.6720
34	sin	0.5592	0.5606	0.5621	0.5635	0.5650	0.5664	0.5678	0.5693	0.5707	0.5721
	cos	0.8290	0.8281	0.8271	0.8261	0.8251	0.8241	0.8231	0.8221	0.8211	0.8202
	tan	0.6745	0.6771	0.6796	0.6822	0.6847	0.6873	0.6899	0.6924	0.6950	0.6976
35	sin	0.5736	0.5750	0.5764	0.5779	0.5793	0.5807	0.5821	0.5835	0.5850	0.5864
	cos	0.8192	0.8181	0.8171	0.8161	0.8151	0.8141	0.8131	0.8121	0.8111	0.8100
	tan	0.7002	0.7028	0.7054	0.7080	0.7107	0.7133	0.7159	0.7186	0.7212	0.7239
36	sin	0.5878	0.5892	0.5906	0.5920	0.5934	0.5948	0.5962	0.5976	0.5990	0.6004
	cos	0.8090	0.8080	0.8070	0.8059	0.8049	0.8039	0.8028	0.8018	0.8007	0.7997
	tan	0.7265	0.7292	0.7319	0.7346	0.7373	0.7400	0.7427	0.7454	0.7481	0.7508
37	sin	0.6018	0.6032	0.6046	0.6060	0.6074	0.6088	0.6101	0.6115	0.6129	0.6143
	cos	0.7986	0.7976	0.7965	0.7955	0.7944	0.7934	0.7923	0.7912	0.7902	0.7891
	tan	0.7536	0.7563	0.7590	0.7618	0.7646	0.7673	0.7701	0.7729	0.7757	0.7785
38	sin	0.6157	0.6170	0.6184	0.6198	0.6211	0.6225	0.6239	0.6252	0.6266	0.6280
	cos	0.7880	0.7869	0.7859	0.7848	0.7837	0.7826	0.7815	0.7804	0.7793	0.7782
	tan	0.7813	0.7841	0.7869	0.7898	0.7926	0.7954	0.7983	0.8012	0.8040	0.8069
39	sin	0.6293	0.6307	0.6320	0.6334	0.6347	0.6361	0.6374	0.6388	0.6401	0.6414
	cos	0.7771	0.7760	0.7749	0.7738	0.7727	0.7716	0.7705	0.7694	0.7683	0.7672
	tan	0.8098	0.8127	0.8156	0.8185	0.8214	0.8243	0.8273	0.8302	0.8332	0.8361
40	sin	0.6428	0.6441	0.6455	0.6468	0.6481	0.6494	0.6508	0.6521	0.6534	0.6547
	cos	0.7660	0.7649	0.7638	0.7627	0.7615	0.7604	0.7593	0.7581	0.7570	0.7559
	tan	0.8391	0.8421	0.8451	0.8481	0.8511	0.8541	0.8571	0.8601	0.8632	0.8662
41	sin	0.6561	0.6574	0.6587	0.6600	0.6613	0.6626	0.6639	0.6652	0.6665	0.6678
	cos	0.7547	0.7536	0.7524	0.7513	0.7501	0.7490	0.7478	0.7466	0.7455	0.7443
	tan	0.8693	0.8724	0.8754	0.8785	0.8816	0.8847	0.8878	0.8910	0.8941	0.8972
42	sin	0.6691	0.6704	0.6717	0.6730	0.6743	0.6756	0.6769	0.6782	0.6794	0.6807
	cos	0.7431	0.7420	0.7408	0.7396	0.7385	0.7373	0.7361	0.7349	0.7337	0.7325
	tan	0.9004	0.9036	0.9067	0.9099	0.9131	0.9163	0.9195	0.9228	0.9260	0.9293
43	sin	0.6820	0.6833	0.6845	0.6858	0.6871	0.6884	0.6896	0.6909	0.6921	0.6934
	cos	0.7314	0.7302	0.7290	0.7278	0.7266	0.7254	0.7242	0.7230	0.7218	0.7206
	tan	0.9325	0.9358	0.9391	0.9424	0.9457	0.9490	0.9523	0.9556	0.9590	0.9623
44	sin	0.6947	0.6959	0.6972	0.6984	0.6997	0.7009	0.7022	0.7034	0.7046	0.7059
	cos	0.7193	0.7181	0.7169	0.7157	0.7145	0.7133	0.7120	0.7108	0.7096	0.7083
	tan	0.9657	0.9691	0.9725	0.9759	0.9793	0.9827	0.9861	0.9896	0.9930	0.9965
Degs.	Function	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′

Natural Sines, Cosines and Tangents

45°-59.9°

Degs.	Function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
45	sin	0.7071	0.7083	0.7096	0.7108	0.7120	0.7133	0.7145	0.7157	0.7169	0.7181
	cos	0.7071	0.7059	0.7046	0.7034	0.7022	0.7009	0.6997	0.6984	0.6972	0.6959
	tan	1.0000	1.0035	1.0070	1.0105	1.0141	1.0176	1.0212	1.0247	1.0283	1.0319
46	sin	0.7193	0.7206	0.7218	0.7230	0.7242	0.7254	0.7266	0.7278	0.7290	0.7302
	cos	0.6947	0.6934	0.6921	0.6909	0.6896	0.6884	0.6871	0.6858	0.6845	0.6833
	tan	1.0355	1.0392	1.0428	1.0464	1.0501	1.0538	1.0575	1.0612	1.0649	1.0686
47	sin	0.7314	0.7325	0.7337	0.7349	0.7361	0.7373	0.7385	0.7396	0.7408	0.7420
	cos	0.6820	0.6807	0.6794	0.6782	0.6769	0.6756	0.6743	0.6730	0.6717	0.6704
	tan	1.0724	1.0761	1.0799	1.0837	1.0875	1.0913	1.0951	1.0990	1.1028	1.1067
48	sin	0.7431	0.7443	0.7455	0.7466	0.7478	0.7490	0.7501	0.7513	0.7524	0.7536
	cos	0.6691	0.6678	0.6665	0.6652	0.6639	0.6626	0.6613	0.6600	0.6587	0.6574
	tan	1.1106	1.1145	1.1184	1.1224	1.1263	1.1303	1.1343	1.1383	1.1423	1.1463
49	sin	0.7547	0.7559	0.7570	0.7581	0.7593	0.7604	0.7615	0.7627	0.7638	0.7649
	cos	0.6561	0.6547	0.6534	0.6521	0.6508	0.6494	0.6481	0.6468	0.6455	0.6441
	tan	1.1504	1.1544	1.1585	1.1626	1.1667	1.1708	1.1750	1.1792	1.1833	1.1875
50	sin	0.7660	0.7672	0.7683	0.7694	0.7705	0.7716	0.7727	0.7738	0.7749	0.7760
	cos	0.6428	0.6414	0.6401	0.6388	0.6374	0.6361	0.6347	0.6334	0.6320	0.6307
	tan	1.1918	1.1960	1.2002	1.2045	1.2088	1.2131	1.2174	1.2218	1.2261	1.2305
51	sin	0.7771	0.7782	0.7793	0.7804	0.7815	0.7826	0.7837	0.7848	0.7859	0.7869
	cos	0.6293	0.6280	0.6266	0.6252	0.6239	0.6225	0.6211	0.6198	0.6184	0.6170
	tan	1.2349	1.2393	1.2437	1.2482	1.2527	1.2572	1.2617	1.2662	1.2708	1.2753
52	sin	0.7880	0.7891	0.7902	0.7912	0.7923	0.7934	0.7944	0.7955	0.7965	0.7976
	cos	0.6157	0.6143	0.6129	0.6115	0.6101	0.6088	0.6074	0.6060	0.6046	0.6032
	tan	1.2799	1.2846	1.2892	1.2938	1.2985	1.3032	1.3079	1.3127	1.3175	1.3222
53	sin	0.7986	0.7997	0.8007	0.8018	0.8028	0.8039	0.8049	0.8059	0.8070	0.8080
	cos	0.6018	0.6004	0.5990	0.5976	0.5962	0.5948	0.5934	0.5920	0.5906	0.5892
	tan	1.3270	1.3319	1.3367	1.3416	1.3465	1.3514	1.3564	1.3613	1.3663	1.3713
54	sin	0.8090	0.8100	0.8111	0.8121	0.8131	0.8141	0.8151	0.8161	0.8171	0.8181
	cos	0.5878	0.5864	0.5850	0.5835	0.5821	0.5807	0.5793	0.5779	0.5764	0.5750
	tan	1.3764	1.3814	1.3865	1.3916	1.3968	1.4019	1.4071	1.4124	1.4176	1.4229
55	sin	0.8192	0.8202	0.8211	0.8221	0.8231	0.8241	0.8251	0.8261	0.8271	0.8281
	cos	0.5736	0.5721	0.5707	0.5693	0.5678	0.5664	0.5650	0.5635	0.5621	0.5606
	tan	1.4281	1.4335	1.4388	1.4442	1.4496	1.4550	1.4605	1.4659	1.4715	1.4770
56	sin	0.8290	0.8300	0.8310	0.8320	0.8329	0.8339	0.8348	0.8358	0.8368	0.8377
	cos	0.5592	0.5577	0.5563	0.5548	0.5534	0.5519	0.5505	0.5490	0.5476	0.5461
	tan	1.4826	1.4882	1.4938	1.4994	1.5051	1.5108	1.5166	1.5224	1.5282	1.5340
57	sin	0.8387	0.8396	0.8406	0.8415	0.8425	0.8434	0.8443	0.8453	0.8462	0.8471
	cos	0.5446	0.5432	0.5417	0.5402	0.5388	0.5373	0.5358	0.5344	0.5329	0.5314
	tan	1.5399	1.5458	1.5517	1.5577	1.5637	1.5697	1.5757	1.5818	1.5880	1.5941
58	sin	0.8480	0.8490	0.8499	0.8508	0.8517	0.8526	0.8536	0.8545	0.8554	0.8563
	cos	0.5299	0.5284	0.5270	0.5255	0.5240	0.5225	0.5210	0.5195	0.5180	0.5165
	tan	1.6003	1.6066	1.6128	1.6191	1.6255	1.6319	1.6383	1.6447	1.6512	1.6577
59	sin	0.8572	0.8581	0.8590	0.8599	0.8607	0.8616	0.8625	0.8634	0.8643	0.8652
	cos	0.5150	0.5135	0.5120	0.5105	0.5090	0.5075	0.5060	0.5045	0.5030	0.5015
	tan	1.6643	1.6709	1.6775	1.6842	1.6909	1.6977	1.7045	1.7113	1.7182	1.7251
Degs.	Function	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′

Natural Sines, Cosines and Tangents

60°-74.9°

Degs.	Function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
60	sin cos tan	0.8660 0.5000 1.7321	0.8669 0.4985 1.7391	0.8678 0.4970 1.7461	0.8686 0.4955 1.7532	0.8695 0.4939 1.7603	0.8704 0.4924 1.7675	0.8712 0.4909 1.7747	0.8721 0.4894 1.7820	0.8729 0.4879 1.7893	0.8738 0.4863 1.7966
61	sin cos tan	0.8746 0.4848 1.8040	0.8755 0.4833 1.8115	0.8763 0.4818 1.8190	0.8771 0.4802 1.8265	0.8780 0.4787 1.8341	0.8788 0.4772 1.8418	0.8796 0.4756 1.8495	0.8805 0.4741 1.8572	0.8813 0.4726 1.8650	0.8821 0.4710 1.8728
62	sin cos tan	0.8829 0.4695 1.8807	0.8838 0.4679 1.8887	0.8846 0.4664 1.8967	0.8854 0.4648 1.9047	0.8862 0.4633 1.9128	0.8870 0.4617 1.9210	0.8878 0.4602 1.9292	0.8886 0.4586 1.9375	0.8894 0.4571 1.9458	0.8902 0.4555 1.9542
63	sin cos tan	0.8910 0.4540 1.9626	0.8918 0.4524 1.9711	0.8926 0.4509 1.9797	0.8934 0.4493 1.9883	0.8942 0.4478 1.9970	0.8949 0.4462 2.0057	0.8957 0.4446 2.0145	0.8965 0.4431 2.0233	0.8973 0.4415 2.0323	0.8980 0.4399 2.0413
64	sin cos tan	0.8988 0.4384 2.0503	0.8996 0.4368 2.0594	0.9003 0.4352 2.0686	0.9011 0.4337 2.0778	0.9018 0.4321 2.0872	0.9026 0.4305 2.0965	0.9033 0.4289 2.1060	0.9041 0.4274 2.1155	0.9048 0.4258 2.1251	0.9056 0.4242 2.1348
65	sin cos tan	0.9063 0.4226 2.1445	0.9070 0.4210 2.1543	0.9078 0.4195 2.1642	0.9085 0.4179 2.1742	0.9092 0.4163 2.1842	0.9100 0.4147 2.1943	0.9107 0.4131 2.2045	0.9114 0.4115 2.2148	0.9121 0.4099 2.2251	0.9128 0.4083 2.2355
66	sin cos tan	0.9135 0.4067 2.2460	0.9143 0.4051 2.2566	0.9150 0.4035 2.2673	0.9157 0.4019 2.2781	0.9164 0.4003 2.2889	0.9171 0.3987 2.2998	0.9178 0.3971 2.3109	0.9184 0.3955 2.3220	0.9191 0.3939 2.3332	0.9198 0.3923 2.3445
67	sin cos tan	0.9205 0.3907 2.3559	0.9212 0.3891 2.3673	0.9219 0.3875 2.3789	0.9225 0.3859 2.3906	0.9232 0.3843 2.4023	0.9239 0.3827 2.4142	0.9245 0.3811 2.4262	0.9252 0.3795 2.4383	0.9259 0.3778 2.4504	0.9265 0.3762 2.4627
68	sin cos tan	0.9272 0.3746 2.4751	0.9278 0.3730 2.4876	0.9285 0.3714 2.5002	0.9291 0.3697 2.5129	0.9298 0.3681 2.5257	0.9304 0.3665 2.5386	0.9311 0.3649 2.5517	0.9317 0.3633 2.5649	0.9323 0.3616 2.5782	0.9330 0.3600 2.5916
69	sin cos tan	0.9336 0.3584 2.6051	0.9342 0.3567 2.6187	0.9348 0.3551 2.6325	0.9354 0.3535 2.6464	0.9361 0.3518 2.6605	0.9367 0.3502 2.6746	0.9373 0.3486 2.6889	0.9379 0.3469 2.7034	0.9385 0.3453 2.7179	0.9391 0.3437 2.7326
70	sin cos tan	0.9397 0.3420 2.7475	0.9403 0.3404 2.7625.	0.9409 0.3387 2.7776	0.9415 0.3371 2.7929	0.9421 0.3355 2.8083	0.9426 0.3338 2.8239	0.9432 0.3322 2.8397	0.9438 0.3305 2.8556	0.9444 0.3289 2.8716	0.9449 0.3272 2.8878
71	sin cos tan	0.9455 0.3256 2.9042	0.9461 0.3239 2.9208	0.9466 0.3223 2.9375	0.9472 0.3206 2.9544	0.9478 0.3190 2.9714	0.9483 0.3173 2.9887	0.9489 0.3156 3.0061	0.9494 0.3140 3.0237	0.9500 0.3123 3.0415	0.9505 0.3107 3.0595
72	sin cos tan	0.9511 0.3090 3.0777	0.9516 0.3074 3.0961	0.9521 0.3057 3.1146	0.9527 0.3040 3.1334	0.9532 0.3024 3.1524	0.9537 0.3007 3.1716	0.9542 0.2990 3.1910	0.9548 0.2974 3.2106	0.9553 0.2957 3.2305	0.9558 0.2940 3.2506
73	sin cos tan	0.9563 0.2924 3.2709	0.9568 0.2907 3.2914	0.9573 0.2890 3.3122	0.9578 0.2874 3.3332	0.9583 0.2857 3.3544	0.9588 0.2840 3.3759	0.9593 0.2823 3.3977	0.9598 0.2807 3.4197	0.9603 0.2790 3.4420	0.9608 0.2773 3.4646
74	sin cos tan	0.9613 0.2756 3.4874	0.9617 0.2740 3.5105	0.9622 0.2723 3.5339	0.9627 0.2706 3.5576	0.9632 0.2689 3.5816	0.9636 0.2672 3.6059	0.9641 0.2656 3.6305	0.9646 0.2639 3.6554	0.9650 0.2622 3.6806	0.9655 0.2605 3.7062
Degs.	Function	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′

Natural Sines, Cosines and Tangents

75°–89.9°

Degs.	Function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
75	sin	0.9659	0.9664	0.9668	0.9673	0.9677	0.9681	0.9686	0.9690	0.9694	0.9699
	cos	0.2588	0.2571	0.2554	0.2538	0.2521	0.2504	0.2487	0.2470	0.2453	0.2436
	tan	3.7321	3.7583	3.7848	3.8118	3.8391	3.8667	3.8947	3.9232	3.9520	3.9812
76	sin	0.9703	0.9707	0.9711	0.9715	0.9720	0.9724	0.9728	0.9732	0.9736	0.9740
	cos	0.2419	0.2402	0.2385	0.2368	0.2351	0.2334	0.2317	0.2300	0.2284	0.2267
	tan	4.0108	4.0408	4.0713	4.1022	4.1335	4.1653	4.1976	4.2303	4.2635	4.2972
77	sin	0.9744	0.9748	0.9751	0.9755	0.9759	0.9763	0.9767	0.9770	0.9774	0.9778
	cos	0.2250	0.2232	0.2215	0.2198	0.2181	0.2164	0.2147	0.2130	0.2113	0.2096
	tan	4.3315	4.3662	4.4015	4.4374	4.4737	4.5107	4.5483	4.5864	4.6252	4.6646
78	sin	0.9781	0.9785	0.9789	0.9792	0.9796	0.9799	0.9803	0.9806	0.9810	0.9813
	cos	0.2079	0.2062	0.2045	0.2028	0.2011	0.1994	0.1977	0.1959	0.1942	0.1925
	tan	4.7046	4.7453	4.7867	4.8288	4.8716	4.9152	4.9594	5.0045	5.0504	5.0970
79	sin	0.9816	0.9820	0.9823	0.9826	0.9829	0.9833	0.9836	0.9839	0.9842	0.9845
	cos	0.1908	0.1891	0.1874	0.1857	0.1840	0.1822	0.1805	0.1788	0.1771	0.1754
	tan	5.1446	5.1929	5.2422	5.2924	5.3435	5.3955	5.4486	5.5026	5.5578	5.6140
80	sin	0.9848	0.9851	0.9854	0.9857	0.9860	0.9863	0.9866	0.9869	0.9871	0.9874
	cos	0.1736	0.1719	0.1702	0.1685	0.1668	0.1650	0.1633	0.1616	0.1599	0.1582
	tan	5.6713	5.7297	5.7894	5.8502	5.9124	5.9758	6.0405	6.1066	6.1742	6.2432
81	sin	0.9877	0.9880	0.9882	0.9885	0.9888	0.9890	0.9893	0.9895	0.9898	0.9900
	cos	0.1564	0.1547	0.1530	0.1513	0.1495	0.1478	0.1461	0.1444	0.1426	0.1409
	tan	6.3138	6.3859	6.4596	6.5350	6.6122	6.6912	6.7720	6.8548	6.9395	7.0264
82	sin	0.9903	0.9905	0.9907	0.9910	0.9912	0.9914	0.9917	0.9919	0.9921	0.9923
	cos	0.1392	0.1374	0.1357	0.1340	0.1323	0.1305	0.1288	0.1271	0.1253	0.1236
	tan	7.1154	7.2066	7.3002	7.3962	7.4947	7.5958	7.6996	7.8062	7.9158	8.0285
83	sin	0.9925	0.9928	0.9930	0.9932	0.9934	0.9936	0.9938	0.9940	0.9942	0.9943
	cos	0.1219	0.1201	0.1184	0.1167	0.1149	0.1132	0.1115	0.1097	0.1080	0.1063
	tan	8.1443	8.2636	8.3863	8.5126	8.6427	8.7769	8.9152	9.0579	9.2052	9.3572
84	sin	0.9945	0.9947	0.9949	0.9951	0.9952	0.9954	0.9956	0.9957	0.9959	0.9960
	cos	0.1045	0.1028	0.1011	0.0993	0.0976	0.0958	0.0941	0.0924	0.0906	0.0889
	tan	9.5144	9.6768	9.8448	10.02	10.20	10.39	58	10.78	10.99	11.20
85	sin	0.9962	0.9963	0.9965	0.9966	0.9968	0.9969	0.9971	0.9972	0.9973	0.9974
	cos	0.0872	0.0854	0.0837	0.0819	0.0802	0.0785	0.0767	0.0750	0.0732	0.0715
	tan	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95
86	sin	0.9976	0.9977	0.9978	0.9979	0.9980	0.9981	0.9982	0.9983	0.9984	0.9985
	cos	0.0698	0.0680	0.0663	0.0645	0.0628	0.0610	0.0593	0.0576	0.0558	0.0541
	tan	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46
87	sin	0.9986	0.9987	0.9988	0.9989	0.9990	0.9990	0.9991	0.9992	0.9993	0.9993
	cos	0.0523	0.0506	0.0488	0.0471	0.0454	0.0436	0.0419	0.0401	0.0384	0.0366
	tan	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27
88	sin	0.9994	0.9995	0.9995	0.9996	0.9996	0.9997	0.9997	0.9997	0.9998	0.9998
	cos	0.0349	0.0332	0.0314	0.0297	0.0279	0.0262	0.0244	0.0227	0.0209	0.0192
	tan	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08
89	sin	0.9998	0.9999	0.9999	0.9999	0.9999	1.000	1.000	1.000	1.000	1.000
	cos	0.0175	0.0157	0.0140	0.0122	0.0105	0.0087	0.0070	0.0052	0.0035	0.0017
	tan	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0
Degs.	**Function**	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′

Squares, Cubes and Roots

Roots of numbers other than those given directly may be found by the following relations:

$$\sqrt{100n} = 10\sqrt{n};\quad \sqrt{1000n} = 10\sqrt{10n};\quad \sqrt{\frac{1}{10}n} = \frac{1}{10}\sqrt{10n};\quad \sqrt{\frac{1}{100}n} = \frac{1}{10}\sqrt{n},$$

$$\sqrt{\frac{1}{1000}n} = \frac{1}{100}\sqrt{10n};\quad \sqrt[3]{1000n} = 10\sqrt[3]{n};\quad \sqrt[3]{10{,}000n} = 10\sqrt[3]{10n};\quad \sqrt[3]{100{,}000n} =$$

$$10\sqrt[3]{100n};\quad \sqrt[3]{\frac{1}{10}n} = \frac{1}{10}\sqrt[3]{100n};\quad \sqrt[3]{\frac{1}{100}n} = \frac{1}{10}\sqrt[3]{10n};\quad \sqrt[3]{\frac{1}{1000}n} = \frac{1}{10}\sqrt[3]{n}.$$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
1	1	1.000 000	3.162 278	1	1.000 000	2.154 435	4.641 589
2	4	1.414 214	4.472 136	8	1.259 921	2.714 418	5.848 035
3	9	1.732 051	5.477 226	27	1.442 250	3.107 233	6.694 330
4	16	2.000 000	6.324 555	64	1.587 401	3.419 952	7.368 063
5	25	2.236 068	7.071 068	125	1.709 976	3.684 031	7.937 005
6	36	2.449 490	7.745 967	216	1.817 121	3.914 868	8.434 327
7	49	2.645 751	8.366 600	343	1.912 931	4.121 285	8.879 040
8	64	2.828 427	8.944 272	512	2.000 000	4.308 869	9.283 178
9	81	3.000 000	9.486 833	729	2.080 084	4.481 405	9.654 894
10	100	3.162 278	10.00000	1 000	2.154 435	4.641 589	10.00000
11	121	3.316 625	10.48809	1 331	2.223 980	4.791 420	10.32280
12	144	3.464 102	10.95445	1 728	2.289 428	4.932 424	10.62659
13	169	3.605 551	11.40175	2 197	2.351 335	5.065 797	10.91393
14	196	3.741 657	11.83216	2 744	2.410 142	5.192 494	11.18689
15	225	3.872 983	12.24745	3 375	2.466 212	5.313 293	11.44714
16	256	4.000 000	12.64911	4 096	2.519 842	5.428 835	11.69607
17	289	4.123 106	13.03840	4 913	2.571 282	5.539 658	11.93483
18	324	4.242 641	13.41641	5 832	2.620 741	5.646 216	12.16440
19	361	4.358 899	13.78405	6 859	2.668 402	5.748 897	12.38562
20	400	4.472 136	14.14214	8 000	2.714 418	5.848 035	12.59921
21	441	4.582 576	14.49138	9 261	2.758 924	5.943 922	12.80579
22	484	4.690 416	14.83240	10 648	2.802 039	6.036 811	13.00591
23	529	4.795 832	15.16575	12 167	2.843 867	6.126 926	13.20006
24	576	4.898 979	15.49193	13 824	2.884 499	6.214 465	13.38866
25	625	5.000 000	15.81139	15 625	2.924 018	6.299 605	13.57209
26	676	5.099 020	16.12452	17 576	2.962 496	6.382 504	13.75069
27	729	5.196 152	16.43168	19 683	3.000 000	6.463 304	13.92477
28	784	5.291 503	16.73320	21 952	3.036 589	6.542 133	14.09460
29	841	5.385 165	17.02939	24 389	3.072 317	6.619 106	14.26043
30	900	5.477 226	17.32051	27 000	3.107 233	6.694 330	14.42250
31	961	5.567 764	17.60682	29 791	3.141 381	6.767 899	14.58100
32	1 024	5.656 854	17.88854	32 768	3.174 802	6.839 904	11.73613
33	1 089	5.744 563	18.16590	35 937	3.207 534	6.910 423	14.88806
34	1 156	5.830 952	18.43909	39 304	3.239 612	6.979 532	15.03695
35	1 225	5.916 080	18.70829	42 875	3.271 066	7.047 299	15.18294
36	1 296	6.000 000	18.97367	46 656	3.301 927	7.113 787	15.32619
37	1 369	6.082 763	19.23538	50 653	3.332 222	7.179 054	15.46680
38	1 444	6.164 414	19.49359	54 872	3.361 975	7.243 156	15.60491
39	1 521	6.244 998	19.74842	59 319	3.391 211	7.306 144	15.74061
40	1 600	6.324 555	20.00000	64 000	3.419 952	7.368 063	15.87401
41	1 681	6.403 124	20.24846	68 921	3.448 217	7.428 959	16.00521
42	1 764	6.480 741	20.49390	74 088	3.476 027	7.488 872	16.13429
43	1 849	6.557 439	20.73644	79 507	3.503 398	7.547 842	16.26133
44	1 936	6.633 250	20.97618	85 184	3.530 348	7.605 905	16.38643
45	2 025	6.708 204	21.21320	91 125	3.556 893	7.663 094	16.50964
46	2 116	6.782 330	21.44761	97 336	3.583 048	7.719 443	16.63103
47	2 209	6.855 655	21.67948	103 823	3.608 826	7.774 980	16.75069
48	2 304	6.928 203	21.90890	110 592	3.634 241	7.829 735	16.86865
49	2 401	7.000 000	22.13594	117 649	3.659 306	7.883 735	16.98499
50	2 500	7.071 068	22.36068	125 000	3.684 031	7.937 005	17.09976

SQUARES, CUBES AND ROOTS

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
50	2 500	7.071 068	22.36068	125 000	3.684 031	7.937 005	17.09976
51	2 601	7.141 428	22.58318	132 651	3.708 430	7.989 570	17.21301
52	2 704	7.211 103	22.80351	140 608	3.732 511	8.041 452	17.32478
53	2 809	7.280 110	23.02173	148 877	3.756 286	8.092 672	17.43513
54	2 916	7.348 469	23.23790	157 464	3.779 763	8.143 253	17.54411
55	3 025	7.416 198	23.45208	166 375	3.802 952	8.193 213	17.65174
56	3 136	7.483 315	23.66432	175 616	3.825 862	8.242 571	17.75808
57	3 249	7.549 834	23.87467	185 193	3.848 501	8.291 344	17.86316
58	3 364	7.615 773	24.08319	195 112	3.870 877	8.339 551	17.96702
59	3 481	7.681 146	24.28992	205 379	3.892 996	8.387 207	18.06969
60	3 600	7.745 967	24.49490	216 000	3.914 868	8.434 327	18.17121
61	3 721	7.810 250	24.69818	226 981	3.936 497	8.480 926	18.27160
62	3 844	7.874 008	24.89980	238 328	3.957 892	8.527 019	18.37091
63	3 969	7.937 254	25.09980	250 047	3.979 057	8.572 619	18.46915
64	4 096	8.000 000	25.29822	262 144	4.000 000	8.617 739	18.56636
65	4 225	8.062 258	25.49510	274 625	4.020 726	8.662 391	18.66256
66	4 356	8.124 038	25.69047	287 496	4.041 240	8.706 588	18.75777
67	4 489	8.185 353	25.88436	300 763	4.061 548	8.750 340	18.85204
68	4 624	8.246 211	26.07681	314 432	4.081 655	8.793 659	18.94536
69	4 761	8.306 624	26.26785	328 509	4.101 566	8.836 556	19.03778
70	4 900	8.366 600	26.45751	343 000	4.121 285	8.879 040	19.12931
71	5 041	8.426 150	26.64583	357 911	4.140 818	8.921 121	19.21997
72	5 184	8.485 281	26.83282	373 248	4.160 168	8.962 809	19.30979
73	5 329	8.544 004	27.01851	389 017	4.179 339	9.004 113	19.39877
74	5 476	8.602 325	27.20294	405 224	4.198 336	9.045 042	19.48695
75	5 625	8.660 254	27.38613	421 875	4.217 163	9.085 603	19.57434
76	5 776	8.717 798	27.56810	438 976	4.235 824	9.125 805	19.66095
77	5 929	8.774 964	27.74887	456 533	4.254 321	9.165 656	19.74681
78	6 084	8.831 761	27.92848	474 552	4.272 659	9.205 164	19.83192
79	6 241	8.888 194	28.10694	493 039	4.290 840	9.244 335	19.91632
80	6 400	8.944 272	28.28427	512 000	4.308 869	9.283 178	20.00000
81	6 561	9 000 000	28.46050	531 441	4.326 749	9.321 698	20.08299
82	6 724	9.055 385	28.63564	551 368	4.344 481	9.359 902	20.16530
83	6 889	9.110 434	28.80972	571 787	4.362 071	9.397 796	20.24694
84	7 056	9.165 151	28.98275	592 704	4.379 519	9.435 388	20.32793
85	7 225	9.219 544	29.15476	614 125	4.396 830	9.472 682	20.40828
86	7 396	9.273 618	29.32576	636 056	4.414 005	9.509 685	20.48800
87	7 569	9.327 379	29.49576	658 503	4.431 048	9.546 403	20.56710
88	7 744	9.380 832	29.66479	681 472	4.447 960	9.582 840	20.64560
89	7 921	9.433 981	29.83287	704 969	4.464 745	9.619 002	20.72351
90	8 100	9.486 833	30.00000	729 000	4.481 405	9.654 894	20.80084
91	8 281	9.539 392	30.16621	753 571	4.497 941	9.690 521	20.87759
92	8 464	9.591 663	30.33150	778 688	4.514 357	9.725 888	20.95379
93	8 649	9.643 651	30.49590	804 357	4.530 655	9.761 000	21.02944
94	8 836	9.695 360	30.65942	830 584	4.546 836	9.795 861	21.10454
95	9 025	9.746 794	30.82207	857 375	4.562 903	9.830 476	21.17912
96	9 216	9.797 959	30.98387	884 736	4.578 857	9.864 848	21.25317
97	9 409	9.848 858	31.14482	912 673	4.594 701	9.898 983	21.32671
98	9 604	9.899 495	31.30495	941 192	4.610 436	9.932 884	21.39975
99	9 801	9.949 874	31.46427	970 299	4.626 065	9.966 555	21.47229
100	10 000	10.00000	31.62278	1 000 000	4.641 589	10.00000	21.54435

Index

A

Advantages of high frequencies............12–1
Airborne antenna systems........13–33 to 13–36
  AEW 13–35
  airborne search 13–34
  general 13–33
  navigational 13–33
Amplidyne 14–22, 23
Amplifier, control 14–24, 25
Amplitude modulation 9–1
Antenna, arrays ....13–22 to 13–28
  current fed 13–19
  directional characteristics 13–1, 16
  distribution of current and voltage.....13–3, 7
  efficiency and gain 13–2
  electric field 13–3, 5, 6
  electromagnetic energy ....13–3 to 13–6
  exciting by radiation fields...........13–4, 21
  feeding with resonant and
    nonresonant lines 13–19, 20, 21
  fields in space 13–5, 6
  folded dipole 13–27
  function 13–1
  half-wave 13–7
  image 13–7
  impedance ....13–7 to 13–10
  impedance matching 13–21
  induction fields 13–4
  length 13–7, 8, 9
  magnetic field 13–3, 5, 6
  Marconi 13–18, 19
  polar diagrams 13–16, 17
  polarization of electromagnetic
    wave ....13–10 to 13–16
  propagation into space...................13–6
  radar systems ....13–33 to 13–38
  reciprocity 13–1
  voltage fed 13–19
  wavelength 13–7
  Yagi 13–27
Artificial lines 10–32
ATR tube 11–28, 29, 30
Automatic frequency control 9–21
Autosyn 14–1

B

Back porch 15–21
Backward-wave oscillator ....12–69 to 12–72
  structure 12–69
  theory 12–69, 70, 71, 72
Bolometer 10–26
Boot-strap driver 12–38, 39
Broadside array 13–22
Buncher grids 12–50
Bunching action 12–44, 45

C

Cardioid array 13–23
Catcher grids 12–50
Cathode-modulated oscillator 12–16, 17
Cavity resonators, description....11–34 to 11–40
  excitation 11–36
  fields 11–34
  resonant frequency variation...........11–37
  uses 11–37, 38
Center frequency 9–3
Characteristic impedance of a coaxial line..10–23
Characteristic impedance of a lossless
  R-F line 10–3
Characteristic impedance of a two-wire
  line 10–21
Charging circuits ....12–33 to 12–40
Choke joint 11–21
Coaxial line 10–22
Colinear array 13–24
Comparison of standing waves..............10–13
Conical scanning 13–32
Connecting elements in array...............13–24

Control amplifier 14–24, 25
Copper losses 10–24; 11–2
Coupling, door knob....................11–18
  loop 11–19
Crossed-field oscillator 12–72
Crystal detection 10–27
Crystal detector 9–12, 13
Current-fed antennas 13–19
Cut-off wavelength 11–6

D

Data transmission system.................14–1
De-emphasis networks ....9–42 to 9–44
Density modulation 12–55
Detection 9–7
Detection, defined 9–1
Detector, crystal 9–12, 13
  diode 9–8, 9
  first 9–16
  grid-leak 9–10
  infinite-impedance 9–11, 12
  plate 9–11
  ratio 9–23
  superregenerative 9–14, 15
Dielectric, losses 10–24; 11–2
Dielectric slabs 13–14, 15
Differential synchro, control
  transformer 14–18, 19
  operation ....14–14 to 14–19
  positioning 14–18, 19
Direct wave 15–21
Directional coupler 11–49
Directivity 13–27
Director 13–25
Discriminator 9–4
Discriminator circuit ....9–17 to 9–21
Dominant mode 11–12
Doppler effect 15–1
  analysis 15–1
  applications 15–2
  continuous-wave system 15–2
  detection 15–2
Driven arrays 13–22
  broadside 13–22
  cardioid 13–23
  colinear 13–24
  end-fire 13–23
Driver hard tube modulator...........12–36, 37
Duplexer, parallel 11–25
  polarization shifting 11–28
  series-parallel 11–25

E

Echo box 11–40
Electric field 11–7; 13–3, 5, 6
Electrical bandwidth 12–47
Electrical plumbing 11–3
Electromagnetic propagation 13–3, 4
Electromagnetic waves, resonant and
  dimensions 13–7
Electron paths 12–22, 23
End-fire array 13–23
Equivalent circuit of R–F line.............10–2

F

Ferrites, applications ....11–42 to 11–45
  cores 11–45
  electron action 11–42
  general 11–40
  gyroscopic action 11–41
  theory 11–41
Fields in space.........................13–5, 6
Filters, bandstop 9–35
  crystal 9–36
  mechanical 9–37
  simple 9–34
Fixed trigger gap........................12–30
Folded dipole 13–27
Frequency, carrier 9–2
  center 9–3
  conversion 9–15, 16
  lower side 9–2
  upper side 9–2
Frequency limitations of oscillators.........12–1
  external circuits 12–3
  overcoming tube limitations..............12–3
  tube construction 12–1
Front porch 15–21

G

Gas-filled tubes, ATR.............11–28, 29, 30
  TR 11–27
Grid-leak detector 9–10
Grid modulated oscillator.............12–16, 17
Ground antenna systems 13–37, 38
  ground search 13–38
  height 13–37
Ground wave 13–7; 15–21, 22
Group velocity 11–15

Guided and unguided electromagnetic waves 11–1, 2, 3

H

Half-wave antenna 13–7
Half-wave R–F section 10–19
Helix 12–66
Hunting 14–25

I

Image antennas 13–17
  theory 13–17
Impedance charts 10–20
Impedance inversion 10–18
Impedance chart for open and shorted lines 10–20
Impedance matching devices.......10–29; 13–21
Importance of losses (in r-f lines).........10–25
Inductance of leads......................12–2
Induction losses 10–25
Infinite impedance detector............9–11, 12
Insulator, metallic 11–4
Interconnection of elements in arrays......13–24
Interlace scanning 15–16
Incident voltage 10–11
Incident wave 10–11
Inertia damper 14–5
Interelectrode capacitance 12–2, 3
Ionosphere 11–1; 13–6, 7; 15–3
Ionospheric scatter 15–3
  antenna equipment 15–5
  diffraction 15–3
  general theory 15–4
  receiving equipment 15–15
  reflection 15–3
  refraction 15–3
  system capabilities 15–5
  transmitting equipment 15–5

K

Kinescope recorders 15–32
Klystron, mixer tube 12–60
  reflex 12–17, 48, 49
  two-cavity 12–49, 50
  three-cavity ....12–51 to 12–54
  versus magnetron 12–59

L

Lecher lines 10–27
Lighthouse tube oscillator ....12–5 to 12–7
Limit of inductance and capacitance........12–3
Limitations of oscillators...........12–1 to 12–4
Line controlled blocking oscillator.....12–39, 40
Line pulsing modulator..................12–35
Losses in R–F lines.....................10–24

M

Magnesyn 14–1
Magnetic amplifiers 14–28
Magnetic field in waveguide...........11–10, 11
Magnetron oscillator, as a transmitter......12–7
  care of 12–43
  cylindrical form 12–23, 24, 25
  description 12–17
  despiking circuit 12–42
  modes of oscillation...............12–26, 27
  modulating ....12–29 to 12–42
  multi-anode transit time.........12–21, 22, 23
  operation 12–18
  paths generated by electrons...........12–19
  plane form 12–21, 22, 23
  plate current-plate voltage curves....12–28, 29
  source of energy......................12–20
  split-anode transit time 12–21
  strapping 12–27
  summary 12–27
  typical transmitter...............12–40, 41, 42
Magnetostriction delay lines.............10–39
Marconi antenna 13–18, 19
Metallic insulator 10–28; 11–4
Microwaves 11–1
Mixer action 9–15, 16
Mixer, crystal 9–16
Modes of operation......................11–12
Modes, numbering system.................11–16
Modulation, amplitude 9–1, 2
  density 12–55
  envelope 9–25
  frequency 9–3
  percentage 9–26
  phase 9–4
Modulator, balanced ....9–30 to 9–34
  reactance tube ....9–4 to 9–7
  velocity 12–55, 56

- Modulators 12–16
  - driver-hard-tube 12–36
  - line-pulsing 12–33
  - types 12–33 to 12–40
- Moving target indicator 15–8
  - block diagram 15–8
  - general 15–8, 9, 10

N

- National Television Systems Committee 15–28

O

- Open end R–F line 10–9
- Oscillator, backward-wave 12–69
  - crossed-field 12–72
  - lighthouse tube 12–5 to 12–7
  - magnetron 12–17 to 12–42
  - push-pull 12–7 to 12–11
  - tuned-grid tuned-cathode 12–9, 10
  - tuned-plate tuned-grid 12–7
  - tuned-plate tuned-grid tuned-cathode 12–11, 12
  - UHF 12–1
  - Ultraudion 12–4, 5
- Overbunching 12–46

P

- Parabolic reflector 13–28
- Parallel circuits 12–30
- Parasitic arrays 13–25
  - director 13–25
  - reflector 13–25
- Perveance 12–51
- Phase modulation 9–4
- Phase velocity 11–15
- Plate detector 9–11
- Plate modulated oscillator 12–16
- Plate tuning 12–7
- Polar diagrams 13–16, 17
  - construction 13–17
- Polarization 13–10 to 13–16
  - circular 13–11 to 13–16
  - elliptical 13–15
  - horizontal 13–10, 11
  - linear 13–10, 11
  - vertical 13–10, 11
- Polarizer 13–12, 13
- Potentiometer and magnet remote indicating system 13–3, 4
- Power factor of transmission lines 10–16
- Pre-emphasis networks 9–42 to 9–44
- Pulse-forming network 12–31
- Push-pull oscillators 12–7 to 12–11
- Pulsing triode oscillators 12–13, 14, 15

R

- Radar, oscillators 12–1 to 12–72
- Radiation 12–3; 13–4, 5
  - directional characteristics 13–16
- Radiation and induction losses 10–25
- Radiation field strength 13–17
- Radiation resistance 13–9
- Radio frequency lines, artificial 10–32 to 10–37
  - characteristic impedance 10–3, 4, 20, 21, 23
  - circuit elements 10–17
  - defined 10–1
  - function 10–1
  - measurements 10–25
  - phase inverting or shifting network 10–31
  - power factor 10–16
  - reflection 10–9 to 10–13
  - sending end impedances 10–15
  - terminations 10–14, 15
  - theory 10–2
  - tuned circuits 10–31
  - types 10–20
  - uses 10–28
  - voltage changes 10–5, 6, 7
- Ratio detector 9–23
- Reactance tube modulator 9–4 to 9–7
- Recovery time 11–28
- Reflected wave 10–11
- Reflection in R–F lines 10–9
- Reflection of radio waves 15–23
- Reflectors 13–28 to 13–33
  - cosecant-squared 13–31
  - cylindrical paraboloid 13–31
  - orange-peel 13–31
  - parabolic 13–28, 29
  - truncated paraboloid 13–30
- Remote indication 14–1
- Resonance and dimensions 13–7
- Resonant cavity 11–34
- Ringing circuit 11–39
- Rotary spark gap 12–31

S

Saturable reactor 14–28
  classification 14–29
  construction 14–29
  operation 14–30
Self-pulsing oscillator 12–13, 14, 15
Selsyn 14–1
Serial circuits 12–30
Servomechanisms ....14–9 to 14–25
Servomotor 14–20
Shielded two-wire line....................10–21
Sideband generation 9–29
Sidebands 9–3, 24
Single-sideband receivers ....9–40 to 9–42
Single-sideband transmission ......9–24 to 9–30
  power comparison 9–27
  power distribution 9–27
Single-sideband transmitters ....9–38 to 9–40
Skin effect 12–3
Sky wave 13–7; 15–21, 22
Spaced two-wire line........................10–21
Standing wave of voltage..............10–10, 13
Superregenerative detector 9–14, 15
Supersonic delay lines, liquid media.......10–37
  solid media 10–38
Synchros, azimuth stabilization....14–25, 26, 27
  connections 14–11, 12
  control transformer 14–18, 19
  current flow 14–12, 13
  differential ....14–14 to 14–18
  electrical position 14–11
  electrical zero 14–6
  operation ....14–5 to 14–14
  potentiometer and magnet remote
    indicating system 14–3, 4
  rotor 14–4, 5
  shaft positioning ....14–2, 14–5 to 14–14
Synchrotie 14–1

T

Television antennas 15–21
  conical 15–24
  dipole with reflector 15–23
  folded dipole 15–24
  stacked 15–24
  Yagi 15–25
Television audio system...................15–33
Television camera tubes...................15–12
  iconoscope 15–12, 13
  image dissector 15–14, 15
  image orthicon 15–12, 13
  vidicon 15–15
Television, closed circuit...................15–30
  applications 15–33, 34
  color 15–34
  distribution amplifiers 15–30
  equipment 15–30
  monitors 15–31
  typical systems 15–32
  video signal distribution equipment.....15–30
Television, color 15–28
  I signal 15–29
  monochrome signal 15–29
  Q signal 15–29
Television controls 15–18
  brilliancy 15–18
  contrast 15–18
  fine tuning 15–18
  hold 15–19
  horizontal 15–19
  vertical 15–19
Television, introduction 15–10
  system essentials 15–11
  system requirements 15–10
Television, propagation 15–21
  direct waves 15–22
  ground waves 15–22
  reflected waves 15–22
  sky waves 15–22
Television receivers 15–18
  controls 15–18
  intercarrier-type 15–18
  theory ....15–19 to 15–21
Television receivers, color.................15–29
  chrominance channel 15–29
  electron gun 15–29
  shadow masks 15–30
Television scanning 15–15
Television signal 15–10
  blanking pulses 15–16
  compound 15–16
  equalizing pulses 15–16
  frequencies 15–17
  standards 15–17
  sync pulses 15–16
  video signals 16–16
Television theory 15–19
Terminations of R–F lines.......10–13 to 10–20
Three-eighth wave section.................10–18
TR tube 11–27
Transductor 14–29
Transit time 12–2

Transmission lines ....10–1 to 10–37; 15–25
Transmission lines as circuit elements.....10–17
Transmission lines as resonant circuits.....10–17
Traveling wave tube ....12–62 to 12–72
Triode oscillators as transmitters...........12–4
Tropospheric scatter 15–5
  antennas 15–8
  atmospheric considerations 15–7
  general theory 15–6
  scatter capabilities 15–7
Tuned-grid tuned-cathode oscillator....12–9, 10
Tuned-plate tuned-grid oscillator..........12–7
Tuned-plate tuned-grid tuned-cathode
  oscillator 12–11, 12
Twisted two-wire line...................10–20

U

UHF antennas 15–26
  corner reflectors 15–27
  cylindrical parabolic reflector...........15–27
  fan dipole 15–26
  rhombic 15–27
Ultraudion oscillator 12–4, 5
Unguided electromagnetic waves......11–1, 2, 3

V

Velocity modulated tubes........12–43 to 12–62
  amplifiers of power....................12–49
  examples 12–47
  local oscillators 12–43
  modes of operation....................12–46
  reflex 12–44
  theory of operation....................12–44
Velocity modulation 12–55
Vestigial sideband transmission........15–17, 18
Voltage, current, and impedance
  inversion 10–18
Voltage changes moving along R–F line.....10–5
Voltage-fed antennas 13–19

W

Ward Leonard Drive.....................14–21
Waveguide, adjustments 11–4, 51
  advantages 11–3
  analogy by electric waves......11–12 to 11–17
  bends 11–20
  boundary conditions 11–9, 10
  circular 11–12, 13
  counting wavelengths 11–17
  coupling 11–3
  crossing angle 11–14
  cut-off frequency.......................11–6
  effect of different frequencies..........11–5, 6
  electric field ....11–7 to 11–10
  electromagnetic fields 11–6, 10
  introducing fields into...........11–17, 18, 19
  joints 11–21
  losses 11–2, 3
  magnetic field 11–10, 11, 12
  matching devices 11–30, 31, 32
  modes of operation....................11–12
  numbering system of modes............11–16
  path of wavefront......................11–14
  R–F system 11–46
  rectangular 11–13
  T-junction 11–23
  test equipment 11–51
  theory 11–4
  transferring electrical energy..........11–1, 2
  terminating 11–32, 33
  twists 11–21
Wavefront 13–10
Wavelength 10–9; 13–7
Waves, incident and reflected..........10–11, 12

Y

Yagi array 13–27

☆ U. S. GOVERNMENT PRINTING OFFICE: 1963—651239/19

8/5/2011
BT 236529 5 35 00

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